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Modeling and Analysis of Small Cell Networks with NOMA: A Stochastic Geometry Approach

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Abstract—In this paper, a stochastic geometry approach is used to model and analyse a downlink small cell network (SCN). Carrier sense multiple access with collision avoidance (CSMA/CA) is used as medium access control (MAC) protocol at the base stations (BSs). To boost spectral efficiency, each active BS deploys both orthogonal multiple access (OMA) and non-orthogonal multiple access (NOMA) to transmit signals to the user equipments (UEs) in its coverage area. The choice between OMA and NOMA mode is based on the density of UEs and aims to improve the sum rate of the BS. If there exist two UEs that sufficiently differ in their channel power gains to the BS, the BS will use NOMA superimposing the signals of the UEs in the power-domain. Otherwise, the BS will operate in OMA mode to transmit the signal of a single UE in its coverage area. Stochastic geometry is used to include the spatial densities of the UEs and BSs in the performance assessment of the SCN. On this basis, analytical expressions for the coverage probability, data rates of the UEs, and sum rate of the BSs of the considered system are derived. Numerical results are provided to illustrate the impact of system parameters on the performance of this SCN with NOMA subject to the spatial densities of the BSs and UEs.

I. INTRODUCTION

The development of 5G and future generation mobile networks strives for providing higher data rates, more capacity, increased reliability, and reduced latency. Due to the uneven spatial distribution of the population, there exists an imbalance in mobile service demands within different areas. Thus, suitable network deployments are needed that can efficiently cope with the spatially dissimilar distributed service demand. Apart from techniques such as millimeter waves and massive multiple-input multiple-output systems, deployment of non-orthogonal multiple access (NOMA), and small cell networks (SCNs) have been shown to support the high capacity demands of 5G and future generation mobile networks [1], [2].

Power-domain NOMA [3] allows users within the same cell to simultaneously access the entire frequency band of the system. In particular, the signals of multiple users are superimposed at the base station (BS) where signals of users experiencing poorer channel conditions are given a higher proportion of the total transmit power. At the user equipments (UEs), successive interference cancellation is used to decode the desired signal of each user. Accordingly, the strongest signals at an UE are successively decoded and cancelled from the received signal treating the weaker signals as interference until its own signal becomes the strongest signal to be decoded.

Small cell networks are able to boost throughput by deploying a large number of picocells and/or femtocells in the

coverage area of a macro BS [4]. The stationary Poisson point process (PPP) is often used to model the distribution of SCNs in a large scale mobile network. However, when analyzing the signal-to-interference-plus-noise ratio subject to the location of UEs and BSs, both the randomness of node locations and randomness associated with the medium access control (MAC) used in the network need to be considered. Carrier sense multiple access (CSMA) MAC is suitable [5] in scenarios where simultaneously transmitting nodes are spatially separated with minimum distance R . To model the concurrent transmitters in CSMA networks, the Matern hard-core point process (HCPP) is one of the best fit point processes of choice [5], [6].

This paper exploits downlink SCNs in which the densities of the BSs and UEs form two independent homogeneous PPPs of density λ and λ_u , respectively. To increase spectrum efficiency, this downlink system aims at operating in NOMA mode whenever possible. Once a BS becomes active, it can serve either a single UE or two UEs simultaneously depending on the distribution and locations of the UEs in its coverage area. To make this choice, the coverage area of a BS specified by radius R is divided into narrower areas bounded by radii $R_1 < R_2 < R$. If at least two UEs exist such that one UE is located in near-distance of less than R_1 from the BS and the other UE is located in far-distance between R_2 to R from the BS, the NOMA mode is used. Otherwise, the BS operates in orthogonal multiple access (OMA) mode to serve a single UE. Stochastic geometry is used to model this SCN with power-domain NOMA and to analyse its performance in terms of coverage probability, data rate of each UE, and sum rate of the BS. The analysis accounts for the probability of each operation mode, signal-to-interference ratio, distance between active BSs, and density of both BSs and UEs.

The rest of the paper is organized as follows. Section II presents the stochastic geometry model of the SCN and derives the probabilities for the BS operating in the OMA and NOMA mode. Analytical expressions of the coverage probability, data rates of the UEs, and sum rate of the BS are derived in Section III. Numerical results are given in Section IV to illustrate the impact of system parameters on the performance of the SCN. Finally, a summary is given in Section V.

II. STOCHASTIC GEOMETRY MODEL OF THE SCN

A. Stochastic Geometry Modeling of BSs

Let us consider downlink SCNs in which BSs follow a PPP Φ with density λ (see Fig. 1). The UEs within the coverage

Fig. 1. SCN topology with BSs operating in OMA and NOMA mode.

area of a BS are assumed to be positioned according to a homogeneous PPP Φ_u with intensity λ_u . Carrier sense multiple access with collision avoidance (CSMA/CA) MAC is used among the SCNs to determine which BS is allowed to transmit at a given time. In particular, based on the Matern hard-core model [6], two BSs located at x and y can simultaneously transmit if they do not belong to the same contention domain.

The operation of the CSMA/CS MAC relies on the knowledge of the available channels at each BS obtained by spectrum sensing at the physical layer in its contention domain. If a BS wants to transmit, it first selects a back-off counter with an initial window contention size and then continuously senses the channel. If the channel is sensed as idle in a time slot, the back-off counter decreases by one but is kept constant otherwise. Once the back-off counter reaches zero, the BS is allowed to transmit and becomes active. This protocol can be modeled by a stochastic geometry approach as follows.

A BS at $x \in \Phi$ is associated with an independent mark m_x uniformly distributed in $[0,1]$ to represent the uniformly distributed back-off time among BSs. Among all available BSs, a BS becomes active when the HCPP condition of having minimum distance R between BSs is satisfied, i.e., it has the smallest mark in the disc $B(x, R)$. The probability p_b that a BS at x with mark $m_x \in [0, 1]$ in the disc $B(x, R)$ is selected is given by

$$p_b = \Pr \{m_x \leq m_y, \forall x, y \in B(x, R) \cap \Phi\} \quad (1)$$

The probability that a BS at x is active, conditioned on the mark m_x , is given as [7]

$$p_b(x|m_x) = E \left[(1 - m_x)^b \right] \quad (2)$$

where b is the number of BSs located in the disc $B(x, R)$, i.e., b is a Poisson random variable with mean $\bar{b} = \lambda \pi R^2$. Because $E[z^b] = \exp(-\bar{b}(1-z))$ for $z > 0$ [8], (2) can be rewritten as

$$p_b(x|m_x) = \exp(-\pi \lambda R^2 m_x) \quad (3)$$

Marginalizing (3) over $m_x \in [0, 1]$ gives

$$p_b = \int_0^1 \exp(-\pi \lambda R^2 m_x) dm_x = \frac{1 - \exp(-\pi \lambda R^2)}{\pi \lambda R^2} \quad (4)$$

As a result, active BSs are modeled as another PPP Φ_b with active density given by

$$\lambda_b = p_b \lambda = \frac{1 - \exp(-\pi \lambda R^2)}{\pi R^2} \quad (5)$$

B. Stochastic Geometry Modeling of UEs

An active BS can serve a single UE (OMA mode) or simultaneously two UEs (NOMA mode) depending on the density and locations of UEs surrounding the BS. Because power-domain NOMA exploits the difference in channel gains between pairs of UEs to the BS, i.e., near-distance UE and far-distance UE, the BS only operates in NOMA mode when it finds pairs of UEs with large enough difference between their distances. For this purpose, the coverage area of a BS is divided into three areas whose boundaries are defined by radius R_1 , R_2 , and R with $R_1 < R_2 < R$ as shown in Fig. 1. Without loss of generality, let BS_i be located in the center of the area with radius R and let A_{i0} , A_{i1} , and A_{i2} denote the areas of the discs $A_{i0} = B(0, R)$, $A_{i1} = B(0, R_1)$, and $A_{i2} = B(0, R) \setminus B(0, R_2)$, respectively. If at least one UE resides in the area A_{i1} and one UE in the area A_{i2} , BS_i will operate in NOMA mode. Then, BS_i distributes the total transmit power with power allocation coefficient $\alpha_{i1} < 0.5$ to the selected near-distance user UE_{i1} in the area A_{i1} and $\alpha_{i2} = 1 - \alpha_{i1}$ to the selected far-distance user UE_{i2} in the area A_{i2} . Otherwise, BS_i will operate in OMA mode and allocate the total power, i.e., $\alpha_{i0} = 1$, to UE_{i0} in the area A_{i0} . Note that time division multiple access (TDMA) is used if several UEs in a particular area intend to communicate with the BS.

C. Probabilities of Being in OMA and NOMA Mode

To obtain the probabilities that a BS operates in OMA and NOMA mode, the probability that at least one UE exists in a particular area A needs to be derived. Because the UEs are homogeneously distributed as PPP Φ_u with density λ_u , the number of UEs in the area A follows a Poisson distribution with mean $\lambda_u A$. As such, the probability that there exist n UEs in the area A is given by

$$P(\Phi_u(A, n)) = \exp(-\lambda_u A) \frac{(\lambda_u A)^n}{n!} \quad (6)$$

The probability that a BS is idle is equivalent to the probability that there is no UE in the area A . Thus, the probability that a BS is idle can be formulated as

$$p_i = P(\Phi_u(A, 0)) = \exp(-\lambda_u \pi R^2) \quad (7)$$

Since BS_i will operate in NOMA mode if there is at least one UE in the area A_{i1} and at least one UE in the area A_{i2} , the probability of this mode is given by

$$p_n = [1 - \exp(-\lambda_u \pi R_1^2)] [1 - \exp(-\lambda_u \pi (R^2 - R_1^2))] \quad (8)$$

Otherwise, BS_i will operate in OMA mode with probability

$$p_o = 1 - p_i - p_n \quad (9)$$

D. Channel Model

Let the downlinks from the BSs to the UEs be subject to pathloss and Rayleigh fading. The channel mean powers on these links are assumed to follow the exponentially decaying pathloss model with distance R as $R^{-\nu}$ where ν is the pathloss exponent. Further, let h_{ik} denote the fading coefficient on the link from BS_{*i*} to the associated UE_{*ik*}, $k = 0, 1, 2$, and h_{ijk} be the fading coefficient on the link from the interfering BS_{*j*} to UE_{*ik*}. Since interference is a main challenge in cellular networks, the signal-to-interference ratio (SIR) becomes more important than the signal-to-noise ratio (SNR). As such, assuming negligible noise compared to the interference in SCNs, the SIR on the downlink from BS_{*i*} to UE_{*ik*} is given by

$$\gamma_{ik} = \frac{P_i \beta_{ik} R_{ik}^{-\nu} X_{ik}}{\sum_{j \in \Phi_b, i=j} P_j R_{ijk}^{-\nu} X_{ijk}} \quad (10)$$

where $X_{ik} = h_{ik}^2$ and $X_{ijk} = h_{ijk}^2$, P_i is the transmit power of BS_{*i*} to UE_{*ik*}, and P_j is the transmit power of BS_{*j*} to UE_{*ik*}. Further, β_{ik} denotes the power allocation coefficient applied by BS_{*i*} to the transmission to UE_{*ik*}. In OMA mode, BS_{*i*} allocates the entire power P_i to UE_{*i0*}, i.e., $\beta_{i0} = \alpha_{i0} = 1$. In NOMA mode, BS_{*i*} simultaneously serves two UEs, i.e., UE_{*i1*} and UE_{*i2*}, using power allocation coefficient $\beta_{ik} = 2\alpha_{ik}$, $k = 1, 2$. In addition, R_{ik} and R_{ijk} are random variables with realisations r_{ik} and r_{ijk} representing the distance from BS_{*i*} to UE_{*ik*}, and BS_{*j*} to UE_{*ik*}, respectively.

Recall that an active BS_{*j*} is spatially positioned according to a PPP Φ_b with intensity λ_b . As UEs always connect to the nearest BS, BS_{*i*} is the closest BS for UE_{*ik*}. In other words, if BS_{*i*} with distance R_{ik} is the closest BS to UE_{*ik*}, then there is no other BS located at a distance less than R_{ik} , i.e.,

$$\Pr\{\Phi_b(|B(0, r_{ik})|) = 0\} = \exp(-\lambda_b \pi r_{ik}^2) \quad (11)$$

Therefore, the distribution of the distance from UE_{*ik*} to the nearest BS_{*i*} is equivalent to the distribution of the so-called first contact. As a consequence, the cumulative distribution function (CDF) and probability density function (PDF) of the distance from UE_{*ik*} to the nearest BS_{*i*} are given by

$$F_{R_{ik}}(r_{ik}) = 1 - \exp(-\lambda_b \pi r_{ik}^2) \quad (12)$$

$$f_{R_{ik}}(r_{ik}) = 2\lambda_b \pi r_{ik} \exp(-\lambda_b \pi r_{ik}^2) \quad (13)$$

Because pathloss has been considered separately, all Rayleigh fading channels are assumed to have unit channel mean power. Then, the CDF and PDF of channel power gain X_{ik} on the link from BS_{*i*} to UE_{*ik*} are given by

$$F_{X_{ik}}(x_{ik}) = 1 - \exp(-x_{ik}) \quad (14)$$

$$f_{X_{ik}}(x_{ik}) = \exp(-x_{ik}) \quad (15)$$

Note that (14) and (15) also apply to channel power gain X_{ijk} .

III. PERFORMANCE ANALYSIS

A. Coverage Probability

As defined in [9], an UE is in coverage of a BS if the SIR is kept above a given threshold. Therefore, the coverage prob-

ability is given by the complementary cumulative distribution function (CCDF) of the SIR of an UE at a given threshold.

1) *BS in OMA mode:* Given (10), the CCDF of the SIR γ_{i0} of UE_{*i0*} randomly located in the area $A_{i0} = B(0, R)$ around BS_{*i*} in OMA mode can be expressed as

$$\bar{F}_{\gamma_{i0}}(\gamma) = \int_0^R \Pr\{\gamma_{i0} \geq \gamma | r_{i0}\} f_{R_{i0}}(r_{i0}) dr_{i0} \quad (16)$$

where $f_{R_{i0}}(r_{i0})$ is the PDF of the distance between the considered UE_{*i0*} and the first contact BS in the disc $B(0, R)$ as given in (13). Let I_{i0} be the total interference from all other BSs to UE_{*i0*} associated with BS_{*i*}, i.e.,

$$I_{i0} = \sum_{j \in \Phi_b, i=j} P_j R_{ij0}^{-\nu} X_{ij0} \quad (17)$$

Then, using (10), we obtain

$$\Pr\{\gamma_{i0} \geq \gamma | r_{i0}\} = \Pr\{X_{i0} \geq \frac{r_{i0}^\nu \gamma}{\beta_{i0} P_i} I_{i0}\} \quad (18)$$

and rewrite (18) as

$$\Pr\{\gamma_{i0} \geq \gamma | r_{i0}\} = \mathbb{E}_{I_{i0}} [1 - F_{X_{i0}}(\frac{r_{i0}^\nu \gamma}{\beta_{i0} P_i} I_{i0})] \quad (19)$$

Substituting (14) into (19), after some modifications, we have

$$\Pr\{\gamma_{i0} \geq \gamma | r_{i0}\} = \mathbb{E}_{I_{i0}} \exp\left(-\frac{r_{i0}^\nu \gamma}{\beta_{i0} P_i} I_{i0}\right) \quad (20)$$

To solve (20), we use the Laplace transform of a variable X :

$$\mathcal{L}_X[s] = \mathbb{E}_X\{\exp(-sX)\} \quad (21)$$

Therefore, $\Pr\{\gamma_{i0} \geq \gamma | r_{i0}\}$ is the Laplace transform of the total interference incurred to UE_{*i0*} at $s = \frac{r_{i0}^\nu \gamma}{\beta_{i0} P_i}$, i.e.,

$$\Pr\{\gamma_{i0} \geq \gamma | r_{i0}\} = \mathcal{L}_{I_{i0}}[s] \Big|_{s=\frac{r_{i0}^\nu \gamma}{\beta_{i0} P_i}} \quad (22)$$

which can be expressed using (17) as

$$\mathcal{L}_{I_{i0}}[s] = \mathbb{E}_{I_{i0}} \left\{ \prod_{j \in \Phi_b, i=j} \exp(-s P_j R_{ij0}^{-\nu} X_{ij0}) \right\} \quad (23)$$

Due to the statistical independence of channel power gains X_{ij0} , $j \in \Phi_b$, (23) can be rewritten as

$$\mathcal{L}_{I_{i0}}[s] = \mathbb{E}_{I_{i0}} \left\{ \prod_{j \in \Phi_b, i=j} \mathbb{E}_{X_{ij0}} \exp(-s P_j R_{ij0}^{-\nu} X_{ij0}) \right\} \quad (24)$$

where

$$\begin{aligned} & \mathbb{E}_{X_{ij0}} \exp(-s P_j R_{ij0}^{-\nu} X_{ij0}) \\ &= \int_0^\infty \exp(-s P_j R_{ij0}^{-\nu} x) f_{X_{ij0}}(x) dx \end{aligned} \quad (25)$$

Substituting (15) into (25) and using [10, eq. (3.381.4)], gives

$$\mathbb{E}_{X_{ij0}} \exp(-s P_j R_{ij0}^{-\nu} X_{ij0}) = \frac{1}{1 + s P_j R_{ij0}^{-\nu}} \quad (26)$$

The Laplace transform of the interference I_{i0} to UE_{*i0*} can then

be written as

$$\mathcal{L}_{I_{i0}}[s] = \mathbb{E}_{I_{i0}} \left\{ \prod_{j \in \Phi_b, i=j} \frac{1}{1 + s P_j R_{ij0}^{-\nu}} \right\} \quad (27)$$

Because BS_i is the closest BS to UE_{i0} , the other active BSs farther away than r_{i0} cause interference to UE_{i0} . Applying the expectation of a product of a function over PPP in area $B(o, r_{i0})^c = B(o, \infty) \setminus B(o, r_{i0})$, we have

$$\mathcal{L}_{I_{i0}}[s] = \exp -\lambda_b \int_{B(o, r_{i0})^c} \frac{1}{1 + s P_j r^{-\nu}} da \quad (28)$$

where da is the element area. Using polar coordinates $da = r dr d\psi$, (28) can be rewrite as

$$\mathcal{L}_{I_{i0}}[s] = \exp -\lambda_b \int_0^{2\pi} d\psi \int_{r_{i0}}^{\infty} \frac{r}{1 + (s P_j)^{-1} r^\nu} dr \quad (29)$$

Applying [10, eq. (3.259.2)] and [10, eq. (3.259.3)] to solve (29), after some modifications, we have

$$\mathcal{L}_{I_{i0}}[s] = \exp \left(\frac{-\lambda_b 2\pi P_j s}{(\nu-2)r_{i0}^{\nu-2}} {}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, -\frac{s P_j}{r_{i0}^\nu} \right] \right) \quad (30)$$

Substituting (30) into (22), and assuming the same transmit power $P_i = P_j$ for all BSs, we obtain the probability

$$\begin{aligned} \Pr \{ \gamma_{i0} \geq \gamma | r_{i0} \} \\ = \exp \left(- \frac{2\pi \lambda_b {}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, -\frac{\gamma}{\beta_{i0}} \right] \gamma r_{i0}^2}{(\nu-2)\beta_{i0}} \right) \end{aligned} \quad (31)$$

Finally, substituting (31) and (13) into (16) and using [10, eq. (3.321.4)] to solve the resulting integral, the CCDF of the SIR γ_{i0} of UE_{i0} when BS_i operates in OMA mode is obtained as

$$\begin{aligned} \bar{F}_{\gamma_{i0}}(\gamma) = \frac{(\nu-2)\beta_{i0}}{2 {}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, \frac{-\gamma}{\beta_{i0}} \right] P_j \gamma + (\nu-2)\beta_{i0}} \\ \times \left[1 - \exp \left(-\pi \lambda_b \left(\frac{{}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, \frac{-\gamma}{\beta_{i0}} \right] \gamma}{(\nu-2)\beta_{i0}} + 1 \right) R^2 \right) \right] \end{aligned} \quad (32)$$

2) *BS in NOMA mode:* Similarly, the CCDF of the SIR γ_{i1} of the near-distance user UE_{i1} located in A_{i1} bounded by radius R_1 when BS_i operates in NOMA mode is obtained as

$$\begin{aligned} \bar{F}_{\gamma_{i1}}(\gamma) = \frac{(\nu-2)\beta_{i1}}{2 {}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, \frac{-\gamma}{\beta_{i1}} \right] \gamma + (\nu-2)\beta_{i1}} \\ \times \left[1 - \exp \left(-\pi \lambda_b \left(\frac{{}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, \frac{-\gamma}{\beta_{i1}} \right] \gamma}{(\nu-2)\beta_{i1}} + 1 \right) R_1^2 \right) \right] \end{aligned} \quad (33)$$

For the far-distance user UE_{i2} located in the area A_{i2} bounded by radius R_2 and R , the CCDF of the SIR γ_{i2} of UE_{i2} when BS_i operates in NOMA mode is formulated as

$$\bar{F}_{\gamma_{i2}}(\gamma) = \int_{R_2}^R \Pr \{ \gamma_{i0} \geq \gamma | r_{i2} \} f_{R_{i2}}(r_{i2}) dr_{i2} \quad (34)$$

where, assuming the same transmit power $P_i = P_j$ for all BSs, we have

$$\begin{aligned} \Pr \{ \gamma_{i2} \geq \gamma | r_{i2} \} \\ = \exp \left(- \frac{2\pi \lambda_b {}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, -\frac{\gamma}{\beta_{i2}} \right] \gamma}{(\nu-2)\beta_{i2}} r_{i2}^2 \right) \end{aligned} \quad (35)$$

Substituting (35) and (13) into (34) and then applying [10, eq. (3.321.4)] to solve the resulting integral, we finally obtain the CCDF of the SIR γ_{i2} of the far-distance user UE_{i2} when BS_i operates in NOMA mode as

$$\begin{aligned} \bar{F}_{\gamma_{i2}}(\gamma) = \frac{(\nu-2)\beta_{i2}}{2 {}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, -\frac{\gamma}{\beta_{i2}} \right] P_j \gamma + (\nu-1)\beta_{i2}} \\ \times \left[\exp \left(-\lambda_b \pi \left(\frac{{}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, -\frac{\gamma}{\beta_{i2}} \right] \gamma}{(\nu-2)\beta_{i2} P_i} + 1 \right) R_2^2 \right) \right. \\ \left. - \exp \left(-\lambda_b \pi \left(\frac{{}_2F_1 \left[1, \frac{\nu-2}{\nu}, \frac{2\nu-2}{\nu}, -\frac{\gamma}{\beta_{i2}} \right] \gamma}{(\nu-2)\beta_{i2}} + 1 \right) R^2 \right) \right] \end{aligned} \quad (36)$$

3) *Coverage Probability:* Given a threshold $\gamma_{\text{th},i}$ for the SIR at UE_{ik} in the area A_{ik} of BS_i , the coverage probability $P_{c,i}$ of UE_{ik} randomly located in A_{ik} of an SCN is given as

$$P_{c,i} = \bar{F}_{\gamma_{ik}}(\gamma_{\text{th},i}), \quad k = 0, 1, 2 \quad (37)$$

B. Sum Rate

Given the probabilities p_0 and p_n that BS_i operates in OMA and NOMA mode, the sum rate of BS_i is given by

$$C_{\text{sum},i} = \frac{p_0 C_{i0} + p_n (C_{i1} + C_{i2})}{p_0 + p_n} \quad (38)$$

where the data rate of UE_{ik} is expressed as

$$C_{ik} = \int_{\gamma_{\text{th},i}}^{\infty} \log_2(1 + \gamma) f_{\gamma_i}(\gamma) d\gamma \quad (39)$$

Applying integration by parts to (39), we obtain

$$C_{ik} = \frac{1}{\ln 2} \left[\ln(1 + \gamma_{\text{th},i}) \bar{F}_{\gamma_{ik}}(\gamma_{\text{th},i}) + \int_{\gamma_{\text{th},i}}^{\infty} \frac{\bar{F}_{\gamma_{ik}}(\gamma)}{1 + \gamma} d\gamma \right] \quad (40)$$

Substituting (32), (33), and (36) into (40), the transmission rate of UE_{ik} in the coverage area of BS_i is obtained. Substituting these results as well as (8) and (9) into (38), we finally obtain the sum rate of BS_i .

IV. NUMERICAL RESULTS

In this section, numerical results are provided to illustrate the impact of system parameters such as the SIR threshold $\gamma_{\text{th},i}$ and UE density λ_u on the coverage probability, data rate, and sum rate of the SCN. As mentioned in the system model, the coverage area of an active BS, BS_i , is divided into three areas whose boundaries are defined by radius R , R_1 , and R_2 . The BS_i either serves a single UE in the area $A_{i0} = B(0, R)$

in OMA mode or two UEs simultaneously in the areas $A_{i1} = B(0, R_1)$ and $A_{i2} = B(0, R) \setminus B(0, R_2)$ in NOMA mode.

In the following, the radii are set to $R_1 = 10$ m, $R_2 = 20$ m, and $R = 30$ m and the pathloss exponent is selected as $\nu = 3$ for an urban area propagation environment. These values are used to quantify the power allocation coefficients α_{i1} and α_{i2} for UE_{i1} and UE_{i2} , respectively, in NOMA mode, i.e.,

$$\alpha_{i1} = \frac{R_2^{-\nu}}{R_1^{-\nu} + R_2^{-\nu}} \quad \text{and} \quad \alpha_{i2} = \frac{R_1^{-\nu}}{R_1^{-\nu} + R_2^{-\nu}}$$

First, we assess the impact of the SIR threshold $\gamma_{th,i}$ on the coverage probability, data rates of the UEs, and data rate of the OMA and NOMA mode. The densities of the BSs and UEs are set to $\lambda = 250$ BSs/km² and $\lambda_u = 300$ UEs/km² [11]. Note that UE_{i0} is the single UE located in the area $A_{i0} = B(0, R)$ of BS_i in OMA mode. Further, UE_{i1} and UE_{i2} are the near-distance and far-distance UE, respectively, located in the area $A_{i1} = B(0, R_1)$ and $A_{i2} = B(0, R) \setminus B(0, R_2)$ of BS_i that simultaneously transmits to both UEs in NOMA mode.

As can be seen from Fig. 2, the coverage probability decreases with the increase of SIR threshold because it becomes decreasingly likely that the SIR exceeds the threshold. In NOMA mode, the coverage probability for the far-distance user UE_{i2} significantly decreases in the high SIR threshold regime compared to the near-distance user UE_{i1} . This behaviour is due to the increased mutual interference in NOMA mode while UE_{i0} in the OMA mode does not experience interference. However, similar high coverage probability is provided to all three UEs as long as the SIR threshold is kept below 5 dB. While the coverage probability for the near-distance user UE_{i1} outperforms that of the far-distance user UE_{i2} in the high SIR threshold regime, slightly better coverage probability is provided to the far-distance user UE_{i2} compared to the near-distance user UE_{i1} in the low SIR threshold regime.

Figs. 3 and 4, respectively, show the impact of the SIR threshold on the data rates of the three UEs, and the data rate of the OMA and NOMA mode. Given that UE_{i0} in OMA mode is not subject to interference but UE_{i1} and UE_{i2} induce mutual interference in NOMA mode, the data rate obtained for UE_{i0} is higher compared to that for UE_{i1} and UE_{i2} over the entire SIR threshold range. Because UE_{i1} and UE_{i2} in NOMA mode transmit simultaneously their total data rate is obtained by accumulating the individual data rates of both UEs as shown in Fig. 4. As a result, the accumulated data rate of the SCN in NOMA mode is higher than that of the OMA mode for the SIR threshold range considered.

Second, we study the impact of the UE density λ_u on the probability of the OMA and NOMA mode being used at the BS and the average data rate accounting for the contribution of the OMA and NOMA mode. Here, the BS density is set to $\lambda = 250$ BSs/km² and the SIR threshold is set to $\gamma_{th,i} = 3$ dB.

Fig. 5 shows that the probability of a BS being idle converges fast to zero as the UE density increases. Further, the probability for the BS operating in OMA mode increases fast with increasing UE density to a maximum value and then decreases with a further increase of the UE density. On the

Fig. 2. Coverage probability in an SCN with three areas A_{i0} , A_{i1} , and A_{i2} .

Fig. 3. Data rate for UE_{i0} in OMA mode, and UE_{i1} , UE_{i2} in NOMA mode.

Fig. 4. Data rates of the SCN operating in OMA and NOMA mode.

other hand, the probability for the BS operating in NOMA mode steadily increases with the increase of UE density. This behaviour is due to the BS using the NOMA mode whenever at least one UE is in the area $A_{i1} = B(0, R_1)$ and at least one UE is in the area $A_{i2} = B(0, R) \setminus B(0, R_2)$. As such, the probability of being in NOMA mode increases, the probability of being in OMA mode decreases, and the probability of the

Fig. 5. Probability of transmission mode being selected at the BS.

Fig. 6. Sum rate at the BS for different coverage radii.

idle mode converges to zero for increased UE density.

Fig. 6 investigates the effect of the UE density on the sum rate of the BS for three different radii around the BS, i.e., $R = 30$ m, 40 m, and 50 m while keeping $R_1 = 10$ m and $R_2 = 20$ m. Recall that the sum rate given in (38) represents the average data rate with respect to the probabilities for the BS being in the OMA and NOMA mode. As can be observed in the figure, the sum rate remains on a rather constant level for the three selected values of radius R when increasing the UE density. This characteristic is attributed to the fact that an increase or decrease of the probability of being in OMA mode is compensated for by a decrease or increase of the probability of being in the NOMA mode. Simultaneously, the probability of being in the idle mode converges fast towards zero with increasing UE density. Because the number of UEs in the coverage area of the BS increases and the interference reduces with larger radius R , the sum rate increases accordingly.

V. SUMMARY

In this paper, the downlink performance of an SCN has been assessed where a BS uses CSMA/CA MAC, and can switch between OMA and NOMA mode. A stochastic geometry approach has been used to model the BSs and UEs allowing to

include their spatial densities into the performance assessment. The selection of operation mode is decided based on the UE density in the coverage area of a BS aiming to improve the sum rate of the BS. A performance analysis has been conducted with analytical expressions derived for the coverage probability, data rates of UEs in OMA and NOMA mode, and sum rate at the BS. On this basis, numerical results have been provided to illustrate the impact of system parameters such as the SIR threshold and UE density on the performance of the SCN. The derived mathematical framework may assist system designers to analyse data rates and sum rates of SCNs subject to the spatial densities of BSs and UEs.

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