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Optimal placement of Charging Stations for Electric Vehicles in large-scale Transportation Networks

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Abstract

This paper presents a new practical approach to optimally allocate charging stations in large-scale transportation networks for electric vehicles (EVs). The problem is of particular importance to meet the charging demand of the growing fleet of alternative fuel vehicles. Considering the limited driving range of EVs, there is need to supply EV owners with accessible charging stations to reduce their range anxiety. The aim of the Route Node Coverage (RNC) problem, which is considered in the current paper, is to find the minimum number of charging stations, and their locations in order to cover the most probable routes in a transportation network. We propose an iterative approximation technique for RNC, where the associated Integer Problem (IP) is solved by exploiting a probabilistic random walk route selection, and thereby taking advantage of the numerical stability and efficiency of the standard IP software packages. Furthermore, our iterative RNC optimization procedure is both pertinent and straightforward to implement in computer coding and the design technique is therefore highly applicable. The proposed optimization technique is applied on the Sioux-Falls test transportation network, and in a large-scale case study covering the southern part of Sweden, where the focus is on reaching the maximum coverage with a minimum number of charging stations. The results are promising and show that the flexibility, smart route selection, and numerical efficiency of the proposed design technique, can pick out strategic locations for charging stations from thousands of possible locations without numerical difficulties.

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1. Introduction

With the continuously growing fleet of alternatively fueled vehicles there is a need to adapt the current transportation infrastructure to meet future needs. Electric vehicles (EVs), including plug-in hybrid electric vehicles (PHEVs)
and battery electric vehicles (BEVs) have been recognized as a promising sustainable approach towards less traffic emissions including greenhouse gases [1].

Considering the positive impacts, two major hindrances should be addressed before EVs may gain full market acceptance; these are limited range and limited access to charging stations [2]. To some extent, the limited driving range of EVs demands a strategic allocation of charging infrastructure along the most probable routes in the network. In addition, starting from each possible node in a transport network, it has to be possible to reach at least one charging station before the vehicle runs out of energy.

The concern that the limited driving range of EVs prevent them from reaching the desired destination before fully depleting its energy source, is referred to as range anxiety, and may cause psychological stress for drivers [3]. Thus, an important infrastructure question for decision-makers and stakeholders regarding how to gain market acceptance for EVs, is the allocation of charging stations. A strategic allocation of the charging infrastructure may not only reduce the range anxiety for EV owners, but also minimize the initial cost of the installment of new charging stations, as well as relieve the load of the electrical power system [4].

The need of strategic and optimal placement of new charging stations is one of the key factors for successful development and the problem has reached substantial attention in the research community. Several methods to optimally allocate charging stations in a transportation network has been proposed in the literature. The proposed methods can be roughly divided into flow-capturing models [5, 6, 7, 8], set-covering models [9, 10], vehicle movement simulation models [11, 12], agent-based models [13], and equilibrium models [14, 15, 16].

In the literature it is acknowledged that an appropriate first step towards acceptance of an electrified transportation fleet is to consider public transportation such as buses and taxis, and on daily basis EV owners may consider round trips (i.e., travelers depart from their origin to the destination, and then go back to their origin) [17, 18]. It is emphasized that our focus in this paper are long distance non-urban trips, i.e., trips where origin and destination are distinct.

In the current paper, we present an iterative approximation technique to solve the so-called Route Node Coverage (RNC) problem, which we apply to identify the optimal location of charging stations for EVs in a road transport network. Given a representable set of routes, the goal of the RNC problem is to find the minimum number of charging stations and their locations, so that any EV in the transportation network is able to reach a charging station before fully depleting its energy source. In the proposed approximation technique, the associated integer problem is solved by exploiting probabilistic self-avoiding random walks, which aim to identify the most probable routes in a transportation network. The identified routes are iteratively generated and added to the integer problem under consideration.

We consider the case where only the link flows, that is, the average number of vehicles traveling along each of the links in the network is known. The link flows are typically collected using a network of sensors, which includes both permanently located sensors and temporary sensors, including pneumatic tubes, that are moved around in the network according to a periodic scheme. Ideally, one would like to have knowledge about the flows between the different relations in the transport network; however, the link flows are typically the only primary data available when trying to find the optimal placement of charging stations. The collected links flows are utilized in order to model driving behavior in the network.

The paper is organized as follows: Section 2 describes the model and presents the problem formulation. In Section 3 the solution algorithm is discussed. For illustration, an example and a case study are given in Section 4. Section 5 concludes this paper and discusses further research directions.

2. Problem formulation

We describe a transportation network by a set of nodes $N$, a set of links $A$, and a set of routes $R$. The number of nodes in the network is denoted $d$. For each route $j \in R$, let $\delta_{ij} = 1$ if node $i \in N$ is contained in the route $j$, and $\delta_{ij} = 0$ otherwise. Mathematically, an allocation of charging stations is defined by a vector $x = (x_i) \in \{0, 1\}^d$, where $x_i = 1$ if a charging station is allocated on the node $i \in N$, and otherwise $x_i = 0$.

Denote by $c(j)$, the travel cost of each route $j \in R$. Typically, the travel cost of a route is the total length or travelling time along the route $j$. 
We say that a route $j \in R$ is *covered* if at least one charging station is placed on one of its contained nodes. More formally, a route $j \in R$ is covered if

$$\sum_{i \in N} \delta_{ij} x_i \geq 1$$

(1)

holds. We now formulate the Route Node Covering (RNC) problem investigated in this paper.

**RNC Problem formulation:** Given a transportation network, find a representative set of routes $R$, and the corresponding minimal set of charging stations such that each route $j \in R$ is covered by a charging station.

The RNC problem corresponds to the optimization problem

$$z = \min \left\{ \sum_{i \in N} x_i : \sum_{i \in N} \delta_{ij} x_i \geq 1, \forall j \in R, x \in \{0, 1\}^d \right\}$$

(2)

It should be noted that the problem $(P)$ would have been unsolvable with all possible routes included as constraints. Even for a rather small network, a complete route enumeration can lead to several millions of constraints in our set covering problem. To avoid route enumeration, we instead claim that it is necessary to identify an appropriate and representable set $R$ of routes, which is far from a trivial task in large transportation networks.

The primary goal with our proposed method is to ensure that any vehicle travelling along the routes $j \in R$ that most likely will be used, must reach at least one charging station, placed in an intersection/junction (node), before it runs out of energy.

### 3. Optimal Placement of Charging Stations

The optimization and optimal placement of charging stations is done by solving a sequence of sub problems $(P^{(k)})$ of our problem $(P)$ with increasing minimal optimal solution based on finite subsets $R_k \subset R$. As mentioned above, the $R_k$:s are iteratively extended by adding routes that are generated using probabilistic self-avoiding random walks in the transportation network. In our route generation process, the maximal travelling cost $c$, which is assumed to be less than the driving range of a typical EV, is the maximal distance or time a vehicle is allowed to travel without passing a node equipped with a charging station. This selection process not only provides the optimization processes with reasonable and probable routes (constraints), the route generation process also puts the foundation for further studying and developing the goal of optimization.

The proposed method to solve our RNC problem is based on integer programming, and the outline of the method can be described with the following basic steps:

0. Initialization: Set $R_k = \emptyset$. Fix $x_i = 1$ if a charging station is already allocated at node $i \in N$, and fix an upper route length bound $c$.
1. Given a reference set $R_k \subset R$, solve the sub problem

$$z^{(k)} = \min \left\{ \sum_{i \in N} x_i : \sum_{i \in N} \delta_{ij} x_i \geq 1, \forall j \in R_k, x \in \{0, 1\}^d \right\}$$

(3)

yielding the solution vector $x^{(k)}$.
2. Define the entering index $R_e$ by route $j$ with $c(j) \leq c$. Stop, if a new $R_e$ cannot be found according to some termination criteria.
3. Define the new reference set by $R_{k+1} = (R_k \cup \{R_e\})$ and go to step 1.
3.1. Selection of Routes

In our examples in Section 4 the reference sets of routes $R_k$ are simulated by the most probable routes, based on known link flows. The constraint index $R_e$ which is chosen to enter the basis $R_k$ is usually defined by a route which does not satisfy the current solution, i.e., a constraint violation. By using the most probable routes in the network, we claim that when no entering index $R_e$ can be found according to some termination criteria, the most commonly used (probable) routes in the network will be covered by at least one charging station, and thus included in $R$. In practice, finding an optimal reference set $R$ is computationally difficult. We use self-avoiding random walks as search method to find a new entering index $R_e$. If a walk passes a node already visited, or reaches a node already equipped with a charging station, i.e., the route is already covered by the current solution, a new random walk is restarted in a randomly chosen node until a new unique entering index $R_e$, violating the constraints in the current sub problem is found. A probabilistic rule determined by link flows is applied at each node to select the next link in the route.

Since routes with maximal travelling cost $c$ is considered, we emphasize that some constraints may not correspond to real-world routes in the network, but rather probable legs of routes.

3.2. Outline of the Convergence

The main idea is to iteratively solve small sub problems of the RNC problem, and continuously further extend the current sub problem to a slightly larger problem. A solution to the RNC problem is not necessarily unique, but continuously striving for a solution with a minimal number of charging stations. The procedure ensures coverage, and localize interesting common junctions within the given transportation network. The purpose of this procedure is to ensure that a charging station is placed in an environment that is of mutual interest to several of the found routes.

The behavior of convergence can be briefly outlined, and the emphasis is on an intuitive understanding for the optimization procedure. Assume that we are given the solution $x^k$ to any sub-problem $P(k)$. If we now add one single constraint where $x^k$ violates the constraint, we can easily provide a solution to iteration $k + 1$ by setting $x_i = 1$ for some $i$ in the left-hand side of the new constraint.

We now establish the following important inequality relations

$$z^{(1)} \leq \ldots \leq z^{(k)} \leq z^{(k+1)} \leq \ldots \leq z. \quad (4)$$

The first inequalities is due to the fact that the optimal solution for problem $(P(k+1))$ is a basic solution for problem $(P(k))$ in integer problem formulation (3). The sequence $(z^{(k)})$ is thus monotonically increasing and upper bounded. Convergence to the optimum value of $z^{(k)}$ is motivated by the last equality where the existence of a finite optimal reference set $R$ is established. We note in this context, that the convergence of the algorithm is based on the assumption that numerical difficulties (such as randomness, size of network, etc.) are avoided due to the presence of a reliable software for the solution. In practice, the optimal value of $z$ will never be reached for large-scaled networks. However, the system yields an appropriate approximation thereof, subject to continuously improvement with the aim of reaching the optimum.

In this presentation, we emphasize on the simplicity and efficiency to find a minimal number of charging stations as well as an approximate solution which cover each found route of this hard problem. Since the procedure is based on a controlled selection of constraints, there are opportunities to add and fulfill requirements that make the procedure of selecting routes to model the transportation network even more realistic.

4. Examples

In this section we provide two examples of our proposed method. In the first example we consider the widely used Sioux-Falls network [19], and in the second example, a large-scale case study is performed on the network of southern Sweden. The optimal solutions of the RNC method were obtained by using the standard software tools MATLAB® [20] and Gurobi [21].
4.1. The Sioux-Falls network

The Sioux-Falls network consisting of 24 nodes and 38 bi-directed links is depicted in Fig. 1. The turning probabilities are calculated from the link flows obtained from the best-known solution to the traffic assignment problem [22]. All nodes are candidate sites for a charging station.

We solved the RNC problem for the Sioux-Falls network with a transportation fleet with various maximal route cost $c$, which in this case is travelling distance. The required minimal number of charging stations for complete coverage is presented in Fig. 2. Example configurations with a minimal number of charging stations for $c = 16.1$, $c = 32.2$, and $c = 48.3$ kilometers are illustrated in Fig. 3. It should be noted that the choice of $c$ was based on an enumeration of possible routes in the network and calculated by the formula $c = 16.09 + 3.22k, k = 0, 1, 2, \ldots$, that is, linear combinations of the longest and shortest link in the network.
Maximal route cost \( c \) in kilometers (km)

Fig. 2: Minimal number of optimum charging stations for the Sioux-Falls network with increasing maximal route cost \( c \). The route cost \( c \) is the maximum distance a vehicle is allowed to travel without passing a charging station.

Fig. 3: Example of charging station allocations for the Sioux-Falls network with route cost parameters \( c = 16.1 \), \( c = 32.2 \), and \( c = 48.3 \) kilometers. Between the node 8 and 9, the longest path of the network is situated, and it is clear that the optimization (as expected) accordingly cover these two or obviously adjacent nodes with charging stations.

4.2. Large-scale case study: The Southern Sweden road network

To demonstrate the capability to find solutions on a large-scale network (a real world network), we consider the network of southern part of Sweden that is presented in Fig. 4. The studied network is one of six traffic regions maintained by the Swedish Transport Administration. The optimization is performed over the blue grid in the network, which consists of 14500 nodes and 34500 links distributed over an area of approximately 80000 square kilometers, spread across eight counties. In the optimization procedure, the neighboring regions have not been considered. The
data was obtained from *The Swedish national road database* (NVDB), provided by the Swedish Transport Administration.

In the simulations, we estimated the range of a typical EV to 150 km and therefore set $c = 75$, which is half of the range. An optimal solution with 66 charging stations and 607 constraints is depicted in Fig. 4. We emphasize that the optimal solution has a macroscopic approach. The allocation of charging stations has a focus outside urban- and metropolitan areas and the optimization has in this regard a focus on the government-controlled transportation network.

![Map of southern Sweden with optimized charging stations](image)

**Fig. 4:** The national road transportation network for the southern Sweden with an optimized placement of 66 charging stations out of 14500 possible locations. The charging stations are in the figure marked with a lightning symbol, and the triangle marks represent urban areas with several charging stations already installed.

5. **Conclusions**

Finding the optimum location of a minimal number of charging stations in a transportation network is a difficult task. The allocation of charging stations within a network should be based on specific goals such as maximizing the coverage given any arbitrary transportation network. The current paper proposes a methodology based on self-avoiding random walks along the links in the network, combined with a probabilistic rule applied at each node (intersection) in the network. The optimal location of charging stations in the network is selected by solving a pruned integer problem.
The proposed method can take into account existing charging infrastructure along the transportation network, and the solution to the problem provides an approximation of the optimal allocation.

The implementation of the proposed modelling approach in the case of the Southern Sweden transportation network shows how charging stations can be located in order to effectively meet the demand of an EV fleet. More specifically, the location of the charging stations deriving from our approach are at nodes that serve significant transportation flows of the study area and through which the drivers can follow alternative routes, in order to avoid congested routes. Moreover, their location within the transportation network can identify a network of paths with significant within-day variations of travel times and serve significant numbers of transportation flow volumes.

Future steps should include research along three main directions. The first one is on the application of alternative solution methods, including the linearization of the product of the variables and the problem formulation as an integer linear programming model. Other approaches such as heuristics and meta heuristics could also be applied. The second direction of further research could focus on the evaluation of the results derived from the location of the charging station with a view to improve other tools and systems that aim to provide real time information to drivers. A third direction of further research includes the optimal allocation considering that charging will most likely occur in the beginning or in the end of a trip, topographical impact of downhill and uphill slopes along the routes, and considering the driving style, traffic levels, etc.

References