Low-Complexity Codebook-Based Beamforming with Four Transmit Antennas and Quantized Feedback Channel

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Abstract—In this paper we propose a low-complexity codebook-based beamforming with four transmit antennas and quantized feedback channel. The codebook design aggregates the effect of power allocation and phase rotation through a simple quantized transmit scheme. The codebook-based beamforming uses the feedback information in order to maximize the instantaneous signal-to-noise ratio (SNR) at the receiver. As a result, the proposed scheme presents an array gain. An SNR analysis is performed and it is used to find the optimal feedback information in the sense of maximizing the instantaneous SNR. A bit error rate (BER) analysis for a quantized feedback channel is also derived and it is used to compare to the results obtained for the proposed scheme under different levels of quantization. Simulations are performed over quasi-static flat Rayleigh fading channels for different closed-loop codebook-based schemes with four transmit antennas and unitary transmission rate. Results illustrate that the proposed scheme achieves full diversity order and outperforms other good schemes in terms of array gain.

I. INTRODUCTION

Quantized transmit beamforming is an interesting real-world solution for multiple-input multiple-output (MIMO) systems since this directional signal can improve the signal-to-noise ratio (SNR) at the receiver enhancing the channel capacity [1] and reducing the symbol or bit error probability of the system [2]. These improvements, obviously, depend on the the amount of feedback available. When full channel state information (CSI) is available to the transmitter, optimal *beamforming* is the best strategy to be considered [3]. In fact, transmit beamforming uses all the transmit antennas and distributes the total average power non-uniformly among them.

Deployment of multiple antennas is normally associated with high cost since multiple antennas usually require an equal number of radio frequency (RF) chains (transmit amplifiers, modulators, etc.), which are expensive and power-consuming. To circumvent this problem, a subset of the available antennas can be selected, thereby reducing the number of RF chains. The amount of feedback required in this case is much smaller.

Such approach is called *transmit antenna selection* (TAS). The most well-known TAS scheme has been proposed by Gore and Paulraj [4] and later, an extension of this idea was presented in [5].

Another efficient way of using the feedback bits is through a codebook-based (CBB) design, where the feedback information is used to select a codeword from a finite codebook [2]. Along the last years, several works have investigated the use of quantized feedback in multiple-antenna CBB systems [6]–[12], which is currently considered in the LTE standard [13].

In [12], an analysis on symbol error rate of maximum ratio transmission, transmit antenna selection, and codebook-based beamforming in correlated Rayleigh fading channels was presented. Zhu et al. have derived a universal upper bound on the average SER of those three schemes for a quantized feedback information.

In [10], a class of extended orthogonal space-time block codes (EO-STBCs) for MIMO channels considering four transmit antennas over quasi-static flat fading channels was proposed. The transmission rate of this scheme is one, since two symbols are transmitted over two symbol periods. In this work, phase information is fed back from the receiver to the transmitter and, based on this information, phases of certain symbols are rotated. As a result, the EO-STBC presented in [10] outperformed previous rate-one closed-loop STBCs with four transmit antennas.

In [11], Choi et al. have proposed an interesting phase-feedback-assisted scheme with four transmit antennas which uses a preprocessor for combining two Alamouti codes in terms of Frobenius norm maximization. They have shown that full diversity is achieved by the combining effect at the preprocessor. Also the phase feedback is utilized to increase the coding gain. Herein, we refer to this scheme as Alamouti-code based scheme (ACBS).

A. Contribution and Objectives

By knowing that the feedback rate is a critical aspect in real-world wireless systems, we propose a low-complexity codebook-based beamforming with four transmit antennas and quantized feedback channel. The feedback index (phases) is used to maximize the instantaneous signal-to-noise ratio (SNR). The preprocessing considered for the proposed scheme is based on the designs presented in [10] and [11].

We also addressed an SNR analysis which is considered to find the optimal (non-quantized) feedback phases in terms of instantaneous signal-to-noise ratio maximization. An upper bound on the average BER performance is obtained, which is based on the analysis presented in [12] and [14]. Another important feature of the proposed scheme is the signal detection which is based on linear processing, leading to a low-complexity receiver.

A quantized feedback analysis of the proposed scheme over quasi-static flat Rayleigh fading channels is also performed. Simulation results illustrate that the proposed scheme outperforms other good phase feedback codebook-based schemes with four transmit antennas.

B. Organization and Some Notations

The rest of the paper is organized as follows. Section II presents the system model. Section III addresses the low-complexity codebook-based beamforming with four transmit antennas and quantized feedback channel. Furthermore an SNR analysis for the proposed scheme is provided. In Section IV, a BER analysis for a quantized feedback channel is derived and used later in Section V, where simulations are taken into account. Finally, Section VI presents some concluding remarks.

Throughout this paper, normal letters represent scalar quantities, boldface lowercase letters indicate vectors, and boldface uppercase letters indicate matrices. The superscripts 'T' and '*' represent the transpose and the complex conjugate operation, $| \cdot |^2$ denotes the modulus squared of a complex number and $\Re\{\cdot\}$ the real part of a complex number.

II. SYSTEM MODEL

Consider a MIMO system with $M_T = 4$ transmit antennas at the transmitter and M_R receive antennas at the receiver. The channel is assumed to be quasi-static, flat Rayleigh fading so that the channel is constant over a frame (τ symbol periods) and vary randomly from one frame to the next. The transmission model consists of *linear processing*, as described in [11]. With some slight modifications, we arrive at

$$\mathbf{Y} = \mathbf{PHI}_{SNR} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ is the $\tau \times M_R$ matrix of the received signals and $\mathbf{P}$ is the $\tau \times M_T$ matrix of processed transmitted signals with unit average energy. Let $\mathcal{CN}(0, \mathbf{R})$ represent the joint p.d.f. (probability density function) of a zero-mean circularly symmetric complex normal random vector with covariance matrix $\mathbf{R}$. Then, $\mathbf{N}$ is the τM_R matrix $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{\tau M_R})$ representing the joint p.d.f. of the i.i.d. (independent and identically distributed) additive Gaussian noise samples with unit variance, $\mathbf{H}$ is the $M_T M_R$ MIMO channel characterized by the p.d.f.

$\mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_T M_R})$, and $\mathbf{I}_{SNR}$ is the $M_R \times M_R$ diagonal matrix, where the i-th diagonal element of this matrix is given by $\sqrt{\rho}$, with $\rho = E_b/N_0$, which means the SNR in each receive antenna, E_b is the average transmitted bit power, and N_0 is the noise variance.

We assume that the information bits are mapped into a base-band unitary average energy constellation, such as PSK or QAM constellations, given rise to Q data symbols $\{s_q\}$, $q = 1, \ldots, Q$, to be transmitted over τ symbol periods. In this paper, we consider only schemes with unitary spatial transmission rate ($R = Q/\tau = 1$). We also assume that the channel coefficients are perfectly estimated at the receiver and that there is a reliable feedback channel through which b bits can be sent to the transmitter.

III. THE PROPOSED SCHEME

We start this section by considering how the transmission of the proposed scheme is performed. After, the linear receiver for M_R receive antennas will be addressed.

A. The Transmitter

The proposed scheme has four transmit antennas, where only one information symbol, s, is transmitted per transmit period. The symbol s is preprocessed by a complex transmission codeword $\mathbf{x}$:

$$\mathbf{p} = \mathbf{x}s \tag{2}$$

where

$$\begin{aligned}
\mathbf{x} &= \left[\mathbf{v}_\varphi \left(\begin{bmatrix} \cos(\theta)/\sqrt{2} & 0 \\ 0 & \sin(\theta)/\sqrt{2} \end{bmatrix} \otimes \mathbf{I}_2 \right) \right], \\
&= \frac{1}{\sqrt{2}} \begin{bmatrix} \cos(\theta)e^{j\varphi_0} \\ \cos(\theta)e^{j\varphi_1} \\ \sin(\theta)e^{j\varphi_0} \\ \sin(\theta)e^{j\varphi_2} \end{bmatrix}^{\mathrm{T}},
\end{aligned}$$

where $\otimes$ is the kronecker product, φ_i, $i = 0, 1, 2$, represents a phase, $\mathbf{I}_2$ is the 2-by-2 identity matrix, and

$$\mathbf{v}_\varphi = \begin{bmatrix} e^{j\varphi_0} & e^{j\varphi_1} & e^{j\varphi_0} & e^{j\varphi_2} \end{bmatrix}.$$

The factor $\frac{1}{\sqrt{2}}$ is used for normalizing the transmit power.

For simplicity of analysis, we assume that $\varphi_0 = 0$. Henceforth we follow with this simplification since φ_0 is used as a reference phase only.

Before that each transmission takes place, the receiver feeds back b bits to the transmitter and this information is used appropriately by the preprocessor for choosing the codeword

$$\mathbf{x}_{\max} \in \mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N],$$

where $N = 2^b$. $\mathbf{x}_{\max}$ is the codeword that maximizes the instantaneous SNR. As we can observe, the codebook is a function of φ_1, φ_2 and θ. Therefore, the codebook is defined based on the sets adopted for those variables.

Next, we present the receiver for M_R receive antennas, the SNR analysis, which is used to find the optimal values for φ_1, φ_2 and θ, and also how the proposed scheme is performed with a quantized feedback.

B. Linear Receiver for M_R receive antennas

From (1) and (2), the received signal $\mathbf{y}$ for M_R receive antennas can be rewritten as

$$\mathbf{y} = s\mathbf{h_p} + \mathbf{n}, \qquad (3)$$

where

$$\begin{aligned}
\mathbf{h_p} &= \mathbf{xH} \\
&= [h_{p1} \ldots h_{pi} \ldots h_{pM_R}],
\end{aligned} \qquad (4)$$

in which

$$\begin{aligned}
h_{pi} = \;& \cos(\theta)h_{1,i} + \cos(\theta)e^{\varphi_1}h_{2,i} \\
& + \sin(\theta)h_{3,i} + \sin(\theta)e^{\varphi_2}h_{4,i}
\end{aligned}$$

and

$$\mathbf{H} = \begin{bmatrix}
h_{1,1} & h_{2,1} & h_{3,1} & h_{4,1} \\
\vdots & \vdots & \vdots & \vdots \\
h_{1,M_R} & h_{2,M_R} & h_{3,M_R} & h_{4,M_R}
\end{bmatrix}^{\mathsf{T}}$$

where $\mathbf{n}$ is the $1 \times M_R$ ($\tau = 1$) additive white Gaussian noise vector, and $h_{i,j}$ denotes the path gain from the i-th transmit antenna to the j-th receive antenna.

Considering the received signal in (3) and employing the maximal ratio combining (MRC) technique, the following linear processing produces the desired inputs to the maximum-likelihood detection:

$$\begin{aligned}
\tilde{s} =\;& y_1 h_{p1}^* + \ldots + y_i h_{pi}^* + \ldots + y_{M_R} h_{pM_R}^* \qquad (5) \\
=\;& s\Bigg[\left(\sum_{i=1}^{M_R}(|h_{1,i}|^2 + |h_{2,i}|^2) + \beta_c \right) \cos(\theta)^2 \\
& + \left(\sum_{i=1}^{M_R}(|h_{3,i}|^2 + |h_{4,i}|^2) + \beta_s \right) \sin(\theta)^2 \\
& + \beta_{cs} \cos(\theta)\sin(\theta) \Bigg]\sqrt{\rho}/2 + \sum_{i=1}^{M_R} h_{pi}^* n_i
\end{aligned}$$

where η_i is the i.i.d. additive Gaussian noise samples with unit variance, $\beta_c = \sum_{i=1}^{M_R}\beta_{ci}$, $\beta_s = \sum_{i=1}^{M_R}\beta_{si}$, $\beta_{cs} = \sum_{i=1}^{M_R}\beta_{csi}$ and

$$\begin{aligned}
\beta_{ci} &= 2\Re\left\{ h_{1,i}h_{2,i}^* e^{-j\varphi_1} \right\} \\
\beta_{si} &= 2\Re\left\{ h_{3,i}h_{4,i}^* e^{-j\varphi_2} \right\} \\
\beta_{csi} &= 2\Re\left\{ (h_{1,i} + h_{2,i}e^{j\varphi_1})(h_{3,i} + h_{4,i}e^{j\varphi_2})^* \right\}
\end{aligned}$$

C. Instantaneous SNR Analysis

For the signal obtained in (5), the instantaneous output SNR is given by

$$\gamma = \left(\frac{g_c \cos^2(\theta) + g_s \sin^2(\theta) + \beta_{cs}\cos(\theta)\sin(\theta)}{2} \right)\rho \qquad (6)$$

where

$$g_c = \left[\sum_{i=1}^{M_R} |h_{1,i}|^2 + |h_{2,i}|^2 + 2\Re\{h_{1,i}h_{2,i}^* e^{-j\varphi_1}\} \right] \cos^2(\theta)$$

$$g_s = \left[\sum_{i=1}^{M_R} |h_{3,i}|^2 + |h_{4,i}|^2 + 2\Re\{h_{3,i}h_{4,i}^* e^{-j\varphi_2}\} \right] \sin^2(\theta)$$

$$\beta_{cs} = \sum_{i=1}^{M_R} 2\Re\left\{ (h_{1,i} + h_{2,i}e^{j\varphi_1})(h_{3,i} + h_{4,i}e^{j\varphi_2})^* \right\}$$

Now, we can determine the optimal φ_1, φ_2, and θ by differentiation of (6), yielding a maximum signal-to-noise ratio and diversity order $4M_R$ ($M_T = 4$). Nevertheless, it is important to mention that full diversity can also be achieved with quantized feedback (as described latter in this paper).

From (6), we observe that there is a dependence among the variables present in this optimization process, making the instantaneous SNR maximization problem hard to solve. Thus, we chose to solve this optimization problem in two steps.

First, we differentiate β_c and β_s with respect to φ_1 and φ_2, respectively, and set them both equal to zero. We can easily verify that those terms are maximized when

$$\varphi_1 = \arctan\left(\sum_{i=1}^{M_R} \frac{\sin(\xi_{1,i} - \xi_{2,i})\alpha_{1,i}\alpha_{2,i}}{\cos(\xi_{1,i} - \xi_{2,i})\alpha_{1,i}\alpha_{2,i}} \right) \qquad (7)$$

$$\varphi_2 = \arctan\left(\sum_{i=1}^{M_R} \frac{\sin(\xi_{3,i} - \xi_{4,i})\alpha_{3,i}\alpha_{4,i}}{\cos(\xi_{3,i} - \xi_{4,i})\alpha_{3,i}\alpha_{4,i}} \right) \qquad (8)$$

where, $h_{a,b} = \alpha_{a,b}\exp\{\xi_{a,b}\}$.

Second, we differentiate (6) with respect to θ. The first and second derivatives are given by

$$\gamma' = 2\kappa\left(\cos(\theta)\sin(\theta)\right) + \beta_{cs}\left(\cos^2(\theta) - \sin^2(\theta)\right) \qquad (9)$$

and

$$\gamma'' = 2\kappa\left(\cos^2(\theta) - \sin^2(\theta)\right) - 4\beta_{cs}\left(\cos(\theta)\sin(\theta)\right), \qquad (10)$$

respectively.

Solving (9) and (10) under the conditions $\gamma' = 0$ and $\gamma'' < 0$, we obtain the following optimal theta phase

$$\theta_{opt} = \arctan\left(\frac{\kappa + \sqrt{\kappa^2 + 2\beta_{cs}^2}}{\beta_{cs}} \right), \qquad (11)$$

with $\kappa = g_s - g_c$.

We decide to omit the term ρ in (9) and (10), since this does not alter the final result.

D. Quantized Feedback

In this section, we present how to perform the proposed scheme with quantized feedback. We assume two uniform finite phase sets: φ_{1q} and $\varphi_{2q} \in [0\ \pi]$ and $\theta_q \in [-\pi/2\ \pi/2]$. Since the receiver needs to feed back the state information about three phases, the transmitter receive, at least, three feedback bits for pre-processing the information properly (see Table I). The receiver computes and compares 2^{b-2} instantaneous SNR and send b feedback bits to the transmitter.

Table I defines the criteria used by the receiver for feeding back the quantized vector. Each vector (or group of phases), presented in each row of Table I, defines a codeword $\mathbf{x}(\varphi_1, \varphi_2, \theta)$. Phases φ_1 and φ_2 are used to ensure that the terms β_c and β_s are positive numbers. As the number of feedback bits is increased, β_c and β_s converge to the ideal (non-quantized) solution. The third phase, i.e. θ, is used to maximize the sum of the terms that compose the instantaneous SNR.

In Table I, we see how the codebooks, $\mathbf{X}_8$ and $\mathbf{X}_{16}$, are designed for $b = 3$ and 4 feedback bits, respectively. The symbol $\sharp$ specifies the "do not care state".

TABLE I.   QUANTIZED FEEDBACK: CODEBOOK DESIGN.

Feedback bits	β_c	β_s	β_{cs}	κ	θ_q	φ_{1q} / φ_{2q}
	>0	>0	>0	$\sharp$	$\frac{\pi}{4}$	0 / 0
	>0	<0	>0	$\sharp$	$\frac{\pi}{4}$	0 / π
	<0	>0	>0	$\sharp$	$\frac{\pi}{4}$	π / 0
	<0	<0	>0	$\sharp$	$\frac{\pi}{4}$	π / π
$b = 3$ bits	>0	>0	<0	$\sharp$	$\frac{-\pi}{4}$	0 / 0
	>0	<0	<0	$\sharp$	$\frac{-\pi}{4}$	0 / π
	<0	>0	<0	$\sharp$	$\frac{-\pi}{4}$	π / 0
	<0	<0	<0	$\sharp$	$\frac{-\pi}{4}$	π / π
	>0	>0	>0	>0	$\frac{3\pi}{8}$	0 / 0
	>0	<0	>0	>0	$\frac{3\pi}{8}$	0 / π
	<0	>0	>0	>0	$\frac{3\pi}{8}$	π / 0
	<0	<0	>0	>0	$\frac{3\pi}{8}$	π / π
	>0	>0	>0	<0	$\frac{\pi}{8}$	0 / 0
	>0	<0	>0	<0	$\frac{\pi}{8}$	0 / π
	<0	>0	>0	<0	$\frac{\pi}{8}$	π / 0
$b = 4$ bits	<0	<0	>0	<0	$\frac{\pi}{8}$	π / π
	>0	>0	<0	>0	$\frac{-\pi}{8}$	0 / 0
	>0	<0	<0	>0	$\frac{-\pi}{8}$	0 / π
	<0	>0	<0	>0	$\frac{-\pi}{8}$	π / 0
	<0	<0	<0	>0	$\frac{-\pi}{8}$	π / π
	>0	>0	<0	<0	$\frac{-3\pi}{8}$	0 / 0
	>0	<0	<0	<0	$\frac{-3\pi}{8}$	0 / π
	<0	>0	<0	<0	$\frac{-3\pi}{8}$	π / 0
	<0	<0	<0	<0	$\frac{-3\pi}{8}$	π / π

IV.   BER ANALYSIS

In this section, we present a BER analysis for a codebook-based beamforming. This analysis is performed for PSK constellations.

The BER of M-ary PSK constellation conditioned on the instantaneous SNR γ can be given by [15]:

$$BER(\gamma) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{C\gamma}{\sin^2(\theta)}\right) d\theta \qquad (12)$$

where M is the constellation size and $C = \sin^2(\pi/M)$.

Taking the expectation over (12), the average BER can be written as

$$\overline{BER}(\gamma) = \mathbb{E}_h\{BER(\gamma)\}, \qquad (13)$$

where $\mathbb{E}_h$ is the expectation operator with respect to h.

Now, by assuming that the transmitter uses feedback information for choosing a codeword $\mathbf{x}$, which belongs to a certain finite codebook $\mathbf{X}$, such that the instantaneous SNR is maximized, we can rewritten (13) as

$$\overline{BER}(\rho) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \mathbb{M}\left(-\frac{C\rho}{\sin^2(\theta)}; \mathbf{X}\right) d\theta \qquad (14)$$

where $\mathbb{M}\left(-\frac{C\rho_x}{\sin^2(\theta)}; \mathbf{X}\right)$ denotes the value of the function

$$\mathbb{M}(t; \mathbf{X}) \triangleq \mathbb{E}\exp\left(-t \max_{\mathbf{x} \in \mathbf{X}} |\mathbf{h_p}|^2\right), t \geq 0 \qquad (15)$$

$$= \mathbb{E}\exp\left(-t \max_{\mathbf{x} \in \mathbf{X}} |\mathbf{xH}|^2\right), t \geq 0$$

As in [12], we consider the following approximation of (15):

$$\beta(\mathbf{X}) = \lim_{t \to \infty} t^{M_T M_R} \mathbb{M}(t; \mathbf{X}) \qquad (16)$$

in which the values assumed by $\beta(t, \mathbf{X})$ depend on the set of codewords in $\mathbf{X}$.

This asymptotic approximation is fundamental for our analysis since an exact expression is very hard to obtain. However, if the codebook is well designed, and that is our case, then the Voronoi cells of the codewords can be approximated by 'spherical caps' on the surface of hypersphere, which simplifies considerably the BER analysis [14].

Based on the results presented in [14] and [12] we have the following approximation of (16)

$$\beta(\mathbf{X}) = C_1(M_T M_R - 1)!(M_T - 1)N$$
$$\times \sum_{n=0}^{M_T-2} \binom{M_T - 2}{n} \frac{(-1)^n (C_2^{-(M_T M_R - 1 - n)} - 1)}{M_T M_R - 1 - n} \qquad (17)$$

where

$$C_1 = M_T M_R \prod_{n=1}^{\min(M_T M_R)} \frac{(\min(M_T M_R) - n)!}{(M_T M_R - 1)!}$$

and

$$C_2 = 1 - N^{\frac{-1}{M_T - 1}}.$$

A. An upper bound on the BER performance

In this section, we continue with the analysis of $\beta(\mathbf{X})$, whose result will be used in (14) culminating in an upper bound on the average BER.

The normalized channel is defined by $\tilde{\mathbf{h}}_p = \frac{\mathbf{h_p}}{|\mathbf{h_p}|}$ where $|\mathbf{h_p}|$ is independent of its direction. Furthermore, since $\mathbf{h_p}$ is Gaussian, $|\mathbf{h_p}|^2$ is a chi-square random variable with $2M_T M_R$ degree of freedom and has the following moment generating function (MGF)

$$\mathbb{E}\exp(-s|\mathbf{h_p}|^2) = (1 + s)^{-M_T M_R} \qquad (18)$$

Applying the MGF to (15), we obtain

$$\mathbb{M}(t; \mathbf{X})$$
$$= \mathbb{E}_{\tilde{\mathbf{h}}_p}\left\{\mathbb{E}_{|\tilde{\mathbf{h}}_p|^2} \exp\left[-(t\Omega|\tilde{\mathbf{h}}_p|^2) \times |\mathbf{h_p}|^2\right]\right\}$$
$$= \mathbb{E}_{\tilde{\mathbf{h}}_p}\left\{((1 + t\Omega)|\tilde{\mathbf{h}}_p|^2)^{-M_T M_R}\right\} \qquad (19)$$

where the subscript on the expectation operator indicates the random variable being averaged over, and Ω is defined as

$$\Omega \triangleq \left(\max_{\mathbf{x} \in \mathbf{X}} |\tilde{\mathbf{h}}_p|^2\right)^{-M_T M_R}. \qquad (20)$$

Hence, we can be rewritten (16) as

$$\beta(\mathbf{X}) = \lim_{t \to \infty} t^{M_T M_R} \mathbb{M}(t; \mathbf{X})$$
$$= \lim_{t \to \infty} t^{M_T M_R} \mathbb{E}_{\tilde{\mathbf{h}}_p}\left\{\left(1 + \Omega^{\frac{-1}{M_T M_R}}\right)^{-M_T M_R}\right\}$$
$$= \mathbb{E}_{\tilde{\mathbf{h}}_p}(\Omega). \qquad (21)$$

For $t > 0$, it is to verify that $(1 + t\Omega^{\frac{-1}{M_T M_R}})^{-M_T M_R}$ is concave with respect to Ω. Thus, applying Jensen's inequality to (19), yielding:

$$\mathbb{M}(t; \mathbf{X}) \leq \left\{1 + t\left[\mathbb{E}_{\tilde{\mathbf{h}}_p}\Omega^{\frac{-1}{M_T M_R}}\right]\right\}^{-M_T M_R}$$
$$= \left\{1 + t\left[\beta(\mathbf{X})^{\frac{-1}{M_T M_R}}\right]\right\}^{-M_T M_R}. \qquad (22)$$

Substituting (22) into (14), an upper bound on the average BER is obtained

$$\overline{BER}(\rho) \le \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \left(1 + \frac{C\rho}{\sin^2(\theta)} \beta(\mathbf{X})^{\frac{-1}{M_T M_R}}\right)^{-M_T M_R} d\theta \tag{23}$$

V. SIMULATION RESULTS

In this section, we present some simulation results to illustrate the performance of the proposed scheme. In order to assess the array gain of the proposed scheme, we also provide the performances obtained by other two closed-loop codebook-based schemes with four transmit antennas: the EO-STBC [10] and the ACBS [11].

In our simulations, we assume that the symbols are mapped to a QPSK constellation and we adopt as a stopping criterion the occurrence of 300 symbol errors per average SNR. We also assume the receiver knows the channel perfectly and sends b bits over an error-free and zero-delay feedback channel. All the schemes considered here are rate one, i.e., for M-PSK constellations we have a transmission rate $R = \frac{Q}{\tau} \log_2(M) = \log_2(M)$. The BER performance for the no-diversity scenario (single-input single-output scheme) is also plotted in Fig. 1, used as a reference curve.

The performance results are compared in terms of BER versus ρ over quasi-static flat Rayleigh fading channels. In Fig. 1 and 2, results are given for $M_T = 4$ transmit antennas and $M_R = 1$ receive antenna. In Fig. 3, results are given for $M_T = 4$ transmit antennas and $M_R = 2$ receive antennas.

Fig. 1 shows the BER performance of the proposed, EO-STBC and ACBS schemes. For this simulations it was assumed an ideal (unquantized) feedback channel. We can observe that the proposed scheme achieves diversity order of $M_T M_R = 4$ (the same order achieved by the other ones) and has a performance gain of about 2.4dB over the EO-STBC and ACBS schemes.
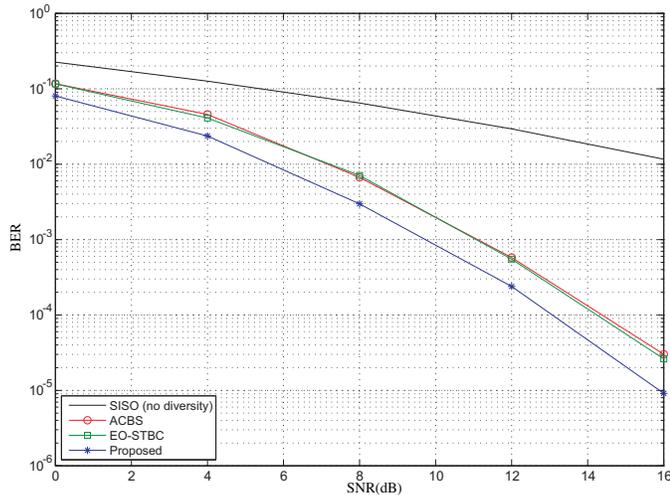
$$\mathbf{X}_8 = \begin{bmatrix} \cos(\pi/4) & \cos(\pi/4) & \sin(\pi/4) & \sin(\pi/4) \\ \cos(\pi/4) & \cos(\pi/4) & \sin(\pi/4) & \sin(\pi/4)e^{j\pi} \\ \cos(\pi/4) & \cos(\pi/4)e^{j\pi} & \sin(\pi/4) & \sin(\pi/4) \\ \cos(\pi/4) & \cos(\pi/4)e^{j\pi} & \sin(\pi/4) & \sin(\pi/4)e^{j\pi} \\ \cos(-\pi/4) & \cos(-\pi/4) & \sin(-\pi/4) & \sin(-\pi/4) \\ \cos(-\pi/4) & \cos(-\pi/4) & \sin(-\pi/4) & \sin(-\pi/4)e^{j\pi} \\ \cos(-\pi/4) & \cos(-\pi/4)e^{j\pi} & \sin(-\pi/4) & \sin(-\pi/4) \\ \cos(-\pi/4) & \cos(-\pi/4)e^{j\pi} & \sin(-\pi/4) & \sin(-\pi/4)e^{j\pi} \end{bmatrix}$$

$$\mathbf{X}_{16} = \begin{bmatrix} \cos(3\pi/8) & \cos(3\pi/8) & \sin(3\pi/8) & \sin(3\pi/8) \\ \cos(3\pi/8) & \cos(3\pi/8) & \sin(3\pi/8) & \sin(3\pi/8)e^{j\pi} \\ \cos(3\pi/8) & \cos(3\pi/8)e^{j\pi} & \sin(3\pi/8) & \sin(3\pi/8) \\ \cos(3\pi/8) & \cos(3\pi/8)e^{j\pi} & \sin(3\pi/8) & \sin(3\pi/8)e^{j\pi} \\ \cos(-3\pi/8) & \cos(-3\pi/8) & \sin(-3\pi/8) & \sin(-3\pi/8) \\ \cos(-3\pi/8) & \cos(-3\pi/8) & \sin(-3\pi/8) & \sin(-3\pi/8)e^{j\pi} \\ \cos(-3\pi/8) & \cos(-3\pi/8)e^{j\pi} & \sin(-3\pi/8) & \sin(-3\pi/8) \\ \cos(-3\pi/8) & \cos(-3\pi/8)e^{j\pi} & \sin(-3\pi/8) & \sin(-3\pi/8)e^{j\pi} \\ \cos(\pi/8) & \cos(\pi/8) & \sin(\pi/8) & \sin(\pi/8) \\ \cos(\pi/8) & \cos(\pi/8) & \sin(\pi/8) & \sin(\pi/8)e^{j\pi} \\ \cos(\pi/8) & \cos(\pi/8)e^{j\pi} & \sin(\pi/8) & \sin(\pi/8) \\ \cos(\pi/8) & \cos(\pi/8)e^{j\pi} & \sin(\pi/8) & \sin(\pi/8)e^{j\pi} \\ \cos(-\pi/8) & \cos(-\pi/8) & \sin(-\pi/8) & \sin(-\pi/8) \\ \cos(-\pi/8) & \cos(-\pi/8) & \sin(-\pi/8) & \sin(-\pi/8)e^{j\pi} \\ \cos(-\pi/8) & \cos(-\pi/8)e^{j\pi} & \sin(-\pi/8) & \sin(-\pi/8) \\ \cos(-\pi/8) & \cos(-\pi/8)e^{j\pi} & \sin(-\pi/8) & \sin(-\pi/8)e^{j\pi} \end{bmatrix}$$

Fig. 2 and 3 show the BER performance of the proposed scheme for $M_R = 1$ and $M_R = 2$, respectively. For these simulations we have considered the codebooks ($\mathbf{X}_8$) for $b = 3$ and ($\mathbf{X}_{16}$) for $b = 4$ (see Table I for more details). Fig. 2 and 3 also present the theoretical upper bounds for $M_R = 1$ and $M_R = 2$, respectively. We note that the proposed and theoretical performances get closer as the number of feedback bits is increased.

Fig. 1. BER performances. M_T=4 transmit antennas, M_R=1 receive antena, and $b = \infty$ feedback bits (unquantized).
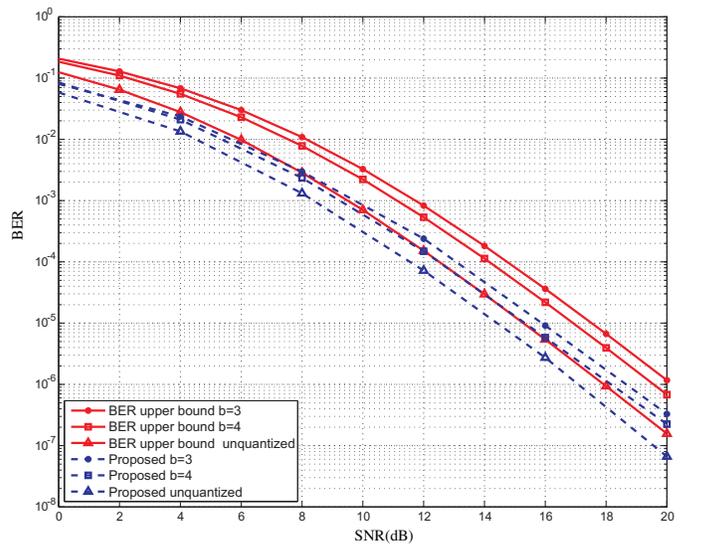

Fig. 2. BER performance: Theoretical upper bound and the performance of the proposed scheme for $M_R = 1$.
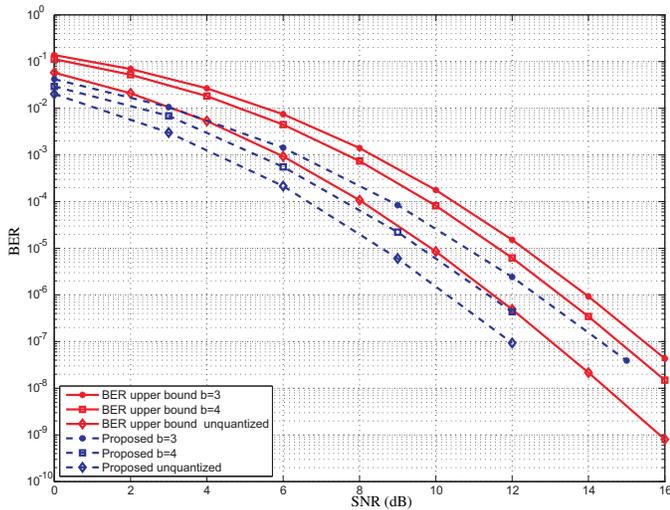
Fig. 3. BER performance: Theoretical upper bound and the performance of the proposed scheme for $M_R = 2$.

VI. CONCLUSIONS AND FINAL REMARKS

In this paper, a low-complexity codebook-based beamforming with four transmit antennas and quantized feedback channel was proposed. The proposed scheme is based on a preprocessing vector that combines the effect of phase rotation and power allocation (suboptimal in this proposal) into a new simple codebook-based transmission scheme.

The proposal can be seen as a code selector scheme, which allows us to evaluate it as a codebook-based design. An instantaneous SNR analysis was addressed in order to find the optimal feedback information. The proposed scheme was evaluated in terms of BER performance through Monte Carlo simulations. Further, a quantized feedback analysis over quasi-static flat Rayleigh fading channels was considered, and an upper bound on the BER performance was derived as well.

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