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Capacity Analysis of Cognitive Radio Relay Networks under Transmission and Interference Power Constraints

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Abstract

This thesis investigates the performance of cognitive radio relay networks (CRRN) in Rayleigh fading channel under various power constraints. Here spectrum sharing approach is considered, whereby a secondary user (SU) may be allowed to transmit simultaneously with a primary user (PU) as long as SU interference to PU remains below a tolerable level. In addition, SU has to meet certain quality of service (QoS) constraints of its own link. To support these QoS constraints, the maximal data rate that can be reliably transmitted with arbitrarily small error of probability is found. It is observed that this capacity is affected by channel quality and interference limit allowed by PU. Ergodic capacity and outage capacity which are two well known capacities, are analysed for CRRN under interference power constraints. This thesis also finds effective capacity for CRRN, a link layer channel model that models the effect of channel fading on queuing behaviour of the link. Effective capacity under interference and secondary transmitter power constraints is also investigated. The way of analysing effective capacity under interference and transmit power constraints is extended to ergodic capacity and outage capacity. Here it is observed that, capacity is affected by the minimum of transmit power and interference power constraints. Monte-Carlo simulations are carried out to support theoretical results obtained in this thesis.

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List of Abbreviations

AWGN	Additive white Gaussian noise
CDF	Cumulative distribution function
CR	Cognitive radio
CRN	Cognitive radio networks
CRRN	Cognitive radio relay networks
CSI	Channel state information
DF	Decode and forward
EC	Effective capacity
i.i.d	Independent and identically distributed
MIMO	Multiple input multiple output
PDF	Probability density function
PU	Primary user
QoS	Quality of service
SNR	Signal to noise ratio
SU	Secondary user

Chapter 1

Introduction

Over the last two decades, the proliferation in the use of internet as well as wireless services has conducted an unprecedented technological evolution in the communications industry. Nowadays cell phones, pocket PCs and laptops have become more essential in modern life. However, such services as wireless broadband internet, mobile multimedia and many other applications have tremendous demands on higher data rates, security measures, location-awareness, energy efficiency and more efficient transmission links.

Providing QoS (Quality of Service) guarantees to various applications is an important objective in designing these high-end, high data rate wireless network devices. Different applications can have very diverse QoS requirements in terms of data rates, delay bounds, delay bound violation probabilities etc. To meet such connection-level QoS, it is necessary for the base station to characterize wireless channels. This task requires characterization of the server/service (i.e., wireless channel modelling) and queueing analysis of the system. However, the existing wireless channel models (e.g., Rayleigh fading model with a specified Doppler spectrum or finite-state Markov chain models) do not explicitly characterize a wireless channel in terms of these QoS measures. To use the existing channel models for QoS support, we first need to estimate the parameters for the channel model and then derive QoS measures from the model, using queueing analysis. This two-step approach is complex [1], and may lead to inaccuracies due to possible approximations in channel modelling and deriving QoS metrics from the models. To overcome this complex approach, a simple link layer channel model, called ‘effective capacity ’is introduced in [2].

As mentioned earlier, due to the rapid growth of wireless communications, demand for radio spectrum has increased. But reports from federal communications commission (FCC) [3] have shown that the spectrum is not optimally utilized. Cognitive radio (CR) technology [4] is considered as a promising paradigm to solve the problem of bandwidth limitation and inefficient spectrum utilization and is gaining much attention now. CR is formalised as a wireless communication system that intelligently utilizes any side information about the activity, channel conditions and codebooks of other nodes with which it shares the spectrum [5].

CR networks (CRN) can be mainly classified as overlay, interweave and underlay networks based on the type of side information. In overlay CRN, both secondary user (SU) and primary user (PU) occupy the spectrum at the same time and SU utilizes the knowledge of PU 's channel state information (CSI) to perform dirty paper coding so that the interference from PU is mitigated [6]. In contrast, in interweave CRN, the SU is allowed to use the spectrum only when it is not occupied by the PU [7]. As such, this technique can be considered as an opportunistic access. In an underlay network, however, the SU simultaneously occupies the spectrum with the PU as long as its interference on the primary network does not cause any harmful interference on the PU [8]. Harmful interference is measured in terms of interference temperature. So SU transmission power should be less than a predefined interference temperature limit. Here underlay approach appears to have many operational advantages [5, 7]. In this thesis, underlay CRN is considered.

Along with higher data rate requirements, future generations of wireless communication requires more reliable transmission links. But due to multipath fading, severe shadowing, path-loss and co-channel interference, communication in single-hop wireless networks has faced some fundamental limits [9]. In order to alleviate the impairment inflicted by wireless channels, multiple-input-multiple-output (MIMO) systems have been proposed to exploit diversity of the channel [10, 11]. Although MIMO systems can unfold their huge benefit in cellular base stations, they may face limitations when it comes to their deployment in mobile handsets. In particular, the typical small-size of mobile handsets makes it impractical to deploy multiple antennas [12]. To overcome this drawback, the concept of cooperative communications has been proposed. The key idea is to form a virtual MIMO antenna array by utilizing a third terminal, a so-called relay node, which assists the direct communication [13, 14]. The transmission between the source and destination nodes is divided into two main phases: i) Broadcasting phase: the source transmits its messages to both relay and destination, and ii) Multiple-access phase: the relay manipulates its received messages from the source before forwarding them to the destination.

As a result, the concept of cooperative communications has gained great attention, inspired by the pioneering works [14, 15]. It has been shown that cooperative communications can achieve significant power savings for extending network life-time, expand the communication range, and keep the implementation complexity low [16–18]. Depending on the relaying operation, the relay can be mainly categorized into two schemes: i) decode-and-forward (DF) and ii) amplify-and-forward (AF), each of which has its own advantages and disadvantages. For the DF scheme, the relay is required to perform an extra operation by decoding the source signal before forwarding it to the destination. In contrast, for the AF scheme, the relay simply amplifies the received message with a scalar gain without performing any signal regeneration, which may cause noise accumulation at the destination. In this thesis DF relaying scheme is used.

1.1 Motivation

Main motivation of doing this thesis for me is to carry out analysis in the area of wireless communications. Analytical results will reduce time and cost that may demand from simulation. Also analysis will help in finding behaviour of parameters in the system. Technically, the motivation of this thesis is to analyse cross-layer design for CR relay networks (CRRN). In particular, what is the behaviour of effective capacity when CRRN is constrained by interference power constraints. I am also interested in finding the behavioural changes when CRRN is restricted by interference and transmission power constraints. At the same time, the thesis also tries to find out whether an increase in the number of relays results in an increase in capacity. Finally I am interested in using the approach of finding effective capacity to outage capacity and ergodic capacity for multi relay network.

Several studies have been done to find capacity of relay channels and in spectrum sharing environment. Capacity of general relay channel with and without feedback is found in [19] . Upper and lower bounds for capacity and power allocation for wireless relay channels in Rayleigh fading environment are presented in [20]. Capacity investigations of additive white Gaussian noise (AWGN) spectrum sharing channels under interference power constraints are presented in [21]. Ergodic capacity and outage capacity for spectrum sharing communication in fading environment are studied in [22]. Ergodic capacity with adaptive transmission and selection combining is found [23]. Exact Outage probability of CR is presented in [24, 25].

1.2 Contribution of the thesis

In this thesis, effective capacity for CRRN under interference power constraints in Rayleigh fading channel is found. Effective capacity under interference and transmit power constraints is also analysed. The analysis is generalised for multiple relays. The treatment of finding capacity in CRRN is new and different from previous approaches. This approach is extended to outage capacity and ergodic capacity in delay insensitive CRRN networks for multiple relays. For all simulation and analysis, Rayleigh as time varying fading channel is considered.

1.3 Outline of the thesis

In the introduction chapter, motivation and contribution of the thesis is provided.Here an attempt is also made to list a few pioneering works in the area of CR and cooperative communications. The remainder of this thesis work is outlined as follows. Chapter 2 gives the background on effective capacity as well as system model of CRRN that is used in this thesis. In Chapter 3, effective capacity for CRRN under interference constraints is presented. Chapter 4, extends the treatment to outage probability and ergodic capacity in CRRN under interference power constraints. Chapter 5, discusses effective capacity for CRRN in interference and transmission power constraints. Chapter 6 presents ergodic capacity and outage probability for CRRN in interference and transmission power constraints. Finally, Chapter 7 concludes the thesis work.

Chapter 2

Background of effective capacity and cooperative communications

In this chapter, a brief background of effective capacity and system model that is used in the thesis are presented. The concepts of effective capacity, modelling channel with link layer objective and link layer channel model advantage over physical layer channel model are reported in section 2.1. System model is explained in section 2.2.

2.1 Effective capacity

In this section, channel modelling using effective capacity and its advantage over existing physical channel model are explained. Effective capacity concepts are presented in [1]. Voracious reader can also find more literature on effective capacity in [2].

Effective capacity is based on the idea of effective bandwidth, which models statistical behaviour of the traffic. Effective bandwidth is the minimum bandwidth that should be allocated to each traffic to maintain QoS constraint maximum delay bound D_{max} in N number of traffics. Consider an arrival process $A(t), t \geq 0$ where $A(t)$ represents amount of source data over the interval $[0, t]$. The asymptotic log-moment generating function of a stationary process $A(t)$, is defined as

$$\Lambda(u) = \lim_{t \to \infty} \frac{1}{t} \log \mathbb{E}[e^{uA(t)}] \tag{2.1}$$

and if log-moment generating function exists, then effective bandwidth function of $A(t)$ is defined as

$$\alpha^{(s)}(\mu) = \frac{\Lambda(u)}{u}, \forall u > 0 \tag{2.2}$$

Consider a queue of infinite buffer size served by a channel of constant service rate r. Let $Q(t)$ be the queue length formed because of mismatch between arrival rate $A(t)$

and service rate $S(t)$. According to [26], the probability of $D(t)$ exceeding $D(\infty)$ is given by

$$\Pr\left\{D(\infty) \geq D(t)\right\} = \gamma^s(r)e^{-\theta_B(r)B} \tag{2.3}$$

where, both $\gamma^s(r)$ and $\theta_B(r)$ are functions of channel capacity r. According to queuing theory, $\gamma^s(r)$ gives the probability that the buffer is non-empty and $\theta_B(r)$ is QoS exponent. The pair of functions $\{\gamma^s(r), \theta_B(r)\}$ model the source. In introducing effective capacity, similar lines are drawn with channel as with source. Concept of effective bandwidth is used in asynchronous transfer mode (ATM) networks. More details on theory of effective bandwidth can be found in [27].

Duality between traffic modelling by $\gamma^s(r), \theta_B(r)$ functions and channel modelling functions $\gamma(\mu), \theta(\mu)$ is used to propose effective capacity. Here μ is the constant source traffic rate. Let $r(t)$ be the instantaneous capacity at time t. So the service provided by the channel is given by

$$\tilde{S}(t) = \int_0^t r(\tau)d\tau \tag{2.4}$$

Here it is assumed that there exists a log-moment generating function i.e.,

$$\Lambda(-u) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}[e^{u\tilde{s}(t)}], \quad \forall \quad u \geq 0 \tag{2.5}$$

Then the effective capacity of $r(t)$ is

$$\alpha(u) = -\frac{\Lambda(-u)}{u}, \quad \forall \quad u \geq 0 \tag{2.6}$$

Substituting (2.5) in (2.6), we get

$$\alpha(u) = -\lim_{t\to\infty} \frac{1}{ut} \log \mathbb{E}[e^{-u\int_0^t r(\tau)d\tau}], \quad \forall \quad u \geq 0 \tag{2.7}$$

If we represent $\alpha(u)$ in discrete form

$$\alpha(u) = -\lim_{N\to\infty} \frac{1}{Nu} \log \mathbb{E}[e^{-u\sum_{n=1}^{N} R[n]}] \tag{2.8}$$

where $R[n], n = 1, 2, ...$ represents stochastic service process which is assumed stationary and ergodic. It can be shown that the probability of $D(t)$ exceeding a delay bound of D_{max} satisfy,

$$\sup_t \Pr\left\{D(t) \geq D_{max}\right\} = \Pr\left\{D(\infty) \geq D_{max}\right\}$$
$$= \gamma(\mu) e^{-\theta(\mu) D_{max}} \tag{2.9}$$

where $\gamma(\mu), \theta(\mu)$ are functions of source rate μ. The function pair $\gamma(\mu), \theta(\mu)$ defines effective capacity channel model.

So effective capacity can be defined as the maximum data rate allowed per user with very low probability of error with link layer channel model. Physical layer channel models are used in predicting physical layer characteristics like bit error rate, frame error rate as a function of signal to noise ratio (SNR) . Once marginal probability density function (PDF) of wireless channel is known, then it is possible to find outage probability, bit error probability or average SNR. But when dealing with multimedia traffic which is packet based network, link layer design changes from circuit based network. From link layer point of view, queuing analysis has to be done when dealing with packet based network. Design objectives of link layer like amout of delay caused, delay probability are difficult to obtain from physical layer channel model. Sometimes it is not possible to obtain delay error probability from PDF. So we need queueing analysis which is required to design appropriate admission control and resource reservation algorithms. We also need source traffic characterization and service characterization. As wireless channels are random in nature, we need statistical traffic characterization. All these resulted in the introduction of link layer channel model. Fig. 2.1 shows that

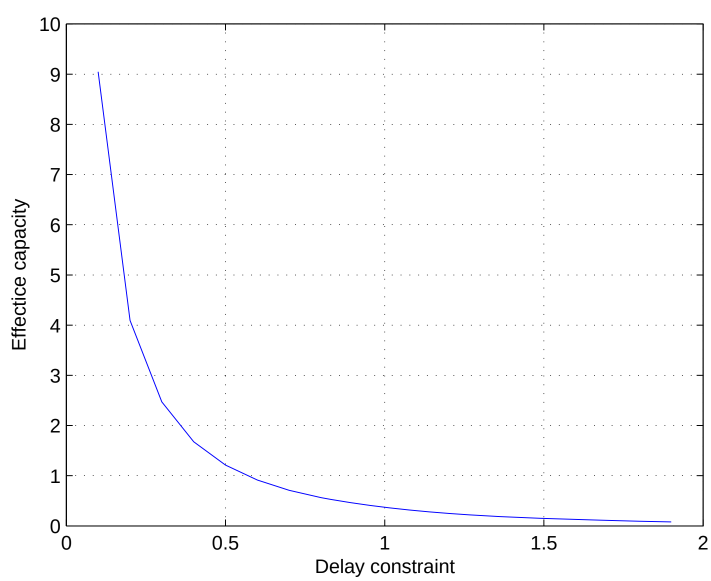

Figure 2.1: Effective capacity vs. QoS delay as seen from (2.8)

the effective capacity $\alpha(\mu)$ decreases with increasing QoS exponent μ, that is, as the

QoS requirement becomes more stringent, the source rate that a wireless channel can support with this QoS guarantee decreases. From (2.9), it is clear that the QoS metric can be easily extracted from the effective capacity (EC) channel model. Once EC model is known, we need channel estimation algorithm. Such an algorithm will estimate the functions $\gamma(\mu), \theta(\mu)$ from channel measurements such as channel capacity $r(t)$. If a channel specifies only PDF and Doppler spectrum then it is difficult to get channel effect on delay probability bound. If higher order statistics are provided then it is possible to calculate but the calculations are highly complex.

$\gamma(\mu), \theta(\mu)$ is the EC channel model, which exists if the log-moment generating function $\lambda(\mu)$ exists. If $r(t)$ is also ergodic, then $\gamma(\mu), \theta(\mu)$ can be estimated by equations 36 to 39 of [2]. Once EC model is found, QoS μ, D_{max}, ϵ can be computed by equation 40 of [2]. The resulting QoS μ, D_{max}, ϵ corresponds to service rate specification ${\lambda_s}^{(c)}, \sigma^{(c)}, \epsilon'$ with ${\lambda_s}^{(c)} = \mu, \sigma^{(c)} = D_{max}, \epsilon' = \epsilon$. The function pair $\gamma(\mu), \theta(\mu)$ corresponds to marginal PDF and Doppler spectrum of underlying physical layer.

2.2 System model

System model of CRRN is introduced here. In this thesis underlay CRRN is considered. In underlay scheme, secondary transmission can coexist with the primary transmissions, however, SUs should know that the interference they caused to the PU is below a predefined threshold. The secondary transmitter communicates to its receiver through relays. I assume multiple relays exist in the network and the relay node which gives the highest achievable rate is used for the communication (best relay selection). The relaying is based on DF technique. The secondary communication is based on dual hop half-duplex. In first hop, the relays listen to the secondary transmitter. In second hop, the relays broadcast signal that they decoded in the first hop. The system model is shown in the Fig. 2.2.

Full channel state information (CSI) is assumed to be available to both transmitter and receiver. It is assumed that relays are also supposed to know information about channel gain between transmitter and relay h_{SR_i}, relay and receiver h_{R_iD}, channel gain between relay and primary receiver h_{R_iP}. Information about channel gain between relay and primary receiver h_{SP} can be obtained from band manager or from the feedback of primary receiver to secondary transmitter. The secondary transmitter analyses the CSI in order to choose relay node to be active in the next time slot. It is assumed that all channel gains are independent and identically distributed (i.i.d) according to gamma distribution with unit variance. The transmitters are assumed to be ideal (free from clock drift, noise etc.). Channel gains are stationary and ergodic random process. Noise power spectral density and received bandwidth are denoted by N_o and B, respectively. In the network, it is assumed that the direct link between secondary transmitter and secondary receiver is weak. Here it is further assumed that the transmission technique has to satisfy certain statistical delay QoS constraint. It is shown that the probability for the queue length of the transmit buffer exceeding a certain threshold x,

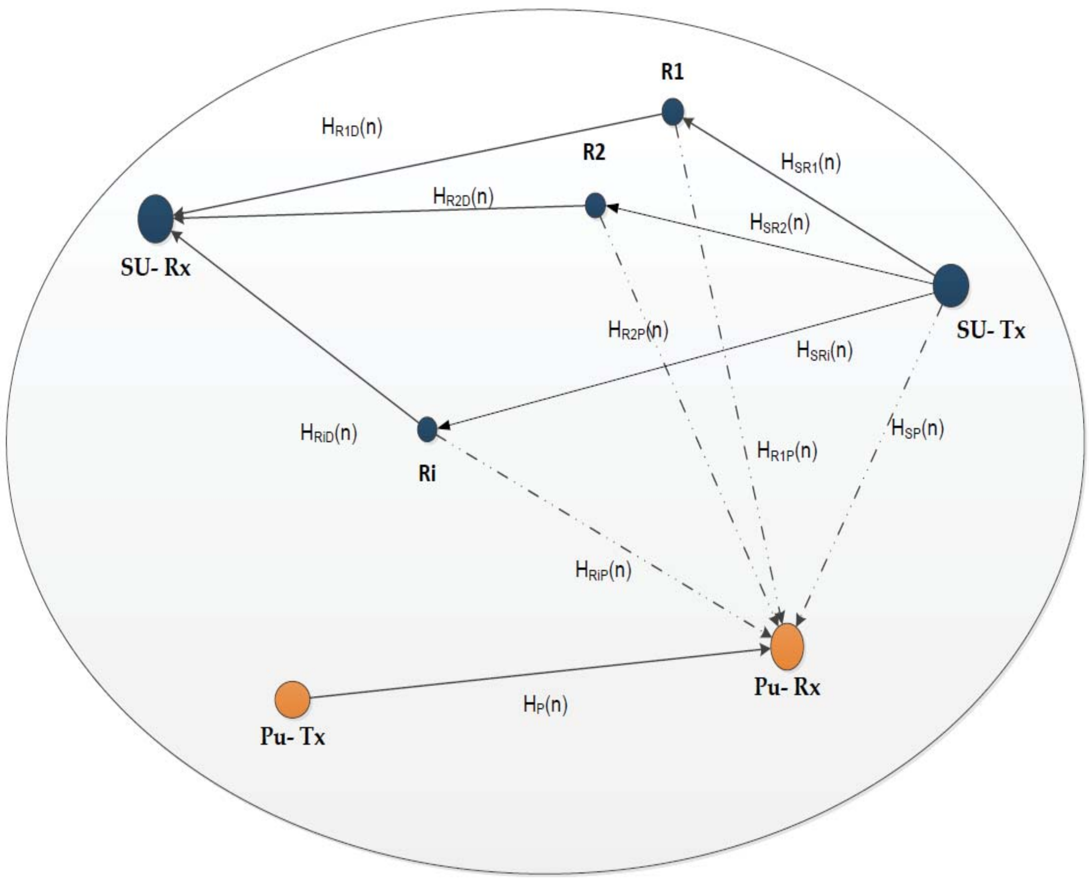

Figure 2.2: System model.

decays exponentially as a function of x. θ as a delay QoS exponent can be defined as

$$\theta = -\lim_{x \to \infty} \frac{\ln(\Pr\{q(\infty) > x\})}{x} \tag{2.10}$$

where $q(n)$ is transmit buffer length at time n. Considering θ as the delay QoS exponent, SU's maximal arrival rate that can be supported is obtained in the following chapter.

Chapter 3

Effective capacity of cognitive radio relay networks under interference power constraints

Here, effective capacity for cognitive radio relay networks (CRRN) under interference constraint is reported. Concept of effective capacity and system model of CRRN can be found in Chapter 2. Effect of primary networks on the performance of spectrum sharing can be studied from [28]. Performance of relay networks under power constraint of multiple primary users is studied in [29].

3.1 Interference power constraint

Transmission power of secondary transmitter and relay transmitters are limited so that their powers do not cross interference threshold. Powers of secondary transmitter and relay as function of channel gains can be related to interference threshold by

$$P(\theta, h_{SR_i}, h_{SP})h_{SP} \leq I_{th} \tag{3.1}$$

$$P(\theta, h_{R_iD}, h_{R_iP})h_{R_iP} \leq I_{th}; \quad i = 1, ..., K. \tag{3.2}$$

where, h_{SP} is channel gain between the secondary transmitter and the primary receiver,
h_{SR_i} is the channel gain between the secondary transmitter and the i^{th} relay,
h_{R_iP} is the channel gain between the i^{th} relay and the primary receiver,
h_{R_iD} is the channel gain between the i^{th} relay and the secondary receiver.

Now we have to relate interference threshold I_{th} and peak primary transmitter power in outage ${P_p}^{out}$. Let R_{min} is the minimum rate allowed by the primary transmitter. Peak power of the primary user in outage ${P_p}^{out}$ can be given as

$$\Pr\{R_p \leq R_{min}\} \leq {P_p}^{out} \tag{3.3}$$

Here R_p is the rate of the primary transmitter. Average power of primary link can be related with $P_p(h_p)$, the input transmit power as function of h_p as

$$\mathbb{E}\{P_p(h_p)\} \leq \bar{P} \tag{3.4}$$

Given μ as the cut-off threshold for the primary transmit power, $P_p(h_p)$ can be related by

$$P_p(h_p) = \mu - \frac{N_o B}{h_P} \tag{3.5}$$

If μ is less than $\frac{N_o B}{h_P}$ then it is not possible for primary receiver to reconstruct data faithfully. We can relate data rate R and power P as $R = ln(1 + \frac{hP}{N_o B})$ where h is channel power gain and $N_o B$ is noise power. Using this relation in (3.3), we can get

$$\Pr\left\{ \ln\left(1 + \frac{P_p(h_P)h_P}{P(\theta, h_{SR_i}, h_{SP})h_{SP} + N_o B}\right) \leq R_{min}, h_P \geq \frac{N_o B}{\mu} \right\} + \Pr\left\{ h_P < \frac{N_o B}{\mu} \right\} \leq P_p{}^{out} \tag{3.6}$$

Here $P(\theta, h_{SR_i}, h_{SP})$ denotes power of SU as function of θ, h_{SR_i} and h_{SP}. When cut off threshold μ is greater than $\frac{N_o B}{h_P}$, SU is allowed to use the spectrum. Mathematically it is, $ln(1 + \frac{P_p(h_P)h_P}{P(\theta, h_{SR_i}, h_{SP})h_{SP} + N_o B}) \leq R_{min}$. Solving for h_P gives

$$h_P \leq \left(\frac{e^{R_{min}} - 1}{\mu}\right)\left(P(\theta, h_{SR_i}, h_{SP})h_{SP} + N_o B\right) + \frac{N_o B}{\mu} \tag{3.7}$$

Let $K_1 = \left(\frac{e^{R_{min}-1}}{\mu}\right)$ and $K_2 = \frac{N_o B}{\mu}$, then (3.6) is simplified as

$$\Pr\{K_2 \leq h_P \leq K_1(P(\theta, h_{SR_i}, h_{SP})h_{SP} + N_o B) + K_2\} + (1 - e^{-K_2}) \leq P_p{}^{out} \tag{3.8}$$

Now solving (3.8) and (3.1), interference power limit I_{th} can be found as

$$I_{th} = -\frac{ln(1 - P_p{}^{out}) + K_2}{K_1} - N_o B \tag{3.9}$$

3.2 Effective capacity analysis

Effective capacity for CRRN with multi relay nodes is analysed in this section. Let $\{R[n], n = 1, 2, \ldots\}$ be the stochastic service process which is stationary and ergodic, then there exists a capacity function

$$\Lambda(-\theta) = \lim_{N\to\infty} \frac{1}{N} \ln \left(\mathbb{E}\left[e^{-\theta \sum\limits_{n=1}^{N} R[n]} \right] \right) \tag{3.10}$$

and effective capacity as given by [2]

$$\begin{aligned} E_c(\theta) &= -\frac{\Lambda(-\theta)}{\theta} \\ &= -\lim_{N\to\infty} \frac{1}{N\theta} \ln \left(\mathbb{E}\left[e^{-\theta \sum\limits_{n=1}^{N} R[n]} \right] \right) \end{aligned} \tag{3.11}$$

where, θ is QoS exponent interpreted as delay constraint and $R[n]$ is data rate of relay channel. As we are considering i.i.d Rayleigh channels, $R[n], n = 1, 2, \ldots$ is uncorrelated and hence effective capacity can be simplified

$$\begin{aligned} E_c(\theta) &= -\lim_{N\to\infty} \frac{1}{N\theta} \ln(\mathbb{E}\{e^{-\theta N R[n]}\}) \\ &= -\lim_{N\to\infty} \frac{1}{N\theta} \ln(e^{N}\, \mathbb{E}\{e^{-\theta R[n]}\}) \\ &= -\frac{1}{\theta} \ln \left(\mathbb{E}\{e^{-\theta R[n]}\} \right) \end{aligned} \tag{3.12}$$

Data rates of secondary transmitter link and relay link in terms of peak power are

$$\begin{aligned} R_{Si}[n] &= \frac{T_f B}{2} \ln \left(1 + \frac{h_{SR_i}[n] P(\theta, h_{SR_i}, h_{SP})}{N_o B} \right) \\ R_{Ri}[n] &= \frac{T_f B}{2} \ln \left(1 + \frac{h_{R_iD}[n] P(\theta, h_{R_iD}, h_{R_iP})}{N_o B} \right) \end{aligned} \tag{3.13}$$

and data rate of the link is $R_i[n] = \min(R_{Si}[n], R_{Ri}[n])$. In terms of interference power I_{th} data rate is

$$R_i[n] = \frac{T_f B}{2} \min \left\{ \ln \left(1 + \frac{h_{SR_i}[n]}{h_{SP}[n]} \frac{I_{th}}{N_o B} \right), \ln \left(1 + \frac{h_{R_iD}[n]}{h_{R_iP}[n]} \frac{I_{th}}{N_o B} \right) \right\} \tag{3.14}$$

In multi relay nodes, the rate of the total channel is the maximum rate of the individual paths i.e.,

$$R[n] = \max\{R_i[n]\}, i = 1, ..., K \tag{3.15}$$

Hence onwards, time index $[n]$ is dropped for simplicity. Now, a closed form expression for effective capacity can be obtained. Let us define new random variable

$Z = \min\left\{\frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_iD}}{h_{R_iP}}\right\}$. The treatment of finding the PDF of Z is different from [22]. The ratio of channel gains $\frac{h_{SR_i}}{h_{SP}}$ is dependent on h_{SP} as channel gain between secondary transmitter and primary receiver is the same for all channel gains between secondary transmitter and relays. So the CDF of Z is

$$F_{Z_i}(z \mid h_{SP}) = \Pr\left(\min\left\{\frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_iD}}{h_{R_iP}}\right\} \leq z \mid h_{SP}\right) \tag{3.16}$$

Here $F_{Z_i}(z \mid h_{SP})$ is CDF of the i^{th} channel path. Let $h_{SP} = X$ and $\Gamma_i = \frac{h_{R_iD}}{h_{R_iP}}$. Substituting h_{SP} and Γ_i in (3.16) gives

$$F_{Z_i}(z \mid h_{SP}) = F_{h_{SR_i}/X}(z \mid h_{SP}) \bigcup F_{\Gamma_i}(z \mid h_{SP}) \tag{3.17}$$

where

$$\begin{aligned} F_{h_{SR_i}/X}(z \mid h_{SP}) &= \Pr(h_{SR_i}/X \leq z \mid h_{SP}) \\ &= 1 - e^{-zX} \end{aligned} \tag{3.18}$$

Γ_i is independent random variable and its CDF is given by

$$F_{\Gamma_i}(z \mid h_{SP}) = 1 - \frac{1}{1+z} \tag{3.19}$$

Substituting (3.19) and (3.18) in (3.17) and after some simplification we get

$$F_{Z_i}(z \mid h_{SP}) = 1 - \frac{e^{-zX}}{1+z} \tag{3.20}$$

For K relays in i.i.d Rayleigh channel, CDF of system is the product of individual CDF s

$$\begin{aligned} F_Z(z \mid h_{SP}) &= \Pr\Big(\max\{Z_1, Z_2, ...Z_K\} < z \mid h_{SP}\Big) \\ &= \prod_{i=1}^{K} \Pr\Big(Z_i < z \mid h_{SP}\Big) \\ &= \left(1 - \frac{e^{-zX}}{1+z}\right)^K = \sum_{l=0}^{K} \binom{K}{l} (-1)^l \left(\frac{e^{-zX}}{1+z}\right)^l \end{aligned} \tag{3.21}$$

Binomial expansion is used in getting (3.21). $F_z(z)$ is defined as

$$\begin{aligned} F_Z(z) &= \int_0^\infty F_Z(z \mid h_{SP}) .p_{h_{SP}}(x)\, dx \\ &= \sum_{l=0}^{K} \binom{K}{l} \frac{(-1)^l}{(1+z)^l} \frac{1}{(zl+1)} \end{aligned} \tag{3.22}$$

Differentiating CDF in (3.22), we obtain PDF

$$p_Z(z) = \sum_{l=1}^{K} \binom{K}{l} (-1)^{l+1} l \left(\frac{1}{(1+z)^l (zl+1)^2} + \frac{1}{(1+z)^{(l+1)} (zl+1)} \right) \quad (3.23)$$

So the effective capacity from (3.12) can be written as

$$\begin{aligned} E_c(\theta) &= -\frac{1}{\theta} \ln \left(\int_0^\infty e^{-\theta R[n]} p_Z(z) \right) \\ &= -\frac{1}{\theta} \ln \Bigg[\int_0^\infty \left(1 + \frac{z I_{th}}{N_o B}\right)^{-\alpha} \sum_{l=1}^{K} \binom{K}{l} (-1)^{l+1} l \times \\ &\quad \left(\frac{1}{(1+z)^l (zl+1)^2} + \frac{1}{(1+z)^{(l+1)} (zl+1)} \right) dz \Bigg] \end{aligned} \quad (3.24)$$

This (3.24) gives effective capacity in integral form. By using partial fraction, we get

$$\begin{aligned} E_c(\theta) = -\frac{1}{\theta} \Bigg(\ln \Bigg[\int_0^\infty \left(1 + \frac{z I_{th}}{N_o B}\right)^{-\alpha} \sum_{l=1}^{K} (-1)^{l+1} \binom{K}{l} l \\ \left(\sum_{n=1}^{l} \frac{(-l)^{n-1} n}{(1-l)^{n+1} (1+z)^{l-n+1}} + (-1)^l \sum_{m=0}^{1} \frac{l^{l+m}}{(1-l)^{l+m} (1+zl)^{2-m}} + \right. \\ \left. \sum_{n=1}^{l+1} \frac{(-l)^{n-1}}{(1-l)^n (1+z)^{l-n+2}} + \frac{(-l)^{l+1}}{(1-l)^{l+1} (1+zl)} \right) dz \Bigg] \Bigg) \end{aligned} \quad (3.25)$$

Using [30, eq.(3.197.5,3.197.1)], (3.25) can be further simplified

$$\begin{aligned} E_c(\theta) = -\frac{1}{\theta} \ln \Bigg[\sum_{l=2}^{K} \binom{K}{l} \sum_{n=1}^{l} (-1)^{n+l} \frac{n l^n}{(1-l)^{n+1} (\alpha + l - n)} \\ \left({}_2F_1\left(\alpha, 1; \alpha + l - n + 1; 1 - \frac{I_{th}}{N_o B}\right) - \sum_{m=0}^{1} \left(\frac{l}{1-l}\right)^{l+m} \frac{1}{(\alpha - m + 1)} \right. \\ {}_2F_1\left(\alpha, 1; \alpha - m + 2; 1 - \frac{I_{th}}{N_o B l}\right) + \sum_{n=1}^{l+1} (-1)^{n+l} \left(\frac{l}{1-l}\right)^n \frac{1}{(\alpha + l - n + 1)} \\ {}_2F_1\left(\alpha, 1; \alpha + l - n + 2; 1 - \frac{I_{th}}{N_o B}\right) + \sum_{l=2}^{K} \left(\frac{l}{(1-l)}\right)^{l+1} \frac{1}{\alpha} \\ \left. {}_2F_1\left(\alpha, 1; \alpha + 1; 1 - \frac{I_{th}}{N_o B l}\right) + \frac{2K}{(\alpha+2)} {}_2F_1\left(\alpha, 1; \alpha + 3; 1 - \frac{I_{th}}{N_o B}\right) \right) \Bigg] \end{aligned} \quad (3.26)$$

where, ${}_2F_1\,(a, b; c; z)$ is Gaussian hyper-geometric function [31, eq.(15.1.1)].

It can be seen that (3.26) is the closed form expression for effective capacity for K relays. This can be verified by substituting $K = 1$ and the resultant equation can be equated to [32, eq.17].

3.3 Numerical results

In this section, numerical results are presented to validate our analytical expressions derived and illustrate the effect of interference power constraints on capacity. All observations are carried out in Rayleigh fading environment. Here for simplicity, we assume $N_oB = 1$ and $T_fB = 1$. In Fig. 3.1, normalised effective capacity versus delay exponent constraint θ is plotted. We observe that effective capacity decreases

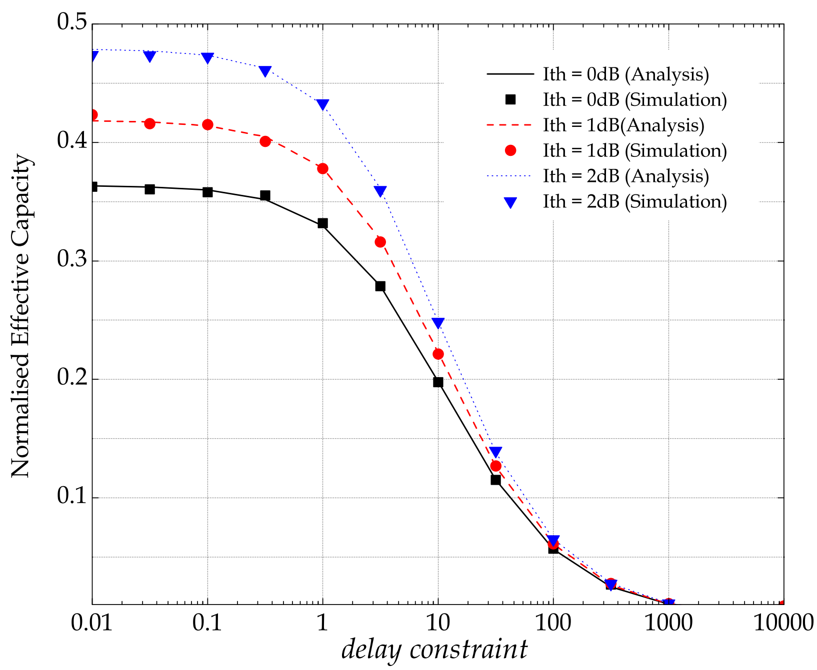

Figure 3.1: Normalised effective capacity vs. QoS delay exponent θ under various interference constraints.

with the increase in delay exponent. This is true as with less stringent constraint, more capacity can be achieved. Secondly, we observe as interference threshold allowed for secondary transmission increases, effective capacity for a given θ increases. One important observation is, higher interference threshold does not result in higher capacity at higher delay exponent θ. For Fig. 3.1, the number of relays used (K) are 2. Fig. 3.2 shows as the number of relays increases in the system, effective capacity increases.

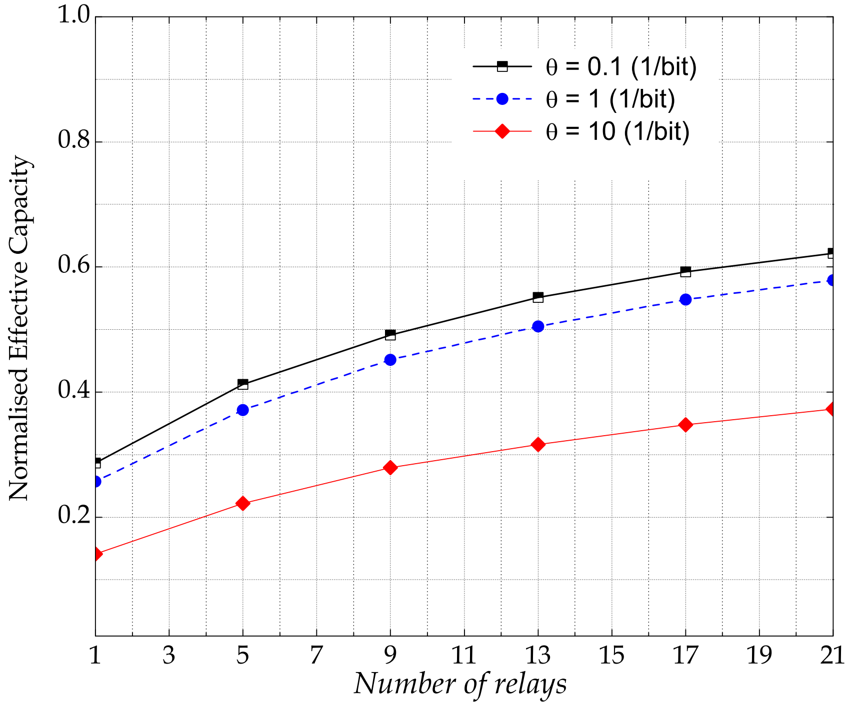

Figure 3.2: Normalised effective capacity vs. number of relays

Chapter 4

Ergodic capacity and Outage probability of cognitive radio relay networks under interference power constraints

Here ergodic and outage capacities of a Rayleigh flat-fading channel are investigated. Ergodic capacity is the maximum on the long-term average rate that can be achieved by fading channel, a capacity metric that is suitable for delay-insensitive applications [33]. Outage capacity is, on other hand, the metric suitable for systems that carry delay-sensitive applications, and is defined as the maximum constant-rate that can be achieved for a certain percentage of time. For further information on theoretic notions pertaining to ergodic capacity and outage capacity under fading channels, the reader is referred to [33] and [34].

4.1 Ergodic capacity analysis

Ergodic capacity is average capacity of the channel for duration of T_f. In this section, Rayleigh fading environment with peak interference-power constraints is considered.

$$\frac{C_{er}}{B} = \max_{h_s,h_p} \left\{ \mathbb{E}_{h_s,h_p} \left(\ln(1 + \frac{P(hs,hp)hs}{N_o B}) \right) \right\} \qquad (4.1)$$
$$s.t.\ P(h_s,h_p)h_p \leq Q_{peak}, \quad \forall\ h_s,h_p.$$

This can be simply written as

$$\frac{C_{er}}{B} = \frac{T_f}{2}\,\mathbb{E}\left[\min\left\{ \frac{C_1}{B}, \frac{C_2}{B} \right\} \right] \qquad (4.2)$$

where, B is bandwidth, $\mathbb{E}$ is expectation operation over $h_{SP}, h_{SR_i}, h_{R_iD}, h_{R_iP}$ and

$$C_1 = \ln\left(1 + \frac{h_{SR_i} I_{th}}{h_{SP} N_o B}\right) \quad \text{and} \quad C_2 = \ln\left(1 + \frac{h_{R_iD} I_{th}}{h_{R_iP} N_o B}\right)$$

Ergodic capacity can be derived using

$$C_{er} = \frac{1}{2}\int_T \ln(1+\alpha x) p_T(x)\, dx \tag{4.3}$$

Here, $T = \min\left(\frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_iD}}{h_{R_iP}}\right)$ and $\alpha = \frac{I_{th}}{N_o B}$. CDF can be obtained from (3.22). Taking partial fraction of CDF in (3.22), we can obtain

$$F_T(x) = \sum_{l=0}^{K} \binom{K}{l} \left(\sum_{n=1}^{l} \frac{(-1)^{l+n-1} l^{n-1}}{(1-l)^n (1+x)^{l-n+1}} + (-1)^{l+k} \left(\frac{l}{1-l}\right)^l \frac{1}{(1+lx)} \right) \tag{4.4}$$

To get PDF $p_T(x)$, differentiate CDF $F_T(x)$ in (4.4). This gives

$$\begin{aligned} p_T(x) = &\sum_{l=1}^{K}\sum_{n=1}^{l} \binom{K}{l} (-1)^{l+n} \frac{l^{n-1}(l-n+1)}{(1-l)^n} \frac{1}{(1+x)^{(l-n+2)}} \\ &+ \sum_{l=1}^{K} \binom{K}{l} (-1)^{l+k+1} \frac{l^{l+1}}{(1-l)^l} \frac{1}{(1+lx)^2} \end{aligned} \tag{4.5}$$

Substituting, (4.5) in (4.3) we have

$$\begin{aligned} C_{er} = \frac{1}{2}\Bigg[&\int_T \ln(1+\alpha x) \sum_{l=1}^{K}\sum_{n=1}^{l} \binom{K}{l} (-1)^{l+n} \frac{l^{n-1}(l-n+1)}{(1-l)^n} \frac{1}{(1+x)^{(l-n+2)}} dx \\ &+ \int_T \ln(1+\alpha x) \sum_{l=1}^{K} \binom{K}{l} (-1)^{l+k+1} \frac{l^{l+1}}{(1-l)^l} \frac{1}{(1+lx)^2} dx \Bigg] \end{aligned} \tag{4.6}$$

Ergodic capacity for K relays in integral form is given as (4.6). By using [30, Eq:4.291.17], (4.6) can be written as

$$\begin{aligned} C_{er} = \frac{1}{2}\Bigg[&\sum_{l=1}^{K}\sum_{n=1}^{l} \binom{K}{l} (-1)^{l+n} (l-n+1) \frac{l^{n-1}}{(1-l)^n} \\ &\left\{ \left(\frac{(n-l)(-1+\frac{1}{\alpha})^{-(l-n+1)} \pi \csc((l-n+2)\pi)}{(l-n)(l-n+1)} \right) + \left(\frac{\alpha_2 F_1(1,1,1-l+n,\alpha)}{(l-n)(l-n+1)} \right) \right\} \\ &+ \sum_{l=1}^{K} \binom{K}{l} (-1)^{l+K+1} \left(\frac{l}{1-l}\right)^l \left(\frac{\alpha \ln(\alpha) - \ln(l)}{(\alpha - l)} \right) \Bigg] \end{aligned} \tag{4.7}$$

It can be observed that, (4.7) is closed form expression for ergodic capacity.

4.2 Outage probability analysis

From Chapter 3, data rate of the i^{th} path is

$$R_i[n] = \frac{T_f B}{2} \min\left\{ \ln(1 + \frac{h_{SR_i}[n]}{h_{SP}[n]} \frac{I_{th}}{N_o B}), \ln(1 + \frac{h_{R_i D}[n]}{h_{R_i P}[n]} \frac{I_{th}}{N_o B}) \right\} \tag{4.8}$$

Here the ratio of channel gains are dependent on h_{SP}. In multi relay nodes, rate of the channel is maximum rate of the individual paths.

$$R[n] = \max\{R_i[n]\}, i = 1, ..., K \tag{4.9}$$

CDF $F_Z\left(z \mid h_{SP}\right)$ is

$$\begin{aligned} F_Z\left(z \mid h_{SP}\right) &= \Pr\{\max(Z_1, Z_2, ..., Z_K) \leq z \mid h_{SP}\} \\ &= \prod_{i=1}^{K} \Pr\{Z_i \leq z \mid h_{SP}\} = \prod_{i=1}^{K} F_{Z_i}(z \mid h_{SP}) \end{aligned} \tag{4.10}$$

CDF $F_{Z_i}\left(z \mid h_{SP}\right)$ can be written as

$$F_{Z_i}\left(z \mid h_{SP}\right) = \Pr\left\{ \min\{\frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_i D}}{h_{R_i P}} \leq z \mid h_{SP}\}\right\} \tag{4.11}$$

Let h_{SP} = X and $\gamma = \frac{h_{R_i D}}{h_{R_i P}}$. Then, we can write

$$F_{Z_i}\left(z \mid h_{SP}\right) = F_{\frac{h_{SR_i}}{X}}(z \mid h_{SP}) \quad \cup \quad F_{Y_i}(z \mid h_{SP}) \tag{4.12}$$

From (3.20), we can get CDF of Z_i as

$$F_{Z_i}\left(z \mid h_{SP}\right) = 1 - \frac{e^{-zX}}{(1+z)} \tag{4.13}$$

When we extend (4.13) to K relays, we get

$$F_Z\left(z \mid h_{SP}\right) = \prod_{i=1}^{K} F_{Z_i}\left(z \mid h_{SP}\right) = \left(1 - \frac{e^{-zX}}{(1+z)}\right)^K \tag{4.14}$$

$F_Z\left(z\right)$ can be obtained as

$$\begin{aligned} F_Z\left(z\right) &= \int_0^\infty F_Z\left(z \mid h_{SP}\right) p_{h_{SP}}\left(x\right) dx = \int_0^\infty \sum_{l=0}^{K} \binom{K}{l} (-1)^l \frac{e^{-zlx}}{(1+z)^l} e^x dx \\ &= \sum_{l=0}^{K} \binom{K}{l} (-1)^l \Big[\frac{1}{(1+z)^l (zl+1)^2} + \frac{1}{(1+z)^{l+1}(zl+1)}\Big] \end{aligned} \tag{4.15}$$

PDF $p_Z(z)$ is obtained by differentiating CDF in (4.15)

$$\begin{aligned} p_Z(z) &= \frac{d}{dx}\left(F_Z(z)\right) \\ &= \sum_{l=1}^{K}\binom{K}{l}(-1)^{(l+1)}l\left(\frac{1}{(1+z)^l(zl+1)^2}+\frac{1}{(1+z)^{l+1}(zl+1)}\right) \end{aligned} \quad (4.16)$$

Now we can calculate outage probability as

$$\begin{aligned} P_{out,CRRN} &= \Pr\{R_n < R_{min}\} \\ &= \Pr\left\{\max\{R_i[n]\} < R_{min}\right\} \\ &= \Pr\left\{1/2 * \ln\left(1+\frac{I_{th}}{N_oB}z\right) < R_{min}\right\} \\ &= \Pr\left\{1+\frac{I_{th}}{N_oB}z < e^{2R_{min}}-1\right\} \\ &= \Pr\left\{z < \frac{N_oB}{I_{th}}(e^{2R_{min}}-1)\right\} \end{aligned} \quad (4.17)$$

Solving (4.17) gives

$$\begin{aligned} P_{out,CRRN} &= \int_0^{\frac{N_oB\beta}{I_{th}}} p_Z(z)\,dz \\ &= \int_0^{\frac{N_oB\beta}{I_{th}}} \sum_{l=0}^{K}\binom{K}{l}(-1)^{(l+1)}l\left(\frac{1}{(1+z)^l(zl+1)^2}+\frac{1}{(1+z)^{l+1}(zl+1)}\right) \end{aligned} \quad (4.18)$$

where $\beta = e^{2R_{min}} - 1$. This is the integral form for outage probability. We have to solve (4.18) to get closed form expression.

$$\begin{aligned} P_{out,CRRN} &= \sum_{l=0}^{K}\binom{K}{l}(-1)^{(l+1)}l\int_0^{\frac{N_oB\beta}{I_{th}}}\left[\left(\frac{1}{(1+z)^l(zl+1)^2}+\frac{1}{(1+z)^{l+1}(zl+1)}\right)\right] \\ &= \sum_{l=0}^{K}\binom{K}{l}(-1)^{(l+1)}\left[1-\frac{l\left(\frac{1+N_oB\beta}{I_{th}}\right)^{-l}}{1+l^2\left(\frac{N_oB\beta}{I_{th}}\right)}\right] \end{aligned} \quad (4.19)$$

$P_{out,CRRN}$ simplification in (4.19) can be obtained by using

$$\int_0^U \frac{1}{(1+x)^{n+1}(1+nx)}+\frac{1}{(1+x)^n(1+nx)^2}dx = \frac{1}{n}-\frac{(1+U)^{-n}}{1+n^2U}$$

For given outage $P_{out,CRRN}$, now maximum supportable rate can be found by numerical calculation in (4.19). Here R_{min} gives capacity that can be supported with outage $P_{out,CRRN}$.

4.3 Numerical results

Here simulation and analytical results are presented for ergodic capacity and outage probability. Fig. 4.1 shows normalised ergodic capacity versus interference constraints. One can observe as interference threshold allowed increases, ergodic capacity increases. At the same time, more number of relays results in increase in ergodic capacity. Fig. 4.2 shows outage probability versus interference constraints. Here we find

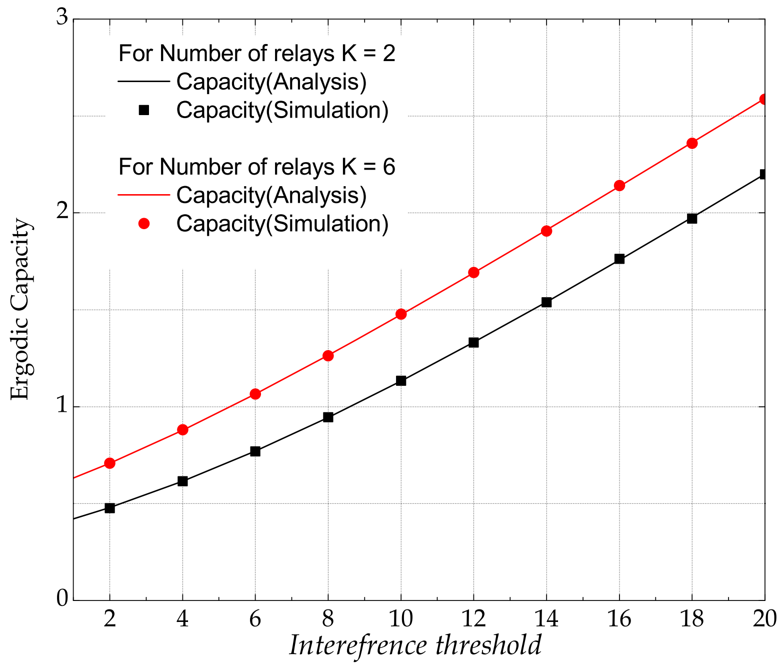

Figure 4.1: Ergodic capacity vs. interference constraints.

that as interference threshold allowed to secondary user increases, outage probability decreases i.e the system being in outage reduces. One can obtain outage capacity, for given outage probability. In (4.19), substituting the allowed outage probability, one can get R_{min} as outage capacity.

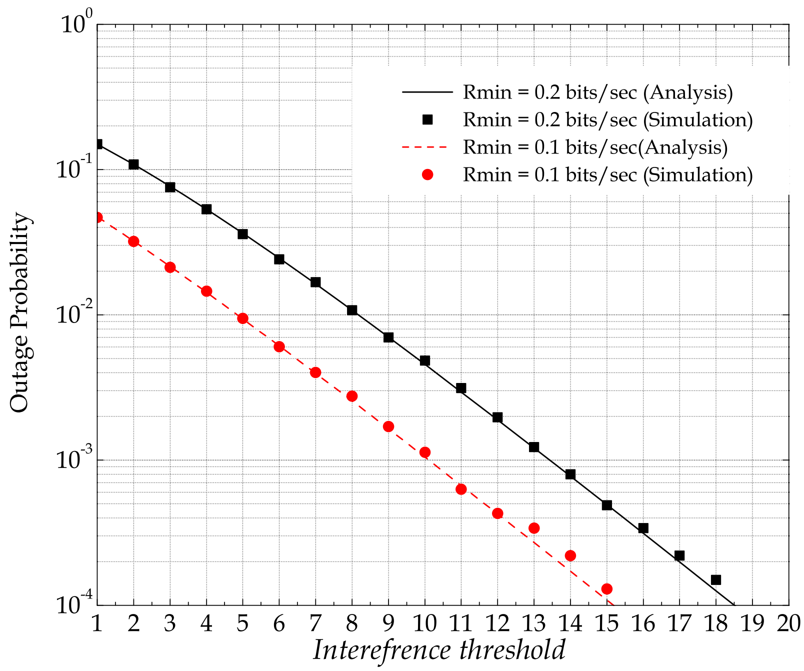

Figure 4.2: Outage probability vs. interference constraints in dB.

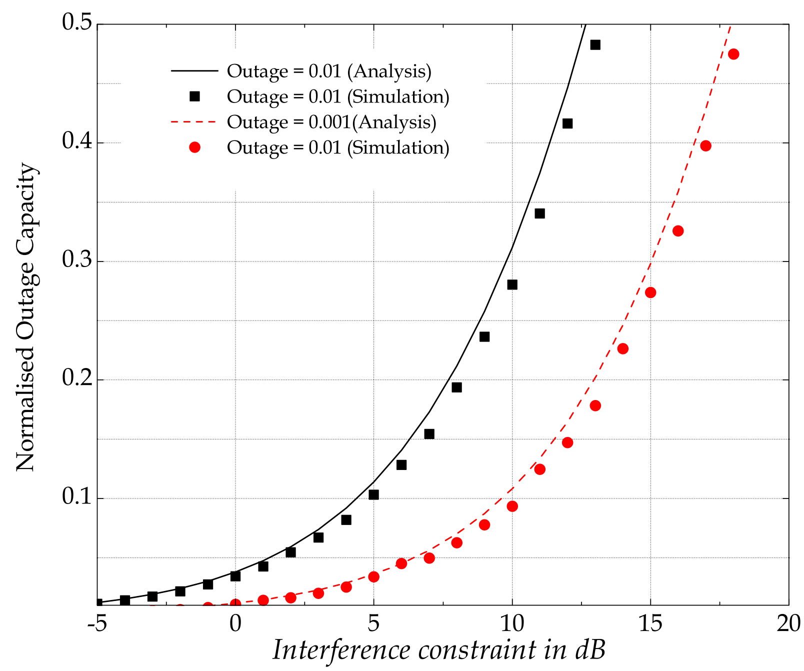

Figure 4.3: Outage capacity vs. interference constraints in dB.

Chapter 5

Effective capacity of cognitive radio relay networks under interference and transmission power constraints

5.1 Introduction

In Chapter 3, effective capacity for interference power constraints is explained. Interference threshold is dependent on primary link. It may also happen that the transmitter or relay cannot transmit with allowed threshold power because of its own transmit power limitation. This is more likely the case with relays, which generally does not have much power to transmit. In this section analysis of effective capacity under both interference and secondary transmit power constraints is carried out. Here too, peak interference power and peak transmit power constraints are considered. Similar approach in finding outage probability for cognitive radio relay networks can be found in [35]. But authors in [35] do not derive exact outage probability.

5.2 Effective capacity analysis

Let P be the maximum transmit power available at secondary transmitter and relays. The secondary transmission is also restricted by interference allowed by primary user. Mathematically

$$\begin{aligned} P_s &\leq \min\left(\frac{I_{th}}{h_{SP}}, P\right) \\ P_r &\leq \min\left(\frac{I_{th}}{h_{R_iP}}, P\right) \end{aligned} \tag{5.1}$$

where, P_s is the secondary transmitter instantaneous power, h_{SP} is the channel power gain between the secondary transmitter and the primary receiver, h_{R_iP} is the channel power gain between the i^{th} relay and the primary receiver. We can find date rate of the secondary link similar to (3.14). i.e.,

$$R_i[n] = \frac{T_f B}{2} \min \Bigg\{ \ln \left(1 + \min\left(\frac{I_{th}}{h_{SP}}, P\right) \frac{h_{SR_i}}{N_o B}\right), \\ \ln \left(1 + \min\left(\frac{I_{th}}{h_{R_iP}}, P\right) \frac{h_{R_iD}}{N_o B}\right) \Bigg\} \tag{5.2}$$

where, B is Bandwidth, T_f is time duration, I_{th} is interference threshold allowed, h_{R_iD} is channel power gain between the i^{th} relay and secondary destination and h_{SR_i} is channel power gain between secondary transmitter and i^{th} relay. In best relay selection network, data rate supported by the channel is maximum rate of the individual paths i.e.,

$$R[n] = \max\{R_i[n]\}, \qquad i = 1, ..., K \tag{5.3}$$

Let us define a new set of random variables U_1, U_2 and Z so that

$$\begin{aligned} U_{1i} &= \min\left[\left(\frac{I_{th}}{h_{SP}}, P\right)\frac{h_{SR_i}}{N_o B}\right] \\ U_{2i} &= \min\left[\left(\frac{I_{th}}{h_{R_iP}}, P\right)\frac{h_{R_iD}}{N_o B}\right] \\ Z_i &= \left\{U_{1i}, U_{2i}\right\} \end{aligned} \tag{5.4}$$

The CDF $F_{Z_i}(z \mid h_{SP})$ is

$$\begin{aligned} F_{Z_i}(z \mid h_{SP}) &= \Pr\left\{ \min\left(U_{1i}, U_{2i}\right) \le z \mid h_{SP}\right\} \\ &= F_{U_{1i}}(z \mid h_{SP}) \cup F_{U_{2i}}(z \mid h_{SP}) \\ &= 1 - \Big[1 - F_{U_{1i}}(z \mid h_{SP})\Big]\Big[1 - F_{U_{2i}}(z \mid h_{SP})\Big] \end{aligned} \tag{5.5}$$

Let us find the CDF of U_{2i} as

$$F_{U_{2i}}(z \mid h_{SP}) = \Pr\left\{ \min\left(\frac{I_{th}}{h_{R_iP}}, P\right)\frac{h_{R_iD}}{N_o B} \le z \mid h_{SP}\right\} \tag{5.6}$$

The CDF in (5.6) can be simplified as

$$F_{U_{2i}}(z \mid h_{SP}) = \begin{cases} \Pr\left\{\frac{I_{th}}{h_{R_iP}}\frac{h_{R_iD}}{N_o B} \le z\right\} & \text{if } \frac{I_{th}}{h_{R_iP}} \le P \\ \Pr\left\{P\frac{h_{R_iD}}{N_o B} \le z\right\} & \text{if } \frac{I_{th}}{h_{R_iP}} > P. \end{cases}$$

i.e.,

$$F_{U_{2i}}(z \mid h_{SP}) = A + B \quad with$$
$$A \doteq \Pr\left\{\frac{I_{th}}{h_{R_iP}}\frac{h_{R_iD}}{N_oB} \leq z, \frac{I_{th}}{h_{R_iP}} \leq P\right\}$$
$$B \doteq \Pr\left\{P\frac{h_{R_iD}}{N_oB} \leq z, \frac{I_{th}}{h_{R_iP}} > P\right\} \tag{5.7}$$

Here B is the CDF of h_{R_iD} and h_{R_iP} which are independent and hence

$$\begin{aligned} B &= F_{h_{R_iD}}\left(\frac{zN_oB}{P}\right)F_{h_{R_iP}}\left(\frac{I_{th}}{P}\right) \\ &= \int_0^{\frac{zN_oB}{P}} e^{-x}dx \int_0^{\frac{I_{th}}{P}} e^{-y}dy \\ &= 1 - e^{\frac{-zN_oB}{P}} - e^{-\frac{I_{th}}{P}} + e^{-\frac{1}{P}(zN_oB+I_{th})} \end{aligned} \tag{5.8}$$

Now A can be solved, by treating ratio of channel gains as dependent variable on h_{R_iP}.

$$A = \int_{\frac{I_{th}}{P}}^{\infty} p_{h_{R_iP}}(y) \int_0^{\frac{zyN_oB}{I_{th}}} p_{h_{R_iD}}(x)\,dxdy = e^{-\frac{I_{th}}{P}} - \frac{e^{-\frac{1}{P}(zN_oB+I_{th})}}{\left(1+\frac{zN_oB}{I_{th}}\right)} \tag{5.9}$$

CDF $F_{U_{2i}}(z \mid h_{SP})$ can be simplified as

$$F_{U_{2i}}(z \mid h_{SP}) = 1 - \beta \tag{5.10}$$

where $$\beta = \left[e^{\frac{-zN_oB}{P}} + \frac{e^{-\frac{1}{P}(zN_oB+I_{th})}}{\left(1+\frac{zN_oB}{I_{th}}\right)} - e^{-\frac{1}{P}(zN_oB+I_{th})}\right] \tag{5.11}$$

The CDF of U_{1i} is

$$F_{U_{1i}}(z \mid h_{SP}) = \Pr\left\{\min\left(\frac{I_{th}}{h_{SP}}, P\right)\frac{h_{SR_i}}{N_oB} \leq z \mid h_{SP}\right\} \tag{5.12}$$

The CDF in (5.12) can be simplified as

$$F_{U_{1i}}(z \mid h_{SP}) = \begin{cases} \Pr\left\{\frac{I_{th}}{h_{SP}}\frac{h_{SR_i}}{N_oB} \leq z\right\} & \text{if } \frac{I_{th}}{h_{SP}} \leq P \\ \Pr\left\{P\frac{h_{SR_i}}{N_oB} \leq z\right\} & \text{if } \frac{I_{th}}{h_{SP}} > P. \end{cases}$$

i.e.,

$$F_{U_{1i}}(z \mid h_{SP}) = \begin{cases} \Pr\left\{h_{SR_i} \leq \frac{zN_oB\alpha}{I_{th}}\right\} & \text{if } h_{SP} \geq \frac{I_{th}}{P} \\ \Pr\left\{h_{SR_i} \leq \frac{zN_oB}{P}\right\} & \text{if } h_{SP} < \frac{I_{th}}{P}. \end{cases} \tag{5.13}$$

Substituting (5.13) and (5.10) in (5.5), we get

$$\begin{aligned} F_{Z_i}(z \mid h_{SP}) &= 1 - (1-(1-\beta))(1 - F_{U_{1i}}(z \mid h_{SP})) \\ &= 1 - \beta(1 - F_{U_{1i}}(z \mid h_{SP})) \end{aligned} \tag{5.14}$$

For K number of relays in i.i.d Rayleigh fading

$$\begin{aligned} F_Z(z \mid h_{SP}) &= \Pr\left\{\max(Z_1, Z_2, ..., Z_i, ..., Z_K) \leq z \mid h_{SP}\right\} \\ &= \left[F_{Z_i}(z \mid h_{SP})\right]^K \\ &= \sum_{l=0}^{K} \binom{K}{l} (-1)^l \beta^l \big(1 - F_{U_{1i}}(z \mid h_{SP})\big)^l \end{aligned} \tag{5.15}$$

Binomial expansion is used to get (5.15). Now we can find $F_Z(z)$ as

$$\begin{aligned} F_Z(z) &= \int_{\alpha=0}^{\infty} F_Z(z \mid h_{SP})\, p_{h_{SP}}(\alpha)\, d\alpha \\ &= \int_{\alpha=0}^{\infty} \sum_{l=0}^{K} \binom{K}{l} (-1)^l \beta^l \big(1 - F_{U_{1i}}(z \mid h_{SP})\big)^l p_{h_{SP}}(\alpha)\, d\alpha \\ &= \sum_{l=0}^{K} \binom{K}{l} (-1)^l \beta^l \Bigg[\int_{\alpha=0}^{\frac{I_{th}}{P}} \big(1 - F_{U_{1i}}(z \mid h_{SP})\big)^l p_{h_{SP}}(\alpha)\, d\alpha + \\ &\int_{\alpha=\frac{I_{th}}{P}}^{\infty} \big(1 - F_{U_{1i}}(z \mid h_{SP})\big)^l p_{h_{SP}}(\alpha)\Bigg] \end{aligned} \tag{5.16}$$

Here, the term β is independent of α so it is moved out of integration. Now substituting (5.13) in (5.16), we can further simplify $F_Z(z)$ as

$$F_Z(z) = \sum_{l=0}^{K} \binom{K}{l} (-1)^l \beta^l \Bigg[\int_{0}^{\frac{I_{th}}{P}} \Big(1 - \int_{0}^{\frac{zN_oB}{P}} e^{-x}dx\Big)^l e^{-\alpha}d\alpha + \int_{\frac{I_{th}}{P}}^{\infty} \Big(1 - \int_{0}^{\frac{zN_oB\alpha}{I_{th}}} e^{-x}dx\Big)^l e^{-\alpha}d\alpha \Bigg] \tag{5.17}$$

Solving integrals and simplifying we get

$$F_Z(z) = \sum_{l=0}^{K} \binom{K}{l} (-1)^l \beta^l \delta \tag{5.18}$$

where

$$\delta = e^{-\frac{zN_oBl}{P}}\big(1 - e^{-\frac{I_{th}}{P}}\big) + \frac{1}{(1 + \frac{zN_oBl}{I_{th}})} e^{\frac{-I_{th}}{P}} \tag{5.19}$$

$$\beta = \left[e^{\frac{-zN_oB}{P}} + \frac{e^{-\frac{1}{P}(zN_oB + I_{th})}}{\big(1 + \frac{zN_oB}{I_{th}}\big)} - e^{-\frac{1}{P}(zN_oB + I_{th})} \right]$$

From (5.18), we can get PDF $p_Z(z)$ by differentiating CDF $F_Z(z)$ i.e.,

$$p_Z(z) = \frac{d}{dx}F_Z(z) = \sum_{l=1}^{\infty} \binom{K}{l} (-1)^l \Big[l\beta^{l-1}\mu\delta + \beta^l \nu \Big] \tag{5.20}$$

where

$$\mu = \frac{d}{dx}\beta = -\frac{N_oB}{P}e^{-\frac{zN_oB}{P}} - \frac{N_oB}{P}\frac{e^{-\frac{1}{P}(zN_oB + I_{th})}}{(1 + \frac{zN_oB}{I_{th}})} - \frac{N_oB}{I_{th}}\frac{e^{-\frac{1}{P}(zN_oB + I_{th})}}{(1 + \frac{zN_oB}{I_{th}})^2} + \frac{N_oB}{P}e^{-\frac{1}{P}(zN_oB + I_{th})} \tag{5.21}$$

and

$$\nu = \frac{d}{dx}\delta = -\frac{N_oBl}{P}e^{-\frac{zN_oBl}{P}}\big(1 - e^{-\frac{I_{th}}{P}}\big) - \frac{N_oBl}{I_{th}}e^{-\frac{I_{th}}{P}}\frac{1}{(1 + \frac{zN_oBl}{I_{th}})^2} \tag{5.22}$$

Now effective capacity in integral form can be given as

$$\begin{aligned} E_c(\theta) &= -\frac{1}{\theta}\ln\left[\int_0^\infty (1+z)^{-\alpha} p_Z(z)\,dz\right] \\ &= -\frac{1}{\theta}\int_0^\infty (1+z)^{-\alpha} \sum_{l=1}^{\infty} \binom{K}{l} (-1)^l \Big[l\beta^{l-1}\mu\delta + \beta^l \nu \Big] \end{aligned} \tag{5.23}$$

To find effective capacity in closed form, let us take partial fraction of $F_Z(z)$ and then differentiate w.r.t z. β^l can be expanded in binomial form as

$$\beta^l = \left[e^{\frac{-zN_oB}{P}} + \frac{e^{-\frac{1}{P}(zN_oB+I_{th})}}{\left(1+\frac{zN_oB}{I_{th}}\right)} - e^{-\frac{1}{P}(zN_oB+I_{th})}\right]^l$$
$$= e^{\frac{-zN_oBl}{P}} \sum_{m=0}^{l}\sum_{n=0}^{m}\binom{l}{m}\binom{m}{n}(-1)^{n+m}e^{-\frac{I_{th}m}{P}}\frac{1}{(1+\frac{zN_oB}{I_{th}})^n} \tag{5.24}$$

Now we can find $\beta^l \times \delta$ as

$$\beta^l \times \delta = R_1 + R_2 \tag{5.25}$$

where

$$R_1 \doteq \beta^l \times e^{-\frac{zN_oBl}{P}}\left(1-e^{-\frac{I_{th}}{P}}\right) \tag{5.26}$$
$$R_2 \doteq \beta^l \times \frac{1}{(1+\frac{zN_oBl}{I_{th}})}e^{-\frac{I_{th}}{P}} \tag{5.27}$$

To get PDF $p_Z(z)$ differentiate $R_1 + R_2$ w.r.t z. Effective capacity can be written as

$$E_c(\theta) = -\frac{1}{\theta}\ln\left[\int_0^\infty \left(1+z\right)^{-\alpha}p_Z(z)\,dz\right]$$
$$= -\frac{1}{\theta}\ln\left[\int_0^\infty \left(1+z\right)^{-\alpha}\sum_{l=1}^{K}\binom{K}{l}(-1)^l\left[\frac{dR_1}{dz}+\frac{dR_2}{dz}\right]\right]$$
$$= -\frac{1}{\theta}\ln\left[O_1 + O_2\right] \tag{5.28}$$

where

$$\frac{dR_1}{dz} = (1-e^{-\frac{I_{th}}{P}})\sum_{m=0}^{l}\sum_{n=0}^{m}\binom{l}{m}\binom{m}{n}(-1)^{n+m}e^{-\frac{I_{th}m}{P}}\times$$
$$\left[-\frac{2N_oBl}{P}\frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^n} - n\frac{NoB}{I_{th}}\frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n+1}}\right] \tag{5.29}$$

$$\frac{dR_2}{dz} = e^{-\frac{I_{th}}{P}}\sum_{m=0}^{l}\sum_{n=0}^{m}\binom{l}{m}\binom{m}{n}(-1)^{n+m}e^{-\frac{I_{th}m}{P}}\times$$
$$\left[-\frac{N_oBl}{P}\frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^{n+1}} - (n+1)\frac{NoBl}{I_{th}}\frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^{n+2}}\right] \tag{5.30}$$

$$O_1 = \int_0^\infty (1+z)^{-\alpha} \sum_{l=1}^{K} \binom{K}{l} (-1)^l \left[\frac{dR_1}{dz}\right] \tag{5.31}$$

$$O_2 = \int_0^\infty (1+z)^{-\alpha} \sum_{l=1}^{K} \binom{K}{l} (-1)^l \left[\frac{dR_2}{dz}\right] \tag{5.32}$$

Substituting (5.29) and (5.30) back into (5.28) and segregating z terms together we have

$$O_1 = \sum_{l=1}^{K} \sum_{m=0}^{l} \sum_{n=0}^{m} \binom{K}{l} \binom{l}{m} \binom{m}{n} (-1)^{l+n+m+1} (1 - e^{-\frac{I_{th}}{P}}) e^{-\frac{I_{th} m}{P}} \left[O_{11} + O_{12}\right] \tag{5.33}$$

where

$$O_{11} = \frac{2N_o Bl}{P} \int_0^\infty (1+z)^{-\alpha} \frac{e^{\frac{-z 2 N_o Bl}{P}}}{(1 + \frac{z N_o B}{I_{th}})^n} dz \tag{5.34}$$

$$O_{12} = n \frac{NoB}{I_{th}} \int_0^\infty (1+z)^{-\alpha} \frac{e^{\frac{-z 2 N_o Bl}{P}}}{(1 + \frac{z N_o B}{I_{th}})^{n+1}} dz \tag{5.35}$$

And

$$O_2 = \sum_{l=1}^{K} \sum_{m=0}^{l} \sum_{n=0}^{m} \binom{K}{l} \binom{l}{m} \binom{m}{n} (-1)^{l+n+m+1} e^{-\frac{I_{th}}{P}} e^{-\frac{I_{th} m}{P}} \left[O_{21} + O_{22}\right] \tag{5.36}$$

where

$$O_{21} = \frac{N_o Bl}{P} \int_0^\infty (1+z)^{-\alpha} \frac{e^{-\frac{z N_o Bl}{P}}}{(1 + \frac{z N_o Bl}{I_{th}})^{n+1}} dz \tag{5.37}$$

$$O_{22} = (n+1) \frac{NoBl}{I_{th}} \int_0^\infty (1+z)^{-\alpha} \frac{e^{-\frac{z N_o Bl}{P}}}{(1 + \frac{z N_o Bl}{I_{th}})^{n+2}} dz \tag{5.38}$$

Now O_{11}, O_{12}, O_{21}, O_{22} can be solved by expressing them in Meijer's G function and then change to Fox H function. Define

$$\mathcal{J} \doteq \int_0^\infty x^\eta (d_1 x + 1)^{-\nu_1} (d_2 x + 1)^{-\nu_2} e^{-\mu x} dx \tag{5.39}$$

To evaluate integral $\mathcal{J}$, first express $(d_1x+1)^{-\nu_1}$ and $(d_2x+1)^{-\nu_2}$ in terms of Meijer's G-function by using [36, eq (8.3.2.21)], and then change to Fox H-function with the help of the identity [36, Eq. (8.4.2.5)]. Expressing $\big(1+z\big)^{-\alpha}$ in terms of Meijer's G-function and then Fox H-function as

$$\begin{aligned}(1+z)^{-\alpha} &= \frac{1}{\Gamma(\alpha)} G_{1,1}^{1,1}\left(z \,\middle|\, \begin{matrix}(1-\alpha)\\(0)\end{matrix}\right)\\ &= \frac{1}{\Gamma(\alpha)} H_{1,1}^{1,1}\left[z \,\middle|\, \begin{matrix}(1-\alpha,1)\\(0,1)\end{matrix}\right]\end{aligned} \tag{5.40}$$

Similarly, expressing $(1+\frac{zN_oB}{I_{th}})^{-n}$ in terms of Meijer's G-function and then Fox H-function as

$$\begin{aligned}\left(1+\frac{zN_oB}{I_{th}}\right)^{-n} &= \frac{1}{\Gamma(n)} G_{1,1}^{1,1}\left(\frac{zN_oB}{I_{th}} \,\middle|\, \begin{matrix}(1-n)\\(0)\end{matrix}\right)\\ &= \frac{1}{\Gamma(n)} H_{1,1}^{1,1}\left[\frac{zN_oB}{I_{th}} \,\middle|\, \begin{matrix}(1-n,1)\\(0,1)\end{matrix}\right]\end{aligned} \tag{5.41}$$

And, expressing $(1+\frac{zN_oB}{I_{th}})^{-(n+1)}$

$$\begin{aligned}\left(1+\frac{zN_oB}{I_{th}}\right)^{-(n+1)} &= \frac{1}{\Gamma(n+1)} G_{1,1}^{1,1}\left(\frac{zN_oB}{I_{th}} \,\middle|\, \begin{matrix}(-n)\\(0)\end{matrix}\right)\\ &= \frac{1}{\Gamma(n+1)} H_{1,1}^{1,1}\left[\frac{zN_oB}{I_{th}} \,\middle|\, \begin{matrix}(-n,1)\\(0,1)\end{matrix}\right]\end{aligned} \tag{5.42}$$

And, expressing $(1+\frac{zN_oBl}{I_{th}})^{-(n+1)}$

$$\begin{aligned}\left(1+\frac{zN_oBl}{I_{th}}\right)^{-(n+1)} &= \frac{1}{\Gamma(n+1)} G_{1,1}^{1,1}\left(\frac{zN_oBl}{I_{th}} \,\middle|\, \begin{matrix}(-n)\\(0)\end{matrix}\right)\\ &= \frac{1}{\Gamma(n+1)} H_{1,1}^{1,1}\left[\frac{zN_oBl}{I_{th}} \,\middle|\, \begin{matrix}(-n,1)\\(0,1)\end{matrix}\right]\end{aligned} \tag{5.43}$$

Expressing $(1+\frac{zN_oBl}{I_{th}})^{-(n+2)}$ as

$$\begin{aligned}\left(1+\frac{zN_oBl}{I_{th}}\right)^{-(n+2)} &= \frac{1}{\Gamma(n+2)} G_{1,1}^{1,1}\left(\frac{zN_oBl}{I_{th}} \,\middle|\, \begin{matrix}(-n-1)\\(0)\end{matrix}\right)\\ &= \frac{1}{\Gamma(n+2)} H_{1,1}^{1,1}\left[\frac{zN_oBl}{I_{th}} \,\middle|\, \begin{matrix}(-n-1,1)\\(0,1)\end{matrix}\right]\end{aligned} \tag{5.44}$$

Now, O_{11} can be solved by using [37, Eq. (2.6.2)]

$$O_{11} = \frac{1}{\Gamma(\alpha)\Gamma(n)} H_{1,(1:1),0,(1:1)}^{1,1,1,1,1}\left[\begin{matrix}\frac{P}{2N_oBl}\\ \frac{P}{2I_{th}l}\end{matrix} \,\middle|\, \begin{matrix}(1,1)\\(1-\alpha,1)\,;(1-n,1)\\ \text{---}\\(0,1)\,;(0,1)\end{matrix}\right] \tag{5.45}$$

Similarly, O_{12} can be written as

$$O_{12} = \frac{nP}{I_{th}l} \times \frac{1}{\Gamma(\alpha)\Gamma(n)} H^{1,1,1,1,1}_{1,(1:1),0,(1:1)} \left[\begin{array}{c|c} \frac{P}{2N_oBl} & (1,1) \\ & (1-\alpha,1)\,;(-n,1) \\ \frac{P}{I_{th}l} & \overline{} \\ & (0,1)\,;(0,1) \end{array} \right] \tag{5.46}$$

And, O_{21}, O_{22} can be written as

$$O_{21} = \frac{1}{\Gamma(\alpha)\Gamma(n+1)} H^{1,1,1,1,1}_{1,(1:1),0,(1:1)} \left[\begin{array}{c|c} \frac{P}{N_oBl} & (1,1) \\ & (1-\alpha,1)\,;(-n,1) \\ \frac{P}{I_{th}} & \overline{} \\ & (0,1)\,;(0,1) \end{array} \right] \tag{5.47}$$

$$O_{22} = \frac{(n+1)P}{I_{th}} \times \frac{1}{\Gamma(\alpha)\Gamma(n+2)} H^{1,1,1,1,1}_{1,(1:1),0,(1:1)} \left[\begin{array}{c|c} \frac{P}{N_oBl} & (1,1) \\ & (1-\alpha,1)\,;(-n-1,1) \\ \frac{P}{I_{th}} & \overline{} \\ & (0,1)\,;(0,1) \end{array} \right] \tag{5.48}$$

By plugging (5.33) and (5.36) with (5.45), (5.46), (5.47) and (5.48) in (5.28) gives closed form expression for effective capacity under interference and transmit power constraints.

5.3 Numerical results

Simulation results obtained for effective capacity for interference power and transmit power constraints matched with analytical results. One can observe that effective capacity increases with interference threshold as long as threshold is less than transmitter power. Here effective capacity without transmit power constraints is also provided to give the insight about the effect of transmit power constraints on capacity.

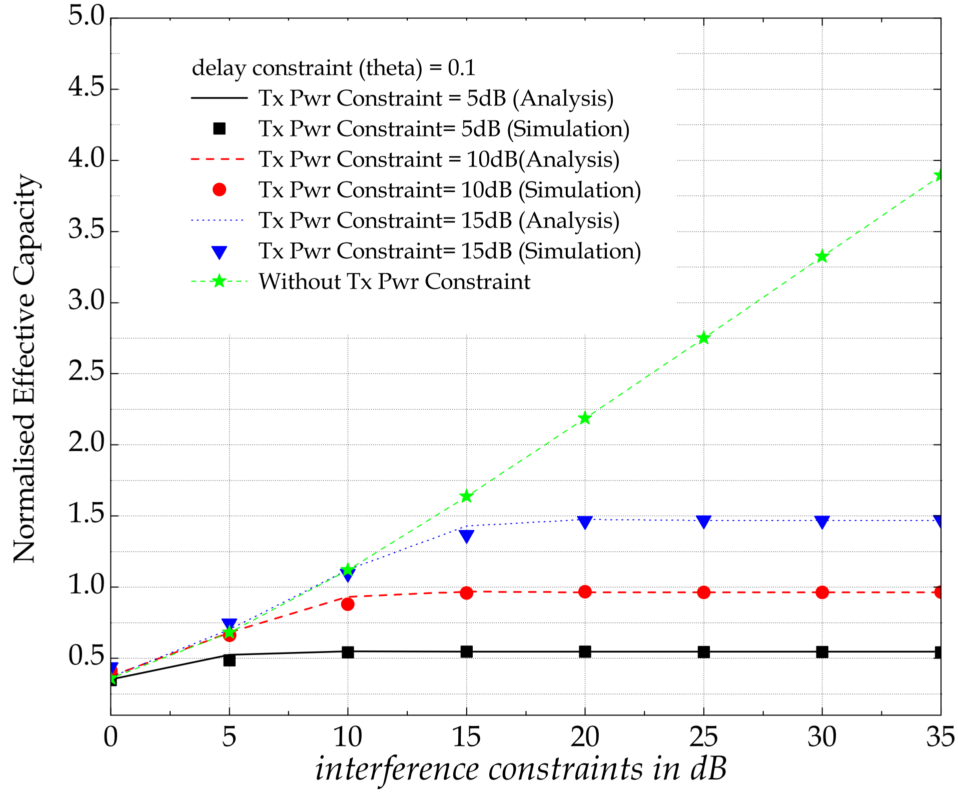

Figure 5.1: Normalised effective capacity vs. interference threshold in dB.

Chapter 6

Ergodic capacity and outage probability of cognitive radio relay networks under interference and transmission power constraints

In Chapter 5, effective capacity for interference power and transmit power constraints is explained. Also in Chapter 4, outage probability and ergodic capacity for interference power constraint are described. In this chapter, both concepts are combined i.e., outage probability and ergodic capacity under interference and transmit power constraints are studied.

6.1 Ergodic capacity analysis

In this section, ergodic capacity for Rayleigh fading environment with interference power and transmit power constraints is considered. From (4.3), ergodic capacity is

$$C_{er} = \frac{1}{2}\int_{T} \ln(1+z)p_T\left(z\right)dz \tag{6.1}$$

where

$$T = \min\left[\min\left(\frac{I_{th}}{h_{R_iP}}, P\right)\frac{h_{R_iD}}{N_oB}, \min\left(\frac{I_{th}}{h_{SP}}, P\right)\frac{h_{SR_i}}{N_oB}\right] \tag{6.2}$$

But from (5.20) and (5.21), $p_T\left(z\right)$ is

$$p_T\left(z\right) = \frac{d}{dz}F_T\left(z\right) = \sum_{l=1}^{\infty}\binom{K}{l}(-1)^l\left[l\beta^{l-1}\mu\delta + \beta^l\nu\right] \tag{6.3}$$

where

$$\beta = \left[e^{-\frac{zN_oB}{P}} + \frac{e^{-\frac{1}{P}(zN_oB+I_{th})}}{\left(1+\frac{zN_oB}{I_{th}}\right)} - e^{-\frac{1}{P}(zN_oB+I_{th})} \right] \tag{6.4}$$

$$\delta = e^{-\frac{zN_oBl}{P}}\left(1-e^{-\frac{I_{th}}{P}}\right) + \frac{1}{\left(1+\frac{zN_oBl}{I_{th}}\right)} e^{-\frac{I_{th}}{P}} \tag{6.5}$$

$$\mu = \frac{d}{dx}\beta = -\frac{N_oB}{P}e^{-\frac{zN_oB}{P}} - \frac{N_oB}{P}\frac{e^{-\frac{1}{P}(zN_oB+I_{th})}}{(1+\frac{zN_oB}{I_{th}})} - \frac{N_oB}{I_{th}}\frac{e^{-\frac{1}{P}(zN_oB+I_{th})}}{(1+\frac{zN_oB}{I_{th}})^2} + \frac{N_oB}{P}e^{-\frac{1}{P}(zN_oB+I_{th})} \tag{6.6}$$

$$\nu = \frac{d}{dx}\delta = -\frac{N_oBl}{P}e^{-\frac{zN_oBl}{P}}\left(1-e^{-\frac{I_{th}}{P}}\right) - \frac{N_oBl}{I_{th}}e^{-\frac{I_{th}}{P}}\frac{1}{(1+\frac{zN_oBl}{I_{th}})^2} \tag{6.7}$$

Substituting (6.3) and (6.4) in (6.1) we have

$$C_{er} = \frac{1}{2}\int_0^{\infty} \ln(1+z) \sum_{l=1}^{\infty} \binom{K}{l} (-1)^l \left[l\beta^{l-1}\mu\delta + \beta^l \nu \right] \tag{6.8}$$

This (6.8) is integral form for ergodic capacity under interference and transmit power constraints. To obtain closed form expression for ergodic capacity, equations (5.24) to (5.28) can be used.

$$C_{er} = \frac{1}{2}\left[O_1 + O_2\right] \tag{6.9}$$

where

$$O_1 = \sum_{l=1}^{K}\sum_{m=0}^{l}\sum_{n=0}^{m} \binom{K}{l}\binom{l}{m}\binom{m}{n} (-1)^{l+n+m+1} (1-e^{-\frac{I_{th}}{P}}) e^{-\frac{I_{th}m}{P}} \left[O_{11} + O_{12}\right] \tag{6.10}$$

$$O_2 = \sum_{l=1}^{K}\sum_{m=0}^{l}\sum_{n=0}^{m} \binom{K}{l}\binom{l}{m}\binom{m}{n} (-1)^{l+n+m+1} e^{-\frac{I_{th}}{P}} e^{-\frac{I_{th}m}{P}} \left[O_{21} + O_{22}\right] \tag{6.11}$$

Here O_{11}, O_{12}, O_{21} and O_{22} are given by

$$O_{11} = \frac{2N_oBl}{P}\int_0^{\infty} \ln(1+z) \frac{e^{\frac{-z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^n} dz \tag{6.12}$$

$$O_{12} = n\frac{NoB}{I_{th}}\int_0^{\infty} \ln(1+z) \frac{e^{\frac{-z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n+1}} dz \tag{6.13}$$

$$O_{21} = \frac{N_oBl}{P}\int_0^\infty \ln(1+z)\frac{e^{\frac{-zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^{n+1}}dz \tag{6.14}$$

$$O_{22} = (n+1)\frac{NoBl}{I_{th}}\int_0^\infty \ln(1+z)\frac{e^{\frac{-zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^{n+2}}dz \tag{6.15}$$

Now expressing, $\ln(1+z)$ in Meijer's G function and then change to Fox H function.

$$\begin{aligned}(\ln(1+z))^{-\alpha} &= G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}(1,1)\\(1,0)\end{matrix}\right)\\ &= H_{2,2}^{2,2}\left[z\,\middle|\,\begin{matrix}(1,1,1)\\(1,0,1)\end{matrix}\right]\end{aligned} \tag{6.16}$$

Fox H functions for $(1+\frac{zN_oB}{I_{th}})^{-n}$, $(1+\frac{zN_oB}{I_{th}})^{-(n+1)}$, $(1+\frac{zN_oBl}{I_{th}})^{-(n+1)}$ and $(1+\frac{zN_oBl}{I_{th}})^{-(n+2)}$ can be obtained from, (5.41), (5.42), (5.43) and (5.44) respectively. Now O_{11} as

$$O_{11} = \frac{1}{\Gamma(n)}H_{1,(2:1),0,(2:1)}^{1,2,1,2,1}\left[\begin{matrix}\frac{P}{2N_oBl}\\ \frac{P}{2I_{th}l}\end{matrix}\,\middle|\,\begin{matrix}(1,1)\\(1,1,1);(1-n,1)\\ \text{——-}\\(1,0,1);(0,1)\end{matrix}\right] \tag{6.17}$$

Similarly, O_{12} as

$$O_{12} = \frac{nP}{2I_{th}l}\frac{1}{\Gamma(n+1)}H_{1,(2:1),0,(2:1)}^{1,2,1,2,1}\left[\begin{matrix}\frac{P}{2N_oBl}\\ \frac{P}{2I_{th}l}\end{matrix}\,\middle|\,\begin{matrix}(1,1)\\(1,1,1);(-n,1)\\ \text{——-}\\(1,0,1);(0,1)\end{matrix}\right] \tag{6.18}$$

And O_{21}, O_{22} as

$$O_{21} = \frac{1}{\Gamma(n+1)}H_{1,(2:1),0,(2:1)}^{1,2,1,2,1}\left[\begin{matrix}\frac{P}{N_oBl}\\ \frac{P}{I_{th}}\end{matrix}\,\middle|\,\begin{matrix}(1,1)\\(1,1,1);(-n,1)\\ \text{——-}\\(1,0,1);(0,1)\end{matrix}\right] \tag{6.19}$$

$$O_{22} = \frac{(n+1)P}{I_{th}}\frac{1}{\Gamma(n+2)}H_{1,(2:1),0,(2:1)}^{1,2,1,2,1}\left[\begin{matrix}\frac{P}{2N_oBl}\\ \frac{P}{2I_{th}}\end{matrix}\,\middle|\,\begin{matrix}(1,1)\\(1,1,1);(-n-1,1)\\ \text{——-}\\(1,0,1);(0,1)\end{matrix}\right] \tag{6.20}$$

By plugging (6.10) in (6.9) with (6.17), (6.18), (6.19) and (6.20) gives closed form expression for ergodic capacity under interference and transmit power constraints.

6.2 Outage probability analysis

Here we derive outage probability under interference and transmit power constraints. Outage probability under interference constraints is discussed in 4. Outage $P_{out,CRRN}$ can be given by (4.17).

$$\begin{aligned} P_{out,CRRN} &= \Pr\{R_n < R_{min}\} \\ &= \Pr\left\{z < \frac{N_oB}{I_{th}}(e^{2R_{min}} - 1)\right\} \end{aligned} \tag{6.21}$$

Let $(e^{2R_{min}} - 1) = \zeta$, then (6.21) as

$$P_{out,CRRN} = \Pr\left\{z < \frac{N_oB}{I_{th}}\zeta\right\} = \int_0^{\frac{N_oB\zeta}{I_{th}}} p_Z(z)\,dz \tag{6.22}$$

But from (6.3) and (6.4)

$$P_{out,CRRN} = \int_0^{\frac{N_oB\zeta}{I_{th}}} \sum_{l=1}^{\infty} \binom{K}{l} (-1)^l \left[l\beta^{l-1}\mu\delta + \beta^l\nu\right] \tag{6.23}$$

This gives outage probability under interference and transmission power constraints in integral form. To get closed form, we use outage probability definition as in (6.22)

$$P_{out,CRRN} = \int_0^{\frac{N_oB\zeta}{I_{th}}} p_Z(z)\,dz = \int_0^{\frac{N_oB\zeta}{I_{th}}} \sum_{l=1}^{K} \binom{K}{l} (-1)^l \left[\frac{dR_1}{dz} + \frac{dR_2}{dz}\right] dz \tag{6.24}$$

where

$$\begin{aligned} \frac{dR_1}{dz} &= (1 - e^{-\frac{I_{th}}{P}}) \sum_{m=0}^{l}\sum_{n=0}^{m} \binom{l}{m}\binom{m}{n} (-1)^{n+m} e^{-\frac{I_{th}m}{P}} \times \\ &\left[-\frac{2N_oBl}{P}\frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^n} - n\frac{NoB}{I_{th}}\frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n+1}}\right] \end{aligned} \tag{6.25}$$

$$\frac{d}{dz}R_2 = \frac{d}{dz}R_{21} + \frac{d}{dz}R_{22} \tag{6.26}$$

$$\begin{aligned} \frac{d}{dz}R_{21} &= e^{-\frac{I_{th}}{P}} \sum_{m=0}^{l}\sum_{n=0}^{m}\sum_{s=1}^{n} \binom{l}{m}\binom{m}{n} (-1)^{n+m+s} e^{-\frac{I_{th}m}{P}} \frac{\left(\frac{NoB}{I_{th}}\right) * \left(\frac{NoBl}{I_{th}}\right)^{s-1}}{\left(\frac{NoB}{I_{th}} - \frac{NoBl}{I_{th}}\right)^s} \times \\ &\left[-\frac{N_oBl}{P}\frac{e^{\frac{-zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+1}} + \frac{N_oB(n-s+1)}{I_{th}}\frac{e^{\frac{-zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+2}}\right] \end{aligned} \tag{6.27}$$

$$\frac{d}{dz}R_{22} = e^{-\frac{I_{th}}{P}} \sum_{m=0}^{l}\sum_{n=0}^{m} \binom{l}{m}\binom{m}{n}(-1)^{m+1} e^{-\frac{I_{th}m}{P}} \frac{(\frac{NoBl}{I_{th}})^n}{\left(\frac{NoB}{I_{th}} - \frac{NoBl}{I_{th}}\right)^n} \times \left[\frac{N_oBl}{P}\frac{e^{\frac{-zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})} + \frac{N_oBl}{I_{th}}\frac{e^{\frac{-zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^2}\right] \tag{6.28}$$

Now, from (6.24)

$$P_{out,CRRN} = O_1 + O_2 + O_3 \tag{6.29}$$

where

$$O_1 = \int_0^\beta \binom{K}{l}(-1)^l \frac{d}{dz}R_1 \tag{6.30}$$

$$O_2 = \int_0^\beta \binom{K}{l}(-1)^l \frac{d}{dz}R_{21} \tag{6.31}$$

$$O_3 = \int_0^\beta \binom{K}{l}(-1)^l \frac{d}{dz}R_{22} \tag{6.32}$$

Define

$$O_{11} = \int_0^\beta \frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^n} \quad \text{and} \quad O_{12} = \int_0^\beta \frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n+1}} \tag{6.33}$$

$$O_{21} = \int_0^\beta \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+1}} \quad \text{and} \quad O_{22} = \int_0^\beta \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+2}} \tag{6.34}$$

$$O_{31} = \int_0^\beta \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})} \quad \text{and} \quad O_{32} = \int_0^\beta \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^2} \tag{6.35}$$

By segregating z terms in O_1, O_2, O_3, we can get O_{11}, O_{12}, O_{21}, O_{22}. O_{11} and O_{12} are part of O_1, O_{21} and O_{22} are parts of O_2 ,O_{31} and O_{32} are part of O_3. Now we can solve (6.33) by using

$$O_{11} = V_1 - V_2 = \int_0^\infty \frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^n}dz - \int_\beta^\infty \frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^n}dz \tag{6.36}$$

Using [31, eq (3.353.1), eq (3.353.2)], O_{11} in (6.36) can be simplified as

$$V_1 \doteq \frac{1}{(n-1)!}\sum_{t=1}^{n-1}(t-1)!\Big(\frac{-2NoBl}{P}\Big)^{n-t-1}\Big(\frac{I_{th}}{N_oB}\Big)^{-t} - \frac{(\frac{-2NoBl}{P})^{n-1}}{(n-1)!}e^{-\frac{2I_{th}l}{P}}Ei(\frac{-2I_{th}l}{P}) \tag{6.37}$$

$$V_2 \doteq \frac{1}{(n-1)!}\sum_{t=1}^{n-1}\frac{(t-1)!}{(n-1)!}\frac{\Big(\frac{2NoBl}{P}\Big)^{n-t-1}}{\Big(\beta+\frac{I_{th}}{NoB}\Big)^{t}} - \frac{(\frac{-2NoBl}{P})^{n-1}}{(n-1)!}e^{-\frac{2I_{th}l}{P}}Ei\Big((\beta+\frac{I_{th}}{N_oB})\frac{-2NoBl}{P}\Big) \tag{6.38}$$

Also

$$O_{12} = U_1 - U_2 = \int_0^\infty \frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n+1}}dz - \int_\beta^\infty \frac{e^{-\frac{z2N_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n+1}}dz \tag{6.39}$$

By using [31, eq (3.353.1), eq (3.353.2)], O_{12} in (6.39) can be simplified as

$$U_1 \doteq \frac{1}{(n)!}\sum_{t=1}^{n}(t-1)!\Big(\frac{-2NoBl}{P}\Big)^{n-t}\Big(\frac{I_{th}}{N_oB}\Big)^{-t} - \frac{(\frac{-2NoBl}{P})^{n}}{(n)!}e^{-\frac{2I_{th}l}{P}}Ei(\frac{-2I_{th}l}{P}) \tag{6.40}$$

$$U_2 \doteq \frac{1}{(n)!}\sum_{t=1}^{n}\frac{(t-1)!}{(n)!}\frac{\Big(\frac{-2NoBl}{P}\Big)^{n-t}}{\Big(\beta+\frac{I_{th}}{NoB}\Big)^{t}} - \frac{(\frac{-2NoBl}{P})^{n}}{(n)!}e^{-\frac{2I_{th}l}{P}}Ei\Big((\beta+\frac{I_{th}}{N_oB})\frac{-2NoBl}{P}\Big) \tag{6.41}$$

Similarly

$$O_{21} = N_1 - N_2 = \int_0^\infty \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+1}}dz - \int_\beta^\infty \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+1}}dz \tag{6.42}$$

And it can be written in closed form as

$$\begin{aligned} N_1 &\doteq \frac{1}{(n-s)!}\sum_{t=1}^{n-s}(t-1)!\Big(\frac{-NoBl}{P}\Big)^{n-s-t}\Big(\frac{I_{th}}{N_oB}\Big)^{-t} \\ &- \frac{(\frac{-NoBl}{P})^{n-s}}{(n-s)!}e^{-\frac{I_{th}l}{P}}Ei(\frac{-I_{th}l}{P}) \end{aligned} \tag{6.43}$$

$$\begin{aligned} N_2 &\doteq \frac{1}{(n-s)!}\sum_{t=1}^{n-s}\frac{(t-1)!}{(n-s)!}\frac{\Big(\frac{-NoBl}{P}\Big)^{n-s-t}}{\Big(\beta+\frac{I_{th}}{NoB}\Big)^{t}} \\ &- \frac{(\frac{-NoBl}{P})^{n-s}}{(n-s)!}e^{-\frac{I_{th}l}{P}}Ei\Big((\beta+\frac{I_{th}}{N_oB})\frac{-NoBl}{P}\Big) \end{aligned} \tag{6.44}$$

$$O_{22} = M_1 - M_2 = \int_0^\infty \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+2}}dz - \int_\beta^\infty \frac{e^{-\frac{zN_oBl}{P}}}{(1+\frac{zN_oB}{I_{th}})^{n-s+2}}dz \tag{6.45}$$

where M_1 and M_2 are

$$\begin{aligned} M_1 &\doteq \frac{1}{(n-s+1)!}\sum_{t=1}^{n-s+1}(t-1)!\Big(\frac{-NoBl}{P}\Big)^{n-s-t+1}\Big(\frac{I_{th}}{N_oB}\Big)^{-t} \\ &- \frac{(\frac{-NoBl}{P})^{n-s+1}}{(n-s+1)!}e^{-\frac{I_{th}l}{P}}Ei(\frac{-I_{th}l}{P}) \end{aligned} \tag{6.46}$$

$$\begin{aligned} M_2 &\doteq \frac{1}{(n-s+1)!}\sum_{t=1}^{n-s+1}\frac{(t-1)!}{(n-s+1)!}\frac{\Big(\frac{-NoBl}{P}\Big)^{n-s-t+1}}{\Big(\beta+\frac{I_{th}}{NoB}\Big)^{t}} \\ &- \frac{(\frac{-NoBl}{P})^{n-s+1}}{(n-s+1)!}e^{-\frac{I_{th}l}{P}}Ei\Big((\beta+\frac{I_{th}}{N_oB})\frac{-NoBl}{P}\Big) \end{aligned} \tag{6.47}$$

We have

$$O_{32} = Q_1 - Q_2 = \int_0^\infty \frac{e^{\frac{zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^2}dz - \int_\beta^\infty \frac{e^{\frac{zN_oBl}{P}}}{(1+\frac{zN_oBl}{I_{th}})^2}dz \tag{6.48}$$

where

$$\begin{aligned} Q_1 &\doteq \frac{NoBl}{I_{th}} + \frac{NoBl}{P}e^{-\frac{I_{th}}{P}}Ei\Big[-\frac{I_{th}}{P}\Big] \\ Q_2 &\doteq \frac{NoBl}{I_{th}}e^{-\frac{NoBl\beta}{P}} + \frac{NoBl}{P}e^{-\frac{I_{th}}{P}}Ei\Big[-(\beta+\frac{I_{th}}{NoBl})\frac{NoBl}{P}\Big] \end{aligned} \tag{6.49}$$

O_{31} can be solved, by using [31, eq (3.352.1)] as

$$O_{31} = e^{-\frac{I_{th}}{P}} \left[Ei(-\frac{\beta NoBl}{P} - \frac{I_{th}}{P}) - Ei(-\frac{I_{th}}{P}) \right] \tag{6.50}$$

Now, outage probability in closed form can be obtained from (6.29) with (6.33), (6.36),(6.39), (6.42),(6.45), (6.46) and (6.48).

6.3 Numerical results

In this section, simulation results are obtained and compared with analytical results. Fig. 6.1 shows normalised ergodic capacity versus interference constraints. One can observe as interference power threshold allowed increases, ergodic capacity increases. But this holds good only when transmit power is more than interference threshold. So capacity is limited by the minimum of interference threshold and transmit power.

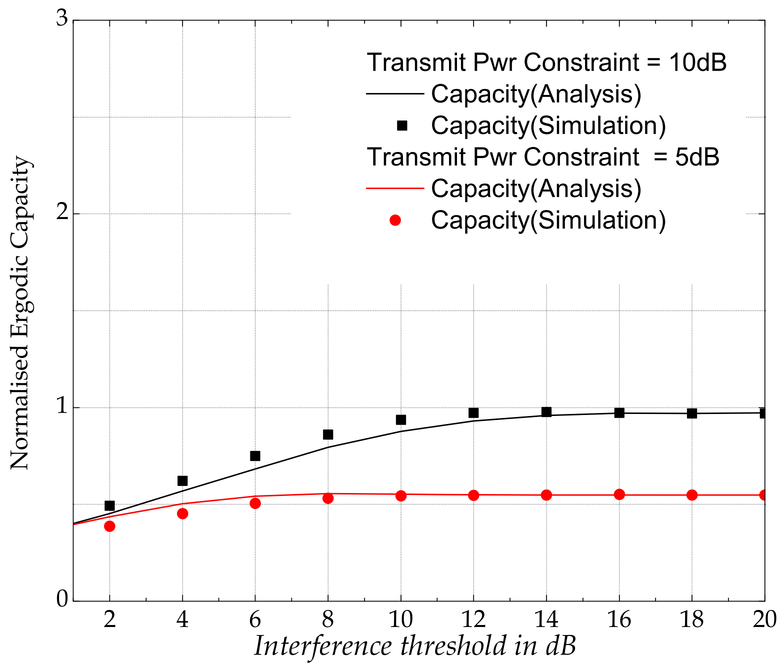

Figure 6.1: Ergodic capacity vs. interference constraints in dB.

Fig. 6.2 shows outage probability versus interference constraints. Here we find that as interference threshold allowed to SU increases, outage probability decreases i.e the system being in outage reduces. One can also observe that the outage probability is minimum of the transmit power and interference power. For simulation purposes $R_{min} = 0.2 bits/sec$ i.e., the system is in outage if the data rate is less than R_{min}.

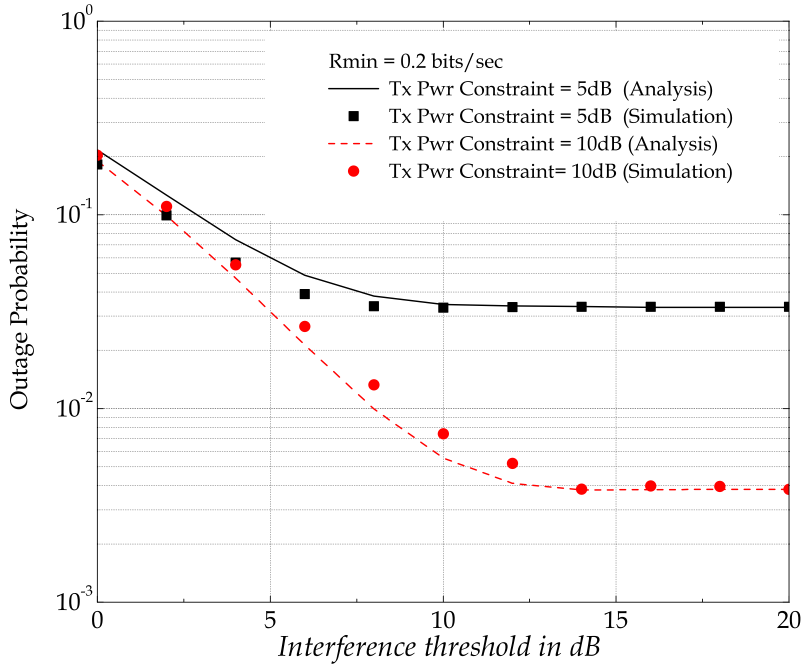

Figure 6.2: Outage probability vs. interference constraints in dB.

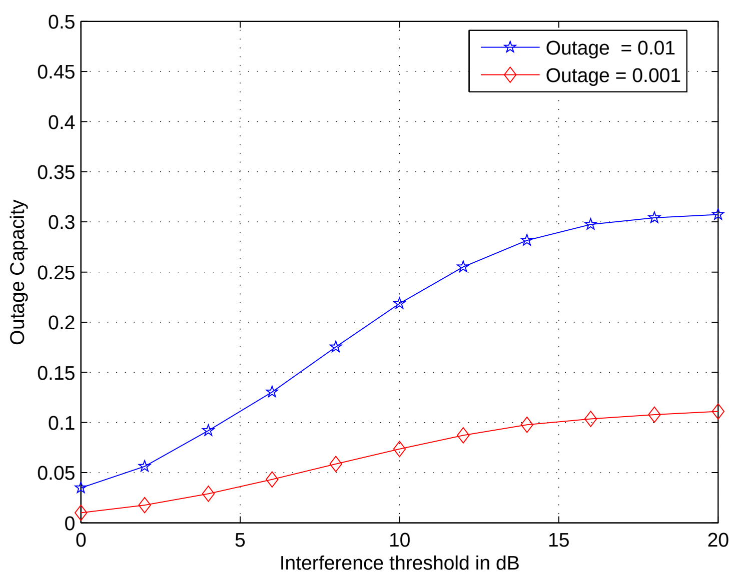

Figure 6.3: Outage capacity vs. interference constraints in dB.

One can obtain outage capacity, for given outage probability. In (6.23), substitute the allowed outage probability, R_{min} gives outage capacity. To calculate outage capacity, numerical analysis is carried out. Here if we compare outage capacity under interference constraints with outage capacity under interference and transmission constraints, one can observe outage capacity is limited by transmit power constraints.

Chapter 7

Conclusions

Cognitive radio technology is an efficient means to improve spectrum utilization and has gained lot of attention in these recent years. Cooperative relay network is a powerful approach to improve the reliability and throughput of wireless network. Effective capacity is a link layer channel model and it models channel as a function of delay constraint. Introduction to effective capacity and small description about CRRN is provided in Chapter 2.

In this thesis, analysis is carried out to find maximum data rate achievable in CRRN considering effective capacity as channel model. Effective capacity analysis under interference allowed by PU is carried out in chapter 3. The analysis is carried out considering that the ratio of channel gains is dependent on channel gain between secondary transmitter and primary receiver. From this analysis, it can be concluded that as interference threshold allowed increases, effective capacity achievable in the network increases. But when delay constraint is relaxed, an increase in interference threshold does not directly benefit capacity.

In chapter 4, ergodic capacity and outage probability for CRRN under interference constraint is analysed. Here, capacity is analysed without considering delay as constraint. The treatment of considering ratio of channel gain is dependent on channel gain between secondary transmitter and primary receiver is extended in this chapter too. From the analysis and simulation one can figure out ergodic capacity increases with an increase in threshold constraint. The number of relays also help in increasing the performance of the system. From outage probability, it can be seen the system being outage decreases with an increase in interference threshold in dB.

In chapter 5, effective capacity is found for CRRN under interference and secondary transmit power constraints. It can be observed that effective capacity increases with an increase in interference threshold as long as interference is less than transmit power constraints. There is no increase in effective capacity, once interference power constraint is more than the transmit power constraints. It can be concluded that effective capacity is influenced by the minimum of interference power and secondary transmit power.

In chapter 6, ergodic capacity and outage probability is analysed. It is found that the ergodic capacity is dependent on minimum of interference threshold and secondary transmit power constraints. Outage probability is also dependent on minimum of transmit power and interference threshold. Here one can observe that the slope of outage capacity when interference power constraint is more than transmit power, is less than outage capacity under interference constraints.

In future, the work can be extended considering imperfect channel state conditions. The work can also be expanded in selection of relay types. Dynamic relay selection is an emerging concept as the number of relays used will change according to channel behaviour. If channel is good, one can save using all relays which in turn saves time and energy. The future work can also be done in finding practical aspects of effective capacity like effect of modulation and coding gain in CRRNs.

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