Pilot study: Modeling of wildfires

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To Madeleine, Emily and Oliver.

Summary

There is presently no wildfire model developed for Swedish conditions, only a fire danger rating system (FWI) has been developed for Swedish conditions.

The demand for a wildfire model has not been great in the past in Sweden but the climate changes now taking place increases the risk of large and intensive wildfires in Sweden. The need for additional and better tools for sizing-up wildfires will be in great demand in the future.

This pre-study is aimed at:
- Presenting what has been done in the wildfire modeling field during the years and mainly the last twenty years.
- Giving recommendations on the continued work with developing a Swedish wildfire model.

The method that was used was literature and article survey.

The study also looks into the required input data for a wildfire model and the input data available at the moment. This issue is highly crucial as the quality of the output of a wildfire model is depending upon the quality of the input data.

During the study, a primitive wildfire model was constructed and refined in order to get an insight in the complexities and problems with developing an operational model.

The following characterization of wildfire models was used during the study:
- Statistical models: based primarily on statistics from earlier or experimental fires. They do not explicitly consider the controlling physical processes.
- Semi-empirical models: based on physical laws, but enhanced with some empirical factors, often by lumping all physical mechanisms for heat transfer together.
- Physical models: based on physical principles and distinguishing between physical mechanisms for heat transfer.

The statistical models make no attempt to involve physical processes, as they are merely a statistical description of test fires. Thus the lack of a physical basis means that statistical models must be used carefully outside the test conditions.

Semi-empirical models are often based on conservation of energy principles but do not make any difference between conduction, convection and radiation heat transfer. The semi-empirical model has low computational requirements and includes variables that are generally easy to measure in the field. So despite the issue with limited accuracy, the speed and simplicity of these models make them useful for operational use.

Physical models have the advantage that they are based on known relationships and thus facilitating their scaling. Thus we can expect that physical models would provide the most accurate predictions and have the widest applicability.

But the work on physical models is suffering of for example the lack of understanding of several processes, such as the characterization of the chemical

processes taking place during combustion, the resulting flame characteristics and the isolation and quantification of physical processes governing heat transfer.

The input data available today are generally not detailed enough for physical models. As a result, a very detailed physical model will still only give imprecise predictions. As better and more detailed input will be available, the use of physical models will be more justified.

A semi-empirical model is recommended being developed in Sweden. This conclusion is based upon the following factors:
- The accuracy of a semi-empirical model is generally much better than for a statistical model, also the use of a semi-empirical model is much wider than the use of a statistical model.
- The amount of work required for developing a semi-empirical model will not differ much from the amount of work required for a statistical model. In both cases a number of test fires will have to be conducted to define and calibrate a number of fuel models representative of Sweden.
- Presently the performance and application of physical models is not at an acceptable level (due to for example the complexity which they are to model and the computational capabilities of the PC's of today) for operational use.

The semi-empirical model for Sweden is recommended to be built upon Swedish conditions (i.e. built upon the type of vegetation found in Sweden) instead of trying to retrofit the local Swedish conditions into an existing model. This would most likely give the best output for Swedish conditions.

A system for better input data - weather and fuel data – should be worked on as well. This could for example take advantage of the results of the very promising "Alarm"-project that is being conducted in western part of Sweden.
Regarding the issue on better fuel data, new technology for satellite images or aerial photos and image classification techniques must be monitored as one major problem to be solved is distinguishing between the canopy fuel and the ground fuel.

For more specific conclusions and reflections, please see the analysis and discussion, and conclusions sections of this report.

Preface

This degree project is the thesis at the MS Programme in Mathematical Modelling and Simulation, at Blekinge Institute of Technology in Sweden.

I would like to extend warm gratitudes towards everyone that has contributed to this project. The people that I would like to thank are without any order of precedence:
- Håkan Lennerstad (Blekinge Institute of Technology), acting as my academic supervisor during the later stages of my thesis.
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- Leif Sandahl (Swedish Rescue Services Agency), providing me with valuable input during my work on my thesis.
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Rickard Hansen

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1. Introduction

The main purposes of this study are:
1. Present what has been done in the wildfire modeling field during the years and mainly the last twenty years.
2. To give recommendations on the continued work with developing a Swedish wildfire model.

The Swedish research into enclosure fires has been highly active during the last twenty years, but the Swedish research into wildfires has been almost non-existant during the same period. Thus there is presently no wildfire model developed for Swedish conditions.

The demand for a wildfire model has not been great in the past in Sweden but the climate changes now taking place increases the risk of large and intensive wildfires in Sweden. The need for additional and better tools for sizing-up wildfires will be in great demand in the future.

The study also looks into the required input data for a wildfire model and the input data available at the moment. This issue is highly crucial as the quality of the output of a wildfire model is depending upon the quality of the input data. The study gives some recommendations with respect to the input data.

During the study, a primitive wildfire model was constructed and refined in order to get an insight in the complexities and problems with developing an operational model. The model is presented in this report.

The issue with modeling spot fires is not included in this study, nor are so called mathematical analogue models.

The main source for this study has mainly been the following scientific journals and other scientific publications:
- Canadian Journal of Forest Research.
- Combustion and Flame.
- International Journal of Wildland Fire
- Combustion, Explosion and Shock Waves.
- USDA Forest Service, research papers.
- ENSIS Bushfire Research.

The following books have been used during the study:
- *Combustion of forest fuels,* Byram G. M. (1959), New York, McGraw-Hill Book Co.
- *Fire in forestry,* vol.1 & 2, Chandler G.C. (1991), New York, John Wiley & Sons

2. Background

2.1 Wildfires

Wildfire is one of the oldest of natural phenomena. Fossil evidences of wildfires can be traced back long in time. Wildfires are also a global issue as every continent is being touched by wildfires – except for Antarctica.

Further back in time fire was used by man to hunt, improve harvests, protection against predators etc. Wildfires have been and will always be an important part of nature, as for example some plant species are depending on wildfires occurring. Today wildfires are considered by many as a growing threat to society. Even though Sweden has been spared from the worst global disasters, the climate changes now taking place increases the risk of large and intense wildfires in Sweden. The need for additional and better tools for sizing-up wildfires will be in great demand in the future.

Fighting wildfires can be done from the ground as well as from the air. Roughly when fighting wildfires from the ground, firelines are constructed or water from hose systems - that are laid out around the fire to prevent it from spreading further – are used. When fighting the wildfire from the air, the wildfire is bombarded by helicopters or airplanes using water or a mixture of water and a chemical additive. Large wildfires generally require large amount of personnel and other resources, take a long time to extinguish and are generally very costly.

Wildfires are driven by extremely complex chemical and physical processes. The interactions of the processes depend on coupling between the weather (atmosphere), topography, fire and the fuel (vegetation). The ability to predict the situation is not being made easier by the fact that one of the factors above – the weather – is highly variable over time.

Among the numerous processes taking place at a wildfire, one will for example find combustion processes, heat transfer processes, atmospheric processes and transport of oxygen.

The three factors – weather, topography and fuel – influencing the fire behaviour of a wildfire, are generally subdivided into the following factors:

Weather:
- Air temperature
- Wind
- Relative humidity
- Precipitation
- Air stability

Topography:
- Inclination of slope
- Direction of slope
- Ravines

Fuel:
- Moisture content
- Size and character of fuel (compact or grinded fuel)
- Heat content

- Amount of fuel
- Continuity of fuel
- Type of fuel

Besides the above factors, the size of the fire will also be an influencing factor as for example larger wildfires will create strong winds.

With respect to decision support tools during a wildfire, one of the great challenges is to model mathematically the complex processes involved in a wildfire.

2.2 Decision support before, during and after a wildfire

Several number of decision support systems exist in the world. They could be divided into the following three groups of subject areas:
- Fire behaviour
- Fire danger rating
- Forest management (for example when assessing the role of fire in the ecosystem)

As maps are very important tools, GIS software is now almost indispensable during the development of a decision support tool for wildfire situations. GIS software is used in all three categories of decision support systems.

Before a wildfire occurs a fire danger rating system could be used to increase readiness in a certain area and predict the risk for a wildfire to occur. A fire behaviour system could be used for pre-planning – for example identifying areas where high intensity fires may occur.
During a wildfire a fire behaviour system could be used to determine where resources should be sent, if additional resources should be called in, the position of the fire front at a certain time, the intensity of the wildfire at a certain time etc.
After a wildfire a forest management system could be used for deciding about re-forestation.
This report will solely deal with fire behaviour systems, as a fire danger rating system has already been in operational use in Sweden for a long time.

Fire behaviour systems have been developed with the objective of increasing our understanding of the phenomena that drives fire spread and heat release rates in order to develop tools to support the incident commander during the suppression operations – such as forecasting fire behaviour during ongoing wildfires – and other fire officials during pre-planning activities.
The desired characteristics of a fire behaviour system could for example be:
- Fast to use (in order to be of operational use)
- Easy to use with respect to required inputs etc.
- Accuracy of predictions.
- Give estimates of reliability of predictions.

2.3 The Swedish fire danger rating system

The purposes of a fire danger rating system are to give the rescue services a possibility to maintain an adequate degree of readiness throughout the dry months, provide decision support when deciding to restrict the use of fire and to inform and warn the public.

Sweden uses a Canadian fire danger rating system called FWI (Fire Weather Index), calibrated for Swedish conditions. The calculations in the system are performed by SMHI and comprises of a grid-system where each grid is 22x22 km in size. The system uses the relative humidity, precipitation, wind speed and air temperature as inputs. The output of FWI consists of a main index value that could be said to describe how big and intensive wildfires we can expect us during the day.

But FWI also consists of several sub-index values that can be used for calculating for example the rate of spread of the wildfire. The Canadian forest fire behaviour prediction (FBP) system is the actual system that is used when calculating the rate of spread. When calculating the rate of spread the so called ISI index value is used – ISI stands for Initial Spread Index and describes how fast the wildfire will spread when it only involves the uppermost layer of finer fuels – together with input data connected to the type of fuel being involved (there are a total of 17 fuel types pre-defined in the system), wind/slope adjustments and adjustments for buildup effects (in order to account for the amount of fuel that is available for combustion within the entire fuel complex) to obtain the rate of spread of the wildfire /1/.
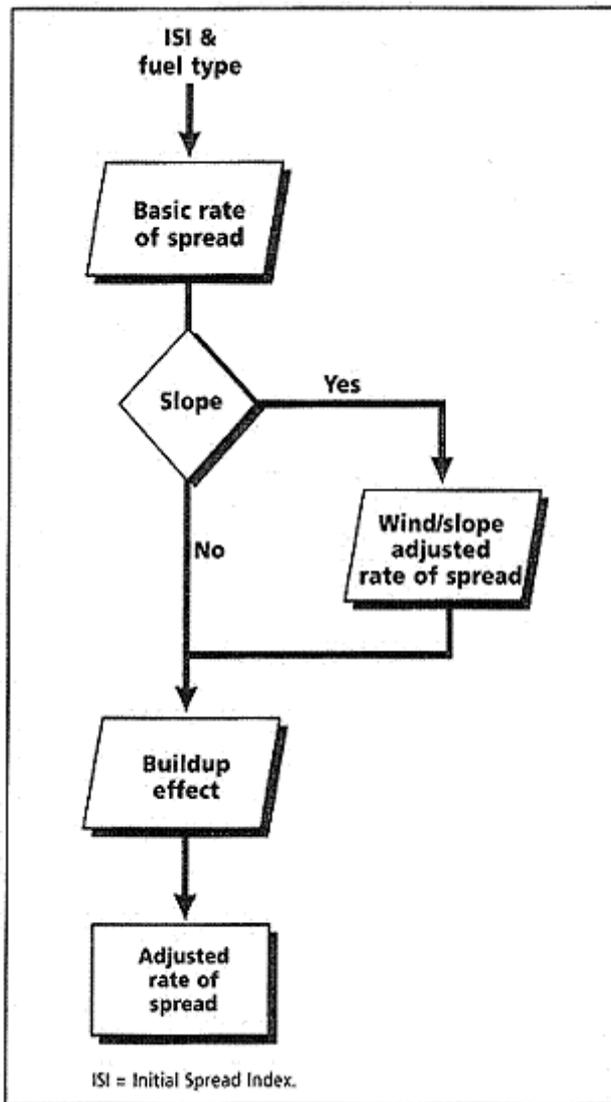
Figure 1. The FBP-system. /1/

The FBP system is not being used in any significant scale in Sweden, largely because it has not been calibrated with Swedish conditions – to be more exact: it has not been calibrated with potential Swedish fuel models.

3. Method

3.1 Litterature and article survey

To find out what has been done and what is being done in the wildfire modeling area around the world, a literature and an article survey was performed. A large amount of articles in scientific publications were found, some literature and a large amount of material on the internet was also found.

Most of the material was from USA, Canada, Australia, Russia and France. No material was found from Sweden (except for material related to fire danger rating systems).

During the survey the strengths and weaknesses of most of the models have been listed and further examined. The following search tools were used during the search:
- LIBRIS
- ELIN @ Blekinge
- Samsök beta
- ebrary
- Google scholar
- Google

The following key-words were used during the search:
- Wildfire
- Forest fire
- Modeling
- Modelling
- Mathematical model
- Mathematical modelling
- Mathematical modeling

The search was mainly aimed at fire behaviour models predicting the spread and intensity of a wildfire. The listing of models in this report is not an attempt to describe all models available. The models listed are a selection of common and often used models.

So called mathematical analogues and simulation models are not included in this report. A mathematical analogue model is a model that is based upon mathematical metaphor – instead of a physical representation of fire spread – that at the same time simulates the spread of a fire. /2/

3.2 Physical model

In order to test the mathematics that is involved in some of the wildfire models found in the literature and articles, a physical model using PDE was constructed and tested. The model was not entirely physical as some parts of it were of semi-empirical nature.

The purpose was also to get some feeling of the complexity of the modeling of wildfires and the computational requirements. For this, the software MATLAB was used.

3.3 MATLAB

MATLAB is an interactive package of program for technical calculations and visualization of data, developed by The MathWorks Inc. MATLAB consist of a large number of pre-defined mathematical functions (which can be used for programming) , the possibility to define your own functions and has a two- or three dimensional graphics to visualize the results.
MATLAB is complemented by a family of application specific solutions called toolboxes, the toolboxes cover areas such as optimization, image processing etc.

4. Modeling of wildfires

4.1 Modeling of wildfires in general

The ongoing work with understanding and modeling the behaviour of wildfires started about 60 years ago with "Analysis of fire spread in light forest fuels" by W.L. Fons /3/. Since then a number of models have been developed.
During the decades right after the second world war, a lot of resources were assigned for the research in wildfire behavior as there were strong connections between this discipline and with large conflagrations connected with mass bombings and nuclear attacks /4/.

The spread of a wildfire can largely be regarded as the ongoing change of the status of fuel cells distributed in space. The process in question is combustion and is limited to vegetal particles either live or dead. The transition from one to next status is a process which includes the production, transfer and absorption of heat progressively and this depends on the environment and the fuel of every point in the terrain.

Three key factors in the behaviour of wildfires are: the fuel, the flames and the heat transfer process, which we discuss next.

The fuel
The fuel bed is composed of combustible and organic matter. Fuel beds can be homogenous of a single type of fuel or they can be composed of discrete elements such as bushes with gaps in between. A botanical classification of pre-defined fuel types (so called fuel models) is used in some models which can assist in modelling the amount of fuel available for combustion and its compactness. To analytically characterize an oven dry, homogenous fuel bed consisting of only one type of fuel, four measurements are sufficient:
- Surface area per unit volume of the individual fuel particles (this will relate to how much heat that can be absorbed per unit volume per unit time during a fire). /5/
- Density of the individual fuel particles.
- Ratio of the density of individual fuel particles to that of the whole bed.
- The depth of the fuel bed.

A fifth measurement is needed for fuel which is not oven dry: the moisture content. The moisture content will depend upon the specific gravity, thus the effect of moisture content will be greater for a dense fuel /5/.
Additional measurements are needed if the bed is composed of more than one fuel type, which of course always is the case in a wildfire.
The thermal characteristics of the fuel will be vital as a heat source is introduced. The minimum set of thermophysical variables would be the conductivity and heat capacity for the fuel and possibly water (the heat capacity of forest fuels will depend upon the temperature and the moisture content of the fuel). With these variables one can calculate the enthalpy of the fuel bed as a function of temperature.

If any chemical processes are to be modeled, at least the heat of reaction, the reaction rate constant and the activation energy – assuming that the Arrhenius law is a reasonable approximation to the reaction rate – are required to be determined. Another factor that may be important is turbulent transport processes within the fuel bed. In this case it would be necessary to determine additional variables describing this type of heat and mass transfer /6/.

The complicated nature of the fuel involved in wildfires is one of the greatest challenges to wildfire modeling.

The flames

Once the enthalpy of any part of the fuel approaches critical values, which occur at temperatures of around 320ºC for cellulosic fuels, pyrolysis begins to occur at considerable rates. The rates of gasification and the chemical species produced during pyrolysis have been identified for particular fuels, but it has proven difficult to come up with a general model applicable to a variety of fuels.

Because of this difficulty, an "ignition temperature" is commonly assumed and one can then calculate the energy required to raise the enthalpy of fuel from ambient to ignition values. It is also assumed that at the ignition temperature, flaming begins to occur and that any additional energy released in flaming combustion is accounted for in an energy balance for the flame, including the amounts of energy transported by convection and radiation. Thus a common assumption is that of a flame of a fixed size at a fixed temperature and with a given emissivity. If a fire occurs when there is no wind then it is usually assumed that convection energy transport only occurs vertically (which is not always the case) /6/.

Heat transfer

There are three basic modes of heat transfer – conductivity, convection and radiation. In general, convection and radiation are the two main driving forces during a wildfire. Which one of them that will dominate, will vary from fire to fire.

The extent to which analytical methods for physically characterizing heat transfer have been used by different users allows one to formally classify models as belonging to one of three types:

- Statistical models: based primarily on statistics from earlier or experimental fires. They do not explicitly consider the controlling physical processes.
- Semi-empirical models: based on physical laws, but enhanced with some empirical factors, often by lumping all physical mechanisms for heat transfer together.
- Physical models: based on physical principles and distinguishing between physical mechanisms for heat transfer.

The classification above of wildfire models, will be used throughout this study. The statistical and semi-empirical models are unable to predict the behaviour of a fire due to changes caused by the fire-atmosphere interaction. This is because the fire shape, size and spread rate are assumed constant for a given fuel, wind and slope. The prediction of local wind and the interaction of the fire and wind are of major importance when predicting fire behaviour, especially in the case of severe fire

behaviour. Fires where strong fire-atmosphere interactions occur gives low credibility to the use of the statistical and semi-empirical fire spread models /7/.

4.2 Statistical models

The statistical models make no attempt to involve physical processes, as they are merely a statistical description of test fires. The model can be calibrated to describe the test fires, but as it does not include any physics it does not contain any generality either. The results can be very succesful in predicting the outcome of similar fires to the test fires and the statistical model has very low computational requirements. But the lack of a physical basis means that statistical models must be used carefully outside the test conditions. As the width of the test fires is small in comparison to the variety of conditions under which natural fires occur, the application of statistical models to wildfires is of questionable value.
Another two disadvantages of statistical models is the large number of fires required to develop the necessary number of homogenous sets and the fact that non-linear relationships among the variables are lost in the necessary linearization process /5/. Below, two statistical models are described: the FBP system and the McArthur model.

4.2.1 FBP system

A total of 495 fires are included in the Canadian FBP system data base, of which 409 are experimental fires and 86 are documented wildfires. Experimental fires are generally small (0.3-3.0 hectares in size) and usually sampled under low-to-moderate fire danger conditions. Data analysis is conducted using simple mathematical models and correlation techniques /1/.

To determine the rate of spread, the FBP system uses the following procedure (also see figure 1 for the overall procedure) /1/:
1. The basic rate of spread is obtained through tables or graphs, using the adequate fuel type and the exising ISI-value (the ISI-value describes how fast the wildfire will spread when it only involves the uppermost layer of finer fuels). See figure 2 for an example of a graph.
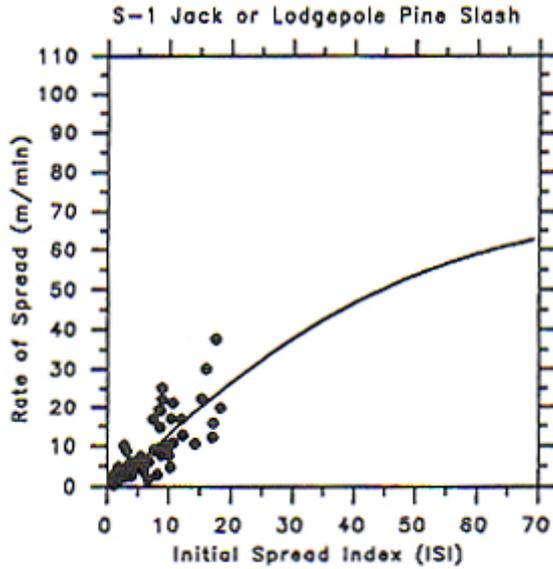
Figure 2. The basic rate of spread-ISI graph for the Jack or Lodgepole pine slash fuel type /1/.

2. If topography is an issue:
    -   Obtaining a spread factor value from a graph. See figure 3 for the graph.
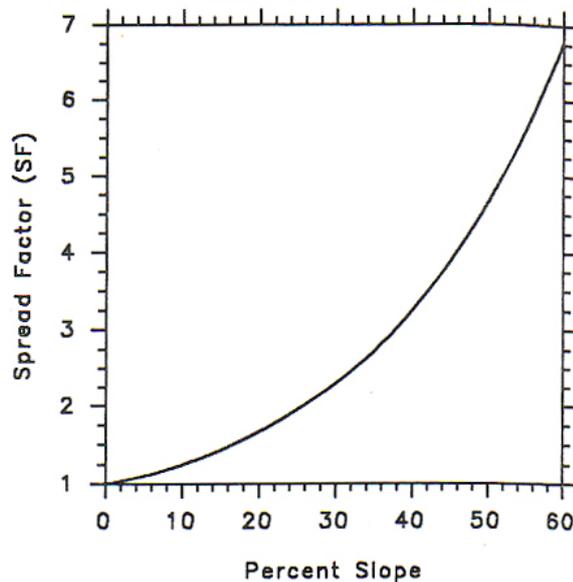

Figure 3. The spread factor-slope graph /1/.

    -   Calculating the ISZ-value (the ISZ-value is the ISI-value when the wind speed is set to zero):
        $$ISZ = 0.208 \cdot f(F) \qquad (1)$$

        Where:

$$f(F) = 91.9 \cdot e^{(-0.1386 \cdot m)} \cdot (1 + \frac{m^{5.31}}{4.93 \cdot 10^7})$$

$$m = \frac{147.2 \cdot (101 - FFMC)}{59.5 + FFMC}$$

FFMC is the Fine Fuel Moisture Code (obtained through the FWI-system).
- The rate of spread at zero wind speed and for the adequate fuel type is obtained through a graph.
- The rate of spread accounting for the topography but setting the wind speed at zero is calculated by multiplying the rate of spread at zero wind speed with the spread factor value.
- Calculating the ISF-value (the ISI value accounting for the topography but setting the wind speed at zero):

$$ISF = \frac{\ln(1 - (\frac{RSF}{a})^{1/c})}{-b} \tag{2}$$

Where:

a, b and c are factors depending upon what fuel type has been chosen.
- Calculate WSE (the equivalent wind speed of the slope):

$$WSE = \frac{\ln(\frac{ISF}{0.208 \cdot F(f)})}{0.05039} \tag{3}$$

- Calculating the the total wind, WSV, using a parallellogram.
- Calculating the final ISI accounting for both the wind and the topography:

$$ISI = 0.208 \cdot f(F) \cdot e^{0.05039 \cdot WSV} \tag{4}$$

- Obtain the final rate of spread – RSI - by using a graph for the corresponding fuel type and final ISI.
3. Adjustments for buildup effects (in order to account for the amount of fuel that is available for combustion within the entire fuel complex), calculating the buildup effect (BE):

$$BE = e^{(50 \cdot \ln(q) \cdot (\frac{1}{BUI} - \frac{1}{BUI_0}))} \tag{5}$$

Where:

BUI_0 and q are factors depending upon what fuel type has been chosen.

4. Calculating the final rate of spread, ROS:

$$ROS = BE \cdot RSI \qquad (6)$$

To determine the predicted head fire intensity, the FBP system uses the following expression:

$$HFI = 300 \cdot TFC \cdot ROS \qquad (7)$$

Where:
HFI is the predicted head fire intensity
TFC is the predicted total fuel consumption
ROS is the predicted head fire rate of spread

The transition of a surface fire to a crown fire is based upon the theory of Van Wagner /8/ and is determined in the following way in the FBP system:
The theory starts with comparing the surface fire intensity to a critical surface fire intensity value. The critical surface fire intensity value is calculated using the height of the live crown base and foliar moisture content. If the intensity of a surface fire is larger than the critical surface fire intensity, then a crown fire will occur.
The critical surface fire intensity – CSI – is calculated using the following equation /1/:

$$CSI = 0.001 \cdot CBH^{1.5} \cdot (460 + 25.9 \cdot FMC)^{1.5} \qquad (8)$$

Where:
CBH is the height to the live crown base
FMC is the foliar moisture content

Besides the predicted head fire rate of spread, the possible transition to a crown fire and the predicted head fire intensity, the FBP system also gives the following outputs:
- Fuel consumption
- Acceleration
- Head fire spread distance
- Back fire rate of spread and spread distance
- Length-to-breadth ratio
- Flank fire rate of spread and spread distance
- Elliptical fire area
- Elliptical fire perimeter
- Perimeter growth rate
- Flank and back fire intensities

As it is a statistical model, it shows poor performance when applied outside its tested conditions.

A major advantage using the FBP system would be the close link to the FWI system, which is used in Sweden. Several sub-index values from the FWI system can be used for calculations in the FBP system. Thus using the FBP system will require less effort when collecting the necessary input data.

4.2.2 McArthur model

The McArthur models are based on a large number of observations - over 800 fires were studied. The models were constructed without any preconceived notions of the functional relationship between variables.
The models are used widely in south-eastern Australia, which contains the grass and forest types in which they were calibrated. Recently the CSIRO Grassland Fire Spread Meter (GSFM) replaced the McArthur model for grass fires in Australia.
As an example, the appropriate relationships for the Mark 4 grassland fire danger meter are /9/:

$$F = 2 \cdot e^{(-23.6+5.01\cdot\ln(C)+0.0281\cdot T -0.226\sqrt{H}+0.633\sqrt{U}\,)} \qquad (9)$$

$$V = 0.13 \cdot F \qquad (10)$$

Where:

C is the degree of curing
T is the air temperature
H is the relative humidity
U is the wind velocity at 10 m height
F is the fire danger index
V is the rate of fire spread

When comparing with actual fires, the Mark 4 meter has a tendency to underpredict the rate of spread /9/.

4.3 Semi-empirical models

As mentioned before the semi-empirical models are often based on conservation of energy principles but do not make any difference between conduction, convection and radiation heat transfer. Also the semi-empirical models lack for example the flame-fuel interactions, which is an essential part of the modeling of wildfires. Thus the output of these models is of limited accuracy.
The semi-empirical models can address horizontal variation of fuel beds, but cannot address three-dimensional structure of fuels. The transition from ground fire to crown fires is thus an example of a fire phenomenon that cannot be analyzed using

this type of model. Instead the semi-empirical models use a different approach in order to evaluate the risk of transition to crown fire, which can be seen in the Rothermel crown fire model described below.

During the development of a semi-empirical model a number of different laboratory experiments and field observations are performed in order to obtain the needed information /10/.

The semi-empirical model has low computational requirements and include variables that are generally easy to measure in the field, compared with a physical model and is aimed at providing fire officers with real-time forecasts of the fire spread rate and intensity of wildfires. So despite the issue with limited accuracy, the speed and simplicity of these models make them useful /11/.

Below a number of semi-empirical models are described.

4.3.1 Rothermel model (1972)

This is the most important semi-empirical model that has been developed. Rothermel performed his experiments over a wide range of environmental and fuel conditions and included effects of wind and slope in his model. The variety of tests makes his model applicable in many different settings of natural wildfires.

The Rothermel model only accounts for ground fire and not for crown fires. Neither is the spread of the fire by firebrands included in the model.

With respect to fuel parameters, both specific fuel parameters for the site in question and pre-defined parameters for the appropriate fuel model can be used.

The model was designed to simulate a fire that has stabilized into a quasi-steady spread condition. Most wildfires begin from a single source and spread outward in the terrain, growing in size and taking shape as an ellipse with the major axis in the direction most favorable to spread (most often the wind direction). When the fire is large enough so that the spread of any part of the fire is independent of influences caused by the opposite side of the fire, it can be assumed to have stabilized into a line fire. A line fire behaves like a reaction wave with progress that is steady over time in uniform fuels /12/.

The Rothermel model is based on a report by Frandsen /13/, who observed that the curved interface between burnt and unburnt fuel implies that the term:

$$\int_{-\infty}^{0} (\partial Q / \partial z)\,dx \qquad\qquad (11)$$

needs to be added to the left-hand side of equation:

$$Q = \rho \cdot h \cdot V \qquad\qquad (12)$$

Where:
Q is the net energy per unit area transported across the surface of fire start
ρ is the fuel density
V is the rate of spread

h is the fuel heat yield

This term represents the effects of the vertical heat flux gradient, the vertical being represented by z and the horizontal direction of the fire front propagation being the x-direction. The Rothermel model defines the reaction intensity – I_R – of the fire, which is defined as the rate of heat production per unit surface area of the fire front. Rothermel assumes that the propagating flux is proportional to the reaction intensity, which is given by:

$$I_R = -\frac{dw}{dt} h \qquad (13)$$

Where:
$\frac{dw}{dt}$ is the fuel mass loss rate per unit area in the fire front

An important concept in Rothermel's model is that both I_R and Q at the fire front can be evaluated independently and correlated in the no-wind and no-slope situation, which is distinguished by the subscript zero in Q_0.
The laboratory results of Rothermel indicate that:

$$Q_0 / I_R = (192 + 7.894 \cdot s)^{-1} \cdot e^{\left[(0.792 + 3.760 \cdot s^{0.5})(b+0.1)\right]} \qquad (14)$$

Where:
s is the fuel surface area to volume ratio
b is the fuel bed packing ratio

Rothermel includes wind and slope in his formulation as noted by the equation:

$$Q = Q_0 (1 + \phi_w + \phi_s) \qquad (15)$$

Where:
ϕ_s is the slope correction
ϕ_w is the wind correction

$$\phi_s = 5.275 \cdot b^{-0.3} \cdot (\tan \phi)^2 \qquad (16)$$

Where:
$\tan \phi$ is the slope of the fuel bed

$$\phi_w = C \cdot U^B \cdot (b / b_0)^{-E} \qquad (17)$$

Where:
B, C and E are parameters given by s (see below)
U is the wind speed

b_0 is the optimum packing ratio

$$B = 0.02526 \cdot s^{0.54} \tag{18}$$

$$C = 7.47 \cdot e^{-0.133 \cdot s^{0.55}} \tag{19}$$

$$E = 0.715 \cdot e^{-3.59 \cdot 10^{-4} \cdot s} \tag{20}$$

/9/

The rate of spread – R - equation is as follows:

$$R = \frac{Q_0 (1 + \phi_w + \phi_s)}{\rho \cdot \varepsilon \cdot Q_{ig}} \tag{21}$$

Where:
ε is the effective heating number (see below for equation)
Q_{ig} is the heat of pre-ignition of the fuel

$$\varepsilon = e^{-138/s} \tag{22}$$

Please observe that the fire spread algorithm of the model assumes uniform fuel type, uniform fuel conditions, uniform slope and uniform wind.

The Rothermel model has been used at several test fires around the world and shown reasonable accuracy. However, the model has shown significant problems when used in more structurally complex systems /14/.

4.3.1.1 Rothermel crown fire model (1991)

Rothermel characterized crown fires into two major types:
- Wind driven fires
- Plume dominated fires

Rothermel uses the nature of the convection column for indicating the type of fire. At a wind driven fire the wind velocity increases with height and the wind not only drives the fire, but bends the convection column sharply in the direction of the wind. At a plume dominated fire, the column is not bent by the wind and can continue to grow as the fire intensity increases.
Rothermel assumes that mainly the wind speed and the dead fine fuel moisture content have an influence on the crown fire spread rate.
The formulation of Rothermel does not include any characterization of the crown fuels.

Rothermel uses the equations provided by Byram /15/ in his model, in order to calculate the power of the fire - P_f - and the power of the wind – P_w.

$$P_f = \frac{g \cdot I}{c_p \cdot T}$$ (23)

Where:
g is the acceleration of gravity
I is the fireline intensity
c_p is the specific heat of air
T is the absolute air temperature

$$P_w = \rho \frac{(V - R)^3}{2}$$ (24)

Where:
ρ is the mass density of air
V is the wind velocity
R is the rate of fire spread

If the power of the fire is larger than for the wind, the fire can be expected to be plume dominated. If the power of the wind is larger than for the fire, the fire can be expected to be wind driven /16/.

Rothermel also used a ratio between observed crown fire rates of eight western wildfires – that showed sustained runs – and predicted spread rates, in order to come up with an average value of 3.34. This value is multiplied with the predicted surface fire rate of spread in order to obtain the rate of spread of the corresponding crown fire /17/.

The Rothermel crown fire model has a tendency to underpredict the rate of spread. One reason for this could be that Rothermel uses a surface fire analysis methodology for crown fires /18/.

4.3.2 BEHAVE and FARSITE

The fire management applications of BEHAVE and FARSITE include several models and modeling systems to simulate fire growth across a two-dimensional landscape.

Rothermel's model (1972) is the basis for the BEHAVE model.
The models in BEHAVE include for example:
- Surface fire spread and intensity
- Size of a point source fire
- Spotting distance
- Crown scorch height
- Probability of ignition from fire brands or from lightning

- Transition from surface to crown fire
- Crown fire spread

FARSITE is a model, built on semi-empirical formulas such as those in the BEHAVE model. It contains for example the Van Wagner model for crown fire initiation (1977) and Rothermel's model for crown fire spread (1991).
Examples on models used in FARSITE:
- Crown fire model
- Acceleration model
- Spotting model
- Fuel moisture model
- Postfrontal combustion model

FARSITE uses the Huygen's principle. The Huygen's principle applied to fire spread models uses the approach that the fire front is composed of an ordered list of vertices. Each vertex on the fire perimeter grows as an elliptical wavelet. The vertices move at certain time steps based on information at that location, plus the relative locations of its neighbouring points.
But basically the Huygen's principle is a question of a statistical model obtained with simulation, even though in some cases the wind or the slope is included in order to get an impression of reality.
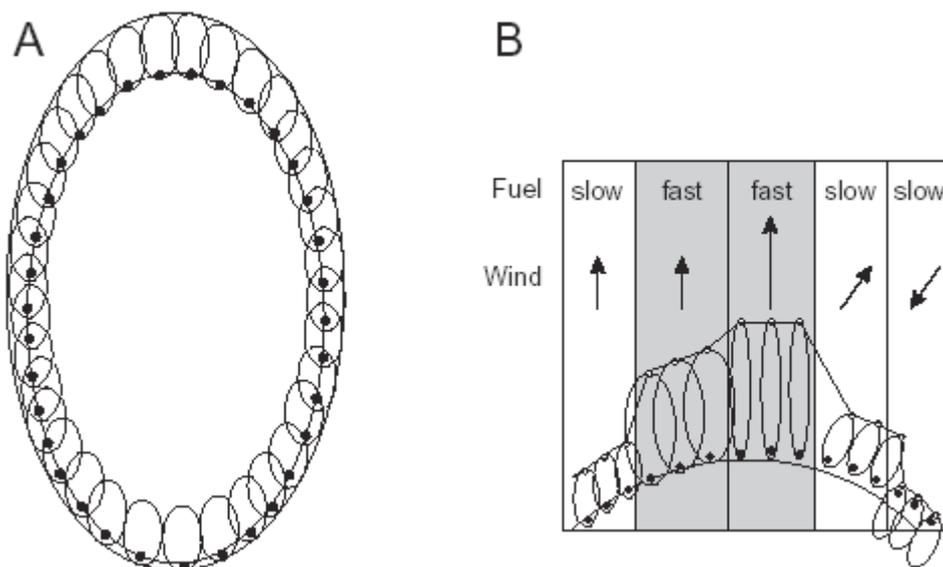

Figure 4. Huygen's principle using elliptical wavelets – each vertex along the fire perimeter grows as an elliptical wavelet/19/.

4.3.3 Van Wagner crown fire initiation and spread models (1977)

Van Wagner developed physical criterias for crown fire initiation and spread.

Van Wagner described the crown layer only in terms of foliar bulk density and moisture content and assumes that the surface fuel layer is clearly separated from the crown by an open trunk space.

Assuming that the ignition of tree crowns only depend upon reaching a certain minimum temperature at the base of the crown layer, Van Wagner uses the following relation for the temperature above ambient, ΔT:

$$\Delta T \propto I^{2/3} / z \qquad (25)$$

Where:
I is the energy output rate per unit length of fire front
z is the height above a line fire where the temperature measurement was taken

Assuming that the relation above gives the ΔT required for crown ignition only at an arbitrary value of the heat of ignition (h) called h_0 and that the actual required temperature rise at the crown base varies with the ratio h/h_0. Thus the left-hand side of the relation above becomes $\Delta T \cdot h/h_0$. Replacing $\Delta T/h_0$ by an empirical quantity C gives us the following equation:

$$I_0 = (Czh)^{3/2} \qquad (26)$$

Where:
I_0 is the critical surface intensity for crown combustion
C is the criterion for initial crown combustion
h is the heat of ignition

If the intensity of the surface fire exceeds I_0, crown combustion should take place.

A simple equaton for the rate of spread could be expressed as:

$$R = \frac{E}{d \cdot h} \qquad (27)$$

Where:
R is the rate of spread
E is the forward heat flux through crown layer
d is the bulk density

When the required heat flux – E – is used in critical sense, it is designated E_0 (i.e. E being sufficiently high to achieve crown spread).
Besides using the required heat flux as a criterion for crown spread, the mass flow rate of fuel into the crown space – S:

$$S = Rd \qquad (28)$$

Where:

S is the mass flow rate through crown layer

As for E_0, the required mass flow rate through crown layer used in critical sense is designated S_0. Below S_0 a solid flame cannot form in the crown space.
The critical spread rate - in order to make crown spread possible – is designated R_0.

Van Wagner divided the crown fires into three different types – based upon the surface intensity, horizontal heat flux and spread rate:

- Passive crown fire. At a passive crown fire, the surface intensity – I – rises above I_0, but the minimum spread rate – R_0 – is not attained.
- Active crown fire. At an active crown fire, the surface intensity – I – rises above I_0 and the minimum spread rate is also attained.
- Independent crown fire. An independent crown fire may occur if the crown phase supplies the whole required horizontal heat flux – E_0 – by itself /8/.

As noted the model of Van Wagner relies solely on the convective heat fluxes for determining crown fuel ignition, the model does not include the radiative contribution. Also the output of the model is questionable with respect to the wind flow effect.

The validation results of Van Wagners models were acceptable /18/.

4.4 Physical models

Most of the physical models are based upon the same concept of a steady-state energy flux relationship being used in Rothermel's model (see equation 12), namely:

$$Q = \rho \cdot h \cdot V$$

A major advantage of the physical models is the fact that they are based on known relationships and thus facilitating their scaling /14/.
But the work on physical models are presently hampered by the lack of understanding of several processes, such as the characterization of the chemical processes taking place during combustion, the resulting flame characteristics, the isolation and quantification of physical processes governing heat transfer and the contribution of each heat-transfer mechanism to the overall energy transmitted to the unburned fuels /20/.
As a result of this, even in physical models there are semi-empirical parts as pure physical models are impossible at the moment due to the enormous complexity of the problem. Another disadvantage with respect to physical models is the fact that the input data of today is not available at an adequate accuracy that would justify the use of physical models. Also some of the fundamental variables included in a physical model are extremely difficult to measure in the field, so they are either assumed or measured in a laboratory environment.

An important part of the development of a physical model is the validation against experimental fires or test fires. Without this, the use of the model is questionable. Below a number of physical models are described.

4.4.1 Albini (1986)

Albini developed a precise formulation of the spread of a line fire by radiative heat transfer. The complete integral equations that describe radiation transfer in a uniform layer - that is assumed to be randomly distributed, thermally-thin and composed of radiometrically black fuel particles - are used to connect the combustion zone and unburnt fuel. The model works with a two-dimensional ignition interface.
The model treats air as transparent, but takes into account the radiative absorption of water vapor.
The model also includes natural convection cooling and re-radiation cooling.

Using non-dimensional variables, the equations are:

$$\left(\mu \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + \eta \frac{\partial}{\partial z} \right) i = C \cdot (\theta - i) \tag{29}$$

$$-\Lambda \frac{\partial Q}{\partial x} = C \left(\frac{1}{\pi} \iint_{4\pi} i \, dw' - 4\theta - 4h(T - T_\infty) / \sigma T_f^4 \right) \tag{30}$$

Where:
μ, v, η are the direction cosines of the ray path described by i
i is the normalized radiation intensity
C is a dimensionless parameter
θ is the normalized radiometric variable
Λ is a dimensionless parameter
Q is the normalized energy density of the fuel
h is the film heat transfer coefficient for convective cooling of a fuel particle
T is the fuel particle temperature
T_∞ is the ambient temperature
T_f is the effective radiometric temperature of burning zone of fuel bed
σ is the Stefan-Boltzmann constant

$$i = (I / I_B) - (T_\infty / T_B)^4 \tag{31}$$

$$\theta = (T / T_B)^4 - (T_\infty / T_B)^4 \tag{32}$$

Where:
I is the radiation intensity
I_B is the radiation intensity from burning zone of fuel bed
T_B is the effective radiometric temperature of burning zone of fuel bed

The term h is expressed in terms of Nusselt number – Nu – and given by:

$$Nu = hD/\lambda \qquad (33)$$

Where:
D is the particle diameter
λ is the thermal conductivity of air

The equations are solved numerically by an iterative process /6/ /21/.

As mentioned before, the model applies only to situations where the radiation is the dominating energy transfer mechanism.

The model agreed fairly well with results from small experimental fires in a laboratory – mainly aimed at verifying the shape of the ignition interface/21/.

In later versions the model has been modified, for example extending the representation of the fuel to multiple levels, where surface fuel, sub-crown fuel and the crown fuel are incorporated into the spread model /22/.

4.4.2 Coupled atmosphere-fire model

The model is designed for inclusion in a coupled fire-atmosphere dynamical code, emphasizing the interaction between the fire plume and the atmosphere.
It uses a continous plane approach – based upon the Huygen's principle – for representing the fire perimeter.
The fluid dynamics model solves equations for momentum, mass, energy, water vapour, and cloud and precipitation variables.
The energy conservation equation being used in the model /11/:

$$\bar{\rho}\frac{\partial \theta}{\partial t} + \overline{\rho v} \cdot \nabla \theta = \nabla \cdot (\bar{\rho} K_H \nabla \theta) \qquad (34)$$

Where:
$\bar{\rho}$ is the mean density at a certain height level
K_H is the eddy mixing coefficient for heat and moisture
Θ is the potential temperature
$\vec{v}$ is the wind vector

The mass conservation equation being used in the model:

$$\nabla \cdot (\overline{\rho v}) = 0 \qquad (35)$$

The model is three-dimensional and uses nested spatial domains. These domains are constructed so that while outer domains resolve atmospheric scales of motion such as fronts and meoscale convective systems, the inner domain resolves fine scale features

such as vortices a few meters wide within firelines. The innermost domain is the one that directly interacts with the fire model. The models are fully coupled so that modeled atmospheric information is used in the spread of the fireline and the heat and moisture from the fire model is released into the modeled atmosphere and thus changing the atmospheric motions.

The rate of spread of the model is determined by using formulas of Rothermel (1972) and formulas of McArthur.

The mass loss rate of the model is determined by using a mass loss parameterization approach provided by Albini /23/. The propagation of the fireline through a fuel cell implies that points within the cell will have been burning for different lengths of time. To determine the fractional mass loss – f_b – over a time step, the time history of the area burned in the fuel cell is determined and integrated to calculate the remaining fuel mass.

The equations describing fuel mass and mass loss are:

$$m(t) = m_0 \cdot f_b(t) \tag{36}$$

or

$$\frac{\partial m}{\partial t} = m_0 \cdot g_b(t) \tag{37}$$

Where:
m_0 is the initial fuel mass
$$g_b(t) = \frac{\partial f_b}{\partial t}$$

Since $f_b(t)$ is an asymptotically decreasing function with increasing time, its time derivatives are reasonably smooth in order to allow for accurate fits using combinations of low order polynomials, exponentials or Pade approximates.

As a fractional mass loss to each fuel cell of the model must be described and as the model area is changing with time, equation (37) cannot be used directly. Instead the time history of the area burned in the fuel cell must be estimated and from that the remaining fuel mass will be calculated. Using equation (37), the following double integral is obtained:

$$F(t) = m_0 \int_0^t \frac{dA(\tau)}{d\tau} \int_0^{t-\tau} g_b(\psi) d\psi d\tau \qquad \text{for } t \leq t_c \tag{38}$$

Where:
$A(\tau)$ is the ignited area of the fuel cell at $t=\tau$
$F(t)$ is the mass of fuel burned at time t
t_c is the time at which all the fuel in the cell is burning

For $t > t_c$ the following equation is applied:

$$F(t) = F(t_c) + m_0 \int\limits_{t_c}^{t} \int\limits_{t_c}^{t-\tau} g_b(\psi) d\psi d\tau \qquad \text{for } t > t_c \qquad (39)$$

A code for crown fire experiments is also built in. Prior to possible crown ignition, the heat flux from surface fire is entirely absorbed by the crown. When the crown is dry, it is ignited at a threshold of 0.17 MW/m². A crown fire in the model is positioned with the ground fire and cannot propagate separately. The dry crown mass equal to the change in fractional area of the ground fire is burned and converted into heat. The heat from the ground and crown for each fuel cell is summed back onto the larger domain and used as surface fluxes in the atmospheric model.
As the model uses mostly semi-empirical formulas, the computational resources can be used mainly to resolve the atmospheric physics.
The model is presently viewed as a research model /24/.

4.4.3 Multiphase models

Models that involve mainly the flame-fuel interaction are called multiphase models. When using the multiphase approach, the vegetation is regarded as fuel particles that are distributed in the air. Particles with similar properties are grouped into families or solid phases (families could for example be: leaves, twigs). To describe the vegetation, the volume fraction of each family is defined - the volume fraction of a family is the fraction of space volume occupied by the particles of a solid phase at a given point of the vegetation layer.
Please observe that this type of model does not include the complete plume-atmosphere interaction.

4.4.3.1 Grishin et al. (1983)

Grishin et al. /25/ proposed fire spread model that considers the flame-fuel interaction – heating, drying, pyrolysis and combustion. The model utilises the conservation of mass, momentum and energy in both the solid and gas phases. The model uses a single spatiotemporal dimension and it also uses first-order Arrhenius reaction chemistry to model pyrolysis and combustion. During the development of the model it was assumed that turbulent transport processes in the vegetation can be modeled using turbulent exchange.
The forest is considered as a multiphase, multistoried, spatially heterogenous medium outside the fire zone. Inside the fire zone, the forest is considered to be a porous-dispersed, seven-phase, two-temperature, single-velocity, reactive medium. The six phases within the combustion zone are: dry organic matter, water in liquid state, solid products of fuel pyrolysis, ash, gas and particles in the dispersed phase.

The heat flux – q – is expressed as:

$$q = \lambda_T \frac{\partial T}{\partial x} \qquad (40)$$

Where:
λ_T is the effective turbulent conductivity or eddy diffusivity
T is the temperature

The energy equation is then as follows:

$$\left(\sum \rho_i \varphi_i c_{pi} + \rho_{cp}\right)\frac{\partial T}{\partial t} + \rho_{cp} W \frac{\partial T}{\partial x} = \frac{\partial T}{\partial x}\left(\lambda_T \frac{\partial T}{\partial x} - H(T - T_\infty)\right) + \text{reaction terms} \qquad (41)$$

Where:
ρ_i is the fuel density
φ_i is the volume fraction
c_{pi} is the specific heat
ρ_{cp} is the gas phase density
W is the relative humidity
H is the enthalpy

The summation term includes four terms which represent dry organic matter, liquid water, condensed pyrolysis products and the mineral composition of the fuel. The convective cooling is included with the term $H(T-T_\infty)$, this emphasizes the fact that the model does not include the hydrodynamic aspects of heat flow, only the combustion /14/.
No information about the performance of the model (with respect to test fires or experimental fires) was found.

4.4.3.2 Grishin et al. (2002)

In 2002 A.M. Grishin and O.V. Shipulina presented a model for the spread of upper crown fires in homogenous forests /26/.
In the model the crown is considered a reactive, multiphase, continous, porous medium, which consists of a dry organic material, water in the liquid-droplet state, carbon-enriched condensed pyrolysis products, mineral components and gas and disperse phases. For simplicity the gas phase is considered to consist of O_2, a combustible gas (mainly CO), and CO_2 and the other inert gases.
In the model the flame above the crown is assumed to radiate as a flat wall located at a certain angle to the crown, whose value depends upon the wind speed above the crown.
No information about the performance of the model (with respect to test fires or experimental fires) was found.

As an example of the complexity of multiphase modeling, the following equations - expressing the conservation laws for the multicomponent reactive medium in the forest crown - are listed (equation 42 through 64).

$$\frac{\partial \rho_5}{\partial t} + \frac{\partial (\rho_5 u)}{\partial x} + \frac{\partial (\rho_5 v)}{\partial y} + \frac{\partial (\rho_5 w)}{\partial z} = Q \tag{42}$$

Where:
ρ_5 is the real density of the gas phase
u, v and w are the speed projections onto the x, y and z-axes
Q is the mass rate of formation of the gas phase

$$\frac{\partial}{\partial t}(\rho_5 u) + \frac{\partial}{\partial x}(\rho_5 u^2) + \frac{\partial}{\partial y}(\rho_5 uv) + \frac{\partial}{\partial z}(\rho_5 uw) = \frac{\partial p}{\partial x} - \rho_5 sc_d u\sqrt{u^2 + v^2 + w^2} + \frac{\partial}{\partial x}\left(\mu_t \frac{\partial u}{\partial x}\right)$$
$$+ \frac{\partial}{\partial y}\left(\mu_t \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu_t \frac{\partial u}{\partial z}\right) \tag{43}$$

Where:
p is the pressure in the flow
s is the specific surface area of the crown phytomass
c_d is the resistance coefficient
μ_t is the dynamic viscosity

$$\frac{\partial}{\partial t}(\rho_5 v) + \frac{\partial}{\partial x}(\rho_5 uv) + \frac{\partial}{\partial y}(\rho_5 v^2) + \frac{\partial}{\partial z}(\rho_5 vw) = -\frac{\partial p}{\partial y} - \rho_5 sc_d v\sqrt{u^2 + v^2 + w^2} + \frac{\partial}{\partial x}\left(\mu_t \frac{\partial v}{\partial x}\right)$$
$$+ \frac{\partial}{\partial y}\left(\mu_t \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu_t \frac{\partial v}{\partial z}\right) \tag{44}$$

$$\frac{\partial}{\partial t}(\rho_5 w) + \frac{\partial}{\partial x}(\rho_5 uw) + \frac{\partial}{\partial y}(\rho_5 vw) + \frac{\partial}{\partial z}\left(\rho_5 w^2\right) = -\frac{\partial p}{\partial z} - \rho_5 sc_d w\sqrt{u^2 + v^2 + w^2} + \frac{\partial}{\partial x}\left(\mu_t \frac{\partial w}{\partial x}\right)$$
$$+ \frac{\partial}{\partial y}\left(\mu_t \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu_t \frac{\partial w}{\partial z}\right) - \rho_5 g \tag{45}$$

$$\frac{\partial}{\partial t}\left(\rho_5 c_{p5} T\right) + \frac{\partial}{\partial x}\left(\rho_5 c_{p5} uT\right) + \frac{\partial}{\partial y}\left(\rho_5 c_{p5} vT\right) + \frac{\partial}{\partial z}\left(\rho_5 c_{p5} wT\right) = \frac{\partial}{\partial x}\left(\lambda_t \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_t \frac{\partial T}{\partial y}\right)$$
$$+ \frac{\partial}{\partial z}\left(\lambda_t \frac{\partial T}{\partial z}\right) + q_5 R_5 (1 - v_5) - \alpha_v (T - T_s) + c_{p5} TQ + k_{s,v} c_{p,s}(T_s - T)(1 - \alpha_c)R_{1s} + k_7 c_{p7}(T_s - T)R_2 \tag{46}$$

Where:
c_{p5} is the specific heat of the gas phase

T is the gas phase temperature

λ_t is the turbulent thermal conductivity

q_5 is the heat effect due to the oxidation of volatile combustible pyrolysis products

R_5 is the mass rate of gas phase oxidation of carbon monoxide

v_5 is the fraction of the heat due to the gas phase oxidation of the gaseous pyrolysis products absorbed by the condensed phase

α_v is the volume heat exchange coefficient

T_s is the solid phase temperature

$k_{s,v}$ is an influence factor (≤ 1)

$c_{p,s}$ is the average heat capacity of the mixture of gaseous pyrolysis products

α_c is the carbon number of the combustible forest materials

R_{1s} is the mass rate of pyrolysis of the dry combustible forest material

k_7 is an influence factor (≤ 1)

$$\sum_{i=1}^{4}\left(\rho_i\varphi_i c_{p,i}\frac{\partial T_s}{\partial t}\right) = q_3 R_{3s} - q_2 R_{2s} + k_s(cU_R - 4\sigma T_s^4) + \alpha_v(T - T_s) + q_5 R_5 v_5 \qquad (47)$$

Where:

ρ_i is the real density of the ith phase of the multiphase reactive medium (where i=1 is the dry organic material; i=2 is the water in the liquid-droplet state; i=3 is the condensed products of pyrolysis of the dry organic material; i=4 is the mineral part; i=5 is the gas phase)

φ_i is the volume fraction of the ith phase of the multiphase reactive medium

$c_{p,i}$ is the specific heat of the ith phase of the multiphase reactive medium

q_3 is the heat effect due to the pyrolysis of carbon burning

R_{3s} is the mass rate of consumption of carbon during combustion

q_2 is the heat effect due to the pyrolysis of water vaporization from combustible forest materials

R_{2s} is the mass rate of water vaporization from the combustible forest material

k_s is the spectral absorption factor

c is the speed of light

U_R is the radiation-flux density

σ is the Stefan-Boltzmann constant

$$\frac{\partial}{\partial t}(\rho_5 C_\alpha) + \frac{\partial}{\partial x}(\rho_5 C_\alpha u) + \frac{\partial}{\partial y}(\rho_5 C_\alpha v) + \frac{\partial}{\partial z}(\rho_5 C_\alpha w) = \frac{\partial}{\partial x}\left(\rho_5 D_t\frac{\partial C_\alpha}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho_5 D_t\frac{\partial C_\alpha}{\partial y}\right)$$

$$+ \frac{\partial}{\partial z}\left(\rho_5 D_t\frac{\partial C_\alpha}{\partial z}\right) + R_{5\alpha} \qquad (48)$$

Where:

C_α is the mass concentration of the gas phase components (α=1 is oxygen; α=2 is the combustible pyrolysis products; α=3 is the oxidation products of the combustible pyrolysis components; α=4 is the inert gas phase components, nonreactive pyrolysis products and water vapor), $C_4 = C_4'$ is a constant (where C_4' is the constant concentration of the inert components

$$\sum_{\alpha=1}^{4} C_\alpha = 1$$

D_t is the turbulent diffusivity

$$\frac{\partial}{\partial x}\left(\frac{c}{3k_\Sigma}\frac{\partial U_R}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{c}{3k_\Sigma}\frac{\partial U_R}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{c}{3k_\Sigma}\frac{\partial U_R}{\partial z}\right) - k_s(cU_R - 4\sigma T_s^4) = 0 \qquad (49)$$

Where:

k_Σ is the attenuation coefficient

$$p_e = \rho_5 RT \sum_{\alpha=1}^{4} \frac{C_\alpha}{M_\alpha} \qquad (50)$$

Where:

p_e is the pressure in the unperturbed region

M_α is the molecular weights of individual components

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_{1s} \qquad (51)$$

$$\rho_2 \frac{\partial \varphi_2}{\partial t} = -R_{2s} \qquad (52)$$

$$\rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_{1s} - \frac{M_c}{M_1} R_{3w} \qquad (53)$$

$$\rho_4 \frac{\partial \varphi_4}{\partial t} = 0 \qquad (54)$$

Where:

ρ_1 is the real density of the dry organic material

ρ_2 is the real density of the water in the liquid-droplet state

ρ_3 is the real density of the condensed products of pyrolysis of the dry organic material

ρ_4 is the real density of the mineral part

φ_1 is the volume fraction of the dry organic material

φ_2 is the volume fraction of the water in the liquid-droplet state

φ_3 is the volume fraction of the condensed products of pyrolysis of the dry organic material

φ_4 is the volume fraction of the mineral part

R_{1s} is the mass rate of pyrolysis of the dry combustible forest material

R_{2s} is the mass rate of water vaporization from the combustible forest materials

R_{3w} is the mass rate of O_2 consumption during heterogenous combustion of carbon

$$\varphi_5 = 1 - \sum_{i=1}^{4} \varphi_i \qquad (55)$$

Where:
φ_5 is the volume fraction of the gas phase

$$Q = (1 - \alpha_c)R_{1s} + R_{2s} + \frac{M_c}{M_1}R_{3w} \tag{56}$$

Where:
M_c is the molecular weight of carbon

$$R_{1s} = k_1\rho_1\varphi_1 e^{\left(-\frac{E_1}{RT}\right)} \tag{57}$$

$$R_{2s} = k_2\rho_2\varphi_2 T^{-0.5} e^{\left(-\frac{E_2}{RT}\right)} \tag{58}$$

$$R_{3w} = k_3 s_\sigma \rho_5 C_1 \varphi_3 e^{\left(-\frac{E_3}{RT}\right)} \tag{59}$$

Where:
k_1, k_2 and k_3 are the preexponents of chemical reactions
E_1, E_2 and E_3 are the activation energies of chemical reactions
s_σ is the effective specific surface area of carbon

$$R_5 = k_5 M_2 T^{-2.25} e^{\left(-\frac{E_5}{RT}\right)} \times \begin{cases} x_1^{0.25} x_2, & x_1 \geq 0.05 \\ x_1 x_2, & x_1 < 0.05 \end{cases} \tag{60}$$

$$x_i = \frac{C_i}{\sum_{k=1}^{4}\frac{C_k}{M_k}M_i} \tag{61}$$

$$R_{51} = -R_{3w} - \frac{R_5 M_1}{2M_2} \tag{62}$$

Where:
R_{51} is the mass rate of consumption of the gas phase components

$$R_{52} = (1 - \alpha_c)v_g R_1 - R_5 \tag{63}$$

$$R_{54} = 0 \tag{64}$$

Where:
R_{52} is the mass rate of formation of the gas phase components
v_g is the mass fraction of the combustible gas in the volatile pyrolysis components
/26/.

4.4.3.3 Balbi et al. (1999)

Balbi J.H., Santoni P.A. and Dupuy J.L. presented in 1999 a reaction chemistry model, where they assumed no wind and no slope conditions across a homogenous fuel bed. They also assumed that the combustion reaction takes place at a constant temperature, that the heat of reaction is constant and that the energy released during the combustion is proportional to the quantity of fuel burnt.

The model includes a cooling convective term. It also includes an equivalent diffusion term in order to represent all three heat transfer mechanisms.

The following assumptions were made with respect to chemical combustion reactions:

- combustion occurs at a threshold temperature
- the enthalpy of the reaction has a constant value
- the fuel mass decreases according to an exponential law
- the heat released during combustion is proportional to the quantity of burnt fuel

The following equation for thermal balance is used in the model:

$$\frac{\partial T}{\partial t} = -k(T - T_a) + K\nabla T - Q\frac{\partial \rho_v}{\partial t} \tag{65}$$

$$k = \frac{k^*}{m} \tag{66}$$

$$K = \frac{K^*}{m} \tag{67}$$

$$Q = \frac{Q^*}{m} \tag{68}$$

Where:
T is the temperature of the fuel bed
k^* is the convective cooling coefficient
K^* is the equivalent thermal conductivity
Q^* is the enthalpy of the combustion per unit mass of fuel
m is the thermal mass of the fuel cell
T_a is the ambient temperature
∇ is the Laplacian operator in two-dimensional space
ρ_v is the mass of fuel per unit platform area of the bed

The partial differential equation above is in a single phase contains only diffusion and cooling terms.

The following first-order reaction chemistry equations were used in the model:

$$\begin{cases} \rho_v = \rho_{v0} \\ \rho_v = \rho_{v0}e^{-\alpha(t-t_{th})} \end{cases} \tag{69}$$

(The first equation applies for an unburnt cell and the second equation applies for a cell in which the temperature has reached T_{th} (the threshold temperature))

Where:
ρ_{v0} is the value of ρ_v at time zero
α is the combustion time constant
t_{th} is the time at which $T=T_{th}$

The unknown coefficients listed above (k, K, Q, α and T_{th}) were estimated from an analysis of the time-temperature curves generated during a number of experimental fires that were conducted.
The experimental fires were set and showed good agreement with the proposed model during the growth stage of the fire. No measurements were made during the steady state stage of the fires /27/.

As noted, the model is greatly simplified and was mainly aimed as an initial model to develop further and make it more sophisticated.

4.4.3.4 PIF97

In the PIF97 model two phases of media is considered: gas and solid. The two phases forming a multiphase medium together. The model also assumes that the solid phase is homogenous.

The model is divided into two parts:
- The preheating zone: a combustion zone model that includes the radiative and convective heat transferred to the fuel bed in front of the flaming zone. The model assumes for example that the fuel bed is composed of an initially homogenous distribution of cylindrical particles whose dimensions are derived from experimental data. The fuel particles are assumed to be motionless. The radiative component considers radiation flux from adjacent fuels, the ignition interface, the flame and the ambient fluid surrounding the fuel. The gas phase is assumed to be transparent to radiation. An ignition temperature for solid fuel of 600 K is used.
- The flaming combustion zone: the second part of the model is the fire-caused flow in the flaming combustion zone behind the ignition interface (the ignition interface is defined as the surface where fuel particles achieve the ignition temperature, separating the two zones). This depends on the rate of spread of the interface derived from the combustion part of the model. The temperature of this gas is assumed to be constant at 900 K /28//29/.

The conservation equations of energy for the preheating zone, are defined as follows:

For the solid phase:

$$-R\rho_p\beta\frac{dh_s}{dx'} = \frac{\sigma\beta}{4}\varphi_R - \sigma\beta h_c(T_s - T_g) \tag{70}$$

Where:
R is the rate of spread
ρ_p is the fuel particle density
β is the packing ratio of the fuel bed
h_s is the enthalpy of the solid phase
x′ is the x-coordinate in the moving frame
σ is the surface to volume ratio
φ_R is the radiative heat flux
T_s is the temperature of the solid phase
T_g is the temperature of the gas phase

For the gas phase:

$$(1-\beta)m_I C_{pg}\frac{dT_g}{dx'} = \sigma\beta h_c(T_s - T_g) \tag{71}$$

Where:
m_I is the mass flux density at the ignition interface
C_{pg} is the specific heat of the gas phase

The final equations of the flaming combustion zone are:

$$\rho_{gI}u_I' - \rho_{gF}u_F' + D\frac{\overline{\rho}_g\overline{w}_\delta}{\ddot{a}} = \frac{D}{1-\beta}\frac{\eta}{\tau}\rho_p\beta \tag{72}$$

Where:
ρ_{gI} is the density of the gas phase at the ignition interface
u_I' is the x-component of gas velocity in the moving frame at the ignition interface
ρ_{gF} is the density of the gas phase at the end of the flaming combustion zone
u_F' is the x-component of gas velocity in the moving frame at the end of the flaming combustion zone
D is the flaming combustion zone depth
$\overline{\rho}_g$ is the average gas phase density
$\overline{w}_\delta$ is the average vertical component of the gas velocity
$\ddot{a}$ is the fuel bed depth
η is the fraction of volatile products
τ is the flaming combustion duration

$$\rho_{gI}u_I'^2 - \rho_{gF}u_F'^2 + D\frac{\overline{\rho}_g\overline{w}_\delta u'}{\ddot{a}} = -\frac{D}{1-\beta}\frac{\eta}{\tau}\rho_p\beta R + Dg\sin(\varphi)(\rho_a - \overline{\rho}_g) - \frac{D}{1-\beta}\overline{F}_{Dx} \tag{73}$$

Where:

$\bar{u}'$ is the average x-component of gas velocity in the moving frame

g is the gravity

φ is the heat flux density

ρₐ is the density of the ambient properties

$\bar{F}_{Dx}$ is the average drag force exerted by the gas flow on motionless particles

$$-\rho_{gF} u'_F \varepsilon_w \overline{w}_\delta + D \frac{\overline{\rho}_g \overline{w}_\delta^2}{\ddot{a}} = Dg\cos(\varphi)(\rho_a - \overline{\rho}_g) - \frac{D}{1-\beta}\overline{F}_{Dx} \qquad (74)$$

Where:

ε_w is the profile parameter for w

When finding a numerical solution of the model, the problem is to solve the coupled energy conservation equations of the preheating zone and the system of final equations of the flaming combustion zone. Finding a rate of spread and the mass flux density at the ignition interface, that satisfies all equations.

The model has been refined in later versions to include multiple solid phases and a crown layer.

The model is capable of modeling the behaviour of a wildfire of landscape scale – i.e. hundreds of meters.

When comparing predictions of the model compared to experimental fires, the model showed poor performance with respect to conditions of wind and slope /4/.

4.4.4 WFDS

WFDS is an example of a model that involves both the flame-fuel interaction and the plume-atmosphere interaction.

The main objective of WFDS was to obtain a model that predicts the spread of fire through terrain of both wildland fuel and combustible structures.

WFDS is an extension of the extensively used Fire Dynamics Simulator (FDS). In this model the pyrolysis of the fuel and gas phase combustion are both implemented in a numerical solution of the Navier-Stokes equations (incorporating the effects of buoyancy, convection and turbulence). Computational resources are mainly used for solving the fire combustion and the fire-atmosphere flow over the fire. Combustion is modeled with a mixture fraction based approach, as the resolved computational grids are too large (see below for smallest size) to allow the combustion being modelled with reaction chemistry instead.

The model cannot represent the local weather within the fire area.

WFDS is fully three-dimensional.

The basis of the surface vegetation model in WFDS is the assumption that combustion occurs mainly above the surface fuel bed. This is consistent with flame heights above the fuel bed that are much larger than the height of the fuel bed itself.

With this assumption, two computational grids can be used: one for the gas phase that defines the fire plume and another for the vegetative fuel bed that defines heat transfer and thermal degradation. A method that is similar to the multiphase approach is used to model the changes in the solid phase.

Compared to FIRETEC, WFDS more directly solves the equations that dictate the interaction of the fire/fuel and fire/atmosphere.

WFDS utilises a varying computational grid to resolve volumes as low as 1.6 m × 1.6 m × 1.4 m within a simulation domain in the order of 1.5 km² in area and 200 m high. Outside regions of interest, the grid resolution is decreased to improve computation efficiency – the same approach is generally used in FDS.

The future use of the model will depend upon the ability to depict fine-scale features such as the burning of isolated individual trees /30/.

The energy conservation of the model:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{75}$$

Where:
ρ is the total density of gas
$\vec{v}$ is the total wind velocity vector

The mass conservation equation of the model:

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} - \nabla \cdot \vec{q}_r + \nabla \cdot k \nabla T + \sum_l \nabla \cdot h_l \rho D_l \nabla Y_l \tag{76}$$

Where:
$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$
h is the enthalpy
p is the pressure
k is the thermal conductivity
$\vec{q}_r$ is the radiation heat flux vector
T is the absolute temperature
h_l is the enthalpy of gaseous species l
D_l is the diffusion coefficient of gaseous species l
Y_l is the mass fraction of gaseous species l

The prediction of the model has been compared with results of experimental fires. The model showed fairly well agreement with the test fires, but in one of the cases the model over predicted the spread rate of a flank fire /7/.

The model is capable of modelling the behaviour of a wildfire of landscape scale – i.e. hundreds of meters.

4.4.5 FIRETEC

Same as for WFDS, FIRETEC is a multi-phase model involving both the flame-fuel interaction and the plume-atmosphere interaction.

The model is based on the averaged conservation equations for mass, momentum, energy and chemical species. The reason for using an average approach is the fact that modeling on the landscape scale will lead to very time and resource consuming computations if for example small-scale chemical processes are taken into account.

The model uses a fully compressible gas transport formulation to couple its physics-based wildfire model with the motions of the local atmosphere. The model can be run in a two- or three-dimensional mode.

Some examples of the physical phenomena being studied are the effects of transient wind conditions, non-homogenous terrain, non-uniform fuel beds with non-continous distributions, and different vertical fuel structures.

FIRETEC can also model the fire spread by firebrands, where it uses a probabilistic approach combined with the wind velocities to model it.

As the specification of the fuel in the model is based on the physical properties of the vegetation, the model can be applied to fuels of various type and distribution. Thus the model is not dependent upon predefined fuel models.

The model is combined with the hydrodynamics model – HIGRAD – in order to simulate wildfires using a terrain-following three-dimensional finite volume grid.

The general approach in the development of the model was to divide the variables that occur in relevant conservation of mass, momentum and energy equations into mean and varying parts and then take the averages of these equations.

The variables that are described by the resulting partial differential equations are average physical quantities meant to describe the resulting averaged properties of the gases and solids within the smallest volume where the mean properties are used.

The large set of chemical reactions occuring in a wildfire was simplified to a small set of reactions involving wood pyrolysis and several solid-gas (burning of char in oxygen) and gas-gas reactions (two reactants forming a single final product and no intermediate species).

In the model it is assumed that the rates of the exothermic reactions in areas of active burning are limited by the rate at which reactants can be brought together in their correct proportions (also known as mixing limited).

For the solid fuel as well as its associated moisture, conservation equations are used for the average properties of the substance within a space much larger than the fuel elements themselves, which is referred to as the resolved volume. The average mass of wood and moisture per unit resolved volume are described by the following two conservation equations:

$$\frac{\partial \rho_{wood}}{\partial t} = -N_{wood} F_{wood} \qquad (77)$$

$$\frac{\partial \rho_{water}}{\partial t} = -F_{water} \qquad (78)$$

Where:
N_{wood} is the ratio of the mass of wood to the total mass of the combined products
F_{water} is the rate of change of water within a resolved volume

The conservation equation for the temperature of the solid:

$$\left(c_{p_{water}} \rho_{water} + c_{p_{wood}} \rho_{wood}\right)\frac{\partial T_s}{\partial t} = Q_{rad} + ha_v\left(T_{gas} - T_s\right) - F_{water}\left(H_{water} + c_{p_{water}} T_{boil}\right)$$
$$+ F_{wood}\left(\Theta H_{wood} - c_{p_{wood}} T_{wood} N_{wood}\right) \qquad (79)$$

Where:
$c_{p_{water}}$ is the isobaric heat capacity of water
$c_{p_{wood}}$ is the isobaric heat capacity of wood
T_s is the average temperature of the solid within a resolved volume
Q_{rad} is the net thermal radiation heat flux to the solid at a given location
h is the convective heat exchange coefficient
a_v is the contact area per unit volume between the gas and the solid
T_{gas} is the average temperature of the gas within a resolved volume
H_{water} is the heat energy per unit mass associated with a flux in water
Θ is the fraction of heat released from gas phase combustion that is deposited directly back to the solid
H_{wood} is the heat energy per unit mass associated with a flux in wood
T_{wood} is the average temperature of the wood within a resolved volume

The equation above describes the changes in the specific internal energy of the solid with respect to the temperature of the solid and the densities of the wood and water.

By dividing the local instantaneous variables into mean and varying parts and calculating the averages of the equations, conservation equations for the average density of oxygen, the average density of the combined gas, the average momentum of the combined gas per unit volume in the i direction, and the average potential temperature of the combined gas are formulated. The change over time of oxygen is described in the following equation and includes terms to represent advection (advection is transport in a fluid), turbulent diffusion and depletion due to the combustion of wood pyrolysis products:

$$\frac{\partial \rho_{O_2}}{\partial t} + \frac{\partial u_i \rho_{O_2}}{\partial x_i} = \frac{\partial}{\partial x_j}\left(\sigma \frac{\partial \frac{\rho_{O_2}}{\rho_{gas}}}{\partial x_j}\right) - F_{wood} N_{O_2} \qquad (80)$$

Where:
u_i is the average velocity of the combined gas in the direction i at a given location
ρ_{gas} is the density of the gas within a minimum volume
N_{O_2} is the ratio of mass of the oxygen to the total mass of the combined products

The density of the combined gas is calculated using the following equation (expressing the sum of the conservation equations of all gas phase species present) - taking into account the movement of gases from one location to another and the creation of new gases through the pyrolysis process and the evaporation of water:

$$\frac{\partial \rho_{gas}}{\partial t} + \frac{\partial u_i \rho_{gas}}{\partial x_i} = F_{wood} N_{wood} + F_{water} \tag{81}$$

The overall conservation of momentum is described as:

$$\frac{\partial \rho_{gas} u_i}{\partial t} + \frac{\partial u_i u_j \rho_{gas}}{\partial x_j} = -\frac{\partial R_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} + \rho_{gas} g_i - \frac{3 C_D \rho_{wood} \rho_{gas} |u| u_i}{8 s_s \rho_{wood(micro)}} \tag{82}$$

Where:
R_{ij} is the Reynolds stress tensor
g_i is the acceleration of gravity in the i direction
C_D is the drag coefficient
s_s is the size scale of the local solid structures
$\rho_{wood(micro)}$ is the microscopic density of wood

The terms to the right of the equation above are advection, gradients of the turbulent Reynolds stress, gradients in pressure, byoyancy and drag.

The following equation describes the change of the temperature of the combined gas using advection, turbulent diffusion and source terms. The source terms in the equation represents the combination of convective heat exchange, radiative heat, and an energy source from the chemical reactions:

$$\frac{\partial \rho_{gas} \theta}{\partial t} + \frac{\partial \theta u_j \rho_{gas}}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\sigma \frac{\partial \theta}{\partial x_j} \right) + \frac{\theta}{c_p} \left[\frac{ha_v(T_s - T_{gas}) + Q_{rad,gas} + F_{wood}(1-\Theta)H_{wood}}{T_{gas}} \right]$$
$$\tag{83}$$

Where:
θ is the average potential temperature of the combined gas at a given location
$Q_{rad,gas}$ is the net gain of energy by the gas from thermal radiation

The energy is emitted or absorbed from the gas phase using a probabilistic approach - using distribution function for gaseous temperature and species concentrations. The emitted energy is diffused through the gas phase based on a radiation energy density

that is not dependent upon the temperature of the gas and is absorbed by solid objects based on their surface area per unit volume /10//31/.

FIRETEC was validated against large scale experimental grassfires and showed good performance. In later reviews of the model, the radiant transfer model in FIRETEC has been considered as very approximate /4/.

The model is capable of modelling the behaviour of a wildfire of landscape scale – i.e. hundreds of meters.

4.5 Available input data

Even though the developed wildfire model may be highly sophisticated in itself, it will depend heavily on the quality of the input data.

4.5.1 Weather

The ideal solution would be to use available weather data from the existing FWI-system. With respect to weather data, the following input data can be obtained through FWI:
- Wind speed and direction
- Temperature
- Relative humidity
- Precipitation

But the question – with respect to input data from WPI – is how much consideration is taken with respect to local spatial variations as the grid is rather coarse (22x22 km in size) and with respect to temporal variations as the output data of the FWI-system is provided only once a day.

A very promising project is being conducted in western Sweden. The project is called "Alarm" and allows for better predictions of gas dispersion models, as the available weather input data is updated once every hour from local weather stations /32/. Using the weather data from the "Alarm"-project would clearly result in better wildfire spread predictions.

4.5.2 Topography

Being the most static factor and with the digital material available from Lantmäteriet, the input data with respect to the topography should not pose any significant problem.

4.5.3 Fuel

Even though the temporal variations of the vegetation in an area may not vary as greatly as the weather, the spatial variations could be very extensive and challenging.

There are several methods to classify the vegetation into different fuel types ready to be used for wildfire spread simulations. The methods could for example be: field reconniassance, using satellite images, using aerial photos, using statistical data etc.

Field reconnaissance means going through a landscape on the ground and recording the fuel conditions in notebooks or on paper maps. The main advantage of field reconnaissance is that fuels are mapped from actual conditions observed on the ground, thus the number of errors will be low. The amount of human effort is however great, which makes it very expensive and not very practical. But during the correlation of the other methods – such as satellite images – the field reconnissance could be a very useful tool /33/.

The use of satellite images and aerial photos will greatly simplify the process and save time and resources /34/.
But there are several disadvantages with using satellite images or aerial photos. Aerial photos and satellite images are unable to detect surface fuels because the ground is often obscured by the forest canopy. Also, it is often difficult to distinguish between the various fuel types using satellite images or aerial photos.
Field reconnaissance is an essential part even though satellite images or aerial photos are used. The field reconnaissance in this case is used in order verify and correct the output based upon satellite images and aerial photos.
Also GIS models can be used that examine the relationship between soil types, forest management activities etc /35//36/.

Thus new technology for satellite images or aerial photos and image classification techniques is in great demand in order to accurately map fuels.
The major problem that must be solved is distinguishing between the canopy fuel and the ground fuel, one possible solution could be the use of so called hyperspectral sensed products and active remote sensors. Synthetic Aperture Radar (SAR) and Lidar (propagating pulses of electromagnetic radiation and detecting the reflective backscatter) have shown promising results when estimating biomass, stand volume, and canopy height /33/.

5. A simple physical model

In order to test the mathematics that is involved in some of the wildfire models found in the literature and articles and to get some feeling of the complexity of the modelling of wildfires, a physical model using a PDE was developed.
When working on the physical model I used the model of Balbi et al. as a basis. The reason for my choice was the positive result when comparing the simulated values and the corresponding values of fire experiments, and the potential for improvements.

5.1 Initial model

The initial model was greatly simplified in order to first of all get an operational model that resulted in reasonable output values.
The initial model was simplified in the following aspects:
- 2-dimensional (x,t).
- No wind.
- No slope.
- No reaction chemistry.
- Using constant values whenever possible.

The following improvements were made:
- A term expressing the radiative heat transfer from the flame to the fuel ahead of the flame was added.
- A term expressing the radiative heat loss from the fuel ahead of the flame was added.

Direction of spread

Flame

Burnt fuel   Unburnt fuel
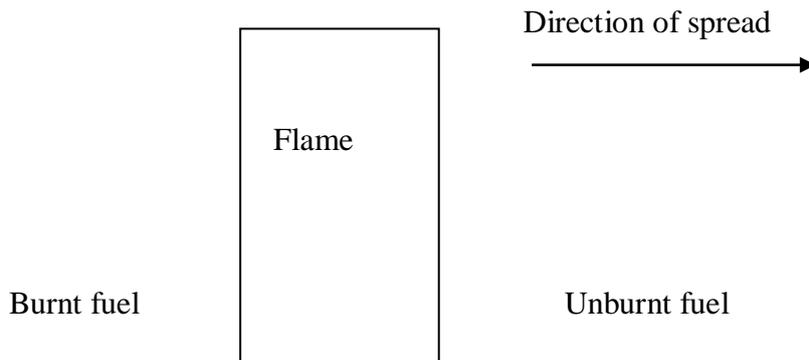
Figure 5. A view of the 2-dimensional model.

The following equation for energy conservation in a fuel element was set up:

$$\rho_f V c_f \frac{\partial T_f}{\partial t} = \rho_f c_f D \frac{\partial^2 T_f}{\partial x^2} + \alpha A_f \sigma (T_{flame}^4 - T_a^4)\phi - T_f \beta h A_f - (1-\phi) A_f \sigma (T_f^4 - T_a^4)$$
(84)

Where:
ρ_f is the fuel density (constant)
c_f is the specific heat of the fuel (constant)
V is the volume of a fuel element (constant)
T_f is the temperature of a fuel element ahead of the fire front
D is the diffusivity coefficient (constant)
α is the absorption coefficient (constant)
A_f is the area of a fuel element (constant)
σ is the Stefan-Boltzmann constant
T_{flame} is the flame temperature (constant)
ϕ is the view factor /37/:

$$\phi = \frac{1}{2\pi}\left[\arctan\left(\frac{1}{x-R(t)}\right) - A \cdot Y \cdot \arctan(A)\right] \tag{85}$$

$$x = L_f \tag{86}$$

$$Y = x - R(t) \tag{87}$$

$$A = \frac{1}{\sqrt{L_f^2 + (x-R(t))^2}} \tag{88}$$

L_f is the flame height (constant)
R(t) is the position of the fire front

β is the packing ratio (constant)
h is the convective cooling coefficient (constant)
A_f is the fuel element area (constant)
T_a is the ambient temperature (constant)

In the calculations of the view factor the flame is assumed to have rectangular shape.

In equation (84) the first term on the right side expresses the convective energy transfer to fuel element. The second term expresses the radiative energy transfer to fuel element. The third term expresses the convective cooling (using a so called Newtonian cooling term) from fuel element. The fourth term expresses the radiative cooling from fuel element.

The following constants were used in the initial model:
ρ_f = 680 kg/m³ (valid for *Pinus Pinaster*) /38/
c_f = 1400 J/kg·K (valid for *Pinus Pinaster*) /29/
V=0.001 m³ (assumed)
D = 14·10⁻⁸ m²/s (which applies to most organic materials) /39/
σ = 5.67·10⁻¹¹ kW/m²·K⁴
T_{flame} = 1200 K /40/

L_f = 0.15 m and 0.3 m
β = 0.029 (valid for *Pinus Pinaster*) /29/
h = 10 W/m²·K (assumed)
A_f = 0.01 m² (using a 0.1×0.1 m grid system)
T_a = 293 K (assumed)

α is calculated using the following equation /41/:

$$\alpha = \frac{\beta \cdot \sigma_s}{4} \tag{89}$$

Where:
σ_s is the surface area-to-volume ratio of the fuel

σ_s = 4285 m⁻¹ (valid for *Pinus Pinaster*) /38/

Resulting in $\alpha \approx 31.07$ m⁻¹

Please observe that *Pinus Pinaster* is a pine specie found in the Mediterranean region, not in the Nordic countries. But the values of *Pinus Pinaster* were used as they were obtainable through literature.

The fire front is assumed to advance to a new fuel element when $T_f \geq 600$ K /5/.
Thus the ignition temperature is assumed to be constant.
At ignition the temperature of the fuel cell is assumed to instantaneously increase to 1200 K.

In the calculation of the radiative heat transfer, the emissivity of the real surface (ε) is set to 1.0 /40/.

In order to solve the equation numerically, the equation was discretized using the forward time centered space scheme (FTCS scheme) /42/.
Using the FTCS scheme, the discretized equation in two dimensions (x,t) is:

$$T_{f,i}^{n+1} = T_{f,i}^n + \frac{\tau \cdot D}{l^2 V}\left[T_{f,i+1}^n + T_{f,i-1}^n - 2 \cdot T_{f,i}^n\right] +$$

$$\frac{1}{\rho_f V c_f l^2}\left[\alpha A_f \sigma(T_{flame}^4 - T_a^4)\phi - T_{f,i}^n \beta h A_f - (1-\phi)A_f \sigma((T_{f,i}^n)^4 - T_a^4)\right]$$

$$\tag{90}$$

Where:
τ is the time step which is set at 1 s.
l is the grid spacing
n is the index indicating the temporal step
i is the index indicating the spatial location of a grid point

The grid spacing – l – is calculated using the following equation:

$$l = \frac{L}{N-1}$$ (91)

Where:
L is the length of one side of the selected terrain section
N is the number of fuel cells along one side

During the initial simulations a length scale of 10 m and a time interval of 35 seconds were used.

When using the FTCS scheme an important part is the verification of the stability. The stability is verified using the following equation:

$$s = \frac{D \cdot \tau}{h^2}$$ (92)

and the following condition:

$$s < 0.5$$

The value of s was found to fulfil the condition above. Thus the scheme is considered as stable.

The following result was received for a flame length of 0.15 m:
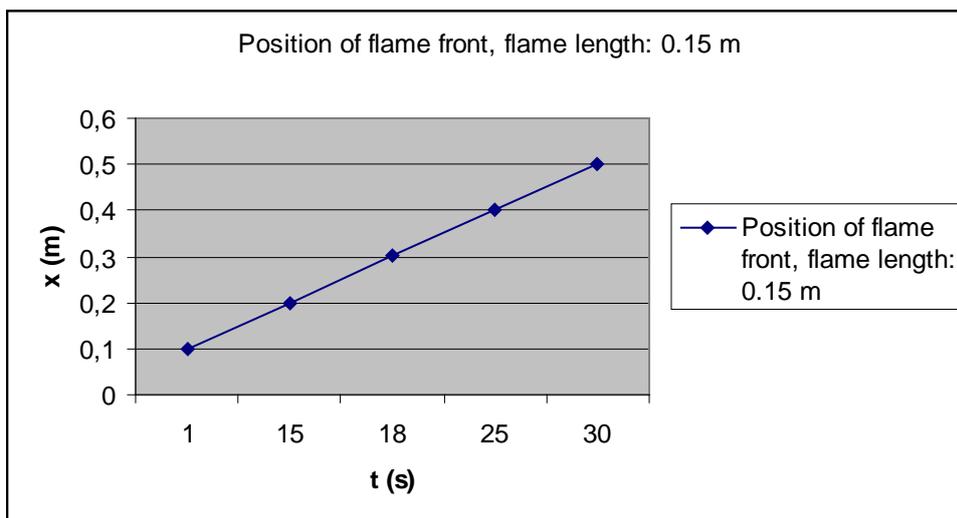
Figure 6. The position of the flame front, flame length: 0.15 m.

From the figure above, the following rate of spread was calculated: ~1 m/min.

The following result was received for a flame length of 0.3 m:
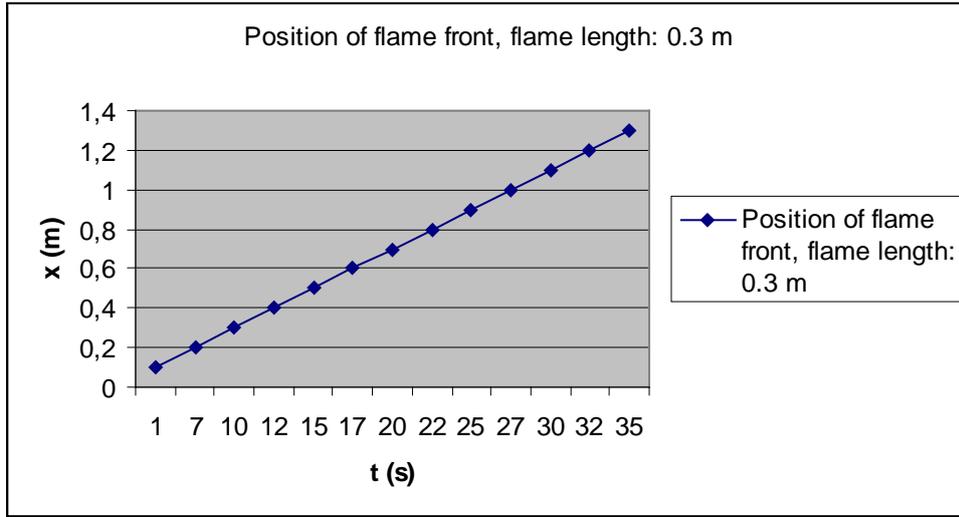
Figure 7. The position of the flame front, flame length: 0.3 m.

From the figure above, the following rate of spread was calculated: ~2.2 m/min.

As noted from the figures and numbers above, a longer flame length will result in a faster rate of spread of the wildfire.
Also note that the rate of spread is constant, which is not realistic as the wildfire will have an acceleration phase at the beginning of the fire before reaching steady state. The constant rate of spread is probably due to the fact that the flame area exposed to the fuel ahead is constant throughout the simulation.

5.2 Three-dimensional model

Equation (84) was modified for the three-dimensional case (x,y,t):

$$\rho_f V c_f \frac{\partial T_f}{\partial t} = \rho_f c_f D \frac{\partial^2 T_f}{\partial x^2} + \rho_f c_f D \frac{\partial^2 T_f}{\partial y^2} + \alpha A_f \sigma (T_{flame}^4 - T_a^4)\phi - T_f \beta h A_f - (1-\phi)A_f \sigma (T_f^4 - T_a^4)$$

(93)

Using the FTCS scheme, the discretized equation in three dimensions (x,y,t) is:

$$T_{f,i,j}^{n+1} = T_{f,i,j}^n + \frac{\tau \cdot D}{l^2 \cdot V}\left[T_{f,i+1,j}^n + T_{f,i-1,j}^n - 2 \cdot T_{f,i,j}^n\right] + \frac{\tau \cdot D}{l^2 \cdot V}\left[T_{f,i,j+1}^n + T_{f,i,j-1}^n - 2 \cdot T_{f,i,j}^n\right] +$$

$$\frac{1}{l^2 \cdot \rho_f \cdot V \cdot c_f}\left[\alpha A_f \sigma (T_{flame}^4 - T_a^4)\phi - T_{f,i,j}^n \beta h A_f - (1-\phi)A_f \sigma ((T_{f,i,j}^n)^4 - T_a^4)\right]$$

(94)

For the three dimensional case the view factor is solved differently compared with the two dimensional case:

$$\phi = \frac{1}{2\pi}\left[\arctan\left(\frac{b}{x-R(t)}\right) - A\cdot\frac{x-R(t)}{b}\cdot\arctan(A)\right] \qquad (95)$$

$$A = \frac{1}{\sqrt{\left(\dfrac{L_f}{b}\right)^2 + \left(\dfrac{x-R(t)}{b}\right)^2}} \qquad (96)$$

Where:
b is the width of the flame (y-direction)

In the calculations of the view factor the flame is assumed to have rectangular shape.

As for the 2-dimensional case the stability of the system will have to be verified. The stability of the 3-dimensional system is verified using the following equation:

$$s_x = s_y = \frac{D\cdot\tau}{h^2} \qquad (97)$$

and the following condition:

$$s_x + s_y \leq 0.5$$

The value of s_x and s_y was found to fulfil the condition above. Thus the scheme is considered as stable.

The following results were received for a flame length of 0.15 m and 0.30 m, respectively using the same values as for the two-dimensional case – the contours marks different temperatures where the red contour marks the 1200 K temperature and thus the flame front (as noted the temperature increases rapidly in the area just adjacent to the flames):
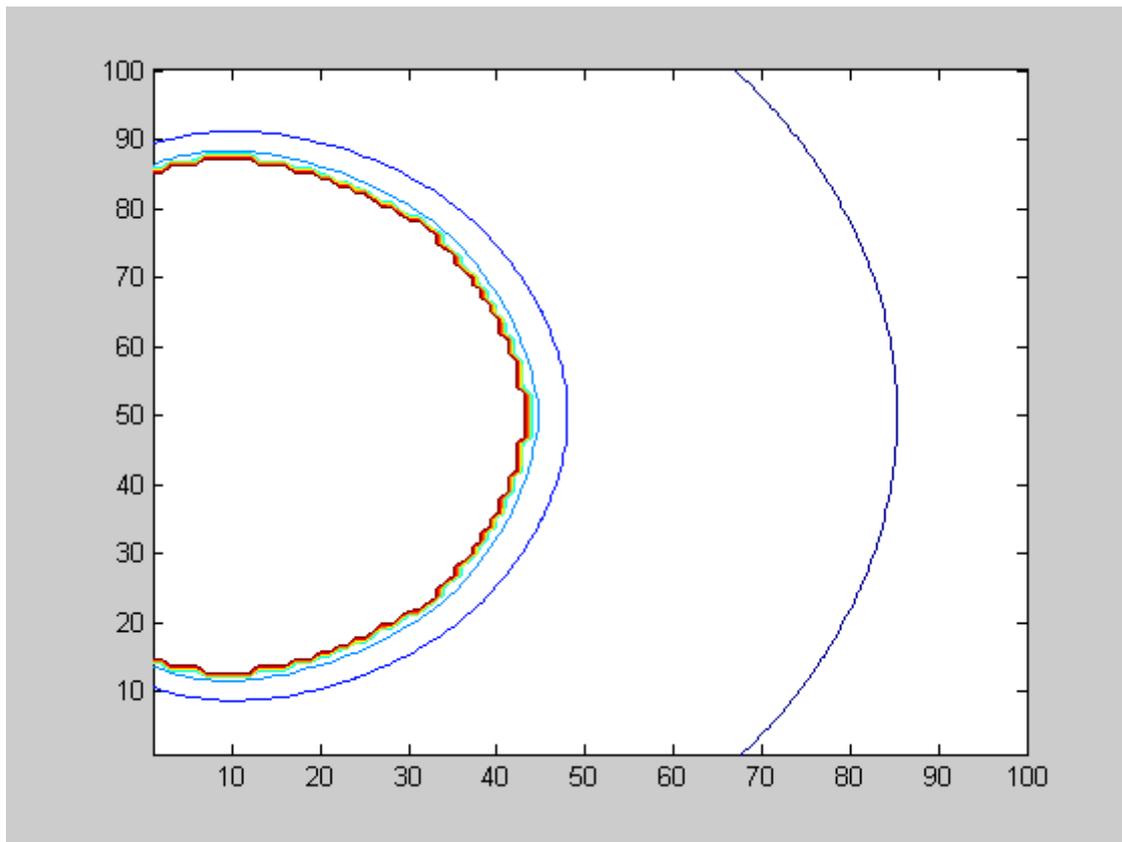
Figure 8. The position of the flame front seen from above (red line) after a time interval of 35 seconds and a flame height of 0.15 m (length scale in figure is 0.1 m).
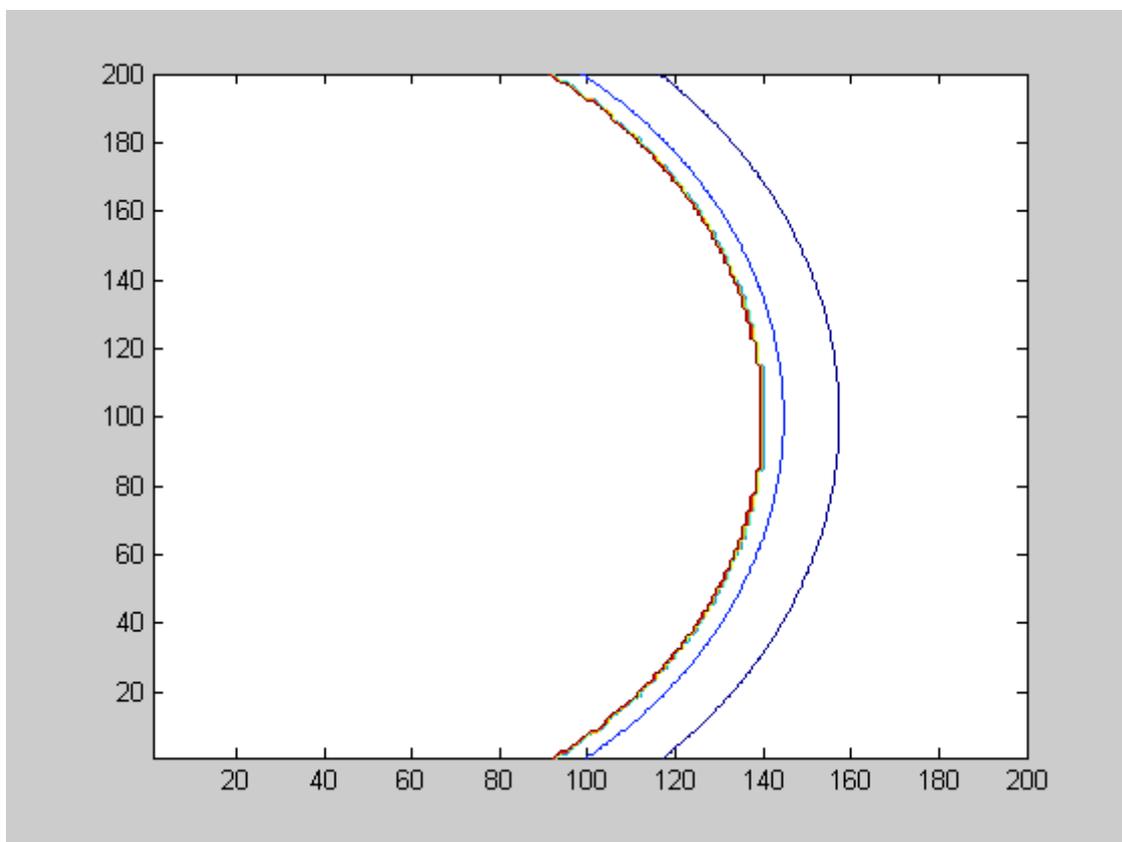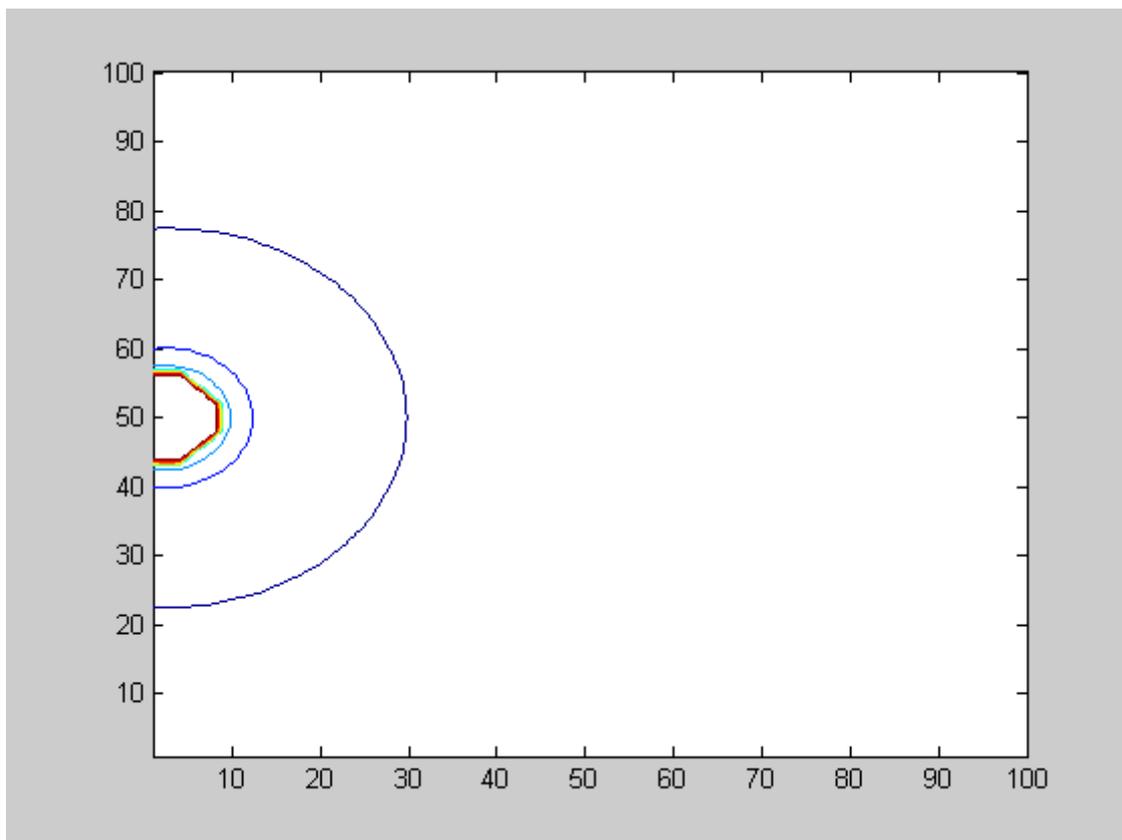
Figure 9. The position of the flame front seen from above (red line) after a time interval of 35 seconds and a flame height of 0.30 m (length scale in figure is 0.1 m).

As noted from the two graphs above, the average rate of spread for a flame height of 0.15 m: ~ 7 m/min and for a flame height of 0.3 m: ~ 23 m/min.

In order to investigate if the above figures correspond well to measured values from test burnings (i.e. to validate the model), measured values from test burnings conducted in northern part of Sweden /43/ were obtained. The test burnings were conducted in terrain sections of 30x30 m in size and with almost no inclination. The following rate of spread was measured at the lowest wind speed (0.5 m/s) in lingonberry type vegetation and with a maximum flame height of 0.8 m: 0.8-4.6 m/min.

When calculating the flame height, Byram's equation /15/ was used as we are focusing on surface fires in this case.

As noted the values of the model clearly overestimates the rate of spread compared with the measured value.

Obviously one of the major drawbacks of the proposed model, is the lack of fuel specific values for Swedish conditions. One approach when validating the values of rate of spread would be to vary the specific heat of the fuel, by doing so some consideration would be taken to Swedish conditions.

5.2.1 Varying the specific heat of the fuel

One approach to account for some of the effects of the moisture content on the forest fuel would be to vary the specific heat of the forest fuel, as the specific heat is a function of the moisture content and the temperature.

Figure 10 describes the specific heat of wood at $100^{\circ}C$ as a function of the moisture content.
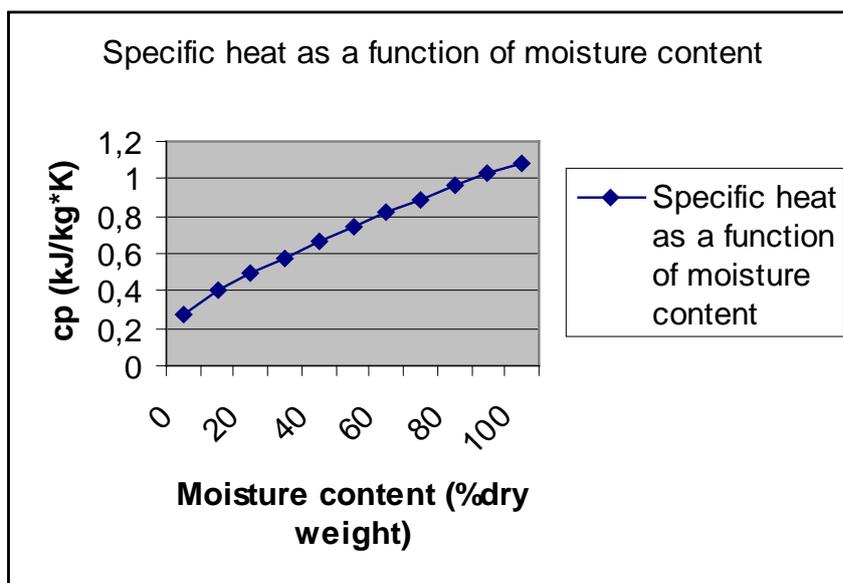
Figure 10. The specific heat of wood (at 100ºC) as a function of the moisture content /5/.

The next step would be to increase the specific heat - using a flame length of 0.8 m - until the simulated rate of spread is equal to the lower limit of the listed interval above (~0.8 m/min) after the initial 60 seconds, as I assumed that the rate of spread will increase as a function of the width of the flame front until steady state is reached. When doing so I found out that the rate of spread (simulated) came close to the upper part of the rate of spread interval (measured) when the specific heat was set to ~13.4 kJ/kg·K. See figure 11.

The value 13.4 kJ/kg·K is clearly not a realistic value as it is too large, a conclusion here is that the error does not solely rest upon the specific heat but on other input parameters as well. As I did not have access to other values that are better fitted for Swedish conditions, I decided to introduce a calibration factor instead of changing the value of the specific heat. This will make the proposed model more semi-empirical but it also demonstrates the difficulty in obtaining adequately accurate input data for a physical model.

Figure 11. The position of the flame front seen from above (red line) after a time interval of 60 seconds and a flame height of 0.80 m (length scale in figure is 0.1 m).

5.2.2 The rate of spread over time

The rate of spread is found to be steady state after approximately 5-10 minutes at a grassfire. In more heavy fuel the time to achieve steady state will obviously be longer /5/.

In order to investigate if the rate of spread was consistent over time, a series of simulations were conducted where a flame height of 0.80 m was used. See appendix 1 showing the simulation results with 1 minute interval.

As the computational time - for a flame height of 0.8 m - very quickly became extremely long I was unable to investigate a possible consistency in rate of spread over time for a flame height of 0.8 m. Instead I decided to verify the consistency using a flame height of 0.1 m. A much smaller flame height will require much smaller computational time. The simulation result over a 10 minute time period is shown in appendix 1. From the result of the simulations, the following graph – expressing the rate of spread as a function of time - was calculated:
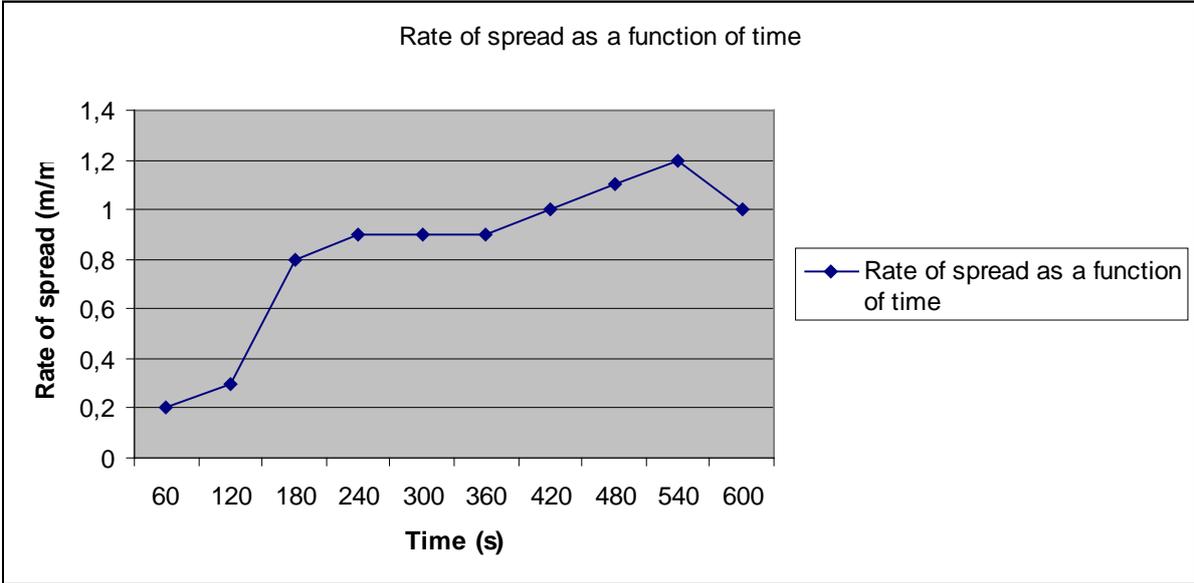
Figure 12. The rate of spread as a function of time.

In figure 12 an accelerating phase is noted at the beginning of the fire and then the rate of spread levels out, thus the result of the simulation shows a consistent rate of spread over time.

5.2.3 Flame tilt due to wind

In order to be able to model the effect of a flame tilt due to wind and be able to validate the model against more cases, the model described in the article *A model for the wind-blown flame from a line fire* by Albini was used /44/. Assuming a constant windspeed, the following equation is used in order to calculate the windspeed at a certain height above the fuelbed (z<10.76 inches):

$$u_a = u \cdot u_a^*$$
(98)

$$u_a^* = 1 + 0.3 \cdot \ln(z/10.76) \tag{99}$$

Where:
u is the windspeed at a height more than 10.76 inches above the ground [feet/min]
u_a^* is the normalized horizontal windspeed
z is the height above the top of the fuelbed [inch]
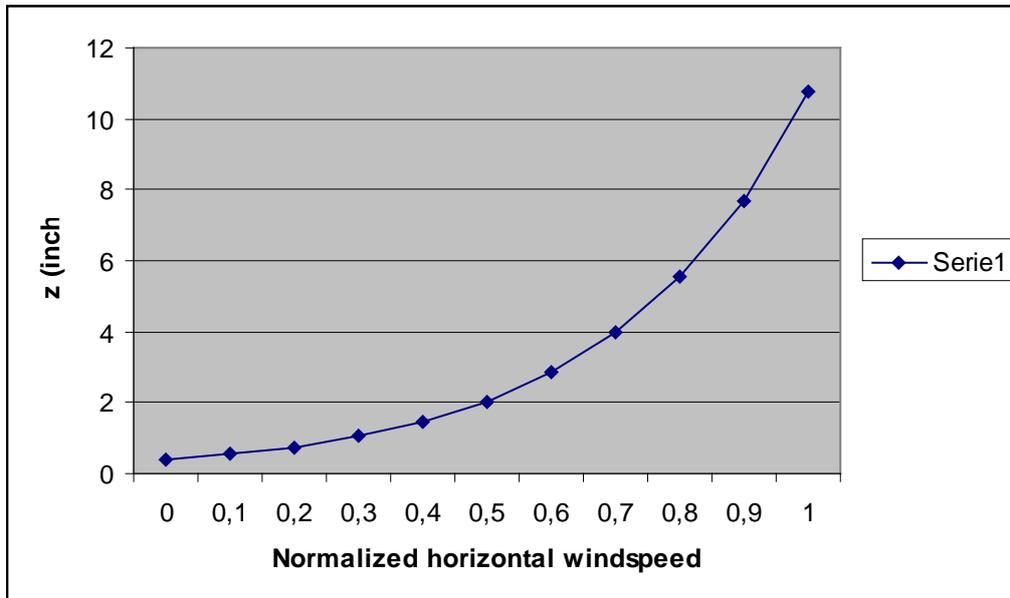
Figure 13. Windspeed profile using the normalized horizontal windspeed /44/.

The calculated windspeed is then inserted in the equation below in order to calculate the flame tilt angle:

$$\tan^2 \alpha = \left(\frac{3}{2}\right) \cdot \frac{u_a^2}{g \cdot H} \tag{100}$$

Where:
α is the flame tilt angle
H is the height of flame tip

The flame tilt angle is then used in the calculations of the view factor in order to account for the influence of wind. The flame tilt angle is also used in order to account for the influence of slope, as the influence of wind and slope on the fire will basically be the same (tilting the flame with respect to the fuel bed).

$$\phi = \frac{1}{2\pi}\left[\arctan\left(\frac{1}{L}\right) + V \cdot (N \cdot \cos\alpha - \alpha \cdot L) \cdot \arctan(V) + \frac{\cos\alpha}{W} \cdot \left[\arctan\left(\frac{N - L \cdot \cos\alpha}{W}\right) + \arctan\left(\frac{L \cdot \cos\alpha}{W}\right)\right]\right] \tag{101}$$

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$$N = \frac{a}{b} \qquad (102)$$

$$L = \frac{c}{b} \qquad (103)$$

$$V = \frac{1}{\sqrt{N^2 + L^2 - 2 \cdot N \cdot L \cdot \cos\alpha}} \qquad (104)$$

$$W = \sqrt{1 + L^2 \cdot \sin^2\alpha} \qquad (105)$$

Where:
$a = L_f$
b is the width of the flame
c is the distance between the flame and the fuel cell

Two cases from the test burnings in northern Sweden /43/ were compared with the results of the simulations in order to investigate the influence of the flame tilt. In both cases the fuel was lingonberry type vegetation in order to keep the fuel type consistent with the earlier simulations.
In the first case the windspeed at 1.5 m height was measured to 1.1 m/s, the maximum flame length was ~1.0 m and the rate of spread was in the interval 1.2-4.1 m/min.
In the second case the windspeed at 1.5 m height was measured to 1.8 m/s, the maximum flame length was ~1.2 m and the rate of spread was in the interval 1.1-2.2 m/min.
In appendix 2 the resulting simulations of the first case are shown for the first two minutes and in appendix 3 the resulting simulation of the second case is shown. Unfortunately, the computational time for the second case at t:120 seconds were extremely long (>20 days) and due to time constraints I decided to stop that simulation.

From the results the following conclusions can be drawn:
- The larger tilt angle, the larger rate of spread.
- The proposed model clearly overpredicts the rate of spread with an increase in tilt angle.

One reason for the high rate of spread is the fact that I have used the view factor for the tilting case in all directions, obviously it only applies in the wind direction. The shape of the wildfire should be an ellipse instead of a circle if being influenced by wind. Further improvements could thus be made to the program.

5.2.4 Varying the fireline intensity

As the flame height corresponds directly with the fireline intensity, the flame height was varied in order to investigate the effect of the fireline intensity on the rate of

spread. The following flame heights were examined: 0.3 m, 0.6 m and 0.9 m. See appendix 4 for the resulting output.

From the output one can see that with an increase in flame height the rate of spread will also increase. Together with the results of the tilt angle influence, a conclusion is thus that the flame radiation is among the main driving forces with respect to the rate of spread.

This is true as long as the angle between the flame and the fuel ahead is below 30 degrees, as the flame in that case does not lift from the fuel /5/. Convection is a dominating force in this case.

When using larger flame lengths, a wider flame front is required in order to attain a constant rate of spread (a constant rate of spread will thus be achieved later in time). This is due to the fact that larger flame lengths will result in larger view factors /5/.

The major drawback of the model is the fact that the flame height – i.e. the fireline intensity – is assumed by the user and is not varied during the simulation. This is clearly not realistic and will have to be worked on when developing the model. A much better approach would be to let the model calculate the flame height, based upon input parameters such as the fuel, wind velocity etc.

5.2.5 Computational time

An Intel(R) Pentium(R) M processor 2.00 GHz, 1.00 GB RAM was used during the simulations.

As an indication of the extent of the computational time, the simulation of the second windspeed case (windspeed at 1.8 m/s and a flame length of 1.2 m) took approximately 3 hours for the first 60 seconds using a grid size of 200x200.

For the first 120 seconds and a grid size of 500x500, the simulation took approximately 90 hours.

Clearly the required computational time indicates that the model is not presently of operational use due to the very long time periods.

6. Analysis and discussion

The statistical models make no attempt to involve physical processes, as they are merely a statistical description of test fires. Thus the lack of a physical basis means that statistical models must be used carefully outside the test conditions.
Another two disadvantages of statistical models is the large number of fires required to develop the necessary number of homogenous sets and the fact that non-linear relationships among the variables are lost in the necessary linearization process.
A major advantage using the statistical model FBP would be the close link to the FWI system. Several sub-index values from the FWI system can be used for calculations in the FBP system, thus if using the FBP system it will require less effort when collecting the necessary input data.
Another advantage would be the low computational requirements of statistical models.

Semi-empirical models are often based on conservation of energy principles but do not make any difference between conduction, convection and radiation heat transfer. The models also lack the flame-fuel interactions and the transition from ground fire to crown fire can not be addressed with this type of model.
The semi-empirical model has low computational requirements and includes variables that are generally easy to measure in the field. So despite the issue with limited accuracy, the speed and simplicity of these models make them useful for operational use.
The Rothermel model is an example of a semi-empirical model that has been used worldwide during several decades, providing valuable decision support at several wildfires. The Rothermel model could very well serve as an example for a Swedish model.

Physical models have the advantage that they are based on known relationships and thus facilitating their scaling. Thus we can expect that physical models would provide the most accurate predictions and have the widest applicability.
But the work on physical models is suffering of for example the lack of understanding of several processes, such as the characterization of the chemical processes taking place during combustion, the resulting flame characteristics and the isolation and quantification of physical processes governing heat transfer. Also the accuracy of a physical model would rely heavily upon the accuracy of the input data, the accuracy and quality of the input data of today is not at an acceptable level. Thus a physical model of operational use is not a reality at the moment.
An idea would be to use a physical model as a basis but set some of the coefficients to zero, making it more of a semi-empirical type of model. As more accurate input data becomes available, the included physical mechanisms are better understood etc. the earlier "inactivated" parts could be included in the model making it more and more into a physical model.

A reflection here would be that based upon the above descriptions of the three types of models, the semi-empirical model would be optimal choice for Sweden as for example the statistical models have severe limitations.

The input data available today are generally not detailed enough for physical models. As a result, a very detailed physical model will still only give imprecise predictions. As better and more detailed input will be available, the use of physical models will be more justified.

The input data that is generally not detailed enough are: weather and fuel.

With respect to weather, an ideal solution would be to use available weather data from the existing FWI-system. But the problems with the existing FWI-system are the coarse grid size and the fact that it is only updated once a day. Thus the question is how well the system is taking the temporal and spatial variations into account.

The project "Alarm" could very well provide the solution on this problem. Providing weather input data that is updated once every hour from local weather stations.

Regarding the fuel, the use of satellite images and aerial photos will greatly simplify the process and save time and resources. But there is still a great demand for new technology for satellite images or aerial photos and image classification techniques, as for example one major problem to be solved is the distinguishing between canopy fuel and the ground fuel. New technology is showing promising results.

With respect to the proposed physical model, even after an attempt to calibrate the model it clearly overestimated the rate of spread compared with measured values. Most probable reasons for the unsuccesful attempt to estimate the rate of spread are for example:

- General lack of accurate and useful input data.
- Model is in some aspects oversimplified (no reaction chemistry involved for example).

A major drawback of the model is the fact that the flame height – i.e. the fireline intensity – is assumed by the user and is not varied during the simulation. This is clearly not realistic and will have to worked on when developing the model.

Further more even for very small terrain sections the computational time for the proposed model became incredible long, thus the proposed model is presently not of operational use.

Further work is needed on the model to make it more accurate and of operational use.

7. Conclusions

7.1 Conclusions

A semi-empirical model is recommended being developed in Sweden. This conclusion is based upon the following factors:
- The accuracy of a semi-empirical model is generally much better than for a statistical model, also the use of a semi-empirical model is much wider than the use of a statistical model.
- The amount of work required for developing a semi-empirical model will not differ much from the amount of work required for a statistical model. In both cases a number of test fires will have to be conducted to define and calibrate a number of fuel models representative of Sweden.
- Presently the performance and application of physical models is not at an acceptable level (due to for example the complexity which they are to model and the computational capabilities of the PC's of today) for operational use.

The semi-empirical model for Sweden is recommended to be built upon Swedish conditions (i.e. built upon the type of vegetation found in Sweden) instead of trying to retrofit the local Swedish conditions into an existing model. This would most likely give the best output for Swedish conditions.

The semi-empirical model could include two sub-models: one model that predicts the rate of spread of a ground fire and the other model would predict the rate of spread of a possible crown fire. The link between the two sub-models should be based upon physical relationships, thus making it more into a physical model.

In parallell with the work on a semi-empirical model, a system for better input data - weather and fuel data – should be worked on as well. This could for example result in more the provision of more localized weather data, updated regularly.

The work on the proposed physical model clearly underlined the need for accurate and useful input data but also one of the major drawbacks of physical models: the very long computational times.

7.2 Further work

If a wildfire model is developed, it is recommended that it is developed in cooperation with corresponding organizations in countries such as for example Norway and Finland. This will be most cost efficient and will not affect the outcome as the vegetation is similar.

A working group should be established including expertise in fire science, ecology, GIS etc. Besides working on potential fuel types, modeling the physical relationships etc. an important part of the work will include the test burnings.
As the development of the model will most likely require a large number of test burnings, it will most likely take several years before the final product will be ready

for operational use. A system for monitoring wildfires – that are not test burnings – are important to update and improve the future wildfire model. Thus in order to obtain input data for calibration and correction of a future Swedish model, vital input data should be included in the swedish incident report form. The work on collecting the input data should start well ahead of the development of the model in order to include temporal variations during a longer interval.

A system for better input data - weather and fuel data – should be worked on as well. This could for example take advantage of the results of the very promising "Alarm"-project that is being conducted in western part of Sweden.
Using the weather data from the "Alarm"-project would provide more localized weather data, which could be updated regularly.
Regarding the issue on better fuel data, new technology for satellite images or aerial photos and image classification techniques must be monitored as one major problem to be solved is distinguishing between the canopy fuel and the ground fuel.

Even though it is decided to go ahead with the work on a semi-empirical model, the ongoing developments on physical models around the world - especially interesting and promising is the work on FIRETEC and WFDS - should be monitored as increased computer capability, refined input data etc. will some day allow for a physical model to be ready for operational use.

8. References

/1/   Hirsch K.G. (1996). *Canadian Forest Fire Behavior Prediction (FBP) System: user's guide.*
        Vancouver: UBC Press

/2/   Sullivan A.L. (2007). *A review of wildland fire spread modelling, 1990-present. 3: Mathematical analogues and simulation models.*
        Ensis Bushfire Research, Kingston, Australia

/3/   Fons W.L. (1946). *Analysis of fire spread in light forest fuels.*
        Journal of Agricultural Research, volume 72, pp 93-121

/4/   Sullivan A.L. (2007). *A review of wildland fire spread modelling, 1990-present. 1: Physical and quasi-physical models.*
        Ensis Bushfire Research, Kingston, Australia

/5/   Chandler G.C. (1991), *Fire in forestry,* vol.1 & 2, New York, John Wiley & Sons

/6/   Weber R.O. (1991). *Modelling fire spread through fuel beds.*
        Prog. Energy Combustion Science, volume 17, pp 67-82

/7/   Mell W. et al. (2006). *A physics-based approach to modelling grassland fires.*
        Forest Ecology and Management, volume 234, supplement 1, page S82

/8/   Van Wagner C.E. (1977). *Conditions for the start and spread of a crown fire.*
        Canadian Journal of Forest Research, volume 7, pp 23-24

/9/   Beer T. (1991). *Bushfire rate-of-spread forecasting: deterministic and statistical approaches to fire modelling.*
        Journal of Forecasting, volume 10, pp 301-317

/10/   Linn R.R. (1997). *A transport model for prediction of wildfire behavior.*
        PhD thesis, Los Alamos National Laboratory

/11/   Sun R. et al. (2006). *An evaluation of fire-plume properties simulated with the fire dynamics simulator (FDS) and the Clark coupled wildfire model.*
        Canadian Journal of Forest Research, volume 36, pp 2894-2908

/12/   Rothermel R.C. (1972). *A mathematical model for predicting fire spread in wildland fuels.*
        USDA Forest Service, Research paper INT-115

/13/   Frandsen W.H. (1971). *Fire spread through porous fuels from the conservation of energy.*
        Combustion and Flame, volume 16, pp 9-16

/14/ Perry G.L.W. (1998). *Current approaches to modelling the spread of wildland fire: a review.*
Progress in Physical Geography, volume 22, pp 222-245

/15/ Byram G.M. (1959). *Combustion of forest fuels.*
New York, McGraw-Hill Book Co.

/16/ Rothermel R.C. (1991). *Crown fire analysis and interpretation.*
USDA Forest Service

/17/ Cruz M.G. et al. (2002). *Predicting crown fire behavior to support forest fire management decision-making.*
In: Viegas D.X., ed. Forest Fire Research and Wildland Fire Safety.
Rotterdam: Millpress Scientific Publications: 1-10

/18/ Pastor E. et al. (2003), *Mathematical models and calculation systems for the study of wildland fire behaviour.*
Progress in Energy and Combustion Science, vol. 29, pp 139-153

/19/ Finney M.A. (2004). *FARSITE: Fire Area Simulator – Model Development and Evaluation.*
USDA Forest Service, Research paper RMRS-RP-4 Revised

/20/ Cruz M.G. et al. (2004). *Modeling the likelihood of crown fire occurrence in conifer forest stands.*
Forest Science, volume 50, pp 640-658

/21/ Albini F.A. (1986). *Wildland Fire Spread By Radiation – a Model Including Fuel Cooling by Natural Convection.*
Combustion Science and Technology, volume 45, pp 101-113

/22/ Albini F. (1996). *Iterative solution of the radiation transport equations governing spread of fire in wildland fuel.*
Combustion, Explosion and Shock Waves, volume 32, pp 534-543

/23/ Albini F.A. (1994). *Program BURNUP: A simulation model of the burning of a large woody natural fuels.*
Montana State University, Research paper INT-92754-GR

/24/ Clark T.L. et al. (2004). *Description of a coupled atmosphere-fire model.*
International Journal of Wildland Fire, volume 13, pp 49-63

/25/ Grishin A.M. et al. (1983). *Mathematical modelling of the spread of high-level forest fires.*
Soviet Physics Doklady, volume 28, pp 328-330

/26/ Grishin A.M. et al. (2002). *Mathematical Model for Spread of Crown Fires in Homogenous Forests and along Openings.*
Combustion, Explosion, and Shock Waves, volume 38, No 6, pp 622-632

/27/ Balbi et al. (1999). *Dynamic Modelling of Fire Spread Across a Fuel Bed.*
International Journal of Wildland Fire, volume 9, pp 275-284

/28/ Morvan D. et al. (2001). *Modeling of fire spread through a forest fuel bed using a multiphase formulation.*
Combustion and flame, volume 127, pp 1981-1994

/29/ Dupuy J-L. et al. (1999). *Fire spread through a porous forest fuel bed: a radiative and convective model including fire-induced flow effects.*
International Journal of Wildland Fire, volume 9, pp 155-172

/30/ Sun R. et al. (2006). *An evaluation of fire-plume properties simulated with the Fire Dynamics Simulator (FDS) and the Clark couples wildfire model.*
Canadian Journal of Forest Research, volume 36, pp 2894-2908

/31/ Linn R. et al. (2002). *Studying wildfire behavior using FIRETEC.*
International Journal of Wildland Fire, volume 11, pp 233-246

/32/ *Unik spridningsmodell integreras med Rib.* Sirenen. August 2007, pp 6

/33/ Keane R.E. et al. (2001), *Mapping wildland fuels for fire management across multiple scales: Integrating remote sensing, GIS, and biophysical modeling.*
International Journal of Wildland Fire, vol. 10, pp 301-319

/34/ Oswald B.P. et al (1999), *Classifying fuels with aerial photography in east Texas.*
International journal of wildland fire, vol 9, pp 109-113

/35/ Weinstein D. et al. (1995), *Fire growth modeling in an integrated GIS environment.*
ESRI conference paper.

/36/ Wimberly M.C. et al. (2003), *A multivariate approach to mapping forest vegetation and fuels using GISdatabases, satellite imagery, and forest inventory plots.*
2nd International Wildland Fire Ecology and Fire Management Congress

/37/ *The SFPE Handbook of Fire Protection Engineering,* third edition, NFPA, Quincy, 2002

/38/ Larini M. et al. (1998), *Multiphase formulation for fire propagation in heterogeneous combustible media.*
International Journal of Heat and Mass Transfer, vol. 41, pp 881-897

/39/   Sandberg D. (1980), *Duff reduction by prescribed underburning in Douglas-Fir.*
Research Paper PNW-272, USDA

/40/   Sullivan A.L. et al. (2003), *A review of radiant heat flux models used in bushfire applications.*
International Journal of Wildland Fire, vol. 12, pp 101-110

/41/   Zhou X. et al. (2005), *Modeling of marginal burning state of fire spread in live Chaparral shrub fuel bed.*
Combustion and Flame, vol. 143, pp 183-198

/42/   Garcia A. L. (2000), *Numerical Methods for Physics.*
Prentice-Hall, New Jersey

/43/   Granström A. et al. (1998), *Utvärdering av det kanadensiska brandrisksystemet.* Statens Räddningsverk, Karlstad

/44/   Albini F.A. (1981), *A model for the wind-blown flame from a line fire.*
Combustion and Flame, vol. 43, pp 155-174

Appendix 1.
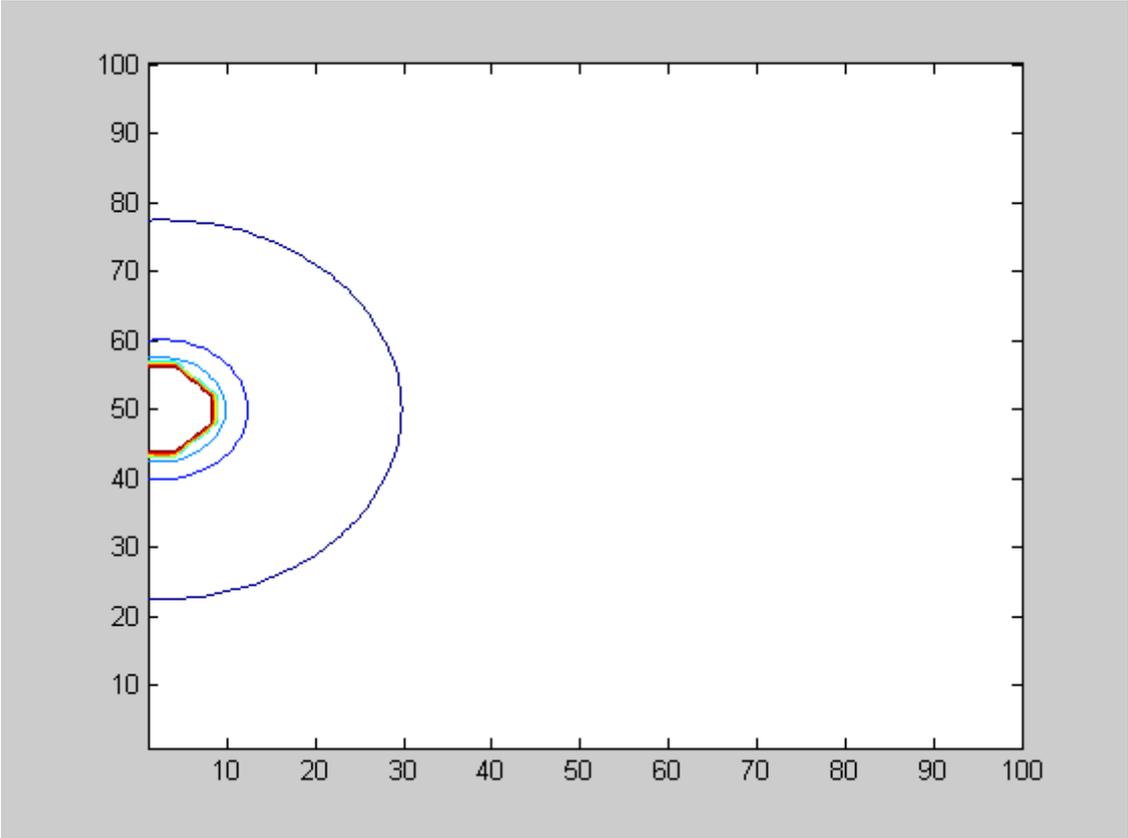
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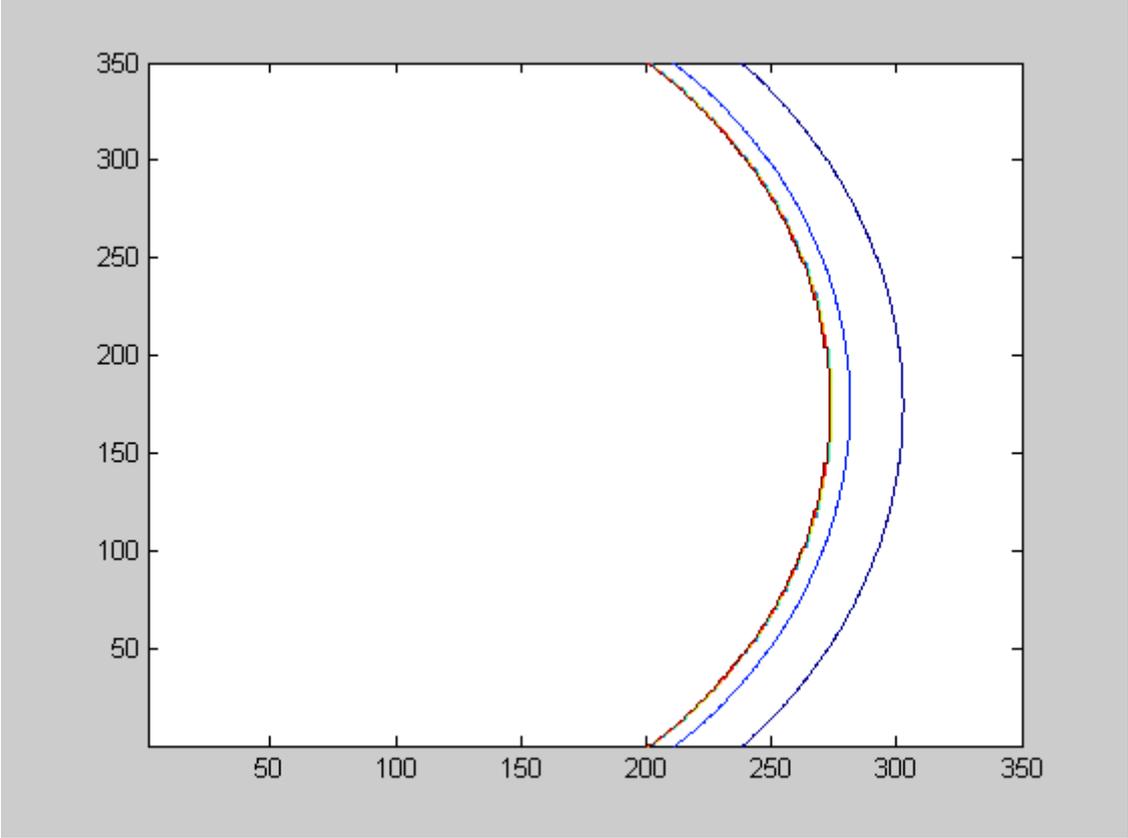
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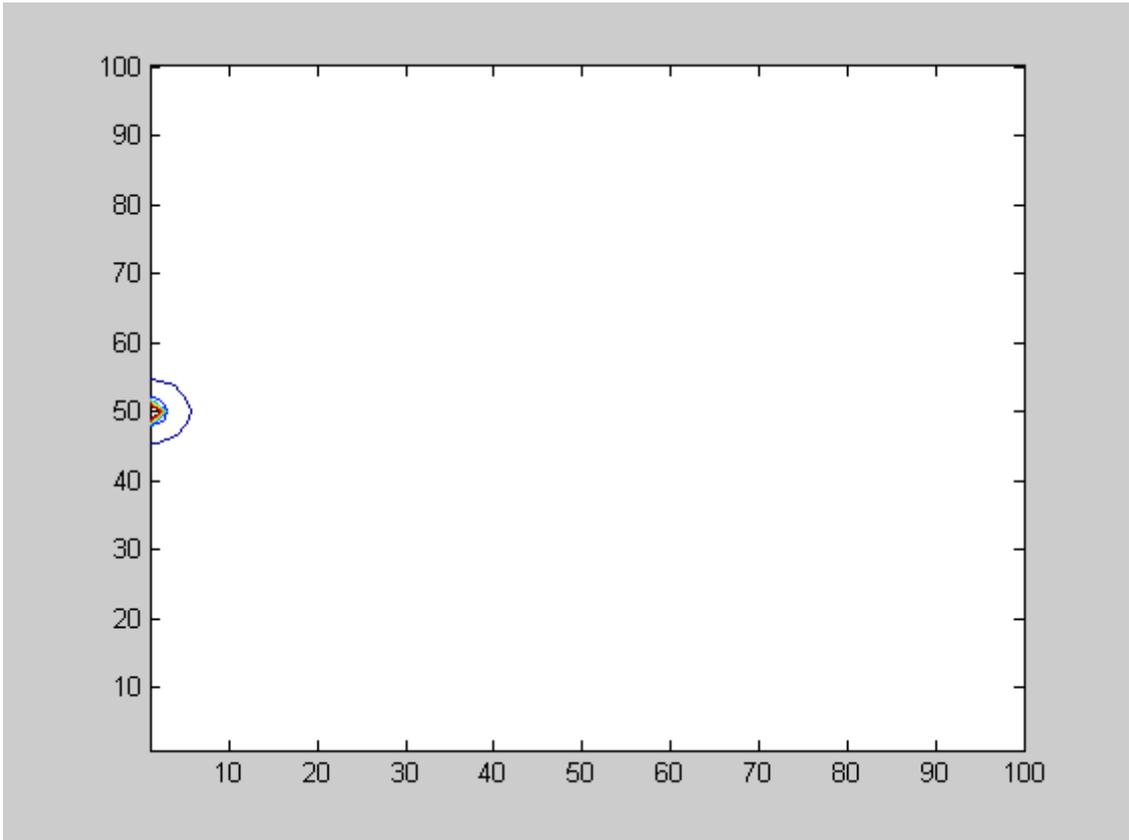
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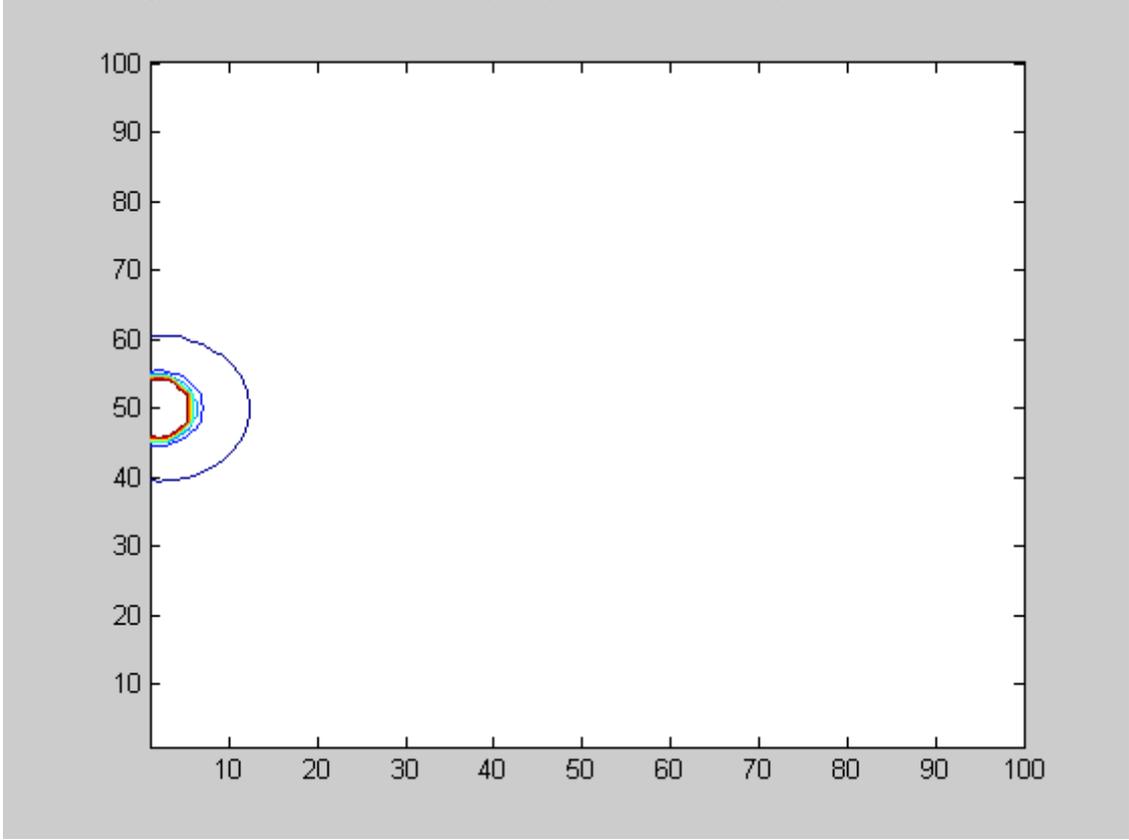
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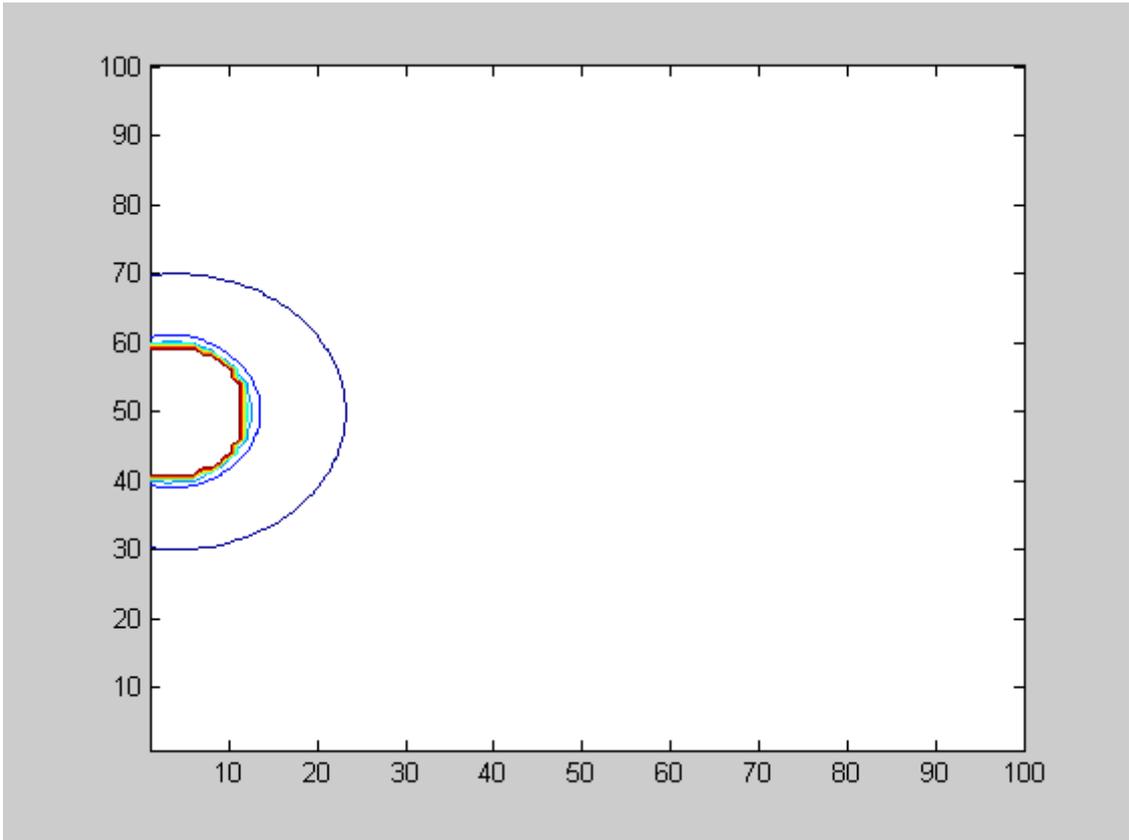
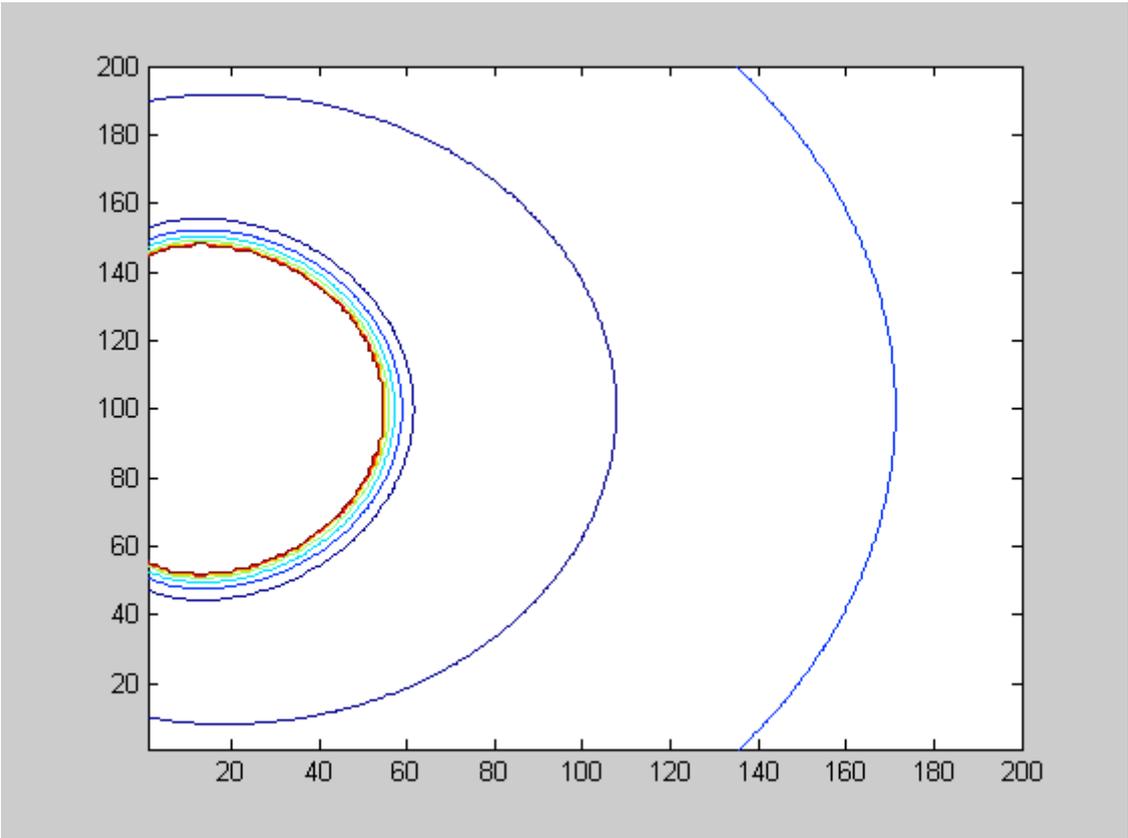Flame height: 0.1 m; t: 180 seconds (length scale is 0.1 m).
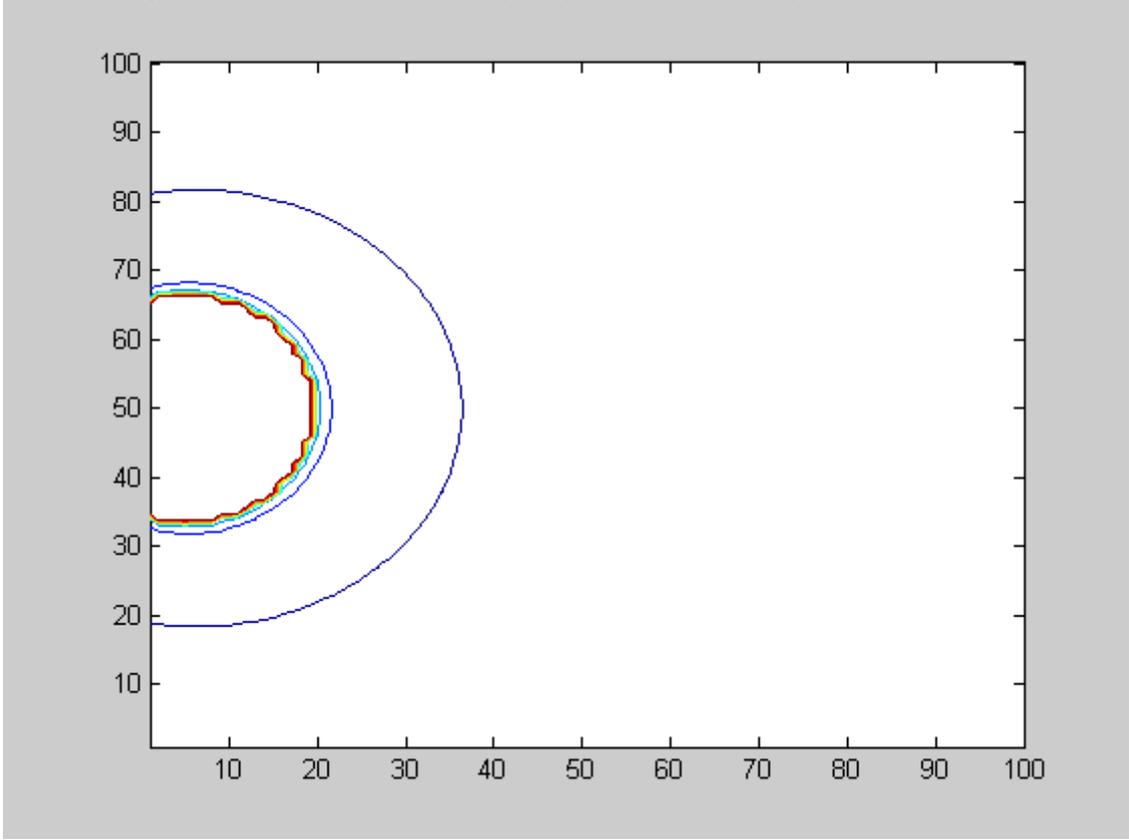
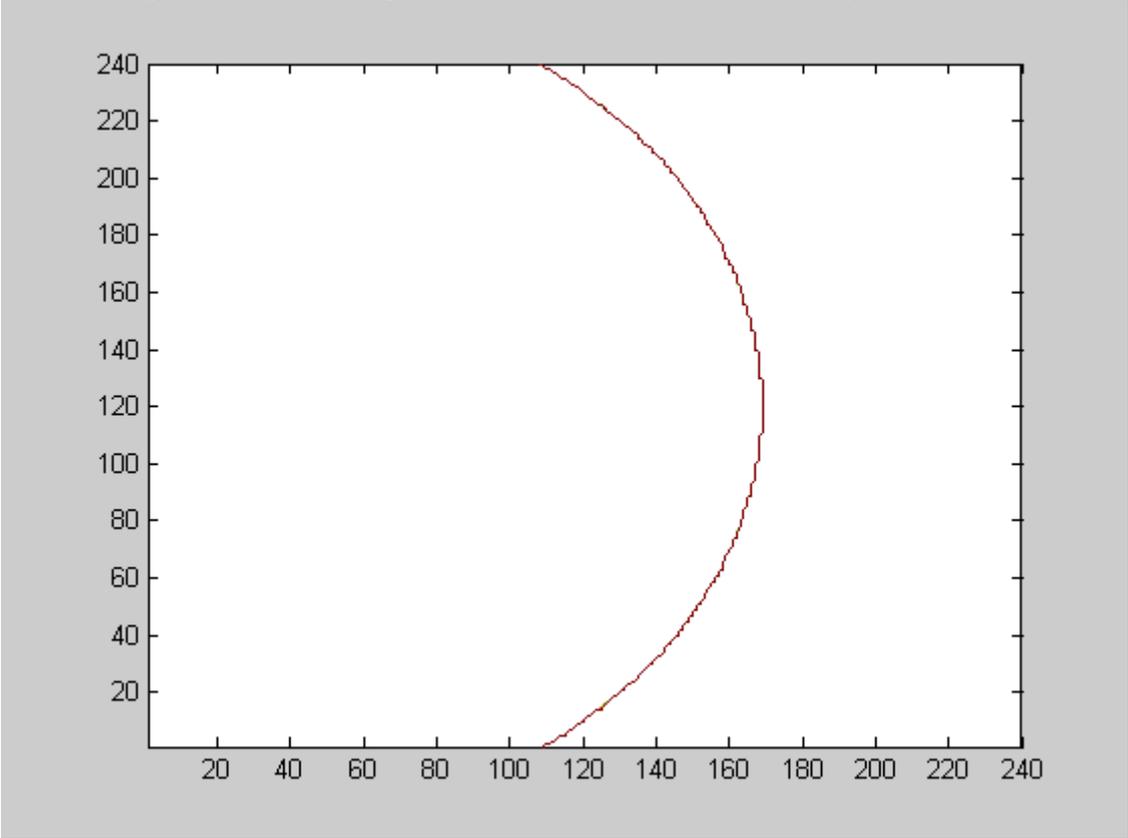Flame height: 0.1 m; t: 240 seconds (length scale is 0.1 m).
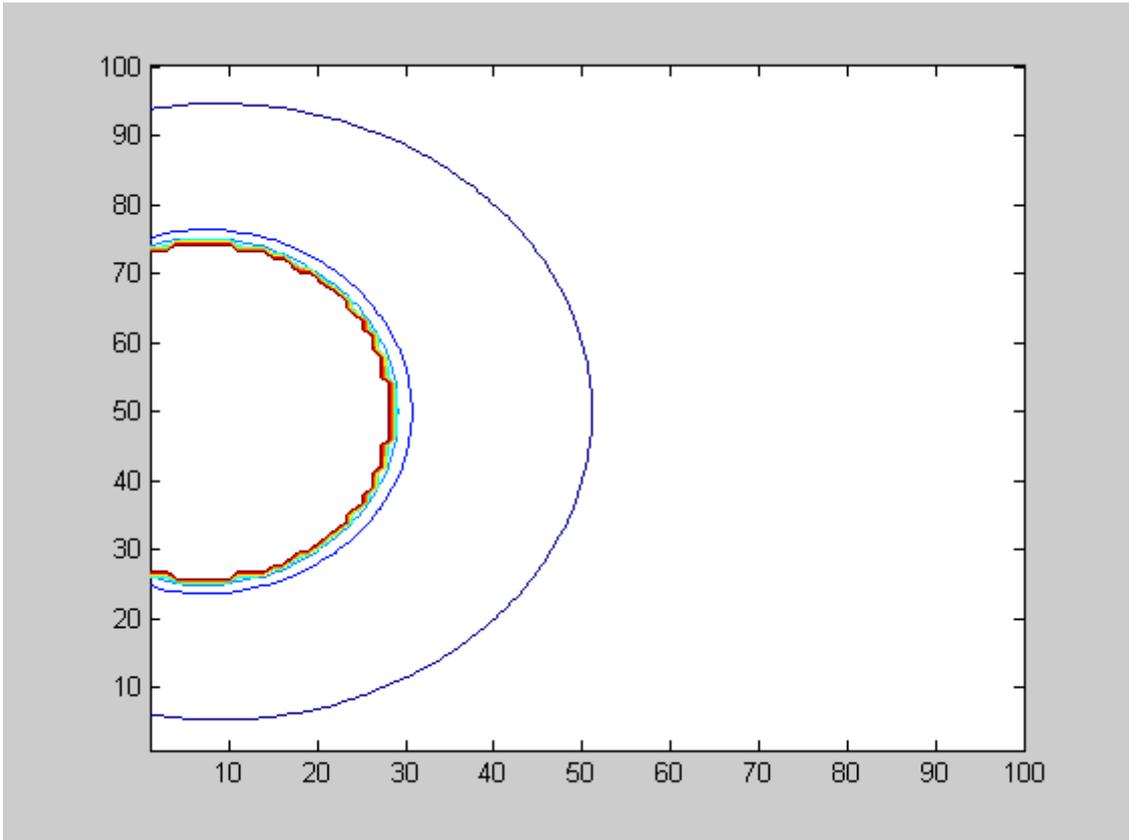
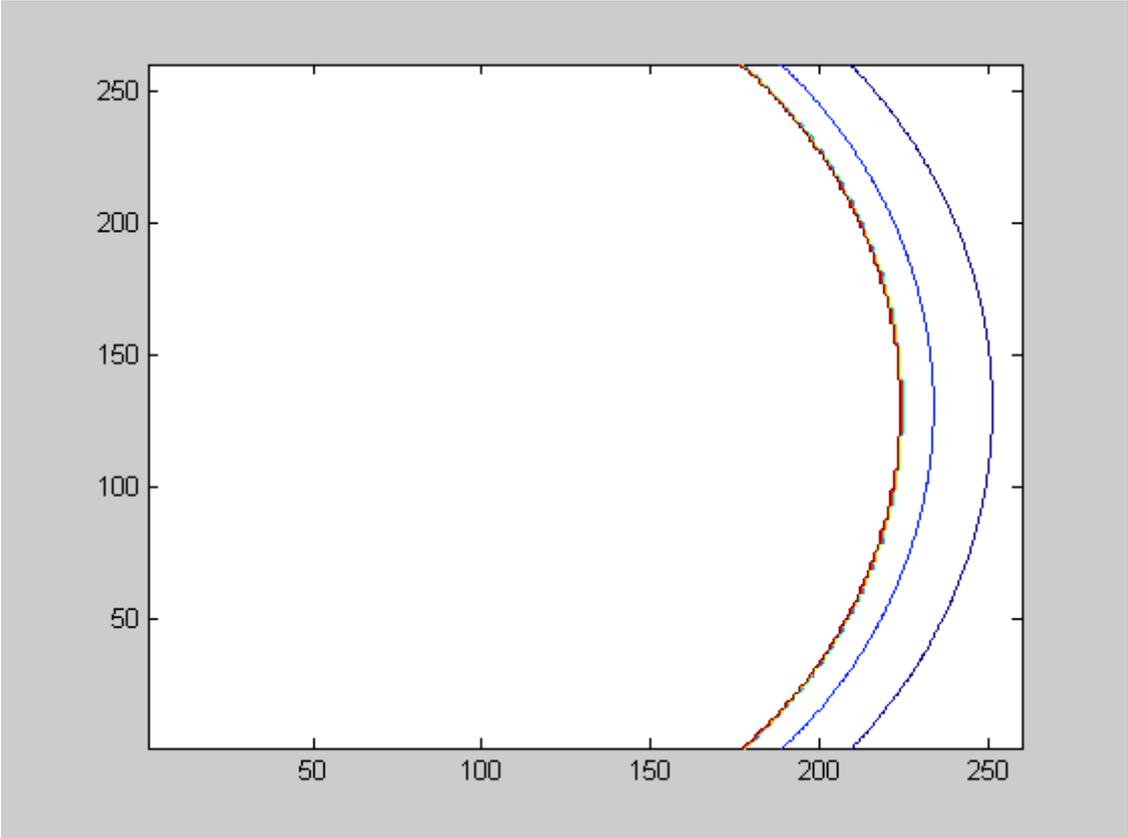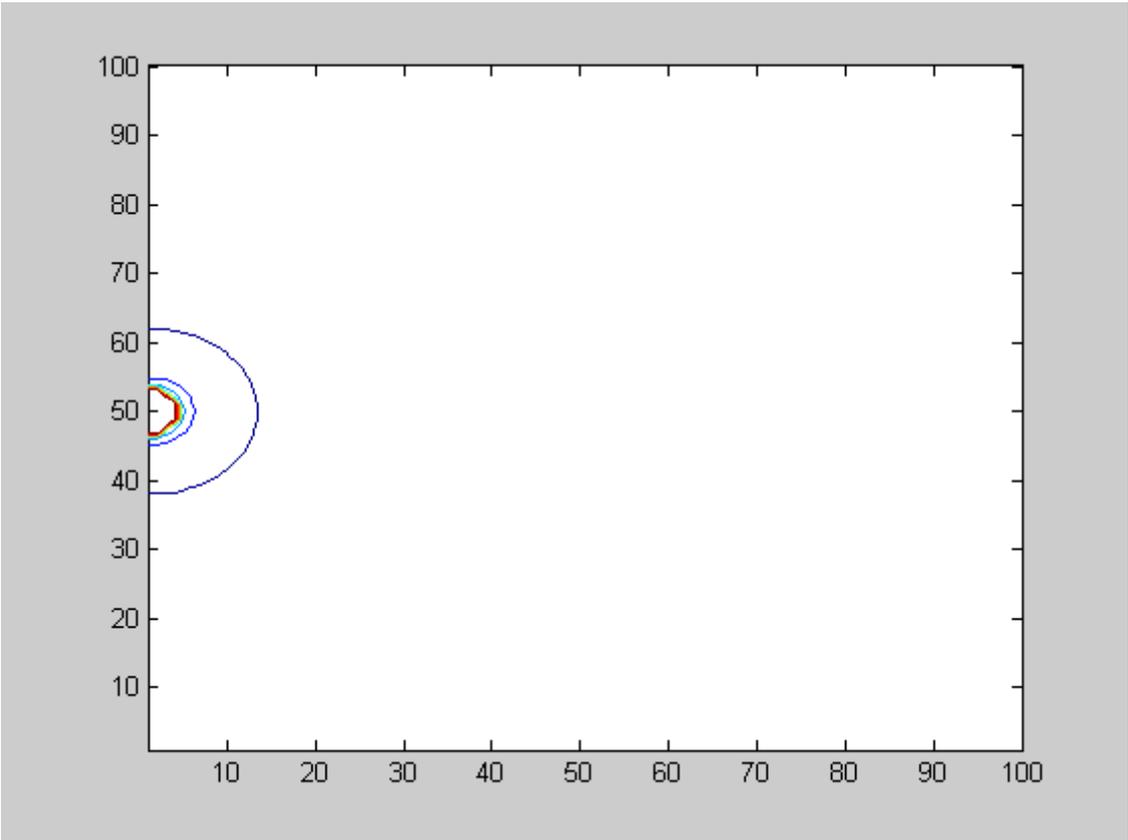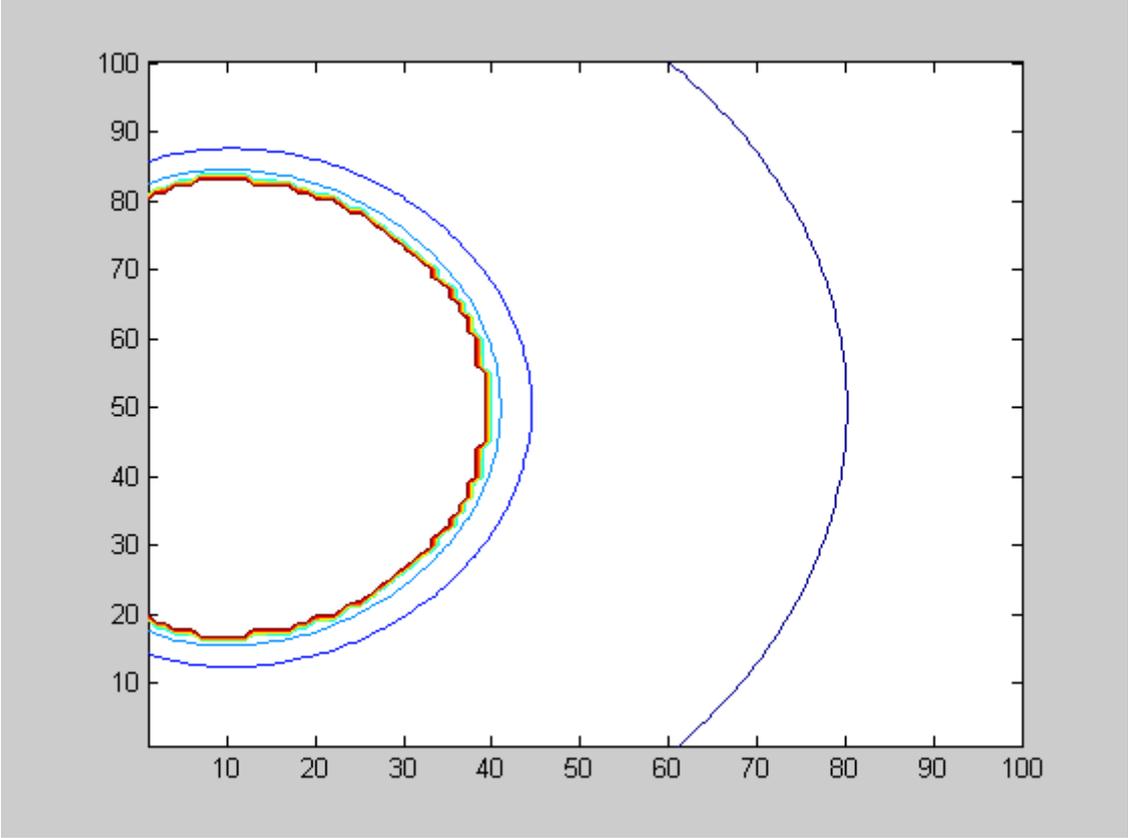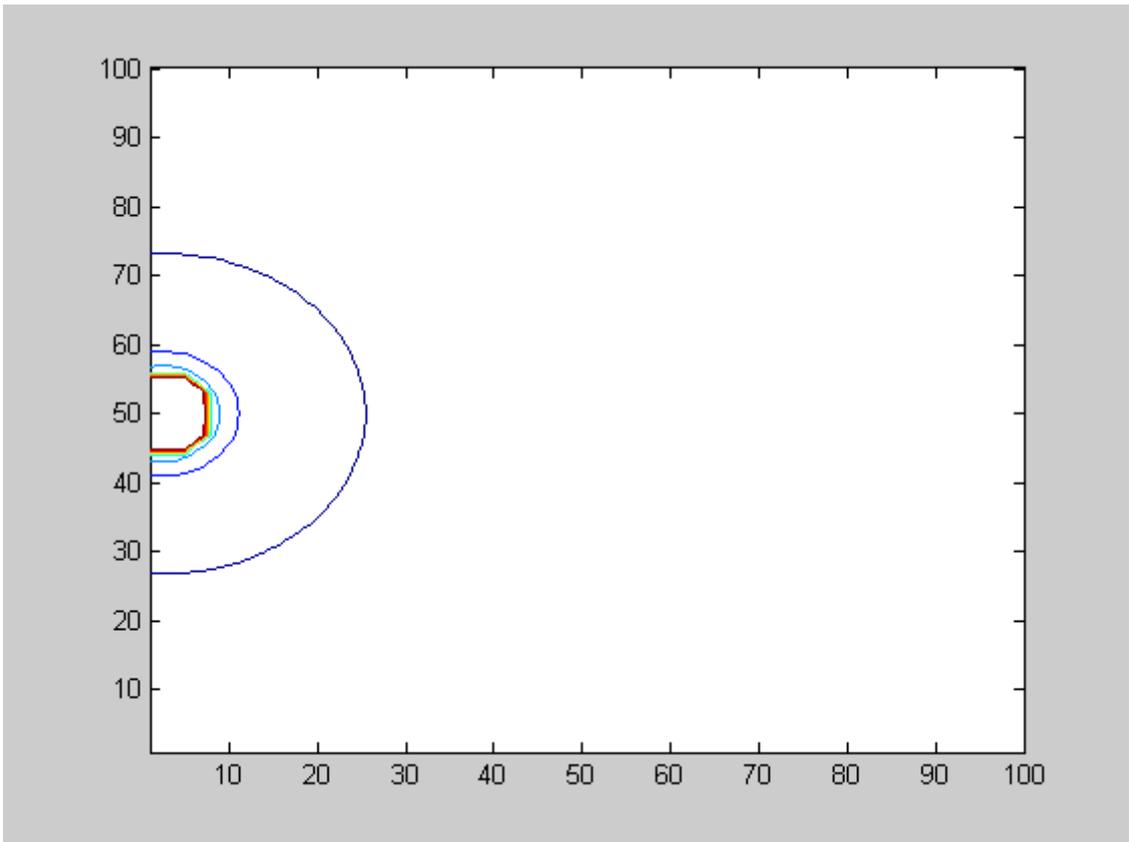Flame height: 0.1 m; t: 300 seconds (length scale is 0.1 m).
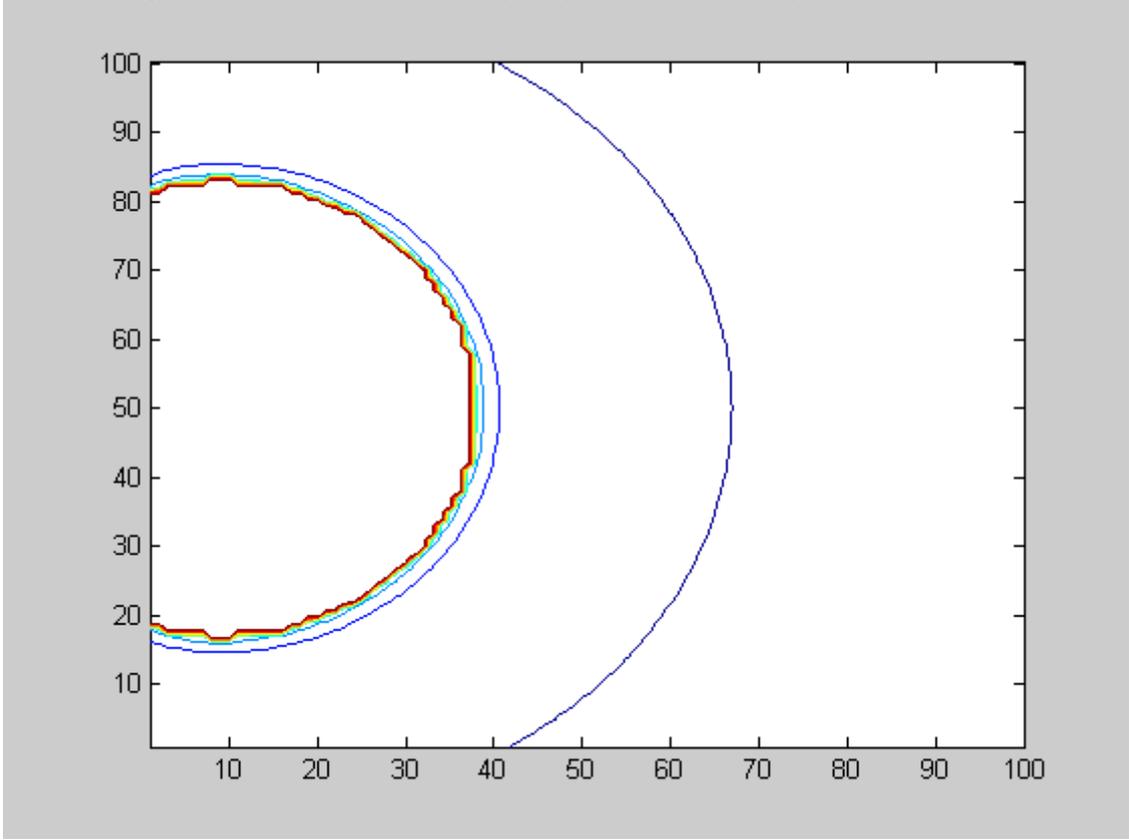
Flame height: 0.1 m; t: 360 seconds (length scale is 0.1 m).
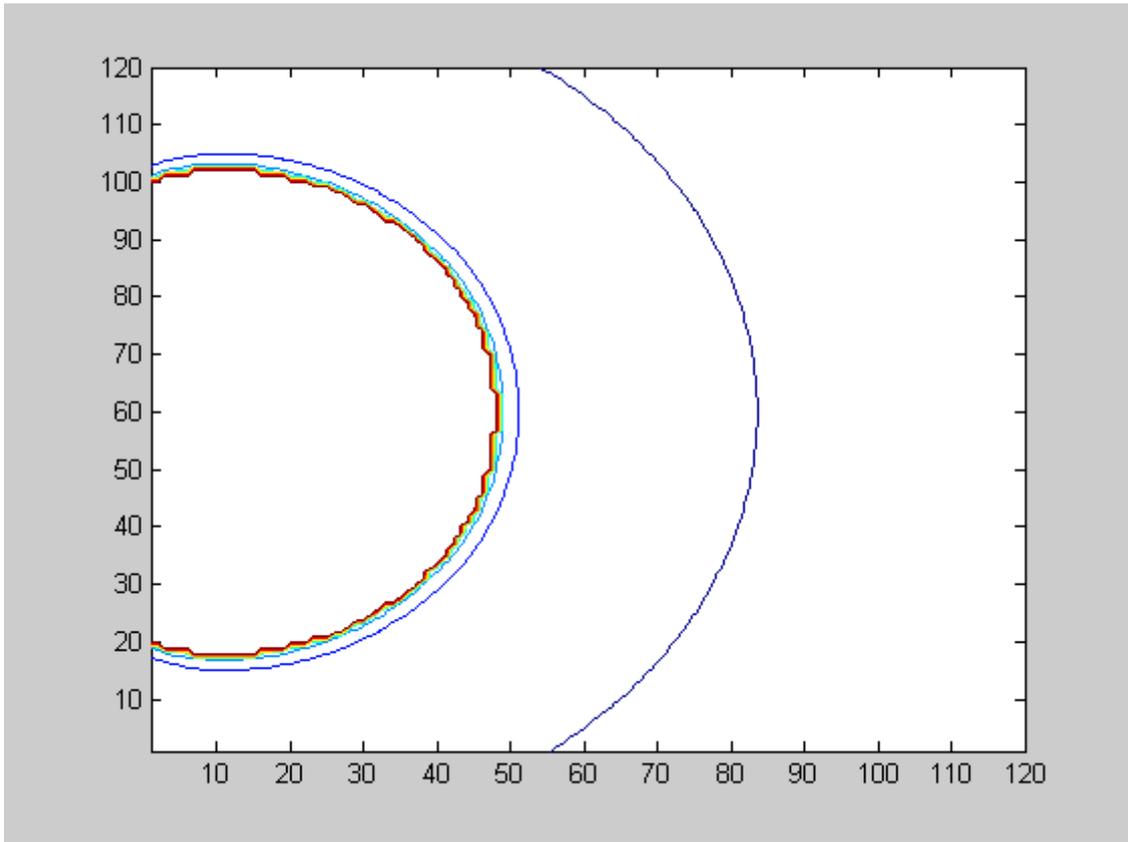
Flame height: 0.1 m; t: 420 seconds (length scale is 0.1 m).
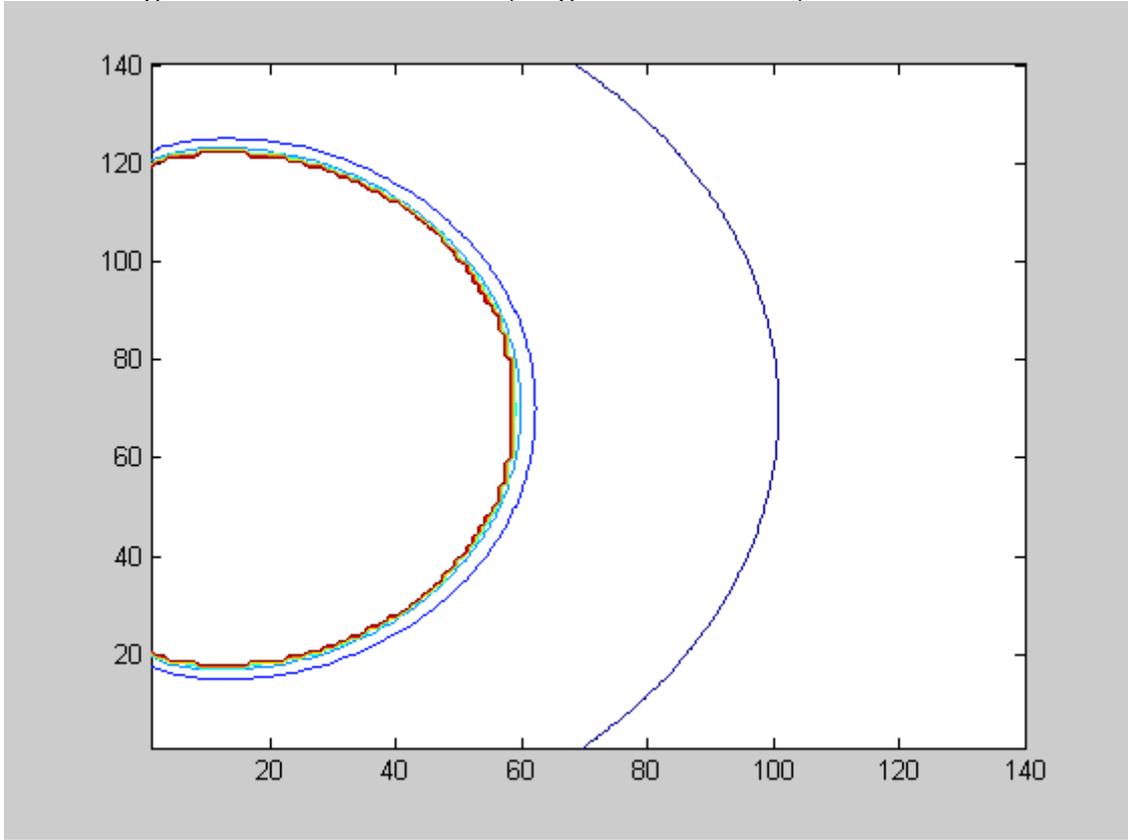
Flame height: 0.1 m; t: 480 seconds (length scale is 0.1 m).
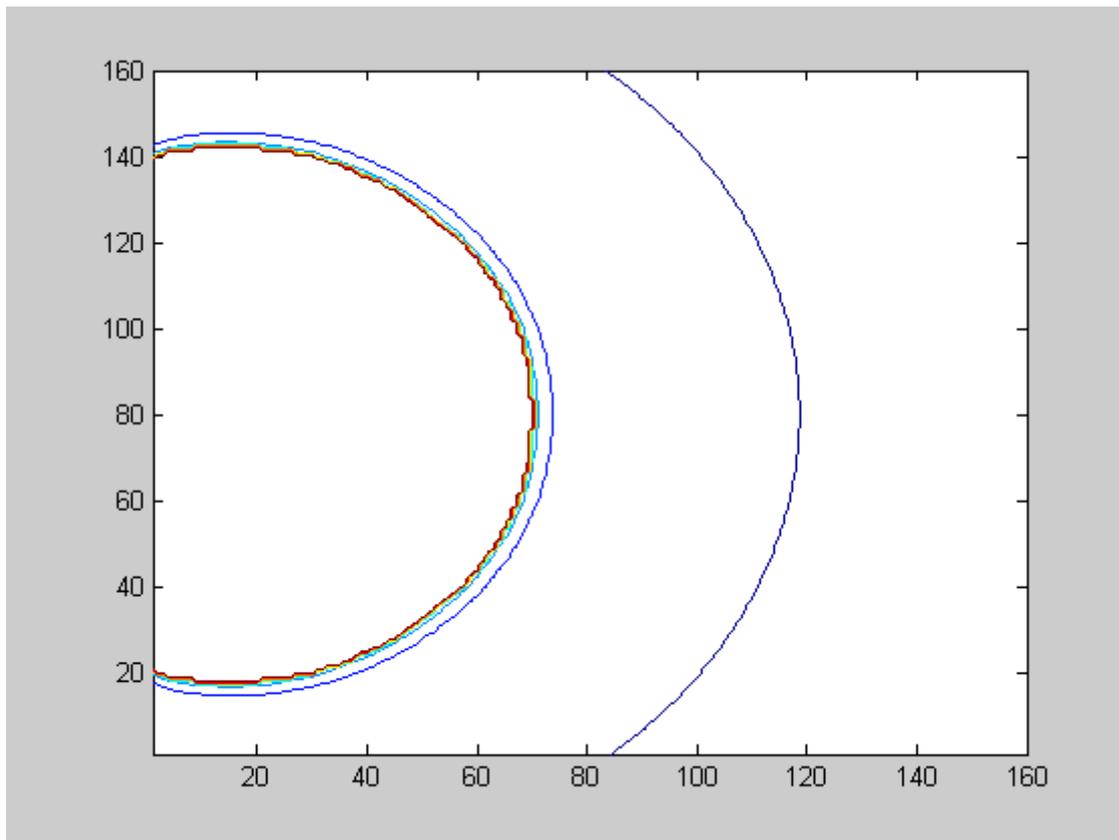
Flame height: 0.1 m; t: 540 seconds (length scale is 0.1 m).
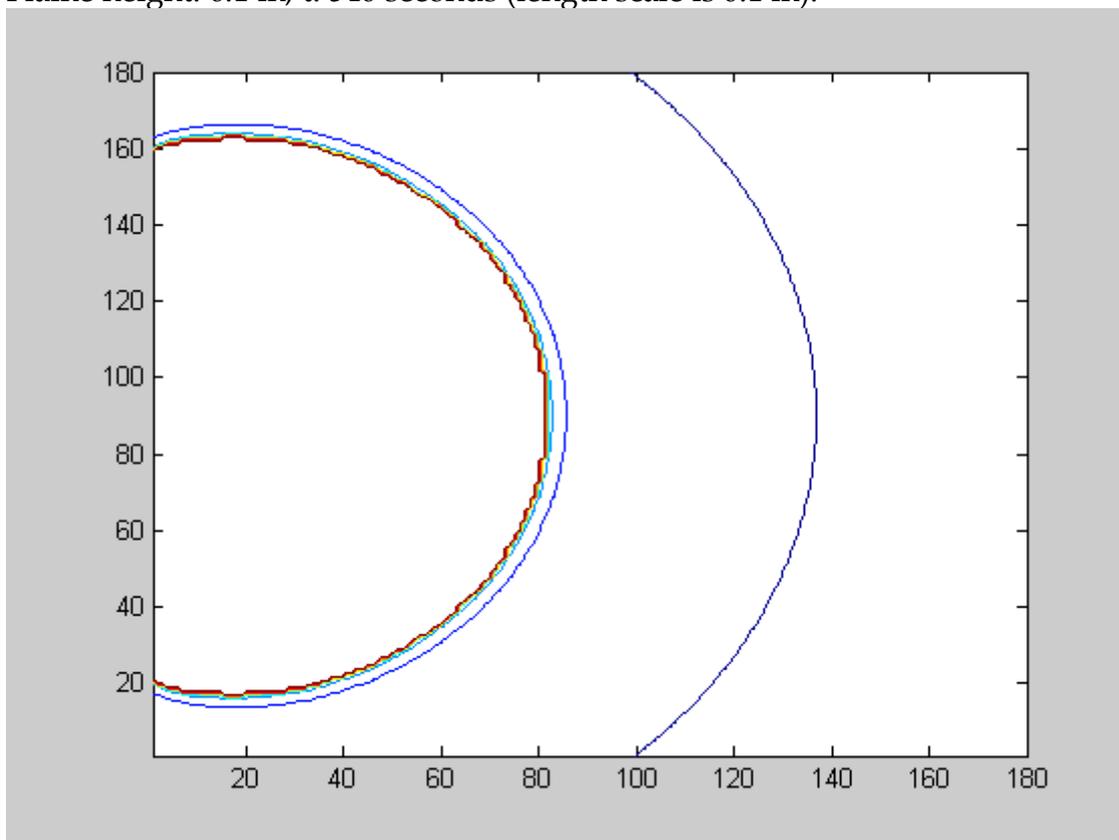
Flame height: 0.1 m; t: 600 seconds (length scale is 0.1 m).

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Appendix 2.

Flame height: 1.0 m; windspeed: 1.1 m/s; t: 60 seconds (length scale is 0.1 m).

t: 120 seconds (length scale is 0.1 m).

Appendix 3.

Flame height: 1.2 m; windspeed: 1.8 m/s; t: 60 seconds (length scale is 0.1 m).
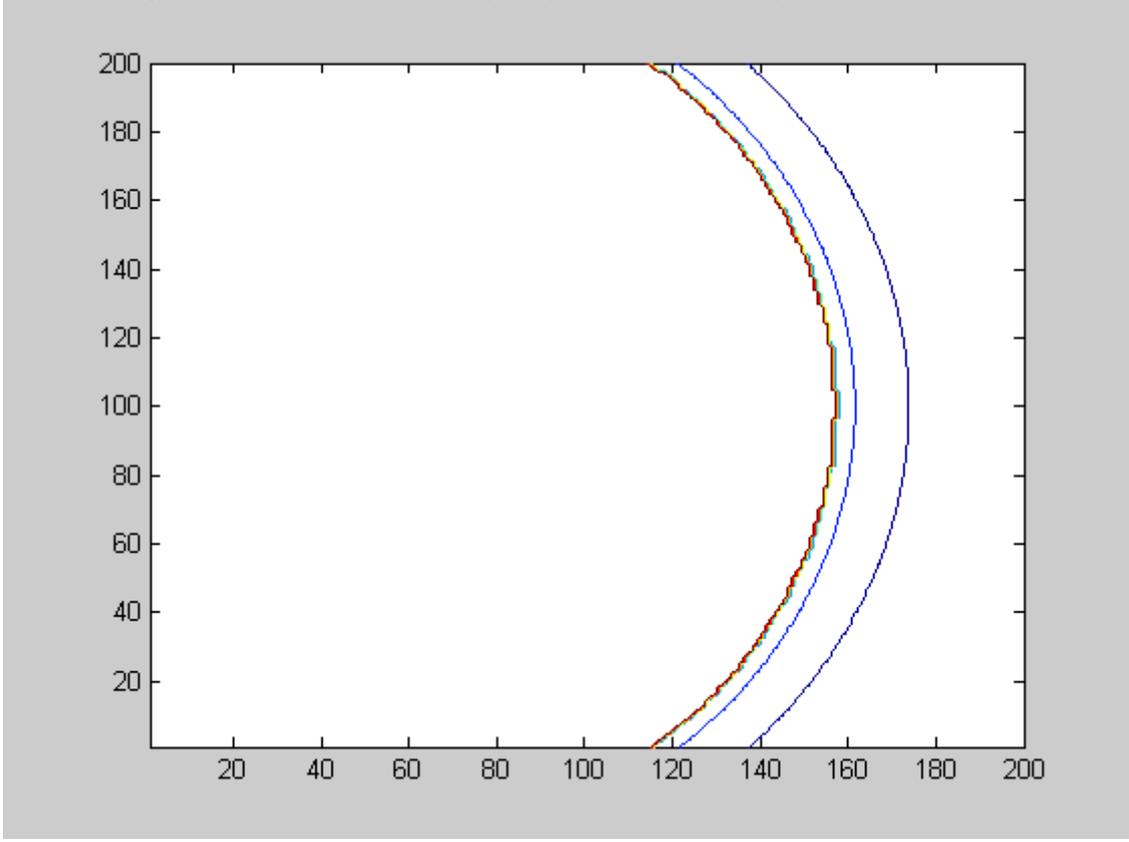
Appendix 4

Flame height: 0.3 m; t: 60 seconds (length scale is 0.1 m).
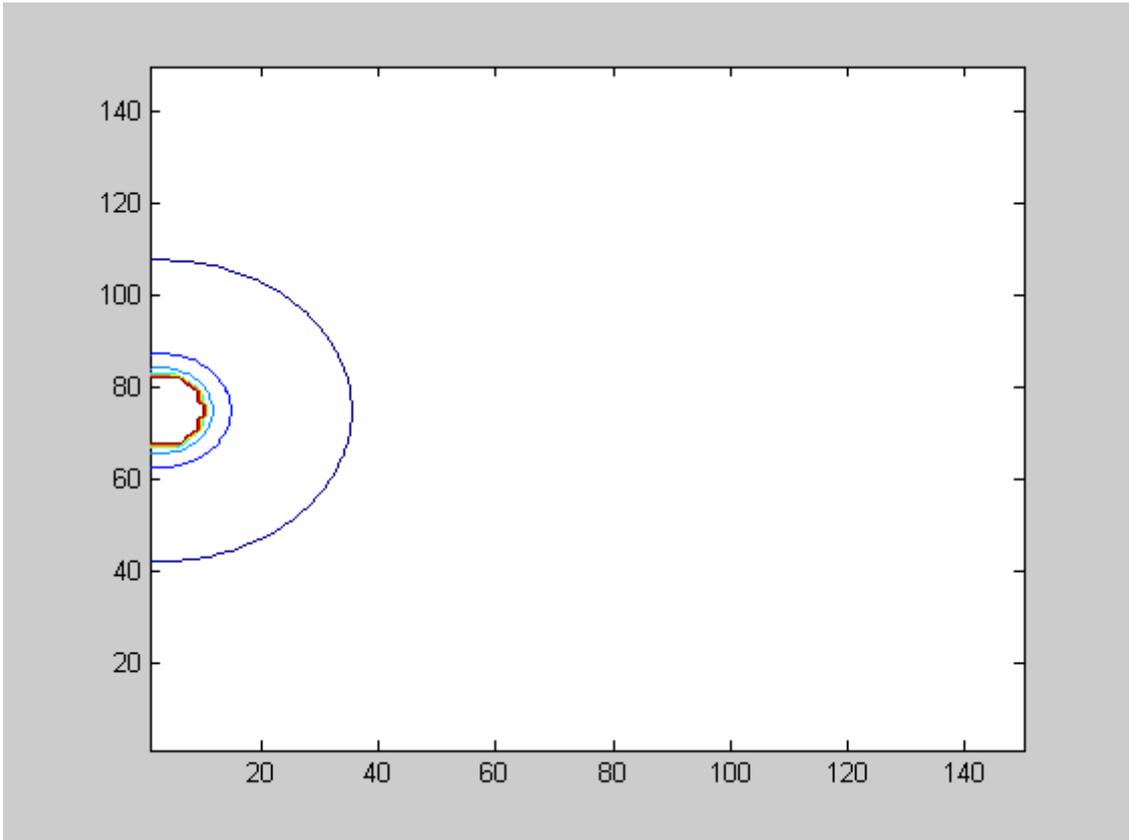

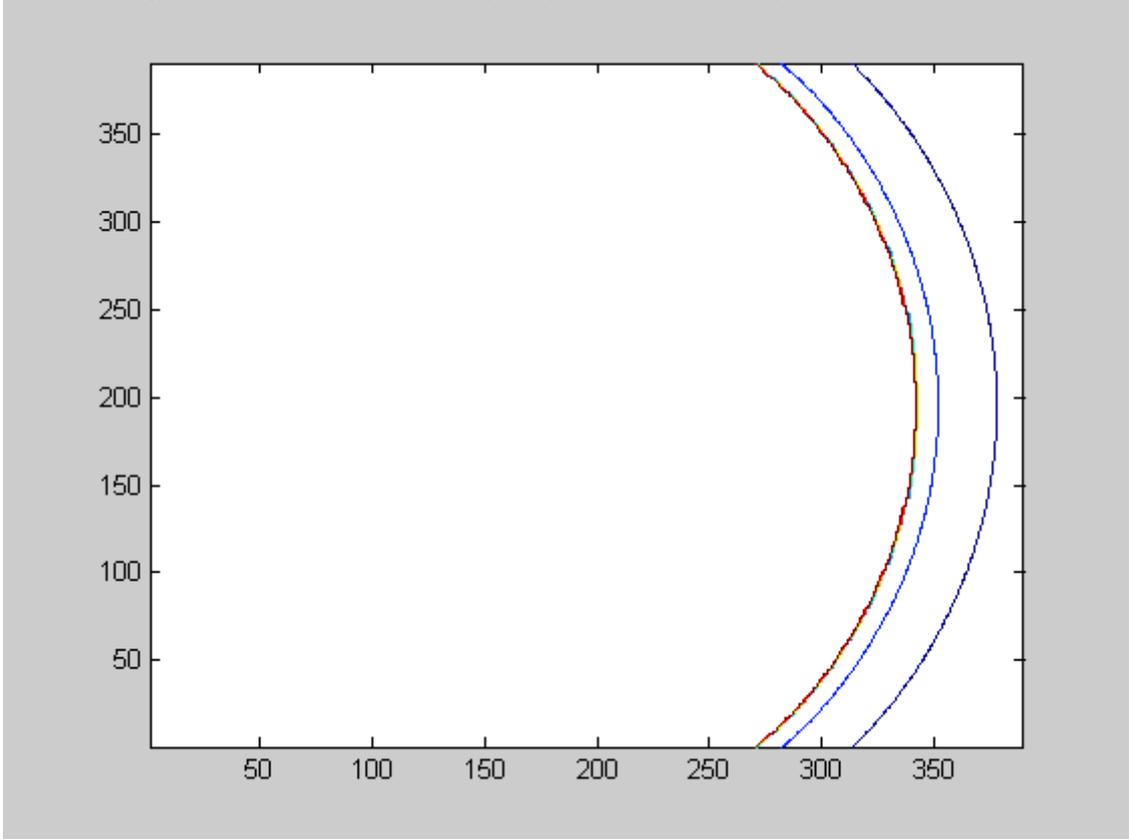Flame height: 0.3 m; t: 120 seconds (length scale is 0.1 m).

Flame height: 0.6 m; t: 60 seconds (length scale is 0.1 m).

Flame height: 0.6 m; t: 120 seconds (length scale is 0.1 m).

Flame height: 0.9 m; t: 60 seconds (length scale is 0.1 m).

Flame height: 0.9 m; t: 120 seconds (length scale is 0.1 m).