RESEARCH ARTICLE

Cognitive amplify-and-forward relay networks with beamforming under primary user power constraint over Nakagami- m fading channels

Hoc Phan*, Hans-Jürgen Zepernick, Trung Q. Duong, Hung Tran and Thi My Chinh Chu

Blekinge Institute of Technology, SE-371 79 Karlskrona, Sweden

ABSTRACT

In this paper, we analyze the performance of cognitive amplify-and-forward (AF) relay networks with beamforming under the peak interference power constraint of the primary user (PU). We focus on the scenario that beamforming is applied at the multi-antenna secondary transmitter and receiver. Also, the secondary relay network operates in channel state information-assisted AF mode, and the signals undergo independent Nakagami- m fading. In particular, closed-form expressions for the outage probability and symbol error rate (SER) of the considered network over Nakagami- m fading are presented. More importantly, asymptotic closed-form expressions for the outage probability and SER are derived. These tractable closed-form expressions for the network performance readily enable us to evaluate and examine the impact of network parameters on the system performance. Specifically, the impact of the number of antennas, the fading severity parameters, the channel mean powers, and the peak interference power is addressed. The asymptotic analysis manifests that the peak interference power constraint imposed on the secondary relay network has no effect on the diversity gain. However, the coding gain is affected by the fading parameters of the links from the primary receiver to the secondary relay network. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS

cognitive radio; beamforming; amplify-and-forward relaying; outage probability; symbol error rate; Nakagami- m fading

*Correspondence

Hoc Phan, Blekinge Institute of Technology, SE-371 79 Karlskrona, Sweden.

E-mail: hoc.phan@bth.se

1. INTRODUCTION

In recent years, studies of cognitive radio networks (CRNs), which have been known as an efficient avenue to enhance the utilization of the limited spectrum, have been well established. Among various aspects of CRNs, the combination of spectrum sharing with cooperative communications has gained increased interest in the literature [1–4]. In particular, bounds on the outage probability of a cognitive relay network using opportunistic relaying (OR) under spectrum sharing constraints were presented in [3]. In addition, the outage probability of cognitive decode-and-forward relay networks utilizing the OR strategy under interference constraints was analyzed in [1]. Taking the outage probability constraint of the primary user (PU) into account, the outage performance of cognitive relay networks using the OR scheme with partial channel state information and experiencing Rayleigh fading has

been investigated in [2]. In [4], an outage analysis of cognitive decode-and-forward relay networks employing OR and being subject to Nakagami- m fading wherein the secondary transmission does not cause harmful interference to the PU has been reported. Recently, an exact analysis for the outage performance of cognitive amplify-and-forward (AF) relay networks under the peak interference power constraint of the PU has been presented in [5].

Beamforming transmission in wireless environments provides substantial advantages such as enhancing the signal reliability, spectral efficiency, and extending the radio coverage of the wireless relay networks [6,7]. In light of this, beamforming has recently gained significant interest in the field of CRNs (see, e.g., [8–11]). Specifically, in [8], the secondary users (SUs) share the same spectrum with the PU while keeping the interference power at the PU below a given threshold. Also, power allocation for beamforming transmission has been developed for CRNs

to guarantee quality of service of the primary network, as the SUs are granted access to the licensed spectrum of the PUs [9–11]. In [9], the issue of joint beamforming and power allocation for CRNs to minimize the total transmit power while guaranteeing quality of service and the signal-to-interference-plus-noise ratio (SINR) requirements of the SUs as well as the tolerant interference of the PU are considered. In addition, the problem of minimizing the total transmit power of the beamforming CRN has been solved, provided that the received interferences at the PUs are kept below a predefined level as well as the SINR at the SU is guaranteed to be greater than a given threshold [10]. Power allocation strategies for cognitive relay networks with multiple sources and a multiple antenna relay are considered in [11], wherein the worst normalized SINR among all destinations is maximized such that total transmit powers do not exceed given thresholds.

Apart from the aforementioned studies on power allocation strategies for cognitive cooperative networks with beamforming transmission, other network performance metrics such as outage probability, symbol error rate (SER), diversity gain, and coding gain are also essential for the system design. Recently, in [12], analytical closed-form expressions for the outage probability and the SER rather than asymptotic evaluations of cognitive AF relay networks with beamforming transmission and feedback delay over Rayleigh fading have been reported. Furthermore, although most of these works consider Rayleigh channels, the more general case of Nakagami fading, covering a broad variety of fading models, has gained less attention. Inspired by all of the above, in this paper, we analyze the performance of a cognitive AF relay network with beamforming transmission subject to the peak interference power constraint of the PU. Our analysis accounts for beamforming transmission being implemented at both the multiple antenna secondary transmitter and receiver. Also, it is assumed that all channels are subject to Nakagami- m fading, and the secondary relay operates in AF mode. Our contributions are summarized as follows:

- Closed-form expressions for the outage probability and SER for a cognitive AF channel state information-assisted relay network with beamforming transmission under the peak interference power constraint at the PU are derived.
- An asymptotic performance analysis, which reveals insights into the cooperative diversity behavior of the considered network, is presented. Our results show that, subject to the peak interference power constraint of the PU, the diversity gain of the secondary network is equal to the minimum of the summation of fading severity in the two hops.
- Utilizing these closed-form expressions, we study the effect of several important network parameters on the system performance. For instance, the impact of fading severity parameter m , the number of antennas at the secondary transmitter and secondary receiver, and

the distances from the PU to the secondary transmitter and secondary relay, on the system performance is examined.

The remainder of this paper is organized as follows. The system model is introduced in Section 2. In Section 3, a closed-form expression for the outage probability is derived. A tight approximation for the SER is presented in Section 4. Furthermore, asymptotic performance is derived in Section 5. Finally, numerical results are provided in Section 6. Concluding remarks are given in Section 7.

Notation: In this paper, lowercase letters stand for vectors. The Frobenius norm of a vector is represented as $\|\cdot\|_F$. $\mathbb{E}\{\cdot\}$ denotes the expectation operator. The superscript $(\cdot)^H$ stands for the complex transpose conjugate. $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X are denoted as $f_X(\cdot)$ and $F_X(\cdot)$, respectively. $C_k^n = [n!/(n-k)!k!]$ is the binomial coefficient. $\Gamma(n)$ denotes the gamma function [13, eq. (8.310.1)], and $\Gamma(n, x)$ is the incomplete gamma function [13, eq. (8.350.2)]. $\mathcal{K}_n(\cdot)$ is the n -th order modified Bessel function of the second kind [13, eq. (8.432.1)]. $\mathcal{W}_{\lambda, \rho}(x)$ represents the Whittaker function [13, eq. (9.222)]. $U(a, b; x)$ is the confluent hypergeometric function [13, eq. (9.211.4)]. Finally, ${}_2F_1(a, b; c; x)$ stands for the Gauss hypergeometric function [13, eq. (9.100)].

2. SYSTEM AND CHANNEL MODELS

Consider a dual-hop spectrum-sharing beamforming relay network consisting of a PU and a secondary relay network as shown in Figure 1. Assume that the secondary relay network comprises of an N_1 -antenna secondary transmitter SU_{TX} , a single-antenna AF relay SU_{AF} , and an N_2 -antenna secondary receiver SU_{RX} . The licensed spectrum of PU can be utilized by SU_{TX} and SU_{AF} provided that both do not cause harmful interference to PU. In addition, all terminals operate over Nakagami- m fading channels, and the direct link from SU_{TX} to SU_{RX} is not applicable because of severe shadowing. In the first hop, SU_{TX} transmits the signal $s(t)$ to SU_{AF} with transmit power P_1 . At SU_{AF} , the received signal is given by

$$y_1(t) = \mathbf{h}_1(t)\mathbf{w}_1^H(t)s(t) + n_1(t) \quad (1)$$

where $\mathbf{w}_1(t) = \mathbf{h}_1(t)/\|\mathbf{h}_1(t)\|_F$ is the beamforming vector at SU_{TX} , $\mathbf{h}_1(t)$ is the $1 \times N_1$ channel coefficient vector of the links from SU_{TX} to SU_{AF} with identical fading severity parameter m_1 and channel mean power Ω_1 , and $n_1(t)$ is the additive white Gaussian noise at SU_{AF} with zero mean and variance N_0 . Assume that the PU is able to tolerate the peak interference power Q . To prevent PU from harmful interference, the maximum transmit power P_1 must satisfy

Figure 1. System model of the considered cognitive beamforming AF relay network [12].

$P_1 = Q/\|\mathbf{h}_3(t)\|_F^2$ where $\mathbf{h}_3(t)$ is the $1 \times N_1$ channel coefficient vector of the links from SU_{TX} to PU with identical fading severity parameter m_3 and channel mean power Ω_3 .

In the second hop, SU_{AF} amplifies its received signal $y_1(t)$ with amplifying gain β and forwards the result to SU_{RX} . Again, the transmit power of SU_{AF} , P_2 , is limited to guarantee the peak interference power constraint for PU as $P_2 = Q/|h_4(t)|^2$, where $h_4(t)$ is the channel coefficient from SU_{AF} to PU with fading severity parameter m_4 and channel mean power Ω_4 . The SU_{RX} will apply beamforming processing for the received signal to generate the signal

$$y_2(t) = \beta \mathbf{w}_2(t) \mathbf{h}_2^H(t) \mathbf{h}_1(t) \mathbf{w}_1^H(t) s(t) + \beta \mathbf{w}_2(t) \mathbf{h}_2^H(t) \mathbf{n}_1(t) + \mathbf{w}_2(t) \mathbf{n}_2^H(t) \quad (2)$$

where $\mathbf{n}_2(t)$ is an $1 \times N_2$ noise vector at SU_{RX} whose elements are complex Gaussian RVs with zero mean and variance N_0 , $\mathbf{w}_2(t) = \mathbf{h}_2(t)/\|\mathbf{h}_2(t)\|_F$ is the beamforming vector at SU_{RX} , and $\mathbf{h}_2(t)$ is the $1 \times N_2$ channel coefficient vector of the links from SU_{AF} to SU_{RX} with identical fading severity parameter m_2 and channel mean power Ω_2 . Given the power constraint at SU_{AF} , that is, $\beta^2(P_1\|\mathbf{h}_1(t)\mathbf{w}_1^H(t)\|^2 + N_0) = P_2$, and the assumption that the signal power is much greater than the noise power, the amplifying gain is selected as $\beta^2 = P_2(P_1\|\mathbf{h}_1\|_F^2)^{-1}$. From (2), we can write the instantaneous signal-to-noise ratio (SNR) as

$$\gamma = \frac{\mathbb{E}\left\{|\beta \mathbf{w}_2(t) \mathbf{h}_2^H(t) \mathbf{h}_1(t) \mathbf{w}_1^H(t) s(t)|^2\right\}}{\mathbb{E}\left\{|\beta \mathbf{w}_2(t) \mathbf{h}_2^H(t) \mathbf{n}_1(t) + \mathbf{w}_2(t) \mathbf{n}_2^H(t)|^2\right\}} \quad (3)$$

Because the noise terms $\mathbf{n}_1(t)$ and $\mathbf{n}_2(t)$ are independently distributed RVs, (3) can be rewritten as

$$\gamma = \frac{\beta^2 \mathbb{E}\{|s(t)|^2\} \|\mathbf{w}_2(t) \mathbf{h}_2^H(t) \mathbf{h}_1(t) \mathbf{w}_1^H(t)\|^2}{\beta^2 \mathbb{E}\{|\mathbf{n}_1(t)|^2\} \|\mathbf{w}_2(t) \mathbf{h}_2^H(t)\|^2 + \mathbb{E}\{|\mathbf{w}_2(t) \mathbf{n}_2^H(t)|^2\}} \quad (4)$$

Note that

$$\begin{aligned} \mathbb{E}\left\{|\mathbf{w}_2(t) \mathbf{n}_2^H(t)|^2\right\} &= \mathbf{w}_2(t) \mathbb{E}\left\{\mathbf{n}_2^H(t) \mathbf{n}_2(t)\right\} \mathbf{w}_2^H(t) \\ &= N_0 \mathbf{w}_2(t) \mathbf{I}_{N_2} \mathbf{w}_2^H(t) = N_0 \end{aligned} \quad (5)$$

Substituting (5) into (4) with some simplifications, we get

$$\gamma = \frac{\beta^2 P_1 \|\mathbf{h}_1(t)\|_F^2 \|\mathbf{h}_2(t)\|_F^2}{\beta^2 N_0 \|\mathbf{h}_2(t)\|_F^2 + N_0} \quad (6)$$

Next, substituting $\beta^2 = P_2(P_1\|\mathbf{h}_1\|_F^2)^{-1}$, $P_1 = Q/\|\mathbf{h}_3(t)\|_F^2$, and $P_2 = Q/|h_4(t)|^2$ into (6), after some manipulations, the instantaneous SNR at SU_{RX} can be formulated as

$$\gamma = \rho \frac{X_1 X_2}{X_1 X_4 + X_2 X_3} \quad (7)$$

where $\rho = Q/N_0$, $X_k = \|\mathbf{h}_k\|_F^2$, $k = 1, 2, 3$, and $X_4 = |h_4|^2$. Because X_1 , X_2 , and X_3 are the summation of independently and identically gamma distributed RVs, they are also gamma distributed RVs with respective parameters $(N_1 m_1, \alpha_1^{-1})$, $(N_2 m_2, \alpha_2^{-1})$, and $(N_1 m_3, \alpha_3^{-1})$, where $\alpha_k = m_k/\Omega_k$. In addition, X_4 is a gamma distributed RV with parameters (m_4, α_4^{-1}) , where $\alpha_4 = m_4/\Omega_4$. In the sequel, analytical and asymptotic closed-form expressions for outage probability and SER are derived.

3. OUTAGE PERFORMANCE ANALYSIS

In this section, the CDF of the instantaneous SNR is derived, which enables us to obtain a closed-form expression for the outage probability. For further calculations, we require the PDF and CDF of a gamma distributed RV X with parameters (m, α^{-1}) , m is a positive integer number, which are given by

$$f_X(x) = \frac{\alpha^m}{\Gamma(m)} x^{m-1} \exp(-\alpha x) \quad (8)$$

$$F_X(x) = 1 - \exp(-\alpha x) \sum_{q=0}^{m-1} \frac{\alpha^q x^q}{q!} \quad (9)$$

It is noted that the instantaneous SNR in (7) is represented as a complicated function of multiple RVs, that is, $\{X_k\}_{k=1}^4$. Therefore, we utilize the total probability theorem to obtain the statistical distribution of the SNR. For convenience, in (7), we define an intermediate RV γ_0 as

$$\gamma_0 = \frac{X_1 X_2}{X_1 X_4 + X_2 X_3} = \frac{\gamma}{\rho} \quad (10)$$

Hence, a closed-form expression for $F_\gamma(\gamma)$ can be found via the CDF of γ_0 given in (10) as $F_\gamma(\gamma) = F_{\gamma_0}(\gamma/\rho)$. From (7), we can express the CDF of γ_0 as

$$F_{\gamma_0}(\gamma) = P\{\gamma_0 < \gamma\} = \int \int \int \int_{\mathcal{D}} f_{X_1 X_2 X_3 X_4}(x_1, x_2, x_3, x_4) dx_1 dx_2 dx_3 dx_4 \quad (11)$$

where $\mathcal{D} = \{(x_1, x_2, x_3, x_4) | x_1 x_2 / (x_1 x_4 + x_2 x_3) < \gamma\}$ represents the integration domain. Because of the statistical independence of RVs $X_k, k \in \{1, \dots, 4\}$, the joint PDF $f_{X_1 X_2 X_3 X_4}(x_1, x_2, x_3, x_4)$ can be given in terms of the product of the PDF of each RV, $f_{X_k}(x_k)$. Furthermore, using the total probability theorem, we have [12]

$$F_{\gamma_0}(\gamma) = \int_0^\infty \int_0^\infty \int_0^\infty P\left\{\frac{X_1 x_2}{X_1 x_4 + x_2 x_3} < \gamma\right\} f_{X_2}(x_2) f_{X_3}(x_3) f_{X_4}(x_4) dx_2 dx_3 dx_4 \quad (12)$$

Theorem 1. Assuming that the fading severity parameters $m_k, k \in \{1, 2, 3, 4\}$, are integers, the CDF of the instantaneous SNR γ in (7) can be formulated in closed-form as

$$P_e = \frac{a\sqrt{b}\rho^{\frac{1}{2}}}{2\sqrt{\pi}} \int_0^\infty F_{\gamma_0}\left(\frac{\gamma}{\rho}\right) \gamma^{-\frac{1}{2}} \exp(-b\rho\gamma) d\gamma \quad (15)$$

where a and b are modulation parameters. In (15), it can be seen that it is not feasible to derive the SER directly from the CDF of the instantaneous SNR, $\gamma = \rho\gamma_0$.

$$\begin{aligned} F_\gamma(\gamma) = 1 - & \sum_{p=0}^{N_1 m_1 - 1} \frac{\alpha_1^p}{p!} \sum_{q=0}^p C_q^p \frac{\alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2)} \frac{\alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{r=0}^{N_2 m_2 - 1} C_r^{N_2 m_2 - 1} \alpha_1^{r-q+1} \\ & \times \Gamma(N_1 m_3 + p + r - q + 1) \Gamma(N_1 m_3 + p) \frac{\Gamma(N_2 m_2 + m_4) \Gamma(N_2 m_2 + m_4 - r + q - 1)}{\Gamma(N_2 m_2 + m_4 + N_1 m_3 + p)} \\ & \times \left(\frac{\alpha_1 \gamma}{\rho} + \alpha_3\right)^{-(N_1 m_3 + p + r - q + 1)} \left(\alpha_4 + \frac{\alpha_2 \gamma}{\rho}\right)^{-(N_2 m_2 + m_4)} \left(\frac{\gamma}{\rho}\right)^{N_2 m_2 + p + r - q + 1} \\ & \times {}_2F_1(N_2 m_2 + m_4, N_1 m_3 + p + r - q + 1, N_2 m_2 + m_4 + N_1 m_3 + p; \varphi(\gamma)) \end{aligned} \quad (13)$$

where $\varphi(\gamma) = [(\alpha_2 \alpha_3 + \alpha_1 \alpha_4) \gamma / \rho + \alpha_3 \alpha_4] / [(\alpha_1 \gamma / \rho + \alpha_3)(\alpha_2 \gamma / \rho + \alpha_4)]$

Proof. See Appendix A. $\square$

Therefore, the outage probability, P_{out} , which is the probability that the instantaneous SNR falls below a pre-defined threshold γ_{th} , can be readily obtained as

$$P_{\text{out}} = F_\gamma(\gamma_{\text{th}}) \quad (14)$$

Note that the closed-form expression for the outage probability in (14) can be particularized into the case of Rayleigh fading ($m = 1$) and (or) the conventional single-antenna cognitive AF relay network ($N_1 = N_2 = 1$).

4. SYMBOL ERROR RATE PERFORMANCE ANALYSIS

In this section, the error performance of the considered network is investigated by deriving a tightly approximated expression for the SER. Specifically, the expression for the SER that is valid for binary phase shift keying or quadrature phase shift keying, binary frequency shift keying, and M -ary phase shift keying is evaluated as [14]

In fact, the moment generating function (MGF) of the reciprocal of the instantaneous SNR can be utilized to evaluate the exact error probability performance in the considered context. This approach has been well studied recently [15–17]. Typically, the error probability can be given in the form of a single-integral expression, for example, [16, eq. (4c); 17, eq. (3)], which can be computed numerically. Similarly, for the considered system, the MGF of the reciprocal of the instantaneous SNR given in (7) can be derived. However, the SER expression may be given in integral form. To overcome this limitation, the CDF of the upper bound on γ_0 , denoted as γ_{0U} , is therefore utilized for the analysis. In particular, this bound is given by [18, eq. (25)]

$$\gamma_{0U} = \min(\gamma_1, \gamma_2) \quad (16)$$

where $\gamma_1 = X_2/X_4$ and $\gamma_2 = X_1/X_3$. It has been known that the bounded SER is very tight as compared with the exact SER in the low and average SNR regimes and converges asymptotically to the exact SER. We then proceed by deriving the CDF of γ_{0U} , which can be calculated as given in the following theorem.

Theorem 2. The CDF of the RV γ_{0U} is given by

$$F_{\gamma_{0U}}(\gamma) = 1 - \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{N_2 m_2 - 1} \frac{\Gamma(m_4 + q) \alpha_2^q \alpha_3^{N_1 m_3}}{q! \Gamma(N_1 m_3)} \\ \times \sum_{r=0}^{N_1 m_1 - 1} \frac{\Gamma(N_1 m_3 + r) \alpha_1^r}{r!} \\ \times \frac{\gamma^{q+r}}{(\alpha_1 \gamma + \alpha_3)^{N_1 m_3 + r} (\alpha_2 \gamma + \alpha_4)^{m_4 + q}} \quad (17)$$

Proof. See Appendix B. $\square$

With the CDF of γ_{0U} at hand, we now obtain a closed-form expression for the SER as in Corollary 1.

Corollary 1. A closed-form expression for the SER is derived as

$$P_e = \frac{a}{2} - \frac{a\sqrt{b}}{2\sqrt{\pi}} \rho^{\frac{1}{2}} \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{N_2 m_2 - 1} \frac{\Gamma(m_4 + q) \alpha_2^q}{q!} \frac{\alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \sum_{r=0}^{N_1 m_1 - 1} \frac{\Gamma(N_1 m_3 + r) \alpha_1^r}{r!} \frac{1}{\alpha_1^{N_1 m_3 + r} \alpha_2^{m_4 + q}} \\ \times \left[\sum_{i=1}^{N_1 m_3 + r} \kappa_{ri} \Gamma\left(q + r + \frac{1}{2}\right) \left(\frac{\alpha_3}{\alpha_1}\right)^{q+r-i+\frac{1}{2}} U\left(q + r + \frac{1}{2}, q + r - i + \frac{3}{2}, b\rho \frac{\alpha_3}{\alpha_1}\right) \right. \\ \left. + \sum_{i=1}^{m_4 + q} \theta_{qi} \Gamma\left(q + r + \frac{1}{2}\right) \left(\frac{\alpha_4}{\alpha_2}\right)^{q+r-i+\frac{1}{2}} U\left(q + r + \frac{1}{2}, q + r - i + \frac{3}{2}, b\rho \frac{\alpha_4}{\alpha_2}\right) \right] \quad (18)$$

Proof. See Appendix C. $\square$

5. ASYMPTOTIC PERFORMANCE ANALYSIS

Although the closed-form expression given in (13) can be used to obtain exact outage performance for the considered network, it is difficult to examine the impact of network parameters on the system performance as well as to identify the diversity and coding gains because of its complicated mathematical representation. To render insights into the spatial diversity, an asymptotic outage probability is provided in this section.

5.1. Asymptotic outage probability

By definition, the expression for the outage probability at high SNR is represented as [19]

$$P_{\text{out}}^{\infty} = (G_c \rho)^{-G_d} + o(\rho^{-G_d}) \quad (19)$$

where G_d and G_c correspond to the diversity and coding gains and $o(g(x))$ denotes the higher order terms of $g(x)$, that is, $\lim_{x \rightarrow x_0} o(x)/g(x) = 0$. In (19), the diversity gain G_d is the absolute value of the slope of the asymptotic outage probability curve as

$$G_d \triangleq \lim_{\rho \rightarrow \infty} \left(-\frac{\log(P_{\text{out}})}{\log(\rho)} \right) \quad (20)$$

The coding gain of the network G_c is defined as the SNR advantage of the asymptotic outage probability curve as compared with the reference curve ρ^{-G_d} . Basically,

the MacLaurin expansion of the CDF of the instantaneous SNR $F_{\gamma}(\gamma)$ at zero is employed to analyze the asymptotic performance. However, it is not feasible to obtain the asymptotic performance from $F_{\gamma}(\gamma)$ because of its complex mathematical representation. Thus, we apply [20, Lemma 1], which concludes that the CDF of RV γ_{0U} owns the same first nonzero coefficient of MacLaurin series as the CDF of γ_0 . The asymptotic outage probability can be obtained as in the following theorem.

Theorem 3. Assuming that $\rho \rightarrow \infty$, the asymptotic expression for the outage probability of the considered network is given by

$$P_{\text{out}}^{\infty} = \begin{cases} \frac{\Gamma(N_1 m_1 + N_1 m_3)}{\Gamma(N_1 m_1 + 1) \Gamma(N_1 m_3)} \frac{\alpha_1^{N_1 m_1}}{\alpha_3^{N_1 m_1}} \left(\frac{\gamma_{th}}{\rho} \right)^{N_1 m_1}, & N_1 m_1 < N_2 m_2 \\ \frac{\Gamma(N_2 m_2 + m_4)}{\Gamma(N_2 m_2 + 1) \Gamma(m_4)} \frac{\alpha_2^{N_2 m_2}}{\alpha_4^{N_2 m_2}} \left(\frac{\gamma_{th}}{\rho} \right)^{N_2 m_2}, & N_1 m_1 > N_2 m_2 \\ \frac{\Gamma(N_1 m_1 + N_1 m_3)}{\Gamma(N_1 m_1 + 1) \Gamma(N_1 m_3)} \frac{\alpha_1^{N_1 m_1}}{\alpha_3^{N_1 m_1}} \left(\frac{\gamma_{th}}{\rho} \right)^{N_1 m_1} + \frac{\Gamma(N_2 m_2 + m_4)}{\Gamma(N_2 m_2 + 1) \Gamma(m_4)} \frac{\alpha_2^{N_2 m_2}}{\alpha_4^{N_2 m_2}} \left(\frac{\gamma_{th}}{\rho} \right)^{N_2 m_2}, & N_1 m_1 = N_2 m_2 \end{cases} \quad (21)$$

Proof. See Appendix D. $\square$

Clearly, from the three cases given in (21), we can see that the diversity gain of the considered network is dominated by the more severe hop of the secondary relay network, that is, $G_d = \min(N_1 m_1, N_2 m_2)$. This means that the presence of the peak interference power constraint of the PU imposed on the secondary relay network does not affect its diversity gain.

5.2. Asymptotic symbol error rate

Corollary 2. Assuming that $\rho \rightarrow \infty$, the asymptotic expression for the SER of the considered network is obtained as

$$P_E^\infty = \begin{cases} \frac{a}{2\sqrt{\pi}b^{N_1 m_1} \rho^{N_1 m_1}} \frac{\Gamma(N_1 m_1 + N_1 m_3) \Gamma(N_1 m_1 + \frac{1}{2}) \alpha_1^{N_1 m_1}}{\Gamma(N_1 m_1 + 1) \Gamma(N_1 m_3) \alpha_3^{N_1 m_1}}, & N_1 m_1 < N_2 m_2 \\ \frac{a}{2\sqrt{\pi}b^{N_2 m_2} \rho^{N_2 m_2}} \frac{\Gamma(N_2 m_2 + m_4) \Gamma(N_2 m_2 + \frac{1}{2}) \alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2 + 1) \Gamma(m_4) \alpha_4^{N_2 m_2}}, & N_1 m_1 > N_2 m_2 \\ \frac{a}{2\sqrt{\pi}b^{N_1 m_1} \rho^{N_1 m_1}} \frac{\Gamma(N_1 m_1 + N_1 m_3) \Gamma(N_1 m_1 + \frac{1}{2}) \alpha_1^{N_1 m_1}}{\Gamma(N_1 m_1 + 1) \Gamma(N_1 m_3) \alpha_3^{N_1 m_1}} \\ + \frac{a}{2\sqrt{\pi}b^{N_2 m_2} \rho^{N_2 m_2}} \frac{\Gamma(N_2 m_2 + m_4) \Gamma(N_2 m_2 + \frac{1}{2}) \alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2 + 1) \Gamma(m_4) \alpha_4^{N_2 m_2}}, & N_1 m_1 = N_2 m_2 \end{cases} \quad (22)$$

Proof. Substituting (D.10) of Appendix C in (15) followed by utilizing [13, eq. (3.381.4)], we obtain the above asymptotic expression for the SER as in (22). $\square$

The closed-form expression given in (22) indicates that the behavior of the asymptotic SER is similar to that of the asymptotic outage probability given in (21). Only the number of antennas at S_{UTX} N_1 and at S_{URX} N_2 along with the fading severity parameters of the two hops of the secondary relay network, m_1 and m_2 , determine the diversity gain.

6. NUMERICAL RESULTS

In this section, numerical results are provided to validate the developed analysis for example scenarios. For brevity, we denote d_1 , d_2 , d_3 , and d_4 as the distances for the links $S_{UTX} \rightarrow S_{UAF}$, $S_{UAF} \rightarrow S_{URX}$, $S_{UTX} \rightarrow PU$, and $S_{UAF} \rightarrow PU$, respectively. Assume that the channel mean power between two arbitrary terminals A and B is attenuated according to the exponential path loss model as d_i^{-L} , $i \in \{1, 2, 3, 4\}$ with $A \in \{S_{UTX}, S_{UAF}\}$ and $B \in \{S_{UAF}, S_{URX}, PU\}$. The path loss exponent is selected as $L = 4$, which is applicable to a highly shadowed urban area. The outage threshold value is set as

$\gamma_{th} = 10$ dB. For all the plots, it can be seen that excellent agreement between analytical results and simulations is achieved. Also, it can be observed that the asymptotic results tightly converge to the analytical results.

Figure 2 shows the outage probability for four different cases of fading channels:

- $\{m_1, m_2\} = \{1, 1\}$: Rayleigh channels.
- $\{m_1, m_2\} = \{2, 2\}$: Fading of the two relay hops is identically severe.
- $\{m_1, m_2\} = \{2, 3\}$: Fading of the first relay hop is more severe than the second relay hop.
- $\{m_1, m_2\} = \{4, 3\}$: Fading of the first relay hop is less severe than the second relay hop.

The number of antennas at S_{UTX} and S_{URX} is set as $N_1 = 2$ and $N_2 = 2$, respectively. The diversity gains as observed in Figure 2 agree with analytical results for the four different scenarios of fading severity relationship in the two hops. As expected, the outage performance will be

Figure 2. Outage probability versus peak interference power-to-noise ratio Q/N_0 for different fading channels.

Figure 3. Outage probability versus peak interference power-to-noise ratio Q/N_0 for different number of antennas at SU_{TX} and SU_{RX} , N_1 and N_2 , respectively.

improved prominently as the minimum value of the fading severities between the two hops, $\min(m_1, m_2)$, increases. This can be deduced from (21), which shows that the diversity gain proportionally increases with $\min(m_1, m_2)$ when the number of antennas at the SU_{TX} and SU_{RX} is identical, $N_1 = N_2$.

Figure 3 depicts the outage probability for different number of antennas at SU_{TX} and SU_{RX} . Pronounced improvement in the outage probability can be achieved as the minimum of the numbers of antennas at SU_{TX} and SU_{RX} , $\min(N_1, N_2)$, increases. This benefit can be attributed to the fact that the diversity gain is proportional to $\min(N_1, N_2)$ for the case of identical fading severity parameters of the two relaying hops, that is, $m_1 = m_2$, as given in (21). In all cases, the diversity gains in Figure 3 agree with the analytical derivations for different cases of fading severity relationship between the two hops.

Figure 4 shows the outage performance for various distances from PU to SU_{TX} and from PU to SU_{AF} , that is, d_3 and d_4 , respectively. As can be seen from the figure, an increase of the distances d_3 and d_4 leads to an improvement in outage probability. This advantage can be explained by the fact that as the PU moves far away from the secondary relay network, the limit of the transmit power of SU_{TX} and SU_{AF} can be increased.

Figure 5 presents the outage probability versus the distance ratio d_2/d_1 for different values of peak interference power Q . For a fixed value of the distance between SU_{TX} and SU_{AF} , d_1 , and $d_2 \leq d_1$, as the distance between SU_{AF} and SU_{RX} , d_2 , decreases, the outage probability almost remains constant. This indicates that when fading condition of the second relaying hop is better than that of the first relaying hop, improvement in fading power at the second hop slightly improves the system performance. For the case of $d_2 \geq d_1$, that is, fading condition of the first relaying hop is more favorable than that of the second relaying

Figure 4. Outage probability for different distances from PU to SU_{TX} and from PU to SU_{AF} , d_3 and d_4 , respectively.

Figure 5. Outage probability versus the ratio of distances from SU_{AF} to SU_{RX} and from SU_{AF} to SU_{TX} , d_2/d_1 , for different values of peak interference power Q .

hop, the outage performance diminishes when fading condition of the second relaying hop becomes worse, that is, d_2 increases. This can be understood from the fact that the worse fading condition of the second relaying hop relative to that of the first hop may lead to this performance degradation.

Figure 6 draws the SER of 8-PSK modulation for different fading channels. It can be seen from Figure 6 that the SER will decrease significantly when the minimum value of the fading severity parameters of the two relaying hops, $\min(m_1, m_2)$, increases. This improvement in the SER performance can be understood from the fact that the diversity gain is proportional to $\min(m_1, m_2)$ when the number of antennas at SU_{TX} and SU_{RX} is identical, $N_1 = N_2$, according to (22).

Figure 7 displays the SER of 8-PSK modulation for different number of antennas at SU_{TX} , N_1 , and at SU_{RX} , N_2 .

Figure 6. SER versus peak interference power-to-noise ratio Q/N_0 for different fading severity conditions.

Figure 7. SER for different number of antennas at SU_{TX} and SU_{RX} , N_1 and N_2 , respectively.

The fading severity parameters of the two relaying hops are fixed as $m_1 = m_2 = 2$. As expected, the SER performance of the considered network obtains substantial enhancement when the minimum of the number of antennas at SU_{TX} and SU_{RX} , $\min(N_1, N_2)$, increases. Theoretically, in this case, the diversity gain of the considered network is proportional to the value $\min(N_1, N_2)$ as can be seen from (22).

Figure 8 draws the SER for different distances from PU to SU_{TX} and from PU to SU_{AF} , that is, d_3 and d_4 , respectively. It can be seen that when PU is placed far away from SU_{TX} and SU_{AF} , that is, d_3 and d_4 increase, the outage probability is enhanced. This benefit results from the fact that the constraint of transmit power of SU_{TX} and SU_{AF} becomes more relaxed.

Figure 9 depicts the SER versus the distance ratio d_2/d_1 for different values of the peak interference power Q . Again, the same characteristic of the performance behavior as for the case of the outage probability is observed.

Figure 8. SER versus peak interference power-to-noise ratio Q/N_0 for different distances from PU to SU_{TX} and from PU to SU_{AF} .

Figure 9. SER versus the ratio of distances from SU_{AF} to SU_{RX} and from SU_{AF} to SU_{TX} , d_2/d_1 , for different values of peak interference power Q .

For each curve, the shifted position between the two floors occurs when the distances from SU_{AF} to SU_{TX} and from SU_{AF} to SU_{RX} are almost the same. High difference of the values of d_1 and d_2 leads to serious unbalance of fading conditions between the two hops of the secondary relay network.

7. CONCLUSIONS

In this paper, we have studied the performance of cognitive beamforming AF relay networks under the peak interference power constraint of the PU. In particular, a closed-form expression for the outage probability and a tight bounded closed-form expression for the SER of the considered network over Nakagami- m fading has been derived. Furthermore, an asymptotic analysis, which provides im-

portant insights into the diversity behavior, has been presented. These closed-form expressions can be utilized to evaluate the performance of cognitive beamforming AF

Noting that $P\{X_1 > [(\gamma x_2 x_3)/(x_2 - \gamma x_4)]\} = 1$, and after several algebraic manipulations, the CDF $F_{\gamma_0}(\gamma)$ can be rewritten as

$$F_{\gamma_0}(\gamma) = \underbrace{\int_0^\infty \int_0^\infty \int_0^{\gamma x_4} f_{X_2}(x_2) f_{X_3}(x_3) f_{X_4}(x_4) dx_2 dx_3 dx_4}_{I} + \underbrace{\int_0^\infty \int_0^\infty \int_0^\infty F_{X_1}(\varphi) f_{X_2}(x_2 + \gamma x_4) f_{X_3}(x_3) f_{X_4}(x_4) dx_2 dx_3 dx_4}_{J} \quad (A.2)$$

relay networks as well as investigate the impact of several important network parameters on the system performance.

where $\varphi = [\gamma(x_2 + \gamma x_4)x_3]/x_2$. It is noted that the first triple integral I in (A.2) can be simplified to the product of three integrals for each variable as

$$I = \int_0^\infty f_{X_3}(x_3) dx_3 \int_0^\infty \left(\int_0^{\gamma x_4} f_{X_2}(x_2) dx_2 \right) f_{X_4}(x_4) dx_4 = \int_0^\infty F_{X_2}(\gamma x_4) f_{X_4}(x_4) dx_4 \quad (A.3)$$

Our analysis can be particularized to the case of conventional cognitive AF relay systems with single-antenna terminals as well as to the case of Rayleigh fading. Finally, using the MGF approach for the reciprocal of the instantaneous SNR, the exact performance in terms of SER can be analyzed for future works.

For notational simplicity in (A.2), let us denote

$$J_1 = \int_0^\infty F_{X_1}(\varphi) f_{X_2}(x_2 + \gamma x_4) dx_2 \quad (A.4)$$

$$J_2 = \int_0^\infty J_1 f_{X_3}(x_3) dx_3 \quad (A.5)$$

$$J = \int_0^\infty J_2 f_{X_4}(x_4) dx_4 \quad (A.6)$$

APPENDIX A: PROOF OF THEOREM 1

In this appendix, we find a closed-form expression for the CDF of γ_0 , $F_{\gamma_0}(\gamma)$, given in (12) in order to identify the CDF of the instantaneous SNR $F_\gamma(\gamma)$. To derive the CDF $F_{\gamma_0}(\gamma)$ given in (12), we adopt the similar approach as in [12]. Specifically, the integration domain can be sepa-

To compute $F_{\gamma_0}(\gamma)$, the integral J_1 is first simplified by substituting the CDF of X_1 and the PDF of X_2 into (A.4), using the binomial theorem, that is, $(a + b)^n = \sum_{k=0}^n C_k^n a^k b^{n-k}$, and followed by some algebraic manipulations, resulting in

$$J_1 = 1 - F_{X_2}(\gamma x_4) - \sum_{p=0}^{N_1 m_1 - 1} \frac{\alpha_1^p}{p!} \sum_{q=0}^p \frac{C_q^p \alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2)} \sum_{r=0}^{N_2 m_2 - 1} C_r^{N_2 m_2 - 1} \gamma^{N_2 m_2 + p + q - r - 1} \times x_3^p x_4^{N_2 m_2 + q - r - 1} \exp(-\alpha_1 \gamma x_3) \exp(-\alpha_2 \gamma x_4) \int_0^\infty x_2^{r-q} \exp\left(-\alpha_2 x_2 - \frac{\alpha_1 \gamma^2 x_3 x_4}{x_2}\right) dx_2 \quad (A.7)$$

rated into two subsets as $X_1 > \frac{\gamma x_2 x_3}{x_2 - \gamma x_4}$ if $x_2 < \gamma x_4$ and $X_1 < \frac{\gamma x_2 x_3}{x_2 - \gamma x_4}$ if $x_2 > \gamma x_4$, which results in

Utilizing [13, eq. (3.471.9)] to solve the remaining integral in (A.7), it is straightforward to show that J_1 in (A.7) can be simplified to

$$F_{\gamma_0}(\gamma) = \int_0^\infty \int_0^\infty \int_0^{\gamma x_4} P\left\{X_1 > \frac{\gamma x_2 x_3}{x_2 - \gamma x_4}\right\} f_{X_2}(x_2) f_{X_3}(x_3) f_{X_4}(x_4) dx_2 dx_3 dx_4 + \int_0^\infty \int_0^\infty \int_{\gamma x_4}^\infty P\left\{X_1 < \frac{\gamma x_2 x_3}{x_2 - \gamma x_4}\right\} f_{X_2}(x_2) f_{X_3}(x_3) f_{X_4}(x_4) dx_2 dx_3 dx_4 \quad (A.1)$$

$$J_1 = 1 - F_{X_2}(\gamma x_4) - 2 \sum_{p=0}^{N_1 m_1 - 1} \frac{\alpha_1^p}{p!} \sum_{q=0}^p C_q^p \frac{\alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2)} \sum_{r=0}^{N_2 m_2 - 1} C_r^{N_2 m_2 - 1} \left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{r-q+1}{2}} \\ \times \gamma^{N_2 m_2 + p} x_4^{\frac{2N_2 m_2 + q - r - 1}{2}} x_3^{\frac{2p + r - q + 1}{2}} \exp(-\alpha_1 \gamma x_3 - \alpha_2 \gamma x_4) \mathcal{K}_{r-q+1}(2\gamma \sqrt{\alpha_1 \alpha_2 x_4 x_3}) \quad (\text{A.8})$$

Then, substituting the expression of J_1 given in (A.8) and the PDF of X_3 into J_2 given in (A.5), rearranging terms, and performing some manipulations leads to the expression for J_2 as

$$J_2 = 1 - F_{X_2}(\gamma x_4) - 2 \sum_{p=0}^{N_1 m_1 - 1} \frac{\alpha_1^p}{p!} \sum_{q=0}^p \frac{C_q^p \alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2)} \sum_{r=0}^{N_2 m_2 - 1} \frac{C_r^{N_2 m_2 - 1} \alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \\ \times \left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{r-q+1}{2}} \gamma^{N_2 m_2 + p} x_4^{\frac{2N_2 m_2 + q - r - 1}{2}} \exp(-\alpha_2 \gamma x_4) \\ \times \int_0^\infty x_3^{\frac{2N_1 m_3 + 2p + r - q + 1}{2}} \exp[-(\alpha_1 \gamma + \alpha_3)x_3] \mathcal{K}_{r-q+1}(2\gamma \sqrt{\alpha_1 \alpha_2 x_4 x_3}) dx_3 \quad (\text{A.9})$$

The remaining integral in (A.9) is solved by applying [13, eq. (6.643.3)] together with some simplifications, which results in

$$J_2 = 1 - F_{X_2}(\gamma x_4) - \sum_{p=0}^{N_1 m_1 - 1} \frac{\alpha_1^p}{p!} \sum_{q=0}^p \frac{C_q^p \alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2)} \sum_{r=0}^{N_2 m_2 - 1} \frac{C_r^{N_2 m_2 - 1} \Gamma(N_1 m_3 + p + r - q + 1) \alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \\ \times \Gamma(N_1 m_3 + p) \left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{r-q+1}{2}} (\alpha_1 \gamma + \alpha_3)^{-\frac{2N_1 m_3 + 2p + r - q}{2}} \frac{\gamma^{N_2 m_2 + p - 1}}{\sqrt{\alpha_1 \alpha_2}} x_4^{\frac{2N_2 m_2 - r + q - 2}{2}} \\ \times \exp\left(-\frac{2\alpha_2 \alpha_3 \gamma + \alpha_1 \alpha_2 \gamma^2}{2(\alpha_1 \gamma + \alpha_3)} x_4\right) \mathcal{W}_{-\frac{2N_1 m_3 + 2p + r - q}{2}, \frac{r-q+1}{2}}\left(\frac{\alpha_1 \alpha_2 \gamma^2 x_4}{\alpha_1 \gamma + \alpha_3}\right) \quad (\text{A.10})$$

To obtain a closed-form expression for the integral J , the closed-form expression for the integral J_2 given in (A.10) is then substituted into (A.6). Rearranging terms as well as performing several algebraic manipulations, the integral J can be rewritten as

$$J = 1 - \int_0^\infty F_{X_2}(\gamma x_4) f_{X_4}(x_4) dx_4 - \sum_{p=0}^{N_1 m_1 - 1} \frac{\alpha_1^p}{p!} \sum_{q=0}^p C_q^p \frac{\alpha_2^{N_2 m_2}}{\Gamma(N_2 m_2)} \frac{\alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \\ \times \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{r=0}^{N_2 m_2 - 1} C_r^{N_2 m_2 - 1} \left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{r-q+1}{2}} \frac{1}{\sqrt{\alpha_1 \alpha_2}} \int_0^\infty x_4^{\frac{2N_2 m_2 + 2m_4 - r + q - 4}{2}} \\ \times \exp\left[-\left(\alpha_4 + \frac{2\alpha_2 \alpha_3 \gamma + \alpha_1 \alpha_2 \gamma^2}{2(\alpha_1 \gamma + \alpha_3)}\right) x_4\right] \mathcal{W}_{-\frac{2N_1 m_3 + 2p + r - q}{2}, \frac{r-q+1}{2}}\left(\frac{\alpha_1 \alpha_2 \gamma^2 x_4}{\alpha_1 \gamma + \alpha_3}\right) dx_4 \quad (\text{A.11})$$

Eventually, adding the first integral I in (A.3) and the second integral J after simplifying the remaining integral in (A.11) in view of [13, eq. (7.621.3)] and using the relationship $F_\gamma(\gamma) = F_{\gamma_0}(\gamma/\rho)$, the closed-form expression for $F_\gamma(\gamma)$ is obtained.

APPENDIX B: PROOF OF THEOREM 2

From the order statistics theory, the CDF of γ_{0U} can be determined as

$$F_{\gamma_{0U}}(\gamma) = 1 - [1 - F_{\gamma_1}(\gamma)][1 - F_{\gamma_2}(\gamma)] \quad (\text{B.1})$$

Hence, we first derive an expression for the CDF of γ_1 , which can be written as

$$F_{\gamma_1}(\gamma) = \Pr \left\{ \frac{X_2}{X_4} < \gamma \right\} = \int_0^\infty F_{X_2}(\gamma x_4) f_{X_4}(x_4) dx_4 \quad (\text{B.2})$$

Substituting the CDF of X_2 and the PDF of X_4 into (D.1) gives

In a similar treatment of deriving the CDF of γ_1 , the CDF of γ_2 , $F_{\gamma_2}(\gamma)$, can be written as

$$F_{\gamma_2}(\gamma) = 1 - \frac{\alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \sum_{r=0}^{N_1 m_1 - 1} \frac{\Gamma(N_1 m_3 + r) \alpha_1^r \gamma^r}{r! (\alpha_1 \gamma + \alpha_3)^{N_1 m_3 + r}} \quad (\text{B.5})$$

Substituting (B.4) and (B.5) in (B.1), the proof is completed.

$$F_{\gamma_1}(\gamma) = 1 - \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{N_2 m_2 - 1} \frac{\alpha_2^q \gamma^q}{q!} \int_0^\infty x_4^{m_4 + q - 1} \exp(-\alpha_2 \gamma x_4 - \alpha_4 x_4) dx_4 \quad (\text{B.3})$$

Applying [13, eq. (3.381.4)] to solve the remaining integral in (D.1), it follows that

$$F_{\gamma_1}(\gamma) = 1 - \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{N_2 m_2 - 1} \frac{\Gamma(m_4 + q)}{q!} \times \frac{\alpha_2^q \gamma^q}{(\alpha_2 \gamma + \alpha_4)^{m_4 + q}} \quad (\text{B.4})$$

APPENDIX C: PROOF OF COROLLARY 1

To prove Corollary 1, we substitute the closed-form expression for the CDF of γ_{0U} given in (17) into (15), followed by rearranging terms and performing algebraic manipulations. Then, the SER can be expressed as

$$P_e = \frac{a\sqrt{b}}{2\sqrt{\pi}} \rho^{\frac{1}{2}} \int_0^\infty \gamma^{-\frac{1}{2}} \exp(-b\rho\gamma) d\gamma - \frac{a\sqrt{b}}{2\sqrt{\pi}} \rho^{\frac{1}{2}} \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{N_2 m_2 - 1} \frac{\Gamma(m_4 + q) \alpha_2^q}{q!} \frac{\alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \times \sum_{r=0}^{N_1 m_1 - 1} \frac{\Gamma(N_1 m_3 + r) \alpha_1^r}{r!} \int_0^\infty \frac{\gamma^{q+r-\frac{1}{2}}}{(\alpha_1 \gamma + \alpha_3)^{N_1 m_3 + r} (\alpha_2 \gamma + \alpha_4)^{m_4 + q}} \exp(-b\rho\gamma) d\gamma \quad (\text{C.1})$$

Clearly, the second remaining integral in (C.1) is not given in tabulated form. Therefore, to solve this integral, we utilize the partial fraction [13, eq. (3.326.2)] as follows:

$$\frac{\gamma^{q+r-\frac{1}{2}}}{(\gamma + \alpha_3 \alpha_1^{-1})^{N_1 m_3 + r} (\gamma + \alpha_4 \alpha_2^{-1})^{m_4 + q}} = \frac{1}{\alpha_1^{N_1 m_3 + r} \alpha_2^{m_4 + q}} \left(\sum_{i=1}^{N_1 m_3 + r} \frac{\kappa_{ri} \gamma^{q+r-\frac{1}{2}}}{(\gamma + \alpha_3 \alpha_1^{-1})^i} + \sum_{i=1}^{m_4 + q} \frac{\theta_{qi} \gamma^{q+r-\frac{1}{2}}}{(\gamma + \alpha_4 \alpha_2^{-1})^i} \right) \quad (\text{C.2})$$

where

$$\kappa_{ri} = \frac{1}{(N_1 m_3 + r - i)!} \left. \frac{d^{N_1 m_3 + r - i}}{d\gamma^{N_1 m_3 + r - i}} \left[\frac{1}{(\gamma + \alpha_4 \alpha_2^{-1})^{m_4 + q}} \right] \right|_{\gamma = -\frac{\alpha_3}{\alpha_1}} \quad (\text{C.3})$$

$$\theta_{qi} = \frac{1}{(m_4 + q - i)!} \frac{d^{m_4+q-i} \left[\frac{1}{(\gamma + \alpha_3 \alpha_1^{-1})^{N_1 m_3 + r}} \right]}{d\gamma^{m_4+q-i}} \Big|_{\gamma = -\frac{\alpha_4}{\alpha_2}} \quad (C.4)$$

Substituting (C.2) into (C.1) and rearranging terms, we get

$$P_e = \frac{a}{2} - \frac{a\sqrt{b}\rho^{\frac{1}{2}}}{2\sqrt{\pi}} \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{N_2 m_2 - 1} \frac{\Gamma(m_4 + q) \alpha_2^q}{q!} \frac{\alpha_3^{N_1 m_3}}{\Gamma(N_1 m_3)} \sum_{r=0}^{N_1 m_1 - 1} \frac{\Gamma(N_1 m_3 + r) \alpha_1^r}{r! \alpha_1^{N_1 m_3 + r} \alpha_2^{m_4 + q}} \times \left(\sum_{i=1}^{N_1 m_3 + r} \kappa_{ri} \int_0^\infty \frac{\gamma^{q+r-\frac{1}{2}}}{(\gamma + \alpha_3 \alpha_1^{-1})^i} \exp(-b\rho\gamma) d\gamma + \sum_{i=1}^{m_4+q} \theta_{qi} \int_0^\infty \frac{\gamma^{q+r-\frac{1}{2}}}{(\gamma + \alpha_4 \alpha_2^{-1})^i} \exp(-b\rho\gamma) d\gamma \right) \quad (C.5)$$

Using [21, eq. (2.3.6.9)] to solve the remaining integrals in (C.5), (18) is now readily found.

APPENDIX D: PROOF OF THEOREM 3

In this appendix, we derive the first nonzero coefficient of the MacLaurin series of $F_{\gamma_{0U}}(\gamma)$. Equation (B.1) indicates that the MacLaurin expansion of $F_{\gamma_{0U}}(\gamma)$ can be found by obtaining the MacLaurin series of $F_{\gamma_i}(\gamma)$, $i = 1, 2$ independently. To this end, we rewrite the CDF $F_{\gamma_1}(\gamma)$ as follows:

$$F_{\gamma_1}(\gamma) = \int_0^\infty F_{X_2}(\phi(x_4, \gamma)) f_{X_4}(x_4) dx_4 \quad (D.1)$$

where $\phi(x_4, \gamma) = \gamma x_4$. Applying the n -th order derivative at zero for the two sides of (D.1) yields

$$F_{\gamma_1}^{(n)}(\gamma) \Big|_{\gamma=0} = \int_0^\infty \frac{\partial^n F_{X_2}(\phi(x_4, \gamma))}{\partial \gamma^n} \Big|_{\gamma=0} f_{X_4}(x_4) dx_4 \quad (D.2)$$

The term $\frac{\partial^n F_{X_2}(\phi(x_4, \gamma))}{\partial \gamma^n} \Big|_{\gamma=0}$ can be rewritten in view of [13, eq. (0.430.1)] as

$$\frac{\partial^n F_{X_2}(\phi(x_4, \gamma))}{\partial \gamma^n} \Big|_{\gamma=0} = \sum_{u=1}^n \sum_{v=0}^{u-1} \frac{C_v^u (-1)^v \phi^v(x_4, \gamma)}{\Gamma(u+1)} \frac{\partial^u F_{X_2}(\phi(x_4, \gamma))}{\partial \phi^u} \frac{\partial^n \phi^{u-v}(x_4, \gamma)}{\partial \gamma^n} \Big|_{\gamma=0} \quad (D.3)$$

Because of the fact that $\phi^v(x_4, \gamma) \Big|_{\gamma=0} \neq 0$ if and only if $v = 0$ as well as

$$\frac{\partial^u F_{X_2}(\phi(x_4, \gamma))}{\partial \phi^u} \Big|_{\gamma=0} = \begin{cases} 0, & u < N_2 m_2 \\ \alpha_2^{N_2 m_2}, & u = N_2 m_2 \end{cases} \quad (D.4)$$

we consider only the case of $n = u = N_2 m_2$ and $v = 0$, leading to

$$\frac{\partial^n (\phi(x_4, \gamma))^{u-v}}{\partial \gamma^n} \Big|_{\gamma=0} = \Gamma(N_2 m_2 + 1) x_4^{N_2 m_2} \quad (D.5)$$

Substituting (D.4) and (D.5) in (D.3) together with some manipulations, we have

$$\frac{\partial^n F_{X_2}(\phi(x_4, \gamma))}{\partial \gamma^n} \Big|_{\gamma=0} = \begin{cases} 0, & n < N_2 m_2 \\ \alpha_2^{N_2 m_2} x_4^{N_2 m_2}, & n = N_2 m_2 \end{cases} \quad (D.6)$$

Again, substituting (D.6) in (D.2) and using [13, eq. (3.381.4)] with some simplifications yield

$$F_{\gamma_1}^{(n)}(\gamma) \Big|_{\gamma=0} = \begin{cases} 0, & n < N_2 m_2 \\ \frac{\Gamma(N_2 m_2 + m_4)}{\Gamma(m_4)} \frac{\alpha_2^{N_2 m_2}}{\alpha_4^{N_2 m_2}}, & n = N_2 m_2 \end{cases} \quad (D.7)$$

Therefore, the MacLaurin expansion of $F_{\gamma_1}(\gamma)$ around zero is found as

$$F_{\gamma_1}(\gamma) \Big|_{\gamma=0} = \frac{\Gamma(N_2 m_2 + m_4)}{\Gamma(N_2 m_2 + 1) \Gamma(m_4)} \frac{\alpha_2^{N_2 m_2}}{\alpha_4^{N_2 m_2}} \gamma^{N_2 m_2} + o(\gamma^{N_2 m_2}) \quad (D.8)$$

Similarly, the MacLaurin expansion of $F_{\gamma_2}(\gamma)$ at zero is given by

$$F_{\gamma_2}(\gamma) \stackrel{\gamma \rightarrow 0}{=} \frac{\Gamma(N_1 m_1 + N_1 m_3)}{\Gamma(N_1 m_1 + 1)\Gamma(N_1 m_3)} \frac{\alpha_1^{N_1 m_1}}{\alpha_3^{N_1 m_1}} \gamma^{N_1 m_1} + o(\gamma^{N_1 m_1}) \quad (\text{D.9})$$

Substituting (D.8) and (D.9) into (B.1) and using the relationship in (7), the MacLaurin expansion of $F_{\gamma}(\gamma)$ is rewritten as

$$F_{\gamma}(\gamma) \stackrel{\gamma \rightarrow 0}{=} 1 - \left[1 - \frac{\Gamma(N_1 m_1 + N_1 m_3)}{\Gamma(N_1 m_1 + 1)\Gamma(N_1 m_3)} \frac{\alpha_1^{N_1 m_1}}{\alpha_3^{N_1 m_1}} \left(\frac{\gamma}{\rho}\right)^{N_1 m_1} + o(\gamma^{N_1 m_1}) \right] \times \left[1 - \frac{\Gamma(N_2 m_2 + m_4)}{\Gamma(N_2 m_2 + 1)\Gamma(m_4)} \frac{\alpha_2^{N_2 m_2}}{\alpha_4^{N_2 m_2}} \left(\frac{\gamma}{\rho}\right)^{N_2 m_2} + o(\gamma^{N_2 m_2}) \right] \quad (\text{D.10})$$

which completes the proof of Theorem 3.

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AUTHORS' BIOGRAPHIES

Hoc Phan received his BS degree in electronics and computer engineering from Da Nang City University of Technology, Vietnam, in 2001, his MS degree in geographic information systems from Ho Chi Minh City University of Technology, Vietnam, in 2005, and his MS degree in electrical and

electronics engineering from Ho Chi Minh City University of Technology, Vietnam, in 2006. In May 2005, he joined the Department of Electrical, Electronics, and Telecommunications of Ho Chi Minh City University of Transport, Vietnam. In April 2009, he joined the Radio Communication Group, Blekinge Institute of Technology, Sweden, as a PhD candidate working towards the PhD degree in telecommunication systems. His current research interests include cooperative communications, relay networks, and cognitive radio networks.

Hans-Jürgen Zepernick received his Dipl-Ing degree from the University of Siegen in 1987 and his Dr-Ing degree from the University of Hagen in 1994. From 1987 to 1989, he was with Siemens AG, Germany. He is currently a professor of radio communications at the Blekinge Institute of Technology, Sweden. Prior to this appointment, he held the positions of Professor of wireless communications at Curtin University of Technology, Deputy Director of the Australian

Telecommunications Research Institute, and Associate Director of the Australian Telecommunications Cooperative Research Centre. His research interests include cooperative communications, cognitive radio networks, mobile multimedia, and perceptual quality assessment.

Trung Q. Duong was born in Hoi An town, Quang Nam province, Vietnam, in 1979. He received his BS degree in electrical engineering from Ho Chi Minh City University of Technology, Vietnam, in 2002, and his MSc degree in computer engineering from Kyung Hee University, South Korea, in 2005.

In April 2004, he joined the Ho Chi Minh City University of Transport, Vietnam, as a teaching assistant. He was a recipient of the Korean Government IT Scholarship Program for International Graduate Students from 2003 to 2007. In December 2007, he joined the Radio Communication Group, Blekinge Institute of Technology, Sweden, as a PhD student. In September 2012, he received the PhD degree in telecommunications systems. He was awarded the Best Paper Award of IEEE Student Paper Contest—IEEE Seoul Section in December 2006 and was listed as the finalist for the Best Paper Award at the IEEE Radio and Wireless Symposium, San Diego, USA, in 2009.

He has been acting as a frequent reviewer for numerous journals/conferences and a TPC member for many conferences including ICC, WCNC, VTC, and PIMRC. He has been a TPC co-chair of the International Conference on Computing, Managements, and Telecommunications 2013 (ComManTel2013). He has been working as a visiting scholar at Polytechnic Institute of New York University from December 2009 to January 2010 and Singapore University of Technology and Design from July 2012 to August 2012. His current research interests include cross-layer design, cooperative communications, and cognitive radio networks.

Hung Tran was born in Hanoi Capital, Vietnam, in 1980. He received his BS degree and MS degree in Information Technology from the Vietnam National University, Hanoi, Vietnam, in 2002 and 2006, respectively. Since 2009, he has been working toward his PhD degree at the School of Computing, Blekinge Institute of Technology, Karlskrona, Sweden. His research interests are the areas of wireless communications, cognitive radio networks, and cooperative communication systems.

Thi My Chinh Chu was born in Ninh Binh province, Vietnam, in November 1981. She received her BS degree in electronics and communications from Hanoi University of Technology, Vietnam, in 2004, and her MSc degree in electronics and communications from Hanoi University of Technology, Vietnam, in 2007. Since 2004, she has been working at the

Radio Broadcasting of Vietnam, the Voice of Vietnam (VOV). In April 2011, she joined the Radio Communications Group, Blekinge Institute of Technology, Sweden, as a PhD candidate. Her current research interests are cooperative communications, cognitive radio networks, and combining cooperative diversity techniques into cognitive radio networks.