The structure of epsilon-strongly group graded rings

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Akademisk avhandling

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Abstract

The development of a general theory of strongly group graded rings was initiated by Dade, Năstăsescu and Van Oystaeyen in the 1980s, and since then numerous structural results have been established. In this thesis we develop a general theory of so-called (nearly) epsilon-strongly group graded rings which were recently introduced by Nystedt, Öinert and Pinedo and which generalize strongly group graded rings. Moreover, we obtain applications to Leavitt path algebras, partial crossed products and algebraic Cuntz-Pimsner rings.

This thesis is based on five scientific papers (A, B, C, D, E).

Papers A and B are concerned with structural properties of epsilon-strongly graded rings. In Paper A, we consider an important construction called the induced quotient group grading. In Paper B, using results from Paper A, we obtain a Hilbert Basis Theorem for epsilon-strongly graded rings.

In Paper C, we study the graded structure of algebraic Cuntz-Pimsner rings. In particular, we obtain a partial characterization of unital strongly graded, epsilon-strongly graded and nearly epsilon-strongly graded algebraic Cuntz-Pimsner rings up to graded isomorphism.

In Paper D, we give a complete characterization of group graded rings that are graded von Neumann regular.

Finally, in Paper E, written in collaboration with Lundström, Öinert and Wagner, we consider prime nearly epsilon-strongly graded rings. Generalizing Passman's work from the 1980s, we give necessary and sufficient conditions for a nearly epsilon-strongly graded ring to be prime.

Keywords: group graded ring, Leavitt path algebra, partial crossed product, Cuntz-Pimsner ring, von Neumann regular ring, non-unital ring