Copyright © IEEE.
Citation for the published paper:

This material is posted here with permission of the IEEE. Such permission of the IEEE does not in any way imply IEEE endorsement of any of BTH's products or services. Internal or personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution must be obtained from the IEEE by sending a blank email message to pubs-permissions@ieee.org.

By choosing to view this document, you agree to all provisions of the copyright laws protecting it.

Cognitive AF Relay Assisting both Primary and Secondary Transmission with Beamforming

Thi My Chinh Chu, Hoc Phan, and Hans-Jürgen Zepernick
Blekinge Institute of Technology, SE-371 79 Karlskrona, Sweden
E-mail: {cch, hph, hjz}@bth.se

Abstract—This paper investigates the system performance of a cognitive relay network with underlay spectrum sharing wherein the relay is exploited to assist both the primary and secondary transmitters in forwarding their signals to the respective destinations. To exploit spatial diversity, beamforming transmission is implemented at the transceivers of the primary and secondary networks. Particularly, exact expressions for the outage probability and symbol error rate (SER) of the primary transmission and tight bounded expressions for the outage probability and SER of the secondary transmission are derived. Furthermore, an asymptotic analysis for the primary network, which is utilized to investigate the diversity and coding gain of the network, is developed. Finally, numerical results are presented to show the benefits of the proposed system.

I. INTRODUCTION

In recent years, incorporating cooperative transmission into cognitive radio networks (CRNs) has attracted great research interest due to its advantages such as high efficiency in spectrum utilization, reliable transmission, and large radio coverage (see [1] and the references therein). Considering the use of relay transmission in CRNs, the work of [2] investigated the outage performance of a cognitive amplify-and-forward (AF) relay network. Furthermore, the authors of [3] studied the integration of a decode-and-forward (DF) relaying scheme into a CRN by examining the outage performance. Moreover, the performance in terms of channel capacity of a cognitive cooperative radio network (CCRN) was analyzed in [4].

Regarding techniques of accessing the licensed spectrum for CRNs, there exist spectrum overlay, spectrum underlay, and interweave (opportunistic) schemes. For instance, in [5], several spectrum sensing techniques and requirements of designing an overlay CRN have been investigated. On the contrary, [6] investigated the outage probability of an underlay CRN under the interference power constraint of the primary user assuming perfect channel state information (CSI) for the interference channel from the cognitive transmitter to the primary receiver. Furthermore, the work of [7] investigated the benefits of using cooperative relays in interweave cognitive radio systems which provide cooperative diversity gain.

Recently, beamforming transmission has been proposed as an efficient technology to improve system performance and has been studied in the area of CRNs. In particular, the work of [8] investigated the implementation of beamforming transmission for underlay cognitive two-way relaying networks. In addition, the authors of [9] considered power allocation strategies for CRNs using beamforming transmission to guarantee the qual-

ity of service (QoS) of the primary system when the secondary users (SUs) are given access to the licensed spectrum of the primary users (PUs). However, in [8], [9], the relays forward the signal of CRNs only. Theoretically, a terminal may receive signals from any neighbor transmitter due to the broadcast nature of wireless communications. Therefore, some recent transmission schemes in which the secondary relays also act as relays for the primary transmission have been proposed. For instance, considering the case of a single-antenna at all terminals of a cognitive AF relay system, it has been shown that letting the secondary relay forward the signals of both the primary and secondary transmitters results in a significant performance improvement for the primary network [10]. The main advantage of these schemes is that the interference incurred by the primary transmitter at the relay can be exploited to increase capacity for the primary network. As a result, the reliability of the primary transmission is enhanced. Further, these infrastructures become more efficient in utilizing spectrum and achieve better performance.

This paper utilizes beamforming transmission in a cognitive relay network wherein the secondary AF relay simultaneously forwards the primary and secondary signals to the respective destinations. All wireless channels are modeled as Nakagami- m fading which comprises a variety of fading models as special cases. We further investigate system performance by deriving exact and asymptotic expressions for the outage probability of the primary and secondary transmissions. Numerical results are presented showing that the primary transmission of the considered network outperforms that of the respective network without the assistance of the relay.

Notation: Vectors are denoted by bold lower case letters and their Frobenius norm is expressed as $\|\cdot\|_F$. Further, the probability density function (PDF) and cumulative distribution function (CDF) of a random variable (RV) X are $f_X(\cdot)$ and $F_X(\cdot)$, respectively. Next, C_k^n represents the binomial coefficient. Additionally, the gamma and incomplete gamma functions, defined in [11, eq. (8.310.1)] and [11, eq. (8.350.2)], are denoted as $\Gamma(n)$ and $\Gamma(n, x)$, respectively. Finally, $U(a, b; x)$ stands for the confluent hypergeometric function [11, eq. (9.211.4)] and $\mathbb{E}\{\cdot\}$ is the expectation operator.

II. SYSTEM AND CHANNEL MODELS

Consider an underlay cognitive relay network which co-exists with a primary network over Nakagami- m fading as depicted in Fig. 1. The secondary network consists of a

Fig. 1. System model of the consider system (SC=Selection combining).

secondary transmitter SU_{TX} with N_1 antennas, a secondary receiver SU_{RX} with N_2 antennas, and a secondary relay SU_R with a single receiver and transmit antenna. In view of the interference power constraint imposed by the primary user, SU_{TX} needs to transmit with relatively low power. Furthermore, we assume that SU_{TX} is located far from SU_{RX} . Therefore, the signal of the direct transmission from SU_{TX} to SU_{RX} can be neglected. In order to achieve spatial diversity for the cognitive network, we deploy beamforming at SU_{TX} and maximum ratio combining at SU_{RX} . The primary network consists of a primary transmitter PU_{TX} equipped with N_3 antennas and a primary receiver PU_{RX} with a single antenna. The communication between PU_{TX} and PU_{RX} is mainly performed through the respective direct link and partially through the assistance of the cognitive relay. To focus the transmit signal towards the single receive antenna of PU_{RX} , we also employ beamforming transmission at PU_{TX} .

Because there is no dedicated feedback channel from SU_{TX} to PU_{RX} , the instantaneous channel power gain of the link from SU_{TX} to PU_{RX} is unavailable at the SU_{TX} . However, SU_{TX} can estimate the average channel gain for this channel based on some relatively stable parameters such as transmission distance and transmit/receive antenna gains. Utilizing this information, the SU_{TX} controls its average transmit power to always meet the interference constraint at the PU_{RX} . With this assumption, the interference from the SU_{TX} to the PU_{RX} can be neglected. Thanks to the assistance of SU_R in forwarding the primary signal, there exists a dedicated feedback channel from PU_{RX} to SU_R , i.e., the instantaneous channel power gain of the link from SU_R to PU_{RX} is available at SU_R . Let x_s and x_p be, respectively, the transmit signals at SU_{TX} and PU_{TX} with corresponding average transmit powers $P_p = E\{|x_p|^2\}$ and $P_s = E\{|x_s|^2\}$. Before transmitting on the i -th antenna, the transmit signal at SU_{TX} is weighted by a complex number $w_{1,i} = h_{1,i}/\|\mathbf{h}_1\|_F$ where $h_{1,i}$ is the channel coefficient of the link from the i -th transmit antenna at SU_{TX} to the receive antenna at SU_R . Here, $\mathbf{h}_1$ is a $1 \times N_1$ fading channel coefficient vector of the channel from SU_{TX} to SU_R whose elements are modeled as Nakagami- m fading with fading severity parameter m_1 and channel mean power Ω_1 . Then, a $1 \times N_1$ beamforming vector $\mathbf{w}_1$ at SU_{TX} is constructed by collecting the weights for all antennas, $\mathbf{w}_1 = \mathbf{h}_1/\|\mathbf{h}_1\|_F$. Similarly, a $1 \times N_3$

beamforming vector at PU_{TX} for the primary transmission is selected as $\mathbf{w}_5 = \mathbf{h}_5/\|\mathbf{h}_5\|_F$, where $\mathbf{h}_5$ is a $1 \times N_3$ fading channel coefficient vector of the channel from PU_{TX} to PU_{RX} whose elements are modeled as Nakagami- m fading with fading severity parameter m_5 and channel mean power Ω_5 . As a consequence, in the first hop, the received signal y_r at SU_R and y_{p1} at PU_{RX} are, respectively, given as

$$y_r = \mathbf{h}_1 \mathbf{w}_1^H x_s + \mathbf{h}_3 \mathbf{w}_5^H x_p + n_r \quad (1)$$

$$y_{p1} = \mathbf{h}_5 \mathbf{w}_5^H x_p + n_{p1} \quad (2)$$

where $\mathbf{h}_3$ is a $1 \times N_3$ fading channel coefficient vector of the channel from PU_{TX} to SU_R whose elements are modeled as Nakagami- m fading with fading severity m_3 and channel mean power Ω_3 . Moreover, n_r and n_{p1} are, respectively, the additive white Gaussian noises (AWGNs) at SU_R and PU_{RX} with zero mean and variance N_0 . At SU_R , the received signal is then amplified with a factor G and forwarded to SU_{RX} . On one hand, SU_R must control its transmit power to keep the interference power from the secondary transmission at PU_{RX} less than a predefined threshold Q . On the other hand, PU_{TX} expects to increase the transmit power to achieve sufficient signal-to-noise ratio (SNR). To compromise these conditions, the amplifying gain G is selected to maintain equilibrium as $E\{|G \mathbf{h}_1 \mathbf{w}_1^H x_s|^2\} = Q/|h_4|^2$, or $G^2 = Q/P_s \|\mathbf{h}_1\|_F^2 |h_4|^2$. Here, h_4 is the channel coefficient from SU_R to PU_{RX} with fading severity parameter m_4 and channel mean power Ω_4 . Therefore, the received signals at SU_{RX} and PU_{RX} are, respectively, given by

$$y_s = G \mathbf{w}_2^H \mathbf{h}_2 \mathbf{h}_1 \mathbf{w}_1^H x_s + G \mathbf{w}_2^H \mathbf{h}_2 \mathbf{h}_3 \mathbf{w}_5^H x_p + G \mathbf{w}_2^H \mathbf{h}_2 n_r + n_s \quad (3)$$

$$y_{p2} = G h_4 \mathbf{h}_1 \mathbf{w}_1^H x_s + G h_4 \mathbf{h}_3 \mathbf{w}_5^H x_p + G h_4 n_r + n_{p2} \quad (4)$$

where n_s and n_{p2} are, respectively, the AWGN at SU_{RX} and PU_{RX} with zero mean and variance N_0 . Next, $\mathbf{h}_2$ is a $1 \times N_2$ fading channel coefficient vector of the channel from SU_R to SU_{RX} whose elements are Nakagami- m fading with fading severity parameter m_2 and channel mean power Ω_2 . Finally, $\mathbf{w}_2$ is a $1 \times N_2$ beamforming vector at SU_{RX} , $\mathbf{w}_2 = \mathbf{h}_2/\|\mathbf{h}_2\|_F$. As observed from the system model in Fig. 1, the primary and secondary transmissions affect each other. In (3), the terms $G \mathbf{w}_2^H \mathbf{h}_2 \mathbf{h}_3 \mathbf{w}_5^H x_p$ and $G \mathbf{w}_2^H \mathbf{h}_2 \mathbf{h}_1 \mathbf{w}_1^H x_s$ are the interference of the secondary and primary networks, respectively. Therefore, the instantaneous signal-to-interference plus noise ratio (SINR) at SU_{RX} , γ_S , can be obtained from (3) as

$$\gamma_S = \frac{X_1 X_2}{a X_2 X_3 + b X_1 X_4 + c X_2} \quad (5)$$

where $X_l = \|\mathbf{h}_l\|_F^2$, $l = 1, 2, 3$ and $X_4 = |h_4|^2$, $a = \frac{P_p}{P_s}$, $b = \frac{N_0}{Q}$, and $c = \frac{N_0}{P_s}$. The instantaneous SNR of the direct link $PU_{TX} \rightarrow PU_{RX}$, γ_{P1} , and the instantaneous SINR of the relaying link $PU_{TX} \rightarrow SU_R \rightarrow PU_{RX}$, γ_{P2} , are, respectively, determined from (2) and (4) as

$$\gamma_{P1} = g X_5 \quad (6)$$

$$\gamma_{P2} = X_3/(d X_1 + f) \quad (7)$$

where $X_5 = \|\mathbf{h}_5\|_F^2$, $d = \frac{P_s}{P_p} \left(1 + \frac{N_0}{Q}\right)$, $f = \frac{N_0}{P_p Q}$, and $g = P_p/N_0$. Given $G^2 = \frac{Q}{P_s \|\mathbf{h}_1\|_F^2 \|\mathbf{h}_4\|^2}$, then X_4 does not appear in (7). It is noted that X_l , $l \in \{1, 2, 3, 5\}$, is formulated as the summation of independent and identically distributed gamma RVs. Thus, X_1 , X_2 , X_3 , and X_5 are, respectively, gamma distributed RVs with parameter sets $(N_1 m_1, \alpha_1^{-1})$, $(N_2 m_2, \alpha_2^{-1})$, $(N_3 m_3, \alpha_3^{-1})$, and $(N_3 m_5, \alpha_5^{-1})$ with $\alpha_l = \frac{m_l}{\Omega_l}$. Note that a gamma distributed RV X with parameters (m, α^{-1}) , where m is a positive integer, has, respectively, the PDF and CDF as

$$f_X(x) = \frac{\alpha^m}{\Gamma(m)} x^{m-1} \exp(-\alpha x) \quad (8)$$

$$F_X(x) = 1 - \exp(-\alpha x) \sum_{p=0}^{m-1} \frac{\alpha^p x^p}{p!} \quad (9)$$

III. OUTAGE PROBABILITY

A. Outage Probability of the Primary Network

Assume that selection combining (SC) is applied at PU_{RX} to select the signal between the relaying and direct transmission of the primary system. Therefore, the instantaneous end-to-end SNR of the primary transmission can be attained as $\gamma_P = \max(\gamma_{P_1}, \gamma_{P_2})$. Utilizing the order statistics theory, the CDF of γ_P is given by

$$F_{\gamma_P}(\gamma) = F_{\gamma_{P_1}}(\gamma) F_{\gamma_{P_2}}(\gamma) \quad (10)$$

From (6) and (7), $F_{\gamma_{P_1}}(\gamma)$ and $F_{\gamma_{P_2}}(\gamma)$ are, respectively, determined as

$$F_{\gamma_{P_1}}(\gamma) = 1 - \exp\left(-\frac{\alpha_5 \gamma}{g}\right) \sum_{k=0}^{N_3 m_5 - 1} \frac{\alpha_5^k \gamma^k}{k! g^k} \quad (11)$$

$$F_{\gamma_{P_2}}(\gamma) = \int_0^\infty F_{X_3}[\gamma(d \cdot x_1 + f)] f_{X_1}(x_1) dx_1 \quad (12)$$

From (9) together with the binomial theorem [11, eq. (1.111)], we obtain an expression for $F_{X_3}[\gamma(d \cdot x_1 + e)]$. Substituting this outcome and the expression of $f_{X_1}(x_1)$ given in (8) into (12), we have

$$\begin{aligned} F_{\gamma_{P_2}}(\gamma) &= 1 - \exp(-f \alpha_3 \gamma) \sum_{l=0}^{N_3 m_3 - 1} \frac{\alpha_3^l \gamma^l}{l!} \sum_{i=0}^l \frac{C_i^l d^i f^{l-i}}{\Gamma(N_1 m_1)} \\ &\times \alpha_1^{N_1 m_1} \int_0^\infty x_1^{N_1 m_1 + i - 1} \exp(-(\alpha_1 + \alpha_3 d \gamma) x_1) dx_1 \end{aligned} \quad (13)$$

Then, applying [11, eq. (3.381.4)] to solve the remaining integral of (13), we obtain $F_{\gamma_{P_2}}(\gamma)$ as

$$\begin{aligned} F_{\gamma_{P_2}}(\gamma) &= 1 - \sum_{l=0}^{N_3 m_3 - 1} \sum_{i=0}^l \frac{C_i^l \Gamma(N_1 m_1 + i) \alpha_1^{N_1 m_1} f^{l-i}}{l! \Gamma(N_1 m_1) \alpha_3^{N_1 m_1 + i - l} d^{N_1 m_1}} \\ &\times \left(\gamma + \frac{\alpha_1}{\alpha_3 d}\right)^{-N_1 m_1 - i} \gamma^l \exp(-f \alpha_3 \gamma) \end{aligned} \quad (14)$$

Substituting (14) and (11) into (10), an exact expression for the CDF of γ_P is given by

$$\begin{aligned} F_{\gamma_P}(\gamma) &= 1 - \sum_{l=0}^{N_3 m_3 - 1} \sum_{i=0}^l \frac{C_i^l \Gamma(N_1 m_1 + i) \alpha_1^{N_1 m_1} f^{l-i}}{l! \Gamma(N_1 m_1) \alpha_3^{N_1 m_1 + i - l} d^{N_1 m_1}} \\ &\times \frac{\gamma^l \exp(-f \alpha_3 \gamma)}{\left(\gamma + \frac{\alpha_1}{\alpha_3 d}\right)^{N_1 m_1 + i}} - \sum_{k=0}^{N_3 m_5 - 1} \frac{\alpha_5^k \gamma^k}{k! g^k} \exp\left(-\alpha_5 \frac{\gamma}{g}\right) \\ &+ \sum_{l=0}^{N_3 m_3 - 1} \sum_{i=0}^l \frac{C_i^l \Gamma(N_1 m_1 + i)}{l! \Gamma(N_1 m_1)} \sum_{k=0}^{N_3 m_5 - 1} \frac{1}{k!} \frac{\alpha_1^{N_1 m_1} \alpha_5^k f^{l-i}}{\alpha_3^{N_1 m_1 + i - l}} \\ &\times \frac{\left(\gamma + \frac{\alpha_1}{\alpha_3 d}\right)^{-N_1 m_1 - i}}{d^{N_1 m_1} g^k} \gamma^{l+k} \exp\left(-\frac{f g \alpha_3 + \alpha_5}{g} \gamma\right) \end{aligned} \quad (15)$$

Outage probability is defined as the probability that the instantaneous SNR falls below a predefined threshold γ_{th}^P . The outage probability of the primary transmission can be calculated with (15) as $P_{out} = F_{\gamma_P}(\gamma_{th}^P)$.

B. Outage Probability of the Secondary Network

It can clearly be seen from (5) that the instantaneous SINR of the secondary transmission is mathematically represented by a complicated function of multiple independent RVs, i.e., X_i , $i = 1, 2, 3, 4$. Thus, finding an exact expression for $F_{\gamma_S}(\gamma)$, from (5) can be very difficult. Taking advantage of [12, eq. (25)], we adopt a tightly upper bounded expression γ_{SU} for γ_S as

$$\gamma_{SU} = \min(\gamma_{S_1}, \gamma_{S_2}) \quad (16)$$

where $\gamma_{S_1} = X_1/(aX_3 + c)$ and $\gamma_{S_2} = X_2/(bX_4)$. Since X_1 , X_2 , X_3 , and X_4 are independent, γ_{S_1} and γ_{S_2} are also mutually independent. As a result, the lower bounded CDF of γ_S can be derived from the CDFs of γ_{S_1} and γ_{S_2} based on the order statistics theory as

$$F_{\gamma_{SU}}(\gamma) = 1 - [1 - F_{\gamma_{S_1}}(\gamma)][1 - F_{\gamma_{S_2}}(\gamma)] \quad (17)$$

where

$$F_{\gamma_{S_1}}(\gamma) = \int_0^\infty F_{X_1}[\gamma(ax_3 + c)] f_{X_3}(x_3) dx_3 \quad (18)$$

$$F_{\gamma_{S_2}}(\gamma) = \int_0^\infty F_{X_2}(b\gamma x_4) f_{X_4}(x_4) dx_4 \quad (19)$$

Using (9) along with the binomial theorem [11, eq. (1.111)], we get $F_{X_1}(\gamma(ax_3 + c))$. Next, substituting this outcome and $f_{X_3}(x_3)$ given in (8) into (18); the expression of $F_{X_2}(b\gamma x_4)$ given in (9) and $f_{X_4}(x_4)$ given in (8) into (19), then applying [11, eq. (3.381.4)] to solve the remaining integrals, expressions for the CDF of γ_{S_1} and γ_{S_2} are obtained as

$$\begin{aligned} F_{\gamma_{S_1}}(\gamma) &= 1 - \exp(-\alpha_1 c \gamma) \sum_{p=0}^{N_1 m_1 - 1} \sum_{i=0}^p \frac{C_i^p}{p!} \frac{c^{p-i} \alpha_3^{N_3 m_3}}{a^{N_3 m_3} \alpha_1^{N_3 m_3 - p + i}} \\ &\times \frac{\Gamma(N_3 m_3 + i)}{\Gamma(N_3 m_3)} \frac{\gamma^p}{(\gamma + \alpha_3/a \alpha_1)^{N_3 m_3 + i}} \end{aligned} \quad (20)$$

$$F_{\gamma_{S_2}}(\gamma) = 1 - \sum_{q=0}^{N_2 m_2 - 1} \frac{1}{q!} \frac{\Gamma(m_4 + q)}{\Gamma(m_4)} \frac{\alpha_4^{m_4}}{\alpha_2^{m_4} b^{m_4}} \frac{\gamma^q}{\left(\gamma + \frac{\alpha_4}{\alpha_2 b}\right)^{m_4 + q}} \quad (21)$$

Finally, substituting (20) and (21) into (17), an expression for the CDF of γ_{SU} can be written as

$$F_{\gamma_{SU}}(\gamma) = 1 - \sum_{p=0}^{N_1 m_1 - 1} \sum_{i=0}^p \sum_{q=0}^{N_2 m_2 - 1} \frac{C_i^p}{p! q!} \frac{\Gamma(m_4 + q)}{\Gamma(m_4)} \times \frac{\Gamma(N_3 m_3 + i)}{\Gamma(N_3 m_3)} \frac{c^{p-i}}{a^{N_3 m_3} b^{m_4}} \frac{\alpha_3^{N_3 m_3} \alpha_4^{m_4}}{\alpha_1^{N_3 m_3 - p + i} \alpha_2^{m_4}} \times \frac{\gamma^{p+q}}{\left(\gamma + \frac{\alpha_3}{a \alpha_1}\right)^{N_3 m_3 + i} \left(\gamma + \frac{\alpha_4}{\alpha_2 b}\right)^{m_4 + q}} \exp(-c \alpha_1 \gamma) \quad (22)$$

Therefore, outage probability of the secondary transmission can be directly obtained by using the outage threshold, γ_{th}^S , as the argument for the expression in (22), $P_{out}^S = F_{\gamma_S}(\gamma_{th}^S)$.

IV. ASYMPTOTIC ANALYSIS FOR THE PRIMARY NETWORK

As can be seen from (15), the exact expression for the outage probability of the primary network is too complicated to render insights into the system performance. Thus, asymptotic expressions for the outage probability and SER in the high SNR regime are derived. In principle, the MacLaurin expansion for the CDF of the end-to-end instantaneous SNR, γ_P , at zero is deployed to analyze the asymptotic performance. Thus, we now adopt the MacLaurin expansion for the CDFs of γ_{P_1} and γ_{P_2} , then substituting these outcomes into (10) to achieve an asymptotic expression for γ_P . From (6), we have $F_{\gamma_{P_1}}(\gamma) = F_{X_5}(\gamma/g)$. Utilizing the MacLaurin expansion, in the high SNR regime, $f_{X_5}(x_5)$ is approximated as

$$f_{X_5}(x_5) = \frac{\alpha_5^{N_3 m_5}}{\Gamma(N_3 m_5)} x_5^{N_3 m_5 - 1} + o(x_5^{N_3 m_5 - 1}) \quad (23)$$

Hence, we obtain an approximation for $F_{X_5}(x_5)$ as

$$F_{X_5}(x_5) = \frac{\alpha_5^{N_3 m_5}}{\Gamma(N_3 m_5 + 1)} x_5^{N_3 m_5} + o(x_5^{N_3 m_5}) \quad (24)$$

Consequently, an asymptotic expression for $F_{\gamma_{P_1}}(\gamma)$ is found as

$$F_{\gamma_{P_1}}^\infty(\gamma) = \frac{\alpha_5^{N_3 m_5}}{\Gamma(N_3 m_5 + 1)} \frac{\gamma^{N_3 m_5}}{g^{N_3 m_5}} \quad (25)$$

In addition, taking the derivative of both sides of (12), the n -th order derivative of $F_{\gamma_{P_2}}(\gamma)$ at zero is given by

$$F_{\gamma_{P_2}}^n(\gamma)|_{\gamma=0} = \int_0^\infty \frac{\partial^n [F_{X_3}(\Psi(x_1, \gamma))]}{\partial \gamma^n} \Big|_{\gamma=0} f_{X_1}(x_1) dx_1 \quad (26)$$

where $\Psi(x_1, \gamma) = \gamma(d \cdot x_1 + e)$. Utilizing the similar approach, an asymptotic expression for $F_{X_3}(x_3)$ in the high SNR regime can be expressed as

$$F_{X_3}(x_3) = \frac{\alpha_3^{N_3 m_3}}{\Gamma(N_3 m_3 + 1)} x_3^{N_3 m_3} \quad (27)$$

Then, applying [11, eq. (0.430.1)], the n -th order derivative of $F_{X_3}(\Psi(x_1, \gamma))$ with respect to γ at zero can be given by

$$\frac{\partial^n [F_{X_3}(\Psi(x_1, \gamma))]}{\partial \gamma^n} \Big|_{\gamma=0} = \sum_{u=1}^n \sum_{v=0}^{u-1} C_v^u \frac{(-1)^v (\Psi(x_1, \gamma))^v}{\Gamma(u+1)} \times \frac{\partial^u F_{X_3}(\Psi(x_1, \gamma))}{\partial \Psi^u} \frac{\partial^n (\Psi(x_1, \gamma))^{u-v}}{\partial \gamma^n} \Big|_{\gamma=0} \quad (28)$$

It can be easily seen that

$$(\Psi(x_1, \gamma))^v \Big|_{\gamma=0} = \begin{cases} (\gamma(dx_1 + e))^v \Big|_{\gamma=0} = 0; & \text{if } v > 0 \\ (\gamma(dx_1 + e))^0 \Big|_{\gamma=0} = 1; & \text{if } v = 0 \end{cases} \quad (29)$$

$$\frac{\partial^u F_{X_3}(\Psi(x_1, \gamma))}{\partial \Psi^u} \Big|_{\gamma=0} = \begin{cases} \alpha_3^{N_3 m_3}; & \text{if } u = N_3 m_3 \\ 0; & \text{if } u < N_3 m_3 \end{cases} \quad (30)$$

Considering the case $v = 0$ and $u = N_3 m_3$, we have

$$\frac{\partial^n (\Psi(x_1, \gamma))^{u-v}}{\partial \gamma^n} \Big|_{\gamma=0} = \begin{cases} \Gamma(N_3 m_3 + 1) \sum_{t=1}^{N_3 m_3} C_t^{N_3 m_3} d^t x_1^t e^{N_3 m_3 - t}; & \text{if } n = N_3 m_3 \\ 0; & \text{if } n < N_3 m_3 \end{cases} \quad (31)$$

By substituting (29), (30), and (31) into (28), the n -th order derivative of $F_{X_3}(\Psi(x_1, \gamma))$ at $\gamma = 0$ is found as

$$\frac{\partial^n [F_{X_3}(\Psi(x_1, \gamma))]}{\partial \gamma^n} \Big|_{\gamma=0} = \begin{cases} \alpha_3^{N_3 m_3} \sum_{t=1}^{N_3 m_3} C_t^{N_3 m_3} d^t x_1^t e^{N_3 m_3 - t} & \text{if } n = u = N_3 m_3 \\ 0 & \text{if } u < N_3 m_3 \end{cases} \quad (32)$$

Putting (8) and (32) into (26) along with the help of [11, eq. (3.381.4)] to solve the remaining integral, we obtain the n -th order derivative of $F_{\gamma_{P_2}}(\gamma)$ at $\gamma = 0$ as

$$F_{\gamma_{P_2}}^{(n)}(\gamma)|_{\gamma=0} = \begin{cases} \sum_{t=1}^{N_3 m_3} C_t^{N_3 m_3} d^t e^{N_3 m_3 - t} \frac{\Gamma(N_1 m_1 + t)}{\Gamma(N_1 m_1)} \frac{\alpha_3^{N_3 m_3}}{\alpha_1^{N_1 m_1}} & \text{if } n = N_3 m_3 \\ 0 & \text{if } n < N_3 m_3 \end{cases} \quad (34)$$

Thus, an asymptotic expression for $F_{\gamma_{P_2}}(\gamma)$ in the high SNR regime is given by

$$F_{\gamma_{P_2}}^\infty(\gamma) = \sum_{t=0}^{N_3 m_3} C_t^{N_3 m_3} d^t e^{N_3 m_3 - t} \frac{\Gamma(N_1 m_1 + t)}{\Gamma(N_1 m_1)} \times \frac{\alpha_3^{N_3 m_3}}{\alpha_1^{N_1 m_1}} \frac{\gamma^{N_3 m_3}}{\Gamma(N_3 m_3 + 1)} \quad (35)$$

Substituting (25) and (35) into (10), an asymptotic expression for the instantaneous SINR of the primary transmission can

be finally established as

$$F_{\gamma_P}^{\infty}(\gamma) = \sum_{t=0}^{N_3 m_3} C_t^{N_3 m_3} \frac{d^t f^{N_3 m_3-t}}{g^{N_3 m_5}} \frac{\Gamma(N_1 m_1 + t)}{\Gamma(N_1 m_1)} \times \frac{\alpha_3^{N_3 m_3} \alpha_5^{N_3 m_5}}{\alpha_1^{N_1 m_1}} \frac{\gamma^{N_3 m_5 + N_3 m_3}}{\Gamma(N_3 m_3 + 1) \Gamma(N_3 m_5 + 1)} \quad (36)$$

Now, we can obtain an asymptotic expression for the outage probability as $P_{out}^{P,\infty} = F_{\gamma_P}^{\infty}(\gamma_{th}^P)$. As pointed out in [13], in order to quantify the diversity gain of the system, we need to transform the expression of the outage probability of the primary system into the following form

$$F_{\gamma_P}^{\infty}(\gamma) = (G_c \rho)^{-G_d} + o(\rho^{-G_d}) \quad (37)$$

where $\rho = \frac{P_p}{N_0}$ denotes the average SNR of the primary system and $o(\rho^{-G_d})$ stands for the higher order terms of $F_{\gamma_P}^{\infty}$, i.e., $\lim_{\rho \rightarrow \infty} \frac{o(\rho^{-G_d})}{F_{\gamma_P}^{\infty}} = 0$. Furthermore, G_d and G_c are the diversity and coding gain of the primary system, respectively. In cooperative systems, G_d is a crucial performance metric which determines the slope of the curve of the outage probability or SER versus average SNR on a log scale. Furthermore, the coding gain of the network, G_c , determines the shift of the asymptotic outage probability curve as compared to the benchmark curve ρ^{-G_d} . It is straightforward to see from (36) that the diversity gain of the primary system is $G_d = N_3 m_5 + N_3 m_3$.

For the secondary network, the performance does not only depend on its transmit power at SU_{TX} but also the interference power threshold Q at PU_{RX} . With a fixed transmit power at SU_{TX} , for low value of Q , the outage probability of the secondary network will decrease corresponding to the increase of Q . However, when the value of Q exceeds a threshold, for which the transmit power of SU_{TX} and SU_R always satisfy the interference constraint, the outage probability of the secondary network is almost invariant to the increase of Q (see Fig. 4 and Fig. 5). Thus, there is no slope of the curve of the outage probability versus Q/N_0 in a log scale. Therefore, we do not investigate asymptotic performance for the secondary system.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present analytical results in comparison with Monte Carlo simulations. Let d_1, d_2, d_3, d_4 , and d_5 be the normalized distances of the links $SU_{TX} \rightarrow SU_R$, $SU_R \rightarrow SU_{RX}$, $PU_{TX} \rightarrow SU_R$, $SU_R \rightarrow PU_{RX}$, and $PU_{TX} \rightarrow PU_{RX}$, respectively. Assume that the path-loss decays with an exponent of 4 for highly shadowed urban environment.

First, we show the impact of fading severity parameters and the number of antennas at PU_{TX} on the primary performance when fixing $d_1 = d_2 = d_5 = 0.5$ and $d_3 = d_4 = 0.5$ in the following cases:

- Case 1: $N_1 = N_2 = 2, N_3 = 2, m_i = 0.5, i \in \{1, \dots, 5\}$
- Case 2: $N_1 = N_2 = 2, N_3 = 2, m_i = 1.0, i \in \{1, \dots, 5\}$
- Case 3: $N_1 = N_2 = 2, N_3 = 3, m_i = 1.0, i \in \{1, \dots, 5\}$

The interference power-to-noise ratio Q/N_0 of PU_{RX} and the average transmit power-to-noise ratio P_s/N_0 of SU_{TX} are selected as $Q/N_0 = 5$ dB and $P_s/N_0 = 10$ dB.

Fig. 2. Outage probability of the primary network for various fading severity parameters and different numbers of antennas at PU_{TX} .

Fig. 3. Outage probability of the primary network with and without the assistance of SU_R .

For all the considered cases, the outage threshold is set as $\gamma_{th}^P = \gamma_{th}^S = 3$ dB. As can be seen from Fig. 2 with the identical antenna configuration at the transceivers of Case 1 and Case 2, $N_1 = N_2 = N_3 = 2$, the primary performance in Case 2 is better than that in Case 1. This is attributed to the fact

Fig. 4. Outage probability of the secondary network for different numbers of antennas at SU_{TX} and SU_{RX} .

Fig. 5. Outage probability of the secondary network with various distances from PU_{TX} to SU_R.

that Case 2 is endowed with more favorable fading conditions, $m_l = 1$, $l \in \{1, 2, 3, 4, 5\}$, compared to Case 1, $m_l = 0, 5$. Further, comparing Case 3 with Case 2, one can observe the effect of the number of antennas at PU_{TX} on the outage probability and SER. Clearly, the outage probability decreases significantly relative to the increase in the number of antennas at the primary transmitter. As expected, the asymptotic curves converge to the analytical results as well as the simulations in the high SNR regime. From these plots, the achievable diversity gains for Case 1, Case 2, and Case 3 are observed as 2, 4, and 6, respectively, which coincide with the asymptotic analysis.

Fig. 3 compares the outage probability of the primary networks with and without the assistance of SU_R. The network parameters are again selected as Case1, Case 2 and Case 3. As expected, in the high SNR regime, the performance of the primary network with the assistance of SU_R outperforms that of the system without the assistance of SU_R.

Fig. 4 exhibits the effect of the antenna configurations at SU_{TX} and SU_{RX} on the secondary performance. For these examples, the average transmit power-to-noise ratios P_p/N_0 at PU_{TX} and P_s/N_0 at SU_{TX} are selected as $P_p/N_0 = P_s/N_0 = 5$ dB. We change the number of antennas N_1 , N_2 while fixing the other parameters, $d_1 = d_2 = d_5 = 0.5$, $d_3 = 1.0$, $N_3 = 2$, and $m_l = 2$, $l \in \{1, \dots, 5\}$. As observed from Fig. 4, when the number of antennas at the secondary transceivers increases, the outage probability of the secondary network decreases. Thus, employing multiple antennas together with beamforming transmission seems to be a suitable solution to improve the system performance of an underlay cognitive AF relay network which suffers from a strict constraint on its transmit power.

Fig. 5 depicts outage probability of the secondary transmission for various values of the distance from PU_{TX} to SU_R, d_3 , when fixing the number of antennas $N_1 = N_2 = 4$, $N_3 = 2$ and distances $d_1 = d_2 = d_4 = d_5 = 0.5$. The average transmit power-to-noise ratios P_p/N_0 and P_s/N_0 are selected as $P_p/N_0 = P_s/N_0 = 5$ dB. It can be seen that when the distance between the primary transmitter and secondary relay become farther away, the secondary performance is improved.

This benefit can be attributed to the fact that the interference power imposed by the primary transmission to the secondary transmission reduces when this distance increases.

VI. CONCLUDING REMARKS

We have deployed beamforming transmission for the primary and secondary networks wherein the secondary relay facilitates both the primary and secondary communications. We have further analyzed the system performance of the network by deriving exact expressions for the outage probability of the primary network and tight bounded expressions for the outage probability of the secondary network. Moreover, an asymptotic analysis has been developed to reveal the diversity gain and coding gain of the primary network. Also, numerical examples have been presented to illustrate the impact of different network parameters on the system performance. The numerical results also indicate that the performance of the primary transmission of the considered network outperforms that of a network in which the secondary relay only assists the secondary transmission.

REFERENCES

- [1] H. Luo, Z. Zhang, and G. Yu, "Cognitive cooperative relaying," in *Proc. IEEE International Conference on Communications Systems*, Singapore, Nov. 2008, pp. 1499–1503.
- [2] T. M. C. Chu, H. Phan, and H.-J. Zepernick, "Performance analysis of MIMO cognitive amplify-and-forward relay networks with orthogonal space-time block codes," *Wirel. Commun. Mob. Comput. DOI: 10.1002/wcm.2449*, Jan. 2014.
- [3] L. Luo, P. Zhang, G. Zhang, and J. Qin, "Outage performance for cognitive relay networks with underlay spectrum sharing," *IEEE Commun. Lett.*, vol. 15, no. 7, pp. 710–712, Jul. 2011.
- [4] S. Kim, W. Choi, Y. Choi, J. Lee, Y. Han, and I. Lee, "Downlink performance analysis of cognitive radio based cellular relay networks," in *Proc. IEEE Cognitive Radio Oriented Wireless Networks and Communications*, Singapore, May 2008, pp. 1–6.
- [5] A. Ghasemi and E. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Commun. Magazine*, vol. 46, no. 4, pp. 32–39, Apr. 2008.
- [6] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless. Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.
- [7] A. A. Naguib, A. Elezabi, and M. Nafie, "A novel scheme for relay cooperation in interweave cognitive radio systems," in *Proc. International Symposium on Applied Sciences in Biomedical and Communication Technologies*, Rome, Italy, Nov. 2010, pp. 1–5.
- [8] K. Jitvanichphaibool, Y.-C. Liang, and R. Zhang, "Beamforming and power control for multi-antenna cognitive two-way relaying," in *Proc. IEEE Wireless Communications and Networking Conference*, Budapest, Hungary, Apr. 2009, pp. 1–6.
- [9] K. Zarifi, A. Ghayeb, and S. Affes, "Jointly optimal source power control and relay matrix design in multipoint-to-multipoint cooperative communication networks," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4313–4330, Sep. 2011.
- [10] T. M. C. Chu, H. Phan, and H.-J. Zepernick, "Amplify-and-forward relay assisting both primary and secondary transmissions in cognitive radio networks over Nakagami- m fading," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Sydney, Australia, Sep. 2012.
- [11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Academic Press, 2007.
- [12] M. D. Renzo, F. Graziosi, and F. Santucci, "A comprehensive framework for performance analysis of dual-hop cooperative wireless systems with fixed-gain relays over generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 5060–5074, 2009.
- [13] Z. Wang and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389–1398, Aug. 2003.