Simulation of Vehicle Response to Throttle Tip-in and Tip-out

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Abstract:
A theoretical model for optimization of the dynamical behavior of a vehicle drive train was suggested and equations of motions derived. A Simulink model was developed to solve the resulting system of equations. Experimental verification showed good agreement with theory.

Keywords:
Simulation, Vehicle, Dynamic, Experimental verification, Tip-in, Tip-out, Shunt and shuffle.
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1 Notation

\begin{itemize}
\item[c] Damping
\item[E] Kinetic energy
\item[F] Force
\item[J] Mass moment of inertia
\item[k] Stiffness
\item[m] Mass
\item[n] Gear ratio
\item[r] Radius
\item[T] Torque
\item[t] Time
\item[U] Potential energy
\item[x] Translational coordinate
\item[y] Translational coordinate
\item[z] Translational coordinate
\item[\theta] Angular displacement
\end{itemize}

Indices

\begin{itemize}
\item[a] Applied
\item[b] Car body
\item[c] Clutch
\item[d] Drive shafts
\item[e] Engine block
\item[f] Flywheel
\item[g] Transmission - gearbox
\item[h] Final drive
\end{itemize}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
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<td>(p)</td>
<td>Rotational movement</td>
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2 Introduction

The rapid development and increasing quality demands of today’s cars force the manufacturers to put time and effort even into what at a first look seems to be a small detail. This detail can in many cases be the reason why a person chooses to buy another car. That is why it is more and more important to predict the behavior of the vehicle and its components long before it will be ready for production.

The aim of this work is to develop a theoretical model that describes the response in a driveline of a front wheel driven car. The model will be used to study a phenomenon called shunt and shuffle. This occurs when driving the car on the lower gears in low speed and then rapidly changes the throttle engagement. This will cause a low frequent fore and aft motion of the car body, which is referred to as shunt and shuffle.

The shuffle is caused by the first dynamic mode of the driveline. It is a harmonic motion of the powertrain-driveline and is initiated by a “beat” or a shunt. Shunt and shuffle can be excited by throttle engagement, Tip-in, or disengagement, Tip-out. It can also occur when rapidly releasing the clutch or by road disturbances and are often described in terms of frequency, damping and first peak overshoot.

In this work, the possibility to describe this phenomenon with a relatively simple model is investigated. The model will then be used to study this phenomenon and particularly the car body and the engine block movement. The displacement directions of interest are the rotation about the driveshafts and the translation along the car. In figure 2.2 the coordinate system for the car is shown.

It is possible to assume a low-order model since shunt and shuffle is related to the first mode of the system. Although the model is to be simple, it still should show good correlation with experimental result.

The experimental investigation to which the model is verified has already been carried out at Volvo. The model will be used for a parameter study of the system.

The car that is studied in this report is the Volvo 850. This car has its engine mounted on a subframe, which is mounted to the car body. The subframe holds not only the engine, but also the complete driveline, which includes the clutch, driveshafts and steering device. The engine, gearbox,
driveshafts and wheels are commonly referred to as powertrain. This is shown in figure 2.1.

References [1], [3] and [4] have been used throughout the report and are not referred to at specific situations.

2.1 Limitations

The model does not consider the effect from tire slip. In order to consider tire slip, the model would need several more degrees of freedom. The vertical motion of the vehicle would have to be calculated to determine the normal forces acting on the wheels. The calculation of the vertical motion requires a complete wheel suspension description for all four wheels, which is beyond the scope of this thesis.

All rotational degrees of freedom of the car body are neglected because they are assumed to not affect the shunt and shuffle phenomenon.
The model does not consider the nonlinear behavior of the damping factors, which depend on rotational speed and frequency. This means that the damping factors in the model are taken from experimental results at a constant frequency that agree with the expected shuffle frequency.

The excitation signal is defined directly as a torque acting at the flywheel instead of using throttle movement as input to the system, which would also require a model of the engine combustion and conversion from translational piston force to crankshaft torque as well. In this case this torque is already known and does not need to be calculated.

The coordinate system referred to in the report is shown in figure 2.2.

Figure 2.2. The coordinate system.

It should be noted that the plots in the report have been scaled by factors in order to retain secrecy.
3 Experimental Investigation

3.1 Introduction

The vehicle behavior during shunt and shuffle is investigated by measuring a number of parameters, which are considered to be of importance for understanding of the phenomenon. The measured quantities of interest for this thesis are the displacement of the engine block and subframe, the torque at the crankshaft and wheels and the acceleration of the car body. It is also necessary to compare different gears and different rotational speeds to a number of altered throttle angles to understand in which region the appearance of shunt and shuffle is most critical.

The displacement of the engine is measured using three displacement gauges. One of them is placed under and two at the top of the engine block. Two gauges are placed at the front of the subframe and all five gauges measure its translational displacement along the x-axis. The translational acceleration along the x-axis of the car is measured in front of the driver’s chair using an accelerometer.

The wheel torque is measured between the brake discs and the rim using two other discs with a beam of known properties. At the beam a full bridge of strain gauges are attached to measure the torsion from which the torque can be calculated.

The measurements are performed with a sampling frequency of 1000 Hz to be able to study the behavior in the time domain with enough accuracy.

The test cycle starts during deceleration with no throttle engagement at a car speed of approximately 20 km/h. After a short time the throttle opens very rapidly to about 5-10° and is kept at that level for several seconds and then it is disengaged rapidly. The total test time is 7 seconds.
3.1.1 Engine Block Movement

The experimental result shows that the engine block simultaneously rotates about the y-axis and moves in x-direction. The translational movement in the x-direction of the engine block and the subframe is shown in figure 3.1. During acceleration the subframe moves forward which also introduces a force at the bottom of the engine block. This force makes the engine block move forward and because the force is acting below the center of gravity of the block it also introduces a torque that makes the block rotate. This torque is added to the total driving torque, also acting on the block.

![Movement, Subframe - Engineblock](image)

*Figure 3.1. Translational movement of subframe and engine block.*
3.1.2 Driveline Lash

Almost every part in the driveline has a small amount of lash. In order to develop a good theoretical model, the lash in the system has to be determined and introduced in the theoretical model.

The lash has been measured for most of the included parts and those measurements show that the lash is greatly dominated by the gearbox.

The amount of lash differs between a new and a used vehicle. Due to wear the model uses values for lash that are estimated for a used vehicle.

The largest amount of lash is in the gearbox. It’s lash and stiffness is shown in figure 3.2 below.

![Gearbox stiffness and lash](image)

**Figure 3.2. Gearbox stiffness and lash.**

The effect of the lash in the driveline can be seen in figure 3.3. When the torque rises from negative to positive values it shows a small delay at zero
torque which depends on the lash. The lash will also have an effect on how fast the torque rises after passing through the lash.

The reason why the level is not exactly zero during lash is that the torque gauge is not correctly calibrated.

3.1.3 Flywheel Torque

The input signal used in the theoretical model is based on an experimental measured torque on the crankshaft in the engine. The measured signal is filtered to pass frequencies below 30 Hz in order to reduce disturbance noise and influence from the high frequency sources, e.g. the cylinders and the combustion. The measured and filtered signal is used to create the torque that is applied to the flywheel in the theoretical model.
Measurement of the torque has been made in another vehicle but since this vehicle is of the same model and measured under similar conditions the signal is fully functional in this case. As seen in figure 3.4 the measured torque step is quite short so it has to be lengthened in order to correlate with the torque signal exciting the system during the rest of the measurements. The slope of the created torque signal is found in the experimental data for the wheel torque. The torque exciting the system decreases proportionally to the mean torque at the wheels since slip is neglected.

![Flywheel Torque](image)

*Figure 3.4. Measured torque and theoretical torque signal.*

The assumed declination of the torque is also seen in the figure where it can be seen that the torque decreases as the engine speed increases.

### 3.1.4 Influence of Different Gears

As seen in figure 3.5 the shuffle frequency is different for different gears and since the engine speed and the amount of throttle engagement is almost
the same at the two measurements, it is also obvious that the amplitude of which the torque changes is greater for lower gears.

**Figure 3.5. Comparisons between first and second gear.**

The reason why the level is not exactly zero during lash is that the torque gauge is not correctly calibrated.
4 Theoretical Model

4.1 Model Description

The main reason for shunt and shuffle is the drivetrain’s torsional movement but also the translational movement of the subframe, engine block and car body has some influence. The model needs to consider both movements because of the interaction between these two movements. The problem is simplified into a model that describes torsional and translational movement and the interaction between them.

The parts included in the model are; engine block, subframe, flywheel, transmission, wheels and car body. After investigating the included parts in the vehicle and the results from the experimental tests, the degrees of freedom for each part is established. The engine block and the subframe are allowed to both rotate and translate in one direction. Flywheel, transmission and wheels are only allowed to rotate and the car body has just a translational movement.

The system uses an input signal, which symbolize the driving torque at the flywheel. This signal is based on a measured torque signal, see section 3.1.3, and is used to excite the flywheel and the engine block in the model. The torque at the wheels is translated into a translational force in the x-direction, which is acting on the subframe. The subframe is attached to the engine block and to the chassi by a spring and a damper.

Figure 4.1 shows a sketch of the theoretical model. In reality the flywheel, gearbox and the wheels will turn during the process. This fact needs not to be considered because it is only the relative rotational movement that is of interest in the model. A spring and a damper model the flexibility and damping in the connection between the rims and the ground.

The force on the engine block due to the car’s and subframe’s translational movement will cause a torque to act on the block. This torque is shown in the sketch as $T_s$.

The output from the model that is of interest is the acceleration of the car body. This is later compared to experimental results from a test with an accelerometer attached to the car body.
Figure 4.1. Theoretical model.
4.2 Lash

The gearbox in the theoretical model needs to consider the influence of lash. The lash occurs when the torque rises from negative to positive values. The determination of the amount of lash for the gearbox is described in chapter 3.

The stiffness between the flywheel and the wheels, $k_{fw}$, is set to zero when the torque rises through the lash. Which means that:

$$k_{fw} = 0, \text{ when } 0 \leq |\theta_w - \theta_f| \leq \text{ radians of lash}$$

$$k_{fw} = k_{fw}, \text{ else}$$

(4.1)

4.3 Gear Box Ratio

The gearbox ratio affects the equivalent stiffness and inertia of the geared shaft, see figure 4.2.

With the speed of shaft 2 equal to $\dot{\theta}_w = n \dot{\theta}_f$, the kinetic energy of the system is, according to e.g. Thomson [2],

$$E = \frac{1}{2} J_f \dot{\theta}_f^2 + \frac{1}{2} J_w n^2 \dot{\theta}_f^2$$

(4.2)

Thus, the equivalent inertia of disc 2 referred to rotation of shaft 1 is $n^2 J_w$. 

Figure 4.2. Geared system.
To determine the equivalent stiffness of shaft 2 referred to rotation of shaft 1, disc 1 and 2 are clamped and a torque is applied to gear wheel 1. Rotating it through an angle $\theta_f$, the twist in shaft 2 will be $\theta_w = n\theta_f$.

The potential energy of the system is then

$$ U = \frac{1}{2} k_{cp} \theta_f^2 + \frac{1}{2} k_{dp} n^2 \theta_f^2 $$

(4.3)

and the equivalent stiffness of shaft 2 referred to rotation of shaft 1 is $n^2k_{dp}$.

The equivalent system can then be illustrated as:

\[ \text{Figure 4.3. Equivalent geared system.} \]

### 4.4 Equations of Motion

The system of equations that describes the model are derived by using Newton’s law of motion, which gives:

#### 4.4.1 Translational Direction

Car body

$$ (x_s - x_b)k_{bsq} + (\dot{x}_s - \dot{x}_b)c_{bsq} + ... $$

$$ (x_e - x_b)k_{beq} + (\dot{x}_e - \dot{x}_b)c_{beq} = m_b \ddot{x}_b $$

(4.4)
Subframe
\[
(x_e - x_s) k_{seq} + (\dot{x}_e - \dot{x}_s) c_{seq} - (x_s - x_b) k_{bsq} - ...
\]
\[
(\dot{x}_s - \dot{x}_b) c_{bsq} + \frac{T_w}{r_w} = m_s \ddot{x}_s
\]

(4.5)

Engine block
\[
-(x_e - x_s) k_{seq} - (\dot{x}_e - \dot{x}_s) c_{seq} - (x_e - x_b) k_{beq} - ...
\]
\[
(\dot{x}_e - \dot{x}_b) c_{beq} = m_e \ddot{x}_e
\]

(4.6)

4.4.2 Rotational Direction

Subframe
\[
(\theta_e - \theta_s) k_{sep} + (\dot{\theta}_e - \dot{\theta}_s) c_{sep} - \theta_s k_{bsp} - ...
\]
\[
\dot{\theta}_s c_{bsp} = J_s \ddot{\theta}_s
\]

(4.7)

Engine block
\[
-(\theta_e - \theta_s) k_{sep} - (\dot{\theta}_e - \dot{\theta}_s) c_{sep} - ...
\]
\[
\theta_e k_{bep} - \dot{\theta}_e c_{bep} - T_s - T_a = J_e \ddot{\theta}_e
\]

(4.8)

Flywheel
\[
-(\theta_f - \theta_w) k_{fw} - (\dot{\theta}_f - \dot{\theta}_w) c_{fw} - T_s + T_a = J_f \ddot{\theta}_f
\]

(4.9)

Wheels
\[
-\theta_w k_i n^2 - \dot{\theta}_w c_i n^2 + (\theta_f - \theta_w) k_{fw} + ...
\]
\[
(\dot{\theta}_f - \dot{\theta}_w) c_{fw} = J_w n^2 \ddot{\theta}_w
\]

(4.10)

\[
k_{fw} = \frac{k_e k_d n_i^2}{k_e + k_d n_i^2}; \quad (k_{fw} = 0 \text{ during lash})
\]

(4.11)
\[ T_w = k_i n_i^2 \dot{\theta}_w + c_i n_i^2 \ddot{\theta}_w \]  
(4.12)

\[ T_s = m_e \ddot{x}_e z_{es} \]  
(4.13)

where \( T_a \) is the applied torque to the system and \( n_i = n_g n_h \). \( z_{es} \) is the distance between center of gravity for engine block and subframe.

### 4.5 Stiffness and Damping

The stiffness and damping factors of the links between each mass are calculated separately from the main model, although some values are taken from tests performed at Volvo. The result is used as an input to the main model.

The theoretical model is built to be able to take into account that the stiffness and damping values shows a non-linear behavior. In this case most values have almost linear behavior, so the calculations are made using constant values for stiffness and damping. The values for damping are taken from test results and are valid for the specific shuffle frequency.

One equivalent rotational and one translational factor for the stiffness and damping replace the rubber mounts that hold the engine and the subframe.

### 4.6 Simulink Model

The resulting system of differential equations is solved using the Matlab toolbox Simulink. The Simulink model is built up by a number of subsystems each representing one part of the 7-DOF model derived. The principal appearance of a subsystem is shown in figure 4.4.
Figure 4.4. Principal subsystem.

The numbered circles are inputs and outputs that connect this system to other subsystems. The in- and out-puts are summed along with the internal forces of the part.

Below in figure 4.5 it is seen that after the torque are summed the result is divided by the mass moment of inertia for the part and acceleration is achieved according to Newton’s second law.

Figure 4.5. Principal subsystem.
This acceleration is integrated once and the resulting velocity is multiplied with the damping constant $c$. The symbol $\frac{1}{s}$ represents integration. The damping needs not to be constant and can easily be replaced by an arbitrary function describing a non-linear damping. This is of course also advantageous when using non-linear stiffness functions, which is multiplied by the integrated velocity. A subsystem for the gearbox lash is also built, which compares the flywheel movement to the movement of the wheels to determine whether the gearbox is to connect the flywheel and the wheels or not.

The rotational and translational systems are connected through the force that causes the car to move forward. This force is achieved by dividing the wheel torque by the wheel radius. A connection is also present between the subframe and the engine block. The engine block experiences a torque coming from the subframe due to the translational force from the wheels as the center of gravity is located above the mountings between the engine block and the subframe.

One of the advantages of using Simulink to solve differential equation systems is that you get a good schematic view of the system. Plotting and saving result vectors is very simple. Another advantage is that the time step can be automatically adjusted during calculations, which saves a lot of computing time. On the other hand it is not so easy to make changes of the system of equations that describe the model as if you, for example, write common Matlab code.

One way of investigating the shunt and shuffle phenomenon would be to use for example ADAMS or a similar program where you build the model “graphically” and where it is not necessary to derive the equations, since ADAMS does it for you. The reason why this is not done in this work is the fact that it is much easier to control your model when using Simulink and have derived the equations on your own. It is also easier to find errors and easier to make parameter changes for parameter studies.
5 Theoretical and Experimental Results

5.1 Wheel Force

The calculated torque (force) at the wheels is compared to the measured torque (force). Figure 5.1 shows the result for the first gear. The agreement is excellent. The shuffle amplitude, frequency and system damping are all similar to experimental results.

Figure 5.1. Wheel force at first gear, calculated and measured.

Figure 5.2 shows the result for the second gear.
Figure 5.2. Wheel force at second gear, calculated and measured.

The amplitude for the first peak, shunt, is satisfactory. The shuffle frequency and the system damping are not quite as good. One reason may be the simplifications made at the gearbox. The model does not consider the difference in mass moment of inertia for different gears in the gearbox.

5.2 Engine Block

Figure 5.3 shows the engine block movement for the first gear, measured at the top of the engine where the torque rod is mounted.
Figure 5.3. Engine block movement at first gear, calculated and measured.

The engine block in the model does not show as much movement as in the experimental results. The reason for this could probably be that the engine block in the model rotates about an axis that does not correspond well enough to reality. The assumed rotational axis used in the model is along the crank axle.
6 Parameter Study

6.1 Parameter Study

In this parameter study the variables are not changed simultaneously. Although it is possible to use the theoretical model for a simultaneous parameter study, it would require a lot of simulation and the presentation of that kind of parameter study is more complicated. The model is tested with different values for the stiffness to see if they can be changed to minimize the shunt and shuffle phenomena. It is particularly the stiffness of the driveshafts and the tires that will affect the result. The result of this study is shown in section 6.2.

Another parameter of interest is the lash in the gearbox. This will also affect the result. With less lash the shunt amplitude will be reduced. How much influence the gearbox lash has is shown in section 6.3.

6.2 Stiffness Study

The model is tested with five different values for the stiffness of the drive shafts, the tires, the upper torque rod and the stiffness between the engine block and the subframe. This is done to see if it is possible to reduce the shuffle amplitude of the acceleration. The diagrams show how the shunt amplitude and the translational motion in the $x$-direction of the engine block vary for different stiffness values. The horizontal axis of each plot is numbered from 0-1.

6.2.1 Drive Shaft Stiffness

Figure 6.1 shows how the shunt amplitude variates during drive shaft stiffness variation.
Figure 6.1. Shunt amplitude during drive shafts stiffness variation.

The amplitude mentioned above refers to the overshoot during shunt.

Figure 6.2 shows the engine block movement during drive shaft stiffness variation.
Figure 6.2. Engine block movement during drive shafts stiffness variation.

It seems that higher drive shaft stiffness reduces the shuffle amplitude. Very low stiffness also gives small amplitudes. Shafts with such low stiffness are however not possible in reality. They would not withstand the engine torque. Even if they did, weak drive shafts may decrease and delay the throttle response and therefore make the car less enjoyable to drive.
6.2.2 Tire Stiffness

Figure 6.3 shows how the shunt amplitude varies during tire stiffness variation.

Figure 6.3. Shunt amplitude during tire stiffness variation.

Figure 6.4 shows the engine block movement during tire stiffness variation.
Figure 6.4. Engine block movement during tire stiffness variation.

Lower tire stiffness appears to reduce the shuffle amplitude. This factor is hard to estimate correctly because it changes with the outside temperature and that is why this factor is difficult to optimize. Furthermore the car must work with different tire stiffness as the stiffness varies between for example summer and winter tires.
6.2.3 Upper Torque Rod Stiffness

Figure 6.5 shows how the shunt amplitude varies during upper torque rod stiffness variation.

![Upper Torque Rod Stiffness Comparison](image)

\textit{Figure 6.5. Shunt amplitude during upper torque rod stiffness variation.}

Figure 6.6 shows the engine block movement during upper torque rod stiffness variation.
Higher stiffness of the upper torque rod seems to reduce the shuffle amplitude. This will also have a major effect on the engine block movement. The problem of using a higher stiffness for the torque rod is that it will probably lead more vibrations into the cabin, since the rod is attached directly to the torpedo wall.
6.2.4 Engine Block - Subframe

Figure 6.7 shows how the shunt amplitude varies during engine block – subframe stiffness variation.

For stiffness values below 0.1 in figure 6.7, calculations do not appear to be valid. Those values are anyway too low to apply in reality.

Figure 6.8 shows the engine block movement during engine block – subframe stiffness variation.
Figure 6.8. Engine block movement during engine block – subframe stiffness variation.

A decreased stiffness between the engine block and the subframe appears to reduce the shuffle amplitude and lower the engine block movement.
6.3 Lash Study

Figure 6.9 shows how the shunt amplitude changes during gearbox lash variation.

![Graph showing shunt amplitude change during gearbox lash variation.]

*Figure 6.9. Shunt amplitude during gearbox lash variation.*

Figure 6.10 shows the engine block movement during gearbox lash variation.
Figure 6.10. Engine block movement during gearbox lash variation.

A minimum of lash gives low shunt amplitudes. The reason is that the included parts in the driveline can accelerate through the lash without any resistance. The larger amount of lash, the larger is this non-resistance distance which gives higher rotational energy for the shunt.
7 Conclusions

The purpose of this thesis was to define a theoretical model for simulation and optimization of a driveline system. Existing experimental results were used to verify the model.

After testing some models of different degrees of freedom it was obvious that a relatively simple model is sufficient for calculations of shunt and shuffle, since this phenomenon mainly occurs due to the first mode of the driveline system. The calculated results show good correlation to the measured results.

The parameter study shows that there are some factors that affect the shunt and shuffle phenomenon. The model seems to give lower shunt amplitude for increased drive shaft stiffness and tire stiffness. The lash in the gearbox has however the greatest effect. The lash should of course be as low as possible.

Further parameter studies should be done where the parameters are changed simultaneously. This could show that there are combinations of stiffness that will reduce the shunt and shuffle behavior significantly.

One way to improve the accuracy of the model would be to consider tire slip rather than divide the driveline into a greater number of parts. To introduce tire slip into the model some parts would need more degrees of freedom. For example, the vertical translation of the wheels would have to be calculated to determine the normal forces, which decide the friction forces between the tires and the ground.
8 References