

Master Thesis  
Electrical Engineering  
MEE09:08



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## **Implementation of a Soft Output Sphere Decoder by Rapid Prototyping Methodology**

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November 2007

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# Abstract

During the last decade there has been tremendous growth in wireless communication, with focus on broadband wireless LAN and mobile phones. To meet the demand for faster wireless access there is a need to change from the traditional Single-Input-Single Output (SISO) antenna systems. There has been significant progress in recent times in developing systems that use multiple antennas at the transmitter and receiver to achieve better performance.

Multiple-input-multiple-output (MIMO) uses multiple antennas in both transmitter and receiver ends to achieve high spectral efficiency. However the implementation of MIMO systems comes at the cost of increased complexity in the receiver design. The detection of vector of symbols drawn from a finite alphabet when transmitted over a MIMO system poses a challenging implementation task.

The aim of this thesis is to use sphere detection algorithm as an efficient means to detect the spatially multiplexed symbols. This thesis discusses the evaluation of the performance and implementation aspects of a Soft output sphere decoder as an efficient detection means for MIMO systems. The implementation is done in MATLAB and C and discusses the prospects of rapid prototyping methodology as an efficient approach towards further implementation.

# Acknowledgements

I would like to thank many people involved in bringing up this course work to this very point. Firstly I would thank my parents who have been a constant source of support; their love and encouragement have brought me this far in life.

I thank Sebastian Caban for inviting me to Vienna to work on my thesis. My sincere thanks to my thesis advisor Christian Mehlfehler for providing me with this topic, sharing his viewpoint, his encouragement and support in pursuing this work. I would further like to express my gratitude to Christian for his extended patience when it came to making the report. The atmosphere at the MIMO Lab was just perfect and I truly had a great experience being part of it for a whole summer.

The time spent in Vienna would not be complete without mentioning the support from the folks met during the stay. Those joyful moments that helped take a breather and later providing with the zeal to work further. A special thanks to Dr.N.Z.A for the unforgettable experience that was Vienna.

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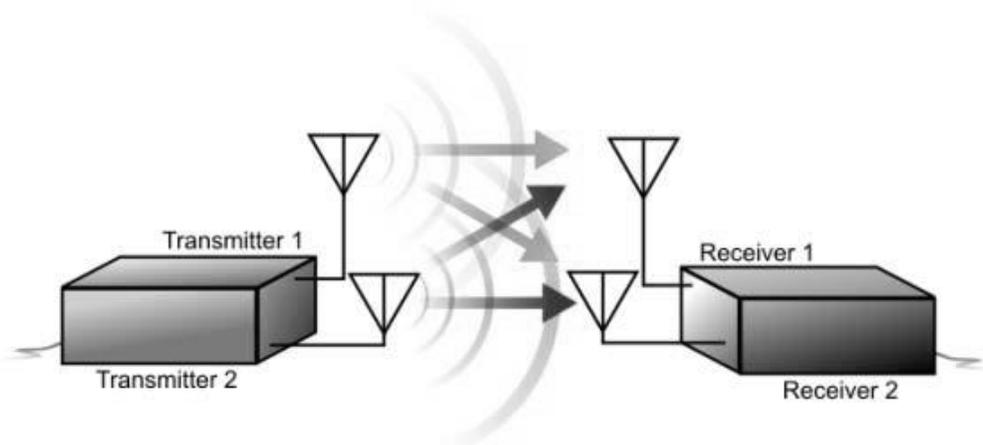
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# CHAPTER 1

## Introduction

### 1.1 Background

Since the early nineties till the recent present, technological advancements concerning Multiple Input Multiple Output (MIMO) wireless communication have undergone unprecedented growth and are still growing. This urge has been spurred by the demand for high speed data communication overcoming problems such as noise and fading. The great deal of attention towards MIMO Wireless systems is mainly due to the promise of increased channel capacity, reliability and extended range. Multiple antennas employed on both sides of the wireless link (as illustrated in Figure 1) offer increased spectral efficiency by transmitting multiple data streams concurrently and in the same frequency band.



**Figure 1: A basic 2 x 2 Communication system.**

Implementation of MIMO Systems has posed wireless engineers significant challenges mainly in the receiver design. All advantages of MIMO systems can only be exploited if soft output, maximum likelihood detectors are implemented.

### 1.1.1 Available detection algorithms

A wide range of algorithms offering various trade-offs between performance and computational complexity has been developed.

*Maximum likelihood detection* (MLD): This detector finds the transmitted signal points by minimising the Euclidean distance. However, such a search is *exponential* in finding the number of constellation points, making MLD unsuitable for practical purposes when aiming at high spectral efficiency.

*Linear detection & Successive Interference Cancellation* (SIC): producing hard or soft decision output and performance is limited by the strongest detected signal, which leads to poor diversity. The main advantage of this detector is the very low complexity compared to the MLD.

*A Posteriori Probability* (APP): is a computationally efficient algorithm with optimal performance when implemented in group detector by soft output *sphere decoder* (SD).

The overall decoding effort is typically constrained by system bandwidth, latency requirements, and limitations on power consumption. Implementing different algorithms, each optimized for a maximum allowed decoding effort and/or a particular system configuration, would entail a considerable hardware overhead.

### 1.1.2 A Possible Solution

Recently sphere detection has emerged as a powerful means of finding the maximum likelihood solution to detection problem for MIMO systems. The computational complexity of the sphere decoder would depend on the symbol constellation size and the number of spatially multiplexed data streams, as well as on the instantaneous MIMO channel realization and the signal-to-noise ratio (SNR). Sphere decoder lowers the computational complexity by limiting the search to the closest lattice point to the received signal within a sphere radius. In a 2-D problem, we can restrict the search by drawing a circle around the received signal just small enough to enclose one signal point and eliminate the search of all points outside the circle. By using the soft output sphere decoder and using the counter hypotheses it is possible to obtain better results.

## **1.2 Objective**

This Masters project aims at evaluating the performance and implementation aspects of a soft output sphere decoder algorithm for a MIMO system using rapid prototyping methodology. The performance gain of using soft output sphere detection is to be discussed while implementing it in MATLAB and C. Further the porting of the code onto a DSP by following the rapid prototyping methodology would be the key objective in this thesis.

# CHAPTER 2

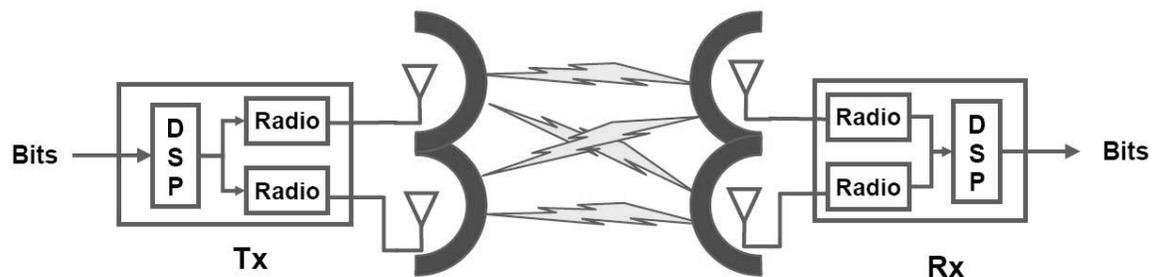
## MIMO-OFDM

This chapter sheds light on MIMO Technology, how MIMO-OFDM has become the best choice for implementing this concept, and the principles of the underlying MIMO-OFDM.

### 2.1 MIMO Overview

Multipath propagation has been part of all wireless communication environments. Traditional radio systems rely on the strong primary signal and employ multipath mitigation techniques to combat multipath propagation and interference. One technique used for this purpose is beamforming technology or smart antenna technology. A smart antenna is able to steer a transmitted beam by accurately controlling the phase of the signal at each element of the antenna array to the receiver. Another technique is receive diversity, which uses multiple antennas at the receiver and combines all receive signal to improve the signal reliability. These techniques help reduce the multipath propagation effect. Additionally, MIMO rather than combating makes use of multipath propagation in its favor to increase throughput, range and reliability.

In MIMO, “Multiple in” refers to simultaneously sending two or more signals at multiple transmitting antennas. “Multiple out” refers to two or more radio signals coming from multiple receiving antennas. Figure 2 shown below provides an overview of a MIMO communication system.



**Figure 2: Overview of a MIMO Communication system.**

This has been known to increase spectral efficiency and link reliability by combining the goodness of Beamforming and Diversity. Common forms of diversity are antenna diversity (or spatial diversity), time diversity (due to Doppler spread) and frequency diversity (due to delay spread). In recent years the use of antenna diversity has become very popular, this is mostly due to the fact that it can be provided without loss in spectral efficiency.

Driven by mobile wireless applications, where it is difficult to deploy multiple antennas in the handset, the use of multiple antennas on the transmit side combined with signal processing and coding has become known under the name of Space Time Coding [3].

MIMO takes advantage of multipath by multiplexing these signals with advanced DSP algorithms to boost wireless bandwidth efficiency and range. MIMO antenna configuration has shown to achieve extraordinary data rates and this technology is known as Spatial Multiplexing. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the receiver can separate these streams, creating parallel channels for free. Spatial multiplexing is very powerful technique for increasing channel capacity at higher SNR.

### **2.1.1 Why MIMO-OFDM**

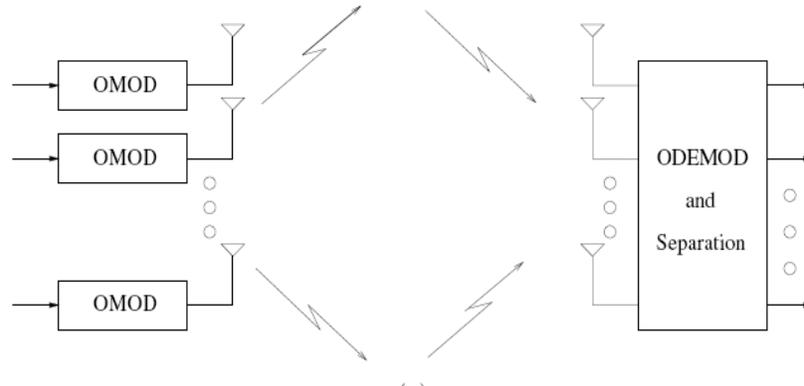
MIMO can be used with any modulation or access technique. Today, many digital radio systems use CDMA (e.g. UMTS) or OFDM (e.g. WLAN, WiMAX). Time division systems transmit bits over a narrowband channel, using time slots to segregate bits for different user or purposes. Code division systems transmit bits over a wideband (spread spectrum) channel, using codes to segregate bits for different user or purposes. OFDM is also a wideband system, but unlike CDMA which spreads the signal continuously over the entire channel, OFDM employs multiple, discrete, lower data rate subchannels. MIMO-OFDM technology has been adopted by the IEEE 802.11 standards group as the foundation for a high throughput amendment to the multimedia wireless fidelity (WiFi) applications.

When it comes to implementation point of view, particularly at higher data rates, MIMO OFDM signals can be processed using relatively straightforward matrix algebra. In other words use of OFDM will significantly reduce the receiver complexity in wireless communication systems. The main motivation for using OFDM in a MIMO channel is the fact that OFDM modulation turns a frequency-selective MIMO channel into a set of

parallel frequency flat MIMO channels. This renders multi-channel equalization particularly simple, since for each OFDM-tone only a constant matrix has to be inverted [4]. Therefore the use of MIMO technology in combination with OFDM, MIMO-OFDM is an attractive solution for future wireless communication systems design

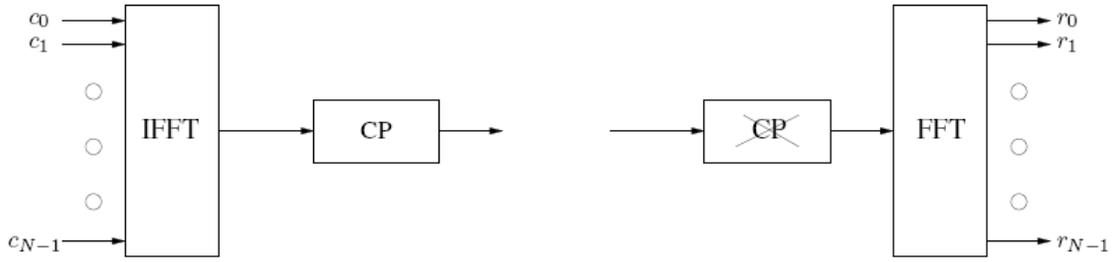
### 2.1.2 Basic principle

OFDM can be viewed as a form of frequency division multiplexing (FDM) with the special property that each tone is orthogonal with every other tone, but it is different from FDM in several ways. On one hand, FDM requires typically the existence of frequency guard bands between the frequencies so that they do not interfere with each other. On the other hand, OFDM allows spectrum of each tone to overlap, and because they are orthogonal, they do not interfere with each other. Furthermore, the overall amount of required spectrum is reduced due to the overlapping of the tones. Figure 3 shown below gives an idea of the process of modulation and demodulation of OFDM signals. OFDM signals are typically generated digitally due to the difficulty in creating large banks of phase lock oscillators and receivers in the analog domain.



**Figure 3: A simple modulation-demodulation block diagram.**

In the transmitter, the incoming data stream is grouped in blocks of  $N$  data symbols, which are the OFDM symbols. They are passed through OFDM modulators which perform IFFT (as illustrated in Figure 4) on blocks of length  $N$ , followed by inserting a cyclic prefix (CP) of length  $N_g$ . The resulting OFDM symbols of length  $N + N_g$  are launched simultaneously from the individual transmit antennas. The CP is essentially a guard interval which serves to eliminate interference between OFDM symbols and turns linear convolution into circular convolution such that the channel is diagonalized by the FFT.



**Figure 4: Model of an OFDM system.**

In the receiver the individual signals are passed through OFDM demodulators which first discard the CP and then perform an N-point FFT (as illustrated in Figure 4). The output of the OFDM demodulator are finally separated and decoded.

## 2.2 Physical layer Implementation Aspects

Consider  $n_T \times n_R$  MIMO system, where  $n_T$  is the number of transmit antennas and  $n_R$  is the number of receive antennas. Each transmitter sends independent data  $[x_1; x_2; \dots; x_{n_T}]$  from different transmit antennas simultaneously and using the same radio channel. At the receiver end, each antenna receives the composite signal from all transmitters represented by  $[y_1; y_2; \dots; y_{n_R}]$ .

The different paths may be represented mathematically as:

$$\begin{aligned}
 y_1 &= h_{11}x_1 + h_{12}x_2 + \dots + h_{1n_T}x_{n_T} \\
 y_2 &= h_{21}x_1 + h_{22}x_2 + \dots + h_{2n_T}x_{n_T} \\
 &\vdots \\
 &\vdots \\
 y_{n_R} &= h_{n_R1}x_1 + h_{n_R2}x_2 + \dots + h_{n_Rn_T}x_{n_T}
 \end{aligned} \tag{2.1}$$

or, in matrix form as

$$[y] = [H][x] \tag{2.2}$$

The  $[H]$  in the equation represents the transfer matrix of the MIMO channel. By carefully designing a MIMO packet and by using advanced DSP techniques in the MIMO decoder, the various independent transmitted datastreams can be recovered. To recover the transmitted datastream  $[x]$  at the  $[y]$ , the MIMO system decoder must first eliminate the

individual channel transfer coefficient  $h_{ii}$  to determine the channel transfer matrix  $[H]$  during the MIMO preamble of the packet. Once the estimated  $[H]$  has been produced, the transmitted datastream  $[x]$  can be reconstructed by multiplying the vector  $[y]$  with the inverse of transfer matrix  $[H]^{-1}$ . This is represented by:

$$x = [H]^{-1}[y] \quad (2.3)$$

The process, in principle, is equivalent to solving a set of  $N$  unknowns with  $N$  linear equations. To ensure that the channel matrix is invertible, MIMO systems require an environment rich in multipath. It is important to note that unlike traditional methods of increasing throughput by increasing bandwidth, MIMO systems can increase throughput without increasing bandwidth. This is accomplished in a MIMO system by exploiting spatial dimensions and increasing the number of signal paths between the transmitters and the receivers [22]. As each independent datastream is transmitted in parallel from separate antennas, the data throughput increases linearly with every pair of antennas added to the MIMO system.

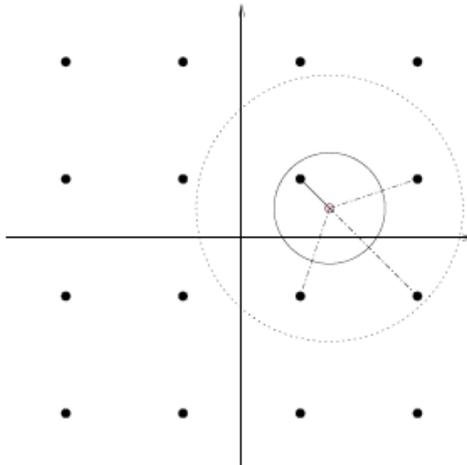
# CHAPTER 3

## Sphere decoding

At the receiver, detection is performed that computes and delivers the estimates of the transmitted data bits. The detector operates on the received signal in each separate transmission symbol interval and produces a number or a set of numbers that represent an estimate of a transmitted Quadrature Amplitude Modulation (QAM) symbol. The decoding methods are used for decision metrics with the aim of making a decision about which bit, 'zero' or 'one' was transmitted. This decision metric can be as simple as hard decision, or more sophisticated, being a soft decision.

### 3.1 Sphere decoder (SD)

Techniques like Maximum likelihood decoding require an exhaustive search over all possible code words used. The computational complexity of these techniques is exponential in the length of the codeword. A promising approach called the sphere decoding algorithm was proposed to lower the computational complexity. The principle of the sphere decoding algorithm is to search the closest lattice point to the received signal within a sphere radius, where each codeword is represented by a lattice point in a lattice field [6]. In 2-D the search can be represented by drawing a circle around the received signal just small enough to enclose one lattice point and eliminate the search of other points outside the circle as shown in Figure 5 below.



**Figure 5: Geometrical representation of the sphere decoding algorithm.**

### 3.1.1 MIMO Channel Model

Consider a MIMO system with  $n_T$  transmit and  $n_R$  receive antennas, with  $n_R \geq n_T$ , denoted as  $n_T \times n_R$ . The transmitted symbols are taken independently from for example a QAM constellation. Let  $n_s$  be the number of bits per alphabet symbol. The coded bit stream is mapped to a  $n_T$  dimensional transmit vector symbol  $s \in \mathcal{O}^{n_T}$ , where  $\mathcal{O}$  stands for the symbol alphabet with cardinality  $|\mathcal{O}| = 2^{n_s}$ .

The channel model is given by complex baseband input-output relation:

$$y = Hs + n \quad (3.1)$$

where  $H$  denotes the  $n_T \times n_R$  channel matrix,

$s = (s_1, s_2, \dots, s_{n_T})^T$  denotes the vector of transmitted symbols,

$n = (n_1, n_2, \dots, n_{n_R})^T$  is the vector of independent and identically distributed

(i.i.d) complex Gaussian noise samples with variance  $\sigma^2 = N_0$

and  $y = (y_1, y_2, \dots, y_{n_R})^T$  is the vector of received symbols.

The main idea behind SD is to reduce the computational complexity of the MLD by searching over only those noiseless received points (defined as  $Hs$ ) that lies within a hyper sphere of radius  $R$  around the received signal  $y$ .

## 3.2 Sphere Decoding Algorithm

The two variants of sphere decoder are hard-output based on the hard decision and the soft-output based on the soft decision. Hard decoding outputs a hard decision as a function of the input, and this form of output is application dependent. It gives uncoded bits at lower complexity. However, the soft decoding outputs a real number in the form of a log-likelihood ratio (LLR). The hard decision does not deliver reliable information while the soft decision would give reliable uncoded bits at the cost of higher complexity.

### 3.2.1 Hard detection

Hard decision is based on finding the minimum Euclidean distance between the received symbol and all allowed points in the constellation map. This method thus involves calculating all cited distances and selecting the received symbol as the point in the constellation map with the smallest Euclidean distance.

The formula to calculate the Euclidean distance is given by Equation:

$$d_E = |y - Hs| \quad (3.2)$$

where  $d_E$  denotes the Euclidean distance.

Thus the knowledge of the channel coefficients or its estimates is needed to implement hard decoding.

To estimate the hard output bits that have been mapped to the symbol vector  $s$  can be obtained by calculating the estimate  $\hat{s}$  from the Euclidean distance as follows:

$$\hat{s} = \arg \min = \arg \min |y - Hs| \quad (3.3)$$

We can obtain the maximum likelihood estimate based on the the conditional pdf of the receive vector  $y$  given the transmitted vector  $s$  as:

$$\hat{s}_{ML} = \arg \max_{s \in \mathcal{O}^{n_T}} \left\{ f(y|s) \frac{1}{2^{n_s n_T}} \right\} = \arg \max_{s \in \mathcal{O}^{n_T}} f(y|s), \quad (3.4)$$

where  $f(y|s)$  is the conditional pdf.

For a system model with additive white Gaussian noise (AWGN) and known channel the conditional pdf is obtained as:

$$f(y|s) = \frac{1}{(\pi\sigma_n^2)^{n_R}} \epsilon^{-\frac{1}{\sigma_n^2} \|y - Hs\|^2}, \quad (3.5)$$

using which the ML solution is obtained as:

$$\hat{s}_{ML} = \arg \min_{s \in \mathcal{O}^{n_T}} \|y - Hs\|^2. \quad (3.6)$$

But this minimisation yields only hard output of the transmit symbol vector. It does not offer reliable information and omits a large part of the information contained in the receive symbol vector.

### 3.2.2 Soft-output SD

In contrast to hard detection, soft detection requires the computation of LLRs for all coded bits to propose a decision metric. These LLRs are calculated for every bit of the symbol. This method gives a better probability whether the transmitted bit was a zero or a one.

The LLR can be defined as:

$$L(x_j|y) = \ln \frac{P(x_j=1|y)}{P(x_j=0|y)}, \quad (3.7)$$

where  $P(x_j = 1|y)$  and  $P(x_j = 0|y)$  are the conditional probabilities for bit  $x_j = 1$  and  $x_j = 0$  respectively conditioned on  $y$ .

Using Baye's theorem and assuming that both cases  $x_j = 1$  and  $x_j = 0$  are equally likely, we can obtain

$$L(x_j|y) = \ln \frac{\sum_{x_j \in \chi_j^{(1)}} p(y|x_j)}{\sum_{x_j \in \chi_j^{(0)}} p(y|x_j)}, \quad (3.8)$$

where  $p(y|b)$  is the conditional probability density function of  $y$  conditioned on vector  $\mathbf{x}$ ,  $\chi_j^{(1)}$  and  $\chi_j^{(0)}$  are disjoint set of vector symbols that have the  $j$ th bit  $x_j$  of the transmit symbol  $\mathbf{s}$  equal to 0 and 1 respectively.

Assuming  $y$  is Gaussian distributed we can write:

$$p(y|x) = \frac{1}{(\pi\sigma_n^2)^{n_R}} e^{-\frac{1}{\sigma_n^2} \|y - Hs(x)\|^2}. \quad (3.9)$$

The Jacobian logarithm can be defined as:

$$\begin{aligned} \text{jac} \ln(a_1, a_2) &\triangleq \ln(e^{a_1} + e^{a_2}) = \max(a_1, a_2) + \ln(1 + e^{-|a_1 - a_2|}) \\ &= \max(a_1, a_2) + r(|a_1 - a_2|) \end{aligned} \quad (3.10)$$

Using (3.9) in (3.8) and using the Jacobian logarithm (3.10) we find the LLR as:

$$\begin{aligned} L(x_j|y) &= \ln \frac{\sum_{x \in \chi_j^{(1)}} p(y|x)}{\sum_{x \in \chi_j^{(0)}} p(y|x)} = \ln \sum_{x \in \chi_j^{(1)}} p(y|x) - \ln \sum_{x \in \chi_j^{(0)}} p(y|x) \\ &= \ln \sum_{x \in \chi_j^{(1)}} e^{-\frac{1}{\sigma_n^2} \|y - Hs\|^2} - \ln \sum_{x \in \chi_j^{(0)}} e^{-\frac{1}{\sigma_n^2} \|y - Hs\|^2} \\ &= \max_{x \in \chi_j^{(1)}} \left\{ -\frac{1}{\sigma_n^2} \|y - Hs\|^2 \right\} - \max_{x \in \chi_j^{(0)}} \left\{ -\frac{1}{\sigma_n^2} \|y - Hs\|^2 \right\} \end{aligned} \quad (3.11)$$

(In order to reduce the computational complexity the max-log approximation is used [1]).

$$L(x_j|y) = \min_{x \in \chi_j^{(1)}} \|y - Hs\|^2 - \min_{x \in \chi_j^{(0)}} \|y - Hs\|^2 \quad (3.12)$$

We rewrite (3.12) as:

$$L(x_j, b) = \min_{s \in \chi_{j,b}^{(1)}} \|y - Hs\|^2 - \min_{s \in \chi_{j,b}^{(0)}} \|y - Hs\|^2 \quad (3.13)$$

where  $\chi_{j,b}^{(1)}$  and  $\chi_{j,b}^{(0)}$  are a disjoint set of vector symbols that have the  $b$ th bit in the label of the  $j$ th scalar symbol equal to 1 and 0, respectively. For each bit, one of the two minima in (3.13) is given by:

$$\lambda^{ML} = \|y - Hs^{ML}\|^2, \text{ where} \quad (3.14)$$

$$s^{ML} = \arg \min_{s \in \mathcal{O}^{\text{NT}}} \|y - Hs\|^2 \quad (3.15)$$

is the maximum likelihood (ML) solution. The other minimum in (3.13) is given by:

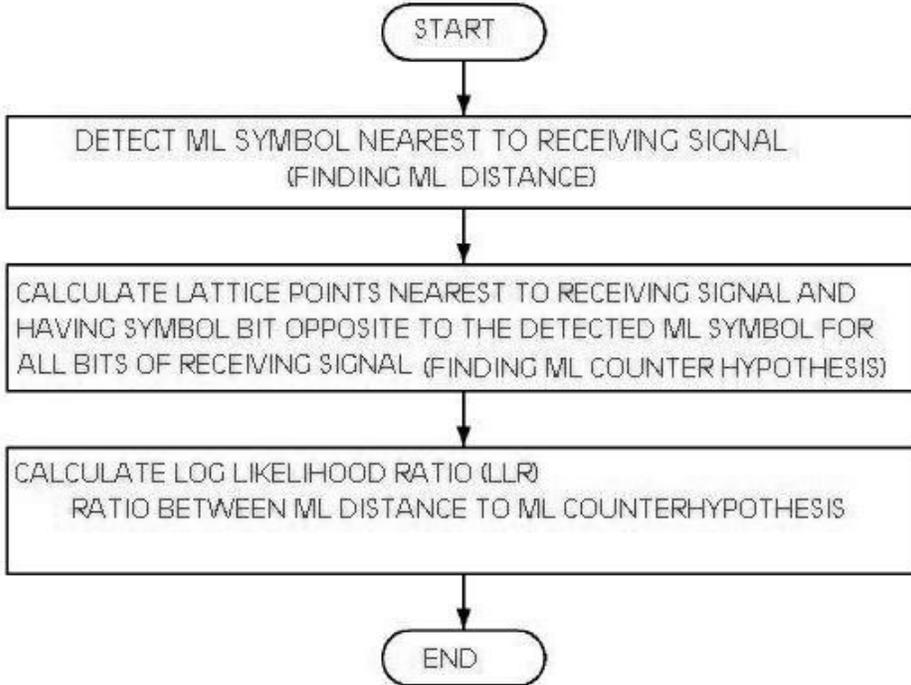
$$\lambda_{j,b}^{ML} = \min_{s \in \chi_{j,b}^{ML}} (x_{j,b}^{ML}) \|y - Hs\|^2 \quad (3.16)$$

where the *counter hypothesis*  $x_{j,b}^{ML}$  denotes the binary complement of the  $b$ th bit in the binary label of the  $j$ th entry of  $s_{ML}$ .

Using (3.14) and (3.16) the max-log LLRs as in (3.13) can be rewritten as:

$$L(x_{j,b}) = \begin{cases} \lambda_{j,b}^{ML} - \lambda_{j,b}^{ML} & \text{for } x_{j,b}^{ML} = 0 \\ \lambda_{j,b}^{ML} - \lambda_{j,b}^{ML} & \text{for } x_{j,b}^{ML} = 1 \end{cases} \quad (3.17)$$

Thus it can be said from (3.17) that efficient max-log APP MIMO detection reduces to efficiently identifying  $s^{ML}, \lambda^{ML}$  and  $\lambda_{j,b}^{ML}$  for  $j = 1, 2, \dots, M$  and  $b = 1, 2, \dots, Q$



**Figure 6: Simple flowchart of the Soft-output sphere decoder algorithm.**

As the next step, **QR**-decomposition is done on the channel matrix **H** is according to  $H = QR$ , where **Q** is unitary, ie  $Q^H Q = I$  and **R** is the upper-triangular with real-valued positive entries on its main diagonal.

Multiplying (3.1) by  $Q^H$  leads to the modified input-output relation:

$$\tilde{y} = Rs + Q^H n \quad \text{with} \quad \tilde{y} = Q^H y \quad (3.18)$$

Noting that  $Q^H n$  has the same statistics as  $n$  gives an equivalent formulation of  $\lambda^{ML}$  and  $\lambda_{j,b}^{\overline{ML}}$  as :

$$\lambda_{ML} = \min_{s \in \mathcal{O}^{n_T}} \|\tilde{y} - Rs\|^2 \quad (3.19)$$

$$\lambda_{j,b}^{\overline{ML}} = \min_{s \in \mathcal{X}_{j,b}^{(\overline{ML})}} \|\tilde{y} - Rs\|^2 \quad (3.20)$$

The computation of the distances can be transformed into a tree search problem and then using the sphere decoding algorithm (as described in the flowchart shown in Figure 6) leads to an efficient computation of LLRs.

Define partial symbol vectors (PSVs) as  $s^{(i)} = [s_i, s_{i+1}, \dots, s_{n_T}]^T$ . PSVs  $s^{(i)}$  are arranged in a tree that has its root just above level  $i = M$  and leaves, on level  $i = 1$ , which corresponds to possible candidate symbol vectors **s** [2].

In the following, the label associated with  $s^{(i)}$  is denoted by  $x^{(i)}$ .

The Euclidean distance is calculated as:

$$d(s) = \|\tilde{y} - Rs\|^2 \quad (3.21)$$

Initialise distance  $d_{M+1} = 0$  and using (3.21) to compute distance for (3.19) and (3.20) is done recursively as  $d(s) = d_1$  with partial Euclidean distances (PEDs)

$$d_i = d_{i+1} + |e_i|^2, \quad \text{where } i = n_T, n_T - 1, \dots, 1 \quad (3.22)$$

and the distance increments (DIs)

$$|e_i|^2 = |\tilde{y}_i - \sum_{j=i}^{n_T} R_{i,j} s_j|^2 \quad (3.23)$$

As the dependence of the PEDs  $d_i$  on the symbol vector  $\mathbf{s}$  is only through  $s^{(i)}$ , ML detection and computation of the max-log LLRs have been transformed into a weighted tree-search problem.

PSVs and PEDs are associated with *nodes* while *branches* correspond to DIs. Each path from root down to a leaf corresponds to a symbol vector  $\mathbf{s} \in \mathcal{O}^{n_T}$ . The leaf associated with the smallest metric in  $\mathcal{O}^{n_T}$  and  $\chi_{j,b}^{(x_{j,b}^{ML})}$  corresponds to the solution of (3.19) and (3.20) respectively.

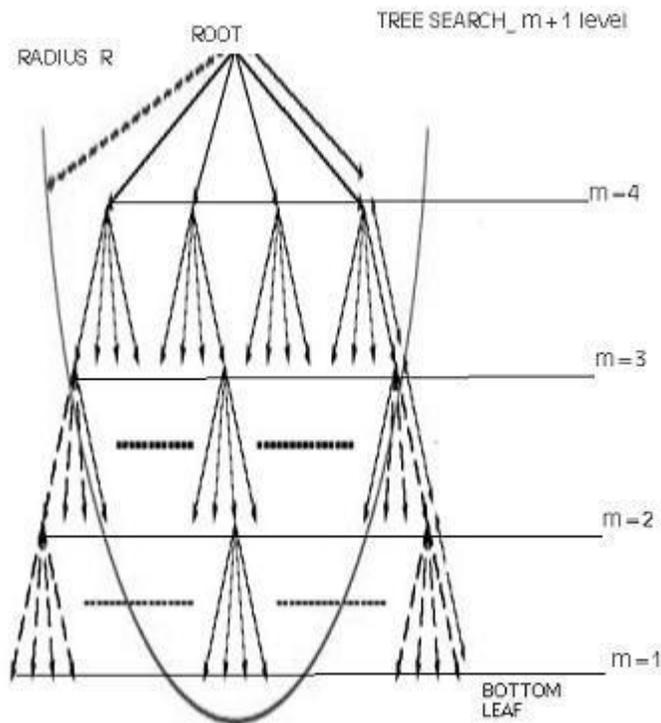
### 3.2.3 Schnorr-Euchner SD (SESD)

#### *Depth First Traversal*

Using SESD no initial radius to limit the search is selected; instead the radius to start the search is initially kept at infinity. Later it updates the radius based on PEDs. Using this, the search is constrained to nodes which lie within the updated radius and traverses the tree depth first, visiting the children of a given node in ascending order of their PEDs.

#### *Radius reduction*

Start the search keeping radius  $r = \infty$  and update the radius according to  $r^2 \leftarrow d(s)$  whenever a leaf  $s$  is reached. This technique avoids the problem to selecting a suitable radius and leads to efficient pruning of the tree. Figure 7 below shows how sphere decoding can be seen as a tree search problem.



**Figure 7: Sphere decoding as a Tree-search problem, with  $m = 4$  as the number of Transmit antennas.**

### 3.2.4 Tree traversal Algorithm

The tree traversal (based on [1]) to find solution to (3.19) and (3.20) can be done in two different ways:

#### *Repeated Tree Search*

The traversal starts by solving (3.19) using SESD and to rerun the SESD to solve (3.20) for each coded bit in the vector symbol. As solving (3.20) by traversing those parts of the tree that has leaves in  $\chi_{j,b}^{(\overline{x_{j,b}^{ML}})}$  has to be done for every coded bit; leading to repeated tree traversals with increased complexity. In repeated tree search when rerunning SESD to determine  $\lambda_{j,b}^{\overline{ML}}$ , the search tree is pruned by forcing the decoder to exclude all nodes and the corresponding subtrees from the search for which  $x_{j,b} = \overline{x_{j,b}^{ML}}$ . However this search leads to repeated traversal of large parts of the tree while finding the counter hypotheses, leading to higher computational complexity.

#### *Single Tree Search*

This is a more efficient tree search that ensures that every node in the tree is visited at most once. This can be accomplished by searching for the ML solution and *all* counter-hypotheses concurrently. The key lies in formulating update rules and a pruning criterion based on a list containing the distance metrics  $\lambda_{ML}$  and  $\lambda_{j,b}^{\overline{ML}}$ .

The algorithm is initialized with  $\lambda^{ML} = \lambda_{j,b}^{\overline{ML}} = \infty (\forall j, b)$ . Whenever leaf with corresponding binary label  $\mathbf{x}$  has been reached, the decoder distinguishes between two cases:

- 1) If a new ML hypothesis is found, i.e.,  $d(x) < \lambda^{ML}$ , all  $\lambda_{j,b}^{\overline{ML}}$  for which  $x_{j,b} = \overline{x_{j,b}^{ML}}$  are set to  $\lambda^{ML}$  followed by the updates  $\lambda^{ML} \leftarrow d(x)$  and  $x^{ML} \leftarrow x$ . In other words, for each bit in the ML hypothesis that is changed in the process of the update, the distance metric of the *former* ML hypothesis becomes the distance metric of the new counter-hypothesis, followed by an update of the ML hypothesis.
- 2) In the case where  $d(x) \geq \lambda^{ML}$ , only the counter-hypotheses have to be checked. For all  $j$  and  $b$  for which  $d(x) < \lambda_{j,b}^{\overline{ML}}$  and  $x_{j,b} = \overline{x_{j,b}^{ML}}$ , the decoder updates  $\lambda_{j,b}^{\overline{ML}} \leftarrow d(x)$

### Pruning criterion

The key aspect of this algorithm is the following pruning criterion. A given node  $s^{(i)}$  on level  $i$  and the subtree originating from that node have the partial binary label  $x^{(i)}$  consisting of the bits  $x_{j,b}$  ( $b = 1, 2, \dots, Q$  and  $j = i, i + 1, \dots, M$ ). The remaining bits  $x_{j,b}$  ( $j = 1, 2, \dots, i - 1$ ) corresponding to the subtree are unknown at this point. The pruning criterion for  $s^{(i)}$  along with its subtree is compiled from two conditions. First, the bits in the partial binary label  $x^{(i)}$  are compared with the corresponding bits in the binary label of the current ML hypothesis. In this comparison, for all  $j, b$  with  $x_{j,b} = x_{j,b}^{\overline{ML}}$ , the corresponding counter-hypotheses  $\lambda_{j,b}^{\overline{ML}}$  might be affected when further searching the node's subtree. Second, *all* counter-hypotheses corresponding to the subtree of  $s^{(i)}$  with the associated distance metrics  $\lambda_{j,b}^{\overline{ML}}$  ( $j = 1, 2, \dots, i - 1$ ) may also be updated since the corresponding bits are not known yet.

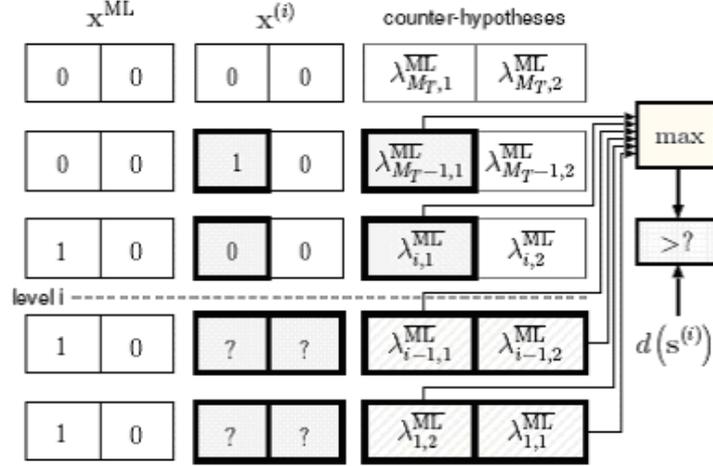
Summarising, the distance metrics which may be affected during further search in the subtree emanating from a node  $s^{(i)}$  are given by the set

$$A = \{a_l\} = \left\{ \lambda_{j,b}^{\overline{ML}} \mid x_{j,b} = x_{j,b}^{\overline{ML}}, j \geq i \right\} \cup \left\{ \lambda_{j,b}^{\overline{ML}} \mid j < i \right\} \quad (3.24)$$

The node  $s^{(i)}$  along with its subtree is pruned if its PED  $d(s^{(i)})$  satisfies

$$d(s^{(i)}) > \max_{a_l \in A} a_l \quad (3.25)$$

This pruning criterion (as illustrated in Figure 8) ensures that the subtree of a given node is explored only if it can lead to an update of either the ML hypothesis or of at least one of the counter-hypotheses. Note that  $\lambda^{ML}$  does not appear in (3.24) as  $\lambda^{ML} \leq \lambda_{j,b}^{\overline{ML}} (\forall j, b)$ .



**Figure 8: Example of a Single Tree Search pruning criterion ( $M = 5$  and two bits per symbol).**

The partial binary label  $x^{(i)}$  determines which counter-hypotheses may be affected during the search of the subtree emanating from the current node (taken from [1]).

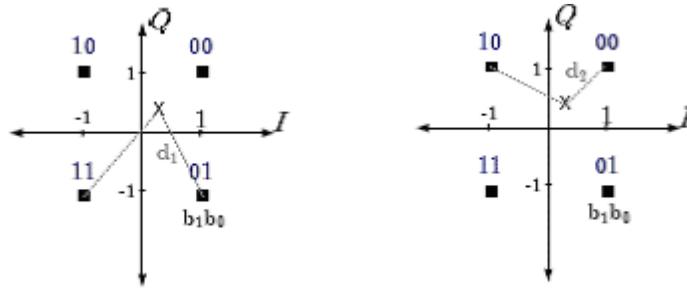
### Interpretation of LLR

To explain the calculation of LLR we rewrite (3.13):

$$LLR(b_k) = \min_{s|b_k=1} \|y - Hs\|^2 - \min_{s|b_k=0} \|y - Hs\|^2 \quad (3.26)$$

As can be seen from (3.26), the LLR is calculated from the minimum distance between the received symbol and all symbols in the constellation map where the bit  $b_k$  is equal to “one”, the minimum distance between the received symbol and the constellation points where the bit  $b_k$  is equal to “zero” in this position [5].

A positive LLR corresponds to a “zero”, and a negative LLR corresponds to a “one”. Hence the larger the LLR is in absolute terms, the higher is the probability that a “zero” or a “one” was transmitted. Consider a 4-QAM constellation map with the received symbol represented with an “x”. Assume the calculation of the LLR for the bit  $b_0$  as shown in Figure 9.

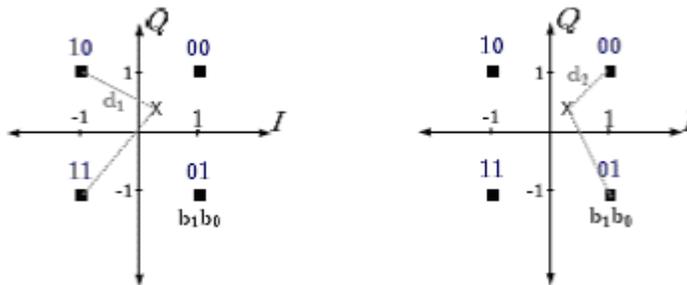


**Figure 9: Example of calculating the LLR for the bit  $b_0$ .**

In subfigure (a), the minimum distance between the received bit and  $b_0 = 1$  is calculated. Subfigure (b) makes the same calculation for  $b_0 = 0$ .

$d_1$  is the minimum distance between the received bit and the points in the constellation map that have a bit equal to “one” in the position of the bit  $b_0$ . The same operation is performed for the bit  $b_0$  equal to “zero”, obtaining the distance  $d_2$ . The numerical values of  $d_1$  and  $d_2$  are 1.56 and 0.92 respectively. The LLR can be calculated as the difference between both distances:  $LLR = d_1 - d_2$ . Replacing the numerical values gives a result of 0.64 for  $LLR(b_0)$ .

Using the same steps to get the LLR value for bit  $b_1$  is shown in Figure 10.



**Figure 10: Example of calculating the LLR for the bit  $b_1$ .**

In this case,  $d_1 = 1.43$ ,  $d_2 = 0.92$ ; which gives an  $LLR(b_1) = 0.51$ .

Since both  $LLR(b_0)$  and  $LLR(b_1)$  have positive values, it can be concluded that the symbol “00” is the one that has been transmitted with the highest probability.

# CHAPTER 4

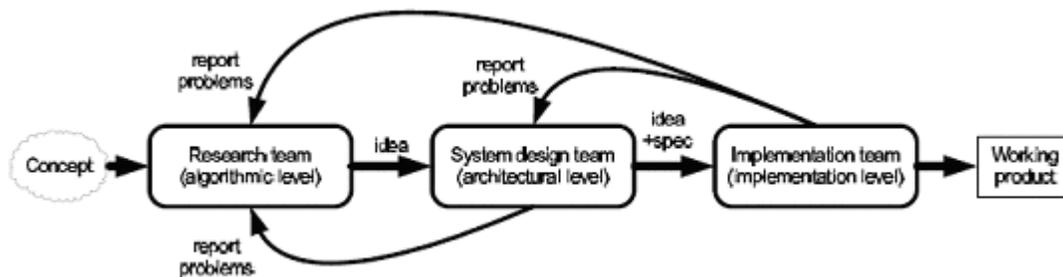
## Rapid prototyping methodology

Digital signal processing (DSP) algorithms are used in an ever increasing number of embedded real-time applications that include systems as simple as consumer products to very sophisticated control systems. Prototyping of these systems include a number of steps, which usually start with a high-level description and simulations using common engineering tools such as MATLAB and Simulink for at least a part of the application. One of the major issues in embedded DSP systems prototyping and implementation is the number of abstraction levels used to express the same functionality. Each of these description levels introduces modification of the algorithms that may influence the final outcome [10].

For example, say we are investigating the performance of highly sophisticated wireless systems involving multiple-transmit and receive antennas. Let us start the simulation of the algorithms using MATLAB that has a highly accurate double-precision numerical environment, which is an ideal tool for this purpose; while many imperfections of the *real-world* neglected [7]. A first evaluation of the performance of a novel idea is usually done mathematically based on very basic, typically simplistic assumptions. Gaining confidence in algorithms developed with theoretical models by means of simulations is an essential part of the early development stage, since wireless channels suffer from a wide variety of effects that are often difficult to model accurately [12]. The next step that aims at the algorithm implementation requires further algorithm modification in order to meet the capabilities of future products (that uses fixed point arithmetic. All this requires repeated verification before the final step of algorithm synthesis for the target platform. Target platforms include technologies such as ASICs, FPGAs, general-purpose processors and Digital Signal Processors (DSPs). The final step involves either transformation into synthesisable hardware description (behavioural or Register Transfer Level, RTL) or machine code of the target processor.

## 4.1 Classical Design Methodology

The design process, leading from concept to realization passes through three general levels of refinement, namely the *algorithmic*, the *architectural* and the *implementation* levels as illustrated in Figure 11 below. In the design process, the teams have necessarily distinct areas of expertise, to tackle each of the three stages. Inherently, each of the teams works with a dedicated set of tools on a system description that is optimized for its work [11].



**Figure 11: Typical product development strategy diagram.**

### *The Research team*

A research team works out new technological ideas to include new features or improve existing ones. The *verification* of such new ideas is typically performed by simulations. Favoured are MATLAB or simply C-programming. The use of graphical tools like Simulink is rarely used in this context since the researchers feel restricted by the formalism and constraints of such tools [9].

### *The System design or Architecture team*

Once the simulation results of the research team are satisfactory, the system design team comes into play. After initial guiding about the newly developed methods from the first team, the design team decides what techniques will be used to map such algorithmic ideas into software and hardware. In particular, which components are mapped into general purpose processors and which require dedicated hardware architectures. During the course of the design team's work, questions arise regarding the feasibility of the future product. These questions are fed back to the research team and typically answered by including more details into the simulations.

### *The Implementation team*

The primary task of the implementation team is to build all required components from outside vendors and to join them with existing parts; they have to make the product work. The implementation team has knowledge of the specific tools that are required to program DSPs, FPGAs and to design ASICs. They need to be taught the innovations of the first team, supplemented by the implementation guidelines of the design team. Further the algorithm implementation requires a permanent co-simulation of the actual implementation code with the original high-level simulation. There is interaction between the two teams and once the implementation team is confident of having a good reference code, they can start building an ASIC or programming DSPs. From here on, co-simulation for the purpose of verification is mostly what defines their work. Hence it is most helpful when a design environment supports co-simulation [9].

## **4.2 Rapid prototyping**

### **4.2.1 Motivation for Rapid prototyping**

As described in the Classical design technique on the design flow that is used to realizing the product; due to severe time constraints, prototypes are often skipped and it is mostly the simulation results and experience from the previous product that is relied upon. However the advantage of prototyping was to reduce the investment risk of the new product in case the new technology would hide unforeseen challenges [7]. Also, one often obtained a flavour of how the new technology needs to be realized and a feeling for how the future product would look like. It could be used to present to the potential customers or to work on potential features long before the final product was available.

Next-generation mobile technology keeps updating with such rapid pace, bringing added pressure on the time to market. Since prototyping is cumbersome work, both the time and cost factor, with the development of the prototype with all features as costly as the final product itself. As a result telecommunication equipment companies often decide to focus more on simulation and trust the expertise of their research team.

## 4.2.2 Key to Rapid prototyping

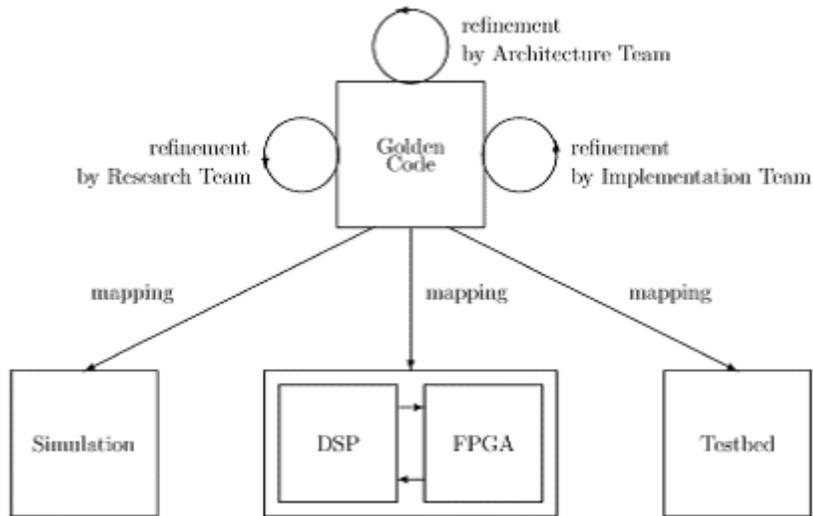
From the classical design approach, it can be seen that the section often exhibits discontinuous phases. Typically, at the time when the design is handled from one team to the next, an entire new set of tools, languages and formalisms is used and the previous code not even used for reference purposes. Further when there is a problem in the code, a team cannot return their design to the previous team as the current code is not *backwards compatible*. Thus the classical approach is a feed-forward structure where the refined product and corresponding responsibility is moved in one direction while necessary expertise and information revealing discussions are only made possible by backward loops.

To make a prototype rapidly could mean to bring out a partial set of features of the final product in a rapid way if not the whole product. In this way only the technical parts that are new to the implementation team are implemented.

Rapid prototyping is in a way similar to product design, requiring a team with almost the same set of skills. The only difference being that fewer people are required with skill sets mainly in electronic design.

The key essentials to make prototyping rapid (and possibly speed up production) can be followed by the Five-Ones Approach [9]:

- one environment
- one automatic documentation by specification
- one forward-backward compatible code revision tool
- one code to be worked on by refinement steps
- one team to improve communication



**Figure 12: Rapid prototyping design flow (taken from [9]).**

This approach is depicted by Figure 12, allows a continuous flow in which the teams start with designs from the previous teams but stay within the same design environment and also allowing for *backward-compatibility* while solving specific problems.

The basic concept of Rapid prototyping is mentioned in this Chapter which highly motivates the implementation of the algorithm and later to perform changes in the implementation by allowing backward compatibility.

## CHAPTER 5

### Implementation in high-level language

The implementation in high-level language tends to produce a less efficient code, but it is still preferred over assembly language programming due to several advantages like:

- It helps the developer or the developers to achieve structure and consistency.
- Complex operations are often easier to describe in a high-level language.
- High-level language development does not require very specialized skills or in-depth knowledge of the target processor. Engineers who are not specialist can readily undertake the development of the high-level language modules sharing the workload of the few specialized engineers who are able to develop in assembly language on a particular platform [18].

#### 5.1 Challenges toward Algorithm Implementation

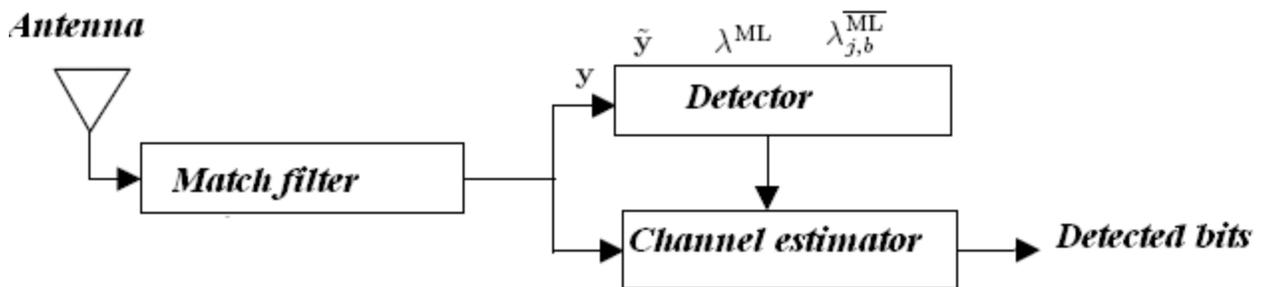
While designing next-generation communication systems with newer features, there is stiff challenge in the development and integration of several computationally-intensive algorithms towards enabling the new features. Designers are faced with two major problems while designing these systems. First, block diagrams and equations compose typical communication systems; however prototype hardware must be programmed in C or assembly, an awkward and error prone means to implement block diagrams and equations. Second, designers develop simulations which execute on a host, while other engineers create hardware prototypes [13]. However, the host and prototype platforms remain isolated from each other; the simulator's power cannot be combined with the real-time constraints of the prototype.

In the design process, the designer must integrate algorithms into a communication system executing on prototype hardware. Each algorithm, usually expressed as a set of equations, must be tested to insure it operates as expected. The algorithm blocks in the block diagram must be checked to verify that each block is properly connected and

operates correctly with its neighbouring blocks. Finally, the resulting block diagram of the system must be translated into a program suitable for execution on the prototype hardware.

For example, Figure 13 shows a simple block of a receiver. Each block is annotated with equations, which specify the algorithms implemented by that block. Simulation is first used to verify the correct operation of each block and then of the entire system. Finally, the system is translated into C or an HDL and then compiled to run on the DSPs or FPGA prototype hardware.

Unfortunately, the languages and design tools available today are largely incompatible with each other and are usually unable to execute both on the host and on the DSPs. The tools and languages used such as MATLAB, Simulink, C and hardware description languages interoperate poorly, and run either only on the host or only on the prototype.



**Figure 13:** A simple receiver, represented as a block diagram with blocks containing algorithm specified by symbols.

Algorithm designers prefer a powerful programming language such as MATLAB which is tailored to the description of equations. Algorithms written in MATLAB, however, cannot directly execute on DSPs, though there are several promising papers in this area [13, 14, and 15]. Communication system designers prefer a block diagram entry and simulation packages such as Simulink. However, like MATLAB, Simulink runs only on the host and its ability to integrate MATLAB into algorithm is very poor. C code written for the DSPs typically uses DSPs-only libraries, preventing it from executing on the host. Integrating C code with MATLAB or with Simulink is a difficult task and requires knowledge of the MATLAB C – MEX interface [13, 16] or the Simulink S-function interface [13, 17].

The monolingual and uni-location nature of today's languages and tools limits the complexity of designs achievable. First, they restrict a designer to the use of only one language for the entire design, though the use of an alternate language for parts of the design is preferable. For example, a MIMO receiver designed in MATLAB implies that all block diagrams must be expressed as text in MATLAB's programming language, rather than representing the diagrams graphically. This text representation obscures the block diagram structure of the system, which also hinders the compiler's ability to understand and optimize the design. Second, today's languages and tools force the designer to rewrite the entire design when moving between languages or locations. Moving a system based on algorithms written in MATLAB from the host to a prototype requires all MATLAB code to be rewritten in C. Finally, modern languages and tools isolate the host-based simulation environment from the DSP-based execution environment. Real-time data acquired by the prototype hardware cannot be easily passed back to the host for analysis; likewise, simulated data generated on the host cannot be processed on the prototype.

## 5.2 Implementation in MATLAB

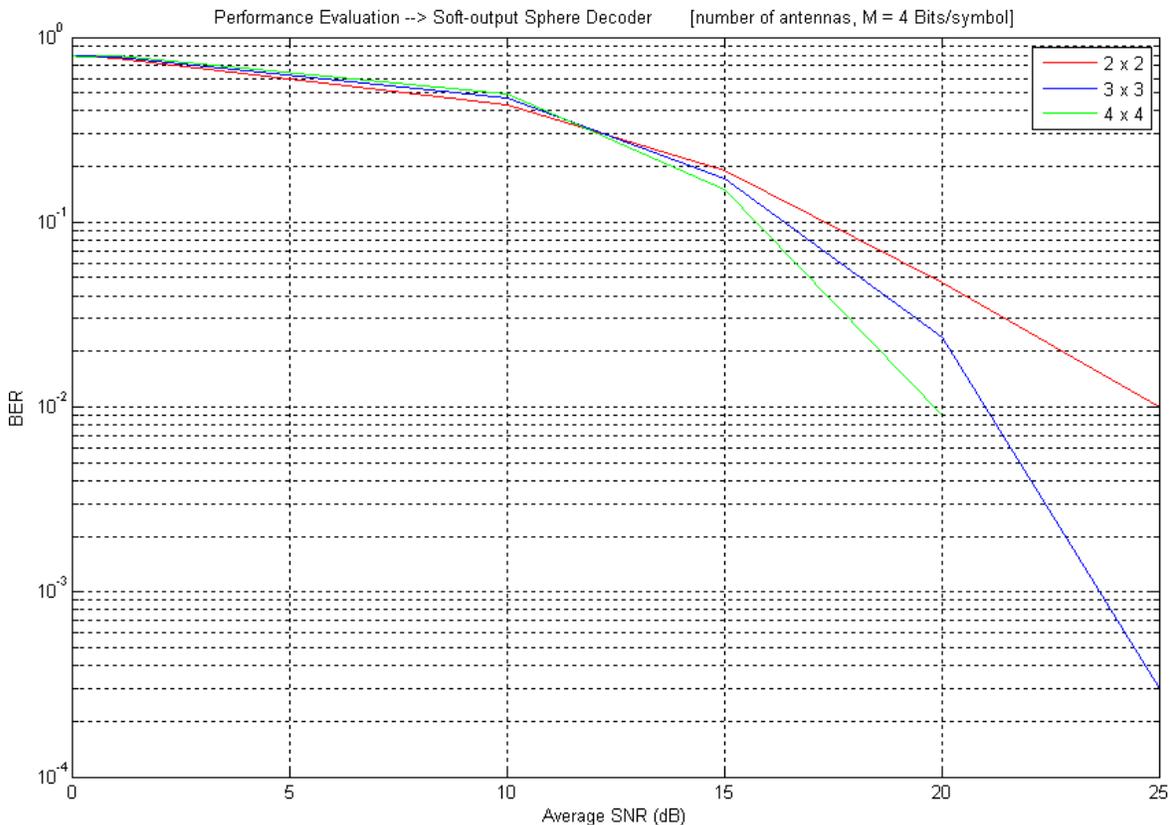
MATLAB [8], a popular tool for DSP engineers, is an excellent candidate for designing algorithms based on a set of equations. MATLAB's language allows these equations to be easily entered in its text-based language. However, as a text-only language, it lacks the ability to graphically represent block diagrams. In addition, it cannot yet produce efficient code for execution on a hardware prototype. MATLAB's programming language contains several features making it uniquely suited for algorithm design as illustrated in Table 1.

**Table 1: Equations used for distance calculation in the Sphere Decoder on the left side of the table and their equivalent in MATLAB on the right side of the table**

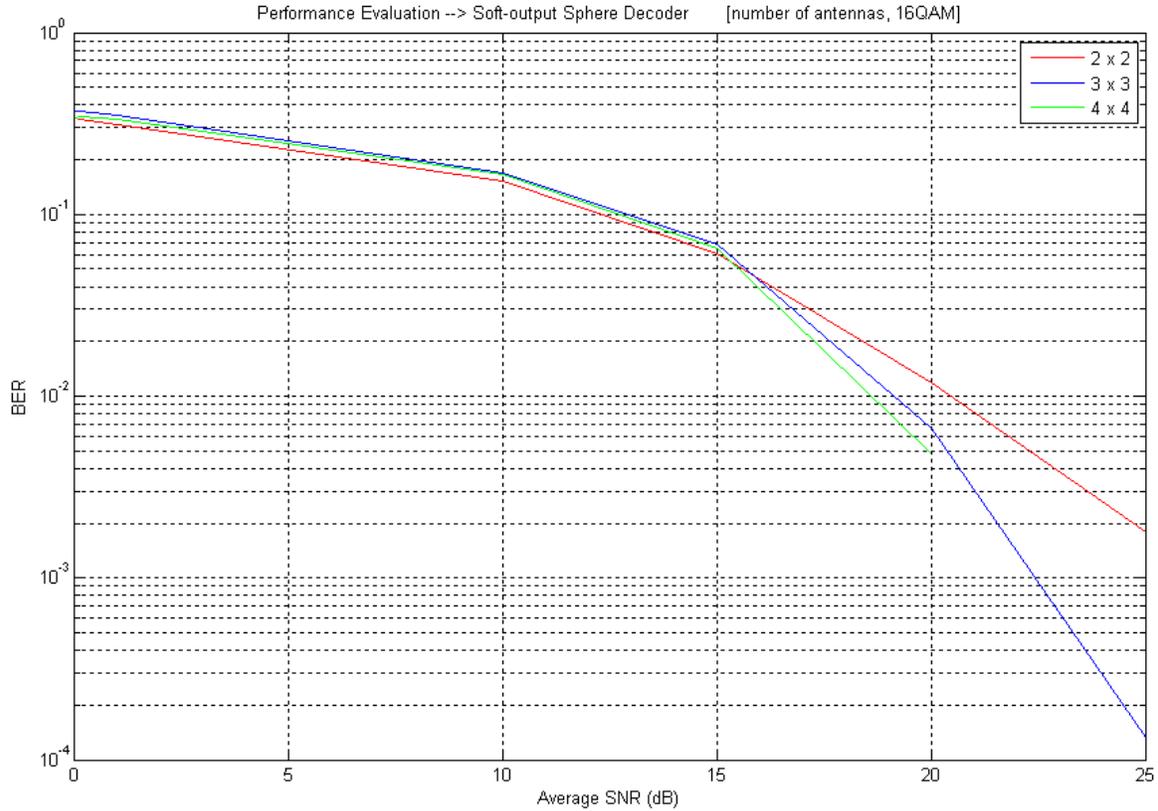
$y = Hs + n$	<code>y = H*s + n</code>
$\tilde{y} = Rs + Q^H n$	<code>[Q,R] = qr(H)</code> <code>y_tilde = R*s + Q'*n</code>
$d(s) = \ \tilde{y} - Rs\ ^2$	<code>dist = abs(y_tilde - R*s)^2</code>
$i = M_T, M_T - 1, \dots, 1$	<code>for i = MT :-1: 1</code>
$d_i = d_{i+1} +  e_i ^2$	<code>dist(curr) = dist(curr+1) +</code> <code>abs(e(curr))^2</code>
	<code>end</code>

Most importantly, MATLAB allows the user to easily enter complex equations, by defining common arithmetic operators such as addition and multiplication for scalars, vectors and matrices over both real and complex numbers. In addition, MATLAB supports a wide range of toolboxes for standard signal processing operations. Its powerful language and flexible plotting features allow the user to quickly develop and debug new algorithms.

The Sphere decoder was implemented in MATLAB – both the hard and soft output was evaluated for performance. Figure 14 below evaluates the performance for a soft output sphere decoder for symbols compared with 16 QAM and 4 Bits per symbol. The plot shows the Bit Error Ratio (BER) with Average Signal to Noise Ratio (SNR) in dB for different antenna configurations. Also the performance is evaluated with bits compared for the same parameters are shown in Figure 15. In both cases it can be seen that at lower SNR different antenna configurations show similar BER till around 15 dB. As the SNR increases further, the higher antenna configuration start to give reduced BER.



**Figure 14: Performance evaluation for soft-output sphere decoder with symbols compared.**



**Figure 15: Performance evaluation for soft-output sphere decoder with the bits compared.**

However, MATLAB's programming language is less suited for describing block diagrams. Unlike synchronous dataflow languages like Simulink, time is not defined in MATLAB. In Simulink, each block specifies the rate at which input data arrives and output data is ready. The knowledge of these rates and the interconnections between blocks enable Simulink to compute an order in which each block must be run. In contrast, MATLAB provides none of these services, which are restrictive when developing new algorithms. Instead, the MATLAB programmer must manually schedule blocks, and write additional code which defines a variable to represent time.

## Summary

Implementing an entire communication system using traditional design methodology suffers from segregation. An idea captured as a rough sketch of a block diagram, with equations noted beside each block, cannot be entered into a computer; the block diagram tool is incompatible with the equation tool. When debugging the system, particularly on embedded hardware, the best analysis and debug tools available on the host must be abandoned for less usable hardware-specific tools designed for the embedded hardware.

Rapid prototyping methodology helps tackle these problems by unifying these classes of tools, providing both block diagram design and equation entry in the same environment. It provides an easy path to migrate from equations coded in MATLAB, which cannot be executed on most embedded hardware platforms, to equations written in C or C++, languages which are supported by most embedded hardware platforms. With each block coded in C, a C program can then be generated and compiled on the embedded hardware. It also enables a development environment that runs partly on the host, with its improved analysis and debug tools, and partly on the embedded hardware.

In this way, the communication system can be build by following the Rapid prototyping methodology. The Sphere Decoder is a part of the MIMO-OFDM WIMAX Receiver that is being implemented using Simulink. This thesis work is restricted to the implementation of the Sphere Decoder by Rapid prototyping methodology. First the sphere decoder is implemented in MATLAB and then verified with a C implementation.

## 5.3 Implementation in ‘C’

When it comes to implementation for embedded applications, the high-level language of choice is C and recently C++. It is usually advantageous to write the higher hierarchical levels of the software applications in C or C++ because it is often the place where the most complexity is found, and at the same time it often represents the less critical portions of the code.

The C programming language produces the highest-performance executables on DSPs. Most modern DSPs provide compilers with excellent support for the C programming language. Also it is almost always necessary to develop some modules of code directly in assembly language. Generally the following factors contribute to the development of code in assembly language:

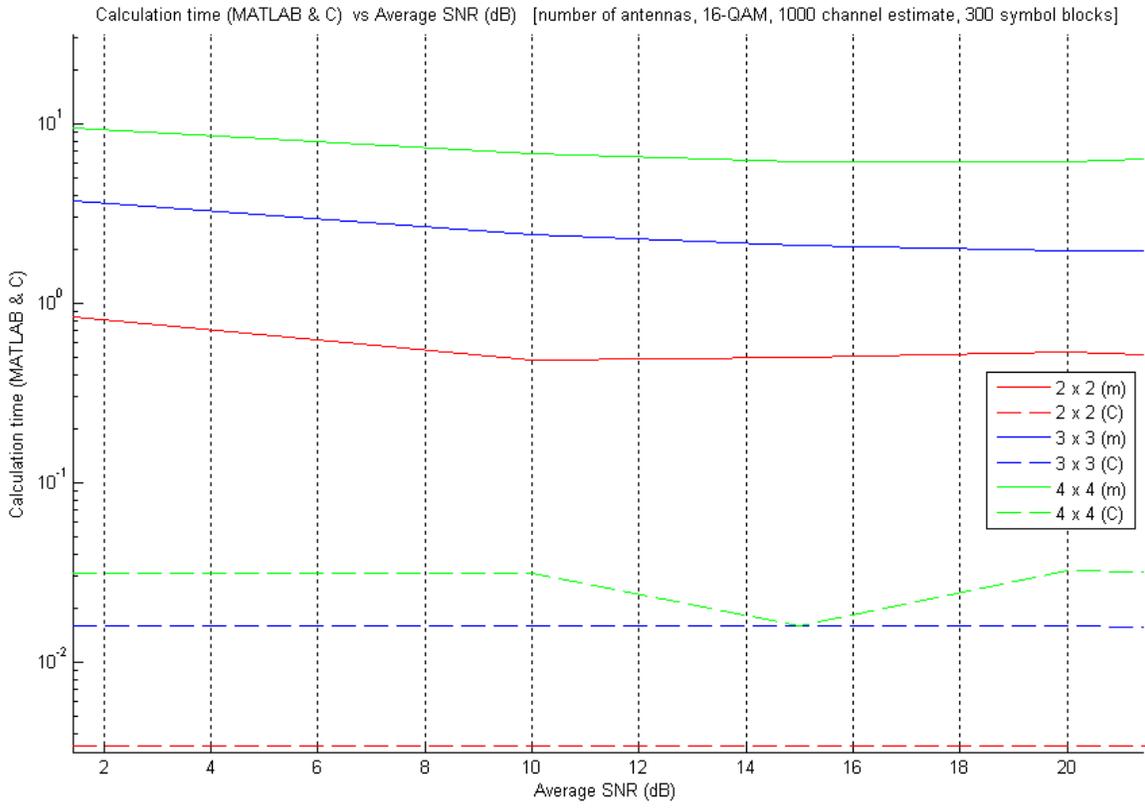
- The C standard is not well defined for many functions that are important in numerical computation and signal processing applications. For instance the overflow behaviour during an integer addition (saturation or rollover) is not defined in the C standard. For certain combinations of compiler and CPU, the behaviour may even change with the context of execution during the same application.
- A code developed in C or C++ is almost always less efficient than the same code [well] developed in assembly language.
- The developer normally chooses to write the code in a high-level language to avoid being concerned with the low-level implementation of the code. High-level languages therefore do not give the developer access to low-level functions of the CPU or its ALU. However, it is precisely by getting access to these low-level functions that the developer is usually able to optimize the efficiency of the code.
- A C or C++ compiler does not have as much information as the developer about the process that has to be implemented. Many details of the process simply cannot be specified in C or C++. For instance when performing a floating point division, the C standard requires that the divisor be tested for zero. In a particular application, the developer may know that a certain divisor can never be zero and be able to skip the test. Usually a C or C++ compiler chooses general solutions that are applicable in a wide range of conditions, rather than specialized solutions that are often the most efficient ones [18].

While writing in assembly language produces the most efficient executables, C compilers provide most of the efficiency necessary for prototyping while significantly reducing the development time spent on implementing a given algorithm. However, unlike MATLAB, the C language poorly supports equation entry, with complex operations involving matrix-vector products, addition and transposition quite difficult to develop and debug.

The Sphere decoder algorithm is rewritten in C and performance is compared with that of MATLAB in terms of time taken for computation. The C generated code can be turned into a compiled MEX file to be called from MATLAB. Using MATLAB C-MEX interface, the complex operations involving transposition etc was done in MATLAB and passed on to MEX file. This MEX wrapper function helps exchange data between MATLAB and C. The result after computation was put back into MATLAB that has a good interface to plot the result.

As the MEX file is already compiled they are quite fast and similar to any other function called in MATLAB. But the disadvantage lies in the process of writing the MEX file which is quite time consuming and error prone. The memory allocation in C being sensitive, the MEX wrapper has to allocate and access the data properly which otherwise would lead to crashes, segmentation violation or incorrect results.

The distinct advantage is the verification of the algorithm in C with a faster execution time. This is due to the MATLAB interpreter having unnecessary memory allocation overhead when compared to the faster C compiled executables. The Figure 16 shows the Calculation time for the algorithm in both MATLAB and C with the Average SNR in dB. This was obtained using different antenna configurations, by averaging over 1000 channel realizations, 16 QAM and with 300 symbol blocks. As evident from Figure 16, C implementation gives a much better performance in terms of computation speed as compared to MATLAB.



**Figure 16: Performance evaluation in terms of computational speed, MATLAB vs. 'C'.**

## CHAPTER 6

# Implementation on Digital Signal Processor

To further investigate the performance of the algorithm as part of a WiMAX Receiver for MIMO systems, the verification of the algorithm on the target platform is the next step. The target platform includes technologies such as ASICs, FPGAs, general purpose processors and DSPs. Higher performance implementations of specific DSP algorithms are increasingly available through implementation within FPGA.

### 6.1 Background

In a Software Defined Radio (SDR), functions that were formerly carried out solely in hardware, such as the generation of the transmitted signal and tuning and detection of received radio signals, are performed by software that controls high-speed signal processors. With the advent of SDR in wireless segments, the usefulness of Field programmable logic (FPGAs) as reprogrammable DSP, has led to SDR taking on increased importance. In traditional software radio systems, the translated and filtered baseband signal is sent into the DSP as a stream of complex samples of a time-domain waveform. The DSP must handle all the demodulation tasks as well as higher-level decisions based on the analysis of the signals being received. In a cell phone base station, the number of DSP tasks grows with each new communication standard. The proliferation of sophisticated digital voice and data protocols require decoding, convolution, framing, error correction and vocoding.

While MIMO combined with OFDM is considered a key to achieve better throughput which involves use of complex signal processing techniques, this comes at the cost of added hardware. These enabling technologies pose significant challenges for OEMs needing to design basestations that are not only scalable and cost-effective but also flexible and re-usable across multiple evolving standards. Wireless system designers need to meet a number of critical requirements which includes processing speed, flexibility and time-to-market. These stringent requirements ultimately drive the hardware platform choice. Some of the major challenges include:

- *Processing Bandwidth:* To support WiMAX systems providing high data rates, the underlying hardware platform must have significant processing bandwidth. Additionally, several advanced signal processing techniques such as Turbo

- coding/decoding and front-end functions including fast Fourier transform/inverse fast Fourier transform (FFT/IFFT), beamforming, MIMO, crest factor reduction (CFR) and digital predistortion (DPD) are computationally intensive and require several billion multiply and accumulate (MAC) operations per second.
- *Flexibility:* As WiMAX and other mobile broadband technologies such as Long Term Evolution (LTE) specifications are being defined and standardised , and still going through numerous revisions before being finalized; there is need to have flexibility and re-programmability in the end product to provide for a standards-agnostic or multi-protocol basestation.
  - *Cost-reduction path:* While WiMAX and LTE standards are expected to stabilize, this will likely lead to a situation where cost of the final product will be more important than flexibility for OEMs and service providers, to remain competitive in the marketplace. Choosing the right hardware platform for prototyping would provide seamless cost-reduction in engineering segment that would otherwise require system re-design [19].

Thus potential advantages to implementing a DSP function within an FPGA include:

- Performance Improvements
- Design Implementation Flexibility
- System Level Integration
- Reduced Cost and Schedule

### **6.1.1 Partitioning between FPGA and DSP**

Signal-processing data path and control operations make up the bulk of the processing load in a wireless basestation. Most architectures implement the system control, configuration and the signal processing data path using a combination of microcontrollers (MCUs), FPGAs and programmable DSPs. The MCUs control the system, while the FPGA and DSP handle the data-flow processing. Systems with light processing demands and control-oriented tasks are implemented in software on a DSP; heavier loads are best implemented in FPGAs that provide significant parallel processing benefits. The combination of DSPs and FPGAs ensures complete flexibility and offers reprogrammability to fix bugs or even support new standard. The exact partitioning between FPGAs and DSPs depends on the processing requirements; system bandwidth as well as system configuration; and the number of transmit and receive antennas [19].

Since the receiver block would require huge amount of bit level manipulations at a fairly high rate, general purpose DSPs are very inefficient to implement those. FPGAs offer the right amount of flexibility at the cost of higher turnaround times for algorithmic modifications. On the other hand DSPs are very well suited for mul/add operations on irregular code as it is common especially for decoder algorithms that rely on matrix inversions. Hence it was decided to implement the decoder algorithm on DSPs.

## 6.2 Digital Signal Processors

### 6.2.1 Introduction

DSPs are specialized microprocessors designed specifically for digital signal processing, generally in real-time computing. It is a special purpose CPU (Central Processing Unit) that provides ultra-fast instruction sequences, such as shift and add, and multiply and add, which are commonly used in math-intensive signal processing applications. They are used for a wide range of applications, from communications and control to speech and image processing. They have become the product of choice for various consumer applications as cellular phones, modems, fax machines, sound cards, high capacity hard disks, digital TVs.

DSPs are concerned primarily with real-time signal processing. Real-time processing means that the processing must keep pace with some external event; whereas non-real time processing has no such timing constraint. The external event to keep pace with is usually the analog input. The basic DSP system would consist of an analog-to-digital converter (ADC) to capture an input signal. The resulting digital representation of the captured signal is then processed by DSPs and then output through a digital-to-analog converter (DAC) [21]. Figure 17 shows the basic DSP system; however the further processing of the signal is done digitally in the receiver application.



**Figure 17: Basic DSP system.**

## **TMS320C6000 Platform of DSPs**

The TMS320C6000 family of processors from the company Texas Instruments (TI) is designed to meet the real-time requirements of high performance DSP. With a performance of up to 2000 million instructions per second (MIPS) at 250 MHz and a complete set of development tools, the TMS320C6000 DSPs offer cost effective solutions to higher-performance DSP programming challenges [23].

The TMS320 family consists of 16-bit and 32-bit fixed and floating-point devices. These DSPs possess the operational flexibility of high-speed controllers and the numerical capacity of array processors. The TMS320 family consists of three supported platforms including the TMS320C2000, TMS320C5000 and TMS320C6000. Within the C6000 platform there are two generations, the TMS320C62x and TMS302C67x. The C62x processors are fixed-point processors whereas the C67x are floating-point processors. These refer to the format used to store and manipulate numbers within the device. The architecture of C6x DSPs is very well suited for numerically intensive calculations. Based on a very-long-instruction word (VLIW) architecture, the C6x is considered to be TI's most powerful processor [20].

Each generation of TMS320 devices uses a core Central Processing Unit (CPU) that is combined with a variety of on-chip memory and peripheral configurations. When memory and peripherals are integrated with a CPU into one chip, the overall system cost is greatly reduced, and circuit board space is reduced.

### **TMS320C6713 DSP**

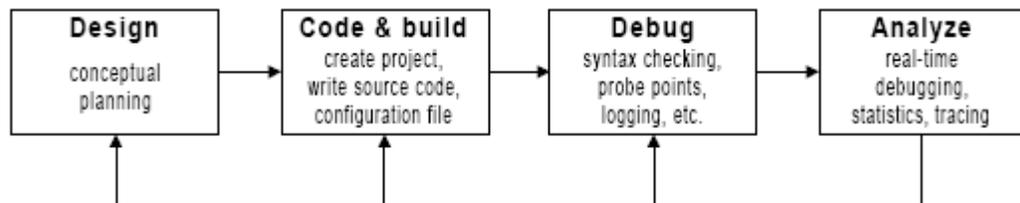
The TMS320C67 DSPs (including the TMS320C713) compose the floating-point DSPs generation in TMS320C6000 DSP platform. The TMS320C6713 is based on the high-performance, advanced VelociTI VLIW architecture developed by TI, making this an excellent choice for multichannel and multifunction applications [23].

It has operational frequency of 225 MHz, and is thus capable to execute up to 1800 MIPS and 1350 million floating-point operations per second (MFLOPS) on a 32-bit word length. It has two sets of 32 general-purpose registers and eight independent functional units capable to fetch up to eight instructions every cycle. These eight functional units are composed of six arithmetic-logic units (ALUs) and of two multiplier units. Other features include Real Time Data Exchange (RTDX) capability and Embedded JTAG support via USB. Software designers can readily target the TMS320C6713 DSPs through TI's robust and comprehensive Code Composer Studio (CCS) development platform.

## 6.2.2 Development Tools

### Code Composer Studio

The CCS provides an integrated development environment (IDE) to incorporate the software tools. CCS includes tools for code generation, such as a C compiler, an assembler, and a linker. Once the generated machine code is loaded and run on the target, the IDE also offers some analysis tools with graphical capabilities to visualize processes running on the DSPs. CCS extends the basic code generation tools with a set of debugging and real time-analysis capabilities. CCS supports all phases of the development cycle shown in Figure 18.



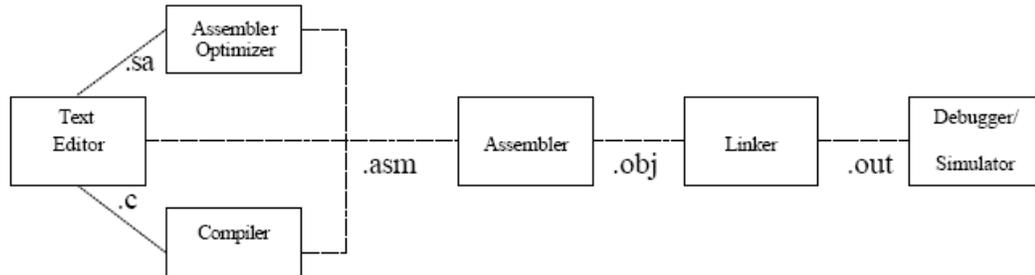
**Figure 18: Different phases of the CCS development cycle.**

CCS works with a project paradigm. Essentially, within CCS it is necessary to create a project for each executable program that is to be created. A project stores all the basic information to build the executable file (“project”.out). In a project the following kind of files can be found:

- *Include files:* Typical header files used in the application development in language.
- *Library files:* User’s library files are included in this folder.
- *Source files:* The different modules that compose the application are included in this folder. These can be developed in C language (.c) or in assembly language (.asm).
- *Cmd files:* Files used by the linker in order to describe the way in which the different sections built in applications are in located in different memory address to run the current application

The C compiler compiles a C source program with extension .c to produce an assembly source file with extension .asm. The assembler assembles an .asm source file to produce a machine language object file with extension .obj. The linker combines object files and

object libraries as input to produce an executable file with extension .out. This executable file can be loaded and run directly on the C6713 processor as shown in Figure 19.



**Figure 19: Process involving creation of an executable file after compiling C file.**

### **The Real-Time Data Exchange (RTDX) protocol**

RTDX allows the transfer of data between a host computer and target devices without interfering with the target application. The data can be analyzed and visualized on the host using the COM interface provided by RTDX. Clients such as Visual Basic, Visual C++, Excel, Labview, MATLAB and others are readily capable of utilizing the COM interface.

RTDX forms a 2-way data pipe between a target application and a host client and vice versa. This pipe can be logically viewed as a collection of one or more thinner virtual (known as channels) through which the data travels. This permits the data to be tagged as a particular virtual pipe (channel) so that various data can be distinguished. These virtual pipes are unidirectional; the data flows from the target application to the host client or from the host client to the target application. The target application sends data to the host by calling functions in the RTDX Target Library. These functions immediately buffer the data and then return. The RTDX Target Library then transmits the buffered data to the host in a way as to not interfere with the target application.

Similarly, a host client can send data to the target. All data to be sent to the target is buffered within the RTDX Host Library. After the RTDX Host Library receives a request for data from the target application, and there is sufficient data in the host buffer to satisfy the request, the data in the host buffer is sent to the target. The data is written to the location requested by the target without interfering with the target application. The host notifies the RTDX Target Library when the operation is complete.

## Link for CCS Development Tools

This supports verification, debugging, visualization and validation of the application running on TI DSPs by establishing real-time bidirectional data transfer links among MATLAB, CCS and TI DSPs hardware. Links are established via RTDX which allows transfer of data to and from DSPs without halting the running application. This tool provides automation of CCS behaviour and full computational and visualization capabilities of MATLAB can be used on real-time data.

Thus using links for RTDX it is possible to:

- Send and retrieve data from memory on the processor.
- Change the operating characteristics of the program.
- Make changes to the algorithm as needed without stopping the program or setting break points in the code.

These tools help ease the job of taking algorithms from the model realm to the real world of the target DSPs on which the algorithm will run.

The next step in the thesis was to port the C implementation of the sphere decoder onto the simulation TI board provided by CCS and by using RTDX link to evaluate performance in MATLAB.

The following RTDX commands were tried using the CCS link in MATLAB to configure the settings:

- *Create* an RTDX link to the desired target and load the program to the processor.

Construct the link to the target board and processor using the command,

$cc = ccsdsp('boardnum',0)$  where *boardnum* defines which board the link accesses while '0' connects the link to the first processor on the board.

- *Configure* channels to communicate with the target.

The task is to open as many channels as required to support data transfer for the development work and also to configure channel buffers to hold data when the data rate from the target exceeds the rate at which MATLAB can capture the data.

*cc.rtdx.configure(1024,4)* defines 4 channels of 1024 bytes each.

*cc.rtdx.open('ichan', 'w')*, opens a channel named as *ichan* with mode set to write

*cc.rtdx.enable('ichan')*, to enable the opened channel.

Similar steps to prepare a read channel.

*cc.rtdx.open('ochan', 'r')*

*cc.rtdx.enable('ochan')*

- *Run* the application on the target, and investigate the results on MATLAB.

*cc.run* or *run(cc)*

Write data to the target using *writemsg*

*writemsg(cc.rtdx, 'ichan', int16(inputdata))*

Read data from the target using *readmsg*

*readmsg(cc.rtdx, 'ochan', 'int16')*

- *Close* the links from to the target and clean up the links and associated debris left from the work

*cc.halt* or *halt(cc)*

*disable(cc.rtdx)*

*close(cc.rtdx, 'all')*, close the read\write channels

*clear cc*, call destructor

### **6.2.3 Fixed point Issues**

To go further with the implementation in MATLAB and C to be ported onto the DSP, the need arises for fixed point implementation. The study requires more time and that brings my thesis study to a conclusion with the results obtained.

# CHAPTER 7

## Conclusion

This thesis gives an insight into MIMO systems and the implementation of the soft output sphere decoder. Initially the performance of a hard output sphere decoder was done in MATLAB and later compared with the soft output decoder for various antenna configurations such as 2x2 systems etc. The hard output was found to give uncoded bits at lower complexity for higher antenna system compared to a soft output which was more accurate in the form of LLR at the cost of higher complexity.

The soft output sphere decoder algorithm was implemented in MATLAB and later the performance evaluated by implementing in C. MATLAB was found to be the ideal tool for such implementation that required complex mathematical operations such as transposition etc. Using the MATLAB C-mex interface the complex mathematical operation was done in MATLAB and passed on to mex file. The C implementation proved to be efficient in terms of computational time for higher antenna configuration.

It was decided to implement further onto a DSP and to use the RTDX protocol for the data transfer. However the implementation required the use of fixed point computation unlike MATLAB that has floating point. The fixed point implementation requires further investigation and due to lack of time the thesis was concluded with the above results. However the fixed point implementation and later the performance evaluation of the algorithm on a DSP would be an interesting future work.

# Abbreviations

ADC	Analog to Digital Converter
APP	A Posteriori Probability
ASIC	Application Specific Integrated Circuit
CCS	Code Composer Studio
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
CPU	Central Processing Unit
DAC	Digital to Analog Converter
DI	Distance Increment
DSP	Digital Signal Processing
DSPs	Digital Signal Processors
FDM	Frequency Division Multiplexing
FPGA	Field Programmable Gate Array
FFT	Fast Fourier Transform
IDE	Integrated Developer Environment
IFFT	Inverse Fast Fourier Transform
LLR	Log Likelihood Ratio
LTE	Long Term Evolution
MLD	Maximum Likelihood Detection
MIMO	Multiple Input Multiple Output
MIPS	Million Instructions Per Second
OFDM	Orthogonal Frequency Division Multiplexing
PED	Partial Euclidean Distance
PSV	Partial Symbol Vector
QAM	Quadrature Amplitude Modulation
RTDX	Real Time Data Exchange
SD	Sphere Decoder
SDR	Software Defined Radio
SIC	Successive Interference Cancellation
SNR	Signal to Noise Ratio
SISO	Single Input Single Output
UMTS	Universal Mobile Telecommunication System
VLW	Very long Instruction word
WLAN	Wireless Local Area Network
WiMAX	Worldwide Interoperability for Microwave Access

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