BLEKINGE TEKNISKA HÖGSKOLA

DESIGNING CUSTOMER REWARDS PROGRAMS FOR REPEATED PURCHASE USING OPTIMIZATION METHODS

Master Thesis IY2542 Spring 2014

MBA Programme

School of Management

Blekinge Institute of Technology

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February 2015

Abstract

A Customer Rewards Program (CRP) or a loyalty program is characterized by customers being rewarded for repeated purchases. The goal of these programs is to increase sales and profit through offering purchase incentives in the form of delayed rewards, as a result influencing customer behavior and in the end increasing company profit. It is an open debate in the literature if such reward programs are truly profitable for the company running it. This thesis therefore proposes a new approach to designing a CRP based on optimization methods with a focus on maximizing the expected profitability of the program. The key finding of the thesis is that such an approach can be applied to CRPs in order to increase the profitability of the companies running such programs. The approach offers promising results towards applicability to specific business segments and as a result towards maximizing the expected profits resulting from customer rewards programs. Since a large-scale CRP may require significant developments on existing infrastructure and can pose high costs through the rewards redeemed during the program, the perspective of being able to maximize profits through fine-tuning the program parameters could be of great help. It can even be crucial to a company considering large investments on necessary developments or aiming to increase the returns on an existing CRP infrastructure.

As part of the thesis we have analyzed the – at the time of writing – running CRP programs of several major companies present on the Hungarian market based on publicly available data in order to evaluate the applicability of the proposed optimization approach to real-world CRP designs. The analysis done on these real-world CRP designs indicates that the proposed optimization approach is applicable to a class of CRP designs witnessed to be popular among companies operating in various business segments.

We have also investigated Hungary's largest pharmacy-chain to gain insight into real-world CRP design considerations. Based on guided interviews conducted with the head of marketing at the aforementioned pharmacy-chain we conclude that the relevant information needed for our proposed optimization approach can be available to a company from business data (e.g. through billing and accounting information) readily available in an integrated IT infrastructure. We also conclude that the proposed theoretical results on CRP design are based on valid mathematical assumptions and correlate to accounting considerations related to the introduction of a CRP program.

Acknowledgements

The authors would like to thank the supervisor, Henrik Sällberg, for fruitful discussions and valuable inputs on the thesis.

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1. Introduction

1.1.Background

A Customer Rewards Program (CRP) is characterized by customers being rewarded for repeated purchase. The goal of these programs is to increase sales and profit through influencing customer behavior and therefore increasing company welfare. Keeping customers over a longer period of time is generally more profitable than attracting new customers, but this is not always the case (Reinartz & Kumar, 2000). Customers cannot be kept at all costs and hence the company needs to find how much to spend on any type of promotion to keep their customers profitable.

Customer Rewards Programs originate in the 1980's when American Airlines launched the first program of this kind, called AAdvantage (Cedrola & Memmo, 2009). From that on a whole family of this type of marketing tool began to spread to other industrial segments as well and became a major asset in customer relationship management (Cedrola & Memmo, 2009). These programs are sometimes called CRPs in the literature and sometimes loyalty programs (LP). The used naming convention depends largely on the authors intentions on emphasizing some aspects of the programs. We use the term Customer Rewards Program most frequently throughout this work, but in some cases we refer to these marketing initiatives as loyalty programs in order to use the same terminology as for example a referred work throughout a section.

In a recent study (Dorotic, H.A.Bijmolt, & Verhoef, 2012) the authors review some important characteristics of loyalty programs discussed in the literature. Referring to previous studies the authors Dorotic, H.A.Bijmolt, & Verhoef (2012) conclude that the final goal of a loyalty program should be to increase purchase frequency, purchase amount and share of wallet (SOW):

"The main purpose of an LP should be to foster and reward members' behavioral and attitudinal loyalty and therefore encourage customer retention and customer share development. Through relationship building, successful LPs encourage a consumer to purchase frequently from the programme provider, increase purchase amounts over time (e.g. cross-purchases, upgrades), and increase the share of wallet (SOW) at the focal provider/brand".

It is important to point out that the authors of the above referred study argue for customer relationship building mainly as a tool to contribute towards the final goal of increasing purchase frequency, purchase amount and share of wallet (SOW). From this aspect it becomes logical to investigate other methods and tools that can contribute towards the same final goal.

Dorotic, H.A.Bijmolt, & Verhoef (2012) also note that customers must (formally) become LP members to obtain benefits, which imply that LPs should be membership-based. This differentiates loyalty programs from other also widespread marketing tools, like discounting campaigns or one time reward promotions ("buy one, get the second for free"). This property of LPs is in close relationship with another aspect: being a long-term initiative. An important consequence (noted in the same study) of becoming formally a member of a program is that it enables the program provider to identify the customers, primarily to manage customer relationships more efficiently through observing customer purchase behavior over a longer time period.

Time is not only a distinguishing aspect of CRPs from other marketing efforts. A CRP setting a time limit of participation provides an incentive for purchase. In most CRPs time can be interpreted as the horizon of the whole program starting at program announcement and in some cases closed after a preannounced fixed time, while in other cases the program is running "forever". Time can obtain an important role in point accumulation too. Some reward programs are designed to include time limits, which within a certain amount should be purchased in order to obtain the reward or keep a special status (membership level). The latter property is referred to as a "limited duration" of a membership level for example (Sällberg, 2010, p. 36).

An important tool in identifying customers participating in a given CRP program is some kind of a customer card, which is most commonly in printed form used to collect stamps or points for example – in general to register customer purchase – or by some sort of an electronic registration like an electronic customer card. As Dorotic, H.A.Bijmolt, & Verhoef (2012) point out "an LP should *reward* members for their loyalty *on the basis of their current or future value to the firm*, usually through the *accumulation* of some form of LP currency *based on purchase* behavior. Cardholders are given discounts, goods, services, personalized offers or preferential treatment".

Regarding reward types, based on (Kim, Shi, & Srinivasan, 2001), most programs offer *efficient rewards* meaning primarily their own rewards (e.g. a hotel chain offering free nights), *inefficient rewards* (these are typically monetary rewards), or a combination of these two. It is also common that a point system is introduced within the program, where the – through purchases accumulated – points serve as a special common currency tradable only within the limits of the program (Sällberg, 2010, p. 33).

Despite the long history of Customer Reward Programs several sources question their effectiveness: e.g. (Dowling & Uncles, 1997); Sharp 2010; (Sharp & Sharp, 1997); (Wright & Sparks, 1999); (Bolton, Kannan, & Bramlett, 2000); and (Cedrola & Memmo, 2009). On the other hand plenty of authors argue for their effectiveness: e.g. (Verhoef, 2003); (Taylor & Neslin, 2005); (Meyer-Waarden, 2007) as it is summarized in Sällberg (2010, p. 30) and in Dorotic, et al. (2012).

Criticizers of loyalty programs point out that the "major reason for the launch of many customer loyalty schemes is competition" (Dowling & Uncles, 1997). These initiatives are motivated by the wish to either take a preemptive measure or respond to other CRP programs already present on the market (ibid). In this regard the same study found that programs initiated by the moves of the competition (assuming a parity band on a competitive market) and which do not enhance product or service value or broaden the availability of these, are less likely to contribute to profitability, since they are most likely counteracted by offers of equal perceived value (ibid). This essentially leads to pushing the market towards a price based competition.

Another study evaluating a large-scale Australian loyalty program, Fly Buys, (Sharp & Sharp, 1997) with statistical methods observed that participating brands did not experience consistent increase in average purchase frequencies. The study concluded that although a general pattern of increased excess loyalty was measured among the participating bands, these results were in statistical terms limited in their effectiveness: "for most [in the Fly Buys program participating brands] it is disappointingly small" (Sharp & Sharp, 1997).

Cedrola & Memmo (2009) also note, that point collection programs have an especially easy-to-copy design Therefore they are in most cases subject to the afore mentioned opportunistic behavior where most customers engage in such loyalty programs (in particularly point earning programs presented often as loyalty cards) for the lower prices these program offer (or other benefits and rewards). There are other studies that noticed this widespread “copycat” phenomenon and even argue that it is the reason why CRPs have spread so much in the first place (Dowling & Uncles, 1997). It must be noted however that, since hedging customers participate in several loyalty programs running in the same timeframe, both the loyalty effect of the programs and their sales increasing effect become questionable (Cedrola & Memmo, 2009) based on (Rubach, 2002) and (Meyer-Waarden, 2007).

On the other hand (Verhoef, 2003) argues that economic incentives result in greater customer retention and in this regard CRPs are effective tools: “If firms strive for immediate results, economically based loyalty programs and direct mailings are preferable”. This study also points out that loyalty programs can have a positive effect on customer share on the long-run, although the short term effects can be rather small.

Meyer-Waarden (2007) also found positive effects of loyalty programs on “lifetime duration and customer SOW at the store level”, although the study also points out that taking the large number of multiple card-holders into consideration these effects may be cancelled out by the competing similar loyalty schemes. This study suggests that behavioral and attitudinal loyalty can be built simultaneously with profitability and that it is reasonable to differentiate between customers: gathering customer data based on purchase behavior, share of wallet (SOW), lifetime duration and loyalty card portfolios can contribute to an effective customer segmentation initiative aimed to differentiate customers based on factors like customers vulnerabilities, defection risk and price sensitiveness. In this regard loyalty schemes are important tools that support behavioral data in order to manage customer heterogeneity and ultimately to create strategic customer segmentation and retention initiatives.

Regarding the controversial effectiveness of loyalty programs Sällberg (2010) concludes that measuring the effects of loyalty programs is difficult because “data used do not allow for measurement of whether customers buy from competing stores as well” - Sällberg (2010, p. 17) based on (Benavent, Crie, & Meyer-Waarden, 2000). Since the data collected during these studies are the results of measurements from selected CRPs, there is no way to tell how the CRP influenced the purchase behavior of the customers towards the products of competition.

It is also noted in the same work that “the use of declarative survey data which reliability problems are well documented” also makes the evaluation of CRPs more difficult (Sällberg, 2010, p. 17) based on Mägi (2003). The survey data used by Mägi (2003) indicates a hedging behavior between customers, meaning that they participate in more than one CRP at a time and try to capture the benefits of all programs. This on one hand reflects on the previous note: there is evidence indicating that customers do not choose exclusively between programs. Assuming this evidence is true; this indicates also that selected measurements from one programs data will not lead to reliable conclusions regarding CRP effectiveness. On the other hand the survey methods reliability is limited since the questions (experimenting) can influence the answers to the survey. It is unclear for example if the observed hedging behavior documented by Mägi (2003) is the result of customers’ incentive to avoid becoming locked-in in a program or simple because the programs are not differentiated enough, or if it is a result of some other reason.

Another reason for mixed results may be "the use of aggregated panel data which fail to account for customer heterogeneity" - (Sällberg, 2010, p. 17) based on (Meyer-Waarden, 2007). Despite the arguments questioning the effectiveness of loyalty programs most companies are not willing to cancel their programs because they fear that they will suffer losses in sales (Dowling & Uncles, 1997) in (Cedrola & Memmo, 2009). Cedrola & Memmo (2009) further argue that a trend of imitation is observable among CRP designs. Innovative companies are giving up on designing new CRP schemes, since their design is easily copied by competition although it requires extensive research and development costs to come up with new designs. As a result most running programs have similar design and offer similar sales incentives to customers. Similar designs result however in reduced effectiveness on motivating customer sales, and simultaneously increase the chance of opportunistic behavior of customers[1] (Cedrola & Memmo, 2009). Other studies also indicate this customer behavior although referring to it as "hedging" (Mägi, 2003)[2].

1.2.Problem discussion

As we could see in the previous section, since the introduction of the first CRP programs, these marketing initiatives have been studied extensively for several reasons. Some studies aimed at primarily exploring and describing the phenomenon itself, while others examined the effectiveness of these programs. Among these studies several sources emphasize "building loyalty" or "creating an incentive for repeated purchase" as the goal of launching a CRP. It seems that these goal definitions are interrelated since "building loyalty" inherently result in "repeated purchase", whereas "repeated purchase" can be interpreted as a form of loyalty.

We believe that the general structure and distinguishing features of a CRP program must be identified in order to argue about the effectiveness of the design with respect to the proposed goal of the given CRP. We have noted some differences in the definitions of CRPs used in the literature. These are, for example, examined and presented in Sällberg (2010, p. 30) and (Dorotic, H.A.Bijmolt, & Verhoef, 2012). We list these definitions below in Table 1.1 in order to get a clear overview.

Beside the definition of the concept a Customer Rewards Programs and their effectiveness, several aspects of CRP design have already been extensively studied, aiming to understand the effects of changing between CRP-design alternatives on customer spending. Such design considerations include, for example, the effects of choosing between a luxury product and a necessity product as a reward (Shafir, Simonson, & Tversky, 1993; Thaler, 1980), the effect of customer effort needed to achieve a reward (Kivetz & Simonson, 2002), the discount effect associated by a customer to the reward value with respect to monetary and product rewards (Odum & Rainaud, 2003), and with respect to high and low value rewards (Estle, S. J.; Green, L.; Myerson, J.; Holt, D. D., 2002).

[1] See the previous studies on hedging behavior mentioned above – the original study of Mägi (2003) for example.

[2] These findings also correlate with the experiences of Pharmanova Zrt., the pharmacy-chain whose CRP design evolution and their related considerations are presented in Section 5.2 and Section 6.2.

Source	Definition
(Sharp & Sharp, 1997)	“Structured marketing efforts which reward and therefore encourage, loyal behavior”
(Leenheer, Bijmolt, Van Heerde, & Smidts, 2007)	“Integrated system of marketing actions, which aims to make member customers more loyal”
(Berry, 1995)	“Schemes devoted to create pricing incentives and developing social aspects of a relationship”
(Shapiro & Varian, 1998)	“Scheme rewarding customers for repeat purchases”
(Johnson, 1998)	“Marketing program designed to increase the lifetime value of customers via a long-term interactive relationship”
(Yi & Jeon, 2003)	“Marketing program design to build customer loyalty by providing incentives to profitable customers”
(Palmer, McMahon-Beattie, & Beggs, 2000)	“Identifiable package of benefits offered to customers which reward repeat purchases”

Table 1.1: ***Definitions of CRP used in literature*** **(Sällberg, 2010)**

Another aspect of CRP design is the concepts of point, redemption and reward functions. A thorough and unified treatment of these functions is given in Sällberg (2010) as part of the discussion on the "incentive structure approach” to CRP design. This work gives insight into important properties of these functions (non-linearity induced by parameters of the respective functions, and its implications on customer preference for these programs for example) and the effects of these properties on the customer purchase incentive. Sällberg (2010) also highlights the importance of some of the properties of the reward and point functions in a CRP design as effective tools to motivate repeated purchase. One of the most important effects of a successful CRP is however the through repeated purchase increased overall profit of the company.

Sällberg (2010) gives the formulation of a ***reward function*** as

$$R = f[P(Sp)] \qquad Eq. (1.1)$$

and refers to this formulation as the *incentive structure approach (it is noted in* (Sällberg, 2010) *that this formulation also exists in previous studies too).* The formula expresses the relationship between rewards (*R*), points (*P*), and spending (*Sp*). The reward (expressed in monetary units) is a function of points (a quantity), which is in turn a function of spending (also expressed in monetary units).

Although the effect of different parameters and resulting function properties on (individual) customer spending is thoroughly discussed in the literature, the relationship between these design choices and the overall profit of the company resulting from the CRP is not formalized explicitly. We believe that in order to effectively design CRPs - with the overall goal to increase expected profitability - this relationship needs to be expressed explicitly as a necessary prerequisite of any further efforts.

Sällberg (2010) starts with the analysis of the *reward function* as a direct function of spending which can be expressed as:

$$R = \alpha \times [Sp(T)]^{\beta} \qquad Eq. (1.2)$$

Where α and β being constant parameters of the expression. This form expresses important assumptions regarding the properties of the reward function: this relationship catches spending as a constant function of time, further it captures the possibility that not all historical spending is valid towards future rewards. Also it can be derived from this form (ibid), that a progressive *(*$\beta > 1$*)* reward function is preferable over a linear *(*$\beta = 1$*)* or a degressive *(*$\beta < 1$*)* reward function (ibid).

The reason why $Eq. (1.1)$ includes points as opposed to $Eq. (1.2)$ is that points have important psychological effects as an incentive towards spending. On the other hand including the point function in $Eq. (1.1)$ has an important consequence on the non-linearity of the reward function, which can be derived from the explicit formulation of the ***point function*** as:

$$P = \gamma \times [Sp(T)]^{\rho} \qquad Eq. (1.3)$$

As before γ, ρ are constant parameters and the formulation captures the points as function of time. Since $Eq. (1.2)$ and $Eq. (1.3)$ have similar form and therefore trajectories, one can also conclude that a progressive point function is preferred over other forms. Intuitively one can argue that a customer prefers more points over lesser since in the end points entitle the customer to rewards.

The explicit relationship between *R* and *P* following from $Eq. (1.1)$ is called the redemption function. As it was mentioned earlier in this section – and also noted in (Sällberg, 2010, pp. 32-39) the parameters α, β, γ, and ρ influence the non-linearity of $Eq. (1.1)$ and as stated in (Sällberg, 2010, pp. 38-39) with the end goal of designing a progressive CRP one must keep in mind that a progressive redemption function and a linear point function can create different purchase incentives than a linear redemption function combined with a progressive point functions due to psychological effects on the customer.

The parameters of point and reward functions determine the properties of the functions and contribute significantly to the overall CRP design. These parameters are subject to *design choices made by the company at the time the CRP is launched*, but in the end *they influence* the purchase incentive of *the individual customer*. Since the company designs its CRP, it is in the position to freely choose reward and point function parameters with the overall goal *to increase profits.*

Although the company sets these parameters at the start of the CRP, it might not be in the position to (easily) change them during the running CRP to react on the actual customer purchase behaviors. Customers may respond negatively to changes in the program even though the company may be entitled to such changes in the contract between the company and the customer. Thus the company is effectively locked-in by its own design choices during the program: company profits may be increased by the CRP, but not necessarily maximized and further corrections are not possible until the end of the program. The high cost of launching a CRP makes it however necessary to *try to maximize the profitability* of the CRP over its lifetime.

1.3.Problem formulation and purpose

In the previous section we arrived at the conclusion that from a financial perspective it is a reasonable goal to try to maximize the profitability of a CRP program over its lifetime due to the high associated costs. Since previous studies developed the foundations of expressing characteristic properties of CRPs as continuous functions, we believe that these results can be developed further to create a model of CRPs based on tools of mathematical optimization, specifically on tools of calculus of variations. Previous studies on similar topics either gave general guidelines to *increase* profits compared to an implicit baseline or developed models to *maximize* profits, but under heavily simplified assumptions about the business context. To the best of our knowledge – although economical applications of calculus of variations and related examples exist in the literature – specifically in the context CRP design these concepts have not yet been applied.

Problem formulation: How can the properties of real-world CRPs be modeled by mathematical optimizations methods in order to develop CRP designs maximizing expected profits with respect to real-world business constraints available?

Proposition: Our proposition is that the toolset of calculus of variations is expressive enough to model real-worlds CRP designs and that the optimization approach can be used to design CRPs with the aim of maximizing expected profits.

Research purpose: *Our purpose is to develop a model capable of describing an abstract CRP design using standard tools of calculus of variations with respect to real-world business constraints and observed CRP properties.* Ideally the model created helps us to identify properties of real-world CRPs that can be described by this mathematical toolset, thus help us in identifying the limitations of our hypothesis.

2. Theory

2.1. Studies related to increasing CRP profit

As already mentioned some research literature question the effectiveness of CRPs as a tool for increasing the profit of a company. There are examples though where CRPs have been proven to benefit the profitability for the company. For example, the article (Berry, 1995) discuss that the profit rises when the company lowers the customer defection rate. It is argued that it has to be the "right" customers that need to be kept to increase profits.

An article, which mathematically models share-of-wallet of customers in the Dutch supermarket industry, is (Leenheer, Bijmolt, Van Heerde, & Smidts, 2007). A small but significant positive effect of SOW is shown in the article and that this might lead to greater profit. The authors however did have problems with estimating the total cost of the CRP program and hence how profitable it is in total is not shown.

Not only delayed rewards and discounts could help in increasing the profitability, e.g. (Melancon, Noble, & Noble, 2011) shows that perceived social rewards (recognition, friendship, and better service) from an organization lead to affective commitment and perceived economic rewards lead to continuance commitment. Since social rewards might be a smaller cost for the company running the CRP this may increase the profit of the program. Based on the above study, social rewards might also even be stronger than economic rewards and be better for a long-term relationship with the customer.

It is important that a CRP is developed to fit the area where it is used. The effectiveness of a certain type of loyalty program varies even within an industry segment. For example, (Palmer, McMahon-Beattie, & Beggs, 2000) show this in a study on how the effectiveness of different loyalty programs can be measured within the hotel industry. The result is that a strategy, which might be considered novel within one part of the hotel sector, may be quite inappropriate to another. The work (Yi & Jeon, 2003) is another article which shows that different reward schemes can be crucial depending on the involvement of the customers towards the company. In high-involvement situations, direct rewards are preferable to indirect rewards. In low-involvement situations, immediate rewards are more effective in building a program's value than delayed rewards.

As we can see from the previous section, general studies include guidelines on how to ***increase*** profits related to CRPs, but doesn't include much on how a company should optimize the CRP itself in a systematic way to receive the ***highest*** profit from it. We believe this is partly because most of the previous research known to us is empirical research, aimed to evaluate the effectiveness of CRPs from one aspect or another. Thus these studies are not suited for guiding companies aiming to launch a CRP in the way of maximizing the resulting profits from it.

Studies following an analytical approach are however less frequent in this field. Further, most analytical studies published employ mathematical tools from game theory and aim to model a specific aspect of customer rewards programs in order to understand their inner workings. We have found these studies interesting for two reasons: They rely on previous empirical findings when formulating their assumptions about the models subjects (in most cases the behavior of the CRPs' participating customers) and since their experimental settings are well defined they give ready-to-use insights into the mechanics of CRP programs. On the other hand they also gave us important methodological guidance on how to address our research goal of using models of mathematical optimization to aid the design of profit maximizing CRPs, which seems to be a topic not covered deeply in the existing literature. Therefore in this section we will present previous studies on loyalty programs of analytical nature.

Kim, Shi, & Srinivasan (2001) focus their research on finding the optimal type and amount of rewards and introduce the concept of light and heavy users. Light users are basically customers who exit the market after one purchase and are thus not subject to reward programs, whereas heavy users purchase in each period and therefore form the basis of reward programs. They found that a firm can exploit a market with a small segment of price sensitive heavy-users and a large segment of price insensitive light-users by offering a reward program with inefficient rewards – rewards with higher unit cost, like cash rewards: "By offering inefficient rewards, the firms are able to commit to weaker competition and, therefore, higher prices." (Kim, Shi, & Srinivasan, 2001). On the other hand when the heavy-user segment is less price-sensitive and constitutes a larger part of the market the companies should offer the most efficient rewards possible (offering their own products for example). They also found that optimal reward amount is in a negative correlation with unit rewards cost - as intuition also suggest: if a reward is cheaper than another one for the company, it should offer more from this reward to obtain better results. As a general conclusion they find that reward programs lower price competition and that companies should adopt reward programs as long their light-user base is not too price sensitive. In case light-users are very price sensitive adopting such programs will increase the chance of price based competition and companies will benefit less from repeated purchases of heavy-users. On the other hand price insensitive light-users will pay in practice for the benefits of the company and heavy users gained through a successful reward program.

As the analytical framework, Kim, Shi, & Srinivasan (2001) used a game theoretical model to differentiate between rewards based on their cost to the firm and benefit for the customer. As a key component of their model they defined efficient rewards as rewards with low unit rewards cost and inefficient rewards as high unit rewards costs to the firm, where they define the unit reward cost as "a firm's cost of offering certain rewards worth one dollar to the customer" (Kim, Shi, & Srinivasan, 2001). We believe that Kim et al's analytical study in this field is an important methodological step in understanding reward programs and highlights the importance of theoretical analytical research in understanding the mechanics behind designing efficient reward programs.

Singh, Jain, & Krishnan (2008) also use a game theoretical framework in order to examine necessary conditions of equilibria (both symmetric and asymmetric) in a duopoly market by modeling customer heterogeneity with respect to customers' liking of the loyalty program and their ability to redeem loyalty rewards of the program. They found that if there is a high probability of customers engaging in further purchases after the first occasion it is beneficial for the firms to take different positions by one offering a reward program and for the other company to compete mostly based on price. The resulting asymmetric equilibrium enables the firms to segment the customers based on the purchase occasions and earn higher profits for both of them.

Beside the above mentioned works the interested reader should also examine the results of (Kopalle & Neslin, 2001), who designed a strategic competitive two-stage game in order to model reward programs and examined the viability of valuation using numerical simulation by modeling customer valuation of the product by a stochastic random variable. Using their game theoretical approach as a tool for analysis they have found support for reward programs being a “fundamentally powerful, multi-period promotions”, although the power of a program is ultimately influenced by the way how customers value future benefits and whether the market on which the company operates is expandable. One interesting finding of this study is that it found evidence for not increasing rewards for underperforming rewards programs can be reasonable if customers discount future benefits heavily. We found this result interesting since it argues against the common sense reaction of increasing rewards for underperforming programs and thus hints that reward value and program performance (profitability) may not be positively correlated. This also suggests that a model is necessary that helps to understand the relationship between reward value and total profitability of the program analytically.

Perhaps the work closest to our proposed approach of analyzing CRPs with mathematical tools is the work of Gandomi & Zolfaghari (2013). This study modeled the effect of customer valuation on a two-staged CRP’s profitability by modeling the expected revenue function of the firm as the goal function of a mathematical optimization problem. We wish to note here that although most studies aimed at understanding the working of CRPs in one way or the other, it can be used to design more profitable customer rewards programs. This is the only work in the literature we know of that set the goal of modeling maximization of profitability in the field of CRPs.

As a major contribution to existing research Gandomi & Zolfaghari (2013) incorporated customer valuation both as a real valued stochastic variable – in order to capture customer heterogeneity - and using deterministic valuation into their optimization models. They also managed to derive analytical solutions for the resulting models, which are crucial for understanding the modeled mechanics of loyalty programs. Since we have this study to be one of the latest of analytic research and modeling approaches aiming primarily to shed some light on the profitability of loyalty programs and thus to be the closest in nature to our proposed approach we would like to outline briefly this work. By doing so we both hope to help the interested reader getting more comfortable with treating loyalty programs as optimization problems and to point out the differences of our work compared to existing similar approaches.

Gandomi & Zolfaghari (2013) model a company selling a good or service over two periods and its associated loyalty program. Those customers who make a purchase in both periods earn a loyalty reward. They assume that a certain proportion of customers will not participate in the second stage. This proportion is modeled as a parameter of the model (γ). The parameter γ can also be thought of the probability a customer enters the second stage. Gandomi & Zolfaghari (2013) are assuming “that customers are forward-looking in their decision, that is, they consider future (period 2) gains or losses when deciding to buy in the current period (period 1)”. To capture this assumption they defined a variable called surplus: the customer’s valuation of the product subtracted by the cost of the product. They further assume that a nonnegative surplus will lead to participation in the second stage of the loyalty program. The surplus, S_1, is defined as

$$S_1 = [v_1 - p_1] + \gamma[v_1 - (p_2 - r)] \qquad \text{Eq. (1)}$$

where v_1 is the valuation of the customer for the product in the first period, p_1 and p_2 are the prices offered in the two different periods, r is the reward value, and γ is the probability a customer will advance to the second period.

Based on Eq. (1) the probability of a customer buying in the first period is

$$\Pi_1 = Pr(S_1 \geq 0) = 1 - F\left(\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right) \quad \text{Eq. (2)}$$

where F represents the Cumulative Distribution Function (CDF) of the first period customer valuation (Gandomi & Zolfaghari, 2013).

According to Gandomi & Zolfaghari's (2013) model a customer purchasing in period 1 will proceed to the second period with the probability of γ. Thus the utility derived from making a purchase for a given customer in stage 2 is:

$$S_2 = v_2 - (p_2 - r) \quad \text{Eq. (3)}$$

This formulation is based on the assumption that the valuation of a customer in the two different periods is not independent. This is supported by the intuition that previous purchase experiences affect current valuation (the probability of a second purchase is conditioned on the fact that the first purchase had a nonnegative surplus) for a detailed derivation see (Gandomi & Zolfaghari, 2013). For our current intentions it is enough to note that in order to model the dependency between v_1 and v_2, $\delta = v_1 - v_2$ is introduced. Further, using Eq. (1) and Eq. (3), the probability of making a purchase in the second period can be expressed as:

$$\Pi_2 = Pr(S_2 \geq 0 \mid S_1 \geq 0) =$$

$$\Pr\left(v_2 \geq p_2 - r \middle| v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right) =$$

$$Pr\left(v_1 \geq p_2 - r - \delta \middle| v_1 \geq \frac{p_1+\gamma(p_2-r)}{1+\gamma}\right) \quad \text{Eq. (4)}$$

Based on the probabilities above and the assumption of a unit market (Gandomi & Zolfaghari, 2013) the expected total profit of the firm can be formulated as:

$$TP = p_1\Pi_1 + \gamma(p_2 - r)\,\Pi_1\Pi_2 + (1-\gamma)p_2\Pi_2^1 \quad \text{Eq. (5)}$$

, which serves as the goal function of the final optimization model of their theoretical problem:

$$\max_{p_1,p_{2,r}} TP = p_1\Pi_1 + \gamma(p_2 - r)\,\Pi_1\Pi_2 + (1-\gamma)p_2\Pi_2^1$$

$$subject\ to:$$

$$p_2 \geq r$$

$$p_1, p_2, r \geq 0 \quad \text{Eq. (6)}$$

Eq. (6) also expresses some necessary constraints like nonnegative prices and reward costs and that the reward should not exceed the second period price (which would lead to certain purchase for a rational customer in the second period).

Gandomi & Zolfaghari (2013) found in their study that in case the company manages to keep its customers satisfied it will not profit from implementing a loyalty program. However if the customers are not satisfied with their first period purchases the introduction of a loyalty program is necessary to maintain profitability. It was also shown that the price offered for first period purchases must be increased proportionally to the reward value and to the probability of return in the second period.

As we can see from previous studies the different analytical approaches addressing different research questions used almost exclusively methods of game theory and were effective tools in highlighting the inner workings of rewards programs. These studies are relevant tools in determining whether a company should engage in a reward program or not, based on specific market properties and have suggestions on designing some parameters of the program in order to create higher profits for the firms – although without the intention of maximizing profits of a CRP design.

These works also highlight that from a research methodological point of view the proposed model should be made as simple as possible initially to be able to derive an analytical solution which is crucial in understanding the underlying mechanics behind the findings of the model. Gandomi & Zolfaghari's (2013) work - and the other related studies mentioned above - also shows that there is need for such "analytical experiments" in the field of CRP programs dominated by empirical studies.

If the model's design and assumptions are based on empirically supported evidence from research on this field – contributing a lot to the real-world credibility of the work – through the clear and rigorous nature of mathematical modeling they help us to get a perhaps more distilled understanding of the working of reward programs than what would be possible with other typical social science research methods.

2.2. Theoretical optimization framework

We have found that since the incentive structure approach works with continuous functions it can be relatively easily extended to a continuous optimization problem. It must be noted however that some changes are necessary in order to achieve this goal: first of all Sällberg (2010) treats rewards by their value to the customer expressed by its cash value to the customer: "*Sp* is expressed as a purchase amount and *R* is expressed as the market value of products for free or the cash value of rebates". This implies that the reward value is captured from the viewpoint of the customer (customer perspective). In order to formalize expected profits however rewards – although still expressed as monetary values – have to be expressed as their cost and the revenue to the firm (i.e. the company's perspective).

Since we wish to capture the profits resulting from the CRP design we will restrain our model of profit to the simplified form of:

$$CRP\ Profit = CRP\ Sales(Reward\ Value) - Reward\ Cost(Spending)$$

This expression tries to capture the relationship between CRP Profit and Customer Spending. It says that the reward cost to the company is a function of customer spending. On the other hand the sales are increased as a result of the customer's perception of reward value. The difference is the profit on the CRP program, the goal of our optimization efforts. The rationale behind launching a CRP is the expectation that the customer perceives the reward value higher than the cost to the company and this induces additional sales related to the CRP.

However this formulation can lead to rather complicated mathematical models in case one wishes to use it as the foundations of designing CRP programs. In order to highlight how fast a simple CRP design becomes a complex (mathematical) problem we use an example of a CRP with an intentionally simple and widespread initial design: "Buy 10 products– get one for free". These types of programs – often used together with some form of loyalty cards – are quite frequent, thus they are a good choice to review some of the complexities in designing CRPs. The example is used primarily to show that as the CRP model becomes more complex, the corresponding mathematical optimization problem evolves too – and thus requires significantly more effort to solve.

As a model of the simple CRP design described above we introduce program launched by a fictional restaurant:

a) A customer receives a card to collect a stamp every time he or she buys a pizza
b) When the customer has collected ten stamps, the next pizza is for free
c) After collecting the reward the customer can typically get a new card and start over again
d) Cards are transferable between customers

Assuming the sales function of pizza is known to the company launching the program, and that it primarily aims to maximize expected profits by changing the parameters of its reward function it would have to solve the following optimizations problem:

$$\max_{10 \geq x \geq 0} S(R) - C(x)$$

Where x represents customer spending, *S(R)* represents the sales function with respect to rewards, R, known to the company and *C(x)* stands for the cost of the reward program. In this example the cost function has the form:

$$C(x) = \begin{cases} c, & if\ x = 10 \\ 0, & if\ x < 10 \end{cases} \quad x \geq 0$$

Where *c* is a positive number representing the cost of one reward, considering the reward is one pizza in our case, you can substitute c = 1 (meaning unit cost of reward). We can see from the reward function that the perhaps most simple case of CRP leads to a discrete optimization problem.

Analyzing these conditions briefly also reveals, that there is no incentive to stay in the CRP when the card is full and handed in to the pizzeria to redeem the reward (in this case the extra pizza). There is a risk that the customer will choose a different restaurant if it also has the same type of CRP (which as previous studies showed is quite often the case, see discussion on imitation of CRP design above or (Cedrola & Memmo, 2009)). To avoid losing customers after reward occasions, customers are often favored by higher value rewards for buying the next ten pizzas, which can be seen as a progressive reward function.

From a practical (administrative) point of view the new increased value reward can be introduced easily to the CRP design through an additional rule:

e) after handing in the first card the customer is given a new type of card which entitles him for *a free pizza and a small soda* after buying the next ten pizzas

The introduction of the new reward requires a more complicated reward function, with two reward values, $c_1(representing\ the\ free\ pizza)\ and\ c_2(representing\ the\ additional\ soda)$:

$$\max S(R) - C(x)$$

$$subject\ to\ 20 \geq x \geq 0,$$

$$C(x) = \begin{cases} c_1 + c_2 & if\ 20 = x \\ c_1 & if\ 20 > x \geq 10 \quad x \geq 0 \\ 0 & if\ x < 10 \end{cases}$$

Through the introduction of a new reward value level, there is an incentive for the customer to keep buying pizzas from the same place. Reward occasions – where new reward values are introduced – are in practice often referred to as the beginning of a new membership level. *(Our program in its current form is similar to membership levels seen in real-life CRPs used in airlines where the customer – after reaching a certain amount of spending – will become a member of the next level, typically a "silver", "gold", or "platinum" member.)*[3]

In typical CRP designs in order to keep his or her membership status the customer needs to buy the ten pizzas within a certain amount of time or he will lose his current status and fall back to the previous level. It is also quite frequent that a loyalty program itself expires after a certain date. This can be modeled for example by introducing the following rule:

f) loyalty cards expire after *timeframe T* (starting from the first purchase)

This can be modeled with the following formulation, encapsulating the time constraint (horizon) as a multi-period problem:

$$\max \sum_{t=1}^{T} S(t,R) - C(t,x_t)$$

$$subject\ to\ 20 \geq x_t \geq 0, t = 1, 2, \dots, T$$

It is important to note, that from a modeling perspective we have now an optimization problem with discrete rewards, i.e. it is a discrete optimization problem.

The current design has several important implications: First, the number of membership levels in a program is limited, because the customers do not respond well for the psychological effect of the new levels after a certain number; Second, in the current design we have implicitly assumed that the company uses its own products as rewards: this is similar to engaging in price based competition with the competition. In practice however it is common that a company uses the products of other (partner) companies as rewards, who cooperate on an exclusive basis, meaning that the offered rewards are not available (as rewards) through other CRP programs. This is reasonable for example if the company wishes to avoid a price-based competition and this design could also lead to the required lock-in effect through offering rewards not available in other restaurants.

[3] We haven't found a general, explicit way of representing membership levels in an optimization formulation though. This will be discussed later during the thesis in Sections 4.3, 5 and 6.

From a modeling perspective this means that if we introduce other products beside pizzas (and sodas) our assumption of unit costs and thus discrete rewards does not hold anymore and we have to work with a continuous reward function to be able to deal with monetary rewards:

$$\max \int_{t=0}^{T} S(t,R) - C\big(t, x(t)\big)dt$$

$$subject\ to\ x(t) > 0, \qquad T \geq\ t \geq 0, \qquad x(0) = 0, \qquad x(T) = B$$

We have relaxed *rule b,* since the continuous reward function allows more flexible designs too and introduced *B* representing the maximum spending counted toward rewards. However from a mathematical perspective it is necessary to consider only convex reward functions since it is a requirement of tractable solution (Boyd & Vandenberghe, 2009).

We have however not yet dealt with the dynamic nature of this problem. Profit does not only depend on current output only, it is influenced by the rate how output changes over time:

$$\max \int_{t=0}^{T} S(t,R) - C\big(t, x(t), x'(t)\big)dt$$

$$subject\ to\ \ x(t) > 0, \qquad T \geq\ t \geq 0, \qquad x(0) = x_0$$

The effect of reward discounting has to be considered to get a financially plausible model. Since we use a continuous reward function we can easily incorporate a monetary discount to catch the time value of money (Kamien & Schwartz, 1991, p. 10):

$$\max \int_{t=0}^{T} e^{-rt}\big[S(t,R) - C\big(t, x(t), x'(t)\big)\big]dt$$

$$subject\ to\ \ x(t) > 0, \qquad T \geq\ t \geq 0, \qquad x(0) = x_0$$

In order to increase the sales effect, the above CRP design can be further complicated by binding the cards to the customers (in practice through negating *rule d*), thus making them a personal property (or at least limiting their use to a certain member group, like a family).

Despite the fact that this program started initially as an overly simplified example of a loyalty card-type CRP design, we can see on this example that taking results of previous studies into consideration – with the goal to maximize expected profits if a typical, therefore realistic design – we arrived to a relatively complex formulation of a convex optimization problem.

During the research on applicable optimizations approaches we have also tried to describe CRPs as dynamic systems to capture the relationship of rewards and the resulting goodwill on actual sales. A CRP can then be seen as a dynamic system where the company is trying to influence or control the customer to keep buying the company's products by giving the customer a kick-back on the purchases being made. A general schematic for a closed loop control system can be found in figure 3.1 (Glad & Ljung, 2000).

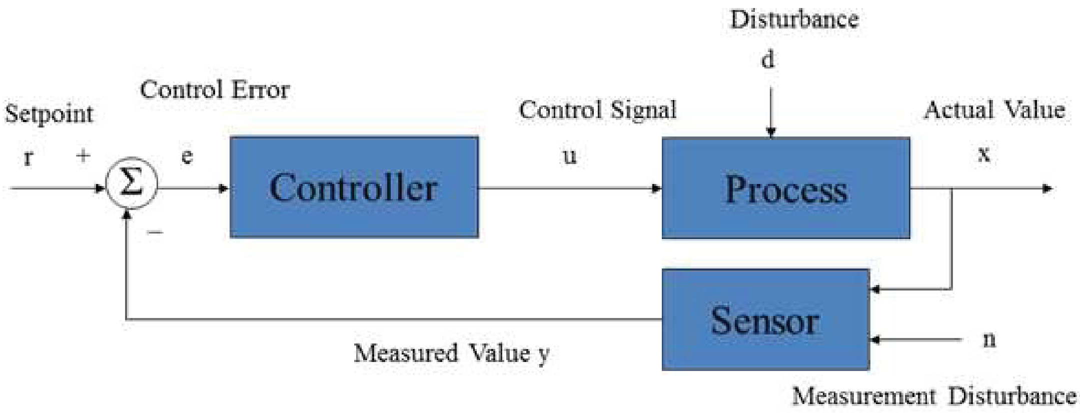

Figure 2.1: ***A general control system description*** **(Glad & Ljung, 2000)**

In the figure the Control Signal (u) would be the cost, C, for the company as described above. The Process includes the model of the influence from the Control Signal to the Actual Value (x), which in this case would be the sales, S, coming from the influence of the CRP, also described above. The goal of the controller is to drive the customer purchase, x, to the level of the Setpoint (r), while the system is being influenced by the disturbances d and n.

In a CRP there are two problems that immediately come to mind to be able to use this type of control system. The first one is how to model the process and the second one is to have a reliable online measurement of the system, or the Sensor in the Figure 3.1. In this thesis we will not look in to the problem of the measurement system since we believe it is mainly a problem of technical nature: Designing an appropriate IT system with an online billing system connected – and therefore it is outside the scope and intentions of the thesis. Instead we will focus on what the company can do if there is a model of the process available, which makes it possible to do off-line optimization of the profitability of the CRP.

3. Method

3.1. Research design

In his work on social science research methodology, Yin identified five components, which have significant importance with respect to social science research design (Yin, 2013, pp. 27-35): (1) a study's questions; (2) its proposition; (3) the unit of analysis; (4) the logic linking the data to the propositions; and (5) the criteria for interpreting the findings. Reflecting these components to this thesis, the study's questions and propositions are discussed in Section 1.3. Considerations regarding the unit of analysis and the closely related issue of choosing the appropriate research method are discussed in the following section, followed by the logic linking of the data to the propositions and the criteria for interpreting the findings. Methodological choices are discussed at the end of this section organized in separate sub-sections.

3.2.Identifying the appropriate research method and techniques

Yin (Yin, 2013, pp. 8-14) advises to identify the appropriate research method based on the questions at hand (among experiment, survey, archival analysis, history and case study as options). Besides these considerations our research was also shaped by practical constraints faced during the early part of our research.

We have closed out archival analysis and history as possible options early since we have found that these methods were inappropriate for our research question and purpose. Reasons for that were that based on previous literature some CRPs are initially designed as "copycats", others are most likely based on in-house (unpublished) best practices of the company or based on literature available to us well. These circumstances together with the rather short history of CRP programs made it unlikely that the aforementioned methods would suit our research goal of developing an abstract model of profit maximizing CRP fitting real-world constraints.

We have tried to make contact to several companies running CRP programs in order to get information about the constraints and issues they are facing when designing CRPs and in general the motivations behind their approach used. One company proved to be exceptionally helpful in giving insights into their CRP business practice. We have decided to conduct an interview with them to gain a better understanding of their practices (see the end of this sub-chapter and subsequent chapters on interview considerations for more details).

The low will from the companies to respond to our request for information has forced us to close out the survey method as a possible research option as well. Yin also recommends the use of the above methods (in particular archival analysis and survey method) when the issue at hand requires the enumeration and subsequent analysis of some properties of the research question or the goal is predictive reasoning about an issue.

Since we had no direct access to business data and process related to running CRPs we had no means to experiment with their manipulation. On the other hand, we wish to develop a model of a CRP based on examining contemporary business use cases. These two properties draw our attention to the case study method (Yin, 2013, p. 11). This choice was also motivated by the wide variety of evidence types possibly available (interview, observations and in our case publicly available online documentation of the CRPs examined during our research) (ibid).

We do not wish to imply however that the final thesis resembles a case study in every aspect. It is perhaps more appropriate to say that we have followed the appropriate guidelines for creating research designs for a case study and general best practices for conducting exploratory social science research. We have applied these guidelines to our goal of developing an abstract CRP model in order to validate our assumptions that the aforementioned tools of mathematical optimizations are applicable for the CRP design process. The resulting thesis depends on the practices proposed by Yin; see for example the subsequent sub-chapter on defining the unit of analysis and section 3.8 on interview design considerations.

In order to examine the extent to which our final research resembles case studies we revert to the technical definition given by Yin in (Yin, 2013, pp. 18-19). According to this definition a case study from a technical point of view is:

> "An empirical inquiry that (a) investigates a contemporary phenomenon in depth and within its real-life context, especially when, (b) the boundaries between phenomenon and context are not clearly evident".

Point a) of the definition above partially applies to our case if we think of CRPs as a contemporary business phenomenon, and since the focus of our thesis is on creating a mathematical model of this phenomenon. One could argue however that we have focused our research on developing an abstract model of the phenomenon rather than focusing on investigating the phenomenon itself in one aspect or another. We believe that the exploratory nature of the thesis justifies this distinction. In point b) Yin also states that the distinction between the contemporary phenomenon examined and its context is not always clear. We will come back to the issue of boundaries in the following sub-chapter.

As mentioned before the available research technique options were limited due to the lack of available company data. Therefore we have chosen techniques that were appropriate to use with readily available public data. Data sources in our case included the previous studies on specific CRPs, publicly available information about running customer relationship programs and Pharmanova the company mentioned earlier. Pharmanova was helpful enough to let us gain some insight into their best practices in form of interviews. Thus we have relied on conducting interviews and analyzing public data from a mathematical modeling point of view as the main techniques used in the thesis.

3.3.Unit of analysis

As it has been stated in Section 1.3 our purpose is to ***develop a model capable of describing an abstract CRP design using standard tools of calculus of variations***. The purpose of this thesis was motivated by our intention to validate the hypothesis that tools of calculus of variations can be used to design CRPs with the aim of maximizing expected profits related to the program. In line with this goal ***the unit of analysis can be defined as the observed set of CRP designs running in Hungary at the time of writing the thesis.*** Through examining the observed CRPs we can get an understanding of their common properties, which in turn will help us determine the applicability of the proposed model in a real-world business context.

When defining the unit of analysis we first considered picking a specific CRP program for a closer examination. We have considered studying our research problem with respect to this CRP as an individual case. The problem with this approach was that the results of the research (naturally) would have been very much tied to the properties of the specific CRP in question. Despite the fact that some design elements are widespread in a given business segment, the field of CRPs as a whole is quite diverse. Choosing a specific design could have threatened the generality of our findings simply because the diversity of CRP properties present in real-world programs would not have been captured by the thorough examination of a single program. As an alternative we considered a multiple case approach, but our efforts in contacting companies running some of the identified programs were in most cases fruitless.

These issues naturally lead to the approach taken in this thesis, namely that we sampled the official description of real-world CRPs (as described in detail later in the thesis) and tried to identify key properties present in their designs. This way we have studied effectively *the properties of the set of sampled CRP designs* instead of conducting a single or multi case study. Defining the unit of analysis as above emphasizes that the thesis focuses on the properties of the CRPs and not on the programs themselves. This is in line with our intentions to validate the applicability of optimization methods in CRP program ***design*** and not on the applicability in calculating the expected profits of any given established program.

3.4. Linking the data to the propositions and criteria for interpreting the findings

As stated in Section 1.3 our proposition is that the toolset of calculus of variations is expressive enough to model real-world CRP designs and that the optimization approach can be used to design CRPs with the aim of maximizing expected profits. Related to that, we stated that we would like to develop a model with the aforementioned tools to demonstrate the benefits and identify the limits of this approach.

We have chosen a modified version of the explanations building technique proposed by Yin (2013, pp. 136-141) to validate this proposition. Yin argues that explanation building can be used as an analytical technique if the researcher studies cases concerning “How?” and “Why” type of questions related to a phenomenon. Our relevant research question is however: “How can the properties of real-world CRPs be modeled by mathematical optimizations methods in order to develop CRP designs maximizing expected profits with respect to real-world business constraints available?” Thus, we are interested in the benefits and limitations of using a model based on the aforementioned tools, rather than explaining casual relationships related to a certain phenomenon. In a sense, since such models not yet exist (to the best of our knowledge) our goal is to make an exploration rather than explanation. Therefore the technique proposed by Yin needs some adjustment to be applicable to our case. In its original form the techniques contains the following steps (Yin, 2013, p. 143):

- Making an initial statement or proposition
- Comparing the findings of an initial case against the proposition
- Revising the statement or proposition
- Comparing other details of the case against the revision
- Comparing the revision to further cases
- Repeating the process if needed

In our case the proposition is that we can use calculus of variations to design CRPs with the aim of maximizing expected profits. We will examine the validity of this proposition by developing a model

CRP. We will compare the properties of our model to insights gained during an interview conducted and data extracted from sampled CRPs.

In order to show how tools of calculus of variations can be used to design CRPs we have to first demonstrate the approach on an abstract model. After the model is developed we can analyze the benefits and limitations of the approach with respect to the business context it is going to be applied in. We have conducted an interview with a company currently running a successful CRP to gather expert knowledge on current CRP design practices.

Based on the interview we will examine how the proposed method contributes to existing CRP design practices. The proposed research question (see above) we believe is two-sided: answering “How?” in this case requires not only identifying applicable mathematical tools, but also identifying the practical issues related to the business context in which we strive for maximizing profits. This step should help to bridge the gap between purely mathematical considerations and practical business problems faced by practitioners. We have come to the conclusion that we cannot apply statistical measures or other commonly suggested approaches to this part of the analysis because of the largely exploratory nature of the issue. Therefore we will enumerate and discuss the identified benefits and limitations of the approach instead.

Sampled CRP data will be used to identify the CRP classes based on identified properties for which the proposed method seems applicable. The goal of the latter process is to identify patterns among the properties identified from sampled CRP for which the model developed is applicable. This way we can clearly identify CRP types for which the proposed method could be useful prior starting actual modeling.

3.5.Selecting relevant methodological literature

We started our research process with reviewing literature both on customer rewards programs, focusing on literature with a comprehensive review of the field (Sällberg, 2010), and on optimization research with focus on standard texts available on state of the art methods. The books (Boyd & Vandenberghe, 2009), (Shawe-Taylor & Cristianini, 2000) were already known to the authors from previous studies and specifically with focus on texts discussing the application of optimizations methods to economic problems (Kamien & Schwartz, 1991). In the end the book of (Kamien & Schwartz, 1991) proved to be the most relevant source on theory, while (Sällberg, 2010) served as the starting point of literature search on the topic of CRPs. We have done thorough research through the extensive sources available from BTH’s online library looking for texts with a similar focus or research methodology to ours. Section 2.1 is the result of this work and primarily the work of Gandomi & Zolfaghari (2013) proved very valuable both as the – we believe – closest source among previous research efforts to our approach and as a practical example in conducting mathematics focused research in the field of customer rewards programs.

We have found the work of Gandomi & Zolfaghari (2013) as a valuable example on structure, format and research design. Following this example – based on existing literature identified during the literature review – we tried to incrementally formulate a model of typical CRP programs formulated as an optimization problem – summed up in the derivation of the model in Section 2.1.

During this process the (Kamien & Schwartz, 1991) book proved to be a valuable resource since it presented several practical examples on the application of optimization methods to economic problems with extensive discussions on different realistic problem formulations based on these methods. These examples helped us in identifying properties of CRPs which could be modeled through optimization methods (because they resemble other economic examples of the book in one aspect or another – the horizon and the dynamic nature of the models for example) and properties which are probably outside the scope of this toolset (modeling of membership levels for example).

3.6.Sampling procedure

We have investigated several programs, identified their key properties based on publicly available sources, and formalized them as optimization problems to be able to discuss their designs with respect to the optimization method approach suggested in this thesis. Our goals was on one hand to identify real-world program structures for which the suggested approach is applicable and on the other hand to sample examples of program designs for which the approach is not suited and identify the reasons for it in order to outline possible future research directions.

During the data collection phase of the study initially we have chosen to select CRPs being run by some of the largest companies in Hungary in a non-random fashion. Reason for that was partially motivated by the intention to select programs with rule-descriptions available online in order to help the validation of our findings by interested readers. Another reason for selecting among the largest programs was that it was clear from the beginning of our research that the approach to be presented in this thesis is most useful for:

a) companies that have a large customer base (in general optimization methods provide more reliable results over a large population – we believe that this consideration is important in this case too, although we tried to look for analytically solvable problem formulations)
b) companies that have an integrated IT infrastructure (an integrated background infrastructure of sales and customer data is necessary to apply the approach in real life)

As a result the programs referred to in this study present a non-random sample of – at the time of writing – running CRPs in one country, from multiple industries. We were not aware of any property of the suggested mathematical approach which would make it non-applicable to certain industrial segments. Therefore the sampled companies vary in the industries they operate in. Also we believe that the fact that the programs are selected from multiple industries strengthens the generality of our findings.

We have also realized during the data collection that the CRPs observed in Hungary follow mostly the same structure. We believe this is due to the presence of the "copycat" phenomenon mentioned in previous studies (Dowling & Uncles, 1997) and discussed in Section 1.1. In fact it was quite hard to select programs that vary in some aspects of their designs. We decided therefore to intentionally limit the sample to a couple of companies which run programs with – from at least one aspect – different designs. Strictly taken this led to the identification of two separate design groups for which our approach seemed applicable. The designs observed in these two groups differed in the horizon of the program and in the number of possible reward redemptions – although these two properties seem to be logically related (discussed later in Section 6). We have decided to include two to three companies in each group from different industries in order to somewhat emphasize that these designs are common among real-life companies and not limited to one company only. Finally we have also identified some other common CRP design elements for which our approach is currently not applicable due to limitations in expressiveness of the proposed optimization formulation. The respective CRP properties are discussed under Section 6.

We hope that the resulting fairly small sample helps the reader to identify the key properties of the observed CRPs and follow our related discussion. We also believe that a larger, redundant sample including all observed CRPs – despite their similar design – and concentrating on sample size and distribution or other statistical measures would have led to false conclusions and therefore would have been more of a distraction from our original intentions to validate the applicability of a new approach to CRP modeling. Putting further efforts into giving a thorough review of CRPs present in Hungary may have given us additional information regarding the cause of popularity of some CRPs, maybe some insight into the extent of popularity, but as already mentioned this research direction was not among our primary intentions.

Related to this issue we realized during the collection of the data that we had no means to reliable measure the frequency of each design property, and based on the result, present a representative distribution of the similar designs. Reason for that is that we had no means to enumerate all CRPs present in the country instead of sampling them. We decided therefore not to include (and discuss the possible implications of) a non-representative distribution of observed CRP designs. Instead we tried to clearly point out this limitation here in order to avoid drawing wrong conclusion based on questionable methodology.

Program properties that currently cannot be modeled with our approach are pointed out as limitations in text in Sections 4.6 and 5.1 (see discussions on membership levels). We have included for comparison the Supershop initiative, a loyalty program with some distinct properties that point us to a limitation not related to the descriptive capabilities of the optimization, but rather to difficulties related to deriving (or validating the existence of) an applicable solution. We believe that the above mentioned loyalty program could be described as an optimization model, but most likely has only numerical solutions if any.

To summarize the result is an intentionally small, non-random sample, designed to represent the observed real-life CRPs including examples which can be fairly straightforwardly modeled by our approach and also cases which show the current limits of this method.

3.7.Collection of data

Based on the sampling procedure described above we have selected several companies who had their CRP's rule set published online on their web pages. We assumed that each program has its important properties published among the rule set accessible to the customers and general public. In order to transform these rules into a comparable form we have studied the rules and identified the respective properties of the CRP programs which are governed by each rule and categorized the programs based on these properties.

For example, most rule sets determine the starting and ending dates of the program, in case there was an explicit ending date, we categorized the respective program as a "fixed horizon program", in cases where there was no ending date set we categorized the program as an "unlimited horizon program". Thus we have assigned two possible category values to the variable "horizon of the program" based on the ending dates although the respective rules governed explicit start and end dates together. We haven't differentiated programs based on their timeframe: in case of two programs running between June – August and March – July, for example, we categorized them only based on the property that the end of the program is set at a fixed date. References to the original rule descriptions are listed as references with date of retrieval pointed out.

After this preprocessing we transformed the extracted properties into the tabular form presented in Section 5.2. The resulting table contains mostly categorical variables like: "point function type: linear". We used the sampled program data as a baseline reference to real-world practice, since we didn't want to evaluate the programs in any way and haven't used derived statistics based on sample population for any purpose, we believe we haven't lost information due this practice (for our purposes).

The resulting tables contains seven variables for the selected companies: *Horizon of program, Point function type, Reward function type, Reward type, Number of redemptions, Point collection medium* and *Number of points required for the reward.* Variables *Horizon of program, Reward type, Number of redemptions, Point collection medium* and *Number of points required for the reward* were directly identified based on the rules observed. Variables *Point function type and Redemption function type* were inferred from other variables as described in detail in Sections 5 and 6.

3.8.Interview design

We tried to check how our approach fits into real-world CRP design procedures by conducting an interview at the largest pharmacy chain in Hungary: Pharmanova Zrt., which has been running CRPs since 2008. Our goal was to examine existing industry practices in order to get an understanding about the business context in which CRPs are executed. We have conducted repeated interviews with the head of marketing at Pharmanova in order to get insight into considerations behind a CRP design.

Pharmanova was chosen because it has been running CRP programs for a long time, their CRP designs were close to our initial assumptions, and they have a significant share on the Hungarian pharmacy market. They also share the properties listed in the previous section expected from the companies sampled. The head of marketing was selected as interviewee since inside the company the marketing department is responsible for the CRP programs. Our data collection problems with respect to accessible company data are partially resolved by the interview, since the interviewee as a direct observer of the events gave important insight into CRP design considerations.

Yin in (Yin, 2013, pp. 106-109) classifies interviews in three classes: in-depth, focused and survey-like interviews. The approach we have taken falls between an in-depth and a focused interview. It resembles in-depth interviews in that we have asked the interview about both the facts of the matter and their opinion on specific aspects of their CRP programs and related issues. Also the interview was held on multiple occasions and during an extensive length of time. On the other hand, the interview was focused in the sense that we concentrated only on issues related to the CRP programs in the company. The interview also resembles focused interviews that in the time of the conversations taking place we already had an established view of some aspects of the topic researched.

As outlined earlier the interview included in this thesis – and presented in a unified form – is the result of multiple interview sessions conducted in person and recorded in handwritten notes and eventually merged in the digital form presented in this thesis (Appendix B) after being reviewed by the company's representatives. To preserve the initial notes we have uploaded them into an online

Dropbox account (Dropbox, 2014) we used as a collaboration tool. The goal of the interview was to get a picture on real-world CRP design considerations in order evaluate how well the proposed approach would fit with current best practices.

The lengthy sessions conducted threatened to unintentionally influence the interviewee by our own opinion on the matter. This was getting more and more obvious during the sessions. To avoid this we have outlined the model we were working on and the modeling issues we were facing only on a high-level. The interview was therefore aware of the purpose of the interview, but had no insight into the technical details of our work which could have interfered with her personal practical experiences. This way we tried to separate personal experience from personal opinion. Also since our goal was to validate the applicability of an – at least to our knowledge – new approach to real-world CRP design it seemed reasonable to focus the conversations on the experience of the company's representative with existing and previous programs at the company. In some cases where additional information was needed on a topic or the answer raised further questions another interview session was held. The presented interview is the merged essence of these iterations.

The most difficult part of the interviews was the presentation of the actual results of the interview. Although Yin advises to record the conversation in some form, it is also stated that recording done by electronic devices can disturb the interviewee. As mentioned earlier we have chosen to take hand-written notes which we have merged into a unified interview and as a final step we have let the interviewee validate the result for inaccuracies. In some case however there were overlaps between some answers. When presenting the interview we have decided to present both questions together and unify the answers (see question 3 "How did you evaluate your CRP program? Did you use economic measures like sales amount or frequency or did you rely on other measures when evaluating the program? ").

Yin also advises to avoid "Why?" questions as a general guideline in order to get unbiased answers. We tried to apply this advice to our interview as well and reformulated our "Why?" to "How?" questions as suggested. We have also tried to avoid "Yes/No" questions in the interview for the same reasons. In most cases we managed to rephrase these questions to an open-ended question instead leaving some additional space for the interviewee to answer. At other occasions we managed corroborate the answer with other questions covering the same issue asked at a different time (see question 3 again) and presenting the answers together as outlined above.

In other cases a "Yes/No" type of question was needed to clarify a previous statement in a clear form: "***How did you decide on whether you should continue the CRP program?"*** and later***: "Have you considered the reward costs too when you were evaluating the CRP?***". Finally "Yes/No" questions were asked in the interview in cases when they were covering factual background information over which the interview had no influence. In these cases the answer can be considered unbiased without respect to the form of the question posed ("***You said that you introduced your first program in 2008. Is this program still running?"*** and ***"Is this program the current loyalty program running in your pharmacies?"***, for example).

Yin argues (Yin, 2013, pp. 106-107) that during the interview the interviewer has the main job of following the initial line of inquiry and asking the actual questions in a conversational manner. In our case we outlined three topics initially we wanted to cover: introduction of the programs at the company; getting an understanding on the evaluation methods used; and design considerations related to the programs (if not apparent from the previous topics). During the interview we focused on these topics asking questions in a more conversational manner and asked additional or rephrased questions at certain areas when it was necessary. Questions 1, 2, 6, and 7 were aimed at getting an initial understanding of the programs at the company. Question 3, 4, and 5 were asked to get an understanding on the evaluation method used at the company related to the loyalty programs. Question 8 was meant to reveal any additional information regarding the design considerations related to the programs. The last two questions, 9 and 10, were needed to clarify the information gained during previous questions.

During the interview some necessary additional considerations related to CRP design became known to us: it turned out that legal regulations can influence heavily the CRP design of a company. Based on our limited insights gained on the topic during the interview this issue is country and industry specific. We decided however to discuss some implications of this issue since it poses a practical limitation to the applicability of the proposed optimization approach too in case a company is operating under similar regulations.

3.9. Optimization method

Optimization methods come naturally in mind if the goal of a technical problem is the maximization or minimization of a function or of the outcome of a process; in our case the technical goal is the maximization of the expected profit of a company resulting from a customer relationship program.

The goal of the thesis is to examine the general applicability and limitations of optimization methods to the field of CRPs (as stated in Section 2.2). However since mathematical optimization is a broad field in itself we had to narrow down our efforts to the discussion of optimization tools that seemed to be most suited for our problem set.

We have shown in Section 2.2 that a CRP program can be quite naturally formulated as an optimization problem and that the complexity of this optimization problem is in direct relationship with the properties of the CRP design modeled. This motivated our efforts to sample some common CRP designs in order to get a better understanding of what kind of loyalty programs can be modeled with optimization tools. During the sampling and data collection process it turned out that it is common that loyalty programs either offer monetary rewards directly or the offered rewards are convertible to monetary rewards easily. This phenomenon narrowed our research to continuous optimization problems for reasons discussed more thoroughly in Sections 5 and 6.

We also realized that arguments formulated in Sällberg (2010, pp. 32-39) regarding the positive effects of progressive function trajectory point us towards convex function shapes which are also commonly researched in the optimization field.

The following simplified form of a profit function (discussed earlier in Section 2.)

$$CRP\ Profit = CRP\ Sales(Reward\ Value) - Reward\ Cost(Spending)$$

gives indication that the problem at hand involves dynamic feedback. This relationship and its implications are discussed in Section 3.2 in detail and Figure 3.1 shows a graphical representation of the same relationship from a control theoretic perspective. We have discussed this property briefly here in order to point out that the process we wish to optimize is dynamic in nature.

After these considerations we have narrowed down our investigation on optimization classes to ones with continuous, convex goal functions and of dynamic nature. These properties together have pointed us to the closely field of calculus of variations where we examined previously researched example economical applications. An important tool in finding a solution to this class of problems in the field of calculus of variations is the Euler equation which is presented in the Appendix of the thesis. We relied on the Euler equation also – following similar problems described in (Kamien & Schwartz, 1991) – during the derivation of the solution to the CRP model presented in Section 3.2.

3.10. Validity and reliability

The main source of reliability for our work is the mathematical toolset and the related proofs, lemmas which we used as the baseline for the suggested approach. In this regard we relied on the comprehensive work presented in the book on methods in optimization and control theory with economical applications: Dynamic Optimization - The Calculus of Variations and Optimal Control in Economics and Management (Kamien & Schwartz, 1991). Since this book is considered a standard text on optimization theory with economical applications, reviewed and referenced many times in scientific literature we took it as a reliable source of theory. The derived solution for the example CRP class in Section 3.2 follows the solution approach presented in the aforementioned book. Since we haven't deviated from the proven theoretical background we can safely assume that the theory is still consistent and the proofs presented originally in (Kamien & Schwartz, 1991) are valid.

Initially we wanted to keep the thesis focused on theory, since developing the modeling approach and checking its consistency with real-world CRP designs consumed most of the time available for the thesis. Therefore we could not validate our proposed approach by computing the optimal parameters given the behavior of the customer base of a real-world CRP as input data. We have found that evaluating the real-world effectiveness of the optimization approach would have required a CRP program designed with the help of this approach from the start, running to the end of its horizon – running for several months in practice – and after that evaluated by comparing to a previous CRP from the same class designed without applying tools from optimization methods (practically with the predecessor of the program under evaluation).

Pharmanova who gave helpful insight into real-world CRP design considerations were running an unlimited horizon CRP, which meant that using their data for evaluation purposes was not available for us: it seemed at this point of the thesis that developing a valid performance evaluation framework for real-world testing the effectiveness of the approach proposed in our thesis would require the amount of time needed for a master thesis in itself. Unfortunately the resources needed were clearly outside the scope of our thesis, but present necessary future research directions. However, we believe that the proposed method would be readily adoptable to practical CRP design processes based on the facts that we relied on existing, proven theory and that CRP classes that can be handled by this approach are common in practice (as we will show in later sections).

We also relied on the existing work done on the incentive structure approach as the baseline of formulating realistic CRP models in the first part of the thesis. During the development of our initial model based on previous work done in the field of CRP research we realized that the research should be restricted to models with analytical solutions since we believe that this solutions class is more useful in understanding the models. In the personal experience of the authors it is an open debate in the industry whether it is better to develop fine-tuned black-box models which promises more accurate results or models that are – although not necessarily – less accurate, but give an insight to the inner workings of a given business model (compare for example the standard regression model discussed in the Managerial Economics book by (Keith, Young, Erfle, 2014) and the treatment of the model parameters as tools for analyzing the business problem with the "black-box" non-linear support vector machines described for example in (Shawe-Taylor & Cristianini, 2000)).

Developing models with analytical solutions can be seen as a limitation of our research from a mathematical perspective. The wide range of numerical optimization problems classes which have tractable solutions are covered in existing literature extensively and have witnessed a rapid developments recently, while only a subset of these problems have analytical solutions - the interested reader can refer to (Boyd & Vandenberghe, 2009) for example which is used as a standard book on state of the art optimization research. However, since our aim was to create a research focused thesis on a less examined aspect of CRP research, it was important to us to develop models which hopefully can serve as the basis of further research and we believe that analytically solvable models are more suited for this goal. Also the complex dynamical, convex nature of the base problem formulation which we have initially developed in Section 2.1 also urged us to focus on analytic solutions.

Our proposed approach's applicability is limited currently to CRP designs which do not contain membership levels. This limitation is related to the used mathematical tools expressiveness. We are unsure whether the proposed models can be developed further in order to be useful also for fine-tuning programs with membership levels. This question should be addressed by further research efforts.

As we realized during the interview some restrictive legal regulations may apply to loyalty programs in specific countries – like as it turned out in Hungary. The respective laws are related to the wish of the regulator to limit promotion efforts on markets where especially strong ethical guidelines should apply like in the case of the pharmacy market. To the best of our knowledge these regulations are not general and are related to the application area in specific countries and not to loyalty programs as specific marketing efforts.

We have taken some modeling assumptions during the development of the example presented in Section 2.2. The most notable are that we worked with continuous, convex functions and specifically used the revenue function developed and discussed also in Section 2.2 for deriving a solution of the aforementioned example. We will discuss the implication of both assumptions in Section 5 and 6.

To sum up the related considerations we have found that the assumption of modeling CRPs with continuous functions is realistic mainly because these functions are used in our models to describe monetary relationships. Also rewards are commonly treated as monetary values (liabilities) in order to support accounting. The convex form was used in order formulate a revenue function forms that describes a realistic customer behavior. However before applying the proposed method in real-life the company considering the methods application should validate that their customer behaves as described by the function. Also considering the application of the proposed revenue function, the specific function parameters should be adjusted to the customer base of the specific CRP. Thus we can conclude that the aforementioned function properties are on one hand necessities for developing an analytical solution, on the other hand they describe realistic customer behavior and are in line with accounting considerations (discussed in more detail in Section 6).

4. Findings

4.1. A model of a CRP as expressed as an optimization problem

We have seen in Section 2.2 that a CRP can be modeled as a dynamic convex optimization problem. Here a more comprehensive theoretical discussion of how this can be done will be shown. We have found that the general form – discussed extensively in (Kamien & Schwartz, 1991) – is both expressive and flexible enough to describe a CRP program. Therefore we rely on the aforementioned work: more specifically we will use example 16.1 from (Kamien & Schwartz, 1991, p. 98) to describe the application of the mathematical theory of optimization in the CRP program design process. In example 16.1 in the book (Kamien & Schwartz, 1991) spending on advertisement is the input that affects the customers' will to buy the company's products. In this thesis it is the rewards given to the customers through the CRP (represented in a unified monetary form) are having a motivating effect on the customers. The motivation for using an infinite horizon optimization problem is due to the fact that a company would like to optimize the profit over the customer's full lifetime or in other words the customer lifetime value (CLV), but this assumption can be relaxed when appropriate.

Let R(t) be the reward value of the CRP perceived by a customer participating in the program (which also can be seen as the "goodwill" a company gets from giving rewards to a customer as defined in (Kamien & Schwartz, 1991)). R(t) can be seen as an abstract way of describing the influence of the CRP on the customer's will to engage in a business relationship with the company: R(t) could express and quantify the effect of many different properties present in a CRP, such as positive perception related to membership levels, points, the effect of a direct dedicated line for ordering with personal assistance, or faster delivery.

Let S(t) = S(R(t)) be the increase in revenue (or sales) a company earns from having the CRP program, or increase in customer spending, and hence the influence on the customer from the CRP rewards R. Different forms of the revenue function, S(R), can be developed. Here only the principle will be discussed and these forms need to be further investigated. How this influences the optimization problem is briefly described below through an example.

A practical model for the "goodwill" function (following the terminology of (Kamien & Schwartz, 1991)) can be a first order dynamical system,

$$R'(t) = C(t) - b * R(t),$$

, where C(t) is the cost of the rewards given to the customers, i.e. the cost for the company for keeping the CRP running[4]. R'(t) is the time derivate of R and b is the rate at which the value

[4] A company using the optimization approach in practice could either rely on the proposed first order dynamical model of "goodwill" or develop another model expressing the company's experiences with respect to its specific customer base. As a third option the company could try to capture this relationship through regression models; these methods are discussed extensively in other works (see for example (Boyd & Vandenberghe, 2009) on engineering or (Keith, Young, Erfle, 2014) for a more practice oriented discussion on economical applications) and the success of their application is based largely on the quality and properties of the available business data. We believe that the discussion of these alternatives and the related difficulties would lead outside the scope and intentions of this thesis. For our purposes - discussing the applicability of our approach - it is enough to rely on a precise mathematical formulation capable of expressing our assumptions on the customer base's behavior.

diminishes over time. The cost C for the company could be different from the reward value R for the customer since, e.g. the marginal cost for giving away a free night in a hotel room is less than what the customer perceives the value to be when he or she gets to spend a night at the hotel for free. A function like this, which also could be describing a point function, says that the greater the reward the greater the influence on the customer goodwill towards the company and that it decreases over time.

Hence we argue that this could be a better way to model the increase in "goodwill" coming from rewards rather than for example an integrating function, i.e. R'(t) = f*C(t). With an integrating function one could model an assumed accumulating motivational effect on the customer: the customer keeps on gaining the same increase in points for each dollar spent in the CRP (since the point function is linear) and perceives every point with the same motivational value (the increase in "goodwill" is also linear). Using linear point functions seems to be a common real-world practice (see Section 5.2). Based on this, one could naturally model the reward cost of the company *C(t)* as a linear function in the optimization model, however assuming as a result also a linear increase in "goodwill" seems to be unnatural for reasons discussed above. We believe that the concept of "goodwill" as first described in (Kamien & Schwartz, 1991) is more applicable in this field.

Given the assumption there is a dynamic system model the company is now to choose how much the cost, C(t), of the customers rewards should be over time to maximize the profit, S(t) - C(t) (i.e. increase in sales by using the CRP minus the CRP cost) over time. The optimization could also be over a specific time period T, which could for example be one year since most CRPs evaluate membership status and validity of points on a yearly basis or launch fixed-term programs in the first place (we will refer to these programs as "fixed-horizon" CRPs throughout the thesis).

Hence, according to (Kamien & Schwartz, 1991, p. 98) the optimization problem can be written as:

$$\max \int_0^\infty e^{-rt}\,[S(t) - C(t)]dt = \max \int_0^\infty e^{-rt}\,[S(R) - R' - bR]dt$$

$$\text{subject to}$$

$$R(0) = R_0 > 0,$$

$$-bR \leq R' \leq \bar{C} - bR$$

, where e^{-rt} is discounting of future profits with rate r, R_0 can be seen as the influence of an initial reward, and $\bar{C}$ is the maximum cost for the rewards the company can spend on its customers at any given time during the time period.

For the optimization problem to be solvable the function need to fulfill S'(0) > 0 and S'' < 0. This means that if the company can model the revenue function S(R) based on its specific customers it should be able to find the optimal reward cost function C(t) to maximize its profits based on a specific type of CRP with a specific function for x.

One simple model for S(t) which fulfills these prerequisite, but has some realistic properties is

$$S(t) = aR(t) - cR(t)^2$$

At the beginning, the influence of the rewards on the customer buying the company's product is high and it decreases the higher the reward becomes, see figure 3.2. Even if this might seem strange at first, this could actually be a viable model for the influence of the reward on sales since the customer might not have any need at all of a product after buying a certain amount. Or maybe stocking the product is not an option for the customer for some reason, no matter how high the reward might be. Also, using this function we could express the realistic assumption that the customer probably has a limited budget and needs to spend his or her money elsewhere.

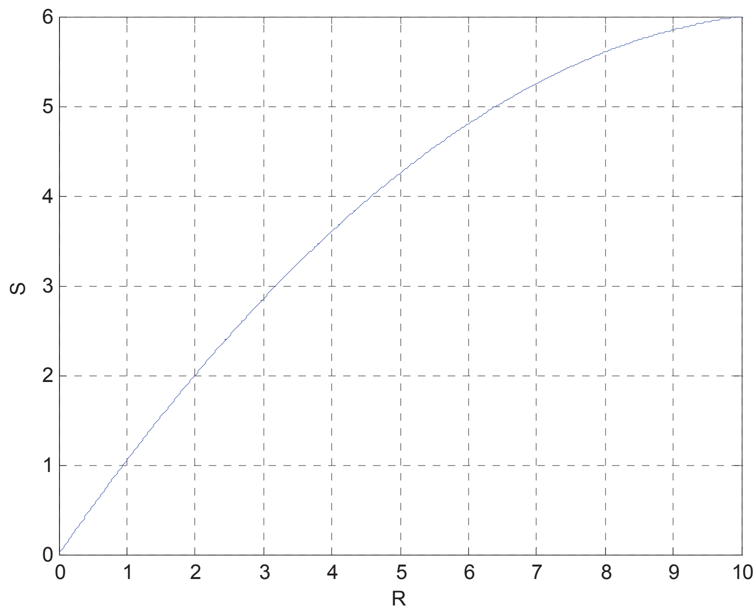

Figure 4.1: *S(t) = aR – cR² (a = 1.1 and c = 0.05) showing that the influence on sales S from the reward R decreases*

The optimization problem (11) has a special form, which is linear in R'

$$\int_0^\infty e^{-rt}\,[S(R) - R' - bR]dt = \int_0^\infty e^{-rt}\,[(a-b)R - cR^2 - R']dt = \int_0^\infty e^{-rt}\,[M(R) + N(R)R']dt$$

Then the Euler equation, see Appendix A, for solving the optimization problem becomes

$$M'(R) + rN(R) = 0$$

Hence the solution to the optimization problem in the example above is then given by the Euler equation

$$(a-b) - 2cR - r = 0$$

And thus the level of goodwill, R(t), the customer needs to be kept at should be constant over time and equal to

$$\hat{R} = (a - b - r)/2\,c$$

to optimize the profit from the CRP. The cost for the company to keep the customer's goodwill constant at this level is $\hat{C}$= b*$\hat{R}$ (If assuming a steady state solution from time equal to zero the right hand side of R'(t) = C(t)-b*R(t) should be equal to zero). Hence if the parameters a, b, c and r in the model are available and it is a good model of the customer behavior we are able to calculate the optimal reward of the CRP to get the optimal profit.

For the example used above with a = 1.1, b = 0.4, c = 0.05, and r = 0 (no discounting) the optimized profit is calculated to be 2.45 at $\hat{R} = 7.0$ and $S = 5.25$ with an optimal steady state cost for the CRP of $\hat{C} = b * \hat{R} = 2.8$, see Figure 3.3.

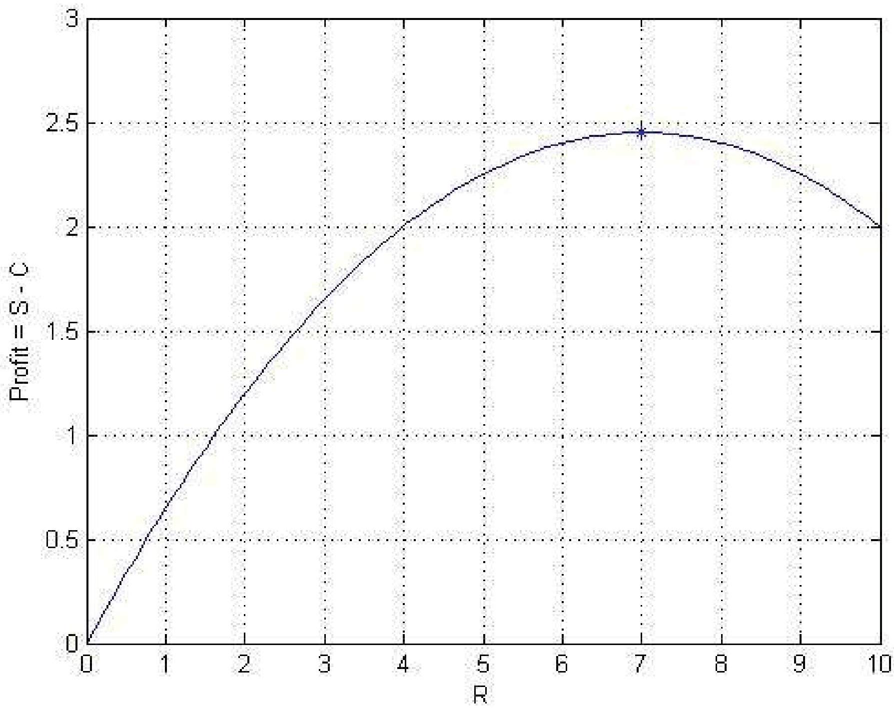

Figure 4.2: ***Optimized profit (*) for the example with a = 1.1, b = 0.3, c = 0.05, and r = 0.***

4.2.Comparing the individual CRP designs investigated

As we have outlined earlier in Section 3.2 Sampling Procedure, we have sampled companies in a non-random fashion. The sampled companies include the biggest companies in their respective fields in Hungary, running country-wide campaigns with high visibility both on regular advertisement channels and on the Internet described below.

Tesco CRP in Central and Eastern Europe

The Brazilian football legend Edson Arantes do Nascimento Pelé, or as he is commonly known just *Pelé*, visited Hungary on 26 March 2014 as the guest of Tesco (Trade magazine, 2014; Promotional Marketing, 2014) in order to launch the promotion of Tesco. The large-scale promotion is running in five countries: Poland, Slovakia, Hungary, Czech Republic and Turkey (Retail Times, 2013) and as Ben Turner, central merchandising manager at Tesco, states in the article "With almost 20 million customers passing through Tesco Poland stores in just one week, one can only imagine the customer reach from a five country programme."

The rules of the program are the following (www.tescohuseg.hu):

- Purchases are to be made between April 3 and July 13 2014 in Tesco stores located in the participating countries.
- 1 loyalty point is given after every 1000 spent forints (tescohuseg.hu), after every 20 zloty in Poland (www.kicklikepele.com/polska), after every 10 Euros in Slovakia, and after every 200 Koruna in the Czech Republic (www.kicklikepele.com/czech).
- Rewards can be redeemed for customers with Tesco Club cards after 25 without Tesco Club cards after 30 points.
- One can redeem the rewards after turning in a with points filled (25 or 30 points see above) point card.
- Rewards are monetary discounts on Pele branded third-party products sold in Tesco stores with different discount amounts for the participating five products.

The rules above describe the following general properties of the CRP design:

- It is a fixed horizon program
- It offers monetary discounts on third-party products as rewards
- The program follows a point function based structure
- Points are collected after purchases and registered on a medium (point card) administrated by the customers
- The person who redeems the reward is not necessarily the same person who collects the points
- The program is not mutually exclusive with other programs
- It has no upper limit on the possible amount of points to collect, rewards to be redeemed or purchases count toward rewards by single customer, but the overall rewards amount redeemable through the whole program is limited.

The CRP of Penny Market

The point collection program of Penny Market has a similar structure as above (Penny Market, 2014). The rules of the program are:

- Purchases are to be made between February 27 and May 14 2014 in the Penny stores located in Hungary.
- 1 loyalty point is given after every 2500 spent forints.
- Rewards can be redeemed after 10 points.
- One can redeem the rewards after turning in a filled point card.
- Rewards are monetary discounts on third party products sold in Penny Market stores with different discount amounts for the participating seven products.

After generalizing the rules of this program we get the same CRP structure as above, the two programs differ only in the respective parameters of the points function, the reward (discount) values and participating product base:

- It is a fixed horizon program
- It offers monetary discounts on third-party products as rewards
- The program follows a point function based structure
- Points are collected after purchases and registered on a medium (point card) administrated by the customers
- The person who redeems the reward is not necessarily the same person who collects the points
- The program is not mutually exclusive with other programs

The CRP program of MOL

MOL Zrt. has a long tradition of different CRP programs usually having monetary discounts on third-party products as rewards. These programs share a similar structure: they are fixed horizon promotions and points can be collected after purchases at MOL gas stations.

The rules of the current program are (MOL Zrt, 2014):

- Purchases are to be made between February 27 and May 14 2014 in MOL gas stations located in Hungary.
- 1 loyalty point is given after every 2000 spent forints.
- After one purchase occasion maximum 30 points can be collected
- In order to redeem the monetary discounts on the third-party products one has to collect 5, 10 or 15 points depending on the product.

This program although shares many similarities to the other programs examined (fixed horizon program with a point-function for monetary reward on third party products) it has some significant differences too: the maximum number of points that can be collected in on purchase is limited and for different products different number of points are needed.

The SuperShop initiative

This is a joint program of eight participating companies: Spar, Interspar, OMW, OBI, Burger King, Union Insurance, Erste Bank and Budapest Taxi (www.supershop.hu). Purchases made at the participating companies count towards points using a linear point function and can be redeemed in around 800 locations (shops of the participating companies) as delayed monetary rewards. This program shares the most similarities with the current program of Pharmanova Zrt.: it is an unlimited horizon program offering monetary discounts based on a linear point function. We included this program because of the many similarities in the comparative analysis, but one should note that the application of our proposed method would be difficult because of the large number of participating companies. In order to use our model one should work with eight different sales functions of the respective companies, although the point function is the same. This property puts this program outside the scope of our current research, but we decided to include the initiative since it closely resembles other properties also available in the Pharmanova CRP design and therefore indicates that the design is not uncommon in the industry.

We have noted based on this non-random sample, that the "copycat" phenomenon described by (Dowling & Uncles, 1997) is also present on the Hungarian market: many programs share similarities, which are most visible when one examines the structure described by the published rules. For our study this means that an optimization model developed for one CRP design can be adapted to other programs with high probability.

We have grouped the sampled CRPs during the data collection phase in two high-level groups. The separation was based on whether the loyalty program design in question can be described by an optimization model. Reason for this division was that for some properties observed by us and also mentioned in previous literature extensively, we currently do not know how to model them with the help of optimization model formulations (a typical example are membership levels). Since we haven't found a suitable representation - to the best of our knowledge - currently we have no means to derive analytic (or calculate numeric) solutions for these cases. The groups of companies, which can be described by an optimization model however are presented in the table and are divided further into two groups.

The first group contains CRPs which have a structure that can be described as an optimization problem, as pointed out in text also solution approaches are covered to similar problems extensively in the book by Kamien and Schwartz (1991). As we will discuss, to this group belong programs with a specific end date. The second group can be modeled the way as described in Section 2.2; these are real-world programs that share many properties with the model outlined in Section 2.2. This also indicates that in case the specific functions related to a company can be described with the same function classes as in Section 2.2 the derived analytical solution would be valid for the respective company too (after substituting the real-world function parameters with values applicable to the customer base of the company). In case of different (but still convex) function classes one could derive the specific solution of the model in an analogous way.

Based on the rules identified for the individual programs during data collection we have extracted seven variables (presented as rows in Table 4.1). These variables and their observed values are discussed below.

Regarding ***reward types*** we have observed that it is common practice for the companies to offer monetary rewards in form of lowering the price of a given product by a certain percentage. Affected products involve in some cases the respective companies' own products in other cases third-party products offered by other companies. For our intentions this means that the rewards can be treated in the same monetary values as customer spending and that the reward functions can be approximated by linear functions in practice. In order to separate directly observed properties from our inferred properties, we have introduced a new row, called ***reward function type***.

Program	Pharmanova	Tesco	Penny Market	Mol	SuperShop
Horizon of program	Unlimited	Limited	Limited	Limited	Unlimited
Point function type	Linear	Linear, accumulating	Linear, accumulating	Linear, accumulating	Multiple linear
Reward function type	Linear	Linear	Linear	Linear	Multiple linear
Reward type	Monetary discount on all products	Monetary discount on a specific product	Monetary discount on a specific product	Monetary discount on a specific product	Monetary discount on all products
Number of redemptions	Unlimited	Limited by the number of products	Limited by the number of products	Limited by the number of products	Unlimited
Point collection medium	Digital Point Card	Paper Card / Digital Point Card	Paper Card	Paper Card	Digital Point Card
Number of points required for the reward	Every point counts as a monetary discount	25/30	10	5/10/15	Every point counts as a monetary discount

Table 4.1: ***Comparison of CRP designs***

Regarding the ***number of points required for a reward***, we observed two in fact very similar rule types. One common approach gives the customer points after a certain amount of spending (meaning a ***linear point function***). For example 20 forints are needed for a single point and every point counts towards rewards as in the case of Pharmanova. The other observed rule type requires accumulating a certain amount of points before the points can be redeemed for rewards (in some cases different amount of points are needed for different products) and points are accumulated linearly after spending in this case too. The linearity of the point function is presented in a different row (***point function type***) to give a clear overview.

It is interesting to note that all programs deal with linear point and reward functions. The difference lies not in the function type rather in the treatment of points, whether the discounts can be applied any time or a certain number of points need to be accumulated. As discussed in previous literature (Sällberg, 2010) this could have a psychological effect on the customer, acting as an incentive of repeated purchase. Unfortunately such psychological considerations are hard to quantify and as a consequence they are hard to model in general with numerical methods. This can be seen as a further limitation.

Beside the possible psychological effects of point accumulation we couldn't identify any reason for this phenomenon (widespread use of linear point function shape) other than simplicity, although we have found arguments for more progressive function trajectories in previous literature (Sällberg, 2010, pp. 32-39). Also linear function shapes are not a prerequisite of using point accumulation as a purchase incentive. We believe that the decision to include linear functions during CRP design was motivated by the need for rules that are easily expressed and communicated through advertisements. One purely mathematical implication of point accumulation is that the point functions becomes stepwise linear. This can however be approximated over the customer's lifetime fairly straightforwardly with purely linear functions in cases this property is used in a mathematical model (we do not express point functions directly in our model as it can be seen from Section 2.2).

We would also like to point out one joint CRP run together by eight participating companies referred to as the Supershop initiative (SuperShop, 2014). Although the Supershop point collection program shares very similar properties with other programs, the fact that it is a coordinated initiative between many partner companies raises questions about the applicability of the proposed optimizations approach from a practical perspective. For example, in order to derive a model one would need to incorporate the effect of eight sales functions for the different companies. The implications on complexity of this modeling problem are outside the scope of the current research, but propose an interesting future research direction. Nevertheless, we present the properties of this program to a limited extent since it is the only unlimited horizon point collection program we are aware of without introducing membership levels, which is a property outside the scope of this approach because of representation issues related to the formulation of membership levels in an optimization context.

The ***point collection medium*** is stated in every rule set, probably because it has a significant technical importance from an accounting perspective. Regarding our intentions the way of administering point collection is rather insignificant. We expect that it is easier to track the relationship between individual spending and aggregated company sales through a digital point collection medium though.

As mentioned briefly in Section 3.3 we have assigned a categorical value (either finite or infinite) to the property ***horizon of the program*** based on whether there is an explicit ending date of the program specified among the corresponding rules. We have assigned finite value in cases where an explicit end date is specified and infinite when there is no end date specified. It must be noted however that even programs without a specified end date will end sometime in practice and that programs with a specific end date can be extended or re-launched with slightly modified rewards, point, or redemption functions. Nevertheless expecting a program to run for a very long time (which is in fact very subjective) can help to derive an analytical solution in certain cases (see for example the derivation of the solution to the problem presented in Section 4.1). As previously mentioned starting a loyalty program without an explicit end date and calculating with an "infinite horizon" is a good approximation of maximizing over the entire Customer Lifetime Value.

Number of redemptions captures an important constraint with respect to the actual timeframe of the program. In case this property has the value "Limited by the number of products" the program will run only as long as there are awards available.

4.3.Findings related to Pharmanova Zrt

Pharmanova Zrt. is the Hungarian representative of the international BENU pharmacy brand (http://www.benu.eu) operated by the German mother company the PHOENIX Group (http://www.phoenixgroup.eu). There are around 700 BENU pharmacies located in seven European countries – Switzerland, the Czech Republic, Lithuania, Latvia, Estonia, the Netherlands and Hungary. The BENU pharmacies have together 5350 employees around Europe, they have 42 million customer contacts in a year and dispense around 114 million drugs. Pharmanova Zrt. operates around 170 pharmacies in Hungary under the BENU brand which makes it the largest pharmacy chain in the country. Pharmanova operates 139 pharmacies and has 28 branch pharmacies under the BENU brand and gives work to 1100 employees, closing year 2013 with gross sales of 53 billion forints - around 176.6 million Euros (Benu Gyógyszertárak, 2014).

The company has been running different customer rewards programs since 2008. Their first CRP program was coupled with an instant sales promotion and the two jointly running initiatives at the time contributed to the company's sales significantly. Since then they have experienced several changes in legislation governing the operation of pharmacies and affecting the running CRPs too, hence they have experience in adopting the designs of the current CRPs to the changed constraints.

We have chosen Pharmanova Zrt. to be included as an industrial reference in this study, primarily because their running CRP's design falls into the design class which can be modeled with the proposed optimizations approach. Another interesting property of Pharmanova Zrt. was that they have changed their CRP designs over the years several times which we believed was important from this studies perspective too.

There is some controversy in the literature about the effectiveness of CRP programs in general as already outlined in Section 1.1. Pharmanova is however considering its loyalty programs successful. They mentioned both financial and attitudinal benefits of the program. It must be noted though that the exact effects of the program on sales were hard to separate from other marketing initiatives. Pharmanova used pre-CRP sales data as a baseline and compared it with sales after the introduction of the program for evaluation purposes. Their initial expectations were a visible increase in sales amount and frequency which they validated through the program evaluation period based on their integrated database systems.

Based on the interview, which can be found in full in Appendix B, we can conclude that the current practice applied at the company for evaluating the CRPs from a profitability perspective is dependent on cost/benefit ratio calculations mainly: concrete considerations included weighing the additional marketing expenses (1-2 % increase in expenses) to the rise in sales. As mentioned earlier the applied methodology could not have been used however to separate sales increase resulting from the CRP and the results of other marketing efforts initiated approximately at the same time as the launch of the CRP.

Assuming that the practice of launching complex marketing campaigns is not uncommon in the industry, we can conclude that this problem related to the separation of a CRP's effect on sales (and profit) probably exists at other companies too. In this regard a significant contribution of the proposed optimization approach could be that it lets the initiating company calculate the expected profits resulting from launching the loyalty program precisely (see the example results presented at the end of Section 4.1). Although there would probably be a difference in the calculated expected profits and realized profits, this property of the proposed modeling approach could definitely help to evaluate the individual effectiveness of the different marketing tools applied in a complex campaign. We believe that this property could also be important since some of the critiques directed at CRPs (Sharp & Sharp, 1997) were also related to the evaluation of CRP programs from a statistical perspective.

In order to measure attitudinal responses from customers the company conducted representative surveys with the help of third-party companies. The fact that they received mainly positive responses reassured them to continue the program despite the problems related to evaluating the effectiveness of individual marketing efforts.

In the case of Pharmanova, changes in the legislative background led to several changes in the CRP program. In its current form it affects only a small portion of the product base and contributes to the overall profits only to a limited extent. Still the company decided to keep the program running because of the positive effects in customer perception towards the company resulting from the CRP. This decision correlates with the findings of Dorotic, H.A.Bijmolt, & Verhoef (2012). The authors of this work state that the main purpose of a loyalty program should be to foster behavioral and attitudinal loyalty towards the company and as a result contribute to an increase in purchase frequency. The positive responses measured by representative surveys are also in line with the findings of a previous study done by Verhoef (2003), where the author argues that economic incentives result in greater customer retention and in this regard CRPs are effective tools.

It must also be noted that on the other hand the company have experienced a serious price-based competition initiated by the launch of the CRP program, which correlates to the findings of existing literature. Dowling & Uncles found in their study (Dowling & Uncles, 1997) that programs initiated as a response on other loyalty programs or as a preemptive measure without an added value or a service can result in pushing the market towards a price based competition.

5. Analysis

In this segment we begin with reviewing the proposed model. As described under Section 3.4 we progress with analyzing the relationship between this model and the observed CRPs and generate the pattern of CRP properties which can be modeled with this approach. We start with the analysis of the interviews implications regarding the model proposed and progress with the discussion on the relationship to other observed CRPs. Finally the relationship of the proposed approach to previous studies is discussed.

5.1. Brief review of the proposed model

A general formulation of a CRP design as an optimization problem was presented in Section 4.1:

$$\max \int_{t=0}^{T} e^{-rt}\left[S(R(t)) - C\big(t, x(t), x'(t)\big)\right] dt$$

$$subject\ to$$

$$x(t) > 0, \qquad T \geq t \geq 0, \qquad x(0) = x_0$$

The shape of $S(R(t))$ can be either inferred from data available to the company or modeled based on assumptions made by field experts at the company. Discounting of future cash-flows is a common financial practice employed independently from business segments and we probably don't make a mistake by assuming that it is applied generally.

The general form can be made more specific by substituting a concrete function for the goodwill function (R(t)) and sales increase $S(R(t))$. We have proposed in Section 4.1 the forms:

$$R'(t) = C(t) - b * R(t)$$
$$\mathrm{S(t) = aR(t) - cR(t)^2}$$

The above function forms are motivated under Section 4.1 and seem to be capable to describe a customer shopping behavior natural to many business segments. We have also shown in Section 4.1 how to derive a concrete solution given the goodwill and sales increase functions above.

5.2. Analysis of the implications of the interview

One important insight of the interview - which is in line with existing literature - is that independently of the effectiveness of the introduced CRP it will most likely induce price based competition on the market. As a result of the copycat phenomenon observed both in previous literature and by practitioners working at Pharmanova Zrt. the effectiveness of the CRP program becomes a key to successful competition in this field for the company. This urges the participants of the competition to apply the most effective methods available during competition.

Related to the issue of effectiveness is the evaluation process applied to judge CRP contribution to overall profits. Based on the interview it seems that it is not possible to clearly separate CRP contribution to sales from other marketing efforts or outside market changes. At Pharmanova the practitioners employ mainly cost-benefit ratio calculations to evaluate the CRP effectiveness. On the one hand there was a clearly observable sales increase (and due to special market properties of the pharmacy market a clearly correlated increase on sales). On the other hand the program required only minor increase in costs mainly because it took advantage of existing (IT and marketing) infrastructure and practices. Thus although the profit contribution of the CRP was not clearly separable from other efforts, since the increase was significant and the costs negligible the company experts voted for running the program on the long-term.

Based on the above remarks we see that there are two separate arguments which are in favor for the proposed method. The pressure from likely increased price competition induced by the introduction of a CRP program urges for methods promising the maximization of profits. The model proposed maximizes profits over sales increase and related costs from the start; therefore it could help to separate the contribution of the program to profitability from other marketing efforts introduced at the same time. It must be also noted that based on the interview CRPs in general effectively make use of existing marketing and IT resources. Therefore there is not much room to increase effectiveness by resolving possible structural or corporate parallelisms resulting from running these programs or cutting down the costs of introduction (which are already fairly low). Effectiveness can be increased under these circumstances by designing the program itself to maximize profits.

We know from the interviews that the company treats points as liabilities. This is important since it implies that rewards (from a financial perspective) are linked to points only and are not linked to the redemption occasions. All points are counted as liabilities and the company counts them as rewards that will have to be paid one time without considering the concrete reward redemption occasion.

In the case of Pharmanova Zrt. almost any amount of points can be turned in as a linear monetary discount on the reward product, but we have witnessed other designs too, where only a certain amount of accumulated points could be turned in for rewards. This practice means that the time of ***reward redemptions becomes irrelevant during modeling.*** However in our model, C(t) the cost function associated to rewards is still modeled as a function of time since the points are allocated over the lifetime of the program. We can neglect the redemption function’s properties during modeling if we apply this accounting perspective when designing CRPs aiming to maximize the ***expected*** profits and count every point as a liability (effectively reward cost) on the respective monetary unit. Since the practice of treating points as liabilities is based on accounting rules we can expect other companies also to follow this praxis. This assumption is strengthened by the empirical data collected: the rules stating that the number of possible reward redemptions is limited implies that these companies either treat rewards as liabilities or wish to limit total reward cost by some other way. From a practical perspective the latter option has the same consequences: reward costs can naturally be linked to the time when points are issued to the customer without neglecting other considerations.

Also when optimizing for expected profits it would be a methodological mistake, we believe, if the programs profitability would depend on the amount of points not turned in for rewards (practically meaning that not all liabilities are needed to be paid). Even if unused points would give a significant contribution to the overall profits applying the above assumption would be equal to counting on the worst case scenario (every reward is to be paid). Thus, this is a reasonable safe assumption. This “worst-case assumption” helps to avoid a methodological mistake – relying on the expectation that a certain amount of liabilities won’t have to be paid to the customers. Even if this accounting practice is not used at other companies it can be applied as part of the design considerations with respect to other CRP programs (including the concrete examples mentioned in this study) as a safety measure.

Further based on the interview we can safely say that that this approach could contribute to the existing CRP practice for the following reason: specifically in the case of Pharmanova the head of marketing stated the company has motivation to keep the CRP program running even if it is not as profitable now that the product group allowed to the program became limited. The reason was that they "wanted to offer a discount to the loyalty program members". This is clearly not a financial reason, but it does only imply that the company is determined to run the program even if it is not as successful, it does not imply that the company would not be interested in making the program more profitable.

We can assume that in industry segments where the relevant product groups are not restricted due to legal constraints and where the nature of customer data is less sensitive the respective companies it would be even more motivated to make their programs as successful as possible from a financial perspective. This question is also closely related to the issue of loyalty program effectiveness evaluation discussed under Section 4.3. Having maximized expected profits as a result of optimized CRP design planning is not the only benefit to the company since the ability to forecast expected profits precisely also helps the program evaluation.

Also the interview implies that the current design process uses mathematical tools only to a limited extent during the CRP design process and for evaluation purposes. As the answers imply a large portion of the design structure is based on practical examples observed to work in other business segments which were supported by initial sensitivity analysis:

> "Our initial design was influenced mainly by other programs offered in other industry segments and as for the parameters we relied on initial calculations of profitability and sensitivity analysis. We observed that point collecting schemes were getting popular and seemed simple enough to do some initial calculations regarding profitability."

We believe that the proposed approach could not only effectively contribute to the design of a CRP program, but it in the case of Pharmanova or other companies running programs without a specific ending date it is essential too, since their program design is especially well suited for optimization over the whole CLV.

About the availability of the needed sales and customer data the last question of the interview, in Appendix B, answers this question. In the case of Pharmanova we can safely assume that the needed data sets are available based on the marketing experts answer: "detailed transactional sales data is still available in our IT system supporting the billing and accounting process. As the result of these transactions we have historical data available on sales amount and frequency for each product and purchase occasion". Since the IT functions originally supported by the collection of these datasets are independent from the presence of a loyalty program we can safely extrapolate this assumption to other companies too with the necessary financial resources to build out the IT infrastructure.

It is also stated in the interview that the CRP design was influenced initially by designs observed in other industries. Previous literature calls this the "copycat" phenomenon and is mentioned in several sources as a limiting aspect of CRP programs (Cedrola & Memmo, 2009), (Dowling & Uncles, 1997) and it is also pointed out by Cedrola & Memmo (2009) that the loyalty effect of the programs and their sales increasing effect becomes questionable as a direct result of the widespread use of similar designs of CRPs. As mentioned earlier the company experienced a price-based competition from the above mentioned effects. As for the spreading of similar designs in their respective industry we point out again that due to changes in legislation the whole drug market was affected by serious efforts aimed at the applicability of CRPs directed at this market's customers. Thus the spreading was effectively stopped by the force of law and we cannot say at what point the spreading of similar CRPs and the resulting price war stopped.

Thus based on the interview we can highlight three issues for which the proposed method offers a solution. The first one is the problem of increased competition resulting from similar competing CRP designs. Unless a company introduces a "dramatic" change in the approach to CRPs in its market segment, the most reasonable step to counter this competition is to maximize the financial effectiveness of the programs. Since the proposed approach makes use of readily available resources like IT and marketing infrastructure – similarly to other CRPs – effectiveness can be further increased by striving for maximizing profits by designing appropriate CRPs.

Singh, Jain, & Krishnan (2008) discuss a game theoretical framework and they found that if there is a high probability of customers engaging in further purchases after the first occasion, it is beneficial for the firms to take different positions by one offering a reward program and for the other company to compete mostly based on price. The resulting asymmetric equilibrium enables the firms to segment the customers based on the purchase occasions and earn higher profits for both companies. It must be noted however, that based on the interview conducted with the representative of Pharmanova the introduction of CRPs on the market triggered a heavy price-based competition rather than resulting in a long-term segmentation of the customer base. The difference in our findings could be due to the fact that the findings of the aforementioned work were the results of the model constructed to describe a duopoly market. The findings of Singh, Jain, & Krishnan seem to be otherwise independent of our work since as a marketing tool the optimization approach proposed could be beneficial to every company considering the launch of a CRP without respect to the strategy followed by the competition.

The second issue is related to the evaluation of the effectiveness of the programs and the separation of the profits from other marketing initiatives. This is inherently handled by the proposed method since it aims to maximize sales increase (additional sales resulting from the introduction of the program) and minimize the related costs at the same time.

As opposed to the incentive structure approach discussed under Section 1.2 we do not model point and reward functions explicitly. This can be seen as moving the focus of modeling from structural analysis to weighing the parameters of the sales calculations. As mentioned before, our model is centered on the concept of "goodwill". We suggested the application of goodwill *as an abstract way of describing the influence of the CRP on the customer's will to engage in a business relationship with the company.* In the above formulation goodwill appears as R(t), influencing the sales S(t). For practical consideration we suggested the application of the form

$$S(t) = aR(t) - cR(t)^2$$

to express the sales increase resulting from the CRP. The explicit expression of the sales increase makes it possible to conduct more precise profitability calculations. In case a the designed with the help of the proposed approach CRP would be launched together with other marketing incentives as described in the interview the profits (costs) could be easily separated by subtracting the calculated expected profits (costs) from the total profits (costs) of simultaneous marketing efforts.

The incentive structure approach on the other hand is centered on the concepts of reward and point functions. It concentrates on CRP properties governed by explicitly formulated rules apparent in real-world CRPs and their incentive on customer purchase without any precise calculations regarding profitability. The optimization approach suggested in this thesis and the concept of "goodwill" helps to model customer behavior from a profit perspective. These differences between the two approaches do not necessarily mean that the methods could not be used together. This will be further discussed under Section 6 Conclusions.

Finally, issues related to accounting considerations are straightforwardly handled by the proposed model. Redemption becomes implicit, since points are treated as liabilities at the moment of issuance (they are effectively reward costs) without respect to the time of redemption. Since points can be treated as monetary units (because they are treated as liabilities), no point-cost conversion is necessary and we make no methodological mistake by optimizing over functions applied over purely monetary units.

5.3.Relationship of the model with the identified CRP properties

Regarding ***Reward types*** we have observed that based on the collected data it is common practice for companies launching large-scale CRPs to offer monetary rewards. This takes form in a price discount on a specific product. Products can be the respective companies' own products or third-party products offered by other companies. As discussed under Findings in both cases the reward can be treated in the same monetary values as customer spending. Since we can express values in the same monetary units throughout the model, which in turn can be closely approximated be continuous functions we can conclude, that the model is applicable in cases where the reward is of a monetary type (or can be directly expressed as a monetary value).

The fact that in some loyalty programs different discount percentages apply to different products somewhat complicates modeling though. One possible approach to treat this problem could be to handle different products with different models. This way separate loyalty programs associated with a given product would be modeled by a separate optimization model. Results of the individual optimizations can be combined by methods of multi-objective optimization (Konaka, Coitb, & Smith, 2006). This approach would however probably lead to an optimization problem with a numerical solution only, thus this option is out of the scope of the proposed approach at its current stage.

The properties ***Type of point and reward function*** have inferred values drawn from other properties and served mainly to illustrate the difference between the Supershop initiative and other programs. Generally the function type (linear or non-linear) can be neglected, since we haven't used these concepts in our model directly. However, they can influence the goodwill function implicitly. This caveat can be handled by carefully validating that for a given customer base an assumed goodwill function type is valid through market research or by approximating the goodwill function based on data drawn from previous CRPs.

Examining this property revealed that there are special cases of CRPs (like SuperShop) which have multiple reward or point functions. At the current stage of our research we cannot model this property with the proposed model. We can conclude that we can use the proposed approach in case there is only one way to earn points, one platform to redeem rewards and using a goodwill function bused on some business assumption instead of approximating the function shape based on previous experience.

Similar remarks can be made for ***Number of points required for a reward*** as for the previous two properties: the model developed does not rely on this property directly and implicit relationships between the numbers of points needed for a reward and the goodwill function can be handled the same way as discussed above.

As already stated under Findings the way of administering point collection and related to that the ***Point collection medium*** is rather insignificant from a modeling perspective.

The parameter T – the upper limit of integration – expresses the ***orizon of the program***: infinite or finite (in the latter case a discrete value in the formulae representing e.g. months or days). Regarding the values of parameter T in Section 4.1 we have shown the possibility of the derivation of an analytical solution for the unlimited case. We have arrived to a possible solution by assuming the applicability of the abstract concept of "goodwill" in order to help to quantify the revenue increase resulting from a CRP design and by proposing a formulation of $S(t, R)$. Different $S(t, R)$-functions will naturally lead to different analytical solutions. The interested reader could refer to the work of Kamien & Schwartz (1991) for a thorough review on the implications of the "horizon of the program" (implications of infinite and finite values for T) on deriving an analytical solution. Our main goal by setting the function type to a concrete form was to show that analytical solutions with concrete result values are derivable for this problem class.

Probably for practical reasons Number of redemptions, Reward type and Horizon of the program seem to have "correlated" values (at least in the case of the observed CRPS and to the limited extent of statistical credibility for drawing such conclusion). In cases the number of redemptions is limited, the reward type is a "monetary discount on a specific product" and the horizon is also limited. In case the number of redemptions is unlimited the respective values of reward type change to "monetary reward on all products" and the horizon observed is unlimited. As mentioned under Section 3.6 Sampling procedure the goal of the sampling was not to draw statistically correct conclusions with respect to the data available. Therefore although we point out this property we do not wish to make further assumptions regarding the possible implications.

5.4.Summary of the analysis of the collected data

To sum this segment of analysis up the proposed model is applicable if the CRP design to be modeled has the following properties summarized in Table 5.1.

Horizon of program	Unlimited
Point function type	Irrelevant (Linear observed)
Reward function type	Irrelevant (Linear observed)
Reward type	Monetary discount on all products
Number of redemptions	Unlimited
Point collection medium	Irrelevant
Number of points required for the reward	Every point counts as a monetary discount

Table 5.1: ***Identified CRP properties***

These values were for example observed by the real-world CRP running at Pharmanova Zrt. Therefore we can conclude that the proposed model can probably be used to design CRPs that resemble already established CRP properties/practices. The proposed model works with continuous functions, which is in line with accounting business practices observed at Pharmanova Zrt. During the derivation of the model and the respective example solution we have assumed the specific form of “goodwill” and sales-increase function (re)stated in the beginning of this chapter. In case this approach would be applied to a given business segment the validity of these functions regarding the given customer base need to be verified.

We have mentioned that reward and point functions are in the observed cases linear. This is probably for practical reasons related to simple and clear CRP rule formulation and communication. Since these functions are not present in our model explicitly we have marked the related property as irrelevant. This does not mean that reward or point function type (linear or non-linear) does not contribute to CRP profits. The contribution is caught however in our model by the goodwill function. Again examination of the applicability of any “goodwill” function shape for a given customer base is a critical part of the proposed approach.

As it was discussed before, we have observed other values for the respective properties presented in table 5.1. As part of a proof of concept model we have only derived the analytical solution for the pattern of values shown above and for the proposed “goodwill” function. Based on our sampled data ***Horizon of program, Reward type*** and ***Number of redemptions*** have in some cases other values. As we have pointed out these values often “change together”. However, as a result of the sampling process discussed in Section 3 Method a statistical correlation between the values was not verified. In case a practitioner would like to use the proposed model for different values (for a limited horizon for example), the model should be adjusted accordingly and most importantly a new solution should be derived for it. Examples for deriving an analytical solution for a finite integrand can be found (among other special cases) in (Kamien & Schwartz, 1991).

Based on the interview conducted we would like to point out that the proposed model could help business practitioners in CRP development by promising a solution to several practical issues. Among these issues we have identified the in previous literature widely discussed “copycat phenomenon” and its implications on profitability. As a second practical argument we have found that by forecasting the expected profits of a loyalty program, the model could help practitioners later separating profits resulting from other marketing initiatives. A third benefit of the model is that it can be naturally used together with current accounting practices.

6. Conclusions

A company planning to launch a CRP could outline the general properties of the loyalty program by the incentive structure approach (Sällberg, 2010). Profitability calculations on the other hand, could be done by applying the optimization techniques suggested in this thesis. As mentioned earlier an important prerequisite of such profitability calculations is some expert knowledge about the shape of the S(t) functions or better how goodwill of customers appears towards the company as the result of certain rewarding schemes.

The company needs to have some initial impressions about customer perception on different reward options. The incentive structure approach can be applied to outline the way rewards are "distributed" throughout the program by comparing point and reward function properties and their implications on purchase incentives. The optimization approach tries to tie the resulting CRP design to customer perception related to reward value (expressed as goodwill) through the reward cost. The proposed approach would help a company to choose how to spend reward costs more wisely with the aim of maximizing expected profits by quantifying the level of goodwill necessary to obtain maximized profits.

In the first part of the thesis we have argued for the need for a profit maximization oriented CRP design method by reviewing related studies (Section 2.1) and proposed a possible approach based on existing achievements on the field of calculus of variations (Section 4.1). We have shown that tools of calculus of variations can be used to design CRPs with the aim of maximizing expected profits first by showing that a realistic CRP can be modeled as a dynamical optimization problem and then by showing that this model falls in an optimization problem class already researched in the literature (Kamien & Schwartz, 1991). The optimization formulation has an analytical solution which has both computational benefits and gives insights into the mechanics of the modeled problem.

The only work in the literature we know of that set the goal of modeling maximization of profitability in the field of CRPs is the study done by Gandomi & Zolfaghari (Profitability of loyalty reward programs: An analytical investigation, 2013). The authors also applied tools of mathematical optimization in their study, although in a different way, by highlighting the importance of analytical solutions in understanding the results of the optimization model. As a major contribution to existing research Gandomi & Zolfaghari (2013) incorporated customer valuation both as a real valued stochastic variable – in order to capture customer heterogeneity - and using deterministic valuation into their optimization models. We have tried to customer valuation by the "goodwill" a continuous function (adopting the idea of Kamien and Schwartz). Our approach can be seen as in this sense being closer to the work of Kopalle & Neslin (2001).

Another difference between the work of Gandomi & Zolfaghari (2013) and the proposed optimization model is that they have created a two period model, whereas time in our approach becomes a parameter – in Section 2.2 we derived the solution of an unlimited horizon example problem, but analytical solution could be also derived for limited horizon designs following and applying the work of Kamien and Schwartz (Dynamic Optimization - The Calculus of Variations and Optimal Control in Economics and Management, 1991). Further Gandomi & Zolfaghari worked with propability describing the chance of a customer entering the second period of the model loyalty program whereas we assumed that the customer are either participatng in the CRP program (are heavy users following the terminology also used by Kim, Shi, & Srinivasan) or one time shoppers outside the scope of our optimization effort.

A necessary requirement for the proposed approach to work is that the company designing the CRP is able to model the influence of the CRP rewards on the sales as a dynamical function. This can be done for example by inferring the functions from historical CRP data or by validating for example the applicability of the function proposed through expert knowledge.

Customer's reward valuation is still a heavily researched field. Kopalle & Neslin found in their study (2001) that not increasing rewards for underperforming rewards programs can be reasonable if customers discount future benefits heavily. We found this result interesting since it argues against the common sense reaction of increasing rewards for underperforming programs and thus hints that reward value and program performance (profitability) may not always be positively correlated. This finding suggests that the customer valuation of certain rewards (reward types) needs to be examined more closely. First of all, it is not without consequences from the perspective of the proposed optimization approach how the customers will value rewards in real-life precisely. Second, the possibility that increasing reward values in certain cases is a suboptimal choice also points interest to methods that help to understand the relationship between rewards and total profitability of the program analytically. Finally our argument in Section 2.2 that the form $R'(t) = C(t) - b * R(t)$ is better suited to describe customer valuation of rewards is correlating with the findings of Kopalle & Neslin.

Another closely related issue is the treatment of rewards. In our work we have argued that most rewards can be quantified as monetary rewards and naturally monetary costs can be coupled to them too. We have found during empirical data collection that – at least on the Hungarian – market there are two common types of rewards both in form of monetary discounts. One type of rewards are applicable to all product offered by the company where the second type is bound to a specific third-party product sold by the company in question for the duration of the CRP. It is a question whether these rewards can be seen as efficient or inefficient in the sense described in (Kim, Shi, & Srinivasan, 2001), since some discounts are applicable to the company's own product base and at other programs the discounts are applicable to third party products offered for a limited time. However the example in the above study for an efficient reward was the company's own product base, whereas for an inefficient reward the direct cash rewards. We believe that the efficient/inefficient classification makes the most sense when it is whether the product on which the discount applies is the company's own product or a third party product the former reward type being more efficient then the latter.

The distinction can be important if we look at the findings of the aforementioned study. The authors have found that the optimal reward type is very much dependent on the size and price sensitivity of the heavy user customer base. It is more reasonable to offer efficient rewards in cases where the customer base has a large segment of price sensitive light users since they will force low prices on the firm. We argue that all reward types observed can be expressed as monetary rewards and thus they are compatible with our optimization approach. This means that in case a reward's valuation is known the method helps to determine the reward amount needed to obtain a level of goodwill resulting in maximized profit with respect to the given reward. The optimization procedure makes no suggestions however, regarding the reward to offer. This is a design choice made in advance. The findings made by Kim, Shi, & Srinivasan imply that these initial considerations on the reward type should be made carefully based on the properties of the customer base in order to obtain maximal profit with respect to all the possible reward types. Thus our proposed optimization approach could be well complemented with the findings of Kim, Shi and Srinivasan.

As already mentioned during the literature review in Section 2.1 the authors used a game theoretical model to differentiate between rewards based on their cost to the firm and the benefit for the customer. As a key component of their model they relied on the definition of efficient rewards and inefficient rewards based on the unit reward cost to the firm, where they define the unit reward cost as “a firm's cost of offering certain rewards worth one dollar to the customer”. A critical component of the proposed optimization approach is the selection of the shape of the S(R(t)) sales function with respect to the given customer base. The sales function used in the optimization model – describing the additional sales resulting from the CRP program – relies on the conept of goodwill also inherently dependent on the customers perception or valuation of the reward offered.

Despite the difficulties we were facing in accessing relevant real-world data (discussed in Section 3 Method) we have argued primarily in Section 5 that the proposed approach of formulating the CRP as a convex, dynamical optimization problem solvable by standard analytical tools used in calculus of variations has the benefits of and contributes to existing theory by:

- Catching the dynamical properties of CRP design with respect to time
- The monetary representation of rewards results in a flexible system capable to describe CRPs both with efficient or inefficient reward types and is in line with existing accounting considerations taken during CRP design
- It is able to deal with both finite and infinite horizon CRP designs
- For the investigated CRP structures (limited or unlimited horizon, point-function based CRP offering monetary rewards) an analytical solution is derivable which has significant computational benefits as opposed to a tractable numerical solution

In Section 4.1 we showed how an analytical solution can be derived for a special case of unlimited horizon CRP designs assuming the proposed function class for *S(t)* holds for the given business segment and referred the reader to existing solutions published in previous literature (Kamien & Schwartz, 1991) which are applicable to other common cases witnessed. Finally limitations of this approach were exemplified partially through the concrete example of the SuperShop initiative presented briefly in Section 4.2. Basically our approach is only capable of dealing with CRP schemes designed within one company – expressing the collaboration of several companies in one loyalty program is outside the scope of our current research. In the same Section and also in Section 3 Method we have discussed the limitations of the approach regarding the expressiveness of the modeling framework and that as a result we haven’t found a general way of describing membership levels although they present a common practice in CRP design.

Unfortunately the short time available to conduct this master thesis has not been enough to be able to verify that the approach works in practice. The reasons for this are partially methodological difficulties in the evaluation of our approach using a previous CRP as the baseline and most importantly the limitations in available time as discussed in Section 3 Method. However we believe that the mathematical proofs already developed in existing literature are reassuring regarding the effectiveness of the proposed method. Still practical limitations as a result of real-world applications need to be identified.

We therefore propose that further efforts should be put in on research on this topic to try to fully understand how mathematical modeling can be done without violating privacy of the company's customers and whether current data collection practices enable the use of the proposed approach without a significant transformation of the available data or a major change in the data acquisition process.

Appendix A

Optimization and the Euler Equation

We have seen in Section 2, Problem formulation and discussion, that a realistic CRP model can be formulated as a dynamic, convex optimization problem quite naturally. In this section the Euler equation – an important concept in the calculus of variations – is derived based on Section 3 in the book by Kamien & Schwartz (1991) changed slightly to put it into the context of CRP design.

In mathematical optimization one seeks properties of the solution to the problem

$$max_{x(t)} \int_{t_0}^{t_1} F\big(t, x(t), x'(t)\big)dt \qquad (1)$$

$$subject\ to \quad x(t_0) = x_0, x(t_1) = x_1 \quad (2)$$

The function F is assumed to be continuous in its three arguments t, x, x' and to have continuous partial derivatives with respect to the second and third, x and x'. Note that while the third argument of F is the time derivative of the second, F is to be viewed as a function of three independent arguments. Thus, if $F(a, b, c) = a^2 + bc - c^2$, then $(t, x, x') = t^2 + xx' - (x')^2$.

The admissible class of functions x(t), among which the maximum is sought, consists of all continuously differentiable functions defined on the interval $[t_0, t_1]$ satisfying the fixed endpoint conditions (2).

Suppose that the function $x^*(t), t_0 \le t \le t_1$, provides the maximum to (1). Let x(t) be some other admissible function. Define the function h(t) to be the deviation between the optimal path x*(t) and the comparison path x(t) at each t:

$$h(t) = x(t) - x^*(t) \quad (3)$$

Since both x* and x must obey (2), we have

$$h(t_0) = 0, \qquad h(t_1) = 0$$

We say the deviation h is admissible if the function *x = x* + h* is admissible.

For any constant *a*, the function *y(t) = x*(t) + ah(t)* will also be admissible since it is continuously differentiable and obeys (2) (since x* does and h is zero at the endpoints).

With the functions *x** and *h* both held fixed, compute the value of the integral in (1) for *y(t)* as a function of the parameter *a.* The result is a function of *a*, say *g(a)*:

$$g(a) = \int_{t_0}^{t_1} F(t, y(t), y'(t))dt = \int_{t_0}^{t_1} F(t, x^*(t) + ah(t), x^{*\prime}(t) + ah'(t))dt \quad (4)$$

Since *x** maximizes (1), the function *g* must assume its maximum at *a = 0*. But this implies that *g'(0) = 0* by the first order necessary condition for a maximum of a function of a single variable. To compute g'(a), first apply the chain rule to the integrand of (4):

$$\frac{dF\Big(t, x^*(t) + ah(t), x^{*\prime}(t) + ah'(t)\Big)}{da} = F_x h(t) + F_{x'} h'(t)$$

, where F_x and $F_{x'}$ denote partial derivatives of *F* with respect to its second and third arguments, respectively, and are evaluated at (t, x*(t) + ah(t), x*'(t) + ah'(t)). Second, Leibnitz's rule for differentiating under an integral is applied to compute g'(a), and the result is evaluated at the point that maximizes g(a), namely, a = 0, yielding

$$g'(0) = \int_{t_0}^{t_1} [F_x(t, x^*(t), x^{*\prime}(t))h(t) + F_{x'}(t, x^*(t), x^{*\prime}(t))h'(t)]dt = 0 \quad (5)$$

The condition that *g'(0)* be zero is necessary since *x** is assumed optimal. Recall that the function *h*, held fixed to this point, was chosen arbitrarily, restricted only to being continuously differentiable and satisfying endpoint conditions (2). The right side of (5) must be zero for any choice of h satisfying these two restrictions.

Expression (5) can be put into a more convenient form by integrating the second term by parts. In $\int u dv = uv - \int v du$, we let $F_{x'}$, play the role of *u* and *h'(t) dt* play *dv*, obtaining

$$\int_{t_0}^{t_1} F_{x'} h' dt = F_{x'} h|_{t_0}^{t_1} - \int_{t_0}^{t_1} h(t) \frac{dF_{x'}}{dt} dt. \quad (6)$$

(Where it is assumed that $\frac{dF_{x'}}{dt}$ exists.) Recall (3) and substitute it into (5):

$$g'(0) = \int_{t_0}^{t_1} [F_x\left(t, x^*(t), x^{*\prime}(t)\right) - \frac{dF_{x'}\left(t, x^*(t), x^{*\prime}(t)\right)}{dt}] h(t)\, dt = 0 \quad (7)$$

Equation (7) must hold if *x** maximizes (1), and it must hold for every continuously differentiable function h that is zero at the endpoints. It will certainly hold if the coefficient of *h(t)* is zero for every t, i.e.

$$F_x\left(t, x^*(t), x^{*\prime}(t)\right) = \frac{dF_{x'}\left(t, x^*(t), x^{*\prime}(t)\right)}{dt}, \qquad t_0 \leq t \leq t_1 \quad (8)$$

Equation (8) is called the Euler equation corresponding to problem (1)—(2).It may be viewed as a generalization of the standard calculus first order necessary conditions *f'(x*) = 0* for a number *x** to maximize the function *f*(x). Indeed if $\frac{dF_{x'}}{dt} = 0$, then (8) reduces to that standard calculus condition.

In fact, (7) holds for the entire class of functions *h* only if (8) holds:

Lemma 1 (Kamien & Schwartz, 1991). Suppose that *g(t)* is a given, continuous function defined on $[t_0, t_1]$. If

$$\int_{t_0}^{t_1} g(t)h(t)dt = 0 \quad (9)$$

for every continuous function h(t) defined on $[t_0, t_1]$ and satisfying (3), then g(t) = 0 for $t_0 \leq t \leq t_1$.

PROOF (Kamien & Schwartz, 1991). Suppose the conclusion is not true, so *g(t)* is nonzero, say positive, for some *t*. Then, since *g* is continuous, *g(t) > 0* on some interval $[a, b]$ in $[t_0, t_1]$. We construct a particular *h(t)* satisfying the conditions of Lemma 1, namely,

$$h(t) = \begin{cases} (t-a)(b-t), & a \le t \le b, \\ 0, & elsewhere. \end{cases}$$

Compute

$$\int_{t_0}^{t_1} g(t)h(t)dt = \int_b^a g(t)(t-a)(b-t)dt > 0,$$

since the integrand is positive. This contradicts the hypothesis that (9) holds for every function *h* in the specified class. A similar argument shows that *"g(t) < 0 for some t"* leads to a contradiction. Supposition that the conclusion of the lemma was false while the hypotheses were true led to an inconsistency, verifying the lemma.

Applying Lemma 1 to (7) indicates that (8) must hold. Equation (8), the Euler equation, is a fundamental necessary condition for optimality of the function x*(t) in the problem (1)-(2). It is important to note the Euler equation must hold for each t in the interval $[t_0, t_1]$. Equally important is the fact that $\frac{dF_{x'}}{dt}$ is the total derivative with respect to t. In interpreting this notation, it must be remembered that the partial derivative $F_{x'}\big(t, x(t), x'(t)\big)$ is itself a function of three variables. This function is to be differentiated with respect to t. The total rate of change of the value of the function $F_{x'}$ with advance in t is due, not only to change in t itself, but also the concomitant changes in x and x'. Apply the chain rule to compute

$$\frac{dF_{x'}}{dt} = F_{x't} + F_{x'x}x' + F_{x'x'}x''$$

, noting that each of F's arguments depends on t. Subscripts indicate partial differentiation. Thus the Euler equation (8) can be written

$$F_x = F_{x't} + F_{x'x}x' + F_{x'x'}x'' \,,\ t_0 \le t \le t_1 \qquad (10)$$

where the partial derivatives are all evaluated at (t, x*(t), x*'(t)) and x' = x'(t), x" = x"(t). The Euler equation is now a second order differential equation for x(t), to be solved with the two boundary conditions (2).

Appendix B

Interview with head of marketing at Pharmanova on the evolution of the CRP and its outside constraints

The following guided interview was constructed in order to gain insight into current CRP design best practices and to help to identify the points in current practices where the proposed approach could contribute. The interview conducted with Zsuzsa Abraham, head of marketing at Pharmanova Zrt., presented in its current form is the result of an iterative process aiming to answer primarily the questions discussed below and to clarify the content wherever necessary.

1. ***How long have you been running CRP programs and what kind of programs were these?***

Our first program was introduced in 2008. In this program the customers collected points towards third-party rewards. The rewards were essentially products of our partner companies: cups, pencil crayons etc. These rewards were not part of our own product base as a pharmacy-chain and could have been bought at other retailers too outside the program.

As a pharmacy chain the most significant portion of our sales comes from medicaments sold based on prescriptions (like antibiotics). We also sell medicines which do not require prescriptions (like ordinary painkillers) and other healthcare related products too, we usually call them simply as “other products”. In our first program all three product types counted towards rewards.

2. ***What was the response on this program?***

Our customers liked the approach, but our initial point collecting scheme had only a small, but positive effect on sales. Therefore in 2009 we have introduced a new offer too: After every three prescriptions the customer was entitled to a small monetary reward (discount) from the purchase. These products give approximately 80% of our sales and this addition was an instant success among customers, but unfortunately triggered price based competition on the market too.

3. ***How did you evaluate your CRP program? Did you use economic measures like sales amount or frequency or did you rely on other measures when evaluating the program?***

We mainly used our pre-CRP sales data as a baseline and compared it with sales after the introduction of the program. We observed a large increase both in sales amount and frequency which was in line with our expectations. Since the rewards were given based on the number of prescriptions turned in our pharmacies we expected that both sales frequency and amount should increase as a result of our program. In the pharmacy business products sold based on turned in prescriptions have a state-regulated margin, therefore our profits increased together with our sales.

4. ***How did you decide on whether you should continue the CRP program?***

When evaluating the sales program we had to weigh the benefits against costs of running the program. As the biggest pharmacy chain in Hungary we had a fairly developed IT infrastructure so it didn’t cost much to run the program from an IT perspective. On the other hand, printed advertisement materials distributed through our regular channels - flyers etc. distributed in the pharmacies and directly to postboxes – together with electronic newsletters did not contribute much to our existing marketing budget either. The overall incremental marketing expenses related to our

CRP consisted around 1-2% of our marketing budget. So we decided to run the program despite the difficulties in evaluation of its individual effectiveness.

5. Have you considered the reward costs too when you were evaluating the CRP?

We have compared the monetary reward costs to the profit increase after the introduction of the program and it had also reassuring results regarding the cost/benefit ratio of the program. It must be notated however that the monetary reward system introduced a very aggressive price-based competition fairly quickly on the market which had obviously a negative effect on sales after the initial success.

We have also conducted representative surveys with the help of third-party companies to evaluate customer responses and attitudes towards the program. We received positive responses in overall: most customers perceived the program as a good initiative and as a result they had a positive perception towards the pharmacy chain.

6. You said that you introduced your first program in 2008. Is this program still running?

No, since the legal regulations on pharmacies changed after the introduction we had to change the program. After changes in September 2011 it was not allowed anymore to offer sales incentives on products sold based on prescriptions. After another change in the legal regulations we had to cancel our popular "three prescriptions – instant reward" program too.

At this time we were restructuring our loyalty program completely and came up – based on the success of the previous instant reward program – with a simple system offering an instant 5% discount for the members of the loyalty program. This program was applicable only to products not requiring a prescription.

7. Is this program the current loyalty program running in your pharmacies?

No, we had to modify the program again after another change in regulations in November 2011. From that on loyalty programs could only affect the "other product" group that gives around 10% of our sales. In order to adjust the program to current regulations we re-introduced the point-collection scheme, but at this time we preferred our "other products" group as rewards. This decision was aimed to simplify the distribution of rewards mainly, the distribution of third-party rewards required more administration and logistics.

The new loyalty program had a very simple design:

- Purchases for 20 forints are awarded with 1 Point and 1 Point can be turned in for a 1 forint "discount" on a reward (a product from the "other product" group)
- Reward redemption is only possible against points: mixed point – forint redemption is not allowed because of accounting reasons
- Points are treated as liabilities for accounting reason
- The customer decides how many points she wishes to turn in at one purchase occasion
- Mixed point – forint purchases are possible as part of the CRP, but every purchase must include 5 forints for accounting reasons

From a profitability perspective this program is not as effective (compared to our first design introduced in 2008), but we wanted to offer some discount for our loyalty members.

Also, despite the fact that we have observed sales increase after the introduction of this new (pharmacy store) design, we are not sure how much is to be accounted to our CRP program. The increase was visible, but we have launched other initiatives in 2012 aimed to increase sales effectiveness at the time of the introduction of the CRP too: training for pharmacy personnel and redesigning the shops in order to increase the shopping experience. Since we have launched these programs approximately at the same time it is hard to tell which initiative had bigger effects on overall sales on the mid-term.

8. ***What design considerations did you take when initially introducing the program? Have you had any kind of guidelines when you decided on the number of purchases required for a reward or type and size of reward or were these parameters the results of trial and error?***

Our initial design was influenced mainly by other programs offered in other industry segments and as for the parameters we relied on initial calculations of profitability and sensitivity analysis. We observed that point collecting schemes were getting popular and seemed simple enough to do some initial calculations regarding profitability.

9. ***You said that the company had to change the programs limits because the changes in industry regulations. Were any other aspects of the program changed too?***

The first loyalty program required the customer to fill in a questionnaire and it was helpful in segmenting our customer base in order to get a better idea of the needs and preferences of the different customer groups.

In our current program we collect intentionally less data about customers and their preferences. We try to avoid collecting personal data since the collection and usage of this data is strictly regulated and in our business segment the usage of personal data is very sensitive from an ethical perspective too. Our customers also preferred less administration and since this move was in line with our ethical guidelines it was an easy decision to make.

10. ***What implications does this change have on the collected data?***

Since we do not use a detailed applications form anymore, we now collect less personal data, but otherwise detailed transactional sales data is still available in our IT system supporting the billing and accounting process. As the result of these transactions we have historical data available on sales amount and frequency for each product and purchase occasion.

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