Due to the rapid development of wireless communications together with the inflexibility of the current spectrum allocation policy, radio spectrum becomes more and more exhausted. One of the critical challenges of wireless communication systems is to efficiently utilize the limited frequency resources to be able to support the growing demand of high data rate wireless services. As a promising solution, cognitive radios have been suggested to deal with the scarcity and underutilization of radio spectrum. The basic idea behind cognitive radios is to allow unlicensed users, also called secondary users (SUs), to access the licensed spectrum of primary users (PUs) which improves spectrum utilization. In order to not degrade the performance of the primary networks, SUs have to deploy interference control, interference mitigating, or interference avoidance techniques to minimize the interference incurred at the PUs.

Cognitive radio networks (CRNs) have stimulated a variety of studies on improving spectrum utilization. In this context, this thesis has two main objectives. Firstly, it investigates the performance of single hop CRNs with spectrum sharing and opportunistic spectrum access. Secondly, the thesis analyzes the performance improvements of two hop cognitive radio networks when incorporating advanced radio transmission techniques.

The thesis is divided into three parts consisting of an introduction part and two research parts based on peer-reviewed publications. Fundamental background on radio propagation channels, cognitive radios, and advanced radio transmission techniques are discussed in the introduction. In the first research part, the performance of single hop CRNs is analyzed. Specifically, underlay spectrum access using M/G/1/K queueing approaches is presented in Part I-A while dynamic spectrum access with prioritized traffics is studied in Part I-B. In the second research part, the performance benefits of integrating advanced radio transmission techniques into cognitive cooperative radio networks (CCRNs) are investigated. In particular, opportunistic spectrum access for amplify-and-forward CCRNs is presented in Part II-A where collaborative spectrum sensing is deployed among the SUs to enhance the accuracy of spectrum sensing. In Part II-B, the effect of channel estimation error and feedback delay on the outage probability and symbol error rate (SER) of multiple-input multiple-output CCRNs is investigated. In Part II-C, adaptive modulation and coding is employed for decode-and-forward CCRNs to improve the spectrum efficiency and to avoid buffer overflow at the relay. Finally, a hybrid interweave-underlay spectrum access scheme for a CCRN is proposed in Part II-D. In this work, the dynamic spectrum access of the PUs and SUs is modeled as a Markov chain which then is utilized to evaluate the outage probability, SER, and outage capacity of the CCRN.
On the Performance Assessment of Advanced Cognitive Radio Networks

Thi My Chinh Chu
On the Performance Assessment of Advanced Cognitive Radio Networks

Thi My Chinh Chu

Doctoral Dissertation in Telecommunication Systems
Abstract

Due to the rapid development of wireless communications together with the inflexibility of the current spectrum allocation policy, radio spectrum becomes more and more exhausted. One of the critical challenges of wireless communication systems is to efficiently utilize the limited frequency resources to be able to support the growing demand of high data rate wireless services. As a promising solution, cognitive radios have been suggested to deal with the scarcity and under-utilization of radio spectrum. The basic idea behind cognitive radios is to allow un-licensed users, also called secondary users (SUs), to access the licensed spectrum of primary users (PUs) which improves spectrum utilization. In order to not degrade the performance of the primary networks, SUs have to deploy interference control, interference mitigating, or interference avoidance techniques to minimize the interference incurred at the PUs. Cognitive radio networks (CRNs) have stimulated a variety of studies on improving spectrum utilization. In this context, this thesis has two main objectives. Firstly, it investigates the performance of single hop CRNs with spectrum sharing and opportunistic spectrum access. Secondly, the thesis analyzes the performance improvements of two hop cognitive radio networks when incorporating advanced radio transmission techniques.

The thesis is divided into three parts consisting of an introduction part and two research parts based on peer-reviewed publications. Fundamental background on radio propagation channels, cognitive radios, and advanced radio transmission techniques are discussed in the introduction. In the first research part, the performance of single hop CRNs is analyzed. Specifically, underlay spectrum access using M/G/1/K queueing approaches is presented in Part I-A while dynamic spectrum access with prioritized traffics is studied in Part I-B. In the second research part, the performance benefits of integrating advanced radio transmission techniques into cognitive cooperative radio networks (CCRNs) are investigated. In particular, opportunistic spectrum access for amplify-and-forward CCRNs is presented in Part II-A where collaborative spectrum sensing is deployed among the SUs to enhance the accuracy of spectrum sensing. In Part II-B, the effect of channel estimation error and feedback delay on the outage probability and symbol error rate (SER) of multiple-input multiple-output CCRNs is investigated. In Part II-C, adaptive modulation and coding is employed for decode-and-forward CCRNs to improve the spectrum efficiency and to avoid buffer overflow at the relay. Finally, a hybrid interweave-underlay spectrum access scheme for a CCRN is proposed in Part II-D. In this work, the dynamic spectrum access of the PUs and SUs is modeled as a Markov chain which then is utilized to evaluate the outage probability, SER, and outage capacity of the CCRN.
Preface

This thesis summarizes my research work within the field of cognitive radio networks. Firstly, the performance of single hop CRNs for both spectrum sharing and opportunistic spectrum access is investigated. Secondly, advanced radio transmission techniques are applied to improve the system performance of two hop CRNs. The work has been carried out at the Faculty of Computing, Blekinge Institute of Technology, Karlskrona, Sweden. The thesis consists of an introduction together with two research parts as follows:

Introduction

Part I: Single Hop Cognitive Radio Networks
A On the Performance of Underlay Cognitive Radio Networks Using $M/G/1/K$ Queueing Model
B Dynamic Spectrum Access for Cognitive Radio Networks with Prioritized Traffics

Part II: Two Hop Cognitive Radio Networks
A Opportunistic Spectrum Access for Cognitive Amplify-and-Forward Relay Networks
B MRT/MRC for Cognitive AF Relay Networks under Feedback Delay and Channel Estimation Error
C Adaptive Modulation and Coding with Queue Awareness in Cognitive Incremental Decode-and-Forward Relay Networks
D Hybrid Interweave-Underlay Spectrum Access for Cognitive Cooperative Radio Networks
Acknowledgements

Reminiscing about the last four years, my Ph.D. studies have been a progression with plenty of effort and pleasure. Now, it is the right time to express my sincere gratitude to people who inspired and supported me to pursue this field of science.

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Finally, the most deepest thanks to my mother Hoang Thi Thong, who sacrificed herself for me, which is much more meaningful than what I can ever express. I appreciate my sister Chu Thi Tra Giang for always believing in me and her unmeasurable love. Heartfelt thanks to my little daughter, Tran Dieu Linh, who gives me energy and inspires me to conquer the difficulties and keep working. I am forever grateful to my husband, Tran Dinh Thi, for his acceptance of my academic choice and always standing by me. Last but absolutely not least, special thanks go to the rest of my family for their love and continuous encouragement.

Thi My Chinh Chu
Karlskrona, January 2015
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Part II-B is published as:


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Acronyms

AF   Amplify-and-Forward
ACK  Acknowledgement
AMC  Adaptive Modulation and Coding
AWGN Additive White Gaussian Noise
BER  Bit Error Rate
BFSK Binary Frequency Shift Keying
CCRN Cognitive Cooperative Radio Network
CDF  Cumulative Distribution Function
CEE  Channel Estimation Error
CF   Compress-and-Forward
CR   Cognitive Radio
CR-MAC Cognitive Radio Medium Access Control
CRN  Cognitive Radio Network
CSMA/CA Carrier Sense Multiple Access with Collision Avoidance
CSR  Channel State Receiver
CSI  Channel State Information
CST  Channel State Transmitter
CTMC Continuous Time Markov Chain
CTS  Clear to Send
DF   Decode-and-Forward
DSA  Dynamic Spectrum Access
EF   Estimate-and-Forward
<table>
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<tr>
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<th>Description</th>
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<tr>
<td>EGC</td>
<td>Equal Gain Combining</td>
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<td>FCC</td>
<td>Federal Communications Commission</td>
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<td>FD</td>
<td>Feedback Delay</td>
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<td>FSK</td>
<td>Frequency Shift Keying</td>
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<td>IA</td>
<td>Interference Avoidance</td>
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<td>IC</td>
<td>Interference Control</td>
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<td>IF</td>
<td>Intermediate Frequency</td>
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<td>i.i.d.</td>
<td>independent and identically distributed</td>
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<td>ISA</td>
<td>Interweave Spectrum Access</td>
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<td>LOS</td>
<td>Line-of-Sight</td>
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<td>MAC</td>
<td>Medium Access Control</td>
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<tr>
<td>MCS</td>
<td>Modulation and Coding Scheme</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<td>MISO</td>
<td>Multiple-Input Single-Output</td>
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<td>MRC</td>
<td>Maximum Ratio Combining</td>
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<td>MRT</td>
<td>Maximum Ratio Transmission</td>
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<td>M/D/1</td>
<td>Markovian Arrival, Deterministic Departure, and Single Server</td>
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<td>M/G/1</td>
<td>Markovian Arrival, General Departure, and Single Server</td>
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<td>M/G/1/K</td>
<td>Markovian Arrival, General Departure, Single Server, and Finite Queue Length</td>
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<td>M/M/1</td>
<td>Markovian Arrival, Markovian Departure, and Single Server</td>
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<tr>
<td>NACK</td>
<td>Negative Acknowledgment</td>
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<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<td>OSA</td>
<td>Overlay Spectrum Access</td>
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<tr>
<td>OP</td>
<td>Outage Probability</td>
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<td>PAM</td>
<td>Pulse Amplitude Modulation</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<td>PSK</td>
<td>Phase Shift Keying</td>
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<td>Abbreviation</td>
<td>Description</td>
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<td>PN</td>
<td>Primary Network</td>
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<td>PU</td>
<td>Primary User</td>
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<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QF</td>
<td>Quantize-and-Forward</td>
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<tr>
<td>QoS</td>
<td>Quality-of-Service</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<td>RTS</td>
<td>Request to Send</td>
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<tr>
<td>RV</td>
<td>Random Variable</td>
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<tr>
<td>SC</td>
<td>Selection Combining</td>
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<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
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<tr>
<td>SIMO</td>
<td>Single-Input Multiple-Output</td>
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<tr>
<td>SINR</td>
<td>Signal-to-Interference-plus-Noise Ratio</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>SU</td>
<td>Secondary User</td>
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<tr>
<td>TAS</td>
<td>Transmit Antenna Selection</td>
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<td>TS</td>
<td>Time Slot</td>
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<td>USA</td>
<td>Underlay Spectrum Access</td>
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Introduction

1 Motivation

In the last decades, wireless communication services have been remarkably thrived which has brought a significant change in many fields of our daily life. As societies and economies become global, wireless communications has become an indispensable part of humanity all over the world. Although many wireless systems have been successfully deployed such as mobile cellular systems, wireless local area networks, television broadcasting, and satellite systems, several challenges must be addressed for wireless communication systems in the future.

One of the critical challenges is to efficiently utilize the limited frequency resources. The emergence of new wireless applications as well as the increasing demands on higher data rates of diverse wireless services have led to a serious shortage of radio spectrum [1]. On the other hand, measurement campaigns have shown that many radio spectrum bands are under-utilization [2,3]. This inefficient use of radio spectrum is mainly due to the inflexibility of the current fixed spectrum allocation policy.

In efforts to improve spectrum utilization and to break the spectrum gridlock for future generation wireless communication systems, cognitive radio (CR) was originally introduced by Mitola [4]. In this work, a new spectrum regulation policy was proposed where radio spectrum is considered as an open source for simultaneous users. However, when using a licensed frequency, cognitive radio networks (CRNs) must assure a satisfactory quality-of-service (QoS) for the licensed networks, also called primary networks (PNs). To fulfill this requirement, CR devices must have a cognitive capability such as awareness of the traffic, frequency, bandwidth, power, and modulation of the PNs. In addition, CR devices must obtain knowledge of the surrounding environment. Based on the sensed information, CR devices must be reconfigurable to rapidly adapt the transmission parameters in order to optimize performance while not degrading the performance of the PN [5].

Regarding the mechanism that a CRN utilizes to handle the interference incurred at the PN, CRNs can be classified into three main categories, i.e., in-
terference avoidance CR, interference cancelation CR, and interference control CR. In the interference avoidance CR [6], the secondary users (SUs) are only temporarily allowed to access the spectrum licensed to the primary users (PUs) at a specific time or in particular geographic locations when the licensed spectrum is idle. In interference cancelation CR [7], both the PUs and SUs can concurrently access the spectrum bands as long as the SUs utilize the information of the PU’s codebooks to get rid of the interference at the PUs. In interference control CR [8], during co-existence, the SUs continuously adapt their transmit powers to maintain the interference incurred at the PUs below a predefined threshold.

Besides the constraints imposed by the PNs and complicated regulation issues of CRNs, it is challenging to provide satisfactory QoS to CRNs due to the channel impairments in wireless communications. Main characteristics of radio channels which cause the received signals to fluctuate are multipath propagation, path loss, and interference [9, 10]. Specifically, the variation of the received signal due to multipath propagation happens over very short distances and is called small-scale propagation. On the other hand, the variation of the received signal due to path loss and shadowing occurs over rather large distances which is referred as large-scale propagation [11]. Both these influences lead to a serious performance degradation of wireless communication systems [11].

In order to improve the performance of CRNs, integrating advanced radio transmission techniques into CRNs has recently attracted a lot of attention in the research community. First, cooperative communications [12–15] has been considered as a powerful technique to mitigate the effects of fading channels, extend the radio coverage, and provide reliable communications. In this kind of communications, one or several relays are utilized to process and forward the source signal to the destination. Since the probability that all independent paths simultaneously experience deep fades is relatively low, the transmission reliability of cooperative communications can be improved substantially. At the destination, by combining the independently faded replicas of the source signal that arrive through multiple paths, the system can obtain spatial diversity. In the context of CRNs where radio coverage is often quite short due to transmit power constraints, cooperative communications can be applied to extend the transmission range for CRNs [16–22]. This technique becomes even more beneficial when the relays can assist both the PNs and adjacent CRNs by forwarding their signals [23, 24]. By this approach, relaying transmission not only reduces the mutual interference between the PNs and adjacent CRNs but also improves the performance of both networks.

Another technique to obtain spatial diversity is deploying multiple-input multiple-output (MIMO) antenna arrays at transmitter and/or receiver [25–
Numerous publications have shown that MIMO systems offer significant advantages with respect to capacity and error performance over single-antenna systems. There exist two main categories of MIMO techniques, i.e., spatial multiplexing and spatial diversity [26,27]. Spatial multiplexing aims at increasing the capacity of the system [28] by simultaneously transmitting several data streams through multiple transmit antennas. On the other hand, spatial diversity increases the transmission reliability by sending the same signal into the MIMO channels [29].

In order to increase the spectrum efficiency, we can increase the bit rate by shortening the symbol duration or adapting the transmission parameters to the time-varying environment. In the first method, due to multipath effects caused by reflections, scattering, and diffraction through radio channels, shortening symbol durations can increase inter-symbol interference which results in higher error rates. Furthermore, when wireless communication systems transmit signals with short symbol duration, larger frequency bands are needed. As radio spectrum has become more and more exhausted, bandwidth expansion is definitely undesirable in wireless communications. In the second method, spectrum efficiency is enhanced by utilizing adaptive schemes where certain parameters such as transmit power, transmission rate, and modulation constellation are adjusted to the variation of the fading channels [30–32].

The main target of this thesis is to analyze the performance of CRNs with advanced radio transmission techniques. In the first part, we focus on assessing the performance of both spectrum sharing and opportunistic spectrum access of single hop CRNs. In particular, multi-dimensional and embedded Markov chains are utilized to evaluate the performance of underlay CRNs and interweave CRNs with prioritized traffics. In the second part, we are interested in deploying advanced radio transmission techniques such as cooperative communications, MIMO techniques, adaptive transmission, and hybrid spectrum access for two hop CRNs to achieve an improvement of system performance. In particular, to improve the transmission reliability and to extend the radio coverage, cooperative communications is utilized in the considered CRNs. Furthermore, the MIMO technique is integrated into CRNs to obtain diversity gains. In addition, adaptive modulation and coding is applied for CRNs to obtain benefits in terms of increased spectrum efficiency. Finally, deploying hybrid interweave-underlay spectrum access for CRNs is shown to offer performance improvement over conventional underlay CRNs in terms of outage probability, symbol error rate, and outage capacity.

The remainder of this introduction is organized as follows. Section 2 presents fundamentals of radio propagation channels. Section 3 discusses basic concepts and main functions of cognitive radios. Important advanced radio transmission techniques are introduced in Section 4. Key metrics commonly
used to evaluate the performance of CRNs are discussed in Section 5. In Section 6, the thesis overview is presented. Finally, Section 7 outlines directions for future research that spans beyond the work of this thesis.

2 Overview of Radio Propagation Channels

This section aims at introducing main concepts of radio propagation channels. Specifically, it focuses on the impulse response models and statistical fading models of radio propagation channels since this knowledge is frequently utilized in the research work presented in this thesis.

2.1 Radio Propagation Channel Models

Because of multipath propagation, path loss, shadowing, and movement of objects, radio propagation channels become time variant which causes the received signals to fluctuate and to be unreliable. Multipath propagation occurs when the transmitted radio signals reach the receiving antenna through more than one path due to atmospheric ducting, scattering, reflection, and refraction from obstacles such as hills or buildings located between the transmitter and receiver. Path loss occurs when the transmit power is dissipated with respect to the propagation environment and the distance from the transmitter to the receiver. However, with the same transmission distance from the transmitter, the received signals at different locations are varying due to random shadowing effects.

To represent the effects of a propagation environment on the transmit signal, propagation models are usually utilized. Based on the prediction of the average signal strength at a particular location from the transmitter, propagation models can be classified into two categories [33], i.e., large-scale propagation models and small-scale propagation or fading models.

Large-scale propagation models

Large-scale propagation models predict the average signal strength at an arbitrary distance from the transmitter and are often used to estimate the radio coverage of a transmitter. The gradual variation of the signal results from path loss and shadowing over relatively large distances [11]. In particular, the variations of the signal caused by path loss significantly take place over a large distance from the transmitter to the receiver. However, variations of the signal caused by shadowing happen over distances proportional to the size of the obstacles [11].
Small-scale propagation models

Small-scale propagation or fading models characterize the rapid variations of the signal strength in a close spatial proximity to a particular location. The variation of the signal is due to constructive and destructive combination of multipath signals which arrive at the receiver from different paths. Small-scale fading occurs over rather short distances in the order of the carrier wavelength. The most important effects of small-scale fading are as follows:

- Rapid fluctuation of the amplitude and phase of the received signal
- Random frequency modulation caused by Doppler shifts
- Time dispersion due to multipath propagation delays

2.2 Impulse Response of a Radio Propagation Channel

As mentioned in the previous section, in a radio propagation environment, the transmit signals are typically propagated through multiple paths due to reflections, scattering, and diffractions before reaching the receiver as shown in Fig. 1. As a consequence, the received signals are trains of pulses with each pulse corresponding to a particular path from the transmitter to the receiver.

![Diagram of a radio propagation environment](image)

Figure 1: Example of a radio propagation environment.

According to [33, Chapter 5], the radio frequency (RF) signal $x(t)$ at time instant $t$ of a complex baseband waveform $x_{b}(t)$, after modulation with the
carrier frequency $f_c$, is given in general form as

$$x(t) = \sqrt{2} \, \text{Re}\{x_b(t)e^{j2\pi f_c t}\}$$  \hspace{1cm} (1)$$

where $\text{Re}\{\cdot\}$ denotes the real part of a complex number. Then, multipath propagation channels can be modeled as linear filters with time-varying impulse responses as in [9, Chapter 2] and [33, Chapter 5]. Omitting noise and ignoring co-channel interference, the bandpass signal at the receiver can be expressed as

$$y(t) = \sum_{i=0}^{N-1} a_i(t)x(t - \tau_i(t))$$  \hspace{1cm} (2)$$

where $N$ denotes the total number of multipath components. Further, $a_i(t)$ and $\tau_i(t)$ are, respectively, the real amplitude and propagation delay of the received signal through the $i$-th path at time $t$. Substituting (1) into (2), the received bandpass signal can be rewritten as

$$y(t) = \sqrt{2} \, \text{Re}\left\{\sum_{i=0}^{N-1} a_i(t)e^{-j2\pi f_c \tau_i(t)}x_b(t - \tau_i(t))e^{j2\pi f_c t}\right\}$$  \hspace{1cm} (3)$$

From (3), the equivalent received complex baseband signal $y_b(t)$ after demodulation is obtained as

$$y_b(t) = \sum_{i=0}^{N-1} a_i(t)e^{-j2\pi f_c \tau_i(t)}x_b(t - \tau_i(t))$$  \hspace{1cm} (4)$$

If the received baseband signal is considered as a function of the time-varying baseband channel impulse response $h_b(\tau, t)$ and the baseband transmit signal $x_b(t)$, $y_b(t)$ can be expressed in the form

$$y_b(t) = x_b(t) \otimes h_b(\tau, t) = \int_{-\infty}^{\infty} h_b(\tau, t)x_b(t - \tau)d\tau$$  \hspace{1cm} (5)$$

where $\otimes$ denotes the convolution operator and $\tau$ represents the multipath propagation delay for a fixed value of $t$. From (4) and (5), the equivalent baseband impulse response $h_b(\tau, t)$ of a multipath channel is expressed as [33, (5.12)]

$$h_b(\tau, t) = \sum_{i=0}^{N-1} a_i(t)e^{-j2\pi f_c \tau_i(t)}\delta(\tau - \tau_i(t))$$  \hspace{1cm} (6)$$
where \( \delta(\cdot) \) denotes the Dirac delta function. If the channel impulse response is time invariant, the channel impulse response can be simplified as

\[
h_b(\tau) = \sum_{i=0}^{N-1} a_i e^{-j2\pi f_c \tau_i} \delta(\tau - \tau_i)
\]  

If the signal \( x_b(t) \) is transmitted over a non-frequency selective and slow fading channel, i.e., the channel impulse response is considered as a constant \( h \) during at least one transmission block, the equivalent received bandpass signal \( y_b(t) \) in one signal interval is obtained as [34, Chapter 14]

\[
y_b(t) = h x_b(t) + n(t)
\]

where \( n(t) \) is the additive white Gaussian noise (AWGN) at the receiver.

2.3 Statistical Models of Fading Channels

In radio propagation environments, it is difficult or even impossible to construct a precise deterministic channel model to characterize the effects of multipath propagation on the received signal. Instead, statistical models are usually utilized to represent multipath channels [11, 33, 34]. Depending on the specific type of radio propagation environment, each of the following statistical models, i.e., Rayleigh, Rician, and Nakagami-m fading, can be suitably utilized.

2.3.1 Rayleigh Fading

Rayleigh fading characterizes a propagation environment with a large number of obstacles between the transmitter and the receiver. As such, there is no line-of-sight (LOS) propagation path but there exist many propagation paths through reflections, scattering, and diffractions. The magnitude \( X = |h| \) of the channel impulse response \( h \), also called channel coefficient, follows a Rayleigh distribution. According to [33, Chapter 5], the probability density function (PDF) of \( X \) is defined as

\[
f_X(x) = \begin{cases} 
\frac{2x}{\Omega^2} \exp\left(-\frac{x^2}{\Omega^2}\right) & x \geq 0 \\
0 & x < 0 
\end{cases}
\]

where \( \Omega = E\{|h|^2\} \) is the channel mean power and \( E\{\cdot\} \) denotes the expectation operator. Then, the channel power gain \( Y = |h|^2 \) follows an exponential
distribution with mean $\Omega$. As a consequence, the PDF and cumulative distribution function (CDF) of $Y$ are given by

$$f_Y(y) = \begin{cases} \frac{1}{\Omega} \exp \left( -\frac{y}{\Omega} \right) & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (10)$$

$$F_Y(y) = \begin{cases} 1 - \exp \left( -\frac{y}{\Omega} \right) & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (11)$$

### 2.3.2 Rician Fading

Rician fading represents a propagation environment in which a LOS path between the transmitter and the receiver exists. Usually, the received signal corresponding to the LOS path is dominant as compared to the signal components received from the other paths. As in [33, Chapter 5], the PDF of the channel coefficient of a Rician fading channel can be expressed as

$$f_X(x) = \begin{cases} \frac{2(1+K)x}{\Omega} \exp \left( -K - \frac{(1+K)x^2}{\Omega} \right) I_0 \left( 2x \sqrt{K(1+K) \frac{x}{\Omega}} \right) & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (12)$$

where $I_0(\cdot)$ is the modified Bessel function of 0-th order [35, eq. (8.431)]. Furthermore, the parameter $K$ denotes the Rician factor which represents the power ratio of the LOS component to the non-LOS (NLOS) components. As given in [34], the PDF of the channel power gain $Y$ of a Rician fading channel is expressed as

$$f_Y(y) = \begin{cases} \frac{(1+K)y}{\Omega} \exp \left( -K - \frac{(1+K)y}{\Omega} \right) I_0 \left( 2 \sqrt{K(1+K) y \frac{y}{\Omega}} \right) & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (13)$$

Note that $K$ can assume any value in the range $[0, \infty)$. For $K = 0$, the Rician fading becomes Rayleigh fading since there exists no LOS component. If $K \rightarrow \infty$, the Rician fading becomes a free space environment without multipath components.

### 2.3.3 Nakagami-$m$ Fading

The Nakagami-$m$ fading characterizes a propagation environment where the wavelength of the carrier is proportional to the size of clusters of scatterers. As in [33, 34], the magnitude $X = |h|$ of the channel impulse response $h$ of
a Nakagami-\(m\) channel follows a Nakagami distribution. The PDF of \(X\) is expressed as

\[
f_X(x) = \begin{cases} 
\frac{2m^m x^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{mx^2}{\Omega}\right) & x \geq 0 \\
0 & x < 0
\end{cases}
\]  

(14)

where \(\Gamma(\cdot)\) is the gamma function defined as in [35, eq. (8.310.1)]. The parameter \(m\) denotes the fading severity parameter given in the range from 0.5 to \(\infty\). The fading channel becomes less severe as the value of the fading severity parameter \(m\) becomes larger. Furthermore, the channel power gain \(Y = |h|^2\) follows a gamma distribution, i.e., the PDF and CDF of \(Y\) are, respectively, given by

\[
f_Y(y) = \begin{cases} 
\frac{m^m y^{m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{my \Omega}{m}\right) & y \geq 0 \\
0 & y < 0
\end{cases}
\]  

(15)

\[
F_Y(y) = \begin{cases} 
1 - \frac{\Gamma(m, my/\Omega)}{\Gamma(m)} & y \geq 0 \\
0 & y < 0
\end{cases}
\]  

(16)

where \(\Gamma(\cdot, \cdot)\) is the incomplete gamma function [35, eq. (8.350.2)].

It should be mentioned that Nakagami-\(m\) fading represents the behavior of a variety of empirical propagation environments as special cases by setting the fading severity parameter \(m\) to a particular value [34]. For example, one-sided Gaussian fading is obtained for \(m=0.5\). When setting \(m=1\), we obtain Rayleigh fading. The Nakagami-\(m\) fading model can also closely approximate Rician fading with the relationship between parameter \(m\) of Nakagami fading and parameter \(K\) of Rician fading given as \(m = (K + 1)^2/(2K + 1)\).

### 3 Fundamentals of Cognitive Radios

The rapid development of wireless communications in the last decades has dramatically increased the scarcity of radio spectrum. However, measurement campaigns have shown that the current fixed spectrum allocation policy, coordinated by the Federal Communications Commission (FCC), is inefficient such that many allocated spectrum bands are under-utilized. This necessitates new spectrum allocation policies that can regulate the spectrum assignment in a more flexible and efficient manner. Cognitive radio [4, 5, 36] has been considered as a promising solution to address the inefficient spectrum utilization.

Fig. 2 shows main functions used in CRNs for spectrum management which will be discussed in the sequel. In particular, as illustrated in Fig. 2, spectrum sensing techniques, cognitive radio spectrum access, cognitive radio medium
access control (MAC), and routing in cognitive radio networks are discussed in the following sections.

3.1 Spectrum Sensing Techniques

A CR has formally been defined as a radio technology that allows an SU to adapt its transmitter parameters based on interaction with its environment [4, 5, 36, 37]. Before dynamically adapting the operating mode, SUs must be aware of essential information of the surrounding environment such as locally available radio spectrum and fading conditions. This requirement is referred as cognitive capability and is performed by spectrum sensing techniques [5, 38]. In this section, we will discuss the most well-known categories of spectrum sensing techniques [37, 39, 40].

3.1.1 Indirect Spectrum Sensing

Indirect spectrum sensing, also called primary transmitter detector, is a method in which the power spectrum density of the transmit signal from the primary transmitter is estimated [37, 41]. The three popular approaches of indirect spectrum sensing are discussed as follows:
**Energy detection**

Energy detection is the most common type of indirect spectrum sensing since it is easy to implement [42, 43]. Furthermore, the SU does not need prior knowledge about the primary signal.

Energy detection is performed as follows. Let $x(t)$ be the transmit signal of the primary transmitter and $n(t)$ be the AWGN at the SU. Then, the primary signal received at the SU is given by

$$y(t) = \begin{cases} n(t) & \mathcal{H}_0 \\ h(t)x(t) + n(t) & \mathcal{H}_1 \end{cases}$$

(17)

where $\mathcal{H}_0$ is the null hypothesis that the frequency band is idle while $\mathcal{H}_1$ is the hypothesis that the primary user is occupying the frequency band. Moreover, $h(t)$ is the channel coefficient from the primary transmitter to the local secondary receiver which is performing the spectrum sensing. Let $Y$ be the average energy of the detector at the SU over $N$ samples. Then, $Y$ can be calculated as

$$Y = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2$$

(18)

The decision on the occupancy of the spectrum band is then made by comparing the obtain average detected energy with a predefined threshold $\lambda$. Specifically, if $Y < \lambda$, the detector considers the spectrum band as being idle. Otherwise, the spectrum band is considered as being occupied by PUs.

The performance of the energy detector is sensitive to the noise at the SU, i.e., if the noise power at the SU is high, the energy detector can easily make a wrong decision on the presence of the primary user. To assess the performance of an energy detector, we define $P_D$ as the detection probability that the SU correctly senses the active state of the PU. Furthermore, $P_F$ is defined as the false alarm probability that the SU considers the licensed spectrum as being occupied by the PU, even though the PU is inactive. The detection probability $P_D$ and the false alarm probability $P_F$ of the energy detector can be expressed as

$$P_D = \Pr (Y \geq \lambda | \mathcal{H}_1)$$

(19)

$$P_F = \Pr (Y \geq \lambda | \mathcal{H}_0)$$

(20)

Accordingly, the missed detection probability and the no alarm probability can be obtained as $P_M = 1 - P_D$ and $P_N = 1 - P_F$, respectively. Thus, the
lower the missed detection and false alarm probabilities are, the better the performance of the energy detector.

**Matched filter detection**

When cognitive users have prior knowledge about the primary signal, a matched filter detection is often applied to perform spectrum sensing [44]. The advantage of the matched filter detection is its short sensing time with good performance in an AWGN environment. However, the SU needs to have prior knowledge of the primary signal such as pilot, preamble, training sequence, modulation, or packet format to perform coherent detection [45]. Therefore, in several circumstances, this method is impractical.

It is assumed that the PU simultaneously transmits a pilot signal with its data where the pilot signal is orthogonal and independent to the data. Furthermore, the sensing detector of the SU is assumed to have prior knowledge of the pilot signal and can perform its coherent processing. The principle of the matched filter detection method is described as follows. Let $x_p(t)$ and $x(t)$ be the known pilot signal and the desired signal of the primary transmitter, respectively. Further, $n(t)$ denotes the AWGN at the cognitive user. Then, there occur two hypotheses in the matched filter detection:

$$y(t) = \begin{cases} n(t) & \mathcal{H}_0 \\ \sqrt{\varepsilon} h(t) x_p(t) + \sqrt{1-\varepsilon} h(t) x(t) + n(t) & \mathcal{H}_1 \end{cases}$$ (21)

where $\varepsilon$ is the fraction of power allocated to the pilot signal, e.g., the power of the pilot signal is typically from 1 to 10 percent of the total transmitted power. Then, the output $Y$ of the matched filter of the detector over $N$ samples is obtained as

$$Y = \frac{1}{N} \sum_{n=1}^{N} y(n) \hat{x}_p^*(n)$$ (22)

where $^*$ denotes complex conjugation and $\hat{x}_p(n) = \sqrt{\varepsilon} x_p(n)$. By comparing $Y$ with a predefined threshold $\lambda$, a decision about the occupancy of the spectrum band by the primary user is made.

**Feature detection**

There are some features associated with the primary signal such as modulation rate and carrier frequency which possess cyclostationary characteristics [46, 47]. These features can be distinguished from AWGN since the noise
is generally wide-sense stationary with no correlation. Therefore, cyclostationary features can be utilized to distinguish noise from the primary signal in feature detectors.

The general principle of cyclostationary feature detection is performed as follows. First, the power spectrum density of the primary signal is calculated in the frequency-domain by applying the Fourier transform to the autocorrelation function of the estimated signal in the time-domain. Specifically, the cyclic spectrum autocorrelation function of the received signal is computed as

$$R^\alpha_y(\tau) = E\{y(t)y^*(t-\tau)e^{-j2\pi\alpha t}\}$$  \hspace{1cm} (23)

where $\alpha$ represents the cyclic frequency. Then, the cyclic spectrum density function of the primary signal is expressed as

$$S(f, \alpha) = \sum_{\tau=-\infty}^{\infty} R^\alpha_y(\tau)e^{-j2\pi f \tau}$$  \hspace{1cm} (24)

Since the noise is a non-cyclostationary signal, there is no peak in the cyclic spectrum density function under hypothesis $H_0$. If the cyclic frequency is equal to the frequency of the primary signal under hypothesis $H_1$, the cyclic spectrum density function has peaks. Based on this feature, the feature detection of the cognitive user is able to decide whether the frequency band is occupied by the primary user.

The advantage of the feature detection is that it can distinguish the primary signals from the noise or any interfering signals with different cyclic frequency. Nonetheless, feature detection has higher computational complexity compared to matched filter detection and energy detection.

### 3.1.2 Direct Spectrum Sensing

Direct spectrum sensing, also called primary receiver detector, is a method in which the power spectrum density is estimated based on the leakage signals from the primary receiver within the transmission range of an SU [37,40]. The two most well-known spectrum sensing techniques of direct spectrum sensing are discussed in the following:

**Local oscillator detection**

For further processing, in most wireless communication systems, the receiver signal is often converted to an intermediate frequency (IF) by shifting the carrier frequency [48]. In these systems, a local oscillator is used in order to down convert the RF band to IF band. In particular, the local oscillator is tuned to
a frequency that then is mixed with the incoming RF signal to generate the desired IF signal. In this process, inevitable oscillator leakage signals are produced. These leakage signals eventually come back to the input port and are emitted by the antenna. In this case, a cognitive user can sense these leakage signals to detect the presence of the primary receiver [48]. However, because the local oscillator leakage signals are often very weak, implementation of a local oscillator detection requires a long detection time. Furthermore, the SU needs to be located closely to the PU receiver to be able to detect the weak leakage signals.

**Proactive detection**

In wireless communication systems, feedback channels are widely utilized to deploy closed-loop control such as power control, adaptive modulation, adaptive coding, and automatic repeat request protocols to maintain the quality of the received signals. As such, an SU can monitor the feedback signals of the primary user to detect the presence of the primary transmission [49–52]. In [50,51], closed-loop power control in primary systems has been exploited to detect the presence of the primary receiver. Apart from power control, other closed-loop control messages such as acknowledgement (ACK) and negative acknowledgment (NACK) can be utilized to detect the primary transmission [52].

### 3.1.3 Cooperative Spectrum Sensing

Due to shadowing, multipath fading, and noise uncertainty, local spectrum sensing techniques do not always provide a reliable detection in a certain sensing time. By taking advantage of the spatial and multi-user diversity, cooperative sensing [53,54] has been proposed to improve the detection accuracy, i.e., decrease the missed detection probability to better protect the primary network. This sensing technique also reduces the false alarm probability to enhance the utilization of the idle spectrum. However, cooperative sensing results in additional overhead to exchange sensing data throughout the secondary network. Therefore, major issues of cooperative spectrum sensing are to select proper secondary users to perform sensing in a particular spectrum band and to effectively exchange sensing information between SUs. Based on these requirements, there are two main kinds of cooperative spectrum sensing:

**Centralized cooperative spectrum sensing**

In this type of cooperative spectrum sensing, there are central spectrum sensing controllers such as secondary base stations to manage the collaboration
of spectrum sensing [53]. Specifically, based on geographic locations, the secondary base station decides which SUs shall perform sensing in particular spectrum bands. Then, each SU independently makes decisions on the states of its observation bands and forwards the outcomes to the secondary base station. The secondary base station collects the sensing results from local SUs and makes the final decision about the occupancy of the bands by using a certain decision fusion rule. Finally, the secondary base station informs the SUs about the decision on the status of the frequency bands.

**Distributed cooperative spectrum sensing**

In this type of cooperative spectrum sensing, there is no backbone infrastructure for cooperative spectrum sensing [54]. In particular, each secondary user is responsible for choosing spectrum bands to sense as well as exchanging the local detection results among themselves. This topology of cooperative spectrum sensing is suitable for relaying communications where SUs operating in the same band can monitor the same range of frequencies.

### 3.2 Cognitive Radio Spectrum Access

The task of cognitive radio spectrum access is to coordinate the co-existence of the secondary users with primary users in a specific spectrum band. In general, there exist three approaches of cognitive radio spectrum access as shown in Fig. 3, each of which exploits a different technique to guarantee satisfactory performance for primary networks.

#### 3.2.1 Interweave Spectrum Access

Interweave spectrum access (ISA) [55, 56] is based on interference avoidance to completely get rid of the interference from the secondary transmission to the primary network. In ISA, the transmit powers of the SUs are not constrained under the interference power thresholds imposed by the PUs. However, in order to not interfere with the primary network, the SUs are strictly constrained on time or location when accessing the licensed spectrum. Specifically, the SUs must periodically monitor the radio spectrum bands to detect the occupancy status in the different parts of the spectrum and opportunistically communicate over spectrum holes (see Fig. 3a).

#### 3.2.2 Underlay Spectrum Access

Underlay spectrum access (USA) [57, 58] is based on interference control to protect the primary network. Specifically, in USA, the SU can simultaneously
access the spectrum with the PU provided that the interference from its transmission appears as noise at the primary receiver (see Fig. 3b). Specifically, the secondary user must strictly control its transmit power to meet the interference power constraint imposed by the primary receiver. Therefore, the transmit power of the SU is severely limited which leads to short range communication of the underlay cognitive radio. However, the secondary user does not need to exploit spectrum white spaces or to obtain prior knowledge of the primary codebooks.

3.2.3 Overlay Spectrum Access

Overlay spectrum access (OSA) [59, 60] is based on interference mitigation to reduce the interference from the secondary network to the primary network. Specifically, the SUs in OSA are allowed to concurrently access the frequency bands together with the PUs (see Fig. 3c). However, in order to not degrade the performance of the primary network, the SUs must have knowledge of the primary codebooks which then can be exploited in a variety of ways to either cancel or mitigate the interference at both the secondary and primary receivers. On one hand, the secondary transmitter may use this knowledge to cooperate with the primary user by assigning a part of its transmit power to
relay the primary signal. On the other hand, the knowledge of the primary codebooks can be utilized at the secondary receiver to extract the primary signal out of the received signal. By this approach, the spectrum utilization is improved.

3.3 Cognitive Radio Medium Access Control

Cognitive radio medium access control (CR-MAC) is designed to regulate the spectrum access of SUs while taking account of features of CRNs such as dynamics in spectrum availability and collision avoidance for PUs and SUs.

3.3.1 Centralized CR-MAC

In a centralized CR-MAC, there exists a controller that regulates the spectrum allocation among the SUs. The centralized CR-MAC can be deployed as either frame-level scheme [61] or call-level scheme [62].

In the frame-level scheme, the MAC protocol consists of two subframes, the uplink subframe from the SUs to the central controller and the downlink subframe from the central controller to the SUs. Specifically, the SUs send their requests of bandwidth in the uplink subframe and the central controller assigns portions of bandwidth to the SUs in the downlink subframe.

In the call-level scheme, the central controller regulates allocation of the available spectrum bands to the SUs and performs spectrum handoff when a PU reclaims its spectrum band. In particular, the central controller serves the PU and SU services as calls, i.e., dedicated frequency bands are assigned to the PU and SU calls. However, possessing the spectrum licence, whenever a PU reclaims the spectrum, the related ongoing SU calls should release the frequency bands. Then, the central controller must handoff the replaced SU calls to the available vacant frequency bands. Based on the particular QoS requirements, the central controllers propose suitable policies for SU admission, channel reservation, and handoff the displaced SUs.

3.3.2 Distributed CR-MAC

In the distributed CR-MAC protocol, there is no central controller. Instead, the SUs regulate the spectrum access by themselves through channel negotiation processes. Most of the distributed CR-MACs utilize a common control channel to coordinate multiple access among the SUs [63, 64]. The common control channel can be either an unlicensed spectrum band, e.g., 2.4 GHz, or a dedicated channel licensed to the SUs.

The general procedure of a distributed CR-MAC is described as follows. When an SU wants to transmit signals, it first uses contention mechanisms
such as carrier sense multiple access with collision avoidance (CSMA/CA) to take the control of the common control channel. Specifically, the SU selects a backoff counter with an initial contention window size. Whenever both the common control channel and at least one data channel are idle in a time slot, the counter is decreased by one. Otherwise, if either the common control channel is sensed active or no data channel is sensed idle, the counter is kept unchanged. When a collision occurs, the window size is set to the maximum value. Once the counter of the SU reaches zero, the SU starts the channel negotiation stage by exchanging handshaking messages with the other SUs in the common control channel. In particular, the secondary source first broadcasts a request to send (RTS) packet over the control channel. The secondary destination hears this packet. If it has detected at least one unoccupied licensed channel, it broadcasts a clear to send (CTS) packet to inform the source that it agrees to participate in the communication. Because the RTS and CTS packets are broadcasted over the control channel, other SUs can also overhear them to avoid reserving this control channel. By exchanging these control packets, the hidden terminal problem is effectively reduced since this process can prevent the other SUs from using the control channel simultaneously. Then, the secondary source and destination exchange a channel state transmitter (CST) packet, containing the available data channels at the source, and a channel state receiver (CSR) packet, containing the available channels at the destination. By exchanging the CST and CSR packets, the accuracy of the sensing results can be enhanced significantly. After exchanging the CST/CSR packets, the data transmission takes place over the chosen available bands.

3.4 Routing in Cognitive Radio Networks

The routing protocol is performed in the network layer to select the path route and to decide a time schedule for sending the data packet to avoid conflicting channel usage. Due to the characteristics of radio channels, routing metrics for wireless networks must deal with a number of challenges such as high error rates, high interference, signal variation, mobility of users, and transmit power limitations. Besides the basic challenges inherited from traditional wireless networks, a routing protocol for CRNs is even more challenging due to features of CRNs, e.g., the heterogeneity of spectrum resources and dynamic changes of spectrum availability. Therefore, routing algorithms for CRNs need to be aware of the dynamically available spectrum and the constraints imposed by the primary users. Furthermore, it must ensure performance such as high network capacity and throughput, short latency, and low packet loss for CRNs. In general, there are two approaches for designing routing protocols for CRNs, i.e., spectrum aware routing and statistic QoS control routing.
3.4.1 Spectrum Aware Routing

The spectrum aware routing approach [65, 66] aims at minimizing the interference at the PUs, i.e., the next routing node, among all potential nodes, will be allocated a frequency if the corresponding transmission through this route produces the lowest interference to the PUs. This routing method is also called opportunistic routing since the routing nodes are selected in a probabilistic manner based on the fading conditions. By taking advantage of the weak radio links from the selected nodes to the PUs, the interference from the CRN to the primary network is reduced. The work of [67] proposed a spectrum aware routing by exploiting a local spatial reuse algorithm. A spectrum aware routing within a coverage range was designed in [68] to minimize interference to the primary network while maximizing the throughput of the CRN and minimizing the delay of the CRN.

3.4.2 Statistic QoS Control Routing

In addition to spectrum aware routing, routing is also needed to provide a certain level of QoS for CRNs. To handle this requirement, the QoS statistics of nodes are utilized to assist the routing mechanisms to make a decision. To obtain prior knowledge of the QoS of all available nodes, the statistic history of packet transportation over a specific path needs to be observed. According to the reception history record of the destination node, the source node can estimate the probability of successful packet transmissions over each path in a given set of feasible relaying routes [69]. Based on this information, the routing algorithm selects an efficient route that satisfies the demand of QoS for a particular CRN. This method was also utilized in the original cognitive cycle concept with learning environment capabilities by Mitola [4]. Recently, this statistic QoS control routing approach has been widely applied in cross-layer design to select the optimal physical layer parameters and network layer parameters for CRNs [70, 71].

4 Advanced Radio Transmission Techniques

Subject to constraints imposed by the primary network, it is very challenging to provide satisfactory QoS for CRNs over error-prone radio channels. A powerful solution for this issue is to deploy advanced radio transmission techniques in CRNs such as cooperative communications, multiple-input multiple-output antennas, and adaptive transmission.
4.1 Cooperative Communications

In conventional radio communication systems where only a single link from the transmitter to the receiver is deployed, the transmission is unreliable due to deep fades. However, if the signal is sent over multiple independent paths, then the probability that all paths simultaneously experience deep fades is lower. Cooperative communications was developed to take advantage of this radio propagation characteristic and has gained overwhelming interest by the research community, e.g., [13, 72–83].

In cooperative networks, as shown in Fig. 4, relaying nodes \( R_k, \ k = 1, \ldots, K, \) are deployed to overhear the transmit signal from the source \( S. \) Then, the received signals are processed at the relays and forwarded to the destination \( D. \) At the destination, the independent copies of the transmit signal are combined to strengthen the received signal. In Fig. 4, \( H_{1k} \) and \( H_{2k} \) denote the channel coefficient matrices from the transmit antennas of the source \( S \) to the receive antennas of the relay \( R_k \) and from the transmit antennas of the relay \( R_k \) to the receive antennas of the destination \( D, \) respectively. Further, \( H_{10} \) is the channel coefficient matrix from the transmit antennas of \( S \) to the receive antennas of \( D. \)

In cooperative communications, each transmission cycle from the source to the destination consists of two phases. In the first phase, the source transmits the signal to both the relays and the destination. In the second phase, the relays process and forward the received signals from the source to the destination. The well-known relaying protocols are decode-and-forward (DF), amplify-and-forward (AF), and estimate-and-forward (EF), also called compress-and-forward (CF) or quantize-and-forward (QF) [76].

![Figure 4: Topology of a relay network.](image-url)
4.1.1 Amplify-and-Forward Protocol

In the AF relaying scheme, the relay simply forwards an amplified version of the transmitted signal without performing any further processing. The selection of the gain scalar at the relay depends on the channel state information (CSI) of the first hop from the source to the relay. If the relay is fully aware of the instantaneous channel power gain of the first hop, the relay deploys a variable-gain in each transmission. In this scheme, the gain scalar is adapted to the instantaneous value of the channel power gain from the source to the relay [84–87]. If the relay knows only the statistical distribution of the channel power gain of the first hop, the relay employs a fixed-gain for all transmissions [86]. In this case, the gain scalar is chosen based on the average channel mean power from the source to the relay [88].

At the destination, the replicas of the transmit signal via the direct and relaying links are combined. Specifically, for maximum ratio combining (MRC) or selection combining (SC), the maximum mutual information between the source and the destination are, respectively, obtained as [13]

\[
I_{AF}^{MRC} = \frac{1}{2} \log \left( 1 + \rho |h_0|^2 + \frac{\rho |h_1|^2 \rho |h_2|^2}{\rho |h_1|^2 + \rho |h_2|^2 + 1} \right)
\]

\[
I_{AF}^{SC} = \frac{1}{2} \max \left( \log(1 + \rho |h_0|^2), \log \left( 1 + \frac{\rho |h_1|^2 \rho |h_2|^2}{\rho |h_1|^2 + \rho |h_2|^2 + 1} \right) \right)
\]

where \( h_0, h_1, \) and \( h_2 \) are the channel coefficients of the links from the source to the destination, from the source to the relay, and from the relay to the destination, respectively. Further, \( \rho = P/N_0 \) is the average SNR, i.e., the ratio between the transmit power \( P \) of the source and the noise power \( N_0 \) at each terminal.

A drawback of the AF protocol is that not only the signal from the source is amplified but also the noise is accumulated and amplified at each relay. However, the AF protocol incurs low delay as compared to DF and CF since the AF protocol does not need to perform decoding or quantizing.

4.1.2 Decode-and-Forward Protocol

The DF relaying scheme performs an additional process to decode the signal received from the source. Then, the relay re-encodes the signal, and forwards the resulting signal to the destination [13,14,76,89,90]. According to [13], the DF protocol is further categorized into fixed DF and adaptive DF schemes.

In the fixed DF scheme [13,14,76,89], the relay is required to fully decode the signal received from the source. The communication process of a fixed DF relay network is described as follows. In the first time slot, the source
broadcasts the signal to both the relay and the destination. In the second time slot, the relay decodes the source signal, re-encodes, and forwards the resulting signal to the destination while the source keeps silent. For the fixed DF scheme, the maximum mutual information between the source and the destination is formulated as [13]

$$I_{DF}^F = \frac{1}{2} \min \left( \log(1 + \rho|h_1|^2), \log \left(1 + \rho|h_0|^2 + \rho|h_2|^2\right) \right) \quad (27)$$

It can be seen from (27) that the performance of a fixed DF relaying system is limited by the performance of the transmission from the source to the relay due to the requirement of fully decoding the source signal at the relay.

To overcome this performance limitation of a fixed DF relaying scheme, the adaptive DF relaying scheme has been proposed and utilized in [13, 14, 76, 89–91]. In this scheme, in spite of decoding and forwarding all the source symbols, the relay only participates in communications when it can correctly decode the signal from the source. Thus, only the successfully decoded signals are re-encoded and forwarded to the destination. In particular, if the received SNR at the relay in the first time slot is lower than a predefined threshold, it is assumed that the relay cannot successfully decode the signal from the source. Then, the relay remains silent while the source retransmits the signal in the second time slot. In this case, the received signal at the destination is obtained from the repeated transmissions over the direct link from source to destination. If the received SNR at the relay in the first time slot is higher than a predefined threshold, the relay decodes and then forwards the re-encoded signal to the destination in the second time slot. Given a target rate $R$, the maximum mutual information for the adaptive DF scheme is formulated as [13]

$$I_{DF}^A = \begin{cases} \frac{1}{2} \log(1 + 2\rho|h_0|^2), & |h_1|^2 < (2^{2R} - 1)/\rho \\ \frac{1}{2} \log(1 + \rho(|h_0|^2 + |h_2|^2)), & |h_1|^2 \geq (2^{2R} - 1)/\rho \end{cases} \quad (28)$$

### 4.1.3 Estimate-and-Forward Protocol

In the estimate-and-forward protocol [92–95], the relay quantizes and possibly compresses the received signal from the source. Then, the quantized signal is encoded and transmitted to the destination. In particular, in [94], an entropy constrained scalar quantization was utilized at the relay to estimate the received signal. Furthermore, in [95], an unconstrained minimum mean square error scheme was applied at the relay to estimate the received signal from the source. Then, the relay forwards the estimated signal to the destination. In [92], the received signal at the relay was first transformed by utilizing a
conditional Karhunen-Love transform. Then, the transformed signal streams were separately encoded to new analog signals by using Wyner-Ziv coding. At the destination, the independent versions of the transmit signals, i.e., the signal from the source and the quantized/compressed signal from the relay, were combined by using a specific combining technique.

It has been shown in [89] that the performance of AF relaying systems in the low SNR regime is better than those of the DF and EF relaying systems. However, in the high SNR regime, the performance of the systems associated with the DF and EF protocols is better than that of the AF protocol [89]. Furthermore, the DF protocol is most effectively in conditions where the quality of the channel from the source to the relay is quite good, i.e., the relay is located closely to the source. However, the EF protocol is useful when the relay is close to the destination. Since the AF and DF protocols are widely implemented in practice, the work presented in this thesis focuses on these two schemes.

4.2 MIMO System

Multiple-input multiple-output antennas can exploit the spatial diversity of multipath channels to significantly improve performance for wireless communication systems. Specifically, data rate is increased, transmission reliability is improved, spectral efficiency is enhanced, and radio coverage is extended [82, 96, 97]. MIMO techniques can be classified into two main categories, i.e., spatial multiplexing techniques and spatial diversity techniques.

Spatial multiplexing aims at increasing the bit rate of MIMO systems based on the following key principle. At the transmitter, the information sequence is split into $M$ subsequences in which each subsequence is modulated and transmitted simultaneously in the same frequency band over each transmit antenna. At the receiver, applying a suitable interference cancelation algorithm, the transmitted subsequences are separated and decoded. In this way, the MIMO system can obtain multiplexing gain.

In contrast to spatial multiplexing, the spatial diversity technique predominantly aims at improving error performance or transmission reliability of MIMO systems. By transmitting the same signal on multiple antennas, the MIMO systems can obtain a diversity gain and a coding gain [98]. However, it is very difficult to make a strict distinction between these MIMO techniques due to the trade-off between multiplexing gain with diversity gain and coding gain [26].

With the goal of improving the performance for CRNs, the work in this thesis focuses on spatial diversity techniques. Furthermore, when this technique is deployed in conjunction with adaptive modulation schemes, not only
the error performance of MIMO CRNs is improved but also the bit rate is increased. Based on the transmit and receive techniques, the spatial diversity technique can be further divided into transmit diversity and receive diversity.

### 4.2.1 Transmit Diversity

Transmit diversity techniques are deployed at the transmitters of MIMO or multiple-input single-output (MISO) antenna systems to achieve diversity gain and coding gain. By sending redundant signals over multiple transmit antennas, MIMO systems can achieve diversity gain with respect to the number of transmit antennas. In transmit diversity schemes such as maximum ratio transmission (MRT) [99], the same signals are sent via all the transmit antennas. However, in order to maximize the received SNR, the transmit signal for each antenna is multiplied with a different weight which depends on the fading condition.

Given the same number of transmit antennas, transmit antenna selection (TAS) has also been proven to obtain the same diversity gain for MIMO systems as compared to MRT [100]. In TAS MIMO systems, only a single antenna which maximizes the instantaneous SNR at the receiver is selected for transmission. Therefore, the complexity of the TAS MIMO system is reduced which comes at the cost of slightly decreased coding gain compared to MRT MIMO systems.

### 4.2.2 Receive Diversity

Receive diversity techniques are applied at the receivers of MIMO or single-input multiple-output (SIMO) antenna systems to combine the received signals from all receive antennas to obtain diversity gain. The most well-known combining diversity techniques are equal gain combining (EGC), MRC, and SC. In EGC [101], all the received signals at the receive antennas are multiplied with the same weight. In MRC [102,103], each individual received signal is multiplied with a weight based on the fading condition to maximize the SNR at the receiver. In contrast, in SC [104], only the maximum received signal is selected while all the other weaker signals are discarded. A comparison of these techniques was performed in [101]. This work has shown that, given a fixed number of receive antennas, EGC, MRC, and SC all achieve the same diversity gain but obtain different coding gain for MIMO systems. Among these techniques, MRC obtains the highest coding gain, but this technique requires perfect CSI at the receiver.
4.3 Adaptive Transmission

The basic concept of adaptive transmission is that communication systems adjust their transmission parameters to the variation of the fading channels in order to enhance spectrum efficiency or to improve the error performance [30,105,106]. This can be done by adapting transmit power, transmission rate, modulation constellation size, coding scheme, or their combination. To deploy adaptive transmission, the receiver needs to accurately estimate the channel state information and to reliably feed back this information to the transmitter. The results in [30,106] have shown that if both power and rate of a radio communication system are optimally adapted, the capacity of the system is slightly improved as compared to the capacity of the optimal rate adaptation system with a constant transmit power. This small increase in capacity becomes insignificant when the average transmit power becomes higher. However, when fixing transmission rate and only adapting power, the system obtains the lowest capacity, especially, when the amount of fading is high. Based on the results in [30,106], it can be concluded that rate adaptation is the key adaptive transmission scheme to achieve high spectrum efficiency. Therefore, the work in this thesis focuses on deploying rate adaptation for CRNs.

5 Performance Metrics

Numerous metrics have been utilized to evaluate the performance of radio communication systems. For the physical layer of radio communications, the three performance criteria that have been widely utilized are outage probability, symbol error rate, and channel capacity. The latter consists of ergodic capacity and outage capacity. In CRNs, the performance depends on the techniques deployed for regulating the co-existence between PUs and SUs. Thus, additional performance metrics need to be used to evaluate spectrum accessibility of SUs in CRNs. The most well-known metrics to measure the accessibility are blocking probability and dropping probability.

5.1 Outage Probability

Let us recall that noise and fading are represented as random variables, so the received signal is a random variable. If the instantaneous SNR at the receiver is less than a given threshold, then it is more likely that the receiver cannot successfully recover the transmit signal. This effect becomes more serious in slow fading in which the coherence time of the channel is quite large relative to the symbol interval. Therefore, in case of slow fading, outage probability
is commonly utilized to assess system performance. By definition, outage probability $P_{\text{out}}$ is the probability that the instantaneous SNR is lower than or equal to a predefined threshold $\gamma_{\text{th}}$, i.e.,

$$P_{\text{out}} = Pr\{\gamma_D \leq \gamma_{\text{th}}\} = \int_{0}^{\gamma_{\text{th}}} f_{\gamma_D}(\gamma) d\gamma \quad (29)$$

where $f_{\gamma_D}(\gamma)$ is the PDF of the instantaneous SNR $\gamma_D$.

### 5.2 Symbol Error Rate

Both symbol error rate (SER) [107] and bit error rate (BER) [108] are important performance measures to quantify the reliability of a radio communication link. However, the work in this thesis focuses on analyzing SER performance since the average BER can be obtained from the average SER for different modulation/detection schemes such as phase-shift keying (PSK) and quadrature amplitude modulation (QAM) [109].

For $M$-PSK, binary frequency-shift keying (BFSK) with orthogonal signaling, and pulse amplitude modulation (PAM), the SER over radio channels can be directly derived from the CDF $F_{\gamma_D}(\gamma)$ of the instantaneous SNR $\gamma_D$ as [107]

$$P_E = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-b\gamma}}{\sqrt{\gamma}} F_{\gamma_D}(\gamma) d\gamma \quad (30)$$

where $a$ and $b$ are positive modulation parameters depending on the modulation scheme, e.g., $a = 2, b = \sin^2(\frac{\pi}{M})$ for $M$-ary PSK; $a = 1, b = 0.5$ for BFSK; and $a = 2(M - 1)/M, b = 3/(M^2 - 1)$ for $M$-ary PAM.

For $M$-QAM constellations, the SER over radio channels can be derived from the PDF $f_{\gamma_D}(\gamma)$ of the instantaneous SNR $\gamma_D$ as

$$P_E = \int_{0}^{\infty} P_e(\gamma) f_{\gamma_D}(\gamma) d\gamma \quad (31)$$

Here, $P_e(\gamma)$ is the error probability of $M$-QAM constellations at the SNR $\gamma$ obtained as [109, 110]

$$P_e(\gamma) = aQ(\sqrt{b\gamma}) - c[Q(\sqrt{b\gamma})]^2 \quad (32)$$

where $a = \frac{4(\sqrt{M} - 1)}{\sqrt{M}}, b = 3 \log_2(\frac{M}{M-1}),$ and $c = \frac{4(\sqrt{M} - 1)^2}{M}$. 
5.3 Channel Capacity

In information theory, the channel capacity over radio channels is the maximum transmission rate that a communication system can achieve with arbitrarily small error probability. While Shannon capacity of a radio channel relates to the asymptotically zero error probability, outage capacity is used when the system tolerates a certain error probability.

5.3.1 Ergodic Capacity

Ergodic capacity or Shannon capacity is defined as the maximum mutual information between the input and output of the channel assuming that the communication duration is long enough to experience all channel states. The Shannon capacity of a discrete-time AWGN channel in bits/s is given by

\[ C = B \log_2(1 + \gamma) \]  

(33)

where \( \gamma \) is the SNR of an AWGN channel and \( B \) is the channel bandwidth. Shannon capacity of a radio channel is an average of Shannon capacity for an AWGN channel over the distribution of the received SNR, i.e.,

\[ C = E[B \log_2(1 + \gamma)] = \int_0^\infty B \log_2(1 + \gamma) f_{\gamma_D}(\gamma)d\gamma \]  

(34)

where \( f_{\gamma_D}(\gamma) \) is the PDF of the instantaneous SNR \( \gamma_D \).

Note that Shannon capacity of a given radio channel is considered as an upper bound on the transmission rate that a communication system can achieve with asymptotically zero error probability. In an extremely severe fading environment, to guarantee the asymptotically zero error probability, the transmission rate of the system is very low.

5.3.2 Outage Capacity

In the context of slow fading, outage capacity is often utilized. Outage capacity is defined as the largest rate that a system can transmit over a channel when the system tolerates a certain outage probability. i.e., the outage probability is less than a predefined threshold. The basic premise of outage capacity is that the system can transmit the signal with a high rate. However, the system is only able to successfully decode the signal in good fading conditions but it must tolerate a certain threshold of unsuccessfully decoding in bad fading conditions. By allowing to loose some data in the event of deep fades, the system can maintain a higher data rate than in the case of Shannon capacity where the system is required to correctly decode all data regardless of the fading state.
Let $F_{\gamma_D}^{-1}(\cdot)$ be the inverse function of the complementary CDF of the instantaneous SNR $\gamma_D$. Then, the outage capacity of the system in bits/s with respective to the outage probability threshold $\epsilon$ is obtained from [9, eq. (5.57)] with a transformation of variable as

$$C_\epsilon = B \log_2(1 + F_{\gamma_D}^{-1}(1 - \epsilon))$$

(35)

It is usually difficult to derive the inverse function of the complementary CDF of the instantaneous SNR $\gamma_D$. Therefore, outage capacity is often approximated as [111]

$$C_\epsilon = E\{C\} + \sqrt{2(E\{C^2\} - (E\{C\})^2)}erfc^{-1}\left[2 - \frac{\epsilon}{50}\right]$$

(36)

where $\epsilon$ in (36) is given in percent and $erfc[\cdot]$ is the complementary error function. Furthermore, $E\{C\}$ and $E\{C^2\}$ are the first and second moments of the ergodic capacity, respectively.

### 5.4 Blocking Probability and Dropping Probability

Blocking probability of SUs is the probability that an arrival SU is not allowed to access the spectrum when the system is full [112]. Dropping probability is the probability that an SU is interrupted by an arrival of a higher priority class [112, 113]. Although blocking probability and dropping probability are common performance measures for CRNs, unfortunately, there is no general formula for all types of CRNs. This is because blocking probability and dropping probability depend on the specific network topology, the statistical distribution of the arrival process, and the channel assignment policy applied in the CRN. An effective method to derive expressions for blocking probability and dropping probability is using a continuous-time Markov chain to determine the steady state distribution of each type of traffic class in the system [112–115].

A Markov chain is a random process with the property that, conditioned on its present state, the future state is independent of the past states. By definition, a random process $Y = \{Y_t|t \geq 0\}$ with discrete countable state space $S$ is a continuous-time Markov chain (CTMC) if the following conditions hold for all $j \in S$, and $t, t' \geq 0$ [116]:

- $Pr\{Y_{t+t'} = j|Y_{t''}, t'' \leq t'| = Pr\{Y_{t+t'} = j|Y_t\}$
- $Pr\{Y_{t+t'} = j|Y_t = i = Pr\{Y_t = j|Y_0 = i\}$

The first condition implies that the conditional probability of the future state $Y_{t+t'}$, given the present state $Y_t$ and the past states $Y_{t''}, 0 \leq t'' < t'$, depends
only on the present state and is independent of the past states. The second
condition says that the conditional probability of the future state $Y_{t+t'}$ only
depends on the difference between the observation points and is independent
of the selected reference point $t'$ of observation.

The evolution of a Markov chain is described by its transition probability
$p_{ij} = \Pr\{Y_{t+1} = j | Y_t = i\}$ from State $i$ to State $j$. Based on all the transition
probabilities, the transition matrix $P$ is constructed as an $S \times S$ matrix which
satisfies the two following conditions [117]:

- $P$ has non-negative entries, i.e., $p_{ij} \geq 0$ for all $i, j \in S$
- Sum of all entries of $P$ is equal to one, i.e., $\sum_{i,j \in S} p_{ij} = 1$

Define $\pi$ as the $1 \times S$ steady state vector of the Markov chain whose entries
$\pi_j, j \in S$, satisfy the following conditions [117]:

- $\pi_j \geq 0$ for all $j \in S$ and $\sum_{j \in S} \pi_j = 1$
- $\pi_j = \sum_{i \in S} \pi_i \cdot p_{ij}$ for all $j \in S$

Then, the vector $\pi$ is determined by solving the following equation:

$$\pi = \pi \cdot P \quad (37)$$

The steady-state probability of each state can also be derived as a function
of the arrival rates and departure rates of nodes in the Markov chain. In
particular, a linear equation system consisting of the flow-balance equations
and the normalized equation are constructed. The flow-balance equations
at all nodes of the Markov chain imply that the arrival rate of any node is
always equal to its departure rate. The normalized equation states that the
total probability of all states is always equal to one. By solving the linear
equation system, the steady state probabilities of all states in the Markov
chain are obtained.

6 Thesis Overview

This thesis aims at evaluating the performance of both spectrum sharing and
opportunistic spectrum access for CRNs. Firstly, the queueing performance
of single hop CRNs is assessed by utilizing embedded and multi-dimensional
Markov chains. Secondly, advanced radio transmission techniques are de-
ployed in two hop CRNs to improve the system performance. The impact of
practical factors such as fading conditions, statistical traffic and the interfe-
rence power threshold of the primary network, imperfect spectrum sensing,
and the transmit power limit of the secondary network on the performance of cognitive cooperative radio networks (CCRNs) is also investigated. This thesis consists of two research parts based on three journal articles and three conference papers.

In Part I, the performance of single hop underlay and interweave cognitive radio networks is investigated. Using the embedded Markov chain approach, the throughput, blocking probability, mean packet transmission time, and mean number of packets of underlay CRNs with finite buffer are examined. Then, utilizing a multi-dimensional Markov chain, the blocking and dropping probabilities for different prioritized traffics of CRNs applying dynamic spectrum access (DSA) are analyzed.

In Part II, advanced radio transmission techniques such as cooperative communication, MIMO techniques, adaptive transmission, and hybrid spectrum access are employed in two hop CRNs to improve the system performance. First, opportunistic spectrum access for cognitive AF relay networks is investigated. In this work, collaborative spectrum sensing is deployed among the secondary transmitter, secondary relay, and secondary receiver to enhance the accuracy of spectrum sensing. Then, the effect of feedback delay and channel estimation error on MIMO cognitive AF relay networks utilizing MRT/MRC are examined. Furthermore, an adaptive modulation and coding scheme is applied for cognitive incremental DF relay networks to increase the spectrum efficiency and to avoid the overflow of the queue at the relay. Finally, by considering the impact of the primary arrival rate and the interference constraint of the primary network, a hybrid interweave-underlay spectrum access is deployed for cognitive cooperative radio networks to inhere benefits of both interweave and underlay spectrum access.

6.1 Part I - Single Hop Cognitive Radio Networks

Part I-A: On the Performance of Underlay Cognitive Radio Networks Using $M/G/1/K$ Queueing Model

Motivated by a realistic scenario for cognitive radio systems, we model the underlay CRNs under interference power constraint imposed by the primary network as an $M/G/1/K$ queueing system. The respective embedded Markov chain is provided to analyze several key queueing performance measures. In particular, the equilibrium probabilities of all states are derived and utilized to evaluate throughput, blocking probability, mean packet transmission time, mean number of packets in the system, and mean waiting time of an underlay CRN with Nakagami-$m$ fading channels.
Part I-B: Dynamic Spectrum Access for Cognitive Radio Networks with Prioritized Traffics

In this part, we develop a dynamic spectrum access strategy for cognitive radio networks where prioritized traffic is considered. Assume that there are three classes of traffic, one traffic class of the primary user and two traffic classes of the secondary users, namely, Class 1 and Class 2. The traffic of the primary user has the highest priority, i.e., the primary users can access the spectrum at any time with the largest bandwidth demand. Furthermore, Class 1 has higher access and handoff priority as well as larger bandwidth demand as compared to Class 2. To evaluate the performance of the proposed DSA, we model the state transitions for DSA as a multi-dimensional Markov chain with three-state variables which present the number of packets in the system of the primary users, the secondary Class 1, and secondary Class 2. In particular, the blocking probability and dropping probability of the two secondary traffic classes are assessed.

6.2 PART II - Two Hop Cognitive Radio Networks

Part II-A: Opportunistic Spectrum Access for Cognitive Amplify-and-Forward Relay Networks

In this part, we study the performance of cognitive AF relay networks where the secondary users opportunistically access $M$ licensed bands of the primary users over Nakagami-$m$ fading channels. In order to enhance the accuracy of spectrum sensing and strongly protect the primary users from being interfered by the secondary transmission, collaborative spectrum sensing is deployed among the secondary transmitter, secondary relay, and secondary receiver. In particular, an analytical expression for the capacity of the considered network is derived. Numerical results are provided to show the influence of the arrival rate of the primary users on the channel utilization of licensed bands. Finally, the impact of the number of licensed bands, channel utilization of the primary users, false alarm probability, and transmission distances on the capacity of the considered system are investigated.

Part II-B: MRT/MRC for Cognitive AF Relay Networks Under Feedback Delay and Channel Estimation Error

In this part, we examine the performance of multiple-input multiple-output (MIMO) cognitive amplify-and-forward (AF) relay systems with maximum ratio transmission (MRT). In particular, closed-form expressions in terms of a tight upper bound for outage probability (OP) and symbol error rate (SER)
of the system are derived when considering channel estimation error (CEE) and feedback delay (FD) in our analysis. Through our works, one can see the impact of FD and CEE on the system as well as the benefits of deploying multiple antennas at the transceivers utilizing the spatial diversity of an MRT system. Finally, we also provide a comparison between analytical results and Monte Carlo simulations for some examples to verify our work.

Part II-C: Adaptive Modulation and Coding with Queue Awareness in Cognitive Incremental Decode-and-Forward Relay Networks

This part studies the performance of adaptive modulation and coding in a cognitive incremental decode-and-forward relaying network where a secondary source can directly communicate with a secondary destination or via an intermediate relay. To maximize transmission efficiency, a policy which flexibly switches between the relaying and direct transmission is proposed. In particular, the transmission, which gives higher average transmission efficiency, will be selected for the communication. Specifically, the direct transmission will be chosen if its instantaneous signal-to-noise ratio is higher than one half of that of the relaying transmission. In this case, the appropriate modulation and coding scheme (MCS) of the direct transmission is selected only based on its instantaneous SNR. In the relaying transmission, since the MCS of the transmissions from the source to the relay and from the relay to the destination are implemented independently to each other, buffering of packets at the relay is necessary. To avoid buffer overflow at the relay, the MCS for the relaying transmission is selected by considering both the queue state and the respective instantaneous SNR. Finally, a finite-state Markov chain is modeled to analyze key performance indicators such as outage probability and average transmission efficiency of the cognitive relay network.

Part II-D: Hybrid Interweave-Underlay Spectrum Access for Cognitive Cooperative Radio Networks

In this part, we study a hybrid interweave-underlay spectrum access system that integrates AF relaying. In hybrid spectrum access, the secondary users flexibly switch between interweave and underlay schemes based on the state of the primary users. A continuous-time Markov chain is proposed to model and analyze the spectrum access mechanism of this hybrid cognitive cooperative radio network. Utilizing the proposed Markov model, steady-state probabilities of spectrum access for the hybrid CCRN are derived. Furthermore, we assess performance in terms of outage probability, symbol error rate, and ou-
tage capacity of this CCRN for Nakagami-$m$ fading with integer values of fading severity parameter $m$. Numerical results are provided showing the effect of network parameters on the secondary network performance such as the primary arrival rate, the distances from the secondary transmitters to the primary receiver, the interference power threshold of the primary receiver in underlay mode, and the average transmit signal-to-noise ratio of the secondary network in interweave mode. To show the performance improvement of the CCRN, comparisons for outage probability, SER, and capacity between the conventional underlay scheme and the hybrid scheme are presented. The numerical results show that the hybrid approach outperforms the conventional underlay spectrum access.

7 Future Works

The research interests presented in this thesis are mainly in the field of cognitive radio networks with queueing theory and advanced radio transmission techniques. In particular, first, multi-dimensional and embedded Markov chains are utilized to evaluate the queueing behavior of single hop CRNs. Then, the performance improvement of two hop cognitive radio networks when incorporating advanced radio transmission techniques has been analyzed. The results presented in this thesis have demonstrated the effectiveness of the derived analytical frameworks to evaluate the performance of CRNs in lower layers which can be further developed in the future by the following ways:

Our ongoing research continues to focus on CRNs. However, in future works, we are interested in developing analytical frameworks to assess the system performance related to higher layers as well as proposing MAC, congestion and flow control mechanisms for CCRNs. Optimal power allocation for such CCRNs can be developed to further improve the system performance. Furthermore, developing innovative transmission strategies for hybrid CCRNs will be very interesting and meaningful to enhance the system efficiency. The future research can also be extended by considering the geometric locations of the nodes in CRNs which better reflects to practical scenarios. Some of the research to be pursued in the future are described in the following.

Research related to the MAC layer

Although MAC protocols have been extensively studied in the literature, developing MAC for CCRNs still remains challenging due to the uncertainty of spectrum availability for secondary users. Therefore, it necessitates more research efforts in developing MAC protocols for CCRNs to coordinate channel allocation of SUs. Among distributed and centralized MAC protocols, the
distributed MAC protocol is more favorable since it is able to cope with the drawback of the capacity limitation of the access point in the centralized MAC protocol. To enhance the accuracy of spectrum sensing, cooperative sequential spectrum sensing should be adopted where the secondary users mutually exchange the sensing results. By integrating the information obtained in spectrum sensing at the physical layer into packet scheduling at the MAC layer, the hidden terminal problem can be significantly alleviated. Finally, an analytical framework for the CCRN should be derived to assess the benefits of the novel MAC protocols.

Deploying optimal power allocation in CCRNs

Considering joint optimal powers of a secondary transmitter and secondary relays, a suitable power allocation policy can be proposed for a hybrid interweave-underlay CCRN to improve the system performance. In particular, the transmit powers and spectrum access modes of the secondary transmitter and secondary relays are optimally adapted to the state of the primary user, the fading conditions as well as the position of each terminal in the network. Furthermore, the practical issue of imperfect spectrum sensing at SUs should be considered. Then, the effect of the missed detection and false alarm probabilities need to be taken into account when deriving the optimal powers and spectrum access modes for the secondary transmitter and relay. Finally, the performance for the CCRN should be quantified to evaluate the advantages of the proposed power allocation strategy.

Stochastic geometry approach for CCRNs

A wireless communication network is considered as a set of nodes which are located in a particular location. At a given time instant, several pairs of transmit and receive nodes simultaneously perform communication over their own radio links. However, due to the broadcasting nature of wireless communications, the received signal at a given node may be jammed by the transmit signals from other adjacent transmissions. Therefore, geometrical locations of the nodes in a wireless communication network play a crucial role to determine the signal-to-interference plus noise ratio at a particular receiver. Stochastic geometry provides powerful mathematical and excellent statistical tools for analyzing and optimizing such networks by treating the network as a stationary random model at a particular location and a particular time instant. In the context of MIMO cognitive radio networks, stochastic geometry models become very helpful to optimize transmit and receive beamforming vectors to mitigate the intra-, inter-, and cross-interferences between the primary and cognitive radio networks.
References


Part I-A
PART I-A

On the Performance of Underlay Cognitive Radio Networks Using $M/G/1/K$ Queueing Model
Part I-A is published as:

On the Performance of Underlay Cognitive Radio Networks Using M/G/1/K Queueing Model

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Abstract

Motivated by a realistic scenario for cognitive radio systems, we model the underlay cognitive radio networks (CRNs) under interference power constraint imposed by the primary network as an M/G/1/K queueing system. The respective embedded Markov chain is provided to analyze several key queueing performance measures. In particular, the equilibrium probabilities of all states are derived and utilized to evaluate throughput, blocking probability, mean packet transmission time, mean number of packets in the system, and mean waiting time of an underlay CRN with Nakagami-\(m\) fading channels.

1 Introduction

Inspired by improving the spectrum utilization, cognitive radio networks (CRNs) have been proposed to grant opportunity to unlicensed users, so-called secondary users, to exploit the frequency bands licensed to primary users according to particular policies. In order to not impair the primary networks, secondary users must dynamically adapt their operating parameters such as transmit power, transmission time, and so on. Specifically, secondary transmitters in an overlay CRN suffer from a transmission time constraint; they are permitted to transmit their signals only when the primary users are idle.
In contrast, secondary transmitters in underlay CRNs can operate simultaneously with the primary users; they, however, must limit their transmit power to meet the interference constraint imposed by the primary users [1]. Due to these constraints, the secondary network undergoes higher transmission delay and transmission error as compared to conventional systems.

A great deal of recent studies have focused on analyzing outage probability or symbol error rate for CRNs (see [2, 3], for example). However, only few works concern about queueing behavior such as transmission delay and blocking probability for CRNs. Particularly, [4] has modeled a spectrum sharing system as an M/G/1 queueing system with infinite capacity, Poisson arrival process, and deterministic departure process to quantify system performance. Furthermore, upon the assumption of Poisson arrival and deterministic departure processes, in [5], an M/D/1 infinite queue model for opportunistic spectrum access was investigated as a special case of an M/G/1 queuing system. However, the works of [4, 5] have not considered the time-out transmission issue and queue length of the system, i.e., assume infinite queueing buffer. In fact, queuing systems with time-out transmission are typically considered in communication systems since the transmission time of packets cannot reach infinity [6]. Further, all service facilities have finite resources, thus finite buffers may be a better presentation of a realistic system than infinite buffers.

In this paper, using the embedded Markov chain approach, we investigate and quantify queueing performance for an underlay CRN over Nakagami-\(m\) fading which has a finite buffer at the secondary transmitter. The arrival process follows a Poisson distribution and the transmitter serves packets with a rate being equal to the capacity of the cognitive channel, thus our system is modeled as an M/G/1/K queueing system. Queueing performance measures are investigated when considering time-out transmission. Furthermore, the impact of system parameters including the queue length, arrival rates, and fading parameters on the queueing performance are illustrated.

2 System and Channel Model

We consider an underlay CRN, including a secondary transmitter SU\(_{TX}\) and receiver SU\(_{RX}\), that operates under the interference power constraint of a primary receiver PU\(_{RX}\). Denote \(h_s\) and \(h_p\) as the channel coefficients of the links SU\(_{TX} \to SU_{RX}\) and SU\(_{TX} \to PU_{RX}\), respectively. Assume that \(h_s\) and \(h_p\) are perfectly known to SU\(_{RX}\) and SU\(_{RX}\) is located far away from the primary transmitter PU\(_{TX}\) such that the interference from PU\(_{TX}\) to SU\(_{RX}\) can be neglected as in [7]. Let \(x_s\) be the transmit signal at SU\(_{TX}\) with average
power $P_s$. Then, the received signal at SU$_{RX}$ is given by

$$y_s = h_s x_s + n_s$$  \hspace{1cm} (1)$$

where $n_s$ is the additive white Gaussian noise (AWGN) at SU$_{RX}$ with zero-mean and variance $N_0$. Under the interference power constraint $Q$ of the PU$_{RX}$, the SU$_{TX}$ must control its transmit power to satisfy $P_s \leq Q/|h_p|^2$. Specifically, the SU$_{TX}$ will transmit the signal with average power $P_s = \min\{P_{\text{max}}, Q/|h_p|^2\}$ where $P_{\text{max}}$ is the transmit power limit of SU$_{TX}$. As a result, the instantaneous signal-to-noise ratio (SNR) of the system is found as

$$\gamma_D = \min\left\{\beta X_s, \frac{\mu X_s}{X_p}\right\}$$  \hspace{1cm} (2)$$

where $\beta = P_{\text{max}}/N_0$, $\mu = Q/N_0$, and $X_i = |h_i|^2$, $i \in \{s,p\}$.

Let the maximum number of packets in the cognitive system be $K$, i.e., one packet being served and $K - 1$ packets waiting in the buffer. Due to the finite buffer length, when the system is full, new arriving packets cannot access the system and have to leave without being served. Assume that packet arrivals follow a Poisson distribution with average arrival rate $\lambda$ and packets are transmitted with a rate equal to the capacity of the channel from SU$_{TX}$.
to SU_{RX}. Thus, the M/G/1/K queueing model is deployed to model the considered system as in Fig. 1. Let us define the number of packets in the system at a particular time as the system state at that time. As in the ordinary M/G/1 queueing system, the embedded Markov states shown in Fig. 1 are observed immediately after a service completion.

3 Queueing Performance Analysis

Assume that SU_{TX} transmits the packets in its buffer based on the first-come first-serve (FCFS) principle with processing rate being equal to the channel capacity $C$ of the cognitive network. According to the Shannon theorem, with a bandwidth $B$, the channel capacity is computed as $C = B \log_2(1 + \gamma_D)$ in bits/sec. Assume that the coherence time of all the channels remains constant during each packet transmission interval and varies independently for every packet transmission duration. Consequently, the packet transmission time $T$ is given by

$$T = \frac{1}{b \log_2 (1 + \gamma_D)} \text{ (sec/packet)}$$

where $b = B/N$, and $N$ is the number of bits per packet. From (3), the cumulative distribution function (CDF) of $T$ is

$$F_T(t) = 1 - F_{\gamma_D}(2^{1/(bt)} - 1)$$

By applying the total probability theorem and order statistic theory to (2), the CDF of $\gamma_D$ can be rewritten as $F_{\gamma_D}(\gamma) = \int_0^\infty F_X\left(\frac{\gamma}{x}\right) f_{X_p}(x)dx$. Here, $X = \min\{\beta, \frac{\mu}{\Omega_i}\}$ and $f_V(\cdot)$ denotes the probability density function (PDF) of random variable $V$. After some manipulations, $F_{\gamma_D}(\gamma)$ is further expressed as

$$F_{\gamma_D}(\gamma) = \int_0^\infty \left[1 - H\left(\beta - \frac{\gamma}{x}\right) F_{X_p}\left(\frac{\mu x}{\gamma}\right)\right] f_{X_s}(x)dx$$

$$= 1 - \int_{\gamma/\beta}^\infty F_{X_p}\left(\frac{\mu x}{\gamma}\right) f_{X_s}(x)dx$$

(5)

where $H(\cdot)$ is the Heaviside step function and $X_i, i \in \{s, p\}$, has gamma distribution with parameter set $(m_i, \alpha_i^{-1})$. Here, $m_i$ is fading severity, $\alpha_i = m_i/\Omega_i$, and $\Omega_i$ is channel mean power. Using positive integer values of $m_i$,
expressions for \( f_{X_s}(x) \) and \( F_{X_p}(x) \) are, respectively, given by

\[
f_{X_s}(x) = \frac{\alpha_s^m_s}{\Gamma(m_s)} x^{m_s-1} e^{-\alpha_s x}
\]

\[
F_{X_p}(x) = 1 - \sum_{i=0}^{m_p-1} \frac{\alpha_p^i x^i e^{-\alpha_p x}}{i!}
\]

where \( \Gamma(\cdot) \) denotes the gamma function \([8, eq. (8.310.1)]\). Substituting (6) and (7) into (5) together with the help of \([8, eq. (3.381.1)], [8, eq. (3.381.4)],\) and \([8, eq. (8.352.2)]\), we obtain \( F_{\gamma_D}(\gamma) \). Finally, the CDF of \( T \) can be found by substituting this outcome into (4) as

\[
F_T(t) = \sum_{l=0}^{m_s-1} \frac{\alpha^l_s \theta(t)^l}{l!\beta^l} e^{-\alpha_s \theta(t)/\beta} - \sum_{i=0}^{m_p-1} \frac{\Gamma(m_s + i) \mu^i}{\Gamma(m_s)}
\]

\[
\times e^{-\alpha_p \mu/\beta} \sum_{j=0}^{m_s+i-1} \frac{\alpha_p^j \alpha_s^{j-i}}{j!\beta^j} \left( \frac{\theta(t)^m_s e^{-\alpha_s \theta(t)/\beta}}{(\theta(t) + \alpha_p \mu/\alpha_s)^{m_s+i-j}} \right)
\]

where \( \theta(t) = 2^{1/(bt)} - 1 \). Taking the derivative of \( F_T(t) \) with respect to variable \( t \), the PDF of \( T \) can be given by

\[
f_T(t) = \log(2)(\theta(t) + 1) \left[ \sum_{l=0}^{m_s-1} \frac{\alpha^l_s \theta(t)^l}{l!\beta^l} e^{-\alpha_s \theta(t)/\beta} \right]
\]

\[
- \sum_{l=0}^{m_s-1} \frac{l \alpha^l_s \theta(t)^l e^{-\alpha_s \theta(t)/\beta}}{l!\beta + 1} - \sum_{i=0}^{m_p-1} \frac{\mu^i}{i!} \frac{\Gamma(m_s + i)}{\Gamma(m_s)}
\]

\[
\times e^{-\alpha_p \mu/\beta} \sum_{j=0}^{m_s+i-1} \frac{\alpha_p^j \alpha_s^{j-i}}{j!\beta^j} \left( \frac{\theta(t)^m_s}{(\theta(t) + \alpha_p \mu/\alpha_s)^{m_s+i-j}} \right)
\]

\[
\times e^{-\alpha_s \theta(t)/\beta} \left( -\frac{m_s}{\theta(t)} + \frac{\alpha_s}{\beta} + \frac{m_s + i - j}{\theta(t) + \alpha_p \mu/\alpha_s} \right)
\]

\[
(9)
\]

### 3.1 Packet Transmission Time

In the considered CRN, we take the problem of time-out into account. In particular, if the SU_{RX} successfully receives a packet before a predefined time-out \( t_o \), it will send an acknowledgement (ACK) to SU_{TX}. If SU_{TX} receives an ACK within \( t_o \), it considers the transmitted packet as being successfully received and continues with transmitting the next packet. Otherwise, SU_{TX}
waits until time-out occurs, declares the packet to be dropped and continues transmitting the subsequent packet. Thus, the probability that a packet is being dropped is obtained as

$$P_d = \Pr\{T > t_o\} = 1 - F_T(t_o)$$  \hspace{1cm} (10)$$

Let \(T_o\) be a random variable representing packet transmission time when considering time-out. Because the transmitter will send the next packet whenever time-out occurs, the transmission time of dropped packets is equal to \(t_o\). Hence, the probability that the transmission time is equal to \(t_o\) will be \(f_{T_o}(t_o) = P_d\). Consequently, the PDF of packet transmission time \(T_o\) will be calculated as in (11).

$$f_{T_o}(t) = \begin{cases} 
\log(2) \frac{\beta}{b^2} \left[ \sum_{l=0}^{m_{s}-1} \frac{\alpha_{l+1}^{i} \theta(t)^l}{l! \beta^{l+1}} e^{-\alpha_s \theta(t)/\beta} - \sum_{l=0}^{m_{s}-1} \frac{\alpha_{l}^{i} \theta(t)^l}{l! \beta^{l}} e^{-\alpha_s \theta(t)/\beta} \right] \\
\times e^{-\alpha_s \theta(t)/\beta} - \sum_{i=0}^{m_{p}-1} \frac{\mu_i}{\beta} \Gamma(m_{s}+i) e^{-\alpha_p \mu_i/\beta} \sum_{j=0}^{m_{s}+i-1} \frac{\alpha_{j}^{i} \alpha_{j+1}^{i-i} \theta(t)^j}{j! \beta^{j}} e^{-\alpha_s \theta(t)/\beta} \\
\times \frac{1}{(\theta(t)+\alpha_p \mu/\alpha_s)^{m_{s}+i+2}} e^{-\alpha_s \theta(t)/\beta} \left( -\frac{m_s}{\beta} + \frac{\alpha_s}{\beta} + \sum_{j=0}^{m_{s}+i-j} \frac{1}{j!} \right) \\
0, \hspace{1cm} 0 < t < t_o \\
1 - \sum_{i=0}^{m_{p}-1} \frac{\mu_i}{\beta} e^{-\alpha_s \theta(t)/\beta} + \sum_{i=0}^{m_{p}-1} \frac{\Gamma(m_{s}+i) \mu_i}{\Gamma(m_{s}) \beta^{i}} e^{-\alpha_p \mu_i/\beta} \sum_{j=0}^{m_{s}+i-1} \frac{1}{j!} \\
\times \frac{\alpha_{j}^{i} \alpha_{j+1}^{i-i}}{(\theta(t)+\alpha_p \mu/\alpha_s)^{m_{s}+i+2}} e^{-\alpha_s \theta(t)/\beta} \left( -\frac{m_s}{\beta} + \frac{\alpha_s}{\beta} + \sum_{j=0}^{m_{s}+i-j} \frac{1}{j!} \right), \hspace{1cm} t = t_o \\
0, \hspace{1cm} t > t_o 
\end{cases}$$  \hspace{1cm} (11)$$

As a result, mean packet transmission time \(\overline{T_o}\) can be obtained as

$$\overline{T_o} = \log(2) \left[ \sum_{l=0}^{m_{s}-1} \frac{\alpha_{l+1}^{i} \phi(l, 0)}{l! \beta^{l+1}} - \sum_{l=0}^{m_{s}-1} \frac{\alpha_{l}^{i} \phi(l-1, 0)}{l! \beta^{l}} - \sum_{i=0}^{m_{p}-1} \frac{\mu_i}{i!} \right] \\
\times \frac{\Gamma(m_{s}+i)}{\Gamma(m_{s})} e^{-\alpha_p \mu/\beta} \sum_{j=0}^{m_{s}+i-1} \frac{\alpha_{j}^{i} \alpha_{j+1}^{i-i} \phi(m_{s}-1, m_{s}+i-j) + \frac{\alpha_s}{\beta} \phi(m_{s}, m_{s}+i-j) + (m_{s}+i-j) \phi(m_{s}, m_{s}+i+1-j)}{j! \beta^{j}} \right] + t_o P_d$$  \hspace{1cm} (12)$$

where \(\phi(\delta_1, \delta_2) = \int_{0}^{t_o} \frac{\phi(t+1) \phi(t)^{i-1}}{[\phi(t+\mu \alpha_p/\alpha_s)]^2} e^{-\alpha_s \theta(t)/\beta} \, dt\).
3.2 Queueing Analysis

Let $\alpha(k)$ be the probability of $k$ new packets entering the system during a service time. In view of queueing theory [9], with Poisson arrival processes, we have

$$\alpha(k) = \int_0^\infty \frac{(\lambda t)^k \exp(-\lambda t)}{k!} f_{T_0}(t) \, dt$$  \quad (13)

Substituting (11) in (13) and after simplifications, we obtain

$$\alpha(k) = \frac{\lambda^k \log(2)}{k! b} \left[ \sum_{l=0}^{m_s-1} \frac{\alpha_s^{l+1}}{l! \beta^{l+1}} \psi(l, 0) - \sum_{l=0}^{m_s-1} \frac{l \alpha_s \psi(l-1, 0)}{l! \beta} \right]$$

$$- \sum_{i=0}^{m_p-1} \frac{1}{i!} \frac{\Gamma(m_s + i)}{\Gamma(m_s)} \mu^i e^{-\alpha_p \mu / \beta} \sum_{j=0}^{m_s+i-1} \frac{1}{j! \beta^j} \alpha_p \alpha_s^{j-i}$$

$$\times \left( -m_s \psi(m_s - 1, m_s + i - j) + \frac{\alpha_s \psi(m_s, m_s + i - j)}{\beta} ight)$$

$$+ (m_s + i - j) \psi(m_s, m_s + i + 1 - j) \right] \right] + \frac{(\lambda b)^k e^{-\lambda b}}{k!} P_d$$  \quad (14)

where $\psi(\omega_1, \omega_2) = \int_0^t \frac{e^{-\alpha \mu t + \alpha_s \theta(t)}}{(\theta(t) + \mu \alpha_p / \alpha_s)^{m_s+i}} e^{-[\lambda \beta t + \alpha_s \theta(t)]/\beta} \, dt$. Let $p_{j,k}$ be the transition probability from state $j$ to state $k$ in the embedded Markov chain of Fig. 1. As in [9], the expression for $p_{j,k}$ is summarized as

$$p_{0,k} = \begin{cases} \alpha(k), & 0 \leq k \leq K - 2 \\ \sum_{i=K-1}^{\infty} \alpha(i), & k = K - 1 \end{cases}; \quad j = 0$$  \quad (15)

$$p_{j,k} = \begin{cases} \alpha(k - j + 1), & j - 1 \leq k \leq K - 2 \\ \sum_{i=K-j}^{\infty} \alpha(i), & k = K - 1 \end{cases}; \quad 1 \leq j \leq K - 1$$  \quad (16)

Denote $p_k, k = 0, \ldots, K - 1$, as the equilibrium state probability that $k$ jobs are in the system immediately after a service completion. Note that $p_k$ can be obtained by solving the balance equation system $p_k = \sum_{j=0}^{K-1} p_{j,k} p_j, k =$
0, \ldots, K - 1, [9, eq. (1.8a)] and normalized equation \( \sum_{k=0}^{K-1} p_k = 1, [9, eq. (1.8b)] \).

Since there exist \( K \) system states from 0 to \( K - 1 \) immediately after a service completion, only \( K - 1 \) independent balance equations and normalized equation are needed to calculate \( p_k, k = 0, \ldots, K - 1 \). In our case, we choose the first \( K - 1 \) equations \( p_k = \sum_{j=0}^{K-1} p_j, p_{k+1} = 0 \), and the normalized equation. Substituting (15) and (16) into these equations, the set of \( K \) needed equations is given by

\[
\begin{align*}
\left[ \alpha(0) - 1 \right] p_0 + \alpha(0) p_1 &= 0 & \text{for } k &= 0 \\
\alpha(k) p_0 + \sum_{j=0}^{k-1} \alpha(k - j + 1) p_j + \left[ \alpha(1) - 1 \right] p_k + \alpha(0) p_{k+1} &= 0 & \text{for } k &= 1, 2, \ldots, K - 2 \\
p_0 + p_1 + \ldots + p_{K-1} &= 1
\end{align*}
\]

(17)

Let \( p \) be a \( K \times 1 \) vector of all the equilibrium state probabilities at time instants immediately after a service completion \( p = \left( p_0, p_1, \ldots, p_{K-1} \right)^T \) and \( b \) is a \( K \times 1 \) vector defined as \( b = \left( 0, 0, \ldots, 1 \right)^T \). Furthermore, we denote \( A \) as a \( K \times K \) matrix as follows:

\[
A = \begin{pmatrix}
\alpha(0) - 1 & \alpha(0) & 0 & 0 & \ldots & 0 \\
\alpha(1) & \alpha(1) - 1 & \alpha(0) & 0 & \ldots & 0 \\
& \ddots & \ddots & \ddots & \ddots & \ddots \\
& & & \alpha(K - 2) & \alpha(K - 2) & \ldots & \alpha(2) \\
1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\]

(18)

From the set of \( K \) linear equations in (17), we have \( Ap = b \), or \( p = A^{-1}b \). Thus, all equilibrium state probabilities \( p_k, k = 0, \ldots, K - 1 \), immediately after a service completion can be found. To further analyze system performance, it is necessary to determine the steady state distribution of the system at an arbitrary observation time, i.e., \( \bar{p}_k, k = 0, \ldots, K - 1, K \). Note that the state space now is expanded to state \( K \) when the system is totally full. Using the pasta property of the Poisson arrival process [9], \( \bar{p}_k \) is also the probability that there are \( k \) packets in the system at an arbitrary time and can be
obtained as in [9, eq. (1.18a)] and [9, eq. (1.18b)], i.e.,

\[
\bar{p}_k = \begin{cases} 
\frac{p_k}{p_0 + \lambda T_o}, & 0 \leq k \leq K - 1 \\
1 - \frac{1}{p_0 + \lambda T_o}, & k = K 
\end{cases} 
\] (19)

With the analytical framework given above, we are ready to investigate several important performance measures.

**Blocking probability**: Probability that an arriving packet is rejected from the system when it is full. This can be obtained as in [9, eq. (1.18b)]

\[
P_B = \bar{p}_K = 1 - \frac{1}{p_0 + \lambda T_o} 
\] (20)

**Throughput**: Throughput or effective arrival rate \( \lambda_c \) of the considered cognitive system is computed as [9, eq. (1.1)]

\[
\lambda_c = \lambda(1 - P_B) = \frac{\lambda}{p_0 + \lambda T_o} 
\] (21)

**Channel utilization**: In the examined system, the channel utilization \( \rho \) can be calculated from [9, eq. (1.2b)] as

\[
\rho = \lambda_c T_o = \frac{\lambda T_o}{p_0 + \lambda T_o} 
\] (22)

**Mean number of packets in the system**: Mean number of packets in the system is given by [9, eq. (1.21a)]

\[
N = \sum_{k=0}^{K} k\bar{p}_k = \frac{1}{p_0 + \lambda T_o} \sum_{k=0}^{K-1} kp_k + K \left( 1 - \frac{1}{p_0 + \lambda T_o} \right) 
\] (23)

**Mean time in the system**: Mean time in the system \( W \) can be easily derived by using the Little theorem [9, eq. (1.21b)]

\[
W = \frac{N}{\lambda_c} = \frac{\sum_{k=0}^{K-1} kp_k + K(p_0 + \lambda T_o - 1)}{\lambda} 
\] (24)

### 4 Numerical Results

In this section, numerical results are presented to reveal the impact of network parameters on the performance of the secondary system. The selected parameters are based on the IEEE 802.22 standard for cognitive radio networks [10].
The bandwidth $B$ is selected as 6 MHz, each packet includes $N = 8184$ bits, and the time-out is set to $t_o = 100$ ms. Let $d_{SD}$ and $d_{SP}$ be the normalized distances from SU$_{TX}$ to SU$_{RX}$ and from SU$_{TX}$ to PU$_{RX}$, respectively, and assume that path-loss exponent is 4 for suburban environment.

Fig. 2 plots blocking probability and mean waiting time in the system for various buffer lengths $K$. As we can see, when buffer length increases, blocking probability will decrease rapidly at the expense of increase in the mean waiting time. This is due to the fact that the larger the buffer, the higher the probability that the arriving packets are accepted into the system. However, the accepted packets must wait longer in the buffer before being transmitted.

Fig. 3 depicts mean number of packets in the system and channel utilization for various arrival rates. Under the condition for system stability ($\rho < 1$), when the arrival rate increases, the channel utilization and mean number of packets in the system will increase accordingly. The reason is that the fraction of time the channel is utilized to transmit the secondary packets will increase as the traffic load increases.

Fig. 4 illustrates throughput and mean packet transmission time for various fading severity parameters $m_s$, $m_p$, and link distances from SU$_{TX}$ to SU$_{RX}$. As expected, when the secondary transmission distance increases, the throughput will decrease and mean packet transmission time will increase. Moreover, increase in fading severity of the cognitive channel, i.e., $m_s$ decreases, the throughput will decrease and the packet transmission time will increase.

For all the examples, at low values of the interference power threshold $Q$, when $Q$ increases, throughput will increase, but the other parameters such as packet transmission time, blocking probability, and mean waiting time will decrease. However, these metrics reach a constant level at high values of $Q$ which can be useful for determining the optimal length of the buffer given the other parameters.
Figure 2: Blocking probability and mean waiting time in the system versus interference power-to-noise ratio $Q/N_0$ for various buffer lengths.
Figure 3: Mean number of packets in the system and channel utilization versus interference power-to-noise ratio $Q/N_0$ for various arrival rates.
Figure 4: Throughput and mean packet transmission time versus interference power-to-noise ratio $Q/N_0$ with various fading severity parameters and transmission distances from SU$_{TX}$ to SU$_{RX}$. 
5 Conclusions

In this paper, an embedded Markov chain approach has been adopted to analyze the M/G/1/K queue of underlay CRNs which are subject to Nakagami-\(m\) fading and interference power constraint of the primary user. Specifically, analytical expressions for throughput, blocking probability, packet transmission time, channel utilization, mean number of packets, and mean waiting time have been derived. Numerical results are provided to illustrate the impact of system parameters in terms of buffer length, arrival rate, fading parameter, and transmission distance on the secondary performance.

References


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Dynamic Spectrum Access for Cognitive Radio Networks with Prioritized Traffics

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Abstract

We develop a dynamic spectrum access (DSA) strategy for cognitive radio networks where prioritized traffic is considered. Assume that there are three classes of traffic, one traffic class of the primary user and two traffic classes of the secondary users, namely, Class 1 and Class 2. The traffic of the primary user has the highest priority, i.e., the primary users can access the spectrum at any time with the largest bandwidth demand. Furthermore, Class 1 has higher access and handoff priority as well as larger bandwidth demand as compared to Class 2. To evaluate the performance of the proposed DSA, we model the state transitions for DSA as a multi-dimensional Markov chain with three-state variables which present the number of packets in the system of the primary users, the secondary Class 1, and secondary Class 2. In particular, the blocking probability and dropping probability of the two secondary traffic classes are assessed.

1 Introduction

With emerging broadband applications of wireless communications, radio spectrum resources are nowadays suffering from serious shortage. Inspired by improving the spectrum utilization, cognitive radio networks (CRNs) have been proposed as a means upon which unlicensed users can exploit the spectrum band of licensed users according to particular policies. To safeguard the performance of primary networks (PNs), the CRNs can implement interference control (IC) or interference avoidance (IA) paradigms. In the IC
paradigm, a CRN can simultaneously access the spectrum at the same time with a PN provided that it strictly controls its transmit powers to meet the interference constraint imposed by the PN [1]. In practice, without direct coordination between PN and CRN, it is very difficult to satisfy this interference constraint. In contrast, the IA paradigm allows a CRN to utilize the unused portions of licensed spectrums for transmission. To deploy the IA paradigm, CRNs must be equipped with two main functions, spectrum sensing (SS) and dynamic spectrum access (DSA). The first function deals with detecting the active/idle status of spectrum bands [2] while the second function relates to mechanisms to regulate the accessibility of CRNs over the detected idle channels [3].

A great deal of recent studies has considered different aspects of DSA. Specifically, the state-of-the art of DSA, mainly focusing on discussing application challenges of DSA in the future, was presented in [4]. Furthermore, the work reported in [5] considered channel reservation for the handoff of the secondary users (SUs) in DSA where arriving SUs are allocated a channel only if the number of unoccupied channels is greater than a predefined threshold. On the contrary, the authors of [6] designed a DSA scheme which, depending on the state of the system, can reassign the number of spectrum bands to SUs between a maximum and minimum value to reduce the dropping probability of SUs. However, all the mentioned works are only concerned with DSA without considering the priority of spectrum access and handoff for different classes of the secondary traffic. Taking into account spectrum access priority, several DSA schemes for two classes of secondary services were proposed in [3]. Nevertheless, both kinds of secondary services in [3] are assumed to require the same bandwidth and the spectrum handoff is applied only when the primary user (PU) requests service.

In this paper, we propose a DSA scheme with three kinds of traffic, the traffic of the PN and the Class 1 and Class 2 traffic of the SUs. Possessing the spectrum license, the PN has the highest priority of accessibility, spectrum handoff, and bandwidth demand. The traffic of SUs is allowed to access the subbands if they are not occupied by the primary traffic. However, the secondary traffic Class 1 has higher priority of accessibility, spectrum handoff, and bandwidth demand than the secondary traffic Class 2. We model the state transitions of the DSA as a multi-dimensional Markov chain with three-state variables. This is then utilized to investigate the system performance in terms of blocking probability and dropping probability for the secondary traffic.
2 System Model

Consider a system which consists of $M$ licensed bands belonging to $M$ PUs. Each licensed band is further divided into $N$ subbands such that there exist $M \times N$ subbands that can be shared between the PUs and SUs. Assume that the system has three kinds of traffic, the primary traffic of PU, the secondary Class 1 traffic of SU1 and secondary Class 2 traffic of SU2. Suppose that the arrivals of the primary, secondary Class 1 and Class 2 traffic follow Poisson processes with rates $\lambda_p$, $\lambda_{s1}$, and $\lambda_{s2}$, respectively. Furthermore, the service times of the primary, secondary Class 1 and Class 2 traffic follow exponential distributions with mean service rates of $\mu_p$, $\mu_{s1}$, and $\mu_{s2}$, respectively. Assume that each service of SU1 and SU2, respectively, requests $N_1$ and $N_2$ subbands. This differs from [3] where the bandwidth requirement of SU1 and SU2 is assumed to be identical. In our system, we utilize a centralized DSA as follows.

Whenever a PU requests service, it is assigned one band consisting of $N$ subbands. All SU1s and SU2s associated with these subbands will handoff to other unoccupied subbands. If there are not enough idle subbands to handoff all these SU1s and SU2s, SU2s will be terminated first and then SU1s. When an SU1 requests service, if there exist enough idle subbands, $N_1$ subbands will be assigned to the SU1. Otherwise, if the summation of the number of idle subbands and subbands occupied by SU2s is larger than or equal to $N_1$, the arrival SU1 will utilize $N_1$ subbands and several SU2s may suffer from forced termination. However, if the summation of the number of idle subbands and subbands occupied by SU2s is smaller than $N_1$, then the arrival SU1 will be blocked. Finally, an SU2 arrival is only allowed to access the spectrum when at least $N_2$ unoccupied subbands exist. Since spectrum handoff in our DSA is considered for both PU and SU1 arrivals, it is more general as compared to [3] wherein spectrum handoff only occurs when PUs arrive, i.e., a new arrival SU1 is only allowed to access the spectrum when there exist idle subbands.

3 Markov Chain Model

In this section, we model the proposed DSA scheme as a multi-dimensional Markov chain with three-state variables $(i, j, k)$. Here, $i$, $j$, and $k$ are positive integers which present the number of PUs, SU1s and SU2s occupying the spectrum, respectively. It is required that $iN + jN_1 + kN_2 \leq MN$, thus the
state space $S$ is given by
\[ S = \left\{ (i, j, k) \mid 0 \leq i \leq M; 0 \leq j \leq \left\lfloor \frac{N(M - i)}{N_1} \right\rfloor; 0 \leq k \leq \left\lfloor \frac{N(M - i) - jN_1}{N_2} \right\rfloor \right\} \]

Define $1[]$ as the indication function, i.e., $1[]$ returns 1 if the condition in [ ] is correct; otherwise, the function returns 0. In the considered system, any of the six following events can cause the system to change its state: $PU$ requests or completes its services, $SU_1$ requests or completes its services, or $SU_2$ requests or completes its services. In what follows, we describe all possible state transitions of the system.

### 3.1 Transitions from State $(i, j, k)$ to other States

First of all, we detail six situations that the system transits from State $(i, j, k)$ to other states.

#### 3.1.1 Primary User Requests Service

Depending on the available subbands, when a $PU$ requests a service, one of the three following transitions occurs:

- If the number of idle subbands is greater than or equal to $N$, i.e., $(i + 1)N + jN_1 + kN_2 \leq MN$, the arrival $PU$ will be assigned one band without any $SU_1$ and $SU_2$ being forced to terminate. Accordingly, the state transition with rate $1[(i+1)N+jN_1+kN_2\leq MN] \lambda_p$ is given by
  \[ (i, j, k) \rightarrow (i + 1, j, k) \]  

- If the number of idle subbands is smaller than $N$ and the summation of the number of idle subbands and subbands occupied by $SU_2$s is greater than or equal to $N$, there are $[(N - (M - i)N + jN_1 + kN_2)/N_2]$ $SU_2$s being forced to terminate by the arrival $PU$. Then, the state transition with rate $1[(i+1)N+jN_1+kN_2\geq MN \text{ and } (M-i)N-jN_1\geq N] \lambda_p$ is
  \[ (i, j, k) \rightarrow (i + 1, j, k - [(N - (M - i)N + jN_1 + kN_2)/N_2]) \]  

- If the number of idle subbands is smaller than $N$ and the summation of the number of idle subbands and subbands occupied by $SU_2$s is also smaller than $N$, there are $[(N - (M - i)N + jN_1)/N_1]$ $SU_1$s and $k$ $SU_2$s being forced to terminate by the arrival $PU$. In this case, the
state transition with rate $1_{[(i+1)N+jN_1+kN_2>MN} \text{and } (M-i)N-jN_1<N]}\lambda_p$
is

$$(i, j, k) \rightarrow (i + 1, j - \lceil (N - (M - i)N + jN_1)/N_1 \rceil, 0) \quad (4)$$

### 3.1.2 Primary User Completes Service

When the primary user finishes the service, the state transition is expressed as

$$(i, j, k) \rightarrow (i - 1, j, k) \text{ with rate } 1_{[i>0]}i\mu_p \quad (5)$$

### 3.1.3 SU$_1$ Requests Service

Based on the available unoccupied subbands, the two following situations can happen:

- If the number of idle subbands is greater than or equal to $N_1$, i.e., $iN + (j + 1)N_1 + kN_2 \leq MN$, the arrival SU$_1$ will be assigned $N_1$ subbands without any SU$_2$ being forced to terminate. Then, the state transition is

$$(i, j, k) \rightarrow (i, j + 1, k) \text{ with rate } 1_{[iN+(j+1)N_1+kN_2\leq MN]}\lambda_{s_1} \quad (6)$$

- If the number of unoccupied subbands is smaller than $N_1$ and the sum-
mation of the number of idle subbands and subbands occupied by SU$_2$s is greater than or equal to $N_1$, the arrival SU$_1$ will be assigned $N_1$ subbands conditioned that $\lceil (N_1 - (M - i)N + jN_1 + kN_2)/N_2 \rceil$ SU$_2$s are forced to terminate. In this case, the state transition with rate $1_{[iN+(j+1)N_1+kN_2>MN} \text{and } (M-i)N-(j+1)N_1\geq0]}\lambda_{s_1}$ is given by

$$(i, j, k) \rightarrow (i, j + 1, k - \lceil (N_1 - (M - i)N + jN_1 + kN_2)/N_2 \rceil) \quad (7)$$

### 3.1.4 SU$_1$ Completes Service

When SU$_1$ finishes the service, the state transition is found as

$$(i, j, k) \rightarrow (i, j - 1, k) \text{ with rate } 1_{[j>0]}j\mu_{s_1} \quad (8)$$

### 3.1.5 SU$_2$ Requests Service

An arrival SU$_2$ is allowed to access the spectrum only if the number of available idle subbands is greater than or equal to $N_2$. Therefore, the transition state is given by

$$(i, j, k) \rightarrow (i, j, k + 1) \text{ with rate } 1_{[iN+jN_1+(k+1)N_2\leq MN]}\lambda_{s_2} \quad (9)$$
3.1.6 \( SU_2 \) Completes Service

When an \( SU_2 \) finishes the service, the state transition is expressed as

\[
(i, j, k) \rightarrow (i, j, k - 1) \quad \text{with rate } 1_{[k>0]}k\mu_{s_2}
\]  

(10)

3.2 Transitions from other States to State \((i,j,k)\)

Secondly, we describe the six events which cause the system to transit from other states to State \((i, j, k)\) as follows:

3.2.1 Primary User Requests Service

When a \( PU \) requests a service, based on the available unoccupied subbands, the following three situations can occur:

- If \( iN + jN_1 + kN_2 < MN \) and \( i > 0 \), there is only one possible state wherein the system will change to State \((i, j, k)\) with rate
  \[
  1_{[iN+jN_1+kN_2<MN \text{ and } i>0]}\lambda_p, \quad \text{i.e.,}
  \]
  \[
  (i - 1, j, k) \rightarrow (i, j, k)
  \]  

(11)

- If \( iN + jN_1 + kN_2 = MN \), \( i > 0 \), and \( k > 0 \), there are \( \lceil N/N_2 \rceil + 1 \) situations wherein the system will change to State \((i, j, k)\) with rate
  \[
  1_{[iN+jN_1+kN_2=MN \text{ and } i>0 \text{ and } k>0]}\lambda_p
  \]
  as follows:
  \[
  (i - 1, j, k + k') \rightarrow (i, j, k)
  \]  

(12)

where \( 0 \leq k' \leq \lceil \frac{N}{N_2} \rceil \). Then, there exist \( k' \) \( SU_2 \)s which are forced to terminate by the arrival \( PU \).

- If \( iN + jN_1 + kN_2 = MN \), \( i > 0 \), and \( k = 0 \), the following states can transfer to State \((i, j, k)\) with rate \( 1_{[iN+jN_1+kN_2=MN \text{ and } i>0 \text{ and } k=0]}\lambda_p \) as
  \[
  (i - 1, j + j', k + k') \rightarrow (i, j, k)
  \]  

(13)

where \( 0 \leq j' \leq \lceil \frac{N(M-i)-jN_1}{N_1} \rceil \) and \( 0 \leq k' \leq \lceil \frac{N-N(M-i)+jN_1}{N_2} \rceil \). In this case, \( j' \) \( SU_1 \)s and \( k' \) \( SU_2 \)s suffer from forced termination when the arrival \( PU \) enters the system.
3.2.2 Primary User Completes Service

In this event, there is only one possibility for which the system will transit to State \((i, j, k)\) with rate \(1_{[i+1]N+jN_1+kN_2\leq MN]}(i + 1)\mu_p\), i.e.,

\[
(i + 1, j, k) \rightarrow (i, j, k)
\]  

\[(14)\]

3.2.3 \(SU_1\) Requests Service

Depending on the number of available unoccupied subbands, the two following situations can occur:

- If \(iN + jN_1 + kN_2 < MN\) and \(j > 0\), \(N_1\) subbands will be assigned to the arrival \(SU_1\) without any \(SU_2\) being forced to terminate. Then, there is only one state from which the system will transfer to State \((i, j, k)\) with rate \(1_{[iN+jN_1+kN_2=MN \text{ and } j>0]}\lambda_{s_1}\) as

\[
(i, j - 1, k) \rightarrow (i, j, k)
\]

\[(15)\]

- If \(iN + jN_1 + kN_2 = MN\) and \(j > 0\), there are \([N_1/N_2] + 1\) states from which the system will transit to State \((i, j, k)\) with rate \(1_{[iN+jN_1+kN_2=MN \text{ and } j>0]}\lambda_{s_1}\):

\[
(i, j - 1, k + k') \rightarrow (i, j, k)
\]

\[(16)\]

where \(0 \leq k' \leq [N_1/N_2]\). Then, the arrival \(SU_1\) will be assigned \(N_1\) subbands and \(k'\) \(SU_2\)s will be forced to terminate.

3.2.4 \(SU_1\) Completes Service

When \(SU_1\) finishes the service, the respective transition which causes the system to transit to State \((i, j, k)\) with rate \(1_{[iN+(j+1)N_1+kN_2\leq MN]}(j + 1)\mu_{s_1}\) is

\[
(i, j + 1, k) \rightarrow (i, j, k)
\]

\[(17)\]

3.2.5 \(SU_2\) Requests Service

When \(SU_2\) requests a service, there is only one possible transition to State \((i, j, k)\), i.e.,

\[
(i, j, k - 1) \rightarrow (i, j, k) \text{ with rate } 1_{[iN+jN_1+kN_2\leq MN \text{ and } k>0]}\lambda_{s_2}
\]

\[(18)\]
3.2.6 \( SU_2 \) Completes Service

This event results in the following transition to State \((i, j, k)\):

\[
(i, j, k + 1) \rightarrow (i, j, k) \quad \text{with rate} \quad 1_{[iN + jN_1 + (k+1)N_2 \leq MN]}(k+1)\mu_{s2}
\]  

(19)

If we do not consider spectrum handoff when \(SU_1\) requires service, our DSA can be further simplified. Specifically, if a new arrival \(SU_1\) cannot cause ongoing \(SU_2\)s to be terminated, as in DSA Scheme 1 and 2 of [3], the transitions in (7) and (16) can be omitted. Simplifying our DSA further to reflect DSA Scheme 2 of [3], the two transitions in (3) and (4) can be combined into one transition, i.e. \((i, j, k) \rightarrow (i + 1, j - j', k - k')\) with \((i + 1)N + (j - j')N_1 + (k - k')N_2 = MN, j \geq j'\) and \(k \geq k'\).

3.3 Steady-state Probabilities of States

From (1), the number of states in state space \(S\) is given by

\[
n = \sum_{l=0}^{M-1} \sum_{h=0}^{\left\lfloor \frac{(M-l)N}{N_1} \right\rfloor} \left\lfloor \frac{(M-l)N - hN_1}{N_2} \right\rfloor + 1 + 1
\]

(20)

In order to obtain steady-state probabilities of all states, we need to construct the linear equation system, including the \((n - 1)\) balance equations and normalized equation. Let \(p_{i,j,k}\) be the steady-state probability of State \((i, j, k)\). Then, the normalized equation of the system can be expressed as

\[
\sum_{i=0}^{M} \sum_{j=0}^{\left\lfloor N(M-i)/N_1 \right\rfloor} \sum_{k=0}^{\left\lfloor (N(M-i)-jN_1)/N_2 \right\rfloor} p_{i,j,k} = 1
\]

(21)
Furthermore, from (2)-(19), the balance equation of State \((i, j, k)\) is derived as in (22).

\[-\left(1\left[(i+1)N+jN_1+kN_2\leq MN\right]\lambda p + 1\left[(i+1)N+jN_1+kN_2>MN\right] \lambda p + 1\left[iN+(j+1)N_1+kN_2\leq MN\right] \lambda s_1 + 1\left[iN+(j+1)N_1+kN_2>MN\right] \lambda s_1 + 1\left[iN+jN_1+(k+1)N_2\leq MN\right] \lambda s_2 \right. \]

\[+ \left. 1\left[iN+jN_1+(k+1)N_2>MN\right] \lambda s_2 \right) p_{i,j,k} + 1\left[iN+jN_1+kN_2<MN\right] \lambda p \right] p_{i-1,j,k} + \sum_{i' = 0}^{[N/N_2]} \sum_{j' = 0}^{[N-(N-M-i)+jN_1]/N_2} 1\left[iN+jN_1+kN_2 = MN, i>0, and k>0\right] \lambda p \right] p_{i-1,j,k+k'} + \lambda p p_{i-1,j',k+k'} + 1\left[(i+1)N+jN_1+kN_2\leq MN\right] (i+1) \mu p \right] p_{i+1,j,k} + \lambda s_1 p_{i-1,j-1,k} \]

\[\times \lambda s_1 + 1\left[iN+jN_1+kN_2\leq MN\right] \left[(j+1) \mu s_1 p_{i,j+1,k} + 1\left[iN+jN_1+kN_2\leq MN\right] \left(k+1\right) \mu s_2 p_{i,j,k+1} = 0 \] (22)

Denote \(p\) as an \(n \times 1\) steady-state probability vector which contains the steady-state probabilities of all states in \(S\). Then, the steady-state probability \(p_{i,j,k}\) becomes the \(m^{th}\) component of \(p\). Here \(m\) is calculated as

\[m = \sum_{l=0}^{i-1} \sum_{h=0}^{\lfloor (M-l)N/N_1 \rfloor} \left(\frac{(M-l)N-hN_1}{N_2} + 1\right) + \sum_{q=0}^{j-1} \sum_{h=0}^{\lfloor (M-j)N-qN_1 \rfloor} \left(\frac{(M-i)N-qN_1}{N_2} + 1\right) + k + 1 \] (23)

Let \(Q\) be an \(n \times n\) matrix whose components are defined as follows. From the first row to the \((n - 1)^{th}\) row, each row of \(Q\) represents a balance equation with respect to a specific state of \(S\). For example, the balance equation of State \((i, j, k)\) in (22) is represented by the \(m^{th}\) row of \(Q\). The coefficient of the respective probability \(p_{i',j',k'}\) in (22) is the component at position \((m, m')\).
of $Q$. Here, $m'$ is calculated as

$$m' = \sum_{l=0}^{j'-1} \sum_{h=0}^{(M-1)N/N_1} \left[ \left( \frac{(M-1)N-hN_1}{N_2} \right) + 1 \right]$$

$$+ \sum_{q=0}^{j'-1} \left[ \left( \frac{(M-i')N-qN_1}{N_2} \right) + 1 \right] + k' + 1 \quad (24)$$

Finally, the components of the $n^{th}$ row of $Q$, which presents the normalized equation in (21), are all $"1"$. As a result, the linear equation system is expressed as $Qp = b$ where $b$ is an $n \times 1$ vector, $b = (0, 0, \ldots, 0, 1)^T$. Accordingly, the steady-state probability vector $p$ can be found as

$$p = Q^{-1}b \quad (25)$$

### 4 Performance Analysis

#### 4.1 Blocking Probability

Blocking probability is the probability that the arrival traffic is not allowed to access the spectrum. The arrival $SU_1$ will be blocked if the summation of the number of idle subbands and subbands occupied by $SU_2$s is lower than $N_1$, i.e., the blocking probability $BP_1$ of $SU_1$ is calculated as

$$BP_1 = \sum_{\{(i,j,k)\mid (MN-iN-jN_1) < N_1\}} p_{i,j,k} \quad (26)$$

In addition, the arrival $SU_2$ is allowed to access the spectrum only if the number of available idle subbands is greater than or equal to $N_2$. As a result, the blocking probability $BP_2$ of $SU_2$ is calculated as

$$BP_2 = \sum_{\{(i,j,k)\mid (MN-iN-jN_1-kN_2) < N_2\}} p_{i,j,k} \quad (27)$$

#### 4.2 Dropping Probability

Dropping probability is the probability that a service is interrupted by an arrival of higher priority traffic. Assume that the system is in State $(i,j,k)$. $SU_1$ will suffer from forced termination by an arriving $PU$ if the summation of the number of idle subbands and subbands occupied by $SU_2$s is lower than
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\[ N, \text{i.e., } (MN - iN - jN_1) < N. \] As a consequence, the number of SU_1s being forced to terminate is

\[ a(i, j, k) = \left\lceil \frac{N - (MN - iN - jN_1)}{N_1} \right\rceil \] (28)

Then, the dropping rate of SU_1 from the network is \( \lambda_p a(i, j, k) \). Thus, the dropping probability \( FT_1 \) of SU_1 is

\[ FT_1 = \sum_{\{(i,j,k)\mid a(i,j,k) > 0\}} \frac{\lambda_p a(i, j, k) p_{i,j,k}}{\lambda_{s1}} \] (29)

Furthermore, assume that the system is in State (\( i, j, k \)). SU_2 will suffer from forced termination by an arriving PU if \( (MN - iN - jN_1 - kN_2) < N \) or by an arriving SU_1 if \( (MN - iN - jN_1 - kN_2) < N_1 \). Applying the same approach of calculating \( FT_1 \) to compute \( FT_2 \), the dropping probability of SU_2 is obtained as

\[ FT_2 = \sum_{\{(i,j,k)\mid a_1(i,j,k) > 0\}} \frac{\lambda_p a_1(i, j, k) p_{i,j,k}}{\lambda_{s2}} \]

\[ + \sum_{\{(i,j,k)\mid a_2(i,j,k) > 0\}} \frac{\lambda_{s1} a_2(i, j, k) p_{i,j,k}}{\lambda_{s2}} \] (30)

where \( a_1(i, j, k) = \left\lceil \frac{N - (MN - iN - jN_1 - kN_2)}{N_2} \right\rceil \) and \( a_2(i, j, k) = \left\lceil \frac{N_1 - (MN - iN - jN_1 - kN_2)}{N_2} \right\rceil \)

5 Numerical Results

In this section, numerical results are presented to evaluate performance metrics of the proposed DSA scheme. The parameters are selected as follows. We fix arrival rate of the SU_1 and SU_2 as \( \lambda_{s1} = \lambda_{s2} = 0.4 \) packets/s and assess blocking probabilities and dropping probabilities of the secondary Class 1 and Class 2 traffic versus arrival rate of PU. Assume that the SU_1 will use two subbands, \( N_1 = 2 \), and the SU_2 will utilize one subband, \( N_2 = 1 \), for each transmission. Further, the departure rates of PU, SU_1 and SU_2 are equal to channel capacity of a Rayleigh fading channel.

Fig. 1 shows blocking probability of the SU_1 and SU_2 for various combinations of the number of licensed bands \( M \) and the numbers of subbands \( N \) in each band. It is observed that when arrival rate of the PU increases, blocking probabilities of SU_1 and SU_2 will increase rapidly. However, these blocking probabilities can be reduced by increasing the number of bands or the number
Figure 1: Blocking probability of $SU_1$ and $SU_2$ versus arrival rate $\lambda_p$ of $PU$ for various values of $M$ and $N$. 

$\lambda_{S1} = \lambda_{S2} = 0.4$ packets/s; $N_1 = 2; N_2 = 1$
Figure 2: Dropping probability of \( SU_1 \) and \( SU_2 \) versus arrival rate \( \lambda_p \) of \( PU \) for various values of \( M \) and \( N \).
of subbands in each band. Finally, we can see from Fig. 1 that, with the same selected parameters, the blocking probability of \(SU_1\) is lower as compared to that of \(SU_2\) since \(SU_1\) has higher priority to access the spectrum.

Fig. 2 shows the effect of the primary arrival rate and the number of bands as well as subbands per band on the dropping probabilities of the secondary Class 1 and Class 2. As expected, increasing the arrival rate of the \(PU\) results in an increase of the dropping probabilities of the secondary Class 1 and Class 2. Through the selected examples, there is high potential in reducing the dropping probabilities of Class 1 and Class 2 by increasing the number of bands. Finally, with the same number of bands and subbands per band, the dropping probability of \(SU_1\) is significantly lower as compared to that of \(SU_2\) since \(SU_1\) also has higher priority in spectrum handoff.

6 Conclusions

In this paper, we have proposed a DSA scheme for CRNs, where priorities for the bandwidth, the spectrum access, and spectrum handoff are considered for three types of traffics. We have adopted a multi-dimensional Markov chain with three-state variables to analyze the state transitions of the DSA scheme which enables us to obtain the steady-state distribution of the number of each kind of traffics in the system. Blocking probability and dropping probability have been derived for two classes of secondary traffic. Numerical results have been provided to illustrate the impact of the primary arrival rate, the number of licensed bands as well as the number of subbands per band on the secondary network performance.

References


Part II-A

Opportunistic Spectrum Access for Cognitive Amplify-and-Forward Relay Networks
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Opportunistic Spectrum Access for Cognitive Amplify-and-Forward Relay Networks

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Abstract

In this paper, we study the performance of cognitive amplify-and-forward (AF) relay networks where the secondary users opportunistically access $M$ licensed bands of the primary users over Nakagami-$m$ fading channels. In order to enhance the accuracy of spectrum sensing and strongly protect the primary users from being interfered by the secondary transmission, collaborative spectrum sensing is deployed among the secondary transmitter, secondary relay, and secondary receiver. In particular, an analytical expression for the capacity of the considered network is derived. Numerical results are provided to show the influence of the arrival rate of the primary users on the channel utilization of licensed bands. Finally, the impact of the number of the licensed bands, channel utilization of the primary users, false alarm probability, and transmission distances on the capacity of the considered system are investigated.

1 Introduction

For many years, frequency resources have been managed by governmental agencies in a fixed manner such that the licensee has the solely right to access the allocated frequency band. This policy results in inefficiency in the utilization of the precious frequency resources [1]. With the rapidly increasing demand on wireless services, spectrum now becomes more and more exhausted. To alleviate the severe scarcity of spectrum resources, cognitive radio networks (CRNs) have been developed to allow the secondary users (SUs) to access the licensed channels allocated to the primary users (PUs) in a particular manner [2, 3]. Before utilizing the licensed bands, some rules must be implemented for the secondary transmission to prevent the primary users from being interfered. In underlay spectrum access, the secondary user is permitted to simultaneously operate with the primary transmission provided
that its transmit power is controlled to keep the interference incurred to the primary transmission below a predefined threshold. Doing restriction on the transmit power, the performance of the secondary transmission seems to be degraded. In opportunistic spectrum access, the secondary user has to monitor the licensed bands and is allowed to opportunistically access the licensed channel only if the primary transmission is inactive.

In order to establish communication, pairs of the transmitter and receiver must have common available channels and must be located in a reachable transmission range. To cope with limitation of transmission range, the concept of cooperative communications [4], wherein transmission is established by relaying the transmitted signal from the source to the destination with the assistance of intermediate nodes, is proposed. The integration of cooperative transmission into CRNs has shown significant performance improvements for CRNs, i.e., extending radio coverage as well as achieving spatial diversity gain [5,6]. Investigating the performance of dual-hop underlay cognitive relay systems for spectrum sharing is well developed [7, 8]. However, the dual-hop cognitive cooperative networks for opportunistic spectrum access is still an open problem for studying.

Inspired by all of the above, in this paper, we investigate the performance of a cognitive amplify-and-forward (AF) relay network where the secondary users opportunistically access $M$ licensed bands of the primary users. This network operates over Nakagami-$m$ fading channels which covers a wide range of multipath fading channels by changing its fading parameter $m$. Specifically, we derive an analytical expression for the capacity of the considered network where orthogonal frequency division multiplexing access (OFDMA) is utilized at the physical layer to allow SUs to access multiple continuous/discontinuous vacant bands simultaneously. Furthermore, collaborative spectrum sensing as in [9] is deployed between the secondary transmitter, secondary relay, and secondary receiver to enhance the accuracy of spectrum sensing and to protect the primary user from being interfered by the secondary transmission. Finally, numerical results are provided to show the impact of the system parameters on the capacity of the considered system.

**Notation:** In this paper, the following notations are used. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) $X$ are $f_X(\cdot)$ and $F_X(\cdot)$, respectively. $E\{\cdot\}$ denotes expectation operator. Finally, $U(a, b; x)$ is the confluent hypergeometric function [10, eq. (9.211.4)].
2 System and Channel Model

The considered cognitive AF relay network includes one secondary transmitter SU\textsubscript{TX}, one secondary relay SU\textsubscript{R}, and one secondary receiver SU\textsubscript{RX} as depicted in Fig. 1. Assume that the considered network operates in independent and non-identically distributed (i.n.i.d.) Nakagami-m fading in half-duplex mode. The secondary network co-exists with other primary networks by opportunistically utilizing \( M \) licensed bands of the primary users. In order to enable SUs to simultaneously sense \( M \) licensed channels, each SU is equipped with \( M \) sensors. Since SU\textsubscript{TX}, SU\textsubscript{R}, and SU\textsubscript{RX} have different distances to the PUs, their sensing outcomes may be different. Thus, collaborative spectrum sensing [9], where SUs cooperatively exchange the sensing information, is utilized to improve the accuracy of the overall sensing results. After exchanging the list of detected available bands at SU\textsubscript{TX}, SU\textsubscript{R}, and SU\textsubscript{RX}, the secondary transmitter selects all the unoccupied bands for transmission. For instance, if the list of the unoccupied bands at SU\textsubscript{TX}, SU\textsubscript{R}, and SU\textsubscript{RX} are (i, j); (i, j, h); and (i, j, k, l), respectively, the common channels i and j will be utilized for the secondary transmission. Generally, the vacant channels for the secondary transmission are often discontinuous. To assist SU\textsubscript{TX} and SU\textsubscript{R} to simultaneously access multiple continuous/discontinuous vacant bands, OFDMA is utilized.

Assume that the usage pattern of the primary users in each licensed channel is non-time slotted and follows a independent and identical distribution (see Fig. 2). Here, the state "active" indicates that the channel is occupied by the primary users while the state "idle" indicates that the channel is unoccupied by the primary users. As in [11], the empirical distribution of the idle period of the licensed bands follows either exponential distribution or Hyper-
Erlang distribution. On the other hand, the occupancy duration of a licensed channel has a general distribution since it is affected by the scheduler for traffic load and transmission environment. For a non-time slotted system, the idle duration of the $i$-th licensed channel is assumed to follow the exponential distribution with parameter $\lambda_i$ where $\lambda_i$ is the arrival rate of PUs on the $i$-th channel. Let $T_{0,i}$ be a random variable representing the idle time period of the $i$-th licensed channel, i.e.,

$$f_{T_{0,i}}(t) = \lambda_i \exp(-\lambda_i t)$$ (1)

Thus, the probabilities $p_i(t)$ of having an arrival of primary users during period $t$ will be the CDF of the exponential distribution with parameter $\lambda_i$:

$$p_i(t) = 1 - \exp(-\lambda_i t)$$ (2)

Note that $p_i(t)$ is also the probability that primary users in the $i$-th channel are interfered by secondary transmission. Since secondary users can utilize the $M$ licensed channels, the probability $p(t)$ that a channel among $M$ channels is interfered by the secondary transmission in period $t$ is given by

$$p(t) = 1 - [1 - p_i(t)]^M = 1 - \exp(-M\lambda_i t)$$ (3)

Given a predefined value, $p(t) = P_{th}$, the maximum transmission time $t_{max}$ of a secondary transmission is constrained as $P_{th} = 1 - e^{-M\lambda t_{max}}$, i.e.,

$$t_{max} = \frac{\ln \left( \frac{1}{1-P_{th}} \right)}{\lambda_i M}$$ (4)

Our cognitive relay network will utilize the maximum transmission time $t_{max}$ to communicate. Specifically, the transmission time $t_{max}$ is split into two
equal sub-time slots. The first sub-time slot is utilized to transmit the signal from $SU_{TX}$ to $SU_{R}$ and the other is utilized to forward the signal from $SU_{R}$ to $SU_{RX}$. Assuming flat fading in each licensed channel, all frequency components of the signal in a band will experience the same magnitude of fading. Let $x_s$ be the transmit signal at $SU_{TX}$ with average transmit power $E\{|x_s|^2\} = P_s$. Furthermore, denote $h_1$ as the channel coefficient of the link from $SU_{TX}$ to $SU_{R}$ with fading severity parameter $m_1$ and channel mean power $\Omega_1$. Then, the received signal $y_r$ at $SU_{TX}$ in the first sub-time slot is obtained as

$$y_r = h_1 x_s + n_r$$

where $n_r$ is the additive white Gaussian noise (AWGN) at $SU_{TX}$ with zero mean and variance $N_0$. In the second sub-time slot, $SU_{R}$ amplifies the received signal with a gain $G$ and forwards the resulting signal to $SU_{RX}$. The factor $G$ is selected to guarantee that the transmit powers at $SU_{TX}$ and $SU_{R}$ are the same, i.e., $G^2 = \frac{1}{|h_1|^2}$. Consequently, the received signal at $SU_{RX}$ in the second sub-time slot is expressed as

$$y_d = h_2 G h_1 x_s + h_1 G n_r + n_d$$

where $h_2$ is the channel coefficient of the link from $SU_{R}$ to $SU_{RX}$ with fading severity parameter $m_2$ and channel mean power $\Omega_2$. Further, $n_d$ is the AWGN at $SU_{RX}$ with zero mean and variance $N_0$. Then, the instantaneous SNR at $SU_{RX}$ is obtained as

$$\gamma_s = \frac{P_s |h_1|^2 |h_2|^2}{N_0 |h_1|^2 + |h_2|^2}$$

For brevity, denote $\beta_s = \frac{P_s}{N_0}$, $X_1 = |h_1|^2$, and $X_2 = |h_2|^2$, i.e.,

$$\gamma_s = \beta_s \frac{X_1 X_2}{X_1 + X_2}$$

Here, the CDF and PDF of the channel power gain $X_i$, $i \in \{1, 2\}$, for a Nakagami-$m$ fading channel with fading severity parameter $m_i$ and channel mean power $\Omega_i$ can be provided as

$$f_{X_i}(x_i) = \frac{\alpha_i^{m_i}}{\Gamma(m_i)} x_i^{m_i-1} \exp(-\alpha_i x_i)$$

$$F_{X_i}(x_i) = 1 - \exp(-\alpha_i x_i) \sum_{j=0}^{m_i-1} \frac{\alpha_i^j x_i^j}{j!}$$

where $\alpha_i = m_i/\Omega_i$. 
3 End-to-End Performance Analysis

Let \( V \) be the average number of licensed channels used for one secondary transmission and each channel has a bandwidth \( B \) (Hz). Based on Shannon’s capacity theorem, the channel capacity in (bits/s) of the secondary transmission can be written as

\[
C = E\left\{ \frac{VB}{2} \log_2(1 + \gamma_s) \right\} = \frac{VB}{2 \ln 2} \int_0^\infty \ln(1 + \gamma) f_{\gamma_s}(\gamma) d\gamma
\]  

By applying integration by parts for (11), the ergodic capacity is expressed in terms of the CDF of \( \gamma_s \) as

\[
C = \frac{VB}{2 \ln 2} \int_0^\infty \frac{1}{1 + \gamma} [1 - F_{\gamma_s}(\gamma)] d\gamma
\]  

As in [12, eq. (25)], a bound, denoted as \( \gamma_s^u \), on the instantaneous SNR \( \gamma_s \) in (8) can be derived as

\[
\gamma_s^u = \min(\gamma_1, \gamma_2)
\]  

where \( \gamma_1 = \beta_s X_1 \) and \( \gamma_2 = \beta_s X_2 \). Based on the order statistics theory, the CDF of \( \gamma_s^u \) can be expressed as

\[
F_{\gamma_s^u}(\gamma) = 1 - [1 - F_{\gamma_1}(\gamma)][1 - F_{\gamma_2}(\gamma)]
\]  

From (10), expressions for \( F_{\gamma_1}(\gamma) \) and \( F_{\gamma_2}(\gamma) \) are found as

\[
F_{\gamma_1}(\gamma) = F_{X_1}\left(\frac{\gamma}{\beta_s}\right) = 1 - \exp\left(-\frac{\alpha_1}{\beta_s} \gamma\right) \sum_{p=0}^{m_1-1} \frac{\alpha_1^p}{p!} \frac{\gamma^p}{\beta_s^p}
\]  

\[
F_{\gamma_2}(\gamma) = F_{X_2}\left(\frac{\gamma}{\beta_s}\right) = 1 - \exp\left(-\frac{\alpha_2}{\beta_s} \gamma\right) \sum_{q=0}^{m_2-1} \frac{\alpha_2^q}{q!} \frac{\gamma^q}{\beta_s^q}
\]  

Substituting (15) and (16) into (14), after some manipulations, an expression for \( F_{\gamma_s^u}(\gamma) \) is obtained as

\[
F_{\gamma_s^u}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\alpha_1^p \alpha_2^q}{p! q!} \frac{\beta_s^{p+q} \gamma^{p+q}}{\exp\left(-\frac{\alpha_1 + \alpha_2}{\beta_s} \gamma\right)}
\]  

By substituting (17) into (12) together with the help of [13, eq. (2.3.6.9)] to solve the resulting integral, it follows that

\[ C = \frac{VB}{2\ln 2} \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\alpha_1^p \alpha_2^q}{p!q!} \frac{\Gamma(p+q+1)}{\beta_s^{p+q}} \times U \left( p + q + 1, p + q + 1; \frac{\alpha_1 + \alpha_2}{\beta_s} \right) \]  \hspace{1cm} (18)

Next, we need to calculate the average number of licensed channels used for one secondary transmission \( V \). In this paper, all the secondary users, \( \text{SU}_{\text{TX}}, \text{SU}_{\text{R}}, \text{SU}_{\text{RX}} \), employ energy-detection and use the same decision threshold to detect the vacant channels with a false alarm probability \( P_f \). Since all the terminals in the secondary network collaboratively exchange the sensing results, i.e., the probability of false alarm for the collaborative scheme, denoted \( Q_f \), is obtained as [9]

\[ Q_f = 1 - (1 - P_f)^3 \]  \hspace{1cm} (19)

Let \( p_i \) be the probability that the secondary users consider the \( i \)-th licensed band to be vacant, i.e.,

\[ p_i = (1 - \rho_i)(1 - Q_f) = (1 - \rho_i)(1 - P_f)^3 \]  \hspace{1cm} (20)

where \( \rho_i \) is the channel utilization of the primary users in the \( i \)-th licensed band. Assume that \( M \) licensed bands have identical usage pattern of the primary users, i.e., \( \rho_i = \rho_j = \rho, \forall i, j \in (1, \ldots, M) \). Let \( V \) be a discrete RV representing the number of vacant channels which are utilized in a secondary transmission. Therefore, the probability \( P_v \) that the secondary users consider \( v \) vacant channels in \( M \) licensed channels will follow the binomial distribution as

\[ P_v = \Pr \{ V = v \} = \binom{M}{v} [(1 - \rho)(1 - P_f)^3]^v \times [1 - (1 - \rho)(1 - P_f^3)]^{M-v} \]  \hspace{1cm} (21)

Consequently, the average number of licensed channels used for one secondary transmission \( \bar{V} \) can be obtained as

\[ \bar{V} = \sum_{v=0}^{M} vP_v = M [(1 - \rho)(1 - P_f^3)] \]  \hspace{1cm} (22)
for the primary transmission. Let \( x_p \) be the transmit signal of the primary transmitter with average transmit power \( E\{x_p\} = P_p \). Thus, the signal \( y_p \) at the primary receiver is expressed as

\[
y_p = h_p x_p + n_p
\]  

(23)

where \( h_p \) is the channel coefficient of the link from the primary transmitter to the primary receiver. Furthermore, \( n_p \) is the AWGN at the primary receiver with zero mean and variance \( N_0 \). As a consequence, the instantaneous SNR at the primary receiver is written as

\[
\gamma_p = \beta_p X_p
\]  

(24)

where \( \beta_p = \frac{P_p}{N_0} \) and \( X_p = |h_p|^2 \). Let \( m_p \) and \( \Omega_p \) be the fading severity parameter and the channel mean power of \( X_p \). From (10) and (24), an expression for \( F_{\gamma_p}(\gamma) \) is obtained as

\[
F_{\gamma_p}(\gamma) = 1 - \exp \left( -\frac{\alpha_p \gamma}{\beta_p} \right) \sum_{i=0}^{m_p-1} \frac{1}{i!} \left( \frac{\alpha_p \gamma}{\beta_p} \right)^i
\]  

(25)

By differentiating \( F_{\gamma_p}(\gamma) \) with respect to variable \( \gamma \), the PDF of \( \gamma_p \) is found as

\[
f_{\gamma_p}(\gamma) = \frac{m_p-1}{\beta_p^{i+1}} \exp \left( -\frac{\alpha_p \gamma}{\beta_p} \right) \sum_{i=0}^{m_p-1} \frac{1}{i!} \left( \frac{\alpha_p \gamma}{\beta_p} \right)^i
\]

\[
- \sum_{i=0}^{m_p-1} \frac{i}{i!} \left( \frac{\alpha_p \gamma}{\beta_p} \right)^{i-1} \exp \left( -\frac{\alpha_p \gamma}{\beta_p} \right)
\]  

(26)

Assume that the primary transmitter sends packets in any licensed band with a rate equal to the channel capacity, i.e., \( C = B \log_2 (1 + \gamma_p) \) (bits/s). As a result, the packet transmission time is obtained as

\[
T = \frac{1}{b \log_2 (1 + \gamma_t)} \text{(secs/packet)}
\]  

(27)

Here, \( b = \frac{B}{N} \) and \( N \) is the number of bits per packet. Consequently, we have

\[
f_T(t) = \frac{\log(2)2^{1/(bt)}}{bt^2} f_{\gamma_p}(2^{1/(bt)} - 1)
\]  

(28)
Substituting (26) into (28), after some rearranging of terms, it follows that

$$f_T(t) = \log(2) b \left[ \sum_{i=0}^{m_p-1} \frac{1}{i!} \frac{\alpha_p^i}{\beta_p^{i+1}} \exp \left( \frac{\alpha_p}{\beta_p} \right) \frac{2^{1/(bt)} (2^{1/(bt)} - 1)^i}{t^2} \exp \left( -\frac{\alpha_p 2^{1/(bt)}}{\beta_p} \right) \right]$$

$$- \sum_{i=0}^{m_p-1} \frac{i}{i!} \frac{\alpha_p^i}{\beta_p^{i+1}} \exp \left( \frac{\alpha_p}{\beta_p} \right) \frac{2^{1/(bt)} (2^{1/(bt)} - 1)^{i-1}}{t^2} \exp \left( -\frac{\alpha_p 2^{1/(bt)}}{\beta_p} \right) \right]$$

(29)

Thus, the average packet transmission time of the primary users is obtained as

$$T = E\{T\} = \log(2) b \left[ \sum_{i=0}^{m_p-1} \frac{1}{i!} \frac{\alpha_p^i}{\beta_p^{i+1}} \exp \left( \frac{\alpha_p}{\beta_p} \right) \phi(i) - \sum_{i=0}^{m_p-1} \frac{1}{(i-1)!} \frac{\alpha_p^i}{\beta_p^{i+1}} \exp \left( \frac{\alpha_p}{\beta_p} \right) \phi(i-1) \right]$$

(30)

where $$\phi(i) = \int_0^\infty \frac{2^{1/(bt)} (2^{1/(bt)} - 1)^i}{t^2} \exp \left( -\frac{\alpha_p 2^{1/(bt)}}{\beta_p} \right) dt$$. As a result, the channel utilization of a licensed channel is calculated as [14, eq. (1.12a)]

$$\rho = \lambda_p T$$

(31)

where $$\lambda_p$$ is the arrival rate of the primary users in any licensed band. Substituting (31) into (22), $$\mathcal{V}$$ is determined. Then, substituting this outcome into (18), the expression for the channel capacity can be found.

4 Numerical Results and Discussion

In this section, numerical examples are provided to reveal the impact of the arrival rate of the primary users on the channel utilization of a licensed band. Furthermore, we also demonstrate the effect of the false alarm probability of the secondary users, the number of licensed bands, the channel utilization of a licensed channel, the transmission distances from SU_TX to SU_R and from SU_R to SU_TX on the channel capacity of the secondary network. Assume that each licensed band has bandwidth $$B = 1$$ MHz and each packet contains 1028 bytes. Let $$d_p$$ be the normalized distance from the primary transmitter to the primary receiver which utilizes a licensed channel for the transmission. In addition, denote $$d_1$$, $$d_2$$ as the normalized distances from SU_TX to SU_R and from SU_R to SU_TX, respectively. Assume that the channel mean power is attenuated according to the transmission distance with the exponential path loss model. We set the path loss exponent as $$\nu = 4$$ which represents a highly shadowed urban area. The fading severities in all hops are fixed as $$m_p = m_1 = m_2 = 2$$. 
Fig. 3 illustrates the effect of the arrival rate and the average transmit power-to-noise ratio $P_p/N_0$ of the primary transmitter on the channel utilization of a licensed band. As we can see from Fig. 3, the channel utilization of a licensed channel increases according to the increase of arrival rate of the primary users. However, when the average transmit power-to-noise ratio $P_p/N_0$ of the primary user increases, the channel utilization decreases. This is because the packet transmission time of the primary transmission is adversely proportional to the increasing of the received SNR.

Fig. 4 depicts the capacity of the secondary transmission versus channel utilization of a licensed channel for different transmission distances from SU$_{TX}$ to SU$_R$ and from SU$_R$ to SU$_{TX}$. The average transmit power-to-noise ratio at the primary transmitter and at SU$_{TX}$ are set as $P_p/N_0 = P_s/N_0 = 10$ dB. Furthermore, we fix the false alarm probability at the secondary users and the number of licensed channels as $P_f = 0.01$ and $M = 5$, respectively. As expected, when the channel utilization increases, the opportunity for the secondary network to access the licensed band decreases, leading to decreasing the capacity of the secondary system. Moreover, when the transmission distances between terminals in the secondary network increase, the capacity of the system decreases accordingly.

Fig. 5 shows the impact of the probability of false alarm at the secondary users on the capacity for different number of licensed bands. The transmission distances are fixed as $d_P = d_1 = d_2 = 0.5$ and the average transmit power-to-noise ratio of the primary transmission and the secondary transmission are the same, $P_p/N_0 = P_s/N_0 = 10$ dB. Furthermore, the arrival rate of the primary users on a licensed band is chosen as $\lambda_p = 300$ packets/sec. It is observed from this figure that when the false alarm probability increases, the capacity of the secondary transmission decreases. This is because increasing the false alarm probability leads to a decrease in spectrum access opportunities of the secondary.

Fig. 6 depicts the capacity versus average transmit power-to-noise ratio of the secondary transmission, $P_s/N_0$, for different number of licensed bands. The average transmit power-to-noise ratio of the primary transmission is fixed as $P_p/N_0 = 10$ dB and the arrival rate of the primary users is chosen as $\lambda_p = 300$ packets/sec. Furthermore, the transmission distances are selected as $d_P = d_1 = d_2 = 0.5$ and the probability of false alarm at each secondary user is 0.01. As can be seen from Fig. 6, the higher transmit power $P_s$, the more capacity can be achieved. Moreover, from Fig. 6 and Fig. 5, when the number of licensed bands increases, the capacity of the secondary system increases accordingly since the secondary users have higher opportunity to access the unoccupied bands.
Figure 3: Channel utilization of the primary user on a licensed channel versus arrival rate of the primary user for different transmit power.

Figure 4: Capacity of the secondary network versus channel utilization of the primary user for different transmission distances in cognitive network.
Figure 5: Capacity of the secondary network versus false alarm probability in each terminal of the secondary network for different number of licensed bands.

Figure 6: Capacity of the secondary network versus transmit power of the secondary transmission for different number of licensed bands.
5 Conclusions

In this paper, we have analyzed the capacity of a cognitive AF relay network where the secondary users opportunistically access $M$ licensed bands of the primary users whenever these channels are unoccupied by the primary users. In order to detect the vacant licensed channels, the secondary users have to monitor the licensed channels to decide whether or not a primary signal exists. Collaborative spectrum sensing among the secondary transmitter, secondary relay, and secondary receiver has been deployed to improve the accuracy of sensing and to protect the primary user from being interfered by the secondary transmission. Our presented analysis shows that the capacity of the secondary network significantly increases with increasing the number of licensed bands. However, the capacity is degraded when the probability of false alarm of the secondary users and the channel utilization of the primary users on the licensed channel increases.

References


PART II-B

MRT/MRC for Cognitive AF Relay Networks under Feedback Delay and Channel Estimation Error
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MRT/MRC for Cognitive AF Relay Networks under Feedback Delay and Channel Estimation Error

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Abstract
In this paper, we examine the performance of multiple-input multiple-output (MIMO) cognitive amplify-and-forward (AF) relay systems with maximum ratio transmission (MRT). In particular, closed-form expressions in terms of a tight upper bound for outage probability (OP) and symbol error rate (SER) of the system are derived when considering channel estimation error (CEE) and feedback delay (FD) in our analysis. Through our works, one can see the impact of FD and CEE on the system as well as the benefits of deploying multiple antennas at the transceivers utilizing the spatial diversity of an MRT system. Finally, we also provide a comparison between analytical results and Monte Carlo simulations for some examples to verify our work.

1 Introduction
Two major challenges for a communication system design are how to use frequency resources efficiently and how to overcome multipath fading to guarantee transmission reliability. Recently, studies on cooperative diversity techniques and cognitive radio networks (CRNs) have revealed promising solutions to combat these difficulties. Deploying cooperative relaying has been shown to offer many benefits such as extending coverage, improving throughput and enhancing transmission reliability [1, 2]. Moreover, CRNs permit unlicensed users, also called secondary users, to access the licensed spectrum opportunistically or concurrently as reported in [3]. Therefore, combining cooperative
diversity with CRNs not only improves the efficiency of spectrum usage but also provides higher transmission reliability, e.g. [1–4]. Specifically, the works of [3] presented a brief overview about the cognitive cooperative techniques. Further, [1,2,4] obtained improved throughput of secondary nodes by increasing spatial diversity through a cooperative cognitive approach.

When considering techniques which enable a secondary user to access licensed spectrum, there are two kinds of approaches, spectrum underlay and spectrum overlay. In spectrum overlay, a secondary user is only allowed to use licensed spectrum when the primary user is idle. Whenever the primary user becomes active, the overlay secondary user must switch off its transmission and search for another spectrum hole. Hence, there is no constraint on transmit power at the secondary transmitter in the overlay approach. Instead, spectrum sensing and detecting a spectrum white space are required at the secondary user. Specifically, [5,6] proposed spectrum sharing schemes in decode-and-forward (DF) relay overlay cognitive networks for single relay and multiple relays, respectively. In the underlay approach, both secondary users and primary users can use the same spectrum simultaneously. As such, underlay secondary transmitters must constrain their transmit power to guarantee that the interference at the primary user is kept below a given threshold. For this reason, their coverage is often not large. If a secondary user wants to extend its coverage, it normally cooperates with other relays. In particular, [7] analyzed outage probability (OP) for a DF relay cognitive network while [8] investigated OP for an amplify-and-forward (AF) relay cognitive network. Moreover, [9] proposed a power allocation to enhance the spectral efficiency and increase the bit rate for a network coded cognitive cooperative network (NCCCN) under peak interference constraints. Additionally, [10,11] proposed algorithms to distribute transmit power for beamforming transmission via a multi-relay underlay cognitive radio architecture. Besides beamforming transmission, maximum ratio transmission (MRT) is shown in [12] as a powerful diversity technique. Deploying antenna arrays in maximum ratio transmission has been shown to offer many benefits such as combating the adverse effect of fading, increasing capacity and extending coverage [12]. Thus, MRT seems to be suitable for underlay cognitive transmission which suffers from a very strict constraint on their transmit power. To the best of our knowledge, there is no work focusing on MRT for an underlay cognitive AF relay network.

In this paper, we deploy MRT with two hop AF relaying in an underlay cognitive network. Specifically, we derive a closed-form expression for the cumulative distribution function (CDF) of the instantaneous signal-noise-ratio (SNR). This outcome enables us to obtain a tight upper bound for the outage probability and symbol error rate (SER) of the considered system.

The paper is organized as follow: Section II describes the considered sys-
tem, related concepts and definitions. The system performance in terms of outage probability and symbol error rate are analyzed in Section III. Section IV presents numerical results and discussions of the achieved results. Finally, conclusions are given in Section V.

Notation: In this paper, matrices and vectors are denoted by bold upper and lower case letters, respectively. Next, \( \| \cdot \|_F \) indicates the Frobenius norm and \( \dagger \) stands for transpose conjugate of a vector or matrix. Then, the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable \( X \) are written as \( f_X(.) \) and \( F_X(.) \), respectively. Furthermore, the gamma function in [13, eq. (8.310.1)] and the incomplete gamma function in [13, eq. (8.350.2)] are denoted as \( \Gamma(n,x) \) and \( \Gamma(n) \), respectively. Finally, the confluent hypergeometric function [13, eq. (9.211.4)] is expressed by \( U(a,b;x) \).

2 System and Channel Model

We consider an underlay cognitive AF relay system including \( N_1 \) antennas at the secondary transmitter \( SU_{TX} \), \( N_2 \) antennas at the secondary relay \( SU_{R} \), \( N_3 \) antennas at the secondary receiver \( SU_{RX} \) and \( N_4 \) antennas at the primary receiver \( PU_{RX} \) as shown in Fig. 1. The primary and the secondary users (including \( SU_{TX} \) and \( SU_{R} \)) can simultaneously access the same spectrum as long as the secondary users guarantee that their interference to the primary user is kept below a predefined threshold, \( Q \). In the first hop from \( SU_{TX} \) to \( SU_{R} \), we implement MRT at the \( SU_{TX} \) by multiplying the transmit signal,

![Figure 1: System model for a cognitive MRT AF relay network under channel estimation error and feedback delay.](image)
s(t) with an \( N_1 \times 1 \) transmit weighting vector \( \mathbf{v}_1(t) \) and employ maximum ratio combining (MRC) at the \( SU_R \) by multiplying the received signal with an \( 1 \times N_2 \) receive weighting vector \( \mathbf{w}_1(t) \). As a result, the received signal \( s_r(t) \) at the \( SU_R \) is given by

\[
s_r(t) = \mathbf{w}_1(t) [\mathbf{H}_1(t)\mathbf{v}_1(t)s(t) + \mathbf{n}_1(t)]
\]

where \( \mathbf{H}_1(t) \) is an \( N_2 \times N_1 \) channel coefficient matrix from \( SU_{TX} \) to \( SU_R \) whose elements are independent and identical distributed (i.i.d.) complex Gaussian random variables (RV) with zero mean and variance \( \Omega_1 \), denoted as \( CN(0, \Omega_1) \). Further, \( s(t) \) is the transmit signal at the \( SU_{TX} \) with average power \( E\{|s(t)|^2\} = P_1 \) where \( E\{|\cdot|\} \) stands for an expectation operator. Finally, \( \mathbf{n}_1(t) \) is an \( N_2 \times 1 \) additive white Gaussian noise (AWGN) vector at the \( SU_R \). It is assumed that all elements of \( \mathbf{n}_1(t) \) are i.i.d. complex Gaussian RVs with zero mean and variance \( N_0 \), denoted as \( CN(0, N_0) \). To get maximum signal-to-noise ratio (SNR) at the \( SU_R \), the transmit weighting vector \( \mathbf{v}_1(t) \) is chosen to be the eigenvector \( \mathbf{u}_1(t) \) corresponding to the largest eigenvalue of the Wishart matrix \( \mathbf{H}_1^\dagger(t)\mathbf{H}_1(t) \) and the receive weighting vector \( \mathbf{w}_1(t) \) is selected as \( \mathbf{w}_1(t) = \mathbf{u}_1^\dagger(t)\mathbf{H}_1^\dagger(t) \).

In order to deploy MRT/MRC, \( SU_{TX} \) and \( SU_R \) need the channel state information (CSI) to adjust the weighting vectors. However, the \( SU_{RX} \) can usually not perfectly estimate \( \mathbf{H}_1(t) \) and there always exits feedback delay (FD) from \( SU_R \) to \( SU_{TX} \) in practice. When taking into account the effect of channel estimation error (CEE) and FD, \( \tau \), the channel coefficient matrix at the \( SU_{TX} \) is \( \mathbf{H}_1(t-\tau) \). As a consequence, \( \mathbf{v}_1(t) \) is selected as the eigenvector \( \mathbf{u}_1(t) \) corresponding to the largest eigenvalue \( \lambda_{max} \) of the Wishart matrix \( \mathbf{H}_1^\dagger(t-\tau)\mathbf{H}_1(t-\tau) \) and \( \mathbf{w}_1(t) \) is chosen to be \( \mathbf{w}_1(t) = \mathbf{u}_1^\dagger(t)\mathbf{H}_1^\dagger(t-\tau) \). As mentioned in [14], the relationship between \( \mathbf{H}_1(t) \) and \( \mathbf{H}_1(t-\tau) \) is given by

\[
\mathbf{H}_1(t) = \rho \mathbf{H}_1(t-\tau) + \mathbf{E}(t) + \mathbf{D}(t)
\]

where \( \rho \) denotes the channel correlation coefficient. As in [14], for the Clarkes fading spectrum, \( \rho \) is expressed in terms of FD \( \tau \) and the Doppler frequency \( f_d \) as \( \rho = J_0(2\pi f_d \tau) \) where \( J_0(\cdot) \) is the zero-th order Bessel function of the first kind [13, eq.(8.441.1)]. Further, \( \mathbf{E}(t) \) is an \( N_2 \times N_1 \) CEE matrix whose elements are i.i.d. complex Gaussian RVs, \( CN(0, \sigma^2 \Omega_1) \); \( \sigma^2 \) is the variance of CEE. Finally, \( \mathbf{D}(t) \) stands for an \( N_2 \times N_1 \) error matrix induced by FD whose elements are i.i.d. complex Gaussian RVs, \( CN(0, (1-\sigma^2)(1-\rho^2)\Omega_1) \). For this more practical scenario, the received signal at the \( SU_R \) is given by

\[
\hat{s}_r(t) = \rho \mathbf{u}_1^\dagger(t)\mathbf{H}_1^\dagger(t-\tau)\mathbf{H}_1(t-\tau)\mathbf{u}_1(t)s(t) + \mathbf{u}_1^\dagger(t)\mathbf{H}_1^\dagger(t-\tau)\mathbf{E}(t) \times \mathbf{u}_1(t)s(t) + \mathbf{u}_1(t)\mathbf{D}(t)\mathbf{u}_1(t)s(t) + \mathbf{u}_1^\dagger(t)\mathbf{H}_1^\dagger(t-\tau)\mathbf{n}_1(t)
\]
At the SU$_R$, the received signal is amplified with a factor $G$ and is then forwarded to the SU$_{RX}$. Let $P_2$ be the average transmit signal at SU$_R$, the gain factor $G$ must satisfy $E\{|G\hat{s}_r(t)|^2\} = P_2$ or

$$G^2 \approx \frac{P_2}{\rho^2\lambda_{max}^2 P_1} \quad (4)$$

Due to the interference constraint $Q$ at the PU$_{RX}$, both SU$_{TX}$ and SU$_R$ must control their transmit power $P_1$, and $P_2$, respectively, to meet the power interference constraint at PU$_{RX}$, i.e.,

$$P_1 = \frac{Q}{\|H_3(t)\|_F^2} \quad (5)$$

$$P_2 = \frac{Q}{\|H_4(t)\|_F^2} \quad (6)$$

where $H_3(t)$ stands for an $N_4 \times N_1$ fading channel matrix from SU$_{TX}$ to PU$_{RX}$, and $H_4(t)$ denotes an $N_4 \times N_2$ fading channel matrix from SU$_R$ to PU$_{RX}$. It is assumed that all elements of $H_3(t)$ are i.i.d. complex Gaussian RVs with zero mean and variance $\Omega_3$, $\mathcal{CN}(0, \Omega_3)$; and all elements of $H_4(t)$ are i.i.d. complex Gaussian RVs with zero mean and variance $\Omega_4$, $\mathcal{CN}(0, \Omega_4)$.

In the second hop, we also deploy MRT at the SU$_R$ with an $N_2 \times 1$ transmit weighting vector $v_2(t)$ and MRC at the SU$_{RX}$ with an $1 \times N_3$ receive weighting vector $w_2(t)$. For this hop, $v_2(t)$ is selected to be the eigenvector $u_2(t)$ corresponding to the largest eigenvalue $\lambda_{max}$ of the Wishart matrix $H_3^H(t)H_2(t)$, and $w_2(t)$ is chosen as $w_2(t) = u_1^H(t)H_2^H(t)$). Here, $H_2(t)$ is an $N_3 \times N_2$ fading channel matrix from SU$_R$ to SU$_{RX}$. Consequently, the received signal at SU$_{RX}$ is obtained as

$$s_D(t) =
\begin{align*}
&G_{\rho} u_2^H(t) H_2^H(t) H_2(t) u_2(t) \hat{u}_1^H(t) \hat{H}_1^H(t-\tau) \hat{H}_1(t-\tau) \hat{u}_1(t) s(t) \\
&\quad \text{desired signal} \\
&+ G \ u_2^H(t) \ H_2^H(t) \ H_2(t) \ u_2(t) \hat{u}_1^H(t) \hat{H}_1^H(t-\tau) E(t) \hat{u}_1(t) s(t) \\
&\quad \text{self interference} \\
&+ G \ u_2^H(t) \ H_2^H(t) \ H_2(t) u_2(t) \hat{u}_1^H(t) \hat{H}_1^H(t-\tau) D(t) \hat{u}_1(t) s(t) \\
&\quad \text{self interference} \\
&+ G \ u_2^H(t) \ H_2^H(t) \ H_2(t) u_2(t) \hat{u}_1^H(t) \hat{H}_1^H(t-\tau) \eta_1(t) + u_2^H(t) H_2^H(t) \eta_2(t) \\
&\quad \text{noise}
\end{align*}$$

(7)
Here, $\mathbf{n}_2(t)$ is an $N_3 \times 1$ AWGN vector at the SU whose elements are i.i.d. complex Gaussian RVs, $\mathcal{CN}(0, N_0)$. Let $\lambda_1$ and $\lambda_2$ be the maximum eigenvalues of the complex central Wishart matrices $\mathbf{X}^\dagger \mathbf{X}$ and $\mathbf{Y}^\dagger \mathbf{Y}$, respectively, where $\mathbf{X}$, $\mathbf{Y}$ are $N_2 \times N_1$ and $N_3 \times N_2$ matrices with all standard complex Gaussian elements, $\mathcal{CN}(0, 1)$. Then, the relationships between $\lambda_{\text{max}}_1$ and $\lambda_1$, $\lambda_{\text{max}}_2$ and $\lambda_2$ are given by $\lambda_{\text{max}}_1 = \Omega_1(1 - \sigma^2)\lambda_1$ and $\lambda_{\text{max}}_2 = \Omega_2\lambda_2$. For notational brevity, we utilize $\lambda_3$ to stand for $\|\mathbf{H}_3(t)\|^2_F$ and $\lambda_4$ to denote $\|\mathbf{H}_4(t)\|^2_F$. As a result, the expression for the end-to-end instantaneous SNR of the secondary network is obtained from (4), (5), (6) and (7), as

$$\gamma_D = \frac{\lambda_1\lambda_2}{c\lambda_2 + d\lambda_3 + e\lambda_1\lambda_4}$$

where $c = \frac{(2 - \rho^2)}{\rho^2}$, $d = \frac{N_0}{Q\rho^2\Omega_1(1 - \sigma^2)}$, and $e = \frac{N_0}{Q\Omega_2}$.

### 3 End-to-End Performance Analysis

In this section, we analyze the outage probability (OP) and the symbol error rate (SER) of the considered system. To do so, we need to obtain the cumulative distribution function (CDF) of $\gamma_D$. However, deriving the exact expression of $F_{\gamma_D}(\gamma)$ from (8) is very challenging, so we use another approach. First, we propose a tightly bounded expression for the CDF of the instantaneous SNR, $\gamma_D$, in a similar manner as in [15, eq.(25)]. With this outcome, we will obtain tight expressions for the OP and SER of the considered system.

As mentioned in [14], the probability density function (PDF) of $\lambda_1$ is given by

$$f_{\lambda_1}(\lambda_1) = K_1 \sum_{k_1=1}^{P_1} \sum_{l_1=Q_1-P_1}^{(Q_1+P_1-2k_1)k_1} d_{k_1,l_1} \lambda_1^{l_1} \exp(-k_1\lambda_1)$$

where $P_1 = \min(N_1, N_2)$, $Q_1 = \max(N_1, N_2)$, and $K_1^{-1} = \prod_{i=1}^{P_1} (Q_1 - 1)! (i - 1)!$.

By integrating $f_{\lambda_1}(x)$ with respect to variable $x$ over the interval $(0, \lambda_1)$ and then applying [13, eq.(3.351.2)] to solve the integral, we obtain the CDF of $\lambda_1$ as

$$F_{\lambda_1}(\lambda_1) = 1 - K_1 \sum_{k_1=1}^{P_1} \sum_{l_1=Q_1-P_1}^{(Q_1+P_1-2k_1)k_1} d_{k_1,l_1} l_1! \sum_{m=0}^{l_1} \frac{k_1^m}{m!} \lambda_1^m \exp(-k_1\lambda_1)$$

where $c = \frac{(2 - \rho^2)}{\rho^2}$, $d = \frac{N_0}{Q\rho^2\Omega_1(1 - \sigma^2)}$, and $e = \frac{N_0}{Q\Omega_2}$.
Similarly, the PDF and CDF of $\lambda_2$ are given by

$$f_{\lambda_2}(\lambda_2) = K_2 \sum_{k_2=1}^{P_2} \frac{(Q_2 + P_2 - 2k_2)^k_2}{k_2!} d_{k_2} \lambda_2^{l_2} \exp(-k_2 \lambda_2)$$

(11)

$$F_{\lambda_2}(\lambda_2) = 1 - K_2 \sum_{k_2=1}^{P_2} \frac{(Q_2 + P_2 - 2k_2)^k_2}{k_2!} d_{k_2} \lambda_2^{l_2} \sum_{n=0}^{l_2} \frac{k_2^n}{n!} \lambda_2^n$$

(12)

where $P_2 = \min(N_2, N_3)$, $Q_2 = \max(N_2, N_3)$, and $K_2^{-1} = \prod_{i=1}^{P_2} (Q_2 - 1)! (i - 1)!$.

Since $\lambda_3$ is the Frobenius norm of the channel coefficient matrix from SU_{TX} to SU_{R}, it is a sum of the squares of $N_4 \times N_1$ i.i.d. complex Gaussian RVs with zero mean and variance $\Omega_3$. Thus, $\lambda_3$ is a Gamma random variable with parameter set $(N_1 N_4, \Omega_3)$ whose PDF and CDF are, respectively, written as

$$f_{\lambda_3}(\lambda_3) = \frac{\lambda_3^{N_1 N_4 - 1}}{\Omega_3^{N_1 N_4} \Gamma(N_1 N_4)} \exp\left(-\frac{\lambda_3}{\Omega_3}\right)$$

(13)

$$F_{\lambda_3}(\lambda_3) = 1 - \exp\left(-\frac{\lambda_3}{\Omega_3}\right) \sum_{p=0}^{N_1 N_4 - 1} \frac{\lambda_3^p}{\Omega_3^p} p!$$

(14)

Similarly, $\lambda_4$ is a sum of the squares of $N_4 \times N_2$ i.i.d. complex Gaussian RVs with zero mean and variance $\Omega_4$. Therefore, $\lambda_4$ has Gamma distribution with parameter set $(N_2 N_4, \Omega_4)$ whose PDF, CDF are, respectively, given by

$$f_{\lambda_4}(\lambda_4) = \frac{\lambda_4^{N_2 N_4 - 1}}{\Omega_4^{N_2 N_4} \Gamma(N_2 N_4)} \exp\left(-\frac{\lambda_4}{\Omega_4}\right)$$

(15)

$$F_{\lambda_4}(\lambda_4) = 1 - \exp\left(-\frac{\lambda_4}{\Omega_4}\right) \sum_{q=0}^{N_2 N_4 - 1} \frac{\lambda_4^q}{\Omega_4^q} q!$$

(16)

Now, we approximate $\gamma_D$ as in [15, eq.(25)], i.e, $\gamma_D \approx \min(\gamma_1, \gamma_2)$ where $\gamma_1 = \frac{\lambda_1}{c + d \lambda_3}$ and $\gamma_2 = \frac{\lambda_2}{c \lambda_4}$. Because $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$ are independent, we can apply the order statistics theory to obtain the CDF of $\gamma_D$ as

$$F_{\gamma_D}(\gamma) = 1 - [1 - F_{\gamma_1}(\gamma)][1 - F_{\gamma_2}(\gamma)]$$

(17)

where $F_{\gamma_1}(\gamma)$ and $F_{\gamma_2}(\gamma)$ are given by

$$F_{\gamma_1}(\gamma) = \int_0^\infty f_{\lambda_1}(\gamma(c + d \lambda_3)) f_{\lambda_3}(\lambda_3) d\lambda_3$$

(18)
\[ F_{\gamma_2}(\gamma) = \int_0^\infty F_{\lambda_2}(\gamma e^{\lambda_4}) f_{\lambda_4}(\lambda_4) d\lambda_4 \quad (19) \]

Substituting (10), (13) into (18) and (12), (15) into (19), after some algebraic manipulations, we rewrite \( F_{\gamma_1}(\gamma) \) and \( F_{\gamma_2}(\gamma) \) as

\[
F_{\gamma_1}(\gamma) = 1 - K_1 \sum_{k_1=1}^{P_1} \frac{(Q_1+P_1-2k_1)k_1}{\lambda_3^{N_1N_4}} \sum_{l_1=Q_1-P_1} \frac{d_{k_1,l_1} \Gamma(l_1+1)}{\Gamma(N_1N_4)} \\
\times \sum_{m=0}^{l_1} \frac{\gamma^m}{k_1^{m+1}m!} \sum_{u=0}^{l_1} C_u d^u e^{m-u} \exp(-k_1 c\gamma) \\
\times \int_0^{\lambda_3^{N_1N_4}} \gamma^u e^{\lambda_4+1} \left(-\frac{k_1 \gamma d \Omega_3 + 1}{\Omega_3} \right) d\lambda_3 \quad (20)
\]

\[
F_{\gamma_2}(\gamma) = 1 - K_2 \sum_{k_2=1}^{P_2} \frac{(Q_2+P_2-2k_2)k_2}{\lambda_4^{N_2N_4}} \sum_{l_2=Q_2-P_2} \frac{d_{k_2,l_2} \Gamma(l_2+1)}{\Gamma(N_2N_4)} \\
\times \sum_{m=0}^{l_2} \frac{\gamma^m}{k_2^{m+1}m!} \sum_{n=0}^{l_2} e^n \\
\times \int_0^{\lambda_4^{N_2N_4}} \gamma^n e^{\lambda_4+1} \left(-\frac{k_2 \gamma d \Omega_4 + 1}{\Omega_4} \right) d\lambda_4 \quad (21)
\]

Utilizing [13, eq.(3.351.2)] to solve the integral of (20) and (21), the closed-form expressions for the CDF of \( \gamma_1 \) and \( \gamma_2 \) are acquired as

\[
F_{\gamma_1}(\gamma) = 1 - K_1 \sum_{k_1=1}^{P_1} \frac{(Q_1+P_1-2k_1)k_1}{\lambda_3^{N_1N_4}} \sum_{l_1=Q_1-P_1} \frac{d_{k_1,l_1} \Gamma(l_1+1)}{\Gamma(N_1N_4)} \\
\times \sum_{m=0}^{l_1} \frac{\gamma^m}{k_1^{m+1}m!} \sum_{u=0}^{l_1} C_u d^u e^{m-u} \frac{\Omega_3^u}{\Gamma(N_1N_4)} \gamma^m \exp(-k_1 c\gamma) \quad (22)
\]

\[
F_{\gamma_2}(\gamma) = 1 - K_2 \sum_{k_2=1}^{P_2} \frac{(Q_2+P_2-2k_2)k_2}{\lambda_4^{N_2N_4}} \sum_{l_2=Q_2-P_2} \frac{d_{k_2,l_2} \Gamma(l_2+1)}{\Gamma(N_2N_4)} \\
\times \sum_{m=0}^{l_2} \frac{\gamma^m}{k_2^{m+1}m!} \sum_{n=0}^{l_2} e^n \frac{\Omega_4^n}{\Gamma(N_2N_4)} \gamma^n \exp(-k_2 c\gamma) \quad (23)
\]
By substituting (22) and (23) into (17), the CDF of the instantaneous end-to-end SNR $\gamma_D$ is finally obtained as

$$F_{\gamma_D}(\gamma) = 1 - K_1 K_2 \sum_{k_1=1}^{P_1} \sum_{l_1=Q_1-P_1}^{(Q_1+P_1-2k_1)k_1} d_{k_1,l_1} l_1! \sum_{m=0}^{l_1} \frac{1}{m!} \times \frac{1}{k_1^{l_1-m+1}} \sum_{u=0}^{m} C_u \sum_{k_2=1}^{P_2} \sum_{l_2=Q_2-P_2}^{(Q_2+P_2-2k_2)k_2} d_{k_2,l_2} l_2! \sum_{n=0}^{l_2} \frac{1}{n!} \times \frac{e^{-u \gamma} \Omega^u}{k_2^{l_2-n+1}} \frac{\Gamma(N_1 N_4 + u)}{\Gamma(N_1 N_4)} \frac{\Gamma(N_2 N_4 + n)}{\Gamma(N_2 N_4)} \times \frac{\gamma^{m+n} \exp(-k_1 \gamma)}{(k_2 \gamma \epsilon \Omega_4 + 1)^{N_2 N_4 + n} (k_1 \gamma d \Omega_3 + 1)^{N_1 N_4 + u}}$$

(24)

### 3.1 Outage Probability

Outage probability, the probability that the instantaneous SNR drops below a predefined threshold $\gamma_{th}$, is easily obtained by using $\gamma_{th}$ as argument of the CDF of the instantaneous SNR given in (24), $P_{out} = F_{\gamma_D}(\gamma_{th})$.

### 3.2 Symbol Error Rate

As shown in [16], for several modulation schemes, the expression for SER can be derived directly from $F_{\gamma_D}(\gamma)$ as

$$P_E = \frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_0^\infty F_{\gamma_D}(\gamma) \gamma^{-\frac{1}{2}} e^{-b \gamma} d\gamma$$

(25)

where $a$ and $b$ are modulation parameters (see [16]), i.e., for $M$-PSK, $a = 2, b = \sin^2(\pi/M)$. By substituting (24) into (25), after some simplifications,
the closed-form expression of a tight upper bound for the SEP is rewritten as

\[ P_E = \frac{a \sqrt{b}}{2\sqrt{\pi}} \int_{0}^{\infty} \gamma^{-\frac{1}{2}} \exp(-b\gamma) d\gamma - \frac{a \sqrt{b} K_1 K_2}{2\sqrt{\pi}} \sum_{k_1=1}^{P_1} (Q_1 + P_1 - 2k_1) k_1 \]

\[ \times \sum_{l_1=Q_1 - P_1}^{l_1} \frac{1}{l_1} \sum_{m=0}^{l_1} \frac{C_m^m}{m!} \sum_{u=0}^{m} C_u^m \]

\[ \times \frac{1}{n!} \frac{1}{k_1^{N_1 N_4 + u + l_1 - m + 1}} \frac{1}{k_2^{N_2 N_4 + l_2 + 1}} e^{N_2 N_4 + l_2} d^{N_1 N_4} \]

\[ \times \frac{\Gamma(N_1 N_4 + u)}{\Gamma(N_1 N_4)} \frac{\Gamma(N_2 N_4 + n)}{\Gamma(N_2 N_4)} \]

\[ \times \left[ \sum_{i=1}^{N_2 N_4 + n} \kappa_i \int_{0}^{\infty} \gamma^{m+n-\frac{1}{2}} \exp\left(-\frac{(k_1 c + b)\gamma}{\kappa_2 e^{\Omega_4}}\right) \frac{1}{\gamma + \left(\frac{1}{\kappa_1 e^{\Omega_3}}\right)^i} d\gamma \right] \]

Utilizing [13, eq.(3.351.2)] to calculate the first integral of (26), and then applying the partial fraction in [13, eq.(3.326.2)] to transform the expression in the second integral of (26) into a tabulated form, we have

\[ P_E = \frac{a \sqrt{b} K_1 K_2}{2\sqrt{\pi}} \sum_{k_1=1}^{P_1} (Q_1 + P_1 - 2k_1) k_1 \]

\[ \times \sum_{l_1=Q_1 - P_1}^{l_1} \frac{1}{l_1} \sum_{m=0}^{l_1} \frac{C_m^m}{m!} \sum_{u=0}^{m} C_u^m \]

\[ \times \frac{1}{n!} \frac{1}{k_1^{N_1 N_4 + u + l_1 - m + 1}} \frac{1}{k_2^{N_2 N_4 + l_2 + 1}} e^{N_2 N_4 + l_2} \]

\[ \times \frac{\Gamma(N_1 N_4 + u)}{\Gamma(N_1 N_4)} \frac{\Gamma(N_2 N_4 + n)}{\Gamma(N_2 N_4)} \]

\[ \times \left[ \sum_{i=1}^{N_2 N_4 + n} \kappa_i \int_{0}^{\infty} \gamma^{m+n-\frac{1}{2}} \exp\left(-\frac{(k_1 c + b)\gamma}{\kappa_2 e^{\Omega_4}}\right) \frac{1}{\gamma + \left(\frac{1}{\kappa_1 e^{\Omega_3}}\right)^i} d\gamma \right] \]
where $\kappa_i$ and $\theta_j$ are defined as

$$
\kappa_i = \frac{d^{N_2 N_4 + n-i}}{(N_2 N_4 + n-i)!} \Gamma(N_2 N_4 + n-i) \left[ \left( \gamma + \frac{1}{k_1 d \Omega_3} \right)^{-(N_1 N_4 + u)} \right]_{\gamma = -\frac{1}{k_2 e \Omega_3}}^{\gamma = -\frac{1}{k_1 d \Omega_3}}
$$

(28)

$$
\theta_j = \frac{d^{N_1 N_4 + u-j}}{(N_1 N_4 + u-j)!} \Gamma(N_1 N_4 + u-j) \left[ \left( \gamma + \frac{1}{k_2 e \Omega_4} \right)^{-(N_2 N_4 + n)} \right]_{\gamma = -\frac{1}{k_1 d \Omega_3}}^{\gamma = -\frac{1}{k_1 d \Omega_3}}
$$

(29)

The integrals in (27) are solved by using [17, eq.(2.3.6.9)]. After some algebraic manipulations and re-arranging terms, we finally obtain an expression for the SER as follows:

$$
P_E = \frac{a}{2} - \frac{a \sqrt{b}}{2 \pi} K_1 K_2 \sum_{k_1=1}^{P_1} \sum_{l_1=Q_1-P_1}^{(Q_1+P_1-2k_1)k_1} \frac{d_{k_1,l_1,l_1}!}{l_1!} \sum_{m=0}^{l_1} \frac{1}{m!}
$$

$$
\times \sum_{u=0}^{m} \sum_{k_2=1}^{P_2} \sum_{l_2=Q_2-P_2}^{(Q_2+P_2-2k_2)k_2} \frac{d_{k_2,l_2,l_2}!}{l_2!} \sum_{n=0}^{l_2} \frac{1}{n!} \kappa_i^{N_2 N_4 + u + l_1 - m + 1} \times \frac{e^{m-u} \Gamma(N_1 N_4 + u) \Gamma(N_2 N_4 + n) \Gamma(m + n + \frac{1}{2})}{k_2^{N_2 N_4 + l_2 + 1} e^{N_2 N_4 \Omega_3^2} N_2 N_4 \Omega_4^2 N_1 N_4 \Omega_3^{N_1 N_4} \Gamma(N_1 N_4) \Gamma(N_2 N_4)}
$$

$$
\times \left[ \sum_{i=1}^{N_2 N_4 + n} \kappa_i \frac{U \left( m + n + \frac{1}{2}, m + n + \frac{1}{2} + 1 - i, \frac{k_1 c + b}{k_2 e \Omega_3} \right)}{k_2^{m+n+\frac{1}{2}-i} e^{m+n+\frac{1}{2}-i \Omega_4^{m+n+\frac{1}{2}-i}}}, \theta_j \frac{U \left( m + n + \frac{1}{2}, m + n + \frac{1}{2} + 1 - j, \frac{k_1 c + b}{k_1 d \Omega_3} \right)}{k_1^{m+n+\frac{1}{2}-j} e^{m+n+\frac{1}{2}-j \Omega_3^{m+n+\frac{1}{2}-j}}} \right]
$$

(30)

### 4 Numerical Results and Discussion

In this section, numerical results are presented for the OP and SER of Q-PSK for the examined MRT scheme in a cognitive AF relay network. In all scenarios, average channel power gains of Rayleigh fading channels are selected as $\Omega_1 = 0.5$, $\Omega_2 = 0.7$, $\Omega_3 = 0.8$, $\Omega_4 = 0.6$. First, we examine the effect of the number of transceiver antennas on the system by fixing the variance of CEE $\sigma^2 = 0.01$ and the normalized Doppler frequency $f_d \tau = 0.03$ to scale the channel correlation coefficient $\rho = 0.99$. Furthermore, to demonstrate the impact of CEE and FD on the considered system, we also illustrate the OP
and SER when fixing the number of transceiver antennas while changing the level of channel estimation error and feedback delay.

Fig. 2 and Fig. 3 depict the OP and SER versus average SNR for various antenna configurations in three cases:

- Case 1: \((N_1, N_2, N_3, N_4 = 2, 2, 2, 2)\)
- Case 2: \((N_1, N_2, N_3, N_4 = 2, 3, 2, 3)\)
- Case 3: \((N_1, N_2, N_3, N_4 = 3, 3, 3, 3)\)

As can be seen from these figures, when the number of transceiver antennas increases, the OP and SER of the considered system is improved significantly; which illustrates the benefits of deploying MRT with multiple antennas in a cognitive AF relay system.

Fig. 4 and Fig. 5 plot the OP and SER versus average SNR for various levels of feedback delay and channel estimation error. We consider four examples where the system is equipped with \(N_1 = 3\) transmit antennas at the SU\(_{TX}\), \(N_2 = 3\) receive antennas at the SU\(_{R}\), \(N_3 = 3\) receive antennas at the SU\(_{RX}\), and \(N_4 = 3\) receive antennas at the PU\(_{RX}\). In Case 4, the system has imperfect CSI in both channel estimation error \(\sigma^2 = 0.01\) and feedback delay presented through the channel correlation coefficient \(\rho = 0.9\) (corresponding to \(f_d\tau = 0.1\)). Case 5 has perfect channel estimation \(\sigma^2 = 0\) but having feedback delay with \(\rho = 0.9\) or \(f_d\tau = 0.1\). Furthermore, in Case 6, we illustrate the system performance for channel estimation error \(\sigma^2 = 0.5\) and no feedback delay \(\rho = 1.0\). Finally, we present the OP and SER when SU\(_{R}\) estimates the channel perfectly \(\sigma^2 = 0\), and there is no feedback delay \(\rho = 0\) \((f_d\tau = 0)\) in Case 7. As expected, the best performance can be achieved in Case 7 when having no channel estimation error and no feedback delay (perfect CSI). Conversely, the worst performance is obtained in Case 4 when having both channel estimation error and feedback delay. In the presence of feedback delay \(\rho \neq 1\) as in Case 5 and channel estimation error \(\sigma^2 \neq 0\) as in Case 6, the system performance will be degraded, but not as serious as in Case 4. By comparing Case 4 with Case 5 and Case 6 with Case 7, the effect of CEE on the system will be observed. Further, we can notice that this influence is significant in low SNR and minor in the high SNR. Finally, by comparing Case 4 with Case 6 and Case 5 with Case 7, one can observe the impact of FD on the system performance being remarkable in all SNR range.
MRT/MRC for cognitive AF relay networks under feedback delay and channel estimation error

Figure 2: Outage probability for cognitive AF relay systems with MRT for various transceiver antenna configurations.

Figure 3: SER of Q-PSK for cognitive AF relay systems with MRT for various transceiver antenna configurations.
Figure 4: Outage probability for cognitive AF relay systems with MRT for various levels of channel estimation error and feedback delay.

Figure 5: SER of Q-PSK for cognitive AF relay systems with MRT for various levels of channel estimation error and feedback delay.
5 Conclusion

In this paper, we investigated the performance of an underlay cognitive AF relay system deploying MRT/MRC. More specifically, we have derived closed-form expressions in terms of tight upper bounds for OP and SER in i.i.d. Rayleigh fading channels. Our analysis takes into account the impact of both channel estimation error and feedback delay on the system. The numerical results illustrate the influence of the number of transceiver antennas and the effect of imperfect CSI including channel estimation error and feedback delay on the considered network.

References


Part II-C
Part II-C

Adaptive Modulation and Coding with Queue Awareness in Cognitive Incremental Decode-and-Forward Relay Networks
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Adaptive Modulation and Coding with Queue Awareness in Cognitive Incremental Decode-and-Forward Relay Networks

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Abstract

This paper studies the performance of adaptive modulation and coding in a cognitive incremental decode-and-forward relaying network where a secondary source can directly communicate with a secondary destination or via an intermediate relay. To maximize transmission efficiency, a policy which flexibly switches between the relaying and direct transmission is proposed. In particular, the transmission, which gives higher average transmission efficiency, will be selected for the communication. Specifically, the direct transmission will be chosen if its instantaneous signal-to-noise ratio (SNR) is higher than one half of that of the relaying transmission. In this case, the appropriate modulation and coding scheme (MCS) of the direct transmission is selected only based on its instantaneous SNR. In the relaying transmission, since the MCS of the transmissions from the source to the relay and from the relay to the destination are implemented independently to each other, buffering of packets at the relay is necessary. To avoid buffer overflow at the relay, the MCS for the relaying transmission is selected by considering both the queue state and the respective instantaneous SNR. Finally, a finite-state Markov chain is modeled to analyze key performance indicators such as outage probability and average transmission efficiency of the cognitive relay network.

1 Introduction

The recent thriving in wireless communications has dramatically increased the scarcity of radio spectrum. However, measurement campaigns have shown
that spectrum bands using a fixed spectrum allocation policy are often under-utilized. This necessitates new technologies to improve the efficiency of spectrum utilization. A novel idea of opportunistically utilizing particular bands of the spectrum, called cognitive radios, was introduced by Mitola [1,2]. This spectrum-allocation policy allows cognitive users, also called secondary users (SUs), to coexist with licensed users or primary users (PUs) in the same spectrum band without degrading the performance of the PUs. To satisfy this requirement, the SU, while accessing the spectrum band, must always consider the impact of its transmission on the quality of the PU reception. In the spectrum-sharing context, [3] has introduced the interference temperature concept and the tolerable interference level at the primary receiver. Therein, the SU is considered not to affect the PU’s performance if the interference power at the primary receiver is kept below a predefined acceptable threshold. Because of this rigid spectrum access regulation, the SU must strictly constrain its transmit power and hence communication range in the secondary network is very limited.

In order to overcome this limitation of spectrum-sharing, relaying transmission has been considered as a potential solution to increase the radio coverage [4]. Based on how the signal is processed at the relay, there are two well-known relaying schemes, namely, decode-and-forward (DF) relaying and amplify-and-forward (AF) relaying. In an AF relay system, the signal is simply amplified at the relay and then forwarded to the destination. Thus, not only the desired signal but also both noise and interference are amplified at the relay. On the contrary, in the DF relay system, the received signal from the source is decoded, regenerated at the relay and then transmitted to the destination. Since only the desired signal is forwarded to the destination, DF relaying is suitable for transmission in interference environments. This, in turn, seems applicable to spectrum-sharing cognitive radio networks where the secondary transmitter is required to emit its signals with rather low power to meet the interference power constraint at the primary receiver.

The policies to select the relaying transmission are distinguished as fixed, selection, and incremental relaying [5]. The simplest scheme is the fixed relaying where the relays are always utilized regardless of their performance. Thus, the fixed relaying suffers from a reduction in spectrum efficiency. Based upon fixed relaying, the selection relaying scheme allows a source to select cooperation or non-cooperation with a suitable relay. By continuously comparing the instantaneous signal-to-noise ratios (SNRs) between the links, the source can select the relay which offers the highest SNR to forward the signal to the destination. With incremental relaying, the relaying mode is active only if the instantaneous SNR raises beyond a predefined threshold. Because of limiting the amount of feedback in terms of channel state information (CSI), the
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spectral efficiency of the incremental relaying outperforms that of fixed and selection relaying [5]. An alternative method to enhance spectrum efficiency is to deploy appropriate link adaptation strategies where certain parameters such as transmit power, modulation scheme, code rate or any combination of these parameters are adjusted to the variations of the fading channel [6]. Furthermore, this strategy conserves transmit power to reduce the interference from its transmission to the PU as well as to alleviate the effect of the fading channels. However, since the fading conditions of the transmissions from the source to the relay and from the relay to the destination are independent to each other, the modulation and coding schemes for these transmissions may be different. Therefore, a buffer is necessary at the relay to reduce the dropping rate of packets at the relay.

In this paper, we deploy adaptive modulation and coding (AMC) for cognitive incremental DF relay networks where the communication can be performed by the relaying or direct transmission to maximize transmission efficiency. The switching policy for selecting between the relaying and the direct communication is based on [7], i.e., the relaying mode is chosen only if its instantaneous SNR is two times higher than that of the direct mode. Furthermore, we address the overflow issue of the buffer at the relay by considering the queue state when selecting the modulation and coding scheme (MCS) for the relaying transmission. A finite-state Markov chain model is utilized to analyze the distribution of the number of packets in the buffer of the relay which enables us to evaluate outage probability and transmission efficiency of the network.

Notation: In this paper, bold lower case and upper case letters are used to represent vector and matrix, respectively. Next, the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable $X$ are denoted as $f_X(\cdot)$ and $F_X(\cdot)$, respectively. Furthermore, the ceiling operator and expectation operator are, respectively, expressed as $\lceil \cdot \rceil$ and $E\{\cdot\}$. The binomial operator is represented as $(\cdot)$. Finally, the gamma function [8, eq. (8.310.1)] and the incomplete gamma function [8, eq. (8.350.2)] are represented as $\Gamma(n)$ and $\Gamma(n, x)$, respectively.

2 System and Channel Model

The considered cognitive relay network consists of a secondary source $S$, a secondary relay $R$, and a secondary destination $D$. This network operates under the interference power constraint $Q$ of a primary receiver, $P$, as depicted in Fig. 1. Here, $h_{yz}$, $y \in \{s,r,d\}$, $z \in \{r,d,p\}$, and $y \neq z$, is the channel coefficient of the link from $Y \in \{S,R,D\}$ to $Z \in \{R,D,P\}$, $Y \neq Z$. In order to maximize transmission efficiency in the secondary network, we employ
incremental DF relaying where S can either directly communicate with D or communicate via the relay R. Selection of the operation mode, direct transmission or relaying, is based on the respective instantaneous SNR. Thus, R and D need to estimate their received SNRs and feed them back to S and R to choose the appropriate operation mode. Assume that this network operates under block Nakagami-m fading such that the channels can be considered as constant for the transmit period of each packet. Let $X_{yz}$ be the channel power gain corresponding to the channel coefficient $h_{yz}$. Also, let us denote $P_{\text{max}}$ as the transmit power limit of both S and R. Under the interference power constraint $Q$ of P and transmit power limit, $P_{\text{max}}$, of S and R, the transmit power $P_s$ of S and $P_r$ of R must be controlled as

$$P_s = \min \left( P_{\text{max}}, \frac{Q}{X_{sp}} \right), \quad P_r = \min \left( P_{\text{max}}, \frac{Q}{X_{rp}} \right)$$

(1)

Based on [9, eq. (5)], we can derive the CDF, $F_{\gamma_{yz}}(\gamma)$, and PDF, $f_{\gamma_{yz}}(\gamma)$, of the instantaneous SNR $\gamma_{yz}$ of the link with channel coefficient $h_{yz}$ as (2) and (3), respectively.

$$F_{\gamma_{yz}}(\gamma) = 1 - \sum_{l=0}^{m_{yz}-1} \frac{1}{l!} \frac{\alpha_{yz}^l \gamma^l}{\beta^l} \exp \left( - \frac{\alpha_{yz} \gamma}{\beta} \right) + \sum_{i=0}^{m_{yp}-1} \frac{\alpha_{yp}^i}{i!} \alpha_{yz}^{m_{yz}}$$

$$\times \mu^i \exp \left( - \frac{\alpha_{yp} \mu}{\beta} \right) \frac{\Gamma(m_{yz} + i)}{\Gamma(m_{yz})} \frac{\gamma^{m_{yz}}}{(\alpha_{yz} \gamma + \alpha_{yp} \mu)^{m_{yz}+i}} \sum_{j=0}^{m_{yz}+i-1} \frac{1}{j!}$$

$$\times \frac{(\alpha_{yz} \gamma + \alpha_{yp} \mu)^j}{\beta^j} \exp \left( - \frac{\alpha_{yz} \gamma}{\beta} \right)$$

(2)
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\[
f_{\gamma_{yz}}(\gamma) = \sum_{l=0}^{m_{yz}-1} \frac{\alpha_{yz}^{l+1}}{\beta^{l+1}} \frac{\gamma^l}{l!} \exp \left( -\frac{\alpha_{yz}\gamma}{\beta} \right) - \sum_{l=0}^{m_{yz}-1} \frac{\alpha_{yz}^{l}}{l!} \frac{\gamma^{l-1}}{\beta^l} \exp \left( -\frac{\alpha_{yz}\gamma}{\beta} \right)
\]

\[
\times \exp \left( -\frac{\alpha_{yz}\gamma}{\beta} \right) - \sum_{i=0}^{m_{yp}-1} \frac{\mu^i}{i!} \frac{\Gamma(m_{yz} + i)}{\Gamma(m_{yz})} \exp \left( -\frac{\alpha_{yp}\mu}{\beta} \right)
\]

\[
\times \sum_{j=0}^{m_{yz}+i-1} \frac{\alpha_{yp}^{i} \alpha_{yz}^{j-i}}{j! \beta^j} \frac{\gamma^{m_{yz}}}{\gamma + \alpha_{yp}\mu/\alpha_{yz}} \exp \left( -\frac{\alpha_{yz}\gamma}{\beta} \right)
\]

\[
\times \left( -\frac{m_{yz}}{\gamma} + \frac{\alpha_{yz}}{\beta} + \frac{m_{yz} + i - j}{\gamma + \alpha_{yp}\mu/\alpha_{yz}} \right)
\]

(3)

Here, \( \beta = P_{max}/N_0 \), \( \mu = Q/N_0 \), and \( N_0 \) is the noise power at the secondary relay and destination. Further, \( \alpha_{yz} \) is defined as \( \alpha_{yz} = m_{yz}/\Omega_{yz} \) where \( m_{yz} \) and \( \Omega_{yz} \) are, respectively, the fading severity parameter and channel mean power of the link with channel coefficient \( h_{yz} \). Then, the instantaneous SNR of the DF relaying link can be approximated as in [10, eq. (25)]

\[
\gamma_{sr_d} = \min(\gamma_{sr}, \gamma_{rd})
\]

(4)

Then, the CDF, \( F_{\gamma_{sr_d}}(\gamma) \), of the instantaneous SNR of the relaying link is given by

\[
F_{\gamma_{sr_d}}(\gamma) = 1 - [1 - F_{\gamma_{sr}}(\gamma)] [1 - F_{\gamma_{rd}}(\gamma)]
\]

(5)

where \( F_{\gamma_{sr}}(\gamma) \) and \( F_{\gamma_{rd}}(\gamma) \) are defined in (2) with parameter sets \( (m_{yz}, \alpha_{yz}, m_{yp}, \alpha_{yp}) = (m_{sr}, \alpha_{sr}, m_{sp}, \alpha_{sp}) \) and \( (m_{yz}, \alpha_{yz}, m_{yp}, \alpha_{yp}) = (m_{rd}, \alpha_{rd}, m_{rr}, \alpha_{rp}) \), respectively.

In order to improve transmission efficiency, we apply AMC with five different MCSs as shown in Table 1 [11].

<table>
<thead>
<tr>
<th>Modulation</th>
<th>MCS 1</th>
<th>MCS 2</th>
<th>MCS 3</th>
<th>MCS 4</th>
<th>MCS 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Code</td>
<td>BPSK</td>
<td>QPSK</td>
<td>QPSK</td>
<td>16-QAM</td>
<td>16-QAM</td>
</tr>
<tr>
<td>Rate (bps)</td>
<td>0.50</td>
<td>1.00</td>
<td>1.50</td>
<td>2.25</td>
<td>3.00</td>
</tr>
<tr>
<td>( a_n )</td>
<td>274.72</td>
<td>90.25</td>
<td>67.61</td>
<td>50.12</td>
<td>53.39</td>
</tr>
<tr>
<td>( g_n )</td>
<td>7.99</td>
<td>3.49</td>
<td>1.68</td>
<td>0.66</td>
<td>0.37</td>
</tr>
<tr>
<td>( \gamma_{Tr} )</td>
<td>-1.533</td>
<td>1.049</td>
<td>3.972</td>
<td>7.702</td>
<td>10.249</td>
</tr>
</tbody>
</table>
into 5 regions with the switching thresholds between MCSs given by

\[ \gamma_n = \frac{1}{g_n} \ln \left( \frac{a_n}{P ER_{TG}} \right), \quad n = 1, 2, \ldots, N \]  

(6)

\[ \gamma_{N+1} = \infty \]  

(7)

where \( a_n, g_n \) are predefined constants given in Table I, \( N \) is the number of modes, in our case, \( N = 5 \), and \( P ER_{TG} \) is the target packet error rate. The parameters \( a_n, g_n \) and \( \gamma_{T_n} \) for the \( n \)-th mode are obtained by applying the fitting algorithm given in [12]. Assuming that the \( n \)-th MCS is selected, then there are \( n \) packets transmitted during one time slot, \( T_s \). However, if the instantaneous SNRs of both the direct and relaying links fall below \( \gamma_1 \), the system falls into outage and no transmission takes place. As the transmission extends over two time slots, the overall transmission efficiency of the relaying mode is reduced by half [13]. Thus, the relaying mode is chosen only if the instantaneous SNR over the relaying link, \( \gamma_{srd} \), is two times higher than that of the direct link, \( \gamma_{sd} \), i.e., \( \gamma_{sd} < \gamma_{srd} \frac{1}{2} \).

In the direct communication, i.e., \( \gamma_{sd} \geq \gamma_{srd} \frac{1}{2} \), S chooses an appropriate number of packets with fixed length, encodes and modulates them with a suitable MCS from Table I and then transmits the signal to D while R remains silent. The MCS selection for the direct transmission is only based on the fading condition of the link from S to D. In particular, the \( n \)-th MCS will be assigned to the direct transmission if the instantaneous SNR of the direct transmission \( \gamma_{sd} \) falls into the region \( (\gamma_n, \gamma_{n+1}) \) and the instantaneous SNR of the relaying transmission \( \gamma_{srd} \) is lower than \( \gamma_{2n+1} \). For this reason, when \( \gamma \left( \frac{N}{2} \right) \leq \gamma_{sd} \), there exists only the direct transmission.

In the relaying communication, i.e., \( \gamma_{sd} < \gamma_{srd} \frac{1}{2} \), the MCS for the relaying transmission is selected based on both the fading conditions and the state of the buffer at R. This means that S and R not only adapt their transmission to the fading condition but also to the current state of the buffer in order to avoid buffer overflow.

We first look at the MCS selection at the source S for the transmission from S to R. During the first time slot, S selects an appropriate number of packets, encodes and modulates them by using a suitable MCS and then transmits the result to R. Specifically, the MCS at S is selected based on the fading condition of the channel from S to R and the available vacant positions at the buffer of the relay as follows. Assume that there are \( N_e \) vacant positions available at the buffer of R for a certain time instant. Furthermore, at that time, the instantaneous SNR \( \gamma_{sr} \) of the transmission from S to R is assumed to be able to support up to the \( t \)-th mode, i.e., the instantaneous SNR \( \gamma_{sr} \).
falls into the below region
\[
\begin{cases}
\gamma_t \leq \gamma_{sr} < \gamma_{t+1} & \text{for } t \in \{1, 2, 3, 4\} \\
\gamma_t \leq \gamma_{sr} & \text{for } t = 5
\end{cases}
\] (8)

In this case, the \(n\)-th MCS, \(n = \min(N_e, t)\), is selected for the transmission from S to R. Then, R attempts to demodulate and decode the received signal, places the successfully decoded packets into the buffer which provides \(L\) positions for storing packets. It is assumed that the unsuccessfully decoded packets are simply dropped and the higher layers are responsible for detecting and retransmitting these packets.

We are now looking at the MCS selection at the relay for the transmission from R to D. During the second time slot, R encodes and modulates an appropriate number of packets using the selected MCS and forwards the resulting signal to D. The MCS selection at R is based on the instantaneous SNR \(\gamma_{rd}\) of the transmission from R to D and the available packets in the buffer at the relay. Assume that there are \(N_a\) available packets at the buffer of R and the instantaneous SNR \(\gamma_{rd}\) of the transmission from R to D falls into the region of the \(k\)-th mode, i.e.,
\[
\begin{cases}
\gamma_k \leq \gamma_{rd} < \gamma_{k+1} & \text{for } k \in \{1, 2, 3, 4\} \\
\gamma_k \leq \gamma_{rd} & \text{for } k = 5
\end{cases}
\] (9)

Then, the \(n\)-th MCS, \(n = \min(N_a, k)\), is assigned to the transmission from R to D.

3 Queueing Analysis

Denote \(p_{i,j}\) as the transition probability that the number of packets at the buffer of the relay changes from \(i\) in the current time interval to \(j\) in the next time interval. In order to calculate \(p_{i,j}\), we define two following probabilities.

First, we denote \(a_{k,i}\) as the probability that, given \(i, 0 \leq i \leq L\) packets in its buffer, R forwards \(k, 0 \leq k \leq \min\{i, N\}\), packets to D. It is noted that if \(k < i\), \(a_{k,i}\) is the probability that \(\gamma_k \leq \gamma_{rd} \leq \gamma_{k+1}\). On the contrary, if \(k = i\), \(a_{k,i}\) is the probability that the instantaneous SNR \(\gamma_{rd}\) of the transmission from R to D is sufficient to operate in the \(k\)-th MCS, i.e.,
\[
a_{k,i} = \begin{cases}
F_{\gamma_{rd}}(\gamma_{k+1}) - F_{\gamma_{rd}}(\gamma_k), & k < i \\
1 - F_{\gamma_{rd}}(\gamma_k), & k = i \\
0, & k > i
\end{cases}
\] (10)
Second, we define \( b_{h,q} \) as the probability that \( h \) new decoded packets are put into the buffer conditioned on having \( q \) empty positions in the buffer. This is also the probability that \( R \) successfully decodes the \( h \) packets sent from \( S \) given \( q \) vacant positions in the buffer, i.e.,

\[
b_{h,q} = \sum_{v=h}^{\min(N,q)} p_{h,v,q}
\]

where \( p_{h,v,q} \) is the joint probability that \( S \) transmits \( v \) packets and \( R \) successfully decodes \( h \) packets given \( q \) empty positions in the buffer. Clearly, if \( v < q \), then \( p_{h,v,q} \) is the probability that the instantaneous SNR of the link from \( S \) to \( R \) falls in the region of the \( v \)-th MCS, \( \gamma_v \leq \gamma_{sr} \leq \gamma_{v+1} \), and exactly \( h \) packets are successfully decoded from \( v \) transmitted packets. However, if \( v = q \), then \( p_{h,v,q} \) is the probability that the instantaneous SNR \( \gamma_{sr} \) is sufficient for \( S \) to operate in the \( v \)-th MCS, \( \gamma_{sr} \geq \gamma_v \), and \( h \) packets are successfully decoded from \( v \) transmitted packets. As a result, we have

\[
p_{h,v,q} = \begin{cases} 
\int_{\gamma_v}^{\gamma_{v+1}} s_{h,v}(\gamma) f_{\gamma_{sr}}(\gamma) d\gamma, & v < \min(N,q) \\
\int_{\gamma_v}^{\infty} s_{h,v}(\gamma) f_{\gamma_{sr}}(\gamma) d\gamma, & v = \min(N,q) \\
0, & v > \min(N,q)
\end{cases}
\]

(12)

Here, \( s_{h,v}(\gamma) \) is the probability that \( R \) successfully decodes \( h \) packets from \( v \) transmitted packets from \( S \). Assuming that the probability of packets being successfully decoded is statistical independent from each other, we have

\[
s_{h,v}(\gamma) = \binom{v}{h}[1 - P_{e,v}(\gamma)]^h P_{e,v}^{v-h}(\gamma)
\]

(13)

where, \( P_{e,v}(\gamma) \) is the packet error rate (PER) with the received SNR \( \gamma \) when the transmission operates with the \( v \)-th MCS. As in [12], \( P_{e,v}(\gamma) \) is approximated as

\[
P_{e,v}(\gamma) = \begin{cases} 
1, & 0 < \gamma < \gamma_{Tv} \\
a_v \exp(-g_v \gamma), & \gamma \geq \gamma_{Tv}
\end{cases}
\]

(14)

Substituting (14) into (13), we obtain \( s_{h,v}(\gamma) \). Then, substituting this outcome and \( f_{\gamma_{sr}}(\gamma) \) defined in (3) into (12) together with the help of [8, eq. (3.381.3)],
\( p_{h,v,q} \) is given by

\[
 p_{h,v,q} = \begin{cases} 
 \theta_1(h, v), & v < \min(N, q) \\
 \theta_2(h, v), & v = \min(N, q) \\
 0, & v > \min(N, q) 
\end{cases}
\]  

(15)

where \( \theta_1(h, v) \) and \( \theta_2(h, v) \) are given by (16) and (17), respectively.

\[
\theta_1(h, v) = \left( \frac{v}{h} \right) \sum_{\eta=0}^{h} \left( \frac{h}{\eta} \right) (-1)^v \alpha_v^\eta + v - h \left\{ \sum_{l=0}^{m_{sr}-1} \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} \frac{\Upsilon(l + 1, \eta, v, h)}{l!} - \sum_{l=0}^{m_{sr}-1} \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} \right\} \\
\times \Upsilon(l, \eta, v, h) \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} + \sum_{i=0}^{m_{sr}-1} \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} \exp \left( - \frac{\alpha_{sp} \mu}{\beta} \right) \frac{\Gamma(m_{sr} + i) \alpha_{sr}^{i+1}}{\Gamma(m_{sr})} \sum_{j=0}^{m_{sr}+i-1} \frac{1}{j! \beta^j} \right\} \\
\times \left[ \phi \left( \gamma_v, m_{sr} - 1, \frac{\alpha_{sp} \mu}{\alpha_{sr}}, m_{sr} + i - j, \frac{\alpha_{sr} + \beta(\eta + v - h) g_v}{\beta} \right) \right] - \frac{\alpha_{sr} + \beta(\eta + v - h) g_v}{\beta} \right]\}
\]

(16)

\[
\theta_2(h, v) = \left( \frac{v}{h} \right) \sum_{\eta=0}^{h} \left( \frac{h}{\eta} \right) (-1)^v \alpha_v^\eta + v - h \left\{ \sum_{l=0}^{m_{sr}-1} \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} \frac{\Upsilon(l + 1, \eta, v, h)}{l!} - \sum_{l=0}^{m_{sr}-1} \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} \right\} \\
\times \frac{l \xi(l, \eta, v, h)}{l!} + \sum_{i=0}^{m_{sr}-1} \frac{\alpha_{sr}^{i+1}}{\beta^{l+1}} \exp \left( - \frac{\alpha_{sp} \mu}{\beta} \right) \frac{\Gamma(m_{sr} + i) \alpha_{sr}^{i+1}}{\Gamma(m_{sr})} \sum_{j=0}^{m_{sr}+i-1} \frac{1}{j! \beta^j} \right\} \\
\times \phi \left( \gamma_v, m_{sr} - 1, \frac{\alpha_{sp} \mu}{\alpha_{sr}}, m_{sr} + i - j, \frac{\alpha_{sr} + \beta(\eta + v - h) g_v}{\beta} \right) - \frac{\alpha_{sr} + \beta(\eta + v - h) g_v}{\beta} \right]\}
\]

(17)
Here, \( \Upsilon(a, b, c, d) \), \( \xi(a, b,c) \), and \( \phi(a, b, c, d, e) \) are, respectively, defined as

\[
\Upsilon(a, b, c, d) = \frac{\Gamma \left( a, \frac{a_1 + \beta(b+c-d)g_c}{\beta} \gamma_c \right) \beta^a}{\left\{ \alpha_1 + \beta(b + c - d)g_c \right\}^a} - \frac{\Gamma \left( a, \frac{a_1 + \beta(b+c-d)g_c}{\beta} \gamma_{c+1} \right) \beta^a}{\left\{ \alpha_1 + \beta(b + c - d)g_c \right\}^a}
\]

(18)

\[
\xi(a, b, c, d) = \frac{\Gamma \left( a, \frac{a_1 + \beta(b+c-d)g_c}{\beta} \gamma_c \right) \beta^a}{\left\{ \alpha_1 + \beta(b + c - d)g_c \right\}^a}
\]

(19)

\[
\phi(a, b, c, d, e) = \int_a^\infty \gamma^b \left( \gamma + c \right)^d \exp(-c\gamma) d\gamma
\]

(20)

Substituting (15) into (11), \( b_{h,q} \) is determined.

With the derived expressions of \( b_{h,q} \) and \( a_{k,i} \), we are now ready to calculate the transition probability \( p_{i,j} \) that the number of packets at the buffer of the relay changes from \( i \) in the current time interval to \( j \) in the next time interval. It can be seen that \( p_{i,j} \) is the joint probability that R forwards \( k \), \( 0 \leq k \leq \min \{ i, N \} \), packets to D given \( i \) packets in its buffer, \( 0 \leq i \leq L \), and that R successfully decodes extra \( (j-i+k) \) packets sent from S given \( (L-i+k) \) vacant positions in the buffer, i.e.,

\[
p_{i,j} = \sum_{k=0}^{\min\{i,N\}} a_{k,i} b_{j-k,i+k, L-i+k}
\]

(21)

By substituting the expressions of \( a_{k,i} \) in (10) and \( b_{h,q} \) in (11) into (21), we finally obtain the transition probability that the buffer of the relay changes from \( i \) packets in the current time interval to \( j \) packets in the next interval, \( p_{i,j} \).

Let \( p = (p_0, p_1, \ldots, p_L) \) be a vector which represents the distribution of the number of packets in the buffer of R. Here, each component, \( p_k \) of \( p \), stands for the steady-state probability that there are \( k \in \{0, \ldots, L\} \) packets in the buffer at R, i.e., \( p_k \geq 0 \) and \( \sum_{i=0}^{L} p_i = 1 \). In order to obtain \( p \), we construct an \( (L+1) \times (L+1) \) transition probability matrix \( A \) where the element at the \( i \)-th row and the \( j \)-th column of \( A \) is \( p_{i,j} \). Then, \( p \) is obtained as the solution of the following equation:

\[
pA = p
\]

(22)

Note that \( p \) is a normalized left eigenvector of \( A \) associated with the eigenvalue one. Utilizing the method of eigenvalue decomposition, performed by the support of mathematical software packages, we obtain the left eigenvector of \( A \) corresponding to eigenvalue one. Then, we normalize this vector such that all entries sum up to one to obtain \( p \).
4 Performance Analysis

4.1 Outage Probability

For the considered system utilizing AMC, if the instantaneous SNRs of both the direct and relaying links fall below the switching threshold $\gamma_1$, no transmission occurs. Thus, the outage probability $P_{out}$ of the system is calculated as

$$P_{out} = F_{\gamma_{sd}}(\gamma_1)F_{\gamma_{srd}}(\gamma_1)$$

(23)

where $F_{\gamma_{sd}}(\gamma)$ and $F_{\gamma_{srd}}(\gamma)$ are given in (2) and (5), respectively.

4.2 Average Transmission Efficiency

Recall that incremental relaying is deployed in the considered system. Thus, the average transmission efficiency of the system, i.e., the average number of packets transmitted during one transmission interval, includes average transmission efficiency of the direct and relaying modes. Since packets can be discarded at the relay if they cannot be successfully decoded, the average transmission efficiency of the relaying mode corresponds to the average transmission efficiency of the transmission from R to D [11]. Furthermore, in the relay mode, the transmission period is divided in two equal time slots, one to transmit a signal from S to R and the other to forward a signal from R to D. Thus, the transmission efficiency of the relaying mode is reduced to one half as compared to the average transmission efficiency of the transmission from R to D. As a result, the overall average transmission efficiency of the system is obtained as

$$\eta = \sum_{n=1}^{N} nP_{sd}(n) + \frac{1}{2} \sum_{t=1}^{N} tP_{rd}(t)$$

(24)

where $P_{sd}(n)$ and $P_{rd}(t)$ are the probabilities that the direct transmission operates using the $n$-th MCS and the transmission of the relay-to-destination link operates using the $t$-th MCS. Again, the direct transmission will operate using the $n$-th MCS if its instantaneous SNR $\gamma_{sd}$ falls in the operation region of the $n$-th MCS and is higher than one half of the instantaneous SNR $\gamma_{srd}$ of the relaying transmission. Thus, $P_{sd}(n)$ is given by $P_{sd}(n) = \Pr\{\gamma_n \leq \gamma_{sd} \leq \gamma_{srd}\}$.
\( \gamma_{n+1}, \gamma_{srd} \leq \gamma_{2n+1} \), i.e.,

\[
\begin{align*}
P_{sd}(n) &= \begin{cases} 
F_{\gamma_{srd}}(\gamma_{2n+1}) \left( F_{\gamma_{sd}}(\gamma_{n+1}) - F_{\gamma_{sd}}(\gamma_{n}) \right), & 1 \leq n \leq \left\lceil \frac{N}{2} - 1 \right\rceil \\
F_{\gamma_{sd}}(\gamma_{n+1}) - F_{\gamma_{sd}}(\gamma_{n}), & \left\lceil \frac{N}{2} \right\rceil \leq n \leq (N-1) \\
1 - F_{\gamma_{sd}}(\gamma_{N}), & n = N
\end{cases}
\end{align*}
\]

(25)

On the other hand, the relay transmission will be chosen only if the instantaneous SNR of the direct link, \( \gamma_{sd} \), is lower than one half of the instantaneous SNR of the relaying link, \( \gamma_{srd} \). Specifically, there are two situations that R will operate using the \( t \)-th MCS, \( 0 \leq t \leq 5 \). The first situation occurs when having more than \( t \) packets in the buffer and the instantaneous SNR of the link from R to D, \( \gamma_{rd} \), falls in the operation region of the \( t \)-th MCS, i.e., \( \gamma_{t} \leq \gamma_{rd} \leq \gamma_{t+1} \). The second situation occurs when having exactly \( t \) packets in the buffer of R and the instantaneous SNR of the link from R to D, \( \gamma_{rd} \), is sufficient to support the \( t \)-th MCS, i.e., \( \gamma_{t} \leq \gamma_{rd} \). Therefore, \( P_{rd}(t) \) is given by (26).

Finally, by substituting (26) and (25) into (24), the overall transmission efficiency of the considered system is obtained.

\[
P_{rd}(m) = \begin{cases} 
F_{\gamma_{rd}}(\gamma_{m/2}) \left[ F_{\gamma_{srd}}(\gamma_{m+1}) - F_{\gamma_{srd}}(\gamma_{m}) \right] \left[ (F_{\gamma_{rd}}(\gamma_{m+1}) - F_{\gamma_{rd}}(\gamma_{m})) \right] \\
\sum_{i=m+1}^{L} p_{i} (1 - F_{\gamma_{rd}}(\gamma_{m})) p_{m}, & 1 \leq m \leq \lceil N - 1 \rceil \\
F_{\gamma_{rd}}(\gamma_{m/2}) \left[ 1 - F_{\gamma_{srd}}(\gamma_{m}) \right] \left[ (F_{\gamma_{rd}}(\gamma_{m+1}) - F_{\gamma_{rd}}(\gamma_{m})) \right] \\
\sum_{i=m+1}^{L} p_{i} (1 - F_{\gamma_{rd}}(\gamma_{m})) p_{m}, & m = N
\end{cases}
\]

(26)

5 Numerical Results and Discussions

In this section, we illustrate the effect of the interference power constraint, \( Q \), transmit power limit, \( P_{\text{max}} \), and packet error rate target, \( PER_{TG} \), on the outage probability and transmission efficiency of the considered system. The fading severity parameters of all links are selected as \( m_{sr} = m_{rd} = m_{sd} = m_{sp} = m_{rp} = 2 \). Further, we choose the buffer length at R as \( L = 5 \). Denote \( d_{yz} \), \( y \in \{ s, r, d \} \), \( z \in \{ r, d, p \} \) and \( y \neq z \) in Fig. 1 as the transmission distance of the link with channel coefficient \( h_{yz} \). We assume that the signal power decays relative to these distances with path-loss exponent 4.
Firstly, in Fig. 2, we investigate the impact of the transmission distances on the outage probability of the system. The transmit power limit-to-noise ratio of S and R is fixed as $P_{\text{max}}/N_0 = 10$ dB and the packet error rate target, $\text{PER}_{\text{TG}}$, is $10^{-3}$. As can be seen from Case 1 and Case 3 of Fig. 2, given the same transmission distances, $(d_{sr}, d_{rd}, d_{sd})$, of the secondary network, the further the distances of the interference links to the primary user, $(d_{sp}, d_{rp})$, the lower the outage probability of the secondary network. This is due to the fact that when the distances of the interference links increase, the constraint on the transmit power of S and R can be more relaxed. Furthermore, by fixing the distances of the interference links in Case 2 and Case 3, we can observe that the outage probability decreases with respect to the decrease in the distances between the terminals in the secondary network.

Secondly, in Fig. 3, we show the impact of $\text{PER}_{\text{TG}}$ on the transmission efficiency of the system. For these examples, we have chosen the transmit power limit-to-noise ratio of S and R as $P_{\text{max}}/N_0 = 10$ dB. Furthermore, the transmission distances of the network are selected as $(d_{sr}, d_{rd}, d_{sd}, d_{sp}, d_{rp}) = (0.5, 0.5, 0.7, 0.8, 0.8)$. It can be observed from Fig. 3 that an increase of $\text{PER}_{\text{TG}}$ leads to an increase of the transmission efficiency. This is because, given the other parameters, when transmission rate increases, the transmission efficiency will increase at the cost of also increasing the average PER.

Thirdly, in Fig. 4 and Fig. 5, we make comparisons of outage probability and transmission efficiency for our system utilizing AMC and relay queue awareness with that of a system without AMC, i.e., operating with Mode 3 of Table 1 and that of the direct transmission utilizing AMC. Here, we select Mode 3 of Table 1, to illustrate performance of a system without AMC as an example. The value of $\text{PER}_{\text{TG}}$ and $P_{\text{max}}/N_0$ are fixed at $10^{-3}$ and 10 dB, respectively. Furthermore, the transmission distances of the network are selected as $(d_{sr}, d_{rd}, d_{sd}, d_{sp}, d_{rp}) = (0.5, 0.5, 0.8, 0.8, 0.8)$. As expected, in the whole investigated range of interference power-to-noise ratio regime, the system with AMC together with relay queue awareness policy obtains the best performance.

As a final point, for all the examined scenarios, we can observe that both the outage and transmission efficiency converge to a constant value when $Q/N_0$ goes beyond a certain value, for instance, 15 dB. This is because the transmit power of the secondary network not only is constrained by $Q$, but also is constrained by its transmit power limit $P_{\text{max}}$, $P_s = \min(P_{\text{max}}, Q/X_{sp})$, and $P_r = \min(P_{\text{max}}, Q/X_{rp})$. When $Q$ is large enough, the transmit powers of S and R only depend on $P_{\text{max}}$. In this case, the transmit powers of S and R are fixed at $P_{\text{max}}$. As a result, both the outage probability and transmission efficiency will no longer increase when $Q/N_0$ increases.
Figure 2: Outage probability of the secondary network versus interference power-to-noise ratio $Q/N_0$ for various transmission distances.

Figure 3: Transmission efficiency of the secondary network versus interference power-to-noise ratio $Q/N_0$ for various packet error rate targets.
Adaptive Modulation and Coding with Queue Awareness in Cognitive Incremental Decode-and-Forward Relay Networks

Figure 4: A comparison of outage probability of the considered system utilizing AMC and relay queue awareness with that of a system without AMC and that of the direct transmission utilizing AMC.

Figure 5: A comparison of transmission efficiency of the considered system utilizing AMC and relay queue awareness with that of a system without AMC and that of the direct transmission utilizing AMC.
6 Conclusion

In this paper, we have analyzed the performance of AMC with queue awareness in cognitive incremental DF relaying systems. In order to maximize the overall spectrum efficiency, a suitable operation mode, including direct and relaying transmission with MCSs, is selected. Furthermore, to avoid buffer overflow at R, the MCS in the relaying transmission is chosen subject to both the SNR of the fading channels and the current state of the buffer. A finite-state Markov chain model is applied to analyze the distribution of the number of packets in the buffer of R which enables us to derive expressions for key performance indicators such as outage probability and average transmission efficiency of the network. Based on the obtained analysis, numerical results are provided to reveal performance advantages of the cognitive incremental DF relay network with AMC and queue awareness compared to cognitive relaying without AMC, and conventional direct transmission without AMC.

References


Part II-D
Part II-D

Hybrid Interweave-Underlay Spectrum Access for Cognitive Cooperative Radio Networks
Part II-D is published as:

Abstract

In this paper, we study a hybrid interweave-underlay spectrum access system that integrates amplify-and-forward relaying. In hybrid spectrum access, the secondary users flexibly switch between interweave and underlay schemes based on the state of the primary users. A continuous-time Markov chain is proposed to model and analyze the spectrum access mechanism of this hybrid cognitive cooperative radio network (CCRN). Utilizing the proposed Markov model, steady-state probabilities of spectrum access for the hybrid CCRN are derived. Furthermore, we assess performance in terms of outage probability, symbol error rate (SER), and outage capacity of this CCRN for Nakagami-$m$ fading with integer values of fading severity parameter $m$. Numerical results are provided showing the effect of network parameters on the secondary network performance such as the primary arrival rate, the distances from the secondary transmitters to the primary receiver, the interference power threshold of the primary receiver in underlay mode, and the average transmit signal-to-noise ratio of the secondary network in interweave mode. To show the performance improvement of the CCRN, comparisons for outage probability, SER, and capacity between the conventional underlay scheme and the hybrid scheme are presented. The numerical results show that the hybrid approach outperforms the conventional underlay spectrum access.

1 Introduction

Nowadays, the increasing demand on mobile multimedia leads to serious shortage of frequency spectrum. Despite this shortage, the allocated spectrum
resources are still utilized inefficiently [1], which necessitates studies on efficient spectrum access mechanisms. Accordingly, cognitive radio technology has emerged as a promising approach to alleviate the insufficiency of spectrum utilization (see, e.g., [2–7], and the references therein). In the study of [2], the fundamental concepts for spectrum sharing have been featured. Furthermore, the work of [3] addressed several major functions of cognitive radios such as interference temperature estimation, spectrum hole detection, channel state estimation, transmit power control, and dynamic spectrum access.

As for the spectrum access strategies, there exist three main approaches, i.e., the interweave, underlay, and overlay scheme [8]. In case of interweave spectrum access, secondary users (SUs) are not allowed to cause any interference to the primary network [3]. Therefore, an SU must periodically monitor the radio spectrum to detect spectrum occupancy and only opportunistically communicates over spectrum holes. This approach may reduce effectiveness of spectrum utilization. On the other hand, interweave spectrum access may offer superior system performance, such as outage probability and error probability, as compared to underlay and overlay networks given the same propagation environment [9]. This performance enhancement is attributed to the fact that transmit power of the SU is not bounded by an interference power constraint of the primary receiver. In addition, the received signal of the SU does not suffer from interference from the primary transmission. An alternative approach is known as underlay spectrum access wherein SUs and primary users (PUs) can simultaneously share the same licensed spectrum provided that the secondary transmit power is adjusted to meet the interference power constraint of the primary receiver [4]. Restricting transmit power leads to a significant reduction in channel capacity and radio coverage in the secondary network. On the other hand, effectiveness of spectrum utilization is improved with underlay spectrum access compared to the interweave scheme as spectrum can be utilized at any time. Finally, overlay spectrum access also allows the SUs to concurrently access spectrum with the PUs given that the SUs implement an appropriate technique to mitigate the interference to the primary network [6]. Interference mitigation in overlay spectrum access is based on SUs having knowledge about the primary network beyond spectrum occupancy but requires information such as code books or even messages that are communicated in the primary network.

Inspired by the inherent benefits of the above schemes, hybrid cognitive radio networks (CRNs) have been proposed as a means of improving the performance of secondary networks (see, for instance, [10,11]). In [10], a novel hybrid scheme was introduced, which combines the conventional interweave and underlay schemes for a single hop CRN. Then, an M/M/1 queuing model with Poisson traffic generation is invoked to analyze the average service rate
of SU video service. Furthermore, [11] proposed a power allocation strategy for a single hop hybrid overlay-underlay CRN to maximize channel capacity for the SU by using a suitable interference cancelation technique.

In addition, cooperative communications has been recently incorporated into CRNs aiming at improving system performance of secondary and/or primary networks [12–16]. In light of this, [14] proposed power and channel allocation strategies for a cognitive cooperative radio network (CCRN) to optimize overall system throughput. Moreover, performance analysis in terms of outage probability and symbol error rate for multi-relay CCRNs was addressed in [15]. The use of cooperative communications in hybrid CRNs was studied in [16] in which the secondary users transmit with their power limits while the primary system is idle. However, when the primary system is active, the secondary users must control their transmit powers under the peak interference power constraint of the primary receiver. Nevertheless, the authors of [16] assumed that the probabilities for idle and active states of the primary transmitter are known and that the network suffers from Rayleigh fading when deriving a bound on the outage probability of the hybrid interweave-underlay CCRN. Thus, selection of operation mode in [16] does not take into account the traffic statistics of the primary and secondary network.

Apart from the above mentioned works, in this paper, we study the incorporation of hybrid interweave-underlay spectrum access into cognitive cooperative relay networks as a means of obtaining the inherent benefits of both hybrid interweave-underlay spectrum access as well as spatial diversity gains of cooperative communications. It is noted that, in our study, the traffic characteristics of both secondary and primary users are taken into consideration when modeling the investigated network as well as analyzing the corresponding system performance. Considering these stochastic processes, of course, covers a more general and practical setting as compared to [16]. Regarding the fading environments, it is assumed that the considered network is subject to Nakagami-\(m\) fading that will induce challenging expressions in the mathematical analysis. Nevertheless, this fading model comprises several other environments as special cases by setting the fading severity parameter \(m\) to a particular value, e.g., \(m = 0.5\) represents a one-sided Gaussian distribution and \(m = 1\) models Rayleigh fading. This general model also closely approximates Nakagami-\(q\) (Hoyt) fading and Nakagami-\(n\) (Rice) fading. Further, according to experimental and theoretical results, the Nakagami-\(m\) distribution is the best distribution for modeling urban multipath radio channels.

In order to achieving performance improvements, in our system, a secondary relay is used to assist the communication from a secondary transmitter to a secondary receiver. Clearly, integrating relay transmission into the secondary network as well as considering a more general fading model results
Part II-D

in a demanding mathematical model and a complex analysis framework. On the other hand, our work gives a more general and practical setting of hybrid interweave-underlay cognitive networks as compared to [17]. In summary, major contributions of this paper can be stated as follows:

- We develop a Markov chain to model the dynamic behavior of the considered cognitive radio amplify-and-forward (AF) relay network with hybrid interweave-underlay spectrum access.

- On this basis, the equilibrium probability of each operation mode for the hybrid interweave-underlay cognitive relay system is derived.

- We further develop an analytical framework for evaluating system performance in terms of outage probability, symbol error rate (SER), and outage capacity in case of Nakagami-\(m\) fading.

- We also make a performance comparison between the hybrid CCRN and a conventional cognitive radio network to reveal the superior performance of the hybrid cognitive relay scheme.

- Finally, through our analysis, insights into the impact of network parameters on system performance are revealed. In particular, we consider the impact of the primary arrival rate, the distances from the secondary transmitter to the primary receiver, fading parameters, the interference constraint threshold at the primary receiver in underlay mode, and the average transmit signal-to-noise ratio (SNR) in interweave mode.

The rest of the paper is organized as follows. Section II describes the system model for a hybrid interweave-underlay CCRN and adopts a Markov chain to model the reactions of the SUs to the PU’s transmission. Steady-state probabilities, deduced from the Markov chain, are utilized to analyze system performance in Section III. Section IV provides numerical results. Finally, conclusions are given in Section V.

**Notation:** We use the following notations throughout the paper. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) \(X\) are denoted as \(f_X(\cdot)\) and \(F_X(\cdot)\), respectively. Here, \(\Gamma(n)\) and \(\Gamma(n, x)\) are the gamma function [18, eq. (8.310.1)] and the incomplete gamma function [18, eq. (8.350.2)], respectively. Next, the \(n\)-th order modified Bessel function of the second kind [18, eq. (8.432.1)] is denoted as \(K_n(\cdot)\) and the Whittaker function [18, eq. (9.222)] is represented by \(W_{a,b}(\cdot)\). Furthermore, \(\, _2F_1(a, b; c; x)\) and \(U(a, b; x)\) are, respectively, the Gauss hypergeometric function [18, eq. (9.100)] and confluent hypergeometric
function \[18, \text{eq. (9.211.4)}\]. In addition, \(B(x, y)\) represents the beta function \[18, \text{eq. (8.380.1)}\] and \(E\{\cdot\}\) stands for the expectation operator. Finally, \(C^n_k = \frac{n!}{k!(n-k)!}\) is the binomial coefficient.

2 System and Channel Model

The CCRN that we consider consists of a primary source, \(P_S\), a primary destination, \(P_D\), a secondary source, \(S_S\), an AF relay, \(S_R\), and a secondary destination, \(S_D\) as depicted in Fig. 1. Here, \(h_1, h_2, h_3, h_4, h_5,\) and \(h_6\) are, respectively, the channel coefficients of the links \(S_S \to S_R, S_R \to S_D, S_S \to P_D, S_R \to P_D, P_S \to S_R,\) and \(P_S \to S_D\). Assume that the secondary transmission operates under Nakagami-\(m\) fading and in half-duplex mode, i.e., the secondary relaying transmission occurs in two time slots (TSs). In the first TS, \(S_S\) broadcasts a signal while \(S_R\) amplifies the received signal with a scale factor and forwards it to \(S_D\) in the second TS.

In this system, we deploy hybrid spectrum access for the SUs where \(S_S\) and \(S_R\) flexibly switch between underlay and interweave mode based on the state of the PU. In particular, one TS of the hybrid cognitive transmission always consists of two phases, sensing phase and data transmission phase as in \[19\]. In the first phase, the SU, \(S_S\) or \(S_R\), listens to the spectrum allocated to the PU to detect the state of the PU. In the second phase, the SU adapts its transmit power based on the sensing results. If the spectrum is sensed idle in the first phase, taking advantage of not requiring the interference constraint,
the SU operates in interweave mode with maximum transmit power $P_{\text{max}}$. In contrast, if the PU is active, the SU must switch to underlay mode under the interference power constraint imposed by $P_D$. Since sensing-based spectrum sharing for CRNs has been proposed in Section III.B of [19], in the context of our paper, we do not delve into spectrum sensing for the hybrid system anymore. Instead, we assume that the secondary users can perform perfect spectrum sensing as reported in Section III.B of [19].

It should be mentioned that in case of imperfect spectrum sensing, the cooperative sequential spectrum sensing approach can be employed where $S_S$ and $S_R$, $S_R$ and $S_D$ mutually exchange the sensing results. This approach essentially reduces the missed detection probability and the false alarm probability at each secondary user. Furthermore, given that the missed detection probability and the false alarm probability at each station can be driven to be lower than 0.1 as in [19], after cooperative sequential spectrum sensing, the missed detection and false alarm probabilities of the hybrid relay network are lower than 0.01. Then, the effect of imperfect spectrum sensing on the performance of the considered network may be neglected. It should also be mentioned that the sensing duration is often very small in the order of 1 ms as compared to the time slot duration of, say 100 ms. As such, the secondary users are considered in this work to start transmission at the beginning of each time slot with virtually no delay.

As for the implementation of the hybrid interweave-underlay CCRN, it should be mentioned that the scheme inherits the complexity from the interweave and underlay schemes. In particular, spectrum sensing must be implemented in each time slot and is compulsory for both interweave and hybrid schemes. In an interweave scheme, the SU keeps silent when the PU is active and only opportunistically transmits a signal when the PU does not occupy the spectrum. In a hybrid interweave-underlay CCRN, when the PU does not occupy the spectrum, the SU transmits its signals. However, when the spectrum is occupied by the PU, the SU is still allowed to transmit provided that its transmit power is continuously adapted under the interference power constraint imposed by the PU. Basically, instead of the SU turning off its transmit power when the PU occupies or starts occupying the spectrum, the hybrid CCRN switches to underlay mode. When the SUs operate in underlay mode, they must perform power adaptation under the interference power constraint of the PU. Therefore, the SUs need to obtain channel state information to the primary receivers to control their transmit powers. Thus, compared to the underlay scheme, the interweave-underlay scheme does not increase the complexity in power control when operating in underlay mode. In fact, the hybrid scheme is even simpler as it only must control its transmit power when the PU is active.
However, in some particular scenarios, it is not beneficial to deploy the hybrid spectrum access approach. For instance, when the secondary transmitters are rather far from the primary receiver such that, given a predefined transmit power range of the secondary transmitter, the interference power constraint of the primary receiver is often satisfied. In this case, the secondary network can operate freely in its transmit power range without considering the primary interference power constraint. On the contrary, for the case that the secondary transmitters are located close to the primary receiver, the transmit powers of the SUs in underlay spectrum access mode are severely constrained to be sufficiently low because of the interference power constraint of the primary receiver. Then, the SUs may take advantage of the interweave spectrum access, i.e., not suffering from the interference power constraint when the licensed bands are not occupied by the PU. In this scenario, it is beneficial to apply the hybrid scheme as it not only allows the secondary network to access the spectrum at any time but can also offer performance improvements when opportunistically operating in interweave mode.

In this paper, we apply the hybrid spectrum access scheme for the scenario that the secondary transmitter is located close to the primary receiver. Let $X_3$ be a random variable which represents the channel power gain of the link from the secondary transmitter to the primary receiver. With the predefined interference power constraint $Q$ of the primary receiver and transmit power limit $P_{max}$ of the secondary transmitter, the transmit power of the secondary transmitter in underlay mode is controlled as $\min\{Q/X_3, P_{max}\}$. However, when the secondary transmitter is rather close to the primary receiver, the probability of the event $Q/X_3 > P_{max}$ is very low, i.e., the constraint of $P_{max}$ on the transmit power of the secondary user in underlay mode can be neglected. In this case, the transmit power of the SU in underlay mode is considered to be only constrained by the interference power threshold $Q$ as in [4, 20].

To assess the system performance of the hybrid scheme, we need to obtain the probabilities that the cognitive relay system operates in interweave mode and underlay mode. To solve this issue, it is necessary to propose a suitable model to capture the system dynamics, especially the effect of the PU activities on the selection of operation mode for the SUs. It should be mentioned that the active/inactive periods of PUs are typically much larger than the duration of one time slot [21]. Thus, we can make the assumption that the TS is sufficiently small as compared to the average arrival period of packets. Then, the states of the primary and secondary users can be considered to change continuously over time units of a TS. Assume that the arrival traffics of the primary and secondary systems are modeled as Poisson random processes and that spectrum access durations or departure times of PUs and
SUs are negative-exponentially distributed. As in [9, 17, 22], if arrival and departure traffics are Poisson processes, the spectrum access can be modeled as a continuous-time Markov chain (CTMC).

![Continuous-time Markov chain with six states modeling the spectrum access of the hybrid interweave-underlay cognitive cooperative radio network.](image)

Figure 2: Continuous-time Markov chain with six states modeling the spectrum access of the hybrid interweave-underlay cognitive cooperative radio network.

Let the arrival traffic of $P_S$ be modeled as a Poisson process with arrival rate $\lambda_p$ packets/TS and let the departure traffic of $P_S$ be another Poisson process with departure rate $\mu_p$ packets/TS. Furthermore, the arrival and departure traffics of $S_S$ and $S_R$ shall be modeled as Poisson processes with rates $\lambda_s$, $\mu_s$, and $\lambda_r$, $\mu_r$ packets/TS, respectively. In this CCRN, all departure packets of $S_S$ become arrival packets of $S_R$, i.e., $\mu_s = \lambda_r$. Recall that transmissions of $S_S$ and $S_R$ always occur in different TSs, as is the common case for cooperative communications. Thus, the probability of $S_S$ and $S_R$ simultaneously transmitting signals is equal to zero. We assume that $S_S$ and $S_R$ do not change their operation mode in a time slot, i.e., the system does not change its state during a TS. Even when the primary user requires service during the transmission period of the secondary user, $S_S$ or $S_R$ still keep the operation mode in the current time slot but adapt their operation modes in the next slot, i.e., the system may change its state in a subsequent TS. This assumption is based on the duration of a time slot being rather short and the
probability of the event that a primary user requires service in the middle of a secondary time slot is very small for independent Poisson processes [17]. As such, the effect of the secondary network remaining in its operation mode during a time slot on the performance of the primary network can be neglected. As a result, the spectrum access of this hybrid interweave-underlay CCRN can be modeled as a six state continuous-time Markov chain as shown in Fig. 2.

In the state diagram of the Markov chain shown in Fig. 2, State 0 represents the state that all terminals $P_S, S_S, S_R$ are idle. States $P, S, R$ correspond to the states that only $P_S$, only $S_S$, only $S_R$ operate, respectively. Finally, States $PS$ and $PR$ stand for the states that both $P_S$ and $S_S$, and both $P_S$ and $S_R$ are active, respectively. It can be easily seen that this continuous-time Markov chain converges to a steady-state distribution because it possesses two sufficient conditions for convergence [23, p. 263]. Firstly, the proposed continuous-time Markov chain has a finite state space with six states: \{0, S, R, PR, PS, P\}. Secondly, this continuous-time Markov chain is irreducible because every state of the Markov chain can be reached from other states.

Without loss of generality, it is assumed that the network is initialized as inactive, i.e., the Markov chain resides in State 0. Upon the first access effort of the secondary transmitter, $S_S$, the Markov chain goes from State 0 to State $S$ with transition rate $\lambda_s$. Since only $S_S$ occupies the spectrum, it will operate in interweave mode, i.e., not face the interference power constraint from the primary user. As long as transmission of $S_S$ ends before $P_S$ requests to use the spectrum, the system goes from State $S$ to State $R$ with transition rate $\mu_s$. Because the departure traffic of the secondary transmitter, $S_S$, becomes the arrival traffic with rate $\lambda_r$ of the secondary relay, $S_R$, we have $\lambda_r = \mu_s$. Then, $S_R$ will operate in interweave mode in which only $S_R$ utilizes the spectrum in the second time slot. If $S_R$ completes its communication before a spectrum access of $S_S$ or $P_S$ occurs, the system transits from State $R$ to State 0 with rate $\mu_r$. Otherwise, the system will transit from state $R$ to $P$ with rate $\mu_r + \lambda_p$, from state $R$ to $S$ with rate $\mu_r + \lambda_s$, or from state $R$ to $PS$ with rate $\mu_r + \lambda_s + \lambda_p$.

It is noted that the probability of the transition from state $R$ to $P$ corresponds to the joint probability of two Poisson processes occurring, i.e., $S_R$ completes its communication with rate $\mu_r$ and there is an arrival of the primary user with rate $\lambda_p$ in the current TS. It is also well known that the sum of two independent Poisson random variables results in a Poisson random variable whose transition rate is equal to the summation of the two rates. Thus, the system transits from state $R$ to $P$ with rate $\mu_r + \lambda_p$. A similar line of argument can be utilized to explain the transitions from state $R$ to $S$ with rate $\mu_r + \lambda_s$, or from state $R$ to $PS$ with rate $\mu_r + \lambda_s + \lambda_p$. 
When the system resides in State $S$, i.e., $S_S$ is transmitting in interweave mode, if the primary user $P_S$ requests the spectrum, the system transits from State $S$ to State $PR$ with transition rate $\mu_s + \lambda_p$ at the end of this time slot. At State $PR$, both $S_R$ and $P_S$ simultaneously operate in the spectrum. Therefore, $S_R$ must switch to underlay mode in the second time slot subject to the interference power constraint imposed by $P_D$. Once $S_R$ completes its communication at the end of the second time slot, depending on the arrival and departure processes of the users during this time slot, the system will make one of the following transitions: A transition from State $PR$ to State $P$ with transition rate $\mu_r$, from State $PR$ to State $0$ with transition rate $\mu_r + \mu_p$, from State $PR$ to State $PS$ with transition rate $\mu_r + \lambda_s$, or from State $PR$ to State $S$ with transition rate $\mu_r + \mu_p + \lambda_s$. It is noted that the transition from State $P$ to State $PS$ is assumed to occur with rate $\lambda_s$, instead of $\lambda_s + \lambda_p$. This implicitly corresponds to the event that the primary user is active in the current time slot and still remains active in the subsequent time slot. Similarly, we can explain all remaining transitions that are shown in Fig. 2.

Let $p_0$, $p_P$, $p_S$, $p_R$, $p_{PS}$, and $p_{PR}$ denote the steady-state probabilities for States 0, $P$, $S$, $R$, $PS$, and $PR$ in the proposed Markov chain, respectively. The steady-state probability of each state can be derived based on the flow-balance equations at all nodes of the Markov chain, which states that the arrival rate of any node is always equal to its departure rate, and the normalized equation, which implies that the total probabilities of all states is always equal to one. Consequently, the steady-state probabilities of the six states can be obtained from a linear equation system as in (1).

\[
\begin{align*}
-(2\lambda_s + 2\lambda_p)p_0 + \mu_pp_P + 0p_s + \mu_pp_r + 0p_{PS} + (\mu_r + \mu_p)p_{PR} &= 0 \\
\lambda_pp_0 - (2\mu_p + 2\lambda_p)p_P + 0p_s + (\mu_r + \lambda_p)p_r + 0p_{PS} + \mu_pp_{PR} &= 0 \\
\lambda_sp_0 + (\mu_p + \lambda_s)p_P - (2\mu_s + \lambda_p)p_S + (\mu_r + \lambda_s)p_r + 0p_{PS} + (\mu_p + \mu_r + \lambda_s)p_{PR} &= 0 \\
0p_0 + 0p_p + \mu_sp_s - (4\mu_r + 2\lambda_p + 2\lambda_s)p_r + (\mu_s + \mu_p)p_{PS} + 0p_{PR} &= 0 \\
(\lambda_p + \lambda_s)p_0 + \lambda_sp_p + 0p_s + (\mu_r + \lambda_p + \lambda_s)p_r - (2\mu_s + \mu_p)p_{PS} + (\mu_r + \lambda_s)p_{PR} &= 0 \\
0p_0 + 0p_p + (\mu_s + \lambda_p)p_S + \mu_sp_{PS} - (2\mu_p + 4\mu_r + 2\lambda_s)p_{PR} &= 0 \\
p_0 + p_p + p_s + p_r + p_{PS} + p_{PR} &= 1
\end{align*}
\]  
(1)

It is sufficient to select six independent equations from (1) to obtain six steady-state probabilities. From (1), we can express the first five flow-balance equations and the normalized equation as $A\mathbf{p} = \mathbf{b}$. Here, $\mathbf{p}$ is a column vector $\mathbf{p} = [p_0, p_p, p_s, p_r, p_{PS}, p_{PR}]^T$ which represents the steady-state probabilities of the six above states. Furthermore, $\mathbf{b}$ is a column vector defined as
\[ b = [0, 0, 0, 0, 0, 1]^T \text{ and } A \text{ denotes a } 6 \times 6 \text{ matrix constructed as in (2).} \]

\[
A = \begin{pmatrix}
- (2 \lambda_s + 2 \lambda_p) & \mu_r & 0 & 0 & 0 & (\mu_r + \mu_p) \\
\lambda_p & - (2 \mu_p + 2 \lambda_s) & 0 & \mu_r & 0 & 0 \\
\lambda_s & \mu_p + \lambda_s & - (2 \mu_s + \lambda_p) & \mu_r + \lambda_s & 0 & 0 \\
0 & 0 & \mu_s & - (4 \mu_p + 2 \lambda_p + 2 \lambda_s) & \mu_s + \mu_p & 0 \\
(\lambda_p + \lambda_s) & \lambda_s & 0 & (\mu_s + \lambda_p + \lambda_s) & - (\mu_p + 2 \mu_s) & \mu_r + \lambda_s \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{pmatrix}
\tag{2}
\]

Thus, the six steady-state probabilities \( p_0, p_p, p_s, p_r, p_{ps}, \text{ and } p_{pr} \) are found as \( p = A^{-1} b \). As a result, the probability \( p \) that the secondary transmission occurs is given by the probability that \( S_S \) is active in the first TS and \( S_R \) is active in the second TS, i.e., \( p \) can be calculated as

\[
p = (p_s + p_{ps})(p_r + p_{pr}) \tag{3}
\]

We assume that the secondary users, \( S_S \) and \( S_R \), do not change their operation mode during a time slot but instead adapt the operation mode in the subsequent time slot. Based on the current state at the beginning of each time slot, the secondary user decides whether interweave or underlay mode should be deployed for each TS. When the system is in State S or R, \( S_S \) or \( S_R \) operates in interweave mode. During periods of States PS and PR, \( S_S \) and \( S_R \) must operate in underlay mode. Therefore, there are four scenarios to be distinguished for the transmission modes of the hybrid interweave-underlay AF relay system as follows:

- **Scenario 1:** \( P_S \) is active in both TSs of the secondary transmission.
- **Scenario 2:** \( P_S \) is only active in the first TS of the secondary transmission.
- **Scenario 3:** \( P_S \) is only active in the second TS of the secondary transmission.
- **Scenario 4:** \( P_S \) is inactive in both TSs of the secondary transmission.

Let \( x_i, \ i \in \{1, 2, 3, 4\}, \) be the transmit symbol at \( S_S \) with average power \( P_{s,i} \) and \( n_0 \) is additive white Gaussian noise (AWGN) with zero-mean and variance \( N_0 \) at \( S_R \) and \( S_P \). Further, we denote \( y_{r,i} \) and \( y_{d,i} \) respectively, as the received signals at \( S_R \) and \( S_D \) in Scenario \( i \). Finally, \( G_i \) stands for the gain factor at \( S_R \) in Scenario \( i \). It is assumed that the noise power at the secondary relay \( S_R \) is much lower than its received signal power. Therefore, we can neglect the noise power at \( S_R \) when calculating the gain factor \( G_i \). In what follows, we derive the gain factors and the expressions of the instantaneous SNR as well as the signal-to-interference plus noise ratio (SINR) for all possible scenarios.

**Scenario 1:** \( P_S \) is active in both TSs of the secondary transmission. The probability that this scenario occurs is

\[
p_1 = \frac{P_{ps} P_{pr}}{(p_s + p_{ps})(p_r + p_{pr})} \tag{4}
\]
The received signal $y_{r,1}$ at $S_R$ in Scenario 1 is obtained as

$$y_{r,1} = h_1 x_1 + h_5 x_p + n_0$$

(5)

where $n_0$ is AWGN with zero-mean and variance $N_0$ at $S_R$. In addition, $x_p$ is the transmit signal of the primary transmitter $P_S$, and $h_5 x_p$ is the interference from $P_S$ to $S_R$. As in [24,25], the interference distribution in wireless networks can be approximated as a Gaussian distribution. Thus, we approximate the effect of interference and noise at the relay $h_5 x_p + n_0$ as another Gaussian random variable $n_r$ with zero-mean and variance $N_r$ where $N_r$ is calculated based on the power law decaying path-loss plus the noise at the relay. Then, the received signal at the secondary destination is

$$y_{d,1} = G_1 h_2 (h_1 x_1 + n_r) + h_6 x_p + n_0$$

(6)

where $n_0$ is AWGN with zero-mean and variance $N_0$ at $S_D$ and $h_6 x_p$ is the interference from $P_S$ to $S_R$. Again, $h_6 x_p + n_0$ is approximated as another Gaussian random variable $n_d$ with zero-mean and variance $N_d$, i.e.,

$$y_{d,1} = G_1 h_2 h_1 x_1 + G_1 h_2 n_r + n_d$$

(7)

In this scenario, both $S_S$ and $S_R$ operate in underlay mode under the interference power constraint $Q$ of $P_D$. Thus, the average transmit power $P_{s,1}$ of $S_S$ and the average transmit power $P_{r,1}$ of $S_R$ must be regulated to meet the interference threshold $Q$ of $P_D$, i.e., $P_{s,1} = E\{|x_1|^2\} = Q/|h_3|^2$ and $P_{r,1} = E\{|G_1 h_1 x_1|^2\} = Q/|h_4|^2$. Here, $h_3$ and $h_4$ are the channel coefficients of the links $S_S \rightarrow P_D$ and $S_R \rightarrow P_D$, respectively. Let $X_1$, $X_2$, $X_3$, $X_4$ be the channel power gains of the links $S_S \rightarrow S_R$, $S_R \rightarrow S_D$, $S_S \rightarrow P_D$, $S_R \rightarrow P_D$, respectively, i.e., $X_i = |h_i|^2$, $i \in \{1, 2, 3, 4\}$. Thus, the gain factor $G_1$ at $S_R$ in Scenario 1 is selected as

$$G_1^2 = \frac{X_3}{X_1 X_4}$$

(8)

Defining $\beta_1 = Q/N_r$ and $\beta_2 = Q/N_d$, the SINR $\gamma_1$ of the secondary system in Scenario 1 can be obtained as

$$\gamma_1 = \frac{\beta_1 \beta_2 X_1 X_2}{\beta_1 X_1 X_4 + \beta_2 X_2 X_3}$$

(9)

**Scenario 2:** $P_S$ is active in the first TS and idle in the second TS of the secondary transmission. The probability that this scenario takes place is

$$p_2 = \frac{p_{ps} p_r}{(p_s + p_{ps})(p_r + p_{pr})}$$

(10)
Accordingly, the received signal $y_{d,2}$ at the secondary destination $S_D$ in Scenario 2 is obtained as

$$y_{d,2} = G_2 h_2 h_1 x_2 + G_2 h_2 n_r + n_0$$  \hspace{1cm} (11)$$

Since $P_S$ is active in the first TS and idle in the second TS of the secondary transmission, $S_S$ operates in underlay mode with average transmit power $P_{s,2} = E\{|x_2|^2\} = Q/X_3$ in the first TS. However, $S_R$ operates in interweave mode with average transmit power $P_{r,2} = E\{|G_2 h_1 x_2|^2\} = P_{max}$ in the second TS. Then, the gain factor $G_2$ at $S_R$ in Scenario 2 is chosen as

$$G_2^2 = \frac{P_{max} X_3}{Q X_1}$$  \hspace{1cm} (12)$$

Hence, the instantaneous SINR $\gamma_2$ at $S_D$ in Scenario 2 is given by

$$\gamma_2 = \beta_1 \frac{X_1 X_2}{X_2 X_3 + \beta_3 X_1}$$  \hspace{1cm} (13)$$

where $\beta_3 = Q/(P_{max} N_r)$.

**Scenario 3:** $P_S$ is idle in the first TS and active in the second TS of the secondary transmission. The probability of this scenario is

$$p_3 = \frac{p_s p_{pr}}{(p_s + p_ps)(p_r + p_{pr})}$$  \hspace{1cm} (14)$$

In this scenario, the received signal $y_{d,3}$ at the secondary destination $S_D$ is given as

$$y_{d,3} = G_3 h_2 h_1 x_3 + G_3 h_2 n_0 + n_d$$  \hspace{1cm} (15)$$

Because $P_S$ is idle in the first TS of the secondary transmission, $S_S$ operates in interweave mode with average transmit power $P_{s,3} = E\{|x_3|^2\} = P_{max}$ in the first TS. However, since $P_S$ is active in the second TS of the secondary transmission, $S_R$ operates in underlay mode in the second TS under the interference power constraint $Q$, i.e., $P_{r,3} = E\{|G_3 h_1 x_3|^2\} = Q/X_4$. Thus, in Scenario 3, the gain factor $G_3$ at $S_R$ is chosen as

$$G_3^2 = \frac{Q}{P_{max} X_1 X_4}$$  \hspace{1cm} (16)$$

Then, the instantaneous SINR $\gamma_3$ at $S_D$ in Scenario 3 can be expressed as

$$\gamma_3 = \beta_2 \frac{X_1 X_2}{X_1 X_4 + \beta_4 X_2}$$  \hspace{1cm} (17)$$
where $\beta_4 = N_0 Q / (N_d P_{\text{max}})$.

**Scenario 4**: $P_S$ is inactive in both TSs of the secondary transmission. The probability that Scenario 4 happens is

$$p_4 = \frac{p_s p_r}{(p_s + p_{ps})(p_r + p_{pr})}$$

Accordingly, the received signal at $S_D$ is expressed as

$$y_{d,4} = G h_2 h_1 x_4 + G h_2 n_0 + n_0$$

In this scenario, both $S_S$ and $S_R$ operate in interweave mode with average transmit power $P_{\text{max}}$. Thus, the gain factor $G_4$ is chosen as for the conventional relay system, i.e.,

$$G_4^2 = 1/X_1$$

Consequently, the instantaneous SNR $\gamma_4$ at $S_D$ in Scenario 4 can be found as

$$\gamma_4 = \beta_5 \frac{X_1 X_2}{X_1 + X_2}$$

where $\beta_5 = P_{\text{max}} / N_0$.

In order to support the subsequent performance analysis, we provide the CDF and PDF of $X_i$, $i \in \{1, 2, 3, 4\}$, for a Nakagami-$m$ fading channel with channel mean power $\Omega_i$ and positive integer values of fading severity $m_i$ as

$$f_{X_i}(x_i) = \frac{\alpha_{i,m_i}}{\Gamma(m_i)} x_i^{m_i-1} \exp(-\alpha_i x_i)$$

$$F_{X_i}(x_i) = 1 - \exp(-\alpha_i x_i) \sum_{j=0}^{m_i-1} \frac{\alpha_i^j x_i^j}{j!}$$

where $\alpha_i = m_i / \Omega_i$.

### 3 End-to-End Performance Analysis

In general, the expectation $\mathbb{B}$ of a specific performance metric $B$ can be formulated as in [21]

$$\mathbb{B} = \lim_{T \to \infty} \frac{1}{T} E \left[ \int_0^T B(t) \, dt \right]$$

(24)
where $B(t)$ is the performance metric of the system at time instant $t$. For a discrete state space $A$, we have

$$B = \sum_{i \in A} B(i)p_i$$  \hspace{1cm} (25)

where $i$ denotes any state in $A$, $B(i)$ is the average performance metric of the system in State $i$ and $p_i$ represents the probability that the system falls into State $i$.

### 3.1 Outage Probability Performance

Outage probability is defined as the probability that the instantaneous SNR falls below a predefined threshold $\gamma_{th}$. Applying (25) to the hybrid interweave-underlay CCRN, this metric can be obtained as

$$P_{out} = \sum_{i=1}^{4} p_i P_{i,\text{out}}$$  \hspace{1cm} (26)

where $p_i$, $i \in \{1, 2, 3, 4\}$, is the probability that Scenario $i$ applies and $P_{i,\text{out}} = F_{\gamma_i}(\gamma_{th})$ is the outage probability in Scenario $i$. Hence, our goal is to derive the CDFs of the instantaneous SINRs for Scenarios 1, 2, 3, and the instantaneous SNR for Scenario 4.

**Scenario 1**: In view of (9), the CDF of the instantaneous SINR $\gamma_1$ for Scenario 1 can be expressed as

$$F_{\gamma_1}(\gamma) = \int_{0}^{\gamma} \int_{0}^{\gamma} \int_{0}^{\gamma} \int_{0}^{\gamma} f_{X_1}(x_1)f_{X_2}(x_2)f_{X_3}(x_3)f_{X_4}(x_4)dx_2dx_3dx_4$$

$$+ \int_{0}^{\gamma} \int_{0}^{\gamma} \int_{0}^{\gamma} F_{X_1} \left( \frac{\gamma x_2x_3}{\beta_1x_2 - \gamma x_4} \right) f_{X_2}(x_2)dx_2 f_{X_3}(x_3)dx_3 f_{X_4}(x_4)dx_4$$  \hspace{1cm} (27)

With the expression of $f_{X_i}(x_i)$ in (22) along with the help of [18, eq. (3.381.4)], $I$ can be simplified as

$$I = 1 - \sum_{q=0}^{m_2-1} \frac{\beta_2^{m_4} \alpha_3^2 \alpha_4^{m_4}}{q!} \frac{\Gamma(m_4 + q)}{\Gamma(m_4)} \frac{\gamma^q}{(\alpha_2 \gamma + \beta_2 \alpha_4)^{m_4+q}}$$  \hspace{1cm} (28)
We can rewrite the inner integral $I_1$ given in (27) as

$$I_1 = \frac{1}{\beta_1 \beta_2} \int_0^\infty F_{X_1} \left( \frac{\gamma x_3}{\beta_1} + \frac{\gamma^2 x_3 x_4}{x_2} \right) f_{X_2} \left( \frac{x_2 + \gamma \beta_1 x_4}{\beta_1 \beta_2} \right) \, dx_2 \quad (29)$$

Substituting (22) and (23) into (29) and then using [18, eq. (3.471.9)] to calculate the remaining integral, after some manipulation, an expression for $I_1$ can be written as

$$I_1 = 1 - F_{X_2} \left( \frac{\gamma x_4}{\beta_2} \right) - 2 \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{C_p^r C_r^{m_2-1}}{p! \Gamma(m_2)} x_3 \left( \frac{2p+r-q+1}{\beta_1} \right)^{(2p+r-q+1)/2} \left( \frac{2p+r-q+1}{\beta_2(2m_2-r-q+1)/2} \right)$$

$$\times \exp \left( -\frac{\alpha_1 \gamma x_3}{\beta_1} \right) \exp \left( -\frac{\alpha_2 \gamma x_4}{\beta_2} \right) K_{r-q+1} \left( 2 \sqrt{\frac{\alpha_1 \alpha_2 \gamma^2 x_3 x_4}{\beta_1 \beta_2}} \right) \quad (30)$$

Using (30) and (22) together with the help of [18, eq. (6.643.3)], we can obtain an expression for $I_2$ as

$$I_2 = 1 - F_{X_2} \left( \frac{\gamma x_4}{\beta_2} \right) - \frac{\Gamma(m_3 + p + r - q + 1)}{\Gamma(m_2) \Gamma(m_3)} \frac{1}{\beta_1^{m_3 \alpha_1} \alpha_1^{(2p+r-q)/2} \alpha_2^{(2m_2-r+q-2)/2} \alpha_3^{m_3}} \times \frac{\gamma^{m_2+p-1} x_4^{(2m_2+r+q-2)/2}}{(\alpha_1 \alpha_3)^{(2m_3+2p+r+q)/2}} \exp \left( -\frac{\gamma^2 \alpha_1 \alpha_2 + 2 \alpha_2 \gamma \beta_1 \alpha_3 x_4}{2 \beta_2 (\alpha_1 \gamma + \beta_1 \alpha_3)} \right)$$

$$\times W_{- (2m_2+2p+r-q), 2r-q+1} \left( \frac{\gamma^2 \alpha_1 \alpha_2 x_4}{\beta_2 (\alpha_1 \gamma + \beta_1 \alpha_3)} \right) \quad (31)$$

Having the expression of $I_2$ in (31) and $f_{X_3}(x_3)$ in (22), we can reach an expression for the outer integral $I_3$ through the help of [18, eq. (7.621.3)]. Substituting this outcome and (28) into (27), then utilizing [18, eq. (7.621.3)] to solve the remaining integral, after rearranging terms, the CDF of the in-
stantaneous SINR for Scenario 1 can be formulated as

$$F_{\gamma_1}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{p} C_p^q \sum_{i=0}^{m_2-1} C_i^{m_2-1} \frac{\Gamma(m_4 + m_2)}{\Gamma(m_2) \Gamma(m_3)} \times \frac{\Gamma(m_3 + p + r - q + 1) \Gamma(m_4 + m_2 + q - r - 1) \beta_1^{m_3}}{\Gamma(m_4) \Gamma(m_3 + m_2 + p)} \times \frac{\Gamma(m_3 + p) \alpha_1^{p+r-q+1} \alpha_2^{m_2} \alpha_3^{m_3} \alpha_4^{m_4} \beta_2^{m_4}}{(\alpha_2^2 + \beta_2 \alpha_4)^{m_4+\alpha_2(m_1 + \gamma) + \beta_1 \alpha_3}^{m_3+p+r-q+1}} \times 2F_1 \left( m_4 + m_2, m_3 + p + r - q + 1, m_4 + m_3 + m_2 + p \right)$$

$$+ p; \frac{\beta_2 \alpha_1 \gamma_3 \gamma + \beta_1 \alpha_2 \gamma_3 \gamma + \beta_1 \beta_2 \alpha_3 \gamma}{\alpha_1 \gamma_2 \gamma^2 + \beta_2 \alpha_1 \gamma + \beta_1 \alpha_2 \gamma + \beta_1 \beta_2 \alpha_3 \gamma}$$

(32)

Similarly, we can derive the CDFs of the instantaneous SINRs for Scenarios 2, 3, and the instantaneous SNR for Scenario 4 as follows:

**Scenario 2:**

$$F_{\gamma_2}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{p} C_p^q \sum_{i=0}^{m_2-1} C_i^{m_2-1} \frac{\beta_3^{(2m_3 - 2m_2 + i - q + 2)/2}}{\alpha_1^{m_3} (\gamma + \beta_1 \alpha_3 / \alpha_1)^{2(m_3 + 2p + q - 1)/2}} \frac{\Gamma(m_3 + p) \Gamma(m_4 + m_2 + q - r - 1) \beta_1^{m_3}}{\Gamma(m_4) \Gamma(m_3 + m_2 + p)} \times \gamma^{m_3 + p - 1} \times W_{-2m_3 + 2p + q - 1, i - q + 1} \left( \frac{\alpha_1 \alpha_2 \beta_3 \gamma^2}{\beta_1 (\alpha_1 \gamma + \beta_1 \alpha_3)} \right)$$

(33)

**Scenario 3:**

$$F_{\gamma_3}(\gamma_3) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{p} C_p^q \sum_{i=0}^{m_2-1} C_i^{m_2-1} \frac{\beta_2^{(2m_4 - 2m_2 + i - q + 2)/2}}{\alpha_2^{m_4} \Gamma(m_2) \Gamma(m_4) (\gamma + \alpha_4 \beta_2 / \alpha_2)^{2(m_4 + 2m_2 + q - i - 2)/2}} \times \Gamma(m_4 + m_2 + q - r - 1) \times W_{-2m_4 + 2m_2 + q - i - 2, i - q + 1} \left( \frac{\alpha_1 \alpha_2 \gamma^2 \beta_4}{\beta_2 (\alpha_2 \gamma + \alpha_4 \beta_2)} \right)$$

(34)
Scenario 4:

\[
F_{\gamma_4}(\gamma) = 1 - 2 \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{C_p^q \cdot \alpha_1^{m_2-1-p-q} \cdot \alpha_2^{2p+1-q+1}}{\Gamma(m_2) \cdot \beta_5^{m_2+p}} \cdot \gamma^{m_2+p} \\
\times \frac{\alpha_2^{2m_2-i+q-1}}{2 \sqrt{\pi}} \exp(-\alpha_1 \gamma + \alpha_2 \gamma \beta_5) K_{i-q+1} \left(2 \sqrt{\frac{\alpha_1 \alpha_2 \gamma^2}{\beta_5^2}}\right)
\]

(35)

Using $\gamma_{th}$ as the argument of (32), (33), (34), and (35), we obtain $P_{i,\text{out}} = F_{\gamma_i}(\gamma_{th})$, $i \in \{1, 2, 3, 4\}$. By substituting these outcomes into (26), finally, the outage probability for the secondary transmission is found.

3.2 Symbol Error Rate Performance

Using (25) for the hybrid interweave-underlay CCRN, the SER can be obtained as

\[
P_E = \sum_{i=1}^{4} p_i P_{i,E}
\]

(36)

where $P_{i,E}$ is the SER for Scenario $i$. For each scenario, the SER can be expressed in terms of the instantaneous SINR (or SNR in Scenario 4) $\gamma_i$ as in [26]

\[
P_{i,E} = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^{\infty} F_{\gamma_i}(\gamma) \gamma^{-\frac{1}{2}} e^{-b\gamma} d\gamma
\]

(37)

where $a$ and $b$ are modulation parameters. Since it is too complicated to derive expressions for the SERs of Scenarios 1, 2, and 3 directly from the exact expressions of the CDFs of the instantaneous SINRs $\gamma_1$ in (32), $\gamma_2$ in (33), and $\gamma_3$ in (34), we instead utilize the CDFs of the bounds on $\gamma_1$, $\gamma_2$, and $\gamma_3$ to obtain approximations for the SERs. Applying [27, eq. (25)] and [28, eq. (2)] to (9), (13), and (17), bounds $\gamma_1^u$, $\gamma_2^u$, and $\gamma_3^u$ on $\gamma_1$, $\gamma_2$, and $\gamma_3$ are, respectively, obtained as $\gamma_1^u = \min\{\theta_1, \theta_2\}$, $\gamma_2^u = \min\{\rho_1, \theta_2\}$, and $\gamma_3^u = \min\{\theta_1, \rho_2\}$. Here, $\theta_1 = \beta_2 X_2 / X_4$, $\theta_2 = \beta_1 X_1 / X_3$, $\rho_1 = \beta_1 X_2 / \beta_3$, and $\rho_2 = \beta_2 X_1 / \beta_4$. Thus, the CDFs of $\gamma_1^u$, $\gamma_2^u$, and $\gamma_3^u$ can be, respectively, obtained as

\[
F_{\gamma_1^u}(\gamma) = 1 - [1 - F_{\theta_1}(\gamma)][1 - F_{\theta_2}(\gamma)]
\]

(38)

\[
F_{\gamma_2^u}(\gamma) = 1 - [1 - F_{\rho_1}(\gamma)][1 - F_{\theta_2}(\gamma)]
\]

(39)

\[
F_{\gamma_3^u}(\gamma) = 1 - [1 - F_{\theta_1}(\gamma)][1 - F_{\rho_2}(\gamma)]
\]

(40)
where the CDFs of $\theta_1$, $\theta_2$, $\rho_1$, and $\rho_2$ are given by

$$F_{\theta_1}(\gamma) = \int_0^\infty F_{X_2}(\frac{\gamma x_4}{\beta_2}) f_{X_4}(x_4) dx_4$$

(41)

$$F_{\theta_2}(\gamma) = \int_0^\infty F_{X_1}(\frac{\gamma x_3}{\beta_1}) f_{X_3}(x_3) dx_3$$

(42)

$$F_{\rho_1}(\gamma) = F_{X_2}(\frac{\beta_3 \gamma}{\beta_1})$$

(43)

$$F_{\rho_2}(\gamma) = F_{X_1}(\frac{\beta_4 \gamma}{\beta_2})$$

(44)

Given $f_{X_i}(x_i)$ in (22) and $F_{X_i}(x_i)$ in (23), $i \in \{1, 2, 3, 4\}$, followed by the help of [18, eq. (3.381.4)], we attain expressions for $F_{\theta_1}(\gamma)$, $F_{\theta_2}(\gamma)$, $F_{\rho_1}(\gamma)$, and $F_{\rho_2}(\gamma)$. Substituting these outcomes into (38), (39), and (40), expressions for $F_{\gamma_1}(\gamma)$, $F_{\gamma_2}(\gamma)$, and $F_{\gamma_3}(\gamma)$ are found as

$$F_{\gamma_1}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{1}{p!q!} \frac{\Gamma(m_3 + p) \Gamma(m_4 + q) \beta_1^{m_3}}{\Gamma(m_3) \Gamma(m_4)} \times \frac{\beta_2^{m_4} \alpha_3^{m_3} \alpha_4^{m_4}}{\alpha_1^{m_3} \alpha_2^{m_4}} \times \frac{\gamma^{p+q}}{(\gamma + \frac{\beta_3 \alpha_3}{\alpha_1})^{m_3+p} (\gamma + \frac{\beta_4 \alpha_4}{\alpha_2})^{m_4+q}}$$

(45)

$$F_{\gamma_2}(\gamma) = 1 - \sum_{q=0}^{m_3-1} \frac{1}{q!} \sum_{p=0}^{m_4-1} \frac{1}{p!} \frac{\Gamma(m_3 + p) \alpha_2^{m_3} \beta_1^{m_3-q} \beta_3^{p}}{\alpha_1^{m_3}} \times \frac{\gamma^{p+q}}{(\gamma + \frac{\beta_1 \alpha_3}{\alpha_1})^{m_3+p}} \exp\left(-\frac{\alpha_2 \beta_3 \gamma}{\beta_1}\right)$$

(46)

$$F_{\gamma_3}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\Gamma(m_4 + q) \beta_2^{m_4-p} \beta_4^{p} \alpha_1^{m_4} \alpha_3^{m_4}}{\alpha_2^{m_4}} \times \frac{\gamma^{p+q}}{(\gamma + \frac{\beta_2 \alpha_4}{\alpha_2})^{m_4+q}} \exp\left(-\frac{\alpha_1 \beta_4 \gamma}{\beta_2}\right)$$

(47)

**Scenario 1:** In order to compute the SER for Scenario 1, first, we replace $F_{\gamma_i}(\gamma)$ in (37) by $F_{\gamma_i}(\gamma)$ in (45). Then, we utilize [18, eq. (2.102)] to transform the integral expression into tabulated form. Finally, we apply [18, eq. (3.361.2)] and [29, eq. (2.3.6.9)] to calculate the resulting integrals.
that leads to an expression for the SER of the system for Scenario 1 as

\[
P_{1,E} = \frac{a}{2} - \frac{a\sqrt{b}}{2\sqrt{\pi}} \sum_{p=0}^{m_3-1} \sum_{q=0}^{m_2-1} \frac{\Gamma(m_3 + p) \beta_1^{m_3} \beta_2^{m_4} \alpha_3^{m_3} \alpha_4^{m_4}}{p! q! \alpha_1^{m_3} \alpha_2^{m_4}} \nonumber \\
\times \frac{\Gamma(m_4 + q)}{\Gamma(m_3) \Gamma(m_4)} \Gamma(p + q + \frac{1}{2}) \left[ \sum_{i=1}^{p+q} \left( \frac{\beta_1 \alpha_3}{\alpha_1} \right)^{p+q-i+\frac{1}{2}} \right] \nonumber \\
\times \kappa_{p,i} U \left( \frac{p + q + \frac{1}{2} + p + q - i + \frac{3}{2}, b \frac{\beta_1 \alpha_3}{\alpha_1} \right) + \sum_{j=1}^{m_4+q} \xi_{q,j} \nonumber \\
\times \left( \frac{\beta_2 \alpha_4}{\alpha_2} \right)^{p+q-j+\frac{1}{2}} U \left( p + q + \frac{1}{2}, p + q - j + \frac{3}{2}, b \frac{\beta_2 \alpha_4}{\alpha_2} \right) \right] \tag{48}\nonumber 
\]

where \(\kappa_{p,i}\) and \(\xi_{q,j}\) are partial fraction coefficients

\[
\kappa_{p,i} = \frac{1}{(m_3 + p - i)!} \frac{d^{m_3+p-i} \left[ (\gamma + \frac{\beta_2 \alpha_4}{\alpha_2})^{-m_4-q} \right]}{d\gamma^{m_3+p-i} |_{\gamma = -\frac{\beta_1 \alpha_3}{\alpha_1}}} \nonumber \\
\xi_{q,j} = \frac{1}{(m_4 + q - j)!} \frac{d^{m_4+q-j} \left[ (\gamma + \frac{\beta_1 \alpha_3}{\alpha_1})^{-m_3-p} \right]}{d\gamma^{m_4+q-j} |_{\gamma = -\frac{\beta_2 \alpha_4}{\alpha_2}}} \nonumber 
\]

**Scenario 2:** By replacing \(F_{\gamma_1}(\gamma)\) in (37) by \(F_{\gamma_2}(\gamma)\) in (46), then making use of [18, eq. (3.361.2)] and [29, eq. (2.3.6.9)] to calculate the remaining integrals, an expression for the SER of Scenario 2 can be obtained as

\[
P_{2,E} = \frac{a}{2} - \frac{a\sqrt{b}}{2\sqrt{\pi}} \sum_{p=0}^{m_3-1} \sum_{q=0}^{m_2-1} \frac{1}{p! q! \alpha_1^{m_3} \alpha_2^{m_4}} \frac{\Gamma(m_4 + q) \Gamma \left( p + q + \frac{1}{2} \right)}{\Gamma(m_3)} \nonumber \\
\times \Gamma(m_3 + p) U \left( p + q + \frac{1}{2}, q + \frac{3}{2} - m_3, \frac{\alpha_2 \beta_2 \alpha_3 + \beta_1 b \alpha_3}{\alpha_1} \right) \tag{49}\nonumber 
\]

**Scenario 3:** Similarly, the SER for Scenario 3 is given by

\[
P_{3,E} = \frac{a}{2} - \frac{a\sqrt{b}}{2\sqrt{\pi}} \sum_{p=0}^{m_3-1} \sum_{q=0}^{m_2-1} \frac{1}{p! q! \alpha_1^{m_3} \alpha_2^{m_4}} \frac{\Gamma(m_4 + q) \Gamma \left( p + q + \frac{1}{2} \right)}{\Gamma(m_3)} \beta_2^{\frac{3}{2}} \nonumber \\
\times \frac{\beta_2^{p} \alpha_4^{p+\frac{1}{2}}}{\alpha_2^{p+\frac{1}{2}}} U \left( p + q + \frac{1}{2}, p + \frac{3}{2} - m_4, \frac{\alpha_1 \alpha_3 \gamma + \alpha_4 b \alpha_3}{\alpha_2} \right) \tag{50}\nonumber 
\]

**Scenario 4:** Finally, replacing \(F_{\gamma_1}(\gamma)\) in (37) by \(F_{\gamma_4}(\gamma)\) in (35), followed by using [18, eq. (3.361.2)] and [18, eq. (6.621.3)] to compute the resulting
integrals, an expression for the SER of Scenario 4 can be given by

\[
P_{4,E} = \frac{a}{2} - a\sqrt{b} \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{p} \frac{C_p q^p}{\Gamma(m_2)} \sum_{i=0}^{m_2-1} C_i^{m_2-1} 4^{i-q+1} \beta_5^{1/2} \Gamma(m_2 + p + i - q + \frac{1}{2}) \Gamma(m_2 + p + q - i - \frac{1}{2}) \times \left(\frac{\alpha_1 + \alpha_2 + b \beta_5 + 2\sqrt{\alpha_1 \alpha_2}}{\alpha_1 + \alpha_2 + b \beta_5 + 2\sqrt{\alpha_1 \alpha_2}}\right)^{m_2 + p + i - q + \frac{1}{2}} \times \Gamma(m_2 + p + 1)^2 F_1 \left(m_2 + p + i - q + \frac{3}{2}, i - q + \frac{3}{2}, \frac{1}{2} \alpha_1 + \alpha_2 + b \beta_5 - 2\sqrt{\alpha_1 \alpha_2} \right)^{m_2 + p + 1} \] (51)

Now, we are ready to obtain the average SER of the system by substituting (48), (49), (50), and (51) into (36).

### 3.3 Outage Capacity Performance

Outage capacity \( C_\epsilon \) in (bits/s/Hz) of the hybrid CCRN, i.e. the largest rate of transmission to guarantee that the outage probability is less than \( \epsilon \)%, is derived from (25) as

\[
C_\epsilon = \sum_{i=1}^{4} p_i C_{i,\epsilon} \] (52)

where \( C_{i,\epsilon}, i \in \{1, 2, 3, 4\} \), is the outage capacity per unit bandwidth of Scenario \( i \). In view of [30], a Gaussian approximation for the CDF of the channel capacity has been considered as an efficient method to evaluate outage capacity. Here, we apply the Gaussian approximation for the CDF of the channel capacity \( C_i \) of Scenario \( i \) as follows:

\[
F_{C_i}(c) \approx 1 - \frac{1}{2} \text{erfc} \left[ \frac{c - E\{C_i\}}{\sqrt{2 \left( E\{C_i^2\} - (E\{C_i\})^2 \right)}} \right] \] (53)

where \( \text{erfc}[: \] is the complementary error function. Furthermore, \( E\{C_i\} \) and \( E\{C_i^2\} \) are the first and the second moment of the channel capacity of Scenario \( i \). As in [30], outage capacity \( C_{i,\epsilon} \) of Scenario \( i \) corresponding to \( \epsilon \)% outage probability can be approximated from (53) as

\[
C_{i,\epsilon} = E\{C_i\} + \sqrt{2 \left( E\{C_i^2\} - (E\{C_i\})^2 \right) \text{erfc}^{-1} \left[ 2 - \frac{\epsilon}{50} \right]} \] (54)
In order to derive the outage capacity of Scenario $i$, we need to obtain the first and the second moment of the channel capacity. Based on the Shannon capacity theorem, the instantaneous channel capacity in (bits/s/Hz) of Scenario $i$ is expressed as

$$C_i = \frac{1}{2} \log_2(1 + \gamma_i) = \frac{\ln(1 + \gamma_i)}{2 \ln 2} \quad (55)$$

Utilizing the Taylor series expansion of $\ln(1 + \gamma_i)$ around the mean value $E\{\gamma_i\}$ of the effective SINR (or SNR in Scenario 4) $\gamma_i$, we have

$$\ln(1 + \gamma_i) = \ln(1 + E\{\gamma_i\}) + \sum_{m=1}^{\infty} \frac{(-1)^{m-1} (\gamma_i - E\{\gamma_i\})^m}{m (1 + E\{\gamma_i\})^m}$$

$$\approx \ln(1 + E\{\gamma_i\}) + \frac{\gamma_i - E\{\gamma_i\}}{1 + E\{\gamma_i\}} - \frac{1}{2} \frac{(\gamma_i - E\{\gamma_i\})^2}{(1 + E\{\gamma_i\})^2} \quad (56)$$

Applying the expectation operation for $C_i$ and $C_i^2$, the second-order approximations for $E\{C_i\}$ and $E\{C_i^2\}$ are obtained as

$$E\{C_i\} \approx \frac{1}{2 \ln 2} \left[ \ln(1 + E\{\gamma_i\}) - \frac{E\{\gamma_i^2\} - (E\{\gamma_i\})^2}{2(1 + E\{\gamma_i\})^2} \right] \quad (57)$$

$$E\{C_i^2\} \approx \left( \frac{1}{2 \ln 2} \right)^2 \left[ (\ln(1 + E\{\gamma_i\}))^2 + \frac{E\{\gamma_i^2\} - (E\{\gamma_i\})^2}{(1 + E\{\gamma_i\})^2} \ln \left( \frac{e}{1 + E\{\gamma_i\}} \right) \right] \quad (58)$$

where $e$ is the Napier constant and $E\{\gamma_i^n\}$, $n = 1, 2$, is the $n$-th moment of the effective SINR (or SNR in Scenario 4) $\gamma_i$, i.e.,

$$E\{\gamma_i\} = \int_0^{\infty} \gamma f_{\gamma_i}(\gamma) d\gamma = \int_0^{\infty} [1 - F_{\gamma_i}(\gamma)] d\gamma \quad (59)$$

$$E\{\gamma_i^2\} = \int_0^{\infty} \gamma^2 f_{\gamma_i}(\gamma) d\gamma = 2 \int_0^{\infty} \gamma [1 - F_{\gamma_i}(\gamma)] d\gamma \quad (60)$$

Again, it is noted that calculating $E\{\gamma_i\}$ and $E\{\gamma_i^2\}$ for Scenarios 1, 2, 3, and 4 directly from the exact expressions of the CDFs of $\gamma_1$ in (32), $\gamma_2$ in (33), $\gamma_3$ in (34), and $\gamma_4$ in (35) is intractable. Thus, $E\{\gamma_i\}$ and $E\{\gamma_i^2\}$ are computed from the CDF of the bound on $\gamma_i$, $i \in \{1, 2, 3, 4\}$. The CDFs of the bounds on $\gamma_1$, $\gamma_2$, and $\gamma_3$ are derived in (45), (46), and (47), respectively.
Now, we need to derive the CDF of the bound on $\gamma_4$. Applying [27, eq. (25)] and [28, eq. (2)], from (21), the bound $\gamma_4^u$ on the instantaneous SNR $\gamma_4$ can be found as

$$\gamma_4^u = \min(\chi_1, \chi_2)$$  \hspace{1cm} (61)

where $\chi_1 = \beta_5 X_1$ and $\chi_2 = \beta_5 X_2$. Hence, an expression for the CDF of $\gamma_4^u$ is given by

$$F_{\gamma_4^u}(\gamma) = 1 - [1 - F_{\chi_1}(\gamma)][1 - F_{\chi_2}(\gamma)]$$  \hspace{1cm} (62)

From (23) together with the help of [18, eq. (3.381.4)], we attain expressions for $F_{\chi_1}(\gamma)$ and $F_{\chi_2}(\gamma)$. With these outcomes, $F_{\gamma_4^u}(\gamma)$ is obtained as

$$F_{\gamma_4^u}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\alpha_1^p \alpha_2^q \gamma^{p+q}}{p! q! \beta_5^{p+q}} \exp\left(-\frac{(\alpha_1 + \alpha_2)\gamma}{\beta_5}\right)$$  \hspace{1cm} (63)

**Scenario 1**: In order to calculate $E\{\gamma_1\}$ and $E\{\gamma_1^2\}$, we first replace $F_{\gamma_1}(\gamma)$ in (59) and (60) by $F_{\gamma_4^u}(\gamma)$ in (45). Then, taking advantage of [18, eq. (3.197.1)] to solve the resulting integrals, expressions for $E\{\gamma_1\}$ and $E\{\gamma_1^2\}$ are obtained as

$$E\{\gamma_1\} = \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\Gamma(m_3 + p) \Gamma(m_4 + q)}{\Gamma(m_3) \Gamma(m_4)} B(p + q + 1, m_3 + m_4 - 1) \times \frac{\beta_2^{p+1} \alpha_1^p \alpha_4^{p+1}}{\beta_1^{p+1} \alpha_2^p \alpha_3^{p+1}} F_1\left(m_3 + p, p + q + 1; m_3 + m_4 + q; 1 - \frac{\beta_2 \alpha_1 \alpha_4}{\beta_1 \alpha_2 \alpha_3}\right)$$  \hspace{1cm} (64)

$$E\{\gamma_1^2\} = 2 \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\Gamma(m_3 + p) \Gamma(m_4 + q)}{\Gamma(m_3) \Gamma(m_4)} B(p + q + 2, m_3 + m_4 - 2) \times \frac{\beta_2^{p+2} \alpha_1^p \alpha_4^{p+2}}{\beta_1^{p+2} \alpha_2^p \alpha_3^{p+2}} F_1\left(m_3 + p, p + q + 2; m_3 + m_4 + p + q; 1 - \frac{\beta_2 \alpha_1 \alpha_4}{\beta_1 \alpha_2 \alpha_3}\right)$$  \hspace{1cm} (65)

**Scenario 2**: Replacing $F_{\gamma_1}(\gamma)$ in (59) by $F_{\gamma_2}(\gamma)$ in (46), then applying [29, eq. (2.3.6.9)] to handle the resulting integral, an expression for $E\{\gamma_2\}$ is given by

$$E\{\gamma_2\} = \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\beta_1 \beta_3 \alpha_3^{q+1} \Gamma(m_3 + p) \Gamma(p + q + 1)}{\alpha_1 \alpha_3^{q+1} \Gamma(m_3)} \times U\left(p + q + 1, q + 2 - m_3; \frac{\beta_3 \alpha_2 \alpha_3}{\alpha_1}\right)$$  \hspace{1cm} (66)
Similarly, $E\{\gamma_2^3\}$ is obtained as

$$E\{\gamma_2^3\} = 2 \sum_{p=0}^{m_2-1} \sum_{q=0}^{m_1-1} \frac{\beta_1^2 \beta_2^p \alpha_1^p \alpha_4 \alpha_4^{p+2} \Gamma(m_4 + q) \Gamma(p + q + 2)}{p! q! \alpha_2^{p+1} \alpha_4^{p+2} \Gamma(m_4)} \times U(p + q + 2, p + 3 - m_3, \beta_3 \alpha_2 \alpha_3 / \alpha_1)$$  \hspace{1cm} (67)

**Scenario 3:** Utilizing the approach of calculating $E\{\gamma_2\}$ and $E\{\gamma_2^2\}$, expressions for $E\{\gamma_3\}$ and $E\{\gamma_3^2\}$ can be obtained as

$$E\{\gamma_3\} = \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\beta_2 \beta_4 \alpha_4^{p+1} \Gamma(m_4 + q) \Gamma(p + q + 1)}{p! q! \alpha_2^{p+1} \Gamma(m_4)} \times U(p + q + 1, p + 2 - m_4, \alpha_1 \alpha_4 \beta_4 / \alpha_2)$$  \hspace{1cm} (68)

$$E\{\gamma_3^2\} = 2 \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\beta_2 \beta_4 \alpha_4^{p+2} \Gamma(m_4 + q) \Gamma(p + q + 2)}{p! q! \alpha_2^{p+2} \Gamma(m_4)} \times U(p + q + 2, p + 3 - m_4, \alpha_1 \alpha_4 \beta_4 / \alpha_2)$$  \hspace{1cm} (69)

**Scenario 4:** Finally, replacing $F_{\gamma_i}(\gamma)$ in (59) and (60) by $F_{\gamma_2}^{\ast}(\gamma)$ in (63), then utilizing [18, eq. (3.381.4)] to handle the resulting integrals, we obtain expressions for $E\{\gamma_4\}$ and $E\{\gamma_4^2\}$ as

$$E\{\gamma_4\} = \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\alpha_1^p \alpha_2 \alpha_5 \Gamma(p + q + 1)}{p! q! (\alpha_1 + \alpha_2)^{p+q+1}}$$  \hspace{1cm} (70)

$$E\{\gamma_4^2\} = 2 \sum_{p=0}^{m_1-1} \sum_{q=0}^{m_2-1} \frac{\alpha_1^p \alpha_2 \alpha_5 \Gamma(p + q + 2)}{p! q! (\alpha_1 + \alpha_2)^{p+q+2}}$$  \hspace{1cm} (71)

By substituting $E\{\gamma_i\}$ and $E\{\gamma_i^2\}$ into (57) and (58), expressions for the mean $E\{C_i\}$ and second moment $E\{C_i^2\}$ of channel capacity of Scenario $i$ are obtained. As a result, an expression for outage capacity $C_{i,\epsilon}$ of Scenario $i$ can be easily attained by substituting these outcomes into (54) which leads to an expression for outage capacity $C_{\epsilon}$ of the system.

## 4 Numerical Results

In this section, numerical results are provided to illustrate the impact of system parameters on the performance of the secondary network. For this purpose, we consider arrival rate of the primary network, average transmit SNR
$P_{max}/N_0$ of the secondary network in interweave mode, interference power threshold $Q$ of the primary receiver, and distances from $S_S$ to $P_D$ and from $S_R$ to $P_D$. According to [31,32], spectrum occupancy of the worldwide licensed users in the 25-3400 MHz frequency range is rather low. Specifically, in the range from 470 MHz to 766 MHz and from 880 MHz to 960 MHz, which are used for cellular phone networks, the average fraction of time that primary users utilize the licensed bands is highest with around 0.45. However, other frequency ranges are much lower utilized, e.g., occupancy ratio for the range from 960 MHz to 1525 MHz was only about 0.0236. As a result, the mean occupancy ratio over the whole range from 25 MHz to 3400 MHz was measured as 0.12 [32]. Thus, for the numerical examples, we select arrival rates and departure rates such that the probability for a primary transmission occurring $(p_p+p_{ps}+p_{pr})$ is less than 0.5. Let $d_1, d_2, d_3, d_4, d_5,$ and $d_6$ denote the normalized distances from $S_S$ to $S_R$, $S_R$ to $S_D$, $S_S$ to $P_D$, $S_R$ to $P_D$, $P_S$ to $S_R$, and $P_S$ to $S_D$, respectively. Assume that channel mean powers are attenuated with distance according to the exponential decaying model with path-loss exponent $n = 4$ as for a suburban environment. The fading severity parameters are selected as $m_1 = m_2 = m_3 = m_4 = 3$. We approximate the effect of interference and noise at the relay and the secondary receiver as Gaussian variables $n_r$ and $n_d$ with zero-mean and variance $N_r$ and $N_d$, respectively. Here, $N_r$ and $N_d$ are calculated based on the power law decaying path-loss as well as on the noise at the relay and the secondary receiver, i.e., $N_r = N_0 + d_5^{-n}P_p$ and $N_d = N_0 + d_6^{-n}P_p$ where $P_p$ is the transmit power of $P_S$. The SNR threshold to calculate the outage is fixed at $\gamma_{th} = 3$ dB and the outage probability threshold to calculate the outage capacity is chosen as $\epsilon = 1\%$.

Fig. 3, Fig. 4, and Fig. 5, respectively, depict outage probability, SER, and outage capacity versus interference power-to-noise ratio $Q/N_0$ for various arrival rates $\lambda_p$ at the primary transmitter. The average SNR of the primary network and the secondary network in interweave mode are selected as $P_p/N_0 = 7$ dB and $P_{max}/N_0 = 10$ dB. We assume that the arrival rate of the secondary network is $\lambda_s = 0.4$ packets/TS and that the departure rates of the secondary network and primary network are the same for all examples, i.e., $\mu_p = \mu_s = \mu_r = 0.4$ packets/TS. In a relay network, the departure rate of $S_S$ becomes the arrival rate of $S_R$, i.e., $\lambda_r = \mu_s$. For all examples of Fig. 3, Fig. 4, and Fig. 5, the normalized distances are fixed as $d_1 = d_2 = 0.5$, $d_3 = d_4 = 1.0$, and $d_5 = d_6 = 2.0$. As expected, when the arrival rate of the primary network decreases, the performance of the hybrid system is improved. Specifically, outage probability and SER of the hybrid system decrease, and outage capacity increases. This is due to the fact that as $\lambda_p$ decreases, the idle periods of the primary source increase. Thus, the probability that the CCRN operates in interweave mode without facing the interference constraint increases.
Figure 3: Outage probability of a hybrid interweave-underlay cognitive cooperative radio network versus interference power-to-noise ratio $Q/N_0$ for different arrival rates $\lambda_p$ of the primary user.

Figure 4: SER of 8-PSK modulation for a hybrid interweave-underlay cognitive cooperative radio network versus interference power-to-noise ratio $Q/N_0$ for different arrival rates $\lambda_p$ of the primary user.
Figure 5: Outage capacity for a hybrid interweave-underlay cognitive cooperative radio network versus interference power-to-noise ratio $Q/N_0$ for different arrival rates $\lambda_p$ of the primary user.

Fig. 6, Fig. 7, and Fig. 8, respectively, illustrate outage probability, SER, and outage capacity versus interference power-to-noise ratio $Q/N_0$ for different values of average transmit SNR $P_{\text{max}}/N_0$. The average transmit SNR of the primary network is selected as $P_p/N_0 = 7$ dB. Arrival rates and departure rates at the primary network and secondary network are selected as $\lambda_p = \lambda_s = 0.5$ packets/TS and $\mu_p = \mu_s = \mu_r = 0.5$ packets/TS. In addition, we fix $d_1 = d_2 = 0.5$, $d_3 = d_4 = 1.0$, and $d_5 = d_6 = 2.0$ for all examples in Fig. 6, Fig. 7, and Fig. 8. As can be observed from these figures, outage probability, SER, and outage capacity are improved as average transmit SNR $P_{\text{max}}/N_0$ increases. However, the effect of $P_{\text{max}}/N_0$ on outage probability, SER, and outage capacity is insignificant for low values of $Q/N_0$. At high values of $Q/N_0$, the outage probability, SER, and outage capacity performance are remarkably enhanced when $P_{\text{max}}/N_0$ increases. The reason for this behavior is that the performance of the hybrid system is not only affected by $P_{\text{max}}/N_0$ but also by the interference threshold $Q$ at $P_D$. For high values of $Q$, since the secondary transmission often satisfies this interference constraint, the performance is mainly affected by $P_{\text{max}}/N_0$. As a result, when the average transmit SNR $P_{\text{max}}/N_0$ increases, the performance will improve accordingly.
Figure 6: Outage probability of a hybrid interweave-underlay cognitive cooperative radio network versus interference power-to-noise ratio $Q/N_0$ for different average transmit SNRs $P_{max}/N_0$ in interweave mode.

Figure 7: SER of 8-PSK modulation for a hybrid interweave-underlay cognitive cooperative radio network versus interference power-to-noise ratio $Q/N_0$ for different average transmit SNRs $P_{max}/N_0$ in interweave mode.
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Figure 8: Outage capacity for a hybrid interweave-underlay cognitive cooperative radio network versus interference power-to-noise ratio $Q/N_0$ for different average transmit SNRs $P_{\text{max}}/N_0$ in interweave mode.

Fig. 9, Fig. 10, and Fig. 11, respectively, show comparisons between outage probability, SER, and outage capacity of the hybrid interweave-underlay CCRN and that of the conventional underlay cognitive relay network. To make a fair comparison, in the examples, the parameters of the hybrid and underlay relay networks are chosen the same. Specifically, we fix the normalized transmission distances $d_1 = d_2 = 0.5$, $d_3 = d_4 = 1.0$ and vary distances $d_5$ and $d_6$. Furthermore, we select arrival rates and departure rates at the primary network and secondary network, respectively, as $\lambda_p = \lambda_s = 0.5$ packets/TS and $\mu_p = \mu_s = \mu_r = 0.2$ packets/TS. Finally, the average transmit SNR of the primary network and the secondary network in interweave mode and the interference power-to-noise ratio in underlay mode are selected as $P_p/N_0 = 7$ dB, $P_{\text{max}}/N_0 = 10$ dB, and $Q/N_0 = 5$ dB in all examples. As expected, when the distances $d_5$ (P$_S$ to S$_R$) and $d_6$ (P$_S$ to S$_D$) increase, outage probability and SER decrease while outage capacity increases for both the hybrid scheme and the conventional cognitive scheme. This is because when the distances from P$_S$ to S$_R$ and P$_S$ to S$_D$ become larger, interference power from the primary transmitter to the secondary network becomes smaller. Furthermore, it can be observed that when the arrival rate of the primary network increases, the performance of the hybrid CCRN degrades but still outperforms the underlay scheme. However, the performance of the conventional underlay CCRN
does not depend on the arrival rate of the primary network. This is because the underlay CCRN utilizes always only one spectrum access mode while the hybrid CCRN can adapt its spectrum access mode based on the state of the primary user.

More importantly, it can be seen from Fig. 9, Fig. 10, and Fig. 11 that the performance of the hybrid CCRN always outperforms that of the conventional underlay CCRN with the same transmission distances, fading conditions, arrival and departure rates. This is because the secondary users in the hybrid CCRN can operate with maximum transmit power $P_{max}$ when the primary transmission is idle. The secondary users only control their transmit powers when the primary transmission is active. On the contrary, in the conventional underlay CCRN, the secondary users always adjust their powers to satisfy the interference power constraint of the primary receiver, even when the primary transmission is idle. In case that the secondary network is located closely to the primary receiver, the transmit power of the conventional underlay CCRN is constrained to be much lower than its maximum transmit power $P_{max}$ which leads to performance degradation as compared to the hybrid CCRN.

Figure 9: Comparison of outage probability for underlay cognitive cooperative radio network and hybrid interweave-underlay cognitive cooperative radio network for different distances $d_5$ and $d_6$, respectively, from $P_S$ to $S_R$ and from $P_S$ to $S_D$. 
Figure 10: Comparison of SER of 8-PSK modulation for underlay cognitive cooperative radio network and hybrid interweave-underlay cognitive cooperative radio network for different distances $d_5$ and $d_6$, respectively, from $P_S$ to $S_R$ and from $P_S$ to $S_D$.

Figure 11: Comparison of outage capacity for underlay cognitive cooperative radio network and hybrid interweave-underlay cognitive cooperative radio network for different distances $d_5$ and $d_6$, respectively, from $P_S$ to $S_R$ and from $P_S$ to $S_D$. 
5 Conclusions

In this paper, we have studied a hybrid interweave-underlay spectrum access system with relaying transmission as a means of improving system performance. A continuous-time Markov chain for modeling and analyzing the spectrum access mechanism of this hybrid CCRN has been developed. Upon the proposed Markov model, the steady-state probability of each state of the hybrid network has been obtained. This has enabled us to further develop a rigorous mathematical framework for performance analysis of such a system. In particular, we have assessed system performance in terms of outage probability, SER, and outage capacity over Nakagami-$\ell$ fading by deriving the respective analytical expressions. The numerical results illustrate the effect of system parameters such as primary network arrival rate, distances from the PU to the SUs, interference power threshold of the underlay scheme, and average transmit SNR in interweave mode on the performance of the hybrid CCRN. Finally, the numerical results have also shown that the hybrid approach outperforms the conventional underlay scheme for the settings considered.

References


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ABSTRACT

Due to the rapid development of wireless communications together with the inflexibility of the current spectrum allocation policy, radio spectrum becomes more and more exhausted. One of the critical challenges of wireless communication systems is to efficiently utilize the limited frequency resources to be able to support the growing demand of high data rate wireless services. As a promising solution, cognitive radios have been suggested to deal with the scarcity and under-utilization of radio spectrum. The basic idea behind cognitive radios is to allow unlicensed users, also called secondary users (SUs), to access the licensed spectrum of primary users (PUs) which improves spectrum utilization. In order to not degrade the performance of the primary networks, SUs have to deploy interference control, interference mitigating, or interference avoidance techniques to minimize the interference incurred at the PUs. Cognitive radio networks (CRNs) have stimulated a variety of studies on improving spectrum utilization. In this context, this thesis has two main objectives. Firstly, it investigates the performance of single hop CRNs with spectrum sharing and opportunistic spectrum access. Secondly, the thesis analyzes the performance improvements of two hop cognitive radio networks when incorporating advanced radio transmission techniques.

The thesis is divided into three parts consisting of an introduction part and two research parts based on peer-reviewed publications. Fundamental background on radio propagation channels, cognitive radios, and advanced radio transmission techniques are discussed in the introduction. In the first research part, the performance of single hop CRNs is analyzed. Specifically, underlay spectrum access using M/G/1/K queueing approaches is presented in Part I-A while dynamic spectrum access with prioritized traffics is studied in Part I-B. In the second research part, the performance benefits of integrating advanced radio transmission techniques into cognitive cooperative radio networks (CCRNs) are investigated. In particular, opportunistic spectrum access for amplify-and-forward CCRNs is presented in Part II-A where collaborative spectrum sensing is deployed among the SUs to enhance the accuracy of spectrum sensing. In Part II-B, the effect of channel estimation error and feedback delay on the outage probability and symbol error rate (SER) of multiple-input multiple-output CCRNs is investigated. In Part II-C, adaptive modulation and coding is employed for decode-and-forward CCRNs to improve the spectrum efficiency and to avoid buffer overflow at the relay. Finally, a hybrid interweave-underlay spectrum access scheme for a CCRN is proposed in Part II-D. In this work, the dynamic spectrum access of the PUs and SUs is modeled as a Markov chain which then is utilized to evaluate the outage probability, SER, and outage capacity of the CCRN.