The Fuzzified Quasi-perceptron in Decision Making Concerning Treatments in Necrotizing Fasciitis

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Abstract. In the current paper we mathematically try to support the decision concerning the treatment with hyperbaric oxygen for patients, suffering from necrotizing fasciitis. To accomplish the task, we involve the fuzzified model of a quasi-perceptron, which is our modification of the classical artificial simple neuron. By means of the fuzzification of input signals and output decision levels, we wish to distinguish between decisions "treatment without recommended hyperbaric oxygen" versus "treatment with hyperbaric oxygen". The number of decision levels can be arbitrary in order to extend the decision scale.

Keywords: Fuzzified quasi perceptron; parametric membership functions; necrotizing fasciitis; treatment with hyperbaric oxygen.

1 Introduction

Necrotizing fasciitis (NF) is a rare, but deadly soft tissue infection. The disease is known from Hippocratic times but has been newly rediscovered in modern times as an "infection with flesh eating bacteria" by Jones in 1871. More specifically, the illness was described in 1952 by Wilson, who also renamed these types of infections as necrotizing fasciitis. The NF group contains various types of infections, usually treated with antibiotics and surgery [2]. In some cases, the treatment with hyperbaric oxygen (HBO) is the adjunct of treatments, mentioned above [5]. Blekinge County City hospital in Karlskrona, Sweden, has the possibility of providing HBO; therefore we serve the treatment to patients from the south eastern part of Sweden, suffering from NF.

From the clinical point of view, it will be interesting to identify a group of patients that have a good prognosis of recovery without recommendations for the specialized treatment with HBO versus a group of patients that need the HBO supplement, provided in a specialized health center.

Nevertheless the number of patients is not so high and, due to the disease rarity, it is difficult to make decisions, which routinely can solve the problems of HBO dosing.

The decision, mainly concerning the HBO recommendation in conformity with the patient's health state, is often based on the physicians' experience. To support this choice, we have developed the mathematical model resembling a simple artificial neuron, called the perceptron. The fuzzified form of the perceptron, furnished with some extra complements created by the authors, is expected to constitute a good tool of the theoretical discrimination of the final decision in as many levels as we wish. In this version of the model, we extract only two levels in the practical application of HBO serving.

The decision objects are vectors consisting of values assigned to some essential quantitative and compound qualitative biological parameters. These values have been sampled during the patients' examinations, and their role is to inform of the disease severity.

The classical perceptron model, invented by Frank Rosenblatt [11], is built as an artificial neuron whose activation function is furnished with two integers [1] [11] [13]. The perceptron is a popular network applied to many engineering solutions. Even medical tasks such as diagnosing [14], predicting lung tumor motion [3], operation decision in stomach cancer [8] and others [6], have been handled by this uncomplicated unit. A neuro-fuzzy approach to medical applications has been considered, e.g., in [12].

The first trials of the introduction of fuzziness in the perceptron machinery were made in 1985 [4]. In our modification of the perceptron model proposed, the input data are constructed as membership degrees of patient values of biological markers, when assigning fuzzy sets to these markers. Instead to learn the perceptron to classify the output decisions by training randomly plucked initial weights, we state the importance weights of harmful effects of the biological markers as unchangeable entries following the input data. Last but not the least, the output is predicted as a collection of membership degrees of decision levels, also built as fuzzy sets. This allows determining the optimal decision level, characteristic of the highest degree.

Since the classical perceptron differs from our model, provided with new solutions inserted, then we will call it "the fuzzified quasi-perceptron".

We outline the fuzzified quasi-perceptron model in Section 2. Section 3 contains the descriptions of constructions of entry data, such as signals and weights. The structure of fuzzified output of the quasi-perceptron will be engineered in Section 4. The study, concerning the treatment with HBO, will be tested in Section 5. In the practical study case, we use the data concerning 13 patients (12 men and 1 woman), who were treated in the Blekinge County City Hospital in Karlskrona between 2006 and 2010. We will formulate some concluding remarks in Section 6.

2 The Fuzzified Quasi-perceptron Model

Let us suppose that a disease is characterized by crucial biological markers X_j, $j = 1,\ldots,n$. To these markers, we intend to assign fuzzy sets also named X_j. The set of patients contains P_i objects, $i = 1,\ldots,p$. If a marker value $x_{i,j}$ for symptom X_j is registered for P_i, then the membership degree $\mu_{X_j}(x_{i,j})$ will be assigned to $x_{i,j}$. Let us name $\mu_{X_j}(x_{i,j})$ the input signals. The way of designing membership functions $\mu_{X_j}\colon X_j \to [0,1]$ will be evolved in Section 3.

Next, we introduce the importance weights w_j of symptoms X_j to emerge X_j's harmful influence on the disease course. We suggest the placement of X_j in the sequence

$X_1 \succ ... \succ X_n$, where "$\succ$" stands for the statement "X_j has more dangerous impact on the patient health state than X_g, j, g = 1,…,n. By making this arrangement of symptoms, we deduce that $w_1 > … > w_n$. We also desire that $\sum_{j=1}^{n} w_j = 1$.

The collected input signal s_i for all $x_{i,j}$, characteristic of patient P_i, will be derived as

$$s_i = \sum_{j=1}^{n} \mu_{X_j}(x_{i,j}) \cdot w_j, \ i = 1, \dots, p. \tag{1}$$

We note that $\min_{1 \le i \le n} s_i = 0$ since, for all minimal $\mu_{X_j}(x_{i,j}) = 0$, we obtain

$$s_i = \sum_{j=1}^{n} 0 \cdot w_j = 0, \ i = 1, \dots, p. \tag{2}$$

The maximal value of s_i will reach 1 if, for all maximal $\mu_{X_j}(x_{i,j}) = 1$,

$$s_i = \sum_{j=1}^{n} 1 \cdot w_j = 1 \cdot \sum_{j=1}^{n} w_j = 1 \cdot 1 = 1, \ i = 1, \dots, p. \tag{3}$$

We formulate the equation of an activation function $f: \{s_1, \dots, s_n\} \subset [0,1] \rightarrow [0,1]$ in the form of

$$f(s_i) = (\mu_{L_1}(s_i), \dots, \mu_{L_m}(s_i)), i = 1, \dots p, \tag{4}$$

where L_l, l = 1,…,m, are fuzzy sets assisting decision levels. Sets L_l are restricted over interval [0, 1] due to f's domain. After comparing the membership degrees in (4), we determine the optimal decision level L_l as the level L^* fitted for $L^* = \max_{1 \le l \le m}(\mu_{L_l}(s_i))$.

Figure 1 shows the procedure of the fuzzified quasi-perceptron for two decision levels L_1 and L_2.

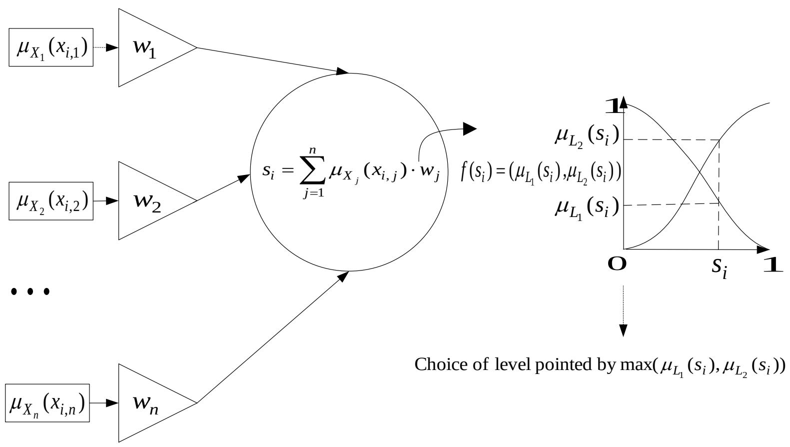

Fig. 1. The action of the quasi-perceptron for two decision levels concerning patient P_i

3 The Design of Input Data

Symptoms X_j are recognized as quantitative and qualitative features. We assign fuzzy sets X_j, $j = 1,\ldots,n$, to both types. As the rising order of symptom characteristics (real values or codes) is associated with the growing states of the disease threat then, as a consequence, the membership functions of X_j will be constructed as ascending functions.

For the measurable symptoms X_j, taking values $x_{i,j}$ in interval $[\alpha_j, \gamma_j]$ continuously, we have prepared a parametric s-function $s(x_{i,j}, \alpha_j, \beta_j, \gamma_j)$ demonstrated by

$$\mu_{X_j}(x_{i,j}) = s(x_{i,j},\alpha_j,\beta_j,\gamma_j) = \begin{cases} 0 & \text{for} \quad x_{i,j} \le \alpha_j, \\ 2\left(\frac{x_{i,j}-\alpha_j}{\gamma_j-\alpha_j}\right)^2 & \text{for} \quad \alpha_j < x_{i,j} \le \beta_j, \\ 1-2\left(\frac{x_{i,j}-\gamma_j}{\gamma_j-\alpha_j}\right)^2 & \text{for} \quad \beta_j < x_{i,j} \le \gamma_j, \\ 1 & \text{for} \quad x_{i,j} > \gamma_j, \end{cases} \tag{5}$$

where $\beta_j = \frac{\alpha_j+\gamma_j}{2}$, $j = 1,\ldots,n$, $i = 1,\ldots,p$.

Example 1

Symptom "*age*"= X_2 is a fuzzy set, constrained by the membership function $s(x_{i,2}, 18, 59, 100)$. For, e.g., $x_{i,2} = 76$, we estimate $\mu_{X_2}(76) = 1-2\left(\frac{76-100}{100-18}\right)^2 = 0.828$ in accordance with the condition 59<76<100.

By adopting the own procedure [7], we intend now to compute the membership degrees for compound qualitative symptoms X_j, characterized by codes $c_{j,0},\ldots,c_{j,k},\ldots,c_{j,z}$, where $c_{j,k+1}=c_{j,k}+1$, $k = 0,\ldots,z-1$, and $c_{j,z}$ is an even integer. Let us first introduce a function $g(c_{j,k})$, which starts with $g(c_{j,0}) = -1$ and terminates with $g(c_{j,z}) = 1$. The length of each of $z-1$ subintervals $I = [c_{j,k}, c_{j,k+1}]$ of X_j, partitioning interval $[-1, 1]$, is equal to $\frac{g(c_{j,z})-g(c_{j,0})}{z-1}$. After concluding that the end of each subinterval is tied to code $c_{j,k+1}$, we derive

$$g(c_{j,k+1}) = g(c_{j,k}) + \frac{g(c_{j,z})-g(c_{j,0})}{z-1} \tag{6}$$

for $k = 0,\ldots,z-1$.

Interval $[-1, 1]$, containing discrete values $g(c_{j,k})$, constitutes a support of fuzzy set X_j assisting the compound qualitative symptom. In order to estimate membership values of $g(c_{j,k})$, where $k = 0,\ldots,z$ actually, we use an s membership function

$$\mu_{X_j}(g(c_{j,k})) = s(g(c_{j,k}),-1,0,1)\,. \tag{7}$$

When studying the properties of Eq. (7), we remark that: the lack of the symptom $g(c_{j,0}) = -1$ possesses membership 0, the critical value of the symptom $g(c_{j,z}) = 1$ is connected to membership 1, whereas the mean value $\frac{g(c_{j,0})+g(c_{j,z})}{2} = 0$ is furnished with membership 0.5. These features of (7) logically agree with the conditions demanded for symptoms coded.

Example 2
The states of symptom "*medical state*" = S_1 are coded as: "*comfortable*" = 0, "*satisfactory*" = 1, "*stable*" = 2, "*critical but stable*" = 3 and "*critical*" = 4. The length of each interval, placed between two adjacent codes, equals $\frac{1-(-1)}{5-1} = 0.5$. Hence, $g(0) = -1$, $g(1) = -1 + 0.5 = -0.5$, $g(2) = 0$, $g(3) = 0.5$ and $g(4) = 1$ due to (6). The degrees, found for $g(k)$, $k = 0,\ldots,4$, are, in accord with (7), numbers: $\mu_{X_1}(g(0)) = 0$, $\mu_{X_1}(g(1)) = 0.125$, $\mu_{X_1}(g(2)) = 0.5$, $\mu_{X_1}(g(3)) = 0.875$ and $\mu_{X_1}(g(4)) = 1$.

In the last part of Section 3, let us solve the problem of assigning the importance weights w_j to symptoms X_j. By "importance" we mean the strength of X_j's adverse and harmful power in the running process of the illness considered. We bring into light another own mathematical algorithm, allowing the estimation of weights [9].

Generally, if we consider n symptoms X_j to find importance weights for them, we wish to arrange them in the sequence $X_1 \succ \ldots \succ X_n$ in accordance with the expert's opinion. We want the sum of all weights w_j, joined to Xj, $j = 1,\ldots,n$, to be 1. Therefore,

$$n \cdot r + (n-1) \cdot r + \ldots + 2 \cdot r + 1 \cdot r = 1, \tag{8}$$

where r is a quotient dependent on n.

Further,

$$w_j = (n - j + 1) \cdot r, \tag{9}$$

for $j = 1,\ldots,n$.

Example 3
The decisive symptoms for the recognition of necrotizing fasciitis are listed in the importance order, decided by a physician, as "*medical state*" = $X_1 \succ$"*age*" = $X_2 \succ$"*risk factors*" = $X_3 \succ$"*crp*" = $X_4 \succ$*wbc* = $X_5 \succ$"*temperature*" = X_6. It should be clarified that "*crp*" stands for C-reactive proteins and "*wbc*" means white blood cells. In conformity with (8), equation $6r+5r+4r+3r+2r+r = 1$ provides r = 0,0476. After employing (9), we receive, in turn for $j = 1,\ldots,6$, the weights w_1 = (6–1+1)0.0476= 0.2856, w_2 = 0.238, w_3 = 0.1904, w_4 = 0.1428, w_5 = 0.0952, w_6= 0.0476.

4 The Theoretical Construction of Fuzzified Output Levels

Due to Eq. (4), we should now generate a collection of output decision fuzzy levels L_l stretched over interval [0, 1], $l = 1,\ldots,m$, to calculate the membership degrees of signal

s_i, $i = 1,\ldots,p$, in each L_l. The largest value $L^* = \max_{1\le l\le m} \mu_{L_l}(s_i)$ points out the optimal decision level assigned to P_i.

Theoretically, m can be either an even or an odd positive arbitrary integer. An own procedure helps us to derive membership functions of L_l. These are dependent only on two parameters, namely, a number of levels and a length of the common set containing all supports of L_l. The proof of Theorem 1, edited by us for odd m values, can be found in [10]. The formulas of L_l's membership functions, containing an even m quantity, have been brought forth for the sake of this paper.

Theorem 1 [10]
Suppose that we want to find membership functions for fuzzy sets $L_1,\ldots,L_m$, where m is an odd positive integer. We assume that supports of constraints $\mu_{L_l}(s_i)$, $l = 1,\ldots,m$, will cover parts of the reference set $L = [\min(L_1),\max(L_m)]$, $s_i \in L$. $E = |L|$ is the length of L.

We divide all L_l in three groups, namely, a family of "*leftmost*" sets $L_1,\ldots, L_{\frac{m-1}{2}}$, the set $L_{\frac{m+1}{2}}$ "*in the middle*" and a collection of "*rightmost*" sets $L_{\frac{m+3}{2}},\ldots,L_m$. To design the membership functions of L_l, the s-class function $s(s_i, m, E)$ will be adopted.

The function of "*in the middle*" = $L_{\frac{m+1}{2}}$ is constructed as

$$\mu_{L_{\frac{m+1}{2}}}(s_i)=\begin{cases} 0 & \text{for} \quad s_i \le \frac{E(m-2)}{2m},\\ 2\left(\frac{s_i-\frac{E(m-2)}{2m}}{\frac{E}{m}}\right)^2 & \text{for} \quad \frac{E(m-2)}{2m} \le s_i \le \frac{E(m-1)}{2m},\\ 1-2\left(\frac{s_i-\frac{E}{2}}{\frac{E}{m}}\right)^2 & \text{for} \quad \frac{E(m-1)}{2m} \le s_i \le \frac{E}{2},\\ 1-2\left(\frac{s_i-\frac{E}{2}}{\frac{E}{m}}\right)^2 & \text{for} \quad \frac{E}{2} \le s_i \le \frac{E(m+1)}{2m},\\ 2\left(\frac{s_i-\frac{E(m+2)}{2m}}{\frac{E}{m}}\right)^2 & \text{for} \quad \frac{E(m+1)}{2m} \le s_i \le \frac{E(m+2)}{2m},\\ 0 & \text{for} \quad s_i \ge \frac{E(m+2)}{2m}. \end{cases} \tag{10}$$

All constraints, characteristic of the "*leftmost*" family of fuzzy sets, will be given by

$$\mu_{L_t}(s_i)=\begin{cases} 1 & \text{for} \quad s_i \le \frac{E(m-1)}{2(m+1)}\delta(t),\\ 1-2\left(\frac{s_i-\frac{E(m-1)}{2(m+1)}\delta(t)}{\frac{E(m-1)}{m(m+1)}\delta(t)}\right)^2 & \text{for} \quad \frac{E(m-1)}{2(m+1)}\delta(t) \le s_i \le \frac{E(m-1)}{2m}\delta(t),\\ 2\left(\frac{s_i-\frac{E(m-1)(m+2)}{2m(m+1)}\delta(t)}{\frac{E(m-1)}{m(m+1)}\delta(t)}\right)^2 & \text{for} \quad \frac{E(m-1)}{2m}\delta(t) \le s_i \le \frac{E(m-1)(m+2)}{2m(m+1)}\delta(t),\\ 0 & \text{for} \quad s_i \ge \frac{E(m-1)(m+2)}{2m(m+1)}\delta(t). \end{cases} \tag{11}$$

where function $\delta(t) = \frac{2}{m-1}\cdot t$, $t = 1,\ldots,\frac{m-1}{2}$.

To generate the "*rightmost*" family of sets $L_{\frac{m+3}{2}},\dots,L_m$, we need to initiate a new function $\varepsilon(t)=1-\frac{2}{m-1}(t-1)$, $t = 1,\dots,\frac{m-1}{2}$. All right functions have a parametric equation

$$\mu_{L_{\frac{m+3}{2}+t-1}}(s_i)=$$

$$\begin{cases} 0 & \text{for} \quad s_i \le E-\frac{E(m-1)(m+2)}{2m(m+1)}\varepsilon(t), \\ 2\left(\frac{s_i-\left(E-\frac{E(m-1)(m+2)}{2m(m+1)}\varepsilon(t)\right)}{\frac{E(m-1)}{m(m+1)}\varepsilon(t)}\right)^2 & \text{for} \quad E-\frac{E(m-1)(m+2)}{2m(m+1)}\varepsilon(t)\le s_i \le E-\frac{E(m-1)}{2m}\varepsilon(t), \\ 1-2\left(\frac{s_i-\left(E-\frac{E(m-1)}{2(m+1)}\varepsilon(t)\right)}{\frac{E(m-1)}{m(m+1)}\varepsilon(t)}\right)^2 & \text{for} \quad E-\frac{E(m-1)}{2m}\varepsilon(t)\le s_i \le E-\frac{E(m-1)}{2(m+1)}\varepsilon(t), \\ 1 & \text{for} \quad s_i \ge E-\frac{E(m-1)}{2(m+1)}\varepsilon(t). \end{cases} \quad (12)$$

Let us emphasize that we use only three equations in order to generate all membership functions of L_l, affected by m and E. The mobile variable in Eqs (11) and (12) is an actual function number t. The procedure can be thus easily computerized.

Theorem 2 (the proof will be provided in the extended version of the paper)
For an even m value we remove the function "*in the middle*" from the model. The restrictions of the "*leftmost*" family of fuzzy sets $L_1,\dots,L_{\frac{m}{2}}$ will be thus recommended as

$$\mu_{L_t}(s_i)=\begin{cases} 1 & \text{for} \quad s_i \le \frac{E(m-2)}{2(m-1)}\delta(t), \\ 1-2\left(\frac{s_i-\frac{E(m-2)}{2(m-1)}\delta(t)}{\frac{E}{(m-1)}\delta(t)}\right)^2 & \text{for} \quad \frac{E(m-2)}{2(m-1)}\delta(t)\le s_i \le \frac{E}{2}\delta(t), \\ 2\left(\frac{s_i-\frac{Em}{2(m-1)}\delta(t)}{\frac{E}{(m-1)}\delta(t)}\right)^2 & \text{for} \quad \frac{E}{2}\delta(t)\le s_i \le \frac{Em}{2(m-1)}\delta(t), \\ 0 & \text{for} \quad s_i \ge \frac{Em}{2(m-1)}\delta(t), \end{cases} \quad (13)$$

where $\delta(t)=\frac{2}{m}\cdot t$, $t = 1,\dots,\frac{m}{2}$.

The common equation of "*rightmost*" family of functions $L_{\frac{m}{2}+1},\dots,L_m$ is affected by the even m value as

$$\mu_{L_{\frac{m}{2}+t}}(s_i)=\begin{cases} 0 & \text{for} \quad s_i \le E-\frac{Em}{2(m-1)}\varepsilon(t), \\ 2\left(\frac{s_i-\left(E-\frac{Em}{2(m-1)}\varepsilon(t)\right)}{\frac{E}{(m-1)}\varepsilon(t)}\right)^2 & \text{for} \quad E-\frac{Em}{2(m-1)}\varepsilon(t)\le s_i \le E-\frac{E}{2}\varepsilon(t), \\ 1-2\left(\frac{s_i-\left(E-\frac{E(m-2)}{2(m-1)}\varepsilon(t)\right)}{\frac{E}{m-1}\varepsilon(t)}\right)^2 & \text{for} \quad E-\frac{E}{2}\varepsilon(t)\le s_i \le E-\frac{E(m-2)}{2(m-1)}\varepsilon(t), \\ 1 & \text{for} \quad s_i \ge E-\frac{E(m-2)}{2(m-1)}\varepsilon(t), \end{cases} \quad (14)$$

for $\varepsilon(t) = 1 - \frac{2}{m}(t-1)$, $t = 1,\ldots,\frac{m}{2}$.

Example 4
We ascertain the propriety of Eqs (10)-(12) in Fig. 2(a) for m = 3 and E = 1, whereas the verification of Eqs (13)-(14) is accomplished in Fig. 2(b), when m = 2 and E = 1.

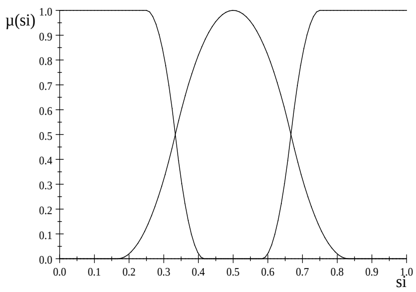

Fig. 2(a). Functions of L_1, L_2, L_3, m=3

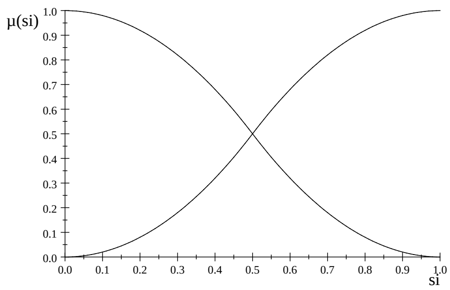

Fig. 2(b). Functions of L_1, L_2, m=2

5 The Decision Concerning the Treatment with HBO

It has already been mentioned in Section 1 that the mathematical apparatus, built in Sections 2-4, will be applied to verify (confirm or deny) a decision concerning the treatment with HBO of patients suffering from necrotizing fasciitis. The data, including the values of crucial clinical markers and the decisions made by physicians, have been sampled for 13 patients (12 men and 1 woman) treated in the Blekinge County City Hospital in Karlskrona, Sweden, between 2006 and 2010.

The clinical symptoms, essential in NF, have been introduced in Example 3. For quantitative symptoms we adapt Eq. (5) as follows:
$\mu_{X_2=\text{"}age\text{"}}(x_{i,2}) = s(x_{i,2}, 18, 59, 100)$, $\mu_{X_4=\text{"}crp\text{"}}(x_{i,4}) = s(x_{i,4}, 0, 250, 500)$,
$\mu_{X_5=\text{"}wbc\text{"}}(x_{i,5}) = s(x_{i,5}, 0, 15, 30)$ and $\mu_{X_6=\text{"}temp.\text{"}}(x_{i,6}) = s(x_{i,6}, 36, 38.5, 41)$.

In Example 2, we have already determined the membership degrees for the coded symptom X_1 = "*medical state*" as: $\mu_{X_1}(g(0)) = 0$, $\mu_{X_1}(g(1)) = 0.125$, $\mu_{X_1}(g(2)) = 0.5$, $\mu_{X_1}(g(3)) = 0.875$ and $\mu_{X_1}(g(4)) = 1$.

We repeat the algorithm for symptom X_3 = "*risk factors*", coded between 0 and 6, where $g(0) = -1$, $g(1) = -0.666$, $g(2) = -0.333$, $g(3) = 0$, $g(4) = 0.333$, $g(5) = 0.666$ and $g(6) = 1$. When fixing $\mu_{X_3=\text{"}risk\ factors\text{"}}(g(k)) = s(g(k), -1, 0, 1)$, $k = 0,\ldots,6$, we list:
$\mu_{X_3}(g(0)) = 0$, $\mu_{X_3}(g(1)) = 0.056$, $\mu_{X_3}(g(2)) = 0.221$, $\mu_{X_3}(g(3)) = 0.5$,
$\mu_{X_3}(g(4)) = 0.779$, $\mu_{X_3}(g(5)) = 0.944$ and $\mu_{X_3}(g(6)) = 1$.

Table 1 contains the clinical data and assigned to them membership degrees, computed in compliance with the membership functions of X_j. The symbol $\mu_{X_j}(x_{i,j})/x_{i,j}$ shows the membership degree of $x_{i,j}$ in X_j before the dash and the $x_{i,j}$ clinical value after the dash, j =1,…,6.

Table 1. Patient clinical data and their membership degrees in necrotizing fasciitis

P_i	$\mu_{X_1}(x_{i,1})/x_{i,1}$	$\mu_{X_2}(x_{i,2})/x_{i,2}$	$\mu_{X_3}(x_{i,3})/x_{i,3}$	$\mu_{X_4}(x_{i,4})/x_{i,4}$	$\mu_{X_5}(x_{i,5})/x_{i,5}$	$\mu_{X_6}(x_{i,6})/x_{i,6}$
P_1	0.125/1	0.058/32	0/0	0.825/352	0.52/15.3	0.003/36.2
P_2	0.5/2	0.828/76	0.5/3	0.566/267	0.493/14.9	0.387/38.2
P_3	0.875/3	0.304/50	0.056/1	0.43/232	0.222/10	0.135/37.3
P_4	0.5/2	0.656/66	0.221/2	0.696/305	0.993/28.2	0.289/37.9
P_5	0.875/3	0.749/71	0/0	0.286/189	0.989/27.8	0.135/37.3
P_6	0.5/2	0.452/57	0/0	0.637/281	0.533/15.5	0.423/38.3
P_7	1/4	0.285/49	0.056/1	0.849/363	0.755/19.5	0.028/36.6
P_8	0.875/3	0.892/81	0.5/3	0.91/394	0.358/12.7	0.205/37.6
P_9	1/4	0.475/58	1/6	0.939/413	0.68/18	0.32/38
P_{10}	0.875/3	0.452/57	0.056/1	0.484/246	0.021/3.1	0/35.8
P_{11}	0.5/2	0.524/60	0.221/2	0.058/85	0.619/16.9	0.289/36.5
P_{12}	0.875/3	0.732/70	0.778/4	0.924/403	0.995/28.5	0.32/38
P_{13}	1/4	0.881/80	0.221/2	0.046/76	0.726/18.9	0.98/40.5

As emerged in Eq. (1), the aggregation of membership degrees $\mu_{X_j}(x_{i,j})$ with weights w_j evaluated in Example 3, j = 1,…,6, will constitute a basis for the calculation of the cumulated entry signal s_i for patient P_i.

Example 5

s_1 = 0.125·0.286+0.058·0.238+0·0.19+0.824·0.1428+0.52·0.095+0.003·0.047 = 0.21676 represents P_1.

In order to interpret two decision states by means of membership degrees in L_1 and L_2, we return to Eqs (13)-(14), for m = 2, L = [0, 1] and E = 1 to derive functions $\mu_{L_1}(s_i) = 1 - s(s_i, 0, 0.5, 1)$ and $\mu_{L_2}(s_i) = s(s_i, 0, 0.5, 1)$. We identify L_1 with the decision about not recommending the treatment with HBO. On the contrary, L_2 confirms the decision about the HBO treatment. We choose the decision characterized by the largest membership degree out of $\mu_{L_1}(s_i)$ and $\mu_{L_2}(s_i)$.

Table 2 collects signals, their membership degrees and the physician's assertion already made.

We note that the fuzzy decisions converge to medical decisions in most of cases. In the future research, we plan to test the model with three decisions levels, where the middle level "wait and see", assigned to values about 0.5, will be often checked to study its tendencies.

Table 2. The comparison of fuzzy decisions (underlined) to decisions made by the physician

P_i	s_i	$\mu_{L_1}(s_i)$ - *without HBO*	$\mu_{L_2}(s_i)$ - *with HBO*	*Physician's decision about treating with HBO*
P_1	0.217	<u>0.906</u>	0.094	*No*
P_2	0.581	0.341	<u>0.659</u>	*Yes*
P_3	0.422	<u>0.643</u>	0.357	*Yes*
P_4	0.548	0.408	<u>0.592</u>	*Yes*
P_5	0.569	0.371	<u>0.629</u>	*Yes*
P_6	0.412	<u>0.660</u>	0.340	*No*
P_7	0.559	0.390	<u>0.610</u>	*Yes*
P_8	0.731	0.144	<u>0.856</u>	*Yes*
P_9	0.802	0.078	<u>0.922</u>	*Yes*
P_{10}	0.439	<u>0.614</u>	0.386	*No*
P_{11}	0.390	<u>0.695</u>	0.305	*No*
P_{12}	0.814	0.069	<u>0.931</u>	*Yes*
P_{13}	0.659	0.232	<u>0.768</u>	*Yes*

6 Conclusions

By suggesting modifications in the classical perceptron, we have adapted it to make decisions concerning curation with hyperbaric oxygen, needed for patients afflicted by necrotizing fasciitis. We have proposed own parametric membership functions of fuzzy sets to be able to fuzzify the input data and output decision levels. We emphasize that the functions are affected only by a number of levels and the length of a set, common for all fuzzy constraints. The functions are derived in the way allowing to determine an arbitrary number of levels, which extends the decision scale of linguistic expressions without making changes in formulas.

The own procedures of estimating the importance weights of symptoms and approximating membership degrees of qualitative symptoms have been added as contributions in imprecise mathematics.

From the mathematical point of view, the results obtained seem to be logically correct.

Necrotizing fasciitis (NF) is a quite rare entity, and there is no widespread consensus regarding neither treatment nor grading. There were several attempts of using laboratory results only to facilitate diagnosis making and grading the disease's severity, but as far as we know, they are not used widely. The idea of combining analysis of numerical parameters, such as body temperature, white blood cell count, age etc. with the non-parametrical estimations, such as medical state etc. is very promising, because it will reflect the real decision making progress. The model, tested above, is based on retrospective analysis of data of patients treated with hyperbaric oxygen (HBO) at the hospital department in Karlskrona, Sweden.

We realize that the present model has weaknesses, mostly if the group used to check the model, has not been very numerous. In spite of this, it seems that we have been successful in selecting essential clinical and biochemical parameters, which has constituted the crucial decision for the correctness of the mathematical model. The results of the perceptron's analysis are not 100% concordant with the real medical decisions, but are quite near to the last ones. This reflects also the reliability of the mathematical technique, and we think that it is a promising beginning of our research work in this direction.

In the further research we will redefine the ordering of importance weights of symptoms more carefully to refine the results. The fuzzified quasi-perceptron can be used to more than two decisions, something, which makes it suitable for decision making in other diseases, characterized by more available treatment options.

To sum up, the authors think that the quasi-perceptron method has shown valuable decisions, when involving into them the commonly used parametric laboratory and clinical data.

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