A NOVEL MULTIPLE–REFERENCE, MULTIPLE–CHANNEL, NORMALIZED FILTERED–X LMS ALGORITHM FOR ACTIVE CONTROL OF PROPELLER–INDUCED NOISE IN AIRCRAFT CABINS

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1. INTRODUCTION

The dominating cabin noise in propeller aircraft consists essentially of strong tonal components at harmonics of the Blade Passage Frequency (BPF) of the propellers. In order to efficiently reduce such low frequency periodic noise, it is advisable to employ an Active Noise Control (ANC) system based on a feedforward controller [1, 2]. This paper presents a set of normalized complex Filtered–X Least–Mean–Square (FX LMS) algorithms [1, 2, 3]. By using different variants of normalization factors the convergence rate, the tracking performance and the steady-state noise attenuation can be improved. The algorithms presented are based either on a single normalization factor for the whole control system (global normalized FX LMS algorithm), or several individual normalization factors (reference–individual FX LMS algorithm or the novel actuator–individual FX LMS algorithm). The evaluation is performed on noise recorded during flight in the cabin of a Dornier 328 (a twin–engine propeller aircraft).

2. THE ALGORITHM

The multiple–reference multiple–channel controller is described with reference to a general case with $R$ reference signals, $H$ harmonics for each reference, $L$ loudspeakers and $M$ microphones. Define the diagonal reference signal matrix $X(n)$, the weight vector $w$ and the matrix $F$ by

$$X(n) = \text{diag} \left\{ x_{11}(n) \mathbf{I}_{1} \ x_{12}(n) \mathbf{I}_{1} \ \cdots \ x_{RH}(n) \mathbf{I}_{1} \right\}$$

$$w = \begin{pmatrix} w_{11}^H & w_{12}^H & \cdots & w_{RH}^H \end{pmatrix}^H, \quad F = \begin{pmatrix} F_{11} & F_{12} & \cdots & F_{RH} \end{pmatrix}$$

(1)

where $x_{rh}(n)$, $w_{rh}$ and $F_{rh}$, each associated with the $r$th reference and the $h$th harmonic, denote the complex scalar reference signal, the $L \times 1$ vector of complex loudspeaker weights and the $M \times L$ matrix of complex acoustic paths respectively. Matrix $\mathbf{I}$ is the $L \times L$ identity matrix and $(\cdot)^H$ denotes the conjugate–transpose. The real valued $M \times 1$ vector $e(n)$ of microphone signals $e_m(n)$ is given by

$$e(n) = d(n) + \Re \{FX(n)w\}$$

(2)
where the real $M \times 1$ vector $d(n)$ contains the primary noise at microphone $m$ and $\Re \{ \cdot \}$ denotes the real part. The objective is to adjust the filter weights so that the squared output signals from the microphones is minimized:

$$J_n = e^T(n)e(n).$$

(3)

The update equation for the weights written in compact matrix form is given by

$$w(n + 1) = w(n) - 2M \frac{\partial J_n}{\partial w} = w(n) - 2MX^H(n)F^He(n)$$

(4)

where $(\cdot)^*$ denotes the complex conjugate and $M$ is a matrix weighting factor of dimension $LRH \times LRH$. The correlation matrices that governs the convergence properties (in a simplified analysis) are given by

$$R = E\{X^H(n)F^HFX(n)\}$$

(5)

where $E\{ \cdot \}$ denotes the expectation operation. The matrix weight factor $M$ of the ordinary normalized LMS algorithm [5] may now be safely chosen as

$$M = \frac{\mu_0}{\text{trace } \{R\}}I = \frac{\mu_0}{\sum_{r=1}^R \sum_{h=1}^H \rho_{rh} \sum_{m=1}^M \sum_{l=1}^L |F_{ml}|^2}I$$

(6)

where $0 < \mu_0 < 1$ and $\rho_{rh} = E\{|x_{rh}(n)|^2\}$. In this case, all the adaptive weights $w_{rh}$ are updated with the same weight factor (global normalization). The weight factor depends on the power of all reference signals and the acoustic paths between all loudspeakers and microphones. If, on the other hand, all the reference signals are assumed to be mutually uncorrelated, the matrix $R$ is block–diagonal, and each diagonal block is given by

$$R_{rh} = \rho_{rh}F^H_{rh}F_{rh}.$$ 

(7)

Now, each reference signal is individually treated, and $M$ can be chosen as

$$M = \text{diag} \left\{ M_{11} M_{12} \cdots M_{RH} \right\}$$

(8)

where

$$M_{rh} = \frac{\mu_0}{\text{trace } \{R_{rh}\}}I = \frac{\mu_0}{\rho_{rh} \sum_{m=1}^M \sum_{l=1}^L |F_{ml}|^2}I.$$ 

(9)

The convergence factors are due to the power of one reference signal only, and on the acoustic paths between all loudspeakers and microphones, cf. (6) and (9).

The novel normalization variant presented is based on an actuator–individual normalization, i.e. each actuator (loudspeaker) has an individual normalization factor for a given reference signal. The matrix weighting factors are given by

$$M_{rh} = \mu_0 \text{diag } \{R_{rh}\}^{-1} = \text{diag} \left\{ \mu_{rhl} \mu_{rh2} \cdots \mu_{rhL} \right\}$$

(10)

where the weight factor for reference signal $x_{rh}(n)$ and loudspeaker $l$ is given by

$$\mu_{rhl} = \frac{\mu_0}{\rho_{rh} \sum_{m=1}^M |F_{ml}|^2}.$$ 

(11)
The significant difference between the reference–individual and actuator–individual normalization factors is that the latter factors depend on the acoustic paths between one loudspeaker and all microphones only, while the others depend on the acoustic paths between all loudspeakers and all microphones, cf. (9) and (11). The actuator–individual weight matrices can be seen as an approximation of the weight matrices of the more efficient Newton’s algorithm [5]:

\[ M_{rh} = \mu_0 R_{rh}^{-1}. \]  

(12)

The correlation matrix \( R_{rh} \) involves no correlation over time. The elements of the matrix reflect the cross–correlation of the reference signals transmitted through different acoustic paths as measured at the microphone positions. For loudspeakers which are spatially separated, the corresponding cross–correlation will be small at all the microphone positions. In applications, such as in aircraft, where the loudspeakers are spatially separated, the correlation matrix \( R_{rh} \) will be diagonally dominant, and may therefore be approximated by its diagonal. A comparison between (10) and (12) shows that (10) is a suitable approximation when the correlation matrix is diagonally dominant. The advantage of using (10) instead of (12) is that no matrix inversion needs to be done: scalar divisions according to (11) are sufficient.

3. EVALUATION

Throughout the evaluation, the normalized convergence factor \( \mu_0 \) was chosen as 1/10 of the value causing instability and the control algorithms were set up to attenuate the BPF up to 4×BPF of the left and right propellers, respectively.

In the steady cruise flight condition, the BPFs of the two propellers were stationary at 105 Hz. Figure 1a shows the normalized mean SPL versus time at BPF for the primary noise and the three algorithmic variants. Tab. 1 summarizes the narrowband mean reduction averaged over the 39 microphones for these same time samples (at approx. 8.5 seconds). The actuator–individual variant exhibited better performance than the other algorithm variants with respect to convergence rate and steady–state noise reduction.

In the non–stationary climb to cruise flight condition the BPFs varied from 110 Hz down to 106 Hz. Figure 1b shows the normalized mean SPL versus time for the primary noise and the different algorithms. The actuator–individual FX LMS algorithm exhibited the best tracking capability, with improved noise attenuation as a result.

4. CONCLUSIONS

The results show that the actuator–individual FX LMS algorithm works well when the correlation matrices are diagonally dominant. The noise reduction obtained is significant and the results indicate that the algorithm shows a good potential for achieving agreement with the optimum noise reduction (least squares solution). The actuator–individual FX LMS algorithm constitutes a good approximation of Newton’s algorithm. The evaluation results also show that the actuator–individual FX LMS algorithm has better tracking performance than the global normalized and reference–individual FX LMS algorithms.
5. ACKNOWLEDGMENTS

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![Figure 1](image)

Figure 1: The normalized mean SPL versus time at BPF: (a) cruise, (b) climb to cruise. Upper solid line: Primary noise. Middle solid line: global normalization. Dashed line: Reference–individual. Lower solid line: Actuator–individual.

<table>
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<th>Control–Algorithm/Method</th>
<th>BPF [dB]</th>
<th>2×BPF [dB]</th>
<th>3×BPF [dB]</th>
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<td>7.7</td>
<td>6.1</td>
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Table 1: The narrowband mean reduction of the primary noise in cruise flight.

References


