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Fuzzy decision-making model with qualitative states and fuzzified outcomes

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Abstract

The classical fuzzy decision-making model is now tested for qualitative compound states-symptoms to select the most efficacious medicine, acting on all symptoms. Instead of terminating the decision procedure in the way comparing values of total utilities of decisions-treatments, we test the aggregated utility values in utility levels. This activity lets us assign a verbally verified utility to each medicine.

Keywords: fuzzy decision-making; utility matrix; qualitative states; total utilities; parametric s-functions; fuzzy utility levels.

1. Introduction

Theoretical fuzzy decision-making models, mostly developed in [1][2][3][4], gave rise to successfully accomplished technical applications. However, there are not so many medical applications to decision-making proposals, especially they are lacking in the domain of pharmacy. Our own trials of selecting the most efficacious medicine, when using fuzzy decision making, can be traced in [5][6][7].

When symptoms, characterizing the disease have a compound qualitative complexion, we still can choose the best acting medicine recommended for an individual patient by employing the own model of designing a utility matrix.

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This matrix constitutes the input system in decision-making [6]. We thus discuss the method that converts information obtained from a questionnaire to entries of the utility matrix. Further, by suggesting the changes in the assumptions of the final part of classical decision-making, we will cause proving total utilities of medicines in the sample of fuzzy sets, standing for levels-terms of efficacy. This makes possible to combine a level of utility with the medicine by using words from a natural language. To this purpose, we need to generate membership functions of fuzzified utility levels.

There already exist some mathematically formalized designs of membership functions [8][9][10], but we propose the own algorithm, which categorizes all membership functions into three parametric families [11]. The activity of the algorithm allows finding membership degrees of the total utility in all effectiveness levels to assign verbal descriptions of the curative power on all symptoms to each decision-medicine.

In Section 2, we introduce the decision-making model to continue the creation of utility matrix entries for qualitative symptoms in Section 3. Total utilities are designed in Section 4. Section 5 exhibits fuzzy outcomes and their action to emerge the best medicine. Some conclusions are parts of Section 6.

2. The outline of decision making model for compound qualitative states and verbal outcomes

Let us recall the definition of a fuzzy set. If X is a collection of objects denoted generically by x, then the fuzzy set A in X is a set of ordered pairs $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x) \in [0,1]$ [12].

Each element x gets a membership degree $\mu_A(x)$, which expresses the strength of the relationship between x and A. Membership degrees, equal to 1, inform about the total relation between the element and the set. The function $\mu_A : X \to [0,1]$ is called "*the membership function*" of A.

The support supp(A) of a fuzzy set A, is a non-fuzzy set of all $x \in X$ such that $\mu_A(x) > 0$ [12].

Let us list the steps of an algorithm, which modifies the classical approach to fuzzy decision-making.

Step 1

We introduce the notions of a space of states $X = \{x_1, \dots, x_m\}$ and a decision space (a space of alternatives) $A = \{a_1, \dots, a_n\}$[2][3][4].

Let us consider a decision model, in which n alternatives $a_1, \dots, a_n \in A$ act as treatments used to cure patients who suffer from a disease. The medicines should influence m states $x_1, \dots, x_m \in X$, which are identified with m symptoms typical of the morbid unit under consideration.

We form a basic utility matrix

$$C = \begin{array}{c} \\ a_1 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{matrix} x_1 & \cdots & x_m \end{matrix} \\ \begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nm} \end{bmatrix} \end{array} \tag{1}$$

where c_{ij} is the utility to decide after connecting a_i to x_j, $i = 1,\dots,n$, $j = 1,\dots,m$. The quantities c_{ij} are assigned to interval [0, 1]. Matrix C collects our primary knowledge about the system, but a purpose of any decision-making method is to select this decision (medicine) among a_i, $i = 1,\dots,n$, that has the greatest power in recovery of the patient from symptoms x_j, $j = 1,\dots,m$. In our approach to decision-making, we even test curative power of medicines by assigning to them expressions constituting the contents of an effectiveness list like, e.g., "little effectiveness" or "large effectiveness".

Let us suppose that all symptoms x_j are of the qualitative nature, i.e., we are able to evaluate their intensities after treatment by regarding several factors included in a questionnaire. The questionnaire can be composed as a collection of questions with alternative answers, provided that each question touches a case of the symptom improvement. We denote symptom x_j affected by a_i by the symbol x_j/a_i. By utilizing the technique, implemented in Section 3, we intend to estimate numbers c_{ij} that will be the entries of utility matrix C.

Step 2

The effectiveness C_{a_i} of each medicine a_i will be evaluated by an aggregation of c_{ij} belonging to row i, $j = 1,\dots,m$.

The importance weights θ_j of symptoms x_j are added to the formula of C_{a_i} to emphasize x_j's harmful influence on the disease course. Due to the professional experience, the physician suggests the placement of x_j in the sequence $x_1 \succ \dots \succ x_m$, where "$\succ$" means "$x_j$ emerges more dangerous impact on the patient health state than x_h, j, $h = 1,\dots,m$. We state $\theta_1 > \ldots > \theta_m$ and want $\sum_{j=1}^{m} \theta_j = 1$.

The collected utility C_{a_i}, estimated via $c_{i,j}$ and θ_j, will be derived for treatment a_i as

$$C_{a_i} = \sum_{j=1}^{m} c_{ij} \cdot \theta_j, i = 1, \ldots, n. \quad (2)$$

We note that the minimal value of C_{a_i} is 0 since, for all minimal $c_{ij} = 0$ in row i, we obtain $C_{a_i} = \sum_{j=1}^{m} 0 \cdot \theta_j = 0$. The maximal value of C_{a_i} will reach 1 if, for all maximal $c_{ij} = 1$ in row i, $C_{a_i} = \sum_{j=1}^{m} 1 \cdot \theta_j = 1 \cdot \sum_{j=1}^{n} \theta_j = 1 \cdot 1 = 1$. Hence, $C_{a_i} \in [0, 1]$, $i = 1,\ldots,n$.

Step 3

We should now generate a list of output utility fuzzy levels L_l, $l = 1,\ldots,\omega$, for C_{a_i}. The supports of L_l will cover parts of [0, 1], as proved in Step 2 ($C_{a_i} \in [0, 1]$). To calculate the membership degrees of utility C_{a_i} in all L_l, we need to derive a formula of the membership function of each L_l. The largest value $\mu_{L_l}(C_{a_i})$, $l = 1,..,\omega$, points out the optimal utility level of medicine a_i. Equation (3) summarizes the action of the algorithm phases as

$$a_i[c_{i1} \cdot \theta_1 \quad \ldots \quad c_{im} \cdot \theta_m] \to C_{a_i} = \sum_{j=1}^{m} c_{ij} \cdot \theta_j \to \begin{matrix} \mu_{L_1}(C_{a_i}) \\ \vdots \\ \mu_{L_\omega}(C_{a_i}) \end{matrix} \to utility\ level\ L^{*}\ for\ a_i \quad (3)$$

Let us now develop the mathematical parts, included in the steps from Section 2.

3. The enumeration of single utilities c_{ij}

The progress in the recovery from a disease is noted by observing the status of symptoms that follow the morbid unit. For the symptoms, which are not measurable or are characterized by levels of a measurable parameter, a verbal description should exist. Let us add a set Q^{x_j/a_i}, formed by the questions, denoted here by $q_p^{x_j/a_i} \in Q^{x_j/a_i}, p = 1, \ldots, Q^{x_j/a_i - last\ question}$ [6]. The symbol $Q^{x_j/a_i - last\ question}$ stands for the number of questions associated with symptom x_j/a_i.

To each of the questions $q_p^{x_j/a_i}$, posed to the person being examined when symptom x_j has been found and treated by medicine a_i, he/she has a possibility of choosing one of several answers regarding an improvement in the health state. These answers are usually furnished with numbers-codes $s_{p,k}^{x_j/a_i} \in \{0, 1, \ldots, \lambda^{(p)}\}$, being non-negative integers. The symbol $\lambda^{(p)}$ (even integer) designates the number of all alternative answers to the question $q_p^{x_j/a_i}$. The codes 0, 1, …, $\lambda^{(p)}$ are hierarchical replacements of the answers from the most negative, which confirm the absolute presence of x_j despite the medication with a_i, to the most positive, revealing the full effectiveness of the drug a_i in the case of x_j.

In the further procedure, one should assign numerical supplements $w_{p,k}^{x_j/a_i}$ to the encoded answers $s_{p,k}^{x_j/a_i}$ [6]. These supplements are suggested to be numbers belonging to the interval [–1, 1]. It is assumed that negative code supplements correspond to negative answers to the question posed, i.e., these replies confirm the occurrence of the symptom (full presence x_j after treatment with a_i, almost full presence, little improvement and the like), with that –1 is assigned to the lack of drug efficiency on the symptom. The positive value of the code supplement gives a suitable positive character, certifying the absence of the symptom in the patient after medication, and the value +1 confirms the entire recovery from the symptom. The code supplement equal to zero is reserved for the case of the lack of the answer or a statement that brings moderate information.

To calculate values for code supplements $w_{p,k}^{x_j/a_i}$, $k = 0,1,\ldots,\lambda^{(p)}$, we construct a function [**7**]

$$w_{p,k}^{x_j/a_i} = w_{p,0}^{x_j/a_i} + k \cdot \frac{w_{p,\lambda^{(p)}}^{x_j/a_i} - w_{p,0}^{x_j/a_i}}{\lambda^{(p)}} = -1 + k \cdot \frac{2}{\lambda^{(p)}}, \quad (4)$$

where $w_{p,0}^{x_j/a_i} = -1$ and $w_{p,\lambda(p)}^{x_j/a_i} = 1$.

The set Q^{x_j/a_i} is a list of terms-questions. Our intention is to concatenate all answers to questions placed in $Q^{x_j/a_i}, p = 1, \dots, Q^{x_j/a_i - last\ question}$, to establish a fuzzy set x_j/a_i depicting the compound qualitative symptom x_j/a_i with the same name. We should thus design a support and a membership function for x_j/a_i.

To form support $[\alpha(x_j/a_i), \gamma(x_j/a_i)]$ of x_j/a_i for all possible combinations of code supplements, we suggest

$$\alpha(x_j/a_i) = \sum_{p=1}^{Q^{x_j/a_i - last\ question}} min_{1 \le k \le \lambda(p)}(w_{p,k}^{x_j/a_i}) \tag{5}$$

and

$$\gamma(x_j/a_i) = \sum_{p=1}^{Q^{x_j/a_i - last\ question}} max_{1 \le k \le \lambda(p)}(w_{p,k}^{x_j/a_i}). \tag{6}$$

The collective information $x(x_j/a_i) \in [\alpha(x_j/a_i), \gamma(x_j/a_i)]$, which samples all code choices made for questions referring to x_j/a_i, is proposed to be

$$x(x_j/a_i) = \sum_{p=1}^{Q^{x_j/a_i - last\ question}} (w_{p,k\ (where\ k\ is\ chosen\ by\ patient}^{x_j/a_i}). \tag{7}$$

The membership function over $[\alpha(x_j/a_i), \gamma(x_j/a_i)]$ is put forward for consideration as the *s*-parametric function

$$y(x(x_j/a_i)) = \mu_{x_j/a_i}(x(x_j/a_i)) = s(x(x_j/a_i), \alpha(x_j/a_i), \beta(x_j/a_i), \gamma(x_j/a_i)), \tag{8}$$

in which $\beta = \frac{\alpha(x_j/a_i)+\gamma(x_j/a_i)}{2}$.

Provided that $x = x(x_j/a_i), \alpha = \alpha(x_j/a_i), \beta = \beta(x_j/a_i), \gamma = \gamma(x_j/a_i)$, the *s*-function is given by the formula

$$s(x,\alpha,\beta,\gamma) = \begin{cases} 0 & for \quad x \le \alpha, \\ 2\left(\frac{x-\alpha}{\gamma-\alpha}\right)^2 & for \quad \alpha \le x \le \beta, \\ 1-2\left(\frac{x-\gamma}{\gamma-\alpha}\right)^2 & for \quad \beta \le x \le \gamma, \\ 1 & for \quad x \ge \gamma. \end{cases} \tag{9}$$

Example 1

We have suggested that the value of the code supplement assigned to the extreme negative answer to $q_p^{x_j/a_i}$ is fixed at –1 and the code supplement of its most positive variant is equal to +1. If the rule is preserved for each question, then the graph of the function (9), stated for five questions characteristic of x_j/a_i and furnished by the equation $y(x(x_j/a_i)) = s(x(x_j/a_i), -5, 0, 5)$, will take the symmetrical form, sketched in Fig. 1.

In the set support we can notice the appearance of the number $\alpha(x_j/a_i)$ = –5; therefore the sum of the minimal code supplements, representing the five answers totally negating the effect of a_i in the case of x_j, has the assigned membership degree equal to zero (no effectiveness). The number $\gamma(x_j/a_i)$ = 5 is a sum of maximally positive code supplements confirming the lack of symptom x_j after the curative process involving a_i and, in consequence, the membership degree associated with $\gamma(x_j/a_i)$, takes the value 1 (full effectiveness). A moderate piece of information or its lack, expressed by $\beta(x_j/a_i)$ takes the membership degree equal to 0.5.

Let us now state the contents of *C* given by (1) by assimilating data from the next example with theoretical assumptions of the technique above [6].

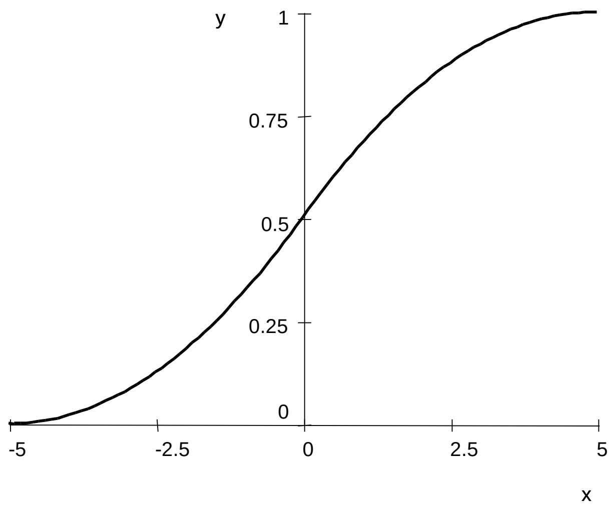

Fig. 1. The membership function of qualitative symptom x_j/a_i recognized by five questions

Example 2

We analyze clinical data that concern the diagnosis "*coronary heart disease*". We consider the most substantial symptoms x_1 = "*pain in chest*", x_2 = "*changes in ECG*" and x_3 = "*increased level of LDL-cholesterol*". The recommended medicines are listed as a_1 = *nitroglycerin*, a_2 = *beta-adrenergic blockade, a_3 = acetylsalicylic acid* (*aspirin*) and a_4 = *statine LDL-reductor*.

By composing a questionnaire, which yields the information of effectiveness of drugs involved in medication, we assign the degree of utility c_{ij} to each x_j/a_i, i = 1, 2, 3, 4, j = 1, 2, 3.

An inquiry of the first considered symptom x_1 – "*pain in chest*", is accomplished by answering, e.g., two questions:

$q_1^{x_1/a_i}$="*How often do you feel pain in chest after cure with a_i?*"

$q_2^{x_1/a_i}$="*How strong is pain in chest when it appears in spite of having taken a_i?*"

The alternative answers to these questions may be formulated, for instance, as follows:

– to the first question $q_1^{x_1/a_i}$

Always	$s_{1,0}^{x_1/a_i} = 0,\ \ w_{1,0}^{x_1/a_i} = -1,$	*Often*	$s_{1,1}^{x_1/a_i} = 1,\ \ w_{1,1}^{x_1/a_i} = -\frac{1}{2},$
From time to time	$s_{1,2}^{x_1/a_i} = 2,\ \ w_{1,2}^{x_1/a_i} = 0,$	*Rarely*	$s_{1,3}^{x_1/a_i} = 3,\ \ w_{1,3}^{x_1/a_i} = \frac{1}{2},$
Never	$s_{1,4}^{x_1/a_i} = 4,\ \ w_{1,4}^{x_1/a_i} = 1,$		

– to the second question $q_2^{x_1/a_i}$

Difficult to breathe	$s_{2,0}^{x_1/a_i} = 0,\ \ w_{2,0}^{x_1/a_i} = -1,$	*Very great*	$s_{2,1}^{x_1/a_i} = 1,\ \ w_{2,1}^{x_1/a_i} = -\frac{2}{3},$
Rather great	$s_{2,2}^{x_1/a_i} = 2,\ \ w_{2,2}^{x_1/a_i} = -\frac{1}{3},$	*I cannot determine*	$s_{2,3}^{x_1/a_i} = 3,\ \ w_{2,3}^{x_1/a_i} = 0,$
Rather week	$s_{2,4}^{x_1/a_i} = 4,\ \ w_{2,4}^{x_1/a_i} = \frac{1}{3},$	*Slightly remarkable*	$s_{2,5}^{x_1/a_i} = 5,\ \ w_{2,5}^{x_1/a_i} = \frac{2}{3},$
I do not feel any	$s_{2,6}^{x_1/a_i} = 6,\ \ w_{2,6}^{x_1/a_i} = 1.$		

We assume that patient P, who has been first treated with a_1, chooses code k =3 to $q_1^{x_1/a_1}$ and code k = 5 to $q_2^{x_1/a_1}$; hence,

$$x(x_1/a_1) = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}, \alpha(x_1/a_1) = (-1) + (-1) = -2, \gamma(x_1/a_1) = 1 + 1 = 2$$

and

$\mu_{x_1/a_1}(x(x_1/a_1)) = s(x(x_1/a_1), -2{,}0{,}2) = 1 - 2\left(\frac{x(x_1/a_1)-2}{2-(-2)}\right)^2$ in view of $x > 0$, that is $c_{11} = \mu_{x_1/a_1}\left(\frac{7}{6}\right) = 0.913$.

After proving a_2, P has selected codes $k = 2$ and $k = 3$ to the questions, which evaluates $c_{21} = \mu_{x_1/a_2}(0+0) = 0.5$. The interview, concerning the effectiveness of a_3 and a_4, provides matrix C with the value of $c_{31} = c_{41} = \mu_{x_1/a_3}\left(\left(-\frac{1}{2}\right) + \left(-\frac{1}{3}\right)\right) = \mu_{x_1/a_4}\left(\left(-\frac{1}{2}\right) + \left(-\frac{1}{3}\right)\right)$ =0.17.

The first column in matrix C is now filled with the numerical substitutes of drug utilities approximated for x_1. Let us emphasize that the qualitative parameter "*pain*" is characterized by the extended information about its character, since the contents of the questionnaire touches every subtle distinction in the patient's report, when comparing to the extreme information of the type "*I have pain*" contrary to "*I do not feel pain*". The list of questions can be arbitrarily extended.

We prepare another questioned characteristics of x_2/a_i and x_3/a_i, i = 1, 2, 3, 4, to complement the entries of C as

$$C = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{array}\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 0.91 & 0.81 & 0.06 \\ 0.5 & 0.5 & 0.27 \\ 0.17 & 0.17 & 0.09 \\ 0.17 & 0.11 & 0.87 \end{bmatrix} \end{array}.$$

4. The construction of utilities C_{a_i}

Apart from c_{ij}, which is the crucial factor of the total utility of a_i, another complement θ_j, called the importance weight of symptom x_j, has been added in Eq. 2. Let us solve the problem of assigning the importance weights θ_j to symptoms x_j. By "importance" we mean the strength of x_j's adverse and harmful power in the running process of the illness diagnosed. We bring into light the own mathematical algorithm, allowing the estimation of weights [7].

Generally, if we consider m symptoms x_j to find importance weights for them, we will wish to arrange the symptoms in the sequence $x_1 \succ \ldots \succ x_m$ in accordance with the expert's opinion. We want the sum of all weights θ_j, assisting x_j, j = 1,...,m, to be 1. Therefore,

$$m \cdot r + (m-1) \cdot r + \cdots + 2 \cdot r + 1 \cdot r = 1 \quad (10)$$

where r is a quotient dependent on m.

Further,

$$\theta_j = (m - j + 1) \cdot r \quad (11)$$

for j = 1,...,m.

Example 3

The physical status of a patient is subjectively better if the pain disappears, which means that a physician tries to release the patient from the symptom x_1 = "*pain in chest*". The next priority is assigned to x_2 = "*changes in ECG*" and finally, we concentrate our attention on getting rid of x_3 = "*increased level of LDL cholesterol*". Due to these recommendations, we state the order of symptoms as $x_1 \succ x_2 \succ x_3$. After solving equation $3r+2r+r = 1$ with respect to r = 0.167, we get $\theta_1 = 0.499, \theta_2 = 0.334$ and $\theta_3 = 0.167$.

In accordance with Eq. (2), we evaluate

$$C_{a_1} = 0.91 \cdot 0.499 + 0.81 \cdot 0.334 + 0.06 \cdot 0.167 = 0.726,$$
$$C_{a_2} = 0.5 \cdot 0.499 + 0.5 \cdot 0.334 + 0.27 \cdot 0.167 = 0.462,$$
$$C_{a_3} = 0.17 \cdot 0.499 + 0.17 \cdot 0.334 + 0.09 \cdot 0.167 = 0.157,$$
$$C_{a_4} = 0.17 \cdot 0.499 + 0.11 \cdot 0.334 + 0.87 \cdot 0.167 = 0.267.$$

We can already rank medicines in the sequence $a_1 \succ a_2 \succ a_4 \succ a_3$, but we also aim to express their efficacy verbally. To achieve this development, we return to assumptions made in Step 3 of the algorithm, sketched in Section 2. This means to carry out a test of C_{a_i} in fuzzy sets, which will replace words coming from the effectiveness list.

5. The generation and the action of fuzzy outcomes L_l

The list of fuzzy outcomes $L = \{L_1, \ldots, L_\omega\}$ is a linguistic list consisting of ω words-expressions. Each word is associated with a fuzzy set. In compliance with [11], ω is a positive odd integer. Furthermore, let E be the length of a common non-fuzzy reference set $[z_{\min}, z_{\max}]$ of real numbers, designed for all membership functions characterizing the fuzzy sets from L, on condition that $z \in [z_{\min}, z_{\max}]$. We now wish to divide the linguistic terms into three groups recognized as a left group, a middle group and a right group. The procedure is described and proved in [11]. Let us emphasize that common formulas of membership functions of fuzzy sets, replacing the list's expressions, are affected by only two parameters, namely, number ω of sets in L and length E. Moreover, we do not need to guess at the boundary values of the sets' supports, as these will be designated by the formulas' effect.

The membership functions assigned to the "*leftmost*" terms are parametric functions, derived in Eq. (12) as

$$\mu_{L_t}(z) = \begin{cases} 1 & for \quad z \le \frac{E(\omega-1)}{2(\omega+1)}\delta(t), \\ 1 - 2\left(\frac{z - \frac{E(\omega-1)}{2(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 & for \quad \frac{E(\omega-1)}{2(\omega+1)}\delta(t) \le z \le \frac{E(\omega-1)}{2\omega}\delta(t), \\ 2\left(\frac{z - \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 & for \quad \frac{E(\omega-1)}{2\omega}\delta(t) \le z \le \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \\ 0 & for \quad z \ge \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \end{cases} \tag{12}$$

where $\delta(t) = \frac{2t}{\omega-1}, t = 1, \ldots, \frac{\omega-1}{2}$, is also a parametric function depending on left function number *t*.

The membership function of the set "*in the middle*" has the form of a clock. It is designed by (13) in the form of

$$\mu_{L_{\frac{\omega+1}{2}}}(z) = \begin{cases} 0 & for \quad z \le \frac{E(\omega-2)}{2\omega}, \\ 2\left(\frac{z - \frac{E(\omega-2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & for \quad \frac{E(\omega-2)}{2\omega} \le z \le \frac{E(\omega-1)}{2\omega}, \\ 1 - 2\left(\frac{z - \frac{E}{2}}{\frac{E}{\omega}}\right)^2 & for \quad \frac{E(\omega-1)}{2\omega} \le z \le \frac{E}{2}, \\ 1 - 2\left(\frac{z - \frac{E}{2}}{\frac{E}{\omega}}\right)^2 & for \quad \frac{E}{2} \le z \le \frac{E(\omega+1)}{2\omega}, \\ 2\left(\frac{z - \frac{E(\omega+2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & for \quad \frac{E(\omega+1)}{2\omega} \le z \le \frac{E(\omega+2)}{2\omega}, \\ 0 & for \quad z \ge \frac{E(\omega+2)}{2\omega}. \end{cases} \tag{13}$$

Finally, the membership functions for the "*rightmost*" sets can be expressed by (14) as

$$\mu_{L_{\frac{\omega+3}{2}+t-1}}(z)=\begin{cases}0 & for \quad z\le E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\varepsilon(t),\\ 2\left(\frac{z-\left(E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\varepsilon(t)}\right)^2 & for \quad E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\varepsilon(t)\le z\le E-\frac{E(\omega-1)}{2\omega}\varepsilon(t),\\ 1-2\left(\frac{z-\left(E-\frac{E(\omega-1)}{2(\omega+1)}\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\varepsilon(t)}\right)^2 & for \quad E-\frac{E(\omega-1)}{2\omega}\varepsilon(t)\le z\le E-\frac{E(\omega-1)}{2(\omega+1)}\varepsilon(t),\\ 1 & for \quad z\ge E-\frac{E(\omega-1)}{2(\omega+1)}\varepsilon(t).\end{cases} \tag{14}$$

A new function $\varepsilon(t)=1-\frac{2(t-1)}{\omega-1}, t=1,\dots,\frac{\omega-1}{2}$, inserts all rightmost functions one by one when setting *t*-function numbers in Eq. (14).

Example 4

Let us propose the utility level list as, e.g., L = {"*none*", "*little*", "*moderate*", "*large*", "*complete*"}. Fuzzy sets, assigned to the terms of *L*, will contain their supports in [0, 1], as C_{a_i} belongs to interval [0, 1], For E = 1, ω = 5, and $z=C_{a_i}$ we set t = 1 and $\delta(1)=0.5$ in Eq. (12) to get

$$\mu_{L_1}(C_{a_i})=\begin{cases}1 & for \quad C_{a_i}\le 0.166,\\ 1-2\left(\frac{C_{a_i}-0.166}{0.067}\right)^2 & for \quad 0.166\le C_{a_i}\le 0.2,\\ 2\left(\frac{C_{a_i}-0.233}{0.067}\right)^2 & for \quad 0.2\le C_{a_i}\le 0.233,\\ 0 & for \quad C_{a_i}\ge 0.233\end{cases} \tag{15}$$

and, for t = 2, $\delta(2)=1$

$$\mu_{L_2}(C_{a_i})=\begin{cases}1 & for \quad C_{a_i}\le 0.334,\\ 1-2\left(\frac{C_{a_i}-0.334}{0.134}\right)^2 & for \quad 0.334\le C_{a_i}\le 0.4,\\ 2\left(\frac{C_{a_i}-0.467}{0.134}\right)^2 & for \quad 0.4\le C_{a_i}\le 0.467,\\ 0 & for \quad C_{a_i}\ge 0.467.\end{cases} \tag{16}$$

The function of L_3 is computed, according to Eq. (13), as

$$\mu_{L_3}(C_{a_i})=\begin{cases}0 & for \quad C_{a_i}\le 0.3,\\ 2\left(\frac{C_{a_i}-0.3}{0.2}\right)^2 & for \quad 0.3\le C_{a_i}\le 0.4,\\ 1-2\left(\frac{C_{a_i}-0.5}{0.2}\right)^2 & for \quad 0.4\le C_{a_i}\le 0.5,\\ 1-2\left(\frac{C_{a_i}-0.5}{0.2}\right)^2 & for \quad 0.5\le C_{a_i}\le 0.6,\\ 2\left(\frac{C_{a_i}-0.7}{0.2}\right)^2 & for \quad 0.6\le C_{a_i}\le 0.7,\\ 0 & for \quad C_{a_i}\ge 0.7.\end{cases} \tag{17}$$

By inserting t = 1, 2 ($\varepsilon(1)=1, \varepsilon(2)=0.5$), in turn, in Eq. (14), we implement the "*rightmost*" functions

$$\mu_{L_4}(C_{a_i})=\begin{cases}0 & for \quad C_{a_i}\le 0.533,\\ 2\left(\frac{C_{a_i}-0.533}{0.134}\right)^2 & for \quad 0.533\le C_{a_i}\le 0.6,\\ 1-2\left(\frac{C_{a_i}-0.667}{0.134}\right)^2 & for \quad 0.6\le C_{a_i}\le 0.667,\\ 1 & for \quad C_{a_i}\ge 0.667\end{cases} \tag{18}$$

and

$$\mu_{L_5}(C_{a_i}) = \begin{cases} 0 & for \quad C_{a_i} \leq 0.768, \\ 2\left(\frac{C_{a_i}-0.767}{0.067}\right)^2 & for \;\; 0.767 \leq C_{a_i} \leq 0.8, \\ 1-2\left(\frac{C_{a_i}-0.833}{0.067}\right)^2 & for \;\; 0.8 \leq C_{a_i} \leq 0.833. \\ 1 & for \quad C_{a_i} \geq 0.833. \end{cases} \tag{19}$$

The membership functions of L_1-L_5 are depicted in Fig. 2.

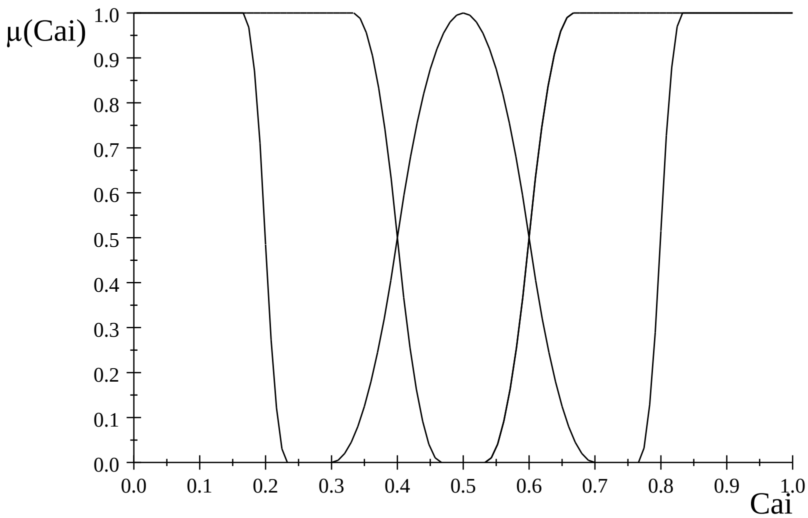

Fig. 2. The membership functions of L_1-L_5

Example 5

We set now the values of utilities C_{a_i}, $i = 1,\ldots,4$, in Eqs (15)-(19) to assess the efficacy of each medicine verbally.

For $C_{a_1} = 0.726$, $\mu_{L_1}(0.726) = \mu_{L_2}(0.726) = \mu_{L_3}(0.726) = 0$. $\mu_{L_4}(0.726) = 1$ since $0.726 > 0.667$. In L_5, $\mu_{L_5}(0.726) = 0$ because $0.726 < 0.767$. The judgment of a_1's influence on all the symptoms is made as "*large*" with membership 1.

By testing $C_{a_2} = 0.462$, we find $\mu_{L_1}(0.462) = 0, \mu_{L_2}(0.462) = 2\left(\frac{0.462-0.467}{0.134}\right)^2 = 0.003, \mu_{L_3}(0.462) = 1 - 2\left(\frac{0.462-0.5}{0.2}\right)^2 = 0.928, \mu_{L_4}(0.462) = 0, \mu_{L_5}(0.462) = 0$. Thus, a_2 affects the symptoms in grade "*medium*" with membership 0.928.

If $C_{a_3} = 0.157$, then $\mu_{L_1}(0.157) = 1$, $\mu_{L_2}(0.157) = 1$, $\mu_{L_3}(0.157) = 0$, $\mu_{L_4}(0.157) = 0$, $\mu_{L_5}(0.157) = 0$. If, in the "*leftmost*" family, the element C_{a_i} is characterized by membership 1 found in more than one set, then we will bind a verbal expression to the set with the narrowest support. This ranks the effectiveness of a_3 as "*none*" with membership 1.

In the end, $C_{a_4} = 0.267$ reveals $\mu_{L_1}(0.267) = 0$, $\mu_{L_2}(0.267) = 1$, $\mu_{L_3}(0.267) = 0$, $\mu_{L_4}(0.267) = 0, \mu_{L_5}(0.267) = 0$. Medicine a_4 acts in the "*little*" grade on the sampling of the symptoms.

6. Conclusions

In this paper, we have proposed some rearrangements in the standard fuzzy decision-making. When states-results are measurable parameters, then it would be not so difficult to fill the utility matrix with measurements of single utilities of pairs (decision-state). In the case of qualitative compound states, examined by a questionnaire, the assignment of entries of the utility matrix is more complicated. Therefore, we have implemented the algorithm, whose acting first samples the information from the questionnaire to convert it later to a membership degree in the fuzzy set assigned to each pair of the type (decision, state). Already in the phase of computing total utilities, we can establish a hierarchy of decisions after placing their values in the descending sequence.

Nevertheless, we would also like to make assignments of verbal utilities concerning decisions. To this purpose, we have planned a list of effectiveness levels, where each level has got the substitute being a fuzzy set. The fuzzy sets have been divided into three groups, which have obtained common parametric equations of membership functions. The total utility of each decision has been tested in all levels to be able to tie the decision to an expression coming from the list. In this way, we have even judged verbally the powers of all decisions.

The model has been tried on the medical example, in which decisions have played a role of medicines, and states have been treated as symptoms. For each medicine, the verbal efficacy strength has been found. This has improved the transparency of decision making by drawing conclusions based not only on the comparison of total utility values. We also want to emphasize that the effects of overlapping fuzzy sets will be more visible, if a list of verbal effectiveness expressions contains more words. This can allocate the total utility in more than one fuzzy set, which leads to richer effects of imprecision.

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