Stress Analysis Validation for Gear Design

Md Asif Adnan
Ahmed Elsayed

Department of Mechanical Engineering
Blekinge Institute of Technology
Karlskrona, Sweden
2018
Master Thesis
MSc in Mechanical Engineering with Emphasis on Structural Mechanics

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China Euro Vehicle Technology AB

Md Asif Adnan            Ahmed Elsayed
mdad16@student.bth.se     ahel16@student.bth.se

Department of Mechanical Engineering
Blekinge Institute of Technology

July 2018
Gothenburg, Sweden
Abstract

Gear stress analysis and understanding the effect of misalignment and microgeometry is important for gear designers and for those who work in gear maintenance. The misalignment can lead to the higher stress acting in one side of the gear tooth and the micro-geometry modification can improve the stress distribution in the gear teeth. In this research, a helical gear pair was modeled using three different software and tools; LDP, KISSsoft and Abaqus. Three different cases were modeled to study the effect of misalignment and microgeometry. Finally, the results from different tools were presented and discussed. It was observed that the tooth contact analysis software resulted in significantly higher stresses than the FE software. The results have been discussed to understand the differences in the cases obtained from the used tools. The results showed how bad is the effect of the misalignment on the gear mesh and the stress distribution and how the microgeometry modifications used to compensate that effect.

Keywords

Finite element analysis, Microgeometry modification, Misalignment, Tooth contact analysis
Acknowledgements

This thesis work was carried out at the Department of Mechanical Engineering, Blekinge Institute of Technology BTH, Karlskrona, Sweden. The work is a cooperation between the Department of Mechanical Engineering, Blekinge Institute of Technology BTH and China Euro Vehicle Technology CEVT, Gothenburg, Sweden, which was initiated in January 2018 under the supervision of Prof. Ansel Berghuvud (BTH) and Dr. Omar D. Mohammed at CEVT.

Here, we would like to show our sincere appreciation to Ansel Berghuvud and Omar D. Mohammed for their continuous engagement and guidance throughout the thesis period.

We would like to show our gratitude to Martin Hedström for providing us the opportunity to implement our thesis work at CEVT, and smoothing our path to complete the task. We would like to thank Jonas Skoglund for his assistance to setup the finite element model and Peter Falk, Viktor Pettersson and Li Junhao for sharing their knowledge in gear design and calculation. We are thankful to Alejandro Martinez for his support with the resources. We also wish to thank the whole Rotating and Structural Parts team for inspiring and helping us in every possible means.

Finally, we wish to thank all members at CEVT for their appreciation and encouragement.

Gothenburg, June 2018
Ahmed Elsayed & Md Asif Adnan
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1 Notations

\( a \) Centre distances
\( b \) Width of the gear tooth or Hertzian half width
\( b_e \) Effective face width
\( d \) Pitch diameter or reference diameter
\( d_a \) Tip diameter of gear
\( d_f \) Root diameter of gear
\( d_1, d_2 \) Diameter of contacting cylinder
\( E_1, E_2 \) Young’s modulus
\( e_k \) Initial separation at point \( k \) in gear 1 and 2
\( F \) Force
\( F_t \) Transmitted tangential load
\( l \) Common length of cylinders in contact
\( m_n \) Normal module
\( m_t \) Transverse module
\( N_1 \) Number of teeth in pinion
\( N_2 \) Number of teeth in gear
\( P_b \) Base pitch
\( P_{dt} \) Transverse diametral pitch
\( p_{max} \) Maximum surface pressure
\( R_b \) Distance of point \( k \) from centreline
\( R_{b\theta} \) Rigid body rotation
\( r_{a1}, r_{a2} \) Tip radius of pinion and gear
\( r_{b1}, r_{b2} \) Base radius of pinion and gear
$T$ Applied torque
$\nu_1, \nu_2$ Poisson’s ratio
$W_{1k}$ Total elastic deformation of point k on gear 1
$W_{2k}$ Total elastic deformation of point k on gear 2
$\mathcal{W}^t$ Tangential load
$x$ Addendum modification coefficient
$Y$ Lewis form factor
$Y_J$ Geometry factor for bending stress
$Z$ Length of line of action
$\alpha_{wt}$ Transverse working pressure angle
$\beta$ Helix angle
$\varepsilon_\alpha$ Transverse contact ratio
$\varepsilon_\beta$ Overlap ratio
$\sigma_F$ Bending stress number
$\sigma_H$ Contact stress
$\theta_1$ Rotation angle of pinion
$\theta_2$ Rotation angle of gear
Abbreviations

AGMA  American Gear Manufacturing Association
DIN   Deutches Institute for Normung
DTE   Dynamic transmission error
FEA   Finite element analysis
FEM   Finite element method
ISO   International Organization for standardization
LDP   Load distribution program
LOA   Line of Action
NVH   Noise vibration and harshness
TE    Transmission error
TCA   Tooth contact analysis
BEM   Boundary element method
2 Introduction

2.1 Background

Gears are a mechanical device, which is used for transmitting mechanical power between two different shafts. They are one of the most important and widely used mechanical devices. Over 1000 years ago, wooden gears in their ancient form were used in water supply. Since the beginning of the 19th century gears played a big role in automotive industry [1]. Gears are also widely used in other different industrial applications such as power plants, turbines and machines.

There are different types of gears depending on the function and the machine layout. For example, spur and helical gears are used for transferring power between two parallel axes, while the bevel gear is used with non-parallel ones [2].

Nowadays with the fast development in automotive industry, different gear transmission concepts have been developed to fulfill the design considerations, testing requirements and customer desires. Gear designers need to satisfy the requirements of acceptable NVH levels, and allowable acting stresses for good durability. Therefore, a lot of research work has been conducted to improve the gear design and related calculations.

2.2 Literature review

One of the important aspects that should be considered when it comes to the gear contact study is the shaft misalignment. Misalignment can be a result of elastic deformation, manufacturing errors and/or assembly errors. Misalignment is unavoidable and can lead to change in the mesh gear stiffness, distributed load and the resulted stresses [3]. Misalignment can cause changes in the contact pattern from the ordinary contact pattern where the maximum contact is acting in the middle of the tooth to the side contact and in result, it can change the maximum acting stresses [4]. Also, it is proved that a very small change in the misalignment can lead to a big change in the stresses acting in a gear tooth [5]. A study on the gear contact is attained
using advanced static finite element analysis with nonlinear gear contact. Through FEA the effects of gear misalignment on these contact conditions are also investigated in [6]. Misalignment also one of the root causes of the speed transmission error in gears along with the manufacturing error. A study to compare the static transmission error (STE) and dynamic transmission error (DTE) is implemented in [7].

It is important to analyze the stresses acting on the gear tooth when it comes to the gear design. Higher repetitive contact stress can lead to surface pitting and scuffing [8]. The same can be said about the stress acting in the gear tooth root [9].

The Micro geometry modification is a way to modify the gear tooth surface by adding or removing material and its purpose is to improve the contact pattern and to compensate for misalignment effect. A study on the effect of manufacturing Micro geometry variations on the gear contact and root stresses has been implemented by D.R.Houser using LDP in [10]. Also, a study on the using of the tooth gear micro geometry modification to reduce the fatigue surface wear damage has been done in [11]. Another study of the effect of the linear tip relief profile modification in the Hertz contact stresses on tooth flank is done in [12]. A model of a spur gear is used to study the effect of intentional tooth profile modifications by using two dimensional FEM [13].

Modelling is a very important process in the automotive industry when it comes to evaluate the chosen design. However, the challenge faced by the designers in the modelling process that needs to be accurate and taking into consideration the real life working conditions. In automotive industry, the helical gears are used with micro geometry modifications to fulfil manufacturing, assembly and durability requirements. There will be always misalignment, and for that reason, some gear calculation software tools such as LDP and KISS soft are used for the calculation of the stresses acting in any gear tooth with considering misalignment effect. That raises the need for a comparison among the different models implemented in different gear calculation tools. In addition, the micro geometry modification is used to improve the contact pattern and compensate for the effect of the misalignment that results in lowering the stress acting and better contact pattern.
2.3 Research questions and hypotheses

The research questions for the current research are

- What are the similarities and differences among the specialized tooth contact analysis tools and finite element software in modelling a helical gear pair in mesh?
- How do the misalignment and micro geometry modification affect the stresses acting in gear tooth and transmission error, and how can those modifications compensate the effect of the misalignment?

The hypotheses are

- The stresses calculated using the tooth contact analysis software might overestimate the stresses compared to finite element software.
- Micro geometry modification to the original design can improve the stress distribution in the gear flank and compensate the effect of the misalignment.
3 Theoretical background of gear design

Gears are mainly used to transmit rotary motion between two shafts. Among different means of mechanical power transmission, gears are most rugged, durable and efficient. Their power transmission efficiency is as high as 98 percent [14]. In addition, as expected the manufacturing cost is also high and increases with the required precision level, which combines high speed, heavy loads and less noise. Incorrect modelling will result to power loss and ultimately damage the components. Therefore, before starting to design it is important to have some basic knowledge of the theory of gear pair teeth in mesh.

3.1 Geometry and nomenclature

Before designing any gear pair, it is important to know the basic terminologies. The nomenclature will be described for the spur gear. Following figure is according to Shigley [15].

Pitch circle is the most important is the pitch circle on which all the calculations are based on. When any gear pair comes into contact, the pitch circle of both the gears is tangent to each other. Circular pitch $p$, is the distance between the corresponding points of adjacent teeth on pitch circle. Addendum and dedendum are the radial distance respectively to top land and
bottomland from the pitch circle. Clearance $c$ is the amount by which the dedendum in a given gear exceeds the addendum of the mating gear. The backlash is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle.

Module $m$ is the ratio of pitch diameter to the number of teeth. It is the index of tooth size in SI unit. Diametral pitch $P$ is the ratio of the number of teeth in gear to the pitch diameter. Pressure angle is defined as the angle between line perpendicular to the line of action and the shared centerline of both gears.

### 3.1.1 Conjugate action

The requirement of gear tooth geometry is to maintain constant angular velocity ratios in each meshing positions, which is not possible in reality because of manufacturing inaccuracy and tooth deflection due to the load applied. The action theoretically satisfies the requirement is called conjugate gear-tooth action. According to Juvinall and Marshek [14] the basic law of conjugate action states that

> “as the gear rotates, the common normal to the surfaces at the point of contact must always intersect the line of centers at the same point $P$, called the pitch point.”

### 3.1.2 Involute profile

The shape of gear tooth can be generated using this. Involute curve can be thought of as a curve created on a straight line, which rolls on a circle without sliding. According to Mägi and Melkersson [16], the advantage of using such a profile is primarily the fact that it fulfills the requirement of constant speed ratio.

### 3.1.3 Line of action

According to Lee [17] When gear pair mates each other, it will generate a rotary motion. When two curved surfaces are pushed against one another, the contact point appears where the two surfaces are tangent. An imaginary line can be formed If the point can be traced starting from coming to contact and leave the contact. This line is called the line of action or pressure line. at any
instance, force will be directed along this line. The intersecting point between line of action and the centerline is called pitch point.

### 3.1.4 Length of line of action

It can be calculated using tip and base radius, center distance and operating transverse pressure angle.

\[
Z = \sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a \sin \alpha_{wt}
\]  

(3.1)

Where

\( r_{a1}, r_{a2} = \text{tip radius of pinion and gear} \)
\( r_{b1}, r_{b2} = \text{base radius of pinion and gear} \)
\( a = \text{operating center distance} \)
\( \alpha_{wt} = \text{transverse working pressure angle} \)

### 3.1.5 Contact ratio

This is one of the most important design aspects that indicates the average number of teeth in contact during the period of engagement and leaving gear pair. Higher contact ratio results to higher tooth stiffness and lower contact and bending stress. For a helical gear pair, total contact ratio is the sum of transverse contact ratio and overlap ratio. Transverse contact ratio is calculated from length of line of action divided by the transverse base pitch.

\[
\varepsilon_{\alpha} = \frac{Z}{\rho_b}
\]  

(3.2)

Where \( \varepsilon_{\alpha} = \text{transverse contact ratio} \) , \( Z = \text{length of line of action} \) and \( \rho_b = \text{base pitch} \).

Overlap ratio is the action in axial direction which spur gear does not have. This ratio increases with the width and helix angle increment.
\[ \varepsilon_\beta = \frac{b \sin \beta}{\pi m_n} \]  \hspace{1cm} (3.3)

Where \( \varepsilon_\beta = \text{overlap ratio} \), \( b = \text{face width} \), \( \beta = \text{helix angle} \) and \( m_n = \text{normal module} \)

And for a helical gear pair total contact ratio, \( \varepsilon_\gamma \) is

\[ \varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \]  \hspace{1cm} (3.4)

3.1.6 Different diameters

Following figure shows different diameters used to define gears geometry. Tip diameter is the highest part of gear tooth and root is the lowest in tooth geometry. Bas diameter is where the involute profile starts to form. There should not be any contact below this diameter and line of action is tangent to this. If the gear can be thought of two rolling cylinders in contact, pitch diameter can be assumed as the diameter of those cylinders[16].

![Figure 3.2 different diameters of a gear tooth](image-url)
3.1.7 Addendum modification coefficient

It is a dimensionless factor to evaluate addendum modification, also known as profile shift coefficient or rack shift coefficient. It quantifies the relationship between the distance from the datum line on the tool to the reference diameter of gear $D_{abs}$ (radial displacement of the tool) and module $m$. This coefficient can be defined separately for pinion and gear [18]. Positive addendum modification ($x > 0$) results in a greater tooth root width and thus in an increase tooth root strength and possibility to avoid or reduce undercut. A negative addendum modification ($x < 0$) will have reverse effect. This is more effective in case of gears with the lower number of teeth.

3.1.8 Gear Types

Different types of gears are used in industry depending on the purpose, application and space. Among all spur gear and helical gears, bevel gear and worm gears are most common types. In this research, the scope is limited to external helical gears used in automotive industry.

3.1.9 Helical gear

In helical gear, the teeth are inclined to the axis of rotation. The inclination is expressed in an angle called helix angle $\beta$. The meshing gear should have the same helix angle but hand of helix must be the opposite. The initial contact in the helical gear pair is a point which extends into a line as the teeth come into more engagement. Unlike the spur gear, the line of contact is diagonal across the face of the teeth. Helical gears can be used for the same application as spur gear but compared to spur gear, helical gears are less noisy because of more gradual engagement of the teeth during meshing and smooth transfer of heavy load in high speed. Sometimes they are also used to transmit motion in non-parallel shafts.
3.2 Microgeometry

To achieve favorable contact and reduce transmission error, material is intentionally removed from the tooth profile and the flank line which is called microgeometry modification in gears. According to Kissling [20], this is the last phase of designing gears and bad choice of macro geometry can never be compensated with good microgeometry modification. But it is not possible to achieve one specific design which will fulfill all the objectives like noise, scuffing, micro pitting, service life etc. This kind of modification is introduced to compensate the manufacturing allowances and shaft misalignment due to assembly.

Three types of modification including profile, lead and bias can be included in the design. First two types will be discussed here according to Zhao [21] as those were used in the research.

Removal of material in tip and root of the teeth profile tip and root relief to eliminate high stresses in the tip and root areas. The relief can be linear or parabolic. It is important where the relief should be started from. That diameter is called tip or root relief start diameter. It can be expressed with rolling angle also.
Profile crowning modification is also expressed as barreling. This allows to gradually remove maximum material at start of active profile (SAP) and end of active profile (EAP). For profile slope modification, no material is removed from SAP, and the specified amount is removed at the EAP.

Lead modification is the modification done to the gear in axial direction. Lead crowning makes the tooth surface slightly convex. It is applied to avoid high local contact at tooth end. Lead slope is to remove defined material from the end of face width and no modification at start of face width.
Figure 3.6 Lead crowning and Lead slope modification

Microgeometry modifications are commonly used by the designers to avoid corner contact and to optimize transmission error and the contact pattern on the gear teeth.

3.3 Misalignment

An involute gear pair without manufacturing error should provide uniform load distribution. However, in the real world, there will always be some manufacturing errors and shaft, bearing, housing will be deflecting under load, which will hinder the conjugate action of gear pair flanks. Because one side of the face width will depart from other teeth and not engage when the teeth mesh each other.

Misalignment can be expressed in different ways. according to Houser [22] misalignment can be divided into three categories. Parallel misalignment or change in center distance, angular misalignment to the plane of action and angular misalignment perpendicular to plane of action.

According to jones [6] misalignment can be decomposed into two linear and two angular forms. Axial and radial are forms of linear and pitch and yaw are the forms of angular.
Involute cylindrical gear sets are very sensitive to misalignment. It leads to discontinuous linear function of transmission error. This results to noise and vibration as well as edge contact and leads to high contact stresses and premature failure because of moving the peak bending stress to the edge of face width [23]. In this study, misalignment in the line of action will be considered which is combination of the angular misalignments.
### 3.4 Gear model data

As stated before the study only considers a pair of helical gear. Following gear pair was used for modeling and simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal module, mm</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>No of teeth</td>
<td>36</td>
<td>53</td>
</tr>
<tr>
<td>Pressure angle</td>
<td></td>
<td>17.5</td>
</tr>
<tr>
<td>Helix angle</td>
<td></td>
<td>28.5</td>
</tr>
<tr>
<td>Hand of Helix</td>
<td>left</td>
<td>right</td>
</tr>
<tr>
<td>Operating center distance, mm</td>
<td></td>
<td>74.041</td>
</tr>
<tr>
<td>Face width, mm</td>
<td>17.7</td>
<td>19</td>
</tr>
</tbody>
</table>

Material was steel with Young’s modulus 207 GPa and Poisson’s ratio 0.3. only elastic deformation was considered so the stress can be calculated using specified Young’s modulus and Poisson’s ratio.

Gear calculation software used for the study are capable for multi-torque analysis but scope was limited to single driving Torque of 160 Nm which is the maximum operating torque for this specific gear pair and speed was 2000 rpm for gear 1.
3.5 Challenges in gear design

To maintain low noise level and run life a designer needs to model the gear by fulfilling some basic criterion discussed below

3.5.1 Contact stress

One main mode of gear tooth failure in Pitting. According to Shigley [15] It can be defined as surface fatigue failure due to many repetitions of high contact stress on tooth flank while the gear pair is rotating against each other. There are different standardized methods to predict contact stress. i.e. ISO, AGMA. But the basis of all the method is the Hertzian contact theory.

When two bodies with curved surface mate contact stress appear. If the bodies are sphere, theoretically the contact area is a point. If the bodies are cylinders, contact area is a line. For both the cases contact stress will be infinite because of negligible contact area and it will initiate yielding. But in real life, the contact area will be deformed and result in finite stress which is called Hertzian stress.

Figure 3.8 simplified definition of Hertzian contact from Karlebo handbook
To calculate the contact stress in gear pair, it is assumed two cylinders are rolling. Contact stress between two cylinders can be expressed from the following equation according to Shigley [15]

\[ p_{\text{max}} = \frac{2F}{\pi al} \]  \hspace{1cm} (3.5)

Where

- \( p_{\text{max}} \) is maximum surface pressure,
- \( F \) force pressing the cylinder,
- \( l \) = common length of the cylinders in contact and \( a = \) half width

\[ a = \frac{2F}{\pi l} \left( \frac{1}{E_1} \left( \frac{1}{d_1} \right) + \frac{1}{E_2} \left( \frac{1}{d_2} \right) \right) \]  \hspace{1cm} (3.6)

Where \( v_1, v_2 = \) poisson's ratio, \( E_1, E_2 = \) young's modulus and \( d_1, d_2 = \) diameter of contacting cylinders.

Stated equations are generalized for any cylinder coming into contact. Equations will be slightly modified to adapt these equations to external gear pair.

\[ \sigma_H = \frac{W^t}{\pi b \cos \alpha} \left( \frac{1}{r_1} \right) + \left( \frac{1}{r_2} \right) \left[ \frac{1 - v_1^2}{E_1} \right] + \left[ \frac{1 - v_2^2}{E_2} \right] \]  \hspace{1cm} (3.7)

Where \( W^t = \) tangential load, \( b = \) common face width and \( r_1, r_2 = \) radius of curvature

and for or any external helical gear pair manufactured with same material contact stress can be expressed.
\[ \sigma_H = \sqrt{\frac{FE}{2\pi(1 - v^2)}} \frac{1}{r_1} + \frac{1}{r_2} \]  

(3.8)

\[ N = \frac{W^t}{\cos \alpha_t \cos \beta_b} \]  

(3.9)

\[ l = \frac{b}{\cos \beta_b} \]  

(3.10)

When no other load increment factor is used, \( W^t \) can be calculated from the drive torque divided by the pitch radius. \( \alpha_t \) is the transverse pressure angle and \( \beta_b \) is the helix angle in base diameter.

### 3.5.2 Bending or tooth root stress

This is the stress in the area that does not come into contact when the gear teeth mate each other. The calculation of bending stress was first presented by Wilfred Lewis in 1892 which is still recognized as the basis for gear tooth bending stress analysis. According to Juvinall and Marshek [14] Lewis made following assumptions

1. The full load is applied to the tip of a single tooth
2. The radial component of the force is negligible
3. The load is distributed uniformly across the full-face width
4. Forces which are due to tooth sliding friction are negligible
5. Stress concentration in the tooth fillet is negligible

The gear tooth everywhere is stronger than the inscribed constant stress parabola, except for the section where tooth profile and parabola are tangent. So, critical bending stress will be in that particular point and can be written

\[ \sigma_b = \frac{W^t}{mbY} \]  

(3.11)

Where

\( m = \text{module} \), \( Y = \text{lewis form factor} \)
Lewis form factor $Y$ is based on the module or diametral pitch, $P$ or module. It’s a function of tooth shape which varies with the number of teeth in the gear. This equation indicates that tooth bending stress varies directly with load $W^t$ and inversely with tooth width, tooth size (module, diametral pitch), shape factor $Y$.

### 3.5.3 Transmission error

Transmission error is one of the main concerns in the field of tooth contact analysis to estimate the noise and vibration from the transmission system. This was first introduced by Harris [24]. Theoretically, a pair of meshing gears with perfectly involute teeth profile should transmit uniform angular motion. But in reality, most gear systems fail to transmit uniform angular motion. This difference between the theoretical and actual angular velocity is the transmission error of the pair in mesh. According to Tharmakulasingam [25] as it is considered a major source of noise after the impact of teeth engagement, doesn’t come directly from the angular speed variation. Torsional acceleration causes vibratory bearing reactions that excite the gearbox casing, which then propagates the noise through the pulsation of the casing wall. According to Wright [26] transmission error can be formulated as

$$ TE = \theta_2 - \theta_1 \left( \frac{N_1}{N_2} \right) $$

Where

$\theta_1 = rotation\ angle\ of\ pinion , \theta_2 = rotation\ angle\ of\ gear$

$N_1 = number\ of\ teeth\ in\ pinion , N_2 = number\ of\ teeth\ in\ gear$
There are different ways to express transmission error including arc sec and μrad but it can be converted into linear displacement with unit μm or μin at either base radius or pitch radius.

There are few types of transmission error explained by Tharmakulasingam [25] including manufacturing transmission error, static transmission error, kinematic transmission error and dynamic transmission error. In this study, scope is limited to static transmission error. Static transmission error (STE) is measured at low speeds to avoid the dynamic effect of the system. It is function of gear geometry and influenced by mesh stiffness, manufacturing error, load and mesh alignment.
4 Gear modelling and simulation

In industry modelling is very important as it saves a lot of money before going to the manufacturing process. Designers can perform simulation and make calculation to ensure that the model is fulfilling the design requirement, but modelling also is a very sensitive operation as the model need to be approximated to the real life running condition with taking into consideration that it should deliver fast and reliable results.

To design gears for transmission or driveline, a designer needs to keep some aspects into considerations. Among them, transmission error, bending stress and surface stress are the most important. Tooth contact analysis is a tool that can perform the calculation for this result. Tooth contact analysis (TCA) was first introduced in early 1960 to do the theoretical analysis of the contact characteristic and running quality. Nowadays, there are several software can perform this analysis in a short period of time and provide reasonable results comparing to finite element analysis. Experimental analysis can be done with trial and error but will be costly, time-consuming and in some cases fatal too. Which opens the door for virtual gear modelling and tooth contact analysis using different gear calculation software. In this research, gear modeling using different software will be discussed. Output from the software will be shown and the result will be discussed and compared.

The studied gear pair will be modelled using three different software. one is Windows LDP developed by Houser, next one is KISSsoft and the finally the finite element software - Abaqus. Three conditions will be considered for the analysis. First contains basic geometry with primary microgeometry modification, second one includes misalignment and third model includes final microgeometry modification to compensate for the effect of misalignment.
4.1 Load distribution program (LDP)

4.1.1 About LDP

Load Distribution Program (LDP) is a program that analyzes single mesh gear pairs using a finite plate compliance calculation in conjunction with the inclusion of Hertzian deflections and deflections of the tooth base [27]. LDP also can use finite element created compliance functions as well as performing finite element calculations of the root stresses.

The program can be used for predicting the load distribution in gear flank. LDP uses AGMA standards in combination with other techniques, e.g. simplex method and FE formulations, to perform the calculations. The advantage of using this approach is fast calculation times, but assumptions regarding the gear geometry are required to use LDP’s approach.

4.1.2 Theory and calculation method

LDP assumes the load distribution to be a function of elasticity of the gear system and errors or modifications on the gear teeth. LDP has two major assumptions the first one is total elastic deformation is the sum of individual elastic deformation and the second one is Elastic deformations are small, thus the tooth contact assumed to remain on the line of contact.

There are several factors affects the LDP calculation of the elastic deformation. Some of this consideration is for the shaft and the bearing modelling and some of them was for the gear modelling itself, in this study not all the assumptions were considered as the model presented in this paper is not considering the shaft and the bearing. The total elastic deformation according to LDP for this study is the sum of three types of deformation considered here-

- Bending and shear deflection of contacting teeth.
- Base rotation and base translation of contacting teeth.
- Local contact deflection.

In LDP, a tapered plate was approximated for bending and shear deformation developed by Yau [28] which uses the Rayleigh Ritz method to compute the deflections.
The base of the gear tooth rotates and translates due to the moment applied and for that the gears assumed to be connected to the wall using springs to the wall as in. This method has one major drawback it can overestimate the rotation compared to finite element method. Local contact deflections are based on an analytical model developed by Weber for gear teeth.

![Diagram](image)

*Figure 4.1 the modelling to calculate base rotation and translation.*

For a helical gear pair, deflections are calculated for spur gear tooth in normal plane and converted to the transverse deflections. Because for helical gear, the solution algorithm calculates all motion and modification in transverse plane.

It is assumed that before the load is applied, the gear tooth flanks have some space between them which is denoted as initial separation in LDP. Any errors or modification in the teeth including shaft misalignment, involute, profile and lead modification will change the location of surface of the teeth and result to positive or negative initial separation.

Elastic deformation and the forces are assumed to be acting on the line of action. LDP considers the sum of elastic deformation and initial separation must be equal to the rigid body displacement. Of the point respect to reference line which is a fixed radial line in the end plane of the shaft where torque is to be applied. Two criteria are proposed in LDP for the mathematical formulation of contact problem. The sum of elastic deformation and initial separation must be greater than or equal to rigid body approach along the line of action.
\[
W_{1k} + W_{2k} + e_k \geq R_{b\theta} \quad (4.1)
\]

Where
- \( W_{1k} \) = total elastic deformation of point \( k \) on gear 1
- \( W_{2k} \) = total elastic deformation of point \( k \) on gear 2
- \( e_k \) = initial separation at point \( k \) between gear 1 and 2
- \( R_{b\theta} \) = rigid body rotation

If the point satisfies above equation that means the point is in contact and force acting on that point is calculated by the condition that summation of all torques acting on a gear body must be zero

\[
\sum_k (F_k + R_b) + T = 0 \quad (4.2)
\]

Where
- \( F_k \) = the discrete force acting at point \( k \)
- \( R_b \) = the distance of point \( k \) from the centerline
- \( T \) = applied torque

The inequality equation can be transformed into equality equation as following

\[
W_{1k} + W_{2k} + e_k - R_{b\theta} - Y_k = 0 \quad (4.3)
\]

\( Y_k \) is called a slack variable, if \( Y_k > 0 \) than the two gears are not in contact at point \( k \) and \( F_k = 0 \). If \( Y_k = 0 \), then the contact exists.

4.1.3 Gear modelling using Windows LDP

To model gear pair LDP has two different approaches. One is to use the Hob geometry and the other one is using the detail. If the dimensions are correct, both approach will produce the same gear pair at the end. It is possible to analyze the pairs for multiple torques in a single run. There is another important consideration regarding corner contact. Corner contact is to check if there is earlier tooth coming in contact or leave contact due to
loading. This is not preferable in practicality. With sufficient micro
geometry modification, the options should yield to very close results.

For a valid geometry, LDP will calculate and provide additional information
including contact ratio, pitch, length of contact, roll angles, diameters, tooth
thickness in different diameters and addendum dedendum coefficient. Visual
representation of gear and hob is also available.

Different results can be achieved from the output module. Among them this
research concentrates mainly on the Contact stress and tooth root stress. there
are two methods to calculate root stress, using Boundary element method and
finite element method. There are also other available output including
transmission error, mesh stiffness, contact length, zone of contact, load
intensity distribution, film thickness distribution, temperature distribution
and microgeometry distribution.

4.1.4 Including microgeometry and misalignment

To include microgeometry modification in LDP, it is required to specify
different modification values. For tip relief it is important to decide whether
the modification is linear or parabolic and the roll angle at start of
modification. Roll angle is nothing but another representation of gear
diameter. Other modification including profile and lead crowning and profile
and lead slope is quite straight forward.

LDP considers the misalignment in line of action. Thus, this is quite similar
to the lead slope of the micro geometry. Positive value indicates removal of
material and negative value means addition of material.

4.2 Contact Analysis using KISSsoft

4.2.1 About KISSsoft

KISSsoft is a tool to perform sizing calculations for machine elements. Other
than different types of gears it can calculate transmission elements including
shafts, bearings, connecting elements, springs etc. in this study, the scope is
limited to contact analysis of cylindrical gear pair. It is possible to compare
the results with respect to different standards such as ISO, AGMA, DIN. In
addition, the program also capable of providing different design and optimization functions.

### 4.2.2 Calculation method

Contact analysis in KISSsoft accumulates both FEM based approach with complex spring models and analytical methods based on stiffness formulas which makes it computationally much faster. It is possible to calculate the stiffness for different tooth profile including involute, cycloidal or trochoidal because the stiffness formula is independent of the shape. It does not take the influence of thin gear rims into mesh stiffness into consideration. Also neglects the deformation of the neighbouring teeth in or out of the mesh.

KISSsoft uses a stiffness model based on Peterson [29] which is ultimately based on Weber/Banaschek. This model calculates deformation during the mesh of gear pair in the plane of line of action. This deformation considers linear bending, linear gear body deformation and nonlinear Hertzian deformation. Other than the Hertzian, rest two components are proportional to external load.

---

**Figure 4.2 Deformation components calculated in KISSsoft [30]**

gear tooth bending $\delta_Z$, gear body deformation $\delta_{R,K}$ and Hertzian flattening $\delta_H$ are calculated using following equations accordingly.
\[ \delta_Z = \frac{F_{bti}}{b} \cos^2 \alpha_{Fy} \frac{1 - \nu^2}{E} \left[ 12 \int_0^{y_p} \frac{(y_p - y)^2}{(2y')^3} dy + \left( \frac{2.4}{1 - \nu} + \tan^2 \alpha_{Fy} \right) \int_0^{y_p} \frac{dy}{2y'} \right] \]

\[ \delta_{RK} = \frac{F_{bti}}{b} \cos^2 \alpha_{Fy} \frac{1 - \nu^2}{E} \left[ \frac{18}{\pi} \frac{y_p^2}{s_{f20}^2} \right. \right.
\[ \left. + \frac{2(1 - 2\nu)}{1 - \nu} \frac{y_p}{s_{f20}^2} + \frac{4.8}{\pi} \left( 1 + \frac{1 - \nu}{2.4} \tan^2 \alpha_{Fy} \right) \right] \]

\[ \delta_H = \frac{F_{bti}}{\pi b_g} \left[ \frac{1 - \nu_1^2}{E_1} \ln \left( \frac{b_{lt}^2}{4t_1^2} \right) + \frac{\nu_1(1 + \nu_1)}{E_1} \right] \]
\[ + \left| \frac{1 - \nu_2^2}{E_2} \ln \left( \frac{b_{lt}^2}{4t_2^2} \right) + \frac{\nu_2(1 + \nu_2)}{E_2} \right| \]

Total deformation \( \delta \) can be expressed

\[ \delta_{total} = \delta_{Z1} + \delta_{RK1} + \delta_{H1,2} + \delta_{Z2} + \delta_{RK2} \]

Then the overall stiffness, \( C_{pet} \) can be formulated

\[ \frac{1}{C_{pet}} = \frac{1}{C_{Z1}} + \frac{1}{C_{RK1}} + \frac{1}{C_{H1,2}} + \frac{1}{C_{Z2}} + \frac{1}{C_{RK2}} \]

To check the real contact condition of a gear pair under load KISSsoft simulates the meshing of the two flanks over the complete meshing cycle. For calculating the stress during the mesh, the contact path under load is calculated. For helical gear pair there are more than one teeth are in contact for any instance and total stiffness in engagement change periodically. A model according to Peterson is used to calculate the stiffness, which includes bending of the teeth, bending of teeth in wheel body, the Hertzian pressure, and the deformation due to shear that is included in bending.
Coupling stiffness can be formulated as-

\[ C_C = 0.04 \times A^2_{sec} \times C_{pet} \]  \hspace{1cm} (4.9)

Where
\( C_{pet} = f(C_Z, C_{RK}) \)
\( C_{pet} = \text{stiffness following Peterson} \)
\( A_{sec} = \text{number of slices} \)
\( C_Z = \text{Stiffness from bending and shear deformation} \)
\( C_{RK} = \text{Stiffness from deformation through rotation in gear blank} \)
\( C_H = \text{hertzian flattening} \)

The gear is cut into several transverse section and stiffness is calculated for the slices. For a helical gear pair beginning and the end of contact of the slices depends on the position of the slice across the tooth width. Total stiffness is calculated by integrating the stiffness function for the slices over the face width.

In the simulation of the meshing, deflection of the teeth is calculated by normal force applied to single tooth divided by the stiffness. As the point of the force applied varies in the height direction, the stiffness will also depend on the meshing position. When the next pair of tooth comes into contact the stiffness increases and that reduce the deflection of the first pair of tooth.
This calculation will provide the stress in root area, Hertzian pressure and the transmission error.

**4.2.3 Tooth root stress calculation in KISSsoft**

The KISSsoft contact analysis determines load distribution across the face width and then calculate the force applied at each individual point of contact in every segment across the face width. The formulas in the standard are then used to determine tooth root stress in the individual segments. According to KISSsoft documentation, tooth root stress in the TCA result is therefore more accurate than the one calculated using the standard.

**4.2.4 Gear modelling using KISSsoft**

To analyze a pair of spur or helical gear, KISSsoft has the calculation module as Cylindrical gear pair. Before running the contact analysis, modelling the correct pair of gear is necessary. To model in KISSsoft, basic geometries including normal module, pressure angle, Helix angle, center distance, number of teeth, face width, profile shift coefficient is required. For helical gear pair hand of helix of gear 1 should be stated also. It is possible to define the tooth profile of the gears according to standard or own input. Input can be factors, length or diameters.

**4.2.5 Including microgeometry and misalignment**

In KISSsoft, All the possible modification in profile and lead direction can be added to the design. There is an advantage over LDP in KISSsoft that it allows to define separate modification for coast and drive side of the gear.

Unlike LDP, KISSsoft has two components to define the misalignment. One is deviation error and the other one is inclination error. Further on this will be in discussion section.

**4.3 Gear modelling using FE software**

The finite element analysis is a numerical method used to solve any differential equation by the discretization of the differential equation and write in the matrix forms [31]. In the last two decades, different software
based on the FE theory has been introduced such as ANSYS, COMSOL and Abaqus. In this research, Abaqus has been used to simulate two gear pairs in mesh and used also to calculate the contact pressure, tooth root stresses and the transmission error.

4.3.1 Parts and material definition

There are two ways that can be used to create parts; the first way is to use the Abaqus part creator, FEM users prefer this method, as they will have all the dimensions and sketches those are being used to create the part. However, with a complicated part geometry, the part creator in Abaqus has limitations and to create a gear with microgeometry modification using this method will be very hard.

The second way that was used here, which is to import the parts from any other software. For that, KISSsoft was used to generate the gear geometry. The microgeometry modification was there for all the cases so the scaling was needed to change the geometry to mm.

It was considered that; the deflections would always be in the elastic region. So, the plastic behavior of the material will not be considered in this study. Steel was used as material with Young’s module $E = 207$ GPa and Poisson ratio of $\nu = 0.3$.

4.3.2 Assembly

The two gear parts will be imported as two dependent instances, mesh on parts, with taking a distance between the two centers, this distance equal to the center distance. Small rotation in one gear may be required to remove any intersection between the gears solid.

4.3.3 Steps

4.3.3.1 Contact step

FEM Contact analysis is based on the clearance and the clearance is the distance between the two surfaces that should come into contact. Before the starting of the analysis, there should be no contact between the two surfaces as the big overclosure will cause in the termination of the analysis and the small overclosure will lead to a very artificial contact stresses[32].
The convergence to a solution in this case should be unlikely and to understand why the unconnected spring in Figure 4.4 can be used as an example to simplify the idea.

\[ F \]
\[ x_2 \]
\[ k_1 \]
\[ x_1 \]

A

B

Figure 4.4 unconnected spring system

By trying to solve this mechanical spring as a two degree of freedom system by considering point A and B displacement and by using newton third law of motion. The two equations of motions will be

\[ k_1(x_2 - x_1) = F \] (4.10)
\[ k_1(x_1 - x_2) = 0 \] (4.11)

And by writing these two equations in matrix form

\[
\begin{bmatrix}
-k_1 & k_1 \\
 k_1 & -k_1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix} F \\ 0 \end{bmatrix}
\] (4.12)

The determinant for that stiffness matrix is equal to zero and that leads to the displacement to become infinity or unsolvable but for the connected case the two equations of motion will be

\[ k_1(x_2) = F = 0 \] (4.13)
\[ x_1 = 0 \] (4.14)

In addition, in the matrix form it come as follows
\[
\begin{bmatrix}
0 & k \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
F \\
0
\end{bmatrix}
\quad (4.15)
\]

And for that a value for the displacement can be obtained and then a value for the acceleration that required to calculate the output forces and then the output stresses.

Therefore, the purpose of this step is to initiate the contact and no stress output required from this step. And to do so, one of the instances should be totally encastered from any translation or rotation in any direction and the other instant should be rotated very small angle to establish the contact.

A check for the contact can be done by printing contact constraint data to the job and then check the contact overclosure for step1 iterations in the job diagnose.

### 4.3.3.2 Load step

The actual system for any two gears in mesh in the gearbox is there is an input shaft that carries the input gear and output shaft that carries the output gear. The load and the boundary conditions applied to that shaft and to simulate that a point is defined in the center of the two gears and the inner surfaces of the gear coupled to it and all the loads and boundary condition is given to this point.

To define a load in Abaqus, the default is that the load is increased until it gives the maximum value at the end of every step and for our analysis. As this paper investigates the contact and the root stress results along the line of action a rotation of at least 40 degrees is required to obtain the contact stress.

To model the two gear a moment is given to one of the instances. in this case, it is the output instances and a rotation in degree is given to the other.
Table 4.1 loads and boundary condition

<table>
<thead>
<tr>
<th>step</th>
<th>Step no.</th>
<th>pinion</th>
<th>gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact step</td>
<td>Step 1</td>
<td>Fully encastred</td>
<td>$\theta_{contact}$</td>
</tr>
<tr>
<td>Load steps</td>
<td>Step 2 to 41</td>
<td>$\theta_{rotation}$</td>
<td>$T_{output}$</td>
</tr>
</tbody>
</table>

This paper investigates the contact stress along the line of action so some angular rotation was given to the pinion to cover at least one teeth full contact. To get that a 40 degree of rotation divided into 40 steps will be applied to the pinion.

4.3.4 Interaction module definition

The contact in gear geometry can be defined using 3 contact pairs and that if the edge will meet surface as follows:

- Surface to surface contact between the two gears outer surfaces.
- Node to surface contact to define the edges that will come into contact in both directions.

In addition, that because the surface-to-surface contact is smoothing the edges so the edge contact with surface that should produce higher stresses will change to surface-to-surface and that will produce a bit less stress as

![Figure 4.5 the real edge geometry in the left compared to the smoothed edge in the right](image)

Both two types of contact will be used using finite sliding contact discretization approach.

But in that model and because of the micro geometry modification, the Tip relief, so there will not be any contact between the edge of one gear and the
surface of the other gear, as a result of that the surface to surface contact will be enough to use.

**Both types of contact have advantage and disadvantages such as**

- The surface-to-surface contact gives a better definition for the two contact surfaces as it considers the whole surface in the discretization of the contact but it smooths the edges and for that reason, it gives a lower contact stress value that the value that should be obtained in the real or the analytical case.
- The surface to surface discretization has a big drawback that it can`t resist penetration and that because it uses the penalty method by default to define the contact pressure overclosure relationship between the master and slave surface, a finer mesh can affect in a better resistance and more accurate stress result.
- The surface to surface formulation is primarily intended for a situation where the direction of contacting surfaces is opposite but in case of a contact that including edge to meet a surface a node to surface discretization is preferable in this case and this is the reason for defining the node to surface discretization.
- The node to surface discretization does not resist penetration of the master surface into the slave surface however; it will resist the penetration of the slave surface into the master surface

![Figure 4.6 penetration of master surface into the slave surface [32]](image)

The contact damping was used in Abaqus to damp any inconsistency in the contact area. The contact damping has some feature such as it has a very high and unrealistic value through the step but the effect is removed at the end of the step. As a result, the output stress from the end of the step is more accurate and realistic. That was the reason behind dividing the analysis into several steps.
4.3.5 The mesh model definition

4.3.5.1 The element types

In Abaqus, the recommended element type is the hex element type. But for the too complex geometry specially in the gear root area, this hard to achieve. Tet quadriaxial element can be used with free mesh. For this purpose, a C3D10I element will be used as it gives a better surface stresses and as the interest here is the stresses at the root area, the model’s element type is the C3D10I.

4.3.5.2 The mesh sizes

The finite element software defines the geometry by discretizing the problem into very small elements. Every element consists of many nodes and the solver is just solving for each node. To compute with good accuracy, a fine mesh is required to catch the micro geometry modification in the gear tooth. However, having a very fine mesh lead to a very high computational time and that is not acceptable for industry. So, a tradeoff between the accuracy and the computational time should be done.

4.4 Verification of the model

As the research is motivated to a comparative study, it is quite important to have the same gear pairs from modelling. Any variation in geometry can cause unidentified effect on the results. Addition to that, to do the finite element analysis, the gears were imported from KISSsoft as solid feature. So once the gear is generated the geometry cannot be edited in Abaqus. Therefore, it was a preliminary challenge to have the same gear pair from both the software. Although there are some differences in modelling and the description of the parameters, but the designer can always check some information to make sure the gear pair from different software are similar and will depict comparable result after contact analysis.

It is very important that the diameters including reference, pitch, tip, base and root are the same in both the tool. If the diameters and the operating transverse pressure angle is the same that will result to same length of line of action. For helical gears total contact ratio which includes profile contact and
overlap ratio is another parameter to consider. Addendum and dedendum length need to be checked also as for the same diameters a different addendum or dedendum can change the involute to have different contact length or different root length. Also, the thickness of the root area need to be checked as this thickness is used by LDP to calculate the involute curve so any change in this thickness lead to a different involute curve. The modelled gear pair from LDP and KISSsoft for this study has similar diameters and contact ratio. Negligible difference was found comparing addendum and dedendum.
5 Results and discussion

In this research, three different cases were studied. First, the basic design of the gear geometry with the ideal running case. Second, the real-life working condition by introducing a misalignment. The last case is the final design with some modification to the lead slope and the lead crown to compensate for the effect of the misalignment.

Misalignment can be a result of elastic deformation, manufacturing errors and/or assembly error [33]. In the current work the misalignment is introduced due to transmission deformation. The used misalignment values were obtained using MASTA modelling software.

Microgeometry modifications is adjusted to compensate the misalignment and to optimize the contact pattern for obtaining the best possible results of stresses and transmission error TE.

5.1 Studied cases

Studied cases are listed below in Table 5.1

Table 5.1 considered cases

<table>
<thead>
<tr>
<th>Case no</th>
<th>Microgeometry modification</th>
<th>Misalignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Basic</td>
<td>No</td>
</tr>
<tr>
<td>Case 2</td>
<td>Basic</td>
<td>Yes</td>
</tr>
<tr>
<td>Case 3</td>
<td>Final</td>
<td>Yes</td>
</tr>
</tbody>
</table>
In *Table 5.2* the micro geometry and misalignment value for three cases is presented. The results were obtained for these three cases from all the studied software and tools –

<table>
<thead>
<tr>
<th>modifications</th>
<th>unit</th>
<th>case 1</th>
<th>case 2</th>
<th>case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>pinion</td>
<td>gear</td>
<td>pinion</td>
</tr>
<tr>
<td>roll angle</td>
<td>degree</td>
<td>30.441</td>
<td>22.833</td>
<td>30.441</td>
</tr>
<tr>
<td>tip relief start diameter, dca</td>
<td>mm</td>
<td>63.2955</td>
<td>88.605</td>
<td>63.2955</td>
</tr>
<tr>
<td>Tip relief value, Caa</td>
<td>μm</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Profile slope, fHα</td>
<td>μm</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Profile crown, Ca</td>
<td>μm</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Lead slope, fHβ</td>
<td>μm</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Lead crown, Cβ</td>
<td>μm</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>misalignment</td>
<td>μm</td>
<td>0</td>
<td>-18.74</td>
<td>-18.74</td>
</tr>
</tbody>
</table>

### 5.2 Results from Abaqus

All the figures in this chapter is normalized to the maximum stress value from case 1 in Abaqus.

#### 5.2.1 The mesh sizes

To decide the element size that suits the model, first task was to check different element size and compare the contact stress obtained from the different mesh sizes with the computational time it took.
Figure 5.1 rough geometry of a gear tooth to explain the mesh seeds

Table 5.3 the different mesh sizes results

<table>
<thead>
<tr>
<th>Model</th>
<th>x element size, mm</th>
<th>w element size, mm</th>
<th>The stress results</th>
<th>Approximate Computation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0.5</td>
<td>0.6</td>
<td>72.1244% of the stress of model 2</td>
<td>16 hours for 40-degree rotation</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.25</td>
<td>0.5</td>
<td>Reference stress</td>
<td>52 hours for 40-degree rotation</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.21</td>
<td>0.4</td>
<td>0.004% increase in the stress from model 2</td>
<td>1 week for 12 degrees</td>
</tr>
</tbody>
</table>

The results of the different element sizes are showed in detail in Table 5.3 the model 1 is not giving an accurate stress result despite its very fast in running as it can be seen it is almost 70% of the value that was obtained from model 2 and that is a very big difference but for the stresses obtained from model 2 it can be said that there is almost no difference between model 2 and model 3 value but the computational time varies too much between them as the third model is taking much longer time than model 3.
5.2.2 The contact and root stress from FEM

For the contact stress results, the scalar value of the Hertzian contact is the value that being showed in all the figures and these values can be found from contact pressure field output data in Abaqus.

The root stress here is the maximum principal tensile stress or the maximum positive stress and can be found in the field output data in Abaqus in the maximum principle stress field output but it must be a vector to show the positive and negative value.

Table 5.4 the contact and the root stress for the three different cases

<table>
<thead>
<tr>
<th>case</th>
<th>Contact stress</th>
<th>Root stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image" alt="Contact Stress 1" /></td>
<td><img src="image" alt="Root Stress 1" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="Contact Stress 2" /></td>
<td><img src="image" alt="Root Stress 2" /></td>
</tr>
</tbody>
</table>
Table 5.4 shows three different cases for contact and root stress results, so as it introduced before case 1 and case 2 both have the same geometry but the difference between them is in case 2, the misalignment is introduced to the system the contact stress for case 1 is the ideal case which will never be true as the introduced misalignment is equivalent to the transmission deformation effect. In case 1, the stress reaches its peak or maximum value in middle of the teeth flank while the value at both ends is much lower. This is due to the contact between the two teeth is happening in mid-region as some crowning was performed in both the profile and lead direction.

The shaft misalignment is expected to affect the contact pattern and shift it towards the start or end of face width. This is due to this shifting that will lead to an increase in the stresses acting in one side of the gear tooth and in the other hand it decreases the acting stress in the other side of the gear tooth and with the increasing misalignment, the contact stress expected to increase and also the contact area will change [34]. There will be an edge contact in the teeth.

The contact stress distribution when the misalignment is introduced to the system. the value of the maximum stress increased by almost 11% of the original value of the maximum stress and it also shifted to one side of the gear. higher contact stress can lead to higher contact fatigue and that can lead to initiate a crack in the tooth surface, that crack can lead to the surface pitting [8].
To compensate for the effect of the misalignment and to return the contact pattern closer to the one when no misalignment, the final microgeometry was introduced on both driving and driven gears.

The contact pattern of case-3 and the maximum stress acting in the gear tooth was in the middle and is higher than the maximum stress of case-1 in Table 5.4 by 5%.

The micro geometry modification introduced to case 3 compensated the effect of the misalignment in case 2 and it reduced the maximum stress if compared to case 2 shifted it to the middle part of the tooth surface. It didn’t compensate fully the effect of the maximum stress value as the maximum stress value is higher than the original value from case 1 by almost 5%.

The same said about the contact stress can be said about the root stress as the maximum stress for case 2 will be shifted to one side with the maximum value for the stress increased by 18.62% as compared to the maximum stress for the case 1.

Again, introducing the micro geometry compensated the effect of the misalignment and the stress distribution for case 3 is the same as case 1, but the maximum stress is still higher almost 10% compared to the maximum stress of the case 1.

The maximum stress and although it’s still higher than the maximum stress distribution for case 1 but it’s better than the 19% increase obtained from case 2.

5.2.3 Transmission error

To calculate the transmission error from the FEM a history output is requested for the rotation of the output reference point and that rotation will be the actual rotation of the gear pairs from the simulation while the theoretical rotation will be calculated from the input rotation to the drive gear.
Transmission error from FEM is introduced. The gear designers are interested in the value that is called peak to peak transmission error or the difference between the maximum and minimum transmission error in one mesh cycle. The peak to peak transmission error is in this study was inconsistent and unreliable.
5.3 Results from LDP

All the values are normalized compared to Abaqus case 1

5.3.1 Contact stress from LDP

Table 5.5 presents contact stress distribution for the three studied cases. It is clear from case two Result how the misalignment shifted the contact towards the end of the face width.

Table 5.5 contact stress distribution for different cases

<table>
<thead>
<tr>
<th>case</th>
<th>3D stress distribution</th>
<th>Contour plot for stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="normalized 3D plot for contact stress distribution" /></td>
<td><img src="image2" alt="normalized contour plot for contact stress distribution" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3" alt="normalized 3D plot for contact stress distribution" /></td>
<td><img src="image4" alt="normalized contour plot for contact stress distribution" /></td>
</tr>
</tbody>
</table>
5.3.2 Tooth root stress from LDP

LDP has two different options to calculate the root stress; one using boundary element method and the other using the finite element method. Both the results presented below. Table 5.6 represents the normalized contour plot for gear 1 tooth root stress and Table 5.7 represents for gear 2 in all the studied cases.

In both tables and for all the cases LDP approximate higher tooth root stress compared to finite element result. For case 1, boundary element method approximates contact pattern similar to FE. However, FEA result from LDP shows some stress concentration closer to one side. Moreover, for case 2, both the method approximated pattern shifting and higher maximum stress value at the end of the face width.

For case 3, maximum stress value is also higher compared to the Abaqus result but the contact pattern represents agreement as the contact shifted back to the original case when no misalignment was introduced.
### Table 5.6 Tooth root stress distribution in gear 1 from LDP

<table>
<thead>
<tr>
<th>case</th>
<th>BEM stress gear 1</th>
<th>FEM stress gear 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="BEM stress plot" /></td>
<td><img src="image2" alt="FEM stress plot" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3" alt="BEM stress plot" /></td>
<td><img src="image4" alt="FEM stress plot" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image5" alt="BEM stress plot" /></td>
<td><img src="image6" alt="FEM stress plot" /></td>
</tr>
</tbody>
</table>
Table 5.7 Tooth root stress distribution in gear 2

<table>
<thead>
<tr>
<th>case</th>
<th>BEM stress gear2</th>
<th>FEM stress gear2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="normalized contour plot for root stress distribution" /></td>
<td><img src="image2" alt="normalized contour plot for root stress distribution" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3" alt="normalized contour plot for root stress distribution" /></td>
<td><img src="image4" alt="normalized contour plot for root stress distribution" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image5" alt="normalized contour plot for root stress distribution" /></td>
<td><img src="image6" alt="normalized contour plot for root stress distribution" /></td>
</tr>
</tbody>
</table>
5.4 Results from KISSsoft

Table 5.8 contact stress distribution from KISSsoft

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Table 5.8 shows the result for stress distribution throughout the teeth from KISSsoft. It is convincing that in case 1, the stress is distributed throughout the teeth but for case 2 the stress is shifted towards corner at the end of face width for both the gears due to inclusion of the misalignment. In case 3 the microgeometry modification compensated the misalignment and contact pattern is better than case 2.

5.5 Comparison and discussion

In Table 5.9 there is visible difference among the results from the different software but if the result for different cases is considered, they followed expected pattern. When the discussion is going on comparing the different software and by considering one case only in the comparison. It can be seen from the table the highest stress results is always coming from KISSsoft and the lowest stress results for is always coming from FEM.
Table 5.9 the comparison between Abaqus, LDP and KISSsoft

<table>
<thead>
<tr>
<th>case</th>
<th>Output</th>
<th>LDP</th>
<th>KISSsoft</th>
<th>Abaqus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Method</td>
<td>gear 1</td>
<td>gear 2</td>
<td>gear 1</td>
</tr>
<tr>
<td>case 1</td>
<td>maximum contact stress (MPa)</td>
<td>1.24</td>
<td>1.38</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Maximum Tooth root stress (MPa)</td>
<td>BEM</td>
<td>2.15</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>2.44</td>
<td>2.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Peak to peak transmission error(μm)</td>
<td>0.2458</td>
<td>0.3438</td>
<td>Inconsistent</td>
</tr>
<tr>
<td>case 2</td>
<td>maximum contact stress (MPa)</td>
<td>1.60</td>
<td>1.87</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>Maximum Tooth root stress (MPa)</td>
<td>BEM</td>
<td>3.24</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>2.75</td>
<td>2.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Peak to peak transmission error(μm)</td>
<td>0.6533</td>
<td>0.4473</td>
<td>Inconsistent</td>
</tr>
<tr>
<td>case 3</td>
<td>maximum contact stress (MPa)</td>
<td>1.26</td>
<td>1.41</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Maximum Tooth root stress (MPa)</td>
<td>BEM</td>
<td>2.19</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>2.44</td>
<td>2.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Peak to peak transmission error(μm)</td>
<td>0.2131</td>
<td>0.3791</td>
<td>inconsistent</td>
</tr>
</tbody>
</table>

From the comparison, it is noticeable that there are some differences among the results from different software. Each tool had separate methods to calculate with some assumptions. To realize why the results are different, it is important to know the theory behind the tools. There are some aspects described below which could affect the results that were general considerations in the tools and some of them were true for specific cases considered in this study.
5.5.1.1 Calculation method

Formulation of the contact problem was not the same for all the software. LDP considers load distribution as a function of elastic deformations and tooth error that was briefly discussed in previous chapter. In KISSsoft, to calculate the stress, path of contact under load is calculated which changes with the total stiffness. This periodically changing stiffness is calculated considering a spring in the line of action.

5.5.1.2 Assumptions

There are some assumptions in LDP to analyse gear pair contact especially when calculating elastic deflection. Among them, one of the most important is elastic deformations are small and remains in the line of action which is not true in case of FE analysis. Another major assumption is made as deflections of any given tooth pair are not influenced by the loads on other tooth pairs which is also not true in case other software. In KISSsoft, total deformation has the effect that the contact point is displaced along the path of contact and length of line of contact is elongated, compared to theoretical.

5.5.1.3 Geometry

To do a comparative study on tooth contact analysis it is quite important to have the same geometry for the gear pair. If the geometry is different that will create different gears with different profiles which will affect the conjugate action and lead to variation in results. In this research gears for FE analysis was imported from KISSsoft. Therefore, the basic geometry of the gear tooth from LDP and KISSsoft was checked and negligible difference was found in addendum and dedendum. But when the geometry for calculation was studied there were some features that could lead to different result from both the software.

5.5.1.4 Teeth Profile

According to LDP, bending of three-dimensional gear tooth comprised of bending, shear, base rotation and deflection which were discussed in the previous chapter. Between Couple of options, tapered plate model was considered in this study to estimate bending and shear deformation. This model approximated gear tooth as a tapered plate as shown in Figure 5.3
Figure 5.3 the tapered plate model created from LDP

$T_r$ and $T_t$ are the tooth thickness respectively at root and tip area was calculated by program depending on input geometry information. Then the gear tooth thickness in different height positions can be calculated by following equation

$$T_y = T_r (1 - \alpha \frac{y}{H}) \quad (5.1)$$

Where, taper ratio, $\alpha$ depends only on tooth root and tip thickness.

$$\alpha = (1 - \frac{T_t}{T_r}) \quad (5.2)$$

Then there is one correction of tooth dimension is done by LDP. For better fitting, tooth thickness at half of the tooth height is calculated using both taper and involute equation. The average value is chosen as thickness at the middle and difference between this thickness and the tapered thickness is added at tip and root. New taper ratio is calculated and this profile is used to calculate the bending and the shear deformation. When compared to 3D model, there can be 20% to 30% too compliant for the bending component.

On the other hand, bending deformation formula in KISSsoft comprises two terms which allow numerical integration in Cartesian coordinate and that will depict the true tooth form. Therefore, KISSsoft can calculate the bending deformation for original tooth shape whether it is involute, trochoidal or cycloidal.
5.5.1.5 Root fillet

Tooth root stress is one of the interests in this research and in many literature, it was found that the bending stress significantly influenced by the Root fillet which is constructed between root and base radius. LDP ignores the fillet and finds the tooth thickness at root assuming an extension of the involute profile from base radius down towards the root radius. This assumption is also not true in the KISSsoft, as it will always have some root fillet either specified by user or calculated by the program.

5.5.1.6 Base radius lower than root radius

Normally root radius of gear is lower than the base radius the involute profile of gear starts from the base radius. However, the studied driving gear has lower base radius compared to root. As LDP calculates tooth root thickness extending the involute towards the root, it will not provide the correct thickness if root radius is higher. In this case, LDP will approximate lower thickness at root that may result to higher stress at root area.

5.5.1.7 Base rotation from LDP

Base rotation deflection and in that model LDP use the fact that the base of a gear tooth rotates due to the moment imposed on it and translates slightly due to the translational force. This model is based on finite element results; however, finite element results have shown that for some cases this method may slightly overestimate the base rotation[35] and that also can lead in a more base deflection and that will lead to more load acting on the gear tooth, thus more stress in the gear tooth.

5.5.1.8 Contact deformation from LDP

Contact deformation and in that one LDP is using the Hertzian contact theory and it is the same method used by FEM to calculate the contact stress. the deformation from the model should agree with the FEM but LDP has a major assumption that “The elastic deformations are small; thus, tooth contact is assumed to remain on the line of contact”. But that assumption is not made by the FEM model and that will affect the load distribution by LDP.
5.5.1.9 Accuracy of root stress in KISSsoft

To calculate the tooth root stress in helical gear, virtual spur gear is used. The calculations give relatively good results for gears with standard tooth profile which is not true for the studied case. According to KISSsoft documentation, the results are inaccurate in some cases which were also true for the modelled gear pair in this research.

- Gears with high helix angle, higher than 25º.
- Gears with big profile shift coefficient.
- Gears with tooth modifications.
- Gears with high contact ratio (deep tooth profiles).

All the above cases affect the results for tooth root stress and justify significant variation from FE result.

5.5.1.10 Transmission error from FEA

The transmission error calculation using FEM is not consistent as it is affected by the small penetration between the driving and the driven gear surfaces. The actual rotation that is calculated from the FEM is changing very slightly with the number of nodes that come into contact and leave the contact for every iteration in Abaqus. That number of nodes is varying slightly so the number of points leaving the contact is not always replaced by the respective number of nodes coming to contact. This change will not lead to a very big change in the rotation but it delivers an error in almost .0001 radians.

The equation (3.12) is used to calculate the linear transmission error in micrometer and that equation is using the term \( r_b \) the base radius which is in range of thousands micro meter, also that equation is considering the difference in rotation between the actual rotation and the theoretical rotation. The difference in rotation can change slightly and may have an error or difference of almost .0001 radians when this value is multiplied by the big value of the base radius which is in mm, that gives a value that differs almost 10 \( \mu m \) and that is not giving a convenient peak to peak transmission error value.
5.5.1.11 Difference in misalignment definition

The misalignment value used in the study was extracted from MASTA where the whole gearbox was modelled. According to MASTA, documentation misalignment is defined in the transverse plane as a displacement along the line of action. If the plane of action is considered, positive misalignment is defined as the gap at the most positive end of the gears in the z direction (in the gears local coordinate system) that there would be if the other ends were just in contact. In this model, the misalignment was negative.

![Figure 5.4 Misalignment definition in MASTA](image)

In LDP, misalignment also considered in the line of action. According to LDP, negative misalignment in the start or end will add extra material from the flank thus the contact shifts to that side. This misalignment value used is composed of two values that can be expressed in vertical and horizontal axis as in equation (5.3) according to Houser [22] where these two coordinates can be seen.
Misalignment along x-direction is called the parallelism error and along y-direction is called skew misalignment. According to Houser, this skew misalignment is far more important compared to the other one. In addition, the misalignment in line of action is calculated with the following formula

\[ M_{LOA} = M_X \cos \alpha + M_Y \sin \alpha \]  

(5.3)

Where \( M_X \) and \( M_Y \) are respectively the two misalignments in x and y direction.

In Abaqus, this misalignment was introduced by rotating the gear around perpendicular axis constructed on a reference plane depicting the plane of line of action.

One problem encountered introducing the same misalignment in KISSsoft as it was considering the two-different component separately as shown in Figure 5.6. Compared to LDP, inclination in KISSsoft is the parallelism error and deviation error is the skew misalignment. However, there were no instruction or document found how equivalent misalignment in the plane of action was calculated.
This uncertainty leads to believe that this could be the reason behind the significant variation in contact stress result for case two from KISSsoft compared to any other software.
6 Conclusion and Future work

6.1 Conclusions

A comparative study has been done for gear tooth contact analysis. Different case consisting different microgeometry and misalignment was modelled using gear calculation tools and finite element software to perform the loaded contact analysis. The result has been discussed from the authors’ point of view. Some conclusion can be drawn from the study -

- The specialized gear calculating tools, which have been used in this research (LDP and KISSsoft), is overestimating contact and bending stress compared to finite element software (Abaqus).
- The shaft misalignment leading to a higher contact and bending stress acting on the gear tooth and for the whole contact pattern to shift to the end of the face width. Transmission error also increases significantly.
- Sufficient micro geometry modification to the design of the gear tooth leads to compensate for the effect of the misalignment and lower the stress and transmission error.
- The peak-to-peak Transmission error from FEM was inconsistent and it differs from the LDP and KISSsoft value.

Contact analysis using Finite element should depict closer to realistic result. But it takes large computational time and depends on the user’s competency who formulates the contact in FE software. In the industry where the designers have several proposals for macro and microgeometry, it is not feasible to model each proposal using dedicated finite element software. But it is important for them to know how much the result will differ. That was the main motivation behind this study.
6.2 Future work

- The scope for this research was limited to only one pair of gear and the maximum load case for that specific pair. As the transmission error result varies with the applied torque, a study can be done for multiple load cases.
- An experimental set up can be proposed to justify the simulation results. That will validate the finite element model and ultimately fortify this study.
- Further investigation is suggested to check the possibility of building a meta-model for contact stress, tooth root stress and transmission error using the results from FE analysis.
- This meta-model can be further used to optimize the Macro geometry along with the Micro geometry for multiple objectives including stresses and transmission error acting on gear tooth within some limitation.
7 References


8 Appendix

The content of the Appendix is non-public due to disclosure agreement. Please contact the authors for further notice on what data may be made available.

8.1 Comparison table with real value

8.2 Abaqus set up and results
8.2.1 Abaqus Set up
8.2.2 Results for case 1
8.2.3 Results for case 2
8.2.4 Results for case 3

8.3 Detailed figures from LDP
8.3.1 Results for Case 1
8.3.2 Results for Case 2
8.3.3 Results for Case 3

8.4 Detailed figure from KISSsoft
8.4.1 Results for Case 1
8.4.2 Results for Case 2
8.4.3 Results for Case 3

8.5 MATLAB code to calculate TE from FE result