Dynamic Characteristics of Exhaust System Hangers

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Abstract:
Comfort in automobiles is greatly affected by vibrations and noise transmitted to the chassis from the engine. This is a study of the dynamic characteristics of exhaust system hangers, which is an important transfer path for vibrations. Theoretical and experimental modal analysis is used to suggest design parameters that increase the natural frequencies to above 450 Hz for two hanger types.

Keywords:
Exhaust System, Hanger, Modal Analysis, Design Parameters, Experimental Verification.

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1 Notation

[A]	Matrix of eigenvectors

[C]	Viscous damping matrix [Ns/m]

{F}	Force vector [N]

f	Frequency [Hz]

I	Area moment of inertia [m^4]

[K]	Stiffness matrix [N/m]

k	Stiffness [N/m]

[M]	Mass matrix [kg]

m	Mass [Kg]

N	Number

{q}	Displacement vector [m]

t	Time [s]

$\{\Lambda\}$	Modal amplitude vector

ξ	Damping ratio

$\{\Phi\}$	Eigenvector

ϕ	Phase angle [rad]

ω	Angular frequency [rad/s]

Indices

i	Number

j	Number

n	Number

2 Introduction

During resent years the demands on product development in the automotive industry has escalated rapidly. Demands for lower production costs and higher comfort have steadily gained in importance. Vibrations and noise must as far as possible be imperceptible to the occupants of the car. Interior vibrations are formed by several different sources but the main exiting source is the engine. Two main types of transfer paths are structure- and airborne vibrations, respectively. The structure borne paths goes through the engine and transmission line mounts and through the exhaust system hangers into the chassis of the car. Therefore it is of great importance that the hangers are designed so that their natural frequencies are higher than the frequencies of the exiting sources acting on the system.

Several papers dealing with the dynamic behaviour of the whole exhaust system have been found, for example references [1-4], but no treatise of the dynamic behaviour of the hangers themselves have been found.

The aim of this work is to study design parameters of the hangers that affect natural frequencies. By this knowledge it will be possible to reach a proper design of the hanger at an early stage of the product development process.

Theoretical and experimental modal analysis will be performed for some existing hanger types. Modification of those as well as some general design guidelines will be suggested.

3 Basic Relations and Limitations

3.1 General Limitations and Demands

The main limitation is a demand from the customers, that is, the automobile manufacturers, that all natural frequencies of the hangers must be above 450 Hz. The vibration sources are assumed to excite the exhaust system up to a frequency around 250 Hz. The customers' demand includes a safety margin of 200 Hz. The hanger must also be able to withstand other loads such as temporary mechanical loads, thermal loads and corrosion.

A number of design parameters are of interest. Those are comprised in the mass m and the stiffness k. The mass is related to material and geometry. The material used in all studied cases is steel (SS 1312). The geometry determines the mass distribution. The stiffness depends on material, geometrical design and connections between the hanger and the exhaust system, that is, the weld bonds. The geometrical limitations of the hanger are that it must be able to fit the standard rubber elements. The rubber elements will not be discussed in this work.

It can also be of interest to achieve as high a damping ratio as possible to decrease the effects of natural frequencies in the hanger. By influencing the damping ratio in the structure it is, as a secondary solution, possible to decrease the effects of natural frequencies that cannot be increased above the wanted level. This is not further discussed in this work.

3.2 Case Studies

Two types of hangers are studied, a U-shaped hanger and a complicated long hanger in two variants. In both cases the hanger rods have a diameter of 10 mm. The hangers are presented in figures 3.1 and 3.2. These cases are chosen because they represent typical hangers and problems.

Figure 3.1. Photo of U-shaped hanger, case 1.

Figure 3.2. Photo of Complicated hangers, case 2a mounted on a bellows and 2b without exhaust pipe.

The aim of the first theoretical models is to achieve a good accordance between theoretical and experimental modal analysis so that they can be used later to study how different properties affect the natural frequencies.

The analyses are made with two different boundary conditions; the hangers free and the hangers connected to different exhaust pipes. The last case is aiming at resemble the real system. The length of the exhaust pipes is further discussed in section 4.2.

4 Dynamic Behaviour

The vibration sources are assumed to excite the exhaust system including the hangers. The vibration of the exhaust system must be considered as an input excitation to the hanger, which implies a base excitation problem. This is further discussed by for example Inman [5].

4.1 Modal Analysis Theory

Modal analyses give information about natural frequencies, mode shapes and damping ratios for the investigated structure. The analyses can be performed both as theoretical calculations on a FE-model and as experimental tests on the real structure. The damping ratios can only be determined experimentally.

The theory that is common for both theoretical and experimental modal analyses is described briefly below. For a description of modal analysis in the calculation software I-DEAS we refer to the interactive documentation *Smart View* in *I-DEAS Master Series 5* [6], and for a description of experimental modal analysis we refer to Maia et al [7].

The equation of motion for an *N* degree of freedom system with viscous damping is

$$[\mathrm{M}]\{\ddot{\mathrm{q}}\}+[\mathrm{C}]\{\dot{\mathrm{q}}\}+[\mathrm{K}]\{\mathrm{q}\}=\{\mathrm{F}(t)\} \qquad (4.1)$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix, {q} is the displacement vector and {F*(t)*} is the excitation force vector. Dots indicate time derivative.

Assuming a free undamped system ({F} = 0, [C] = 0), the equation of motion becomes

$$[\mathrm{M}]\{\ddot{\mathrm{q}}\}+[\mathrm{K}]\{\mathrm{q}\}=\{0\} \qquad (4.2)$$

To find the natural modes and frequencies, the displacement vector is assumed to be harmonic

$$\{q\} = \{\Phi\} e^{i\omega t} \tag{4.3}$$

where {Φ} is the vector of amplitudes at the nodes, ω is the angular frequency and t is time. Substituting (4.3) in (4.2) gives

$$\left[[\mathrm{K}] - \omega^2 [\mathrm{M}]\right]\{\Phi\} = \{0\} \tag{4.4}$$

Equation (4.4) is an eigenvalue problem. The equation has non-trivial solution only when

$$\det \left| [\mathrm{K}] - \omega^2 [\mathrm{M}] \right| = 0 \tag{4.5}$$

Equation (4.5) is called the characteristic equation of the system. The solution of this equation gives the same number of roots for ω^2 as the numbers of degree of freedom of the system. The roots are called eigenvalues. Putting those eigenvalues into equation (4.4) successively gives the corresponding eigenvectors.

Once the eigenvalue problem is solved, it is possible to determine solutions for [C] ≠ [0] and {F(t)} ≠ {0} with the method of mode superposition. The solution vector {q} is then expressed as a linear combination of all eigenvectors of the system, that is

$$\{q(t)\} = \left[\{\Phi\}_1, \{\Phi\}_2, \{\Phi\}_3, \ldots., \{\Phi\}_n\right]\{\Lambda(t)\} = [\mathrm{A}]\{\Lambda(t)\} \tag{4.6}$$

where [A] is a square matrix whose columns are the eigenvectors and {Λ(t)} is the vector of unknown modal amplitudes.

Substituting (4.6) into (4.1) and multiplying the resulting equation by the transpose of [A] gives

$$[A]^T[M][A]\{\ddot{\Lambda}\}+[A]^T[C][A]\{\dot{\Lambda}\}+[A]^T[K][A]\{\Lambda\}=[A]^T\{F\}$$

or

$$[M^*]\{\ddot{\Lambda}\}+[C^*]\{\dot{\Lambda}\}+[K^*]\{\Lambda\}=\{F^*\} \qquad (4.7)$$

where

$$\begin{aligned}
[M^*]&=[A]^T[M][A]\\
[C^*]&=[A]^T[C][A]\\
[K^*]&=[A]^T[K][A]\\
\{F^*\}&=[A]^T\{F\}
\end{aligned} \qquad (4.8)$$

By definition

$$[K]\{\Phi\}_i=\omega_i^2[M]\{\Phi\}_i \qquad (4.9)$$

Also considering that [K] and [M] are symmetric it can be shown that

$$\begin{aligned}
\{\Phi\}_i^T[K]\{\Phi\}_j&=0 \qquad i\neq j\\
\{\Phi\}_i^T[M]\{\Phi\}_j&=0 \qquad i\neq j
\end{aligned} \qquad (4.10)$$

for two different eigenvectors, corresponding to two different natural frequencies ω_i and ω_j. Equation (4.10) shows the weighted orthogonality of the eigenvectors.

Consequently, $[K^*]$ and $[M^*]$ are diagonal ($K_{ij}^* = 0$, $i \neq j$). If also $[C^*]$ can be made diagonal, a system of uncoupled equations is obtained, which is a significant computational advantage when the time response is to be found.

It is therefore common to assume that [C] is proportional to either [K] or [M]. This assumption is usually justified for structural systems because the actual coupling produced by damping is often negligible. Thus, introducing

$$[\mathrm{C}] = 2\xi_i \omega_i [\mathrm{M}] \tag{4.11}$$

where ξ_i is the damping ratio for mode i, and also using equation (4.9) a typical equation for the i:th mode has the form

$$M_{ii}^{*}\ddot{\Lambda}_i + 2\xi_i\omega_i M_{ii}^{*}\dot{\Lambda}_i + \omega_i^2 M_{ii}^{*}\Lambda_i = F_i^{*} \tag{4.12}$$

These ordinary differential equations can now be solved individually by for example some standard time marching procedure for given excitation forces. For a further discussion see for example Heubner et al [8].

4.2 Theoretical Calculations

The theoretical calculations are executed in *I-DEAS Simulation*. Finite Element models are representing the hangers, the exhaust pipes and their connections. *Normal Mode Dynamics* analyses are performed using the *Lanczos* method for the eigenvalue problem.

To find different types of improvements the mode shapes are studied. The improvements are to make the structure stiffer without adding to much mass in the sections with large deformation amplitudes. It is desirable to find the weakest part and make it stiffer. This is found either by visual studies of the mode shapes or by studies of the strain energy distribution in the structure.

The improvements can be divided into three main types; cross-section changes, reinforcements between rod/pipe segments, and more drastic redesigns of the whole hanger. The last type is not presented in this work.

The influence of the flexibility of the hanger connection to the exhaust pipe is studied particularly. An exhaust pipe length, from the edge of the exhaust pipe to the weld bonds, equal to one exhaust pipe diameter is found sufficient to describe this flexibility.

When the mode shape of the system only includes deflections of the hanger (the pipe has none or small motions) the length of the exhaust pipe has little effect on the natural frequency. An example of this is mode 1 for the original hanger case 1, see figure 4.3. When the mode shape includes motions of the exhaust pipe, the length and the mass of the pipe greatly affects the frequency. This is important to be aware of when discussing experimental test methods for approval of the hanger design. It is only the modes with internal hanger deflections that can be effectively affected by changing the hanger design. To get relevant test results also for modes including pipe motions it is essential that the test prototype and its boundary conditions are clearly defined.

Modes including motions of the exhaust pipe can be affected by changing and reinforce the connection between the hanger and the exhaust pipe by for example a total redesign, or reinforcement rods that increases the stiffness according to the same statements as for the hangers. Those methods are not further discussed in this work.

The influence of the pipe length is exemplified and further discussed in section 4.2.2.

4.2.1 Discussion on Finite Element Models

The Finite Element models are successively improved to find the simplest possible meshes that produce results of sufficient accuracy. The improvements are made both by refining meshes and by increasing element orders. Several element categories such as beam-, thin shell-, rigid- and solid elements are used in this study. Different element modelling are performed and evaluated with consideration of their ability to describe the dynamic behaviour of the different hangers and their weld bonds. For what parts the different element categories are used is presented in table 4.1.

Table 4.1. Use of different element categories.

Element category	**Hanger Rods and Pipes**	**Plates**	**Exhaust pipe**	**Weld bonds**	**Degrees of freedom**
Thin shell		X	X	X	6
Beam	X				6
Solid wedge				X	3
Rigid				X	6

Beam elements are used for the rods in the hangers. When studying the dynamic behaviour it is important that the mesh includes enough elements to describe the mass distribution of the rods properly.

For the exhaust pipe and the plate reinforcements, thin shell elements are used. The cross-section of the exhaust pipe will change close to the hanger connection. Therefore it is not appropriate to use beam elements for the exhaust pipe in these studies.

The weld bonds are modelled with thin shell-, solid wedge-, and rigid-elements. Depending on the aim of the analysis, different element categories are able to describe the reality with varying accuracy. The weld bonds are placed between all components in the structure, that is, between thin shell elements and beam elements in various variants. According to table 4.1 those elements have six degrees of freedom. When using solid wedge element for description of the weld bonds the rotational degrees of freedom in the common nodes of the solid wedge- and the beam- or the shell elements must be deleted. These nodes cannot transfer moments, which may be a clear disadvantage. Adjustments of the degrees of freedom are further discussed by for example Sunnersjö [9]. When using thin shell and rigid elements to describe the weld bonds the elements have the same degrees of freedom. They are therefore easier to stitch together with the shell mesh of the exhaust pipe and the beam mesh of the hanger. The stiffness of thin shells can easily be adjusted through the thickness of the elements to simulate the flexibility of the weld bonds, which rigid elements cannot. Thin shell elements are therefore preferred for the weld bonds.

4.2.2 Theoretical analyses and results, case 1

The theoretical models of the original hanger, see figure 4.3, give the natural frequency equal to approximately 170 Hz, which is clearly below the demand of 450 Hz.

Different types of weld bonds modelling are tried out on the original hanger. Their influences on the natural frequencies are small. Results from different types of weld bonds are shown in table 4.2.

Table 4.2. Original hanger, different weld bonds modelling between the hanger and the pipe.

Weld bond alternatives	**Rigid**	**Thin shell**	**Solid wedge**
Welded at two points, four nodes each like the real hanger.	174 Hz	168 Hz	167 Hz
Welded all the way around, all nodes.	184 Hz	181 Hz	179 Hz
Welded at three points, three nodes each.	178 Hz	172 Hz	171 Hz

To increase the first natural frequency to above 450 Hz different types of cross-sections are tried out for the hanger rods. The results show that some kinds of pipe cross-sections are to be preferred, since they are light-weighted and stiff. If a pipe cross-section is used, the hanger must be sealed to prevent dirt and water to gather in the pipe. The cross-section change can be used in combination with other measures.

Two reinforcement types are investigated. A U-shaped reinforcement presented in figure 4.1 and a plate reinforcement, presented in figure 4.2.

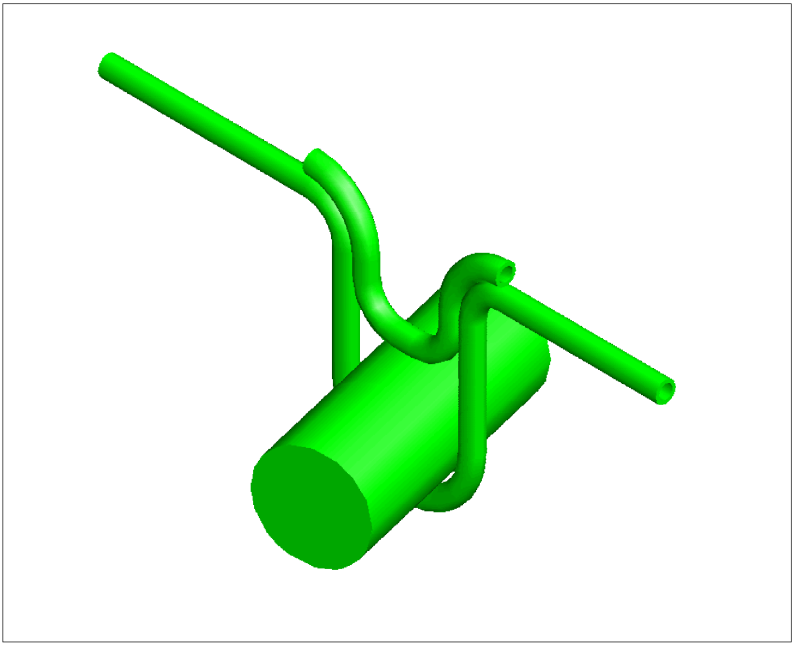

Figure 4.1. U-shaped reinforcement with pipe, case 1.

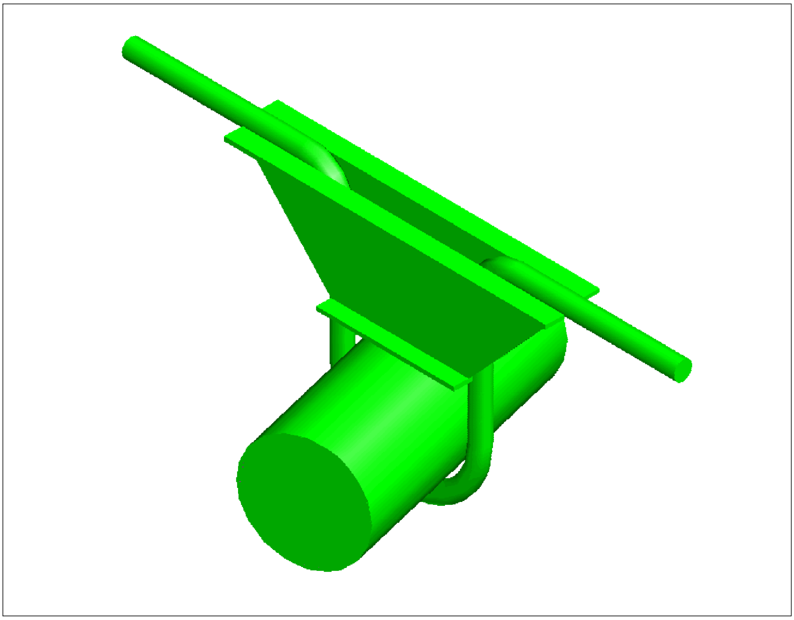

Figure4.2. Plate reinforcement, case 1.

These reinforcement suggestions and the conclusions regarding design parameters stated in section 4.2, were reached through studies of the mode shapes of the original hanger and different improvement alternatives.

The original hanger has the mode shape for the first natural frequency presented in figure 4.3. It clearly shows that the vertical arms are moving towards each other.

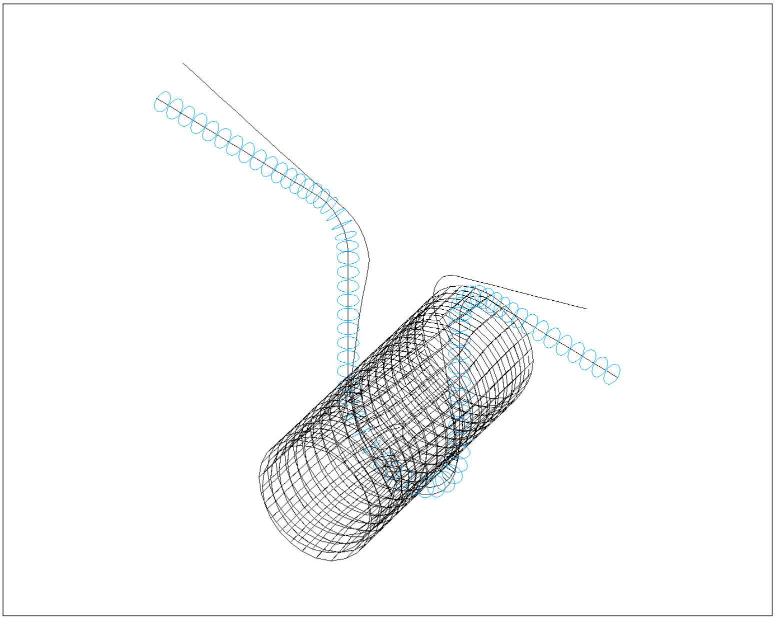

Figure 4.3. First mode shape of the original hanger on an exhaust pipe of length 144 mm.

A first idea for improvement was to put a straight rod between the arms of the hanger, a T-reinforcement as shown in figure 4.4, to prevent the relative motion of the arms.

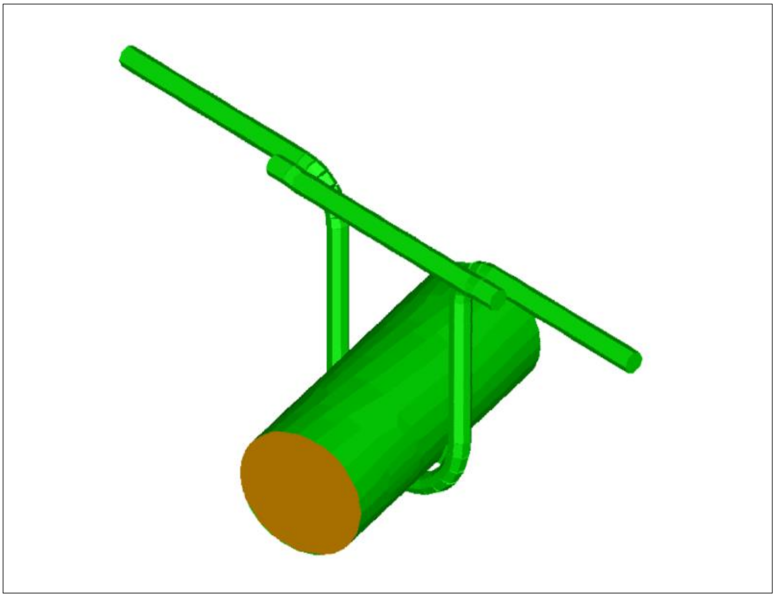

Figure 4.4. T-reinforcement on the original hanger.

The natural frequency increases and the mode shape changes, but this improvement does not fulfil the demand of 450 Hz even with reasonable cross-section changes. This is since the weak part is not stiffened enough and that relatively much mass is added far from the exhaust pipe where the deflections are large.

Other improvements had to be found. From the mode shape seen in figure 4.3, it can be concluded that the arms of the hanger are the section with the largest deflection and the weakest part seams to be the vertical arms close to the exhaust pipe. Considering this it is desirable to achieve higher stiffness in the arms without adding too much mass in the higher sections. This led to the U-reinforcement presented in figure 4.1, and plate reinforcement presented in figure 4.2. These alternatives also give the opportunity to attach the reinforcement to the exhaust pipe between the vertical arms of the hanger, which makes the structure even stiffer.

As seen in table 4.3 these reinforcements are much better. This investigation led to the general statement in section 4.2 on reinforcement strategies.

The result of the U-reinforcement shows that it does not completely fulfil the demand, but by changing the cross-sections of the hanger and the reinforcement, it does. The plate reinforcement shows very satisfying results. Results are given in table 4.3.

Table 4.3. Comparison of results.

Changes of the hanger	**Reinforcement type**	**Exhaust pipe length**	**First natural frequency**
None (Original hanger, rod 10 mm)	None	144 mm	165-175 Hz
None (Original hanger, rod 10 mm)	T-reinforcement	144 mm	330 Hz
Pipe cross-section 12/2 mm	None	144 mm	226 Hz
None (Original hanger, rod 10 mm)	U-reinforcement, rod 10 mm	144 mm	375 Hz
Pipe cross-section 12/2 mm	U-reinforcement, pipe cross-section 12/2 mm	144 mm	470-480 Hz
Pipe cross-section 15/1.5 mm (Prototype)	U-reinforcement, pipe cross-section 15/1.5 mm	144 mm 208 mm	591 Hz 464 Hz
None (Original hanger, rod 10 mm)	Plate reinforcement 2 mm	144 mm	725-735 Hz

The conclusion regarding the influence of the length of the exhaust pipe, stated in section 4.2, was drawn by comparing the first mode shape of the original hanger, with an exhaust pipe length of 144 mm and 500 mm, respectively, with the corresponding results of the U-reinforced alternative. The results are given in table 4.4.

Table 4.4. First natural frequencies found during examination of influence of different exhaust pipe length.

Hanger type	**Pipe length 144 mm**	**Pipe length 500 mm**
Rod 10 mm, original hanger	173 Hz	168 Hz
Rod 10 mm, U-reinforcement	375 Hz	246 Hz

As seen in table 4.4 the length of the pipe has different significance for the two alternatives. For the original hanger the exhaust pipe length has very little influence. This is because for both the short pipe alternative and the long pipe alternative, the mode shape includes almost exclusively internal hanger deformations. As the pipe does not move its length have no significance, see figure 4.3 and 4.6.

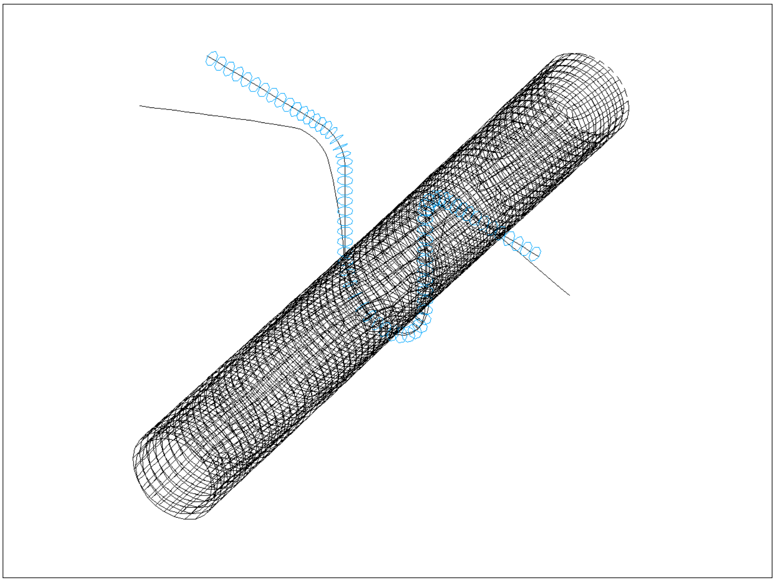

Figure 4.6. First modes shape of original hanger on an exhaust pipe of length 500 mm.

For the U-reinforced hanger it is seen in figure 4.7 and 4.8 that the exhaust pipe length has high significance. This is because for both the short pipe alternative and the long pipe alternative, the mode shape now includes significant pipe motions and the hanger moves almost like a rigid body. Of course the mass and the length of the exhaust pipe is then significant.

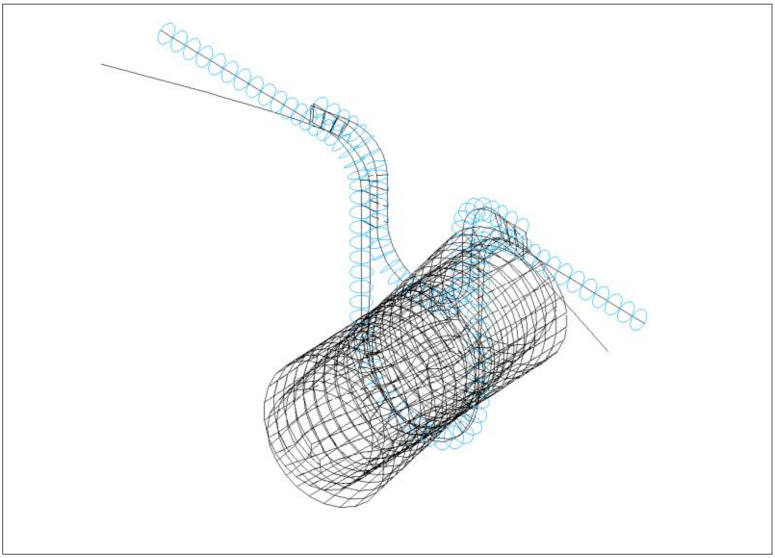

Figure 4.7. First mode shape of the U-reinforced hanger on an exhaust pipe of length 144 mm.

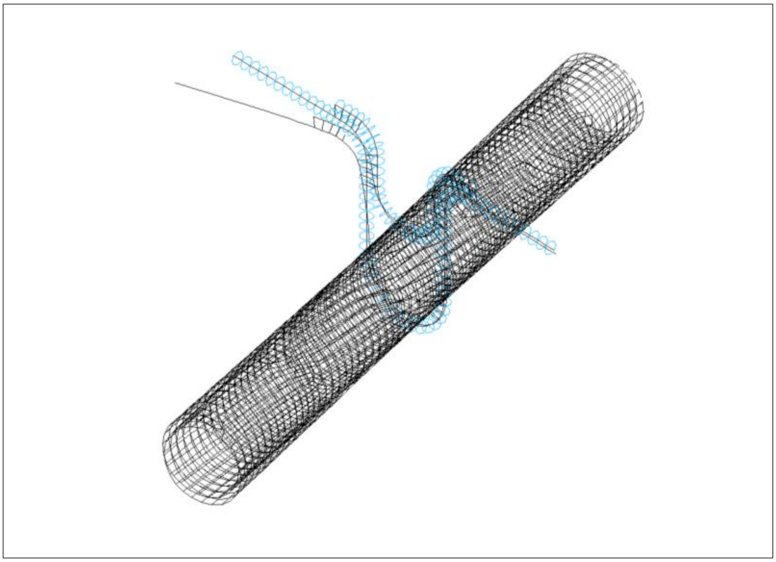

Figure 4.8. First mode shape of the U-reinforced hanger on an exhaust pipe of length 500 mm.

This investigation led to the statement in section 4.2 on testing methods for approval of hanger design.

A study besides the original aim was made to determine the effect of changing the length and the height of the horizontal arms on the original hanger. The results show that shortening the arms from 122 and 95 mm to 40 mm gives an increased natural frequency from 170 Hz to 280 Hz. If the height from the centreline is decreased from 86 to 20 mm the frequency increases from 170 Hz to 260 Hz. If both measures are made at the same time the natural frequency increases to 670 Hz.

4.2.3 Discussion on case 1

From the studies of the U-shaped hanger, it is found that

- A very important parameter is the cross-section of the hanger, which has great influence on the natural frequencies. As high area moment of inertia, and as low mass per length, as possible should be aimed at to achieve a high natural frequency
- The length of the exhaust pipe can greatly affect the natural frequencies if the mode includes more than internal hanger deflections. Therefore it is of great importance that the test prototype (how much of the exhaust pipe that is included) and its boundary conditions are clearly defined when discussing test models. It is only the modes with internal hanger deflections that can be effectively affected by changing the hanger design. The modes including deflections of the exhaust pipe can only be influenced by general changes of the hanger's connection to the exhaust pipe or by changes of the exhaust pipe.
- Different reinforcement types can be adapted. The two most efficient ones are to add plates or rods/pipes in proper places in the structure of the hangers.
- The different methods to mesh the weld bonds do not make any great difference. Rigid elements give a stiffer structure than reality. Wedge and thin shell elements give a weaker structure, but the stiffness can be adjusted to reality by changing the element's physical properties. As the investigations aim at as high natural frequencies as possible, a structure known to be slightly weaker than reality is preferable since it will give results including a safety margin. Solid wedge elements imply disadvantages regarding degrees of freedom, possibilities to transfer

moments and they are not so easy to stitch together with other element types.

- The number of weld bonds and their placement are not important as long as the strength is high enough and no section of the hanger can move relative to the exhaust pipe or other parts of the hanger.
- Before designing an improvement it is important to investigate the mode shape for the frequency of current interest. It gives information about the weakest section of the hanger, where the stiffness is to be increased, and which part that has the largest deflections, where it is very important not to increase the mass since it tends to greatly decrease the natural frequency. It is also possible to decide to which extent the exhaust pipe is included in the mode shape.
- An essential parameter is the length and the height of the horizontal arms. Longer and higher arms imply a lower natural frequency. This is a parameter that ought to be considered at a very early design stage.

4.2.4 Theoretical analyses and results, case 2

Case 2 is divided into two different variants, presented in figure 3.2. Case 2a is the original hanger and case 2 b is an already improved variant of the same hanger. The objective of this case study is to confirm the conclusions from case 1, improve case 2a further, and try to find new interesting design parameters.

Similar types of reinforcements as in case 1 are theoretically investigated for case 2a. The rods are changed to pipes of 15/2 mm and 2 mm plate reinforcements is added. The plate is placed vertically between the arms and horizontally 2/3 between the arms, see figure 4.9.

The hangers are studied both without exhaust pipe and with an exhaust pipe of length 300 mm. Results from theoretical calculations are given in table 4.5 and 4.6.

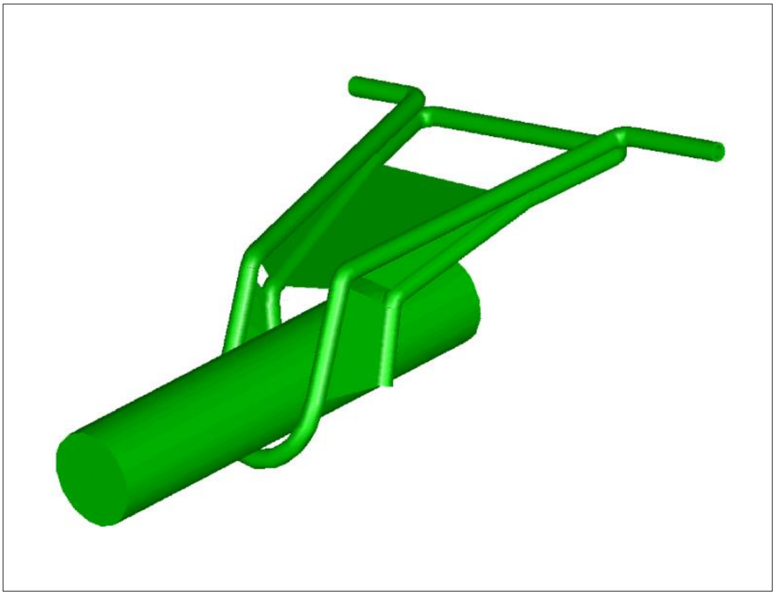

Figure 4.9. Improved variant of case 2a.

Table 4.5. Theoretical results case 2, without exhaust pipe.

Case	**First natural frequency**
2 a	106 Hz
2 a, pipe 15/2 mm	181 Hz
2 a, pipe 15/2 mm and 2 mm plate reinforcement	450 Hz
2 b	560 Hz

Table 4.6. Theoretical results Case 2, with exhaust pipe of 300 mm.

Case	**First natural frequency**
2 a	167 Hz
2 a, pipe 15/2 mm	259 Hz
2 a, pipe 15/2 mm and 2 mm plate reinforcement	482 Hz
2 b	334 Hz

4.2.5 Discussion on case 2

The same conclusions can be drawn for case 2 as for case 1. In addition the following new conclusion is drawn:

- The effects of the differences in nodal degrees of freedom, discussed in section 4.2.1, are more obvious in this case. Thin shell and rigid elements are found to be better alternatives than solid wedge elements.

4.3 Experimental Verification

The experimental analyses are performed on the hangers in order to establish the reliability of the theoretical modal analyses. The same procedure is repeated on the prototypes to confirm the expected results and achieve a complete approach.

4.3.1 Experimental Procedure

The experiments are performed using *I-DEAS Test*, HP-measuring system, excitation hammer and accelerometers. To simulate the free-free condition the test parts are suspended in rubber bands. The hanger is excited at several excitation points and the accelerometer is placed in one measure-point. The accelerometer is melt-glued to the structure and placed in points with expected large deflections to achieve good test results. This gives an increased mass in the test structure that is necessary to consider when comparing test results with theoretical results. By this procedure a number of frequency response functions are achieved and it is possible to get the mode shapes from *I-DEAS Test*. The experimentally determined mode shapes are used to confirm the finite element models.

In this study it is primarily the first natural frequency that is of interest. Therefore the mode shapes are not determined experimentally for every measurement. The unknown natural frequency is determined by one measurement when knowledge of good measurement points is achieved.

4.3.2 Experimental results

The experimental modal tests give the first natural frequencies presented in table 4.7. Theoretical results are presented in the same table for comparison.

Table 4.7. Experimentally determined natural frequencies compared with theoretical results.

Case	Pipe length	Experimental results	Theoretical results
1	144 mm	170 Hz	168 Hz
2a, no exhaust pipe	-	108 Hz	106 Hz
2a	300 mm	171 Hz	167 Hz
2b no exhaust pipe	-	522 Hz	560 Hz
2b	300 mm	381 Hz	350 Hz

The agreement between theoretical and experimental results is good.

4.4 Prototypes of case 1 improvements

Two prototypes are made for the reinforcement of case 1.

- A reinforcement pipe of 15/1.5 mm together with a cross-section change of the original hanger rod to a pipe of 15/1.5 mm, according to figure 4.10.
- A plate reinforcement applied to the original hanger, according to figure 4.11.

Figure 4.10. Photo of prototype with pipe reinforcement.

Figure 4.11. Photo of prototype with plate reinforcement.

The aim is to verify the theories experimentally and achieve a complete approach. The results for the prototypes are presented in table 4.8.

Table 4.8. Theoretical and experimental result for prototypes.

Type	**Pipe length**	**Experimental results**	**Theoretical results**
Pipe and U-reinforcement	208 mm	486 Hz	464 Hz
Plate reinforcement	144 mm	820 Hz	807 Hz

The agreement between theoretical and experimental results is good.

5 Conclusions

The aim of this work was to study design parameters of exhaust system hangers that affect natural frequencies. By this knowledge it will be possible to reach a proper design of the hanger at an early stage in the product development process.

Two general parameters affecting the natural frequencies are stiffness and mass distribution. Changes of these parameters can be reached through changes of several different physical properties. One of the most important ones is the cross-section; light-weighted stiff cross-sections are desirable. Another essential modification is different types of reinforcements, for example pipe- and plate reinforcements. When designing reinforcements the mode shape of the structure must be studied. The reinforcements are to make the structure stiffer in the weak sections without adding unnecessary mass to moving sections. It is very important to keep the mass as low as possible in the parts of the structure that have large displacements at the natural frequency.

It is important to clearly define the test prototype and its boundary conditions to get relevant test results also for modes including exhaust pipe motions. It is only the modes with internal hanger deflections that can be effectively affected by changing the hanger design.

The Finite Element models for the theoretical modal analysis are preferably made with elements having the same degrees of freedom.

For the two hanger types studied it was shown that rather small design changes made it possible to satisfy the customers' demand of a first natural frequency above 450 Hz. By the general reasoning presented in this work it should be possible to fulfil this demand also for many other hanger types without a total redesign.

6 References

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