The Choquet Integral Applied to Ranking Therapies in Radiation Cystitis

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Abstract. We modify the classical fuzzy decision making model by adopting the concept of the Choquet integral as a measure of the therapy utility, when proving different treatments in radiation cystitis. The objective is to rank therapies as a sequence, commencing with the most efficacious remedy.

Keywords: Utility matrix, parametric membership functions, weights of importance, utilities of therapies, Choquet integral.

1 Introduction

Radiation cystitis is in general rarely occurring, which makes it very difficult to study in a large group of clinical trials. Most available data about radiation cystitis treatment come from a small number of descriptive studies or from expert opinions [2, 4]. As clinical data are considered to have low quality then physicians, who are still facing patients with a disease hugely influencing quality of life, mostly base on their own experience.

We thus want to test fuzzy decision-making model, regarded as a valuable tool, to help in selecting a patient-tailored treatment in radiation cystitis [12].

Theoretical fuzzy decision-making models [14–15], possessing the utility matrix filled with distinct utilities of pairs (decision, object-state), give rise to own trials of successfully accomplished applications concerning the item of medication [8, 12]. After interpreting pairs (decision, object-state) as (therapy, symptom), we intend to prove decision-making based on the Choquet integral to extract the optimal treatment in radiation cystitis. We wish to confront the results, obtained by the Choquet integral technique, with another model made for the purpose of radiation cystitis in [12].

In Section 2, we recall the basic data inserted into the model of fuzzy decision making. Section 3 provides weights of importance, regarding the symptoms' priorities to retreat. The discussion about the enumeration of utilities is accomplished in Section 4. The concepts of Choquet integrals, playing role of total utilities of therapies, are proved in Section 5, whereas some conclusions are added to Section 6.

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2 Basic Data in Fuzzy Decision Making

We introduce the notions of a space of states $X = \{x_1,...,x_m\}$ and a decision space (a space of alternatives) $A = \{a_1,...,a_n\}$. We consider a decision model, in which n alternatives $a_1,...,a_n \in A$ act as therapies used to treat patients who suffer from radiation cystitis. The therapies should influence m states $x_1,...,x_m \in X$, identified with m symptoms typical of the morbid unit considered.

When a decision maker applies therapy $a_i \in A$, $i = 1,…,n$, to symptom $x_j \in X$, $j = 1,...,m$, then a utility of treating x_j by a_i is determined. In order to sample all distinct utilities, assigned to pairs (a_i, x_j), we estimate the total utility $\boldsymbol{U_{a_i}}$ as [12, 14–15]

$$U_{a_i} = \text{aggregation of } u_{ij}, i = 1,...,n, j = 1,...,m \; , \tag{1}$$

in which each element u_{ij} is a utility following from the decision a_i with the result x_j. The most efficacious therapy will be associated with the highest value of $\boldsymbol{U_{a_i}}$.

Example 1
The symptoms, selected as the most decisive for radiation cystitis, are listed as x_1 = urgency, x_2 = dysuria, x_3 = urinary bladder pain, x_4 = macrohaematuria, x_5 = urine retention.

The treatments are extracted among a_1 = alum irrigation, a_2 = formalin instillation, a_3 = D-glucosamine, a_4 = oestrogens, a_5 = cystodiathermy, a_6 = interruption of internal illiac arteries, a_7 = bilateral percutaneous nephrostomy, a_8 = ileal diversion (with cystectomy), a_9 = pentoxyfilline and a_{10} = hyperbaric oxygen.

3 The Importance Weights Assisting Symptoms

The purpose of this section is to add other factors having impact on the solution of the decision-making model. We wish the model to be furnished with extraction of the most efficacious treatment, provided that the particular emphasis is also concentrated on assigning differing degrees of importance to states-symptoms [8, 12, 14–15].

Let us associate with each symptom x_j, $j = 1,…,m$, a non negative number that indicates its power or importance in the decision according to the rule: the higher the number is, the more important role of x_j's retreat will be regarded. We assign $w_1,…,w_m$ as powers-weights to $x_1,…,x_m$. A procedure for obtaining a ratio scale of importance for a group of m symptoms is developed by the authors as a novelty.

Generally, if we consider m symptoms X_j to find importance weights for them, we want to place them in the sequence $X_1 > X_2 > ... > X_m$ in accordance with the expert's opinion. We explain that the symbol ">" stands for "more important than". We wish the sum of all weights W_j, tied to Xj, $j = 1,...,m$, to be 1 as to

$$m \cdot r + (m-1) \cdot r + \cdots + 2 \cdot r + 1 \cdot r = 1, \tag{2}$$

where r is a quotient depending on m. The weights W_j constitute a new order of old w_j, due to the sequence $X_1>X_2>...>X_m$.

Further,

$$W_j = (m - j + 1) \cdot r \tag{3}$$

for $j = 1,...,m$.

Example 2
The physician intends to release the patient from the symptoms with the following priorities: x_4 – priority 1, x_3 – priority 2, x_5 = priority 3, x_2 – priority 4 and x_1 – priority 5.

We thus reorder the sequence of the symptoms x_j from Ex.1, named X_j now, in a new placement

$X_1 = x_4 > X_2 = x_3 > X_3 = x_5 > X_4 = x_2 > X_5 = x_1$.

For $m = 5$, due to (2), the equation $5 \cdot r + 4 \cdot r + 3 \cdot r + 2 \cdot r + 1 \cdot r = 1$ provides $r = 0.066$. We get
$W_1 = (5 - 1 + 1) \cdot 0.066 = 0.33$, $W_2 = (5 - 2 + 1) \cdot 0.066 = 0.264$,
$W_3 = (5 - 3 + 1) \cdot 0.066 = 0.198$, $W_4 = (5 - 4 + 1) \cdot 0.066 = 0.132$,
$W_5 = (5 - 5 + 1) \cdot 0.066$.

The common utility U_{a_i} of therapy a_i is now computed as

$$U_{a_i} = \sum_{j=1}^{m} U_{ij} \cdot W_j \tag{4}$$

where U_{ij} are the utilities of using a_i to treat X_j, $j = 1,\dots,m$. We emphasize that U_{ij} replace u_{ij}, introduced in (1), in compliance with weighed symptom order $X_1>X_2>...>X_m$.

To find a numerical expression for the utility U_{ij}, verbally decided by a physician for each pair (a_i, X_j), we suggest the analysis of the process following the next session.

4 Creation of Numerical Expressions for Utilities

We first want the utility, estimating the remission of symptom X_j after treating it by a_i, $i = 1,\dots,n$, $j = 1,..,m$, to be verbally expressed in order to facilitate the communication with a professional adviser. We generate a linguistic list named L = "Utility of applying a_i to X_j". L becomes stated, e.g., as [12]

"Utility U_{ij} of applying a_i to X_j" = {N_1 ="*none*", N_2 = "*almost none*", N_3 = "*very little*", N_4 = "*little*", N_5 = "*rather little*", N_6 = "*moderate*", N_7 = "*rather large*", N_8 = "*large*", N_9 = "*very large*", N_{10} = "*almost complete*", N_{11} = "*complete*"}.

Each expression N_q, $q = 1,..,11$, will be replaced by a fuzzy set, also named N_q.

By linking the therapy to the symptom for pair (a_i, X_j), a physician selects expression N_q from the list due to his experience. It means that $U_{ij} = N_q$ in practice. We, in turn, should assign a numerical representative to fuzzy set N_q, assisting each verbal description.

In order not to generate the boundaries of fuzzy sets N_q in an ad hoc manner, we suppose that $L = \{N_1, \dots, N_\omega\}$ is a general linguistic list consisting of ω words. Each word is associated with a fuzzy set. The number ω is a positive odd integer. Furthermore, let E be the length of a set R, designed for all restrictions of the fuzzy sets from L, provided that $z \in R$. We now wish to divide the linguistic terms into three groups recognized as a left group, a middle group and a right group. Albeit the trials of generating membership functions with modifiers for linguistic terms were already accomplished [1, 6], we propose the authors' own procedure of adopting parametric s-functions whose derivations can be followed in [9–11].

The membership functions, assigned to the leftmost terms, are prepared by (5) as

$$\mu_{N_t}(z) = \begin{cases} 1 & \text{for} \quad z \le \frac{E(\omega-1)}{2(\omega+1)}\delta(t), \\ 1 - 2\left(\frac{z - \frac{E(\omega-1)}{2(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 & \text{for} \quad \frac{E(\omega-1)}{2(\omega+1)}\delta(t) \le z \le \frac{E(\omega-1)}{2\omega}\delta(t), \\ 2\left(\frac{z - \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 & \text{for} \quad \frac{E(\omega-1)}{2\omega}\delta(t) \le z \le \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \\ 0 & \text{for} \quad z \ge \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \end{cases} \tag{5}$$

where $\delta(t) = \frac{2t}{\omega-1}, t = 1, \dots, \frac{\omega-1}{2}$ is a function depending on left function number t.

The membership function in the middle is expanded by (6) in the form of

$$\mu_{N_{\frac{\omega+1}{2}}}(z) = \begin{cases} 0 & \text{for} \quad z \le \frac{E(\omega-2)}{2\omega}, \\ 2\left(\frac{z - \frac{E(\omega-2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega-2)}{2\omega} \le z \le \frac{E(\omega-1)}{2\omega}, \\ 1 - 2\left(\frac{z - \frac{E}{2}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega-1)}{2\omega} \le z \le \frac{E}{2}, \\ 1 - 2\left(\frac{z - \frac{E}{2}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E}{2} \le z \le \frac{E(\omega+1)}{2\omega}, \\ 2\left(\frac{z - \frac{E(\omega+2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega+1)}{2\omega} \le z \le \frac{E(\omega+2)}{2\omega}, \\ 0 & \text{for} \quad z \ge \frac{E(\omega+2)}{2\omega}. \end{cases} \tag{6}$$

Finally, the membership functions on the right-hand side can be yielded by (7) as

$$\mu_{N_{\frac{\omega+3}{2}+t-1}}(z) = \begin{cases} 0 & \text{for} \quad z \le E - \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)} \cdot \varepsilon(t), \\ 2\left(\frac{z-\left(E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\cdot\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\cdot\varepsilon(t)}\right)^2 & \text{for} \quad E - \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)} \cdot \varepsilon(t) \le z \le E - \frac{E(\omega-1)}{2\omega} \cdot \varepsilon(t), \\ 1-2\left(\frac{z-\left(E-\frac{E(\omega-1)}{2(\omega+1)}\cdot\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\cdot\varepsilon(t)}\right)^2 & \text{for} \quad E - \frac{E(\omega-1)}{2\omega} \cdot \varepsilon(t) \le z \le E - \frac{E(\omega-1)}{2(\omega+1)} \cdot \varepsilon(t), \\ 1 & \text{for} \quad z \ge E - \frac{E(\omega-1)}{2(\omega+1)} \cdot \varepsilon(t). \end{cases} \tag{7}$$

A new function $\varepsilon(t) = 1 - \frac{2(t-1)}{\omega-1}, t = 1, \dots, \frac{\omega-1}{2}$ allows creating all rightmost functions one by one, when setting t-values in (7).

For $\omega = 11$ and $E = 100$ (the typical length of the reference set [0, 100] tested in medical investigations of frequencies) we derive membership functions of N_q, $q = 1,\dots,11$, by employing formulas (5)–(7).

In order to determine the most adequate representative for each N_q, $q = 1,\dots,11$, we first introduce another fuzzy set "total over [0, 100]" with the membership function

$$\mu_{\text{"total over [0,100]"}}(z) = \begin{cases} 0 & \text{for} \quad z \le 0, \\ 2\left(\frac{z}{100}\right)^2 & \text{for} \quad 0 \le z \le 50, \\ 1-\left(\frac{z-100}{100}\right)^2 & \text{for} \quad 50 \le z \le 100, \\ 1 & \text{for} \quad z \ge 100. \end{cases} \tag{8}$$

The membership degrees, being the second coordinates of intersection points between membership functions of each N_q and "total over [0, 100]", will constitute the numerical assignments of the terms allocated in L.

Figure 1 collects the graphs of N_q, $q = 1,\dots,11$, and "total over [0, 100]".

We thus connect N_1 to 0.0054, N_2 to 0.042, N_3 to 0.156, N_4 to 0.274, N_5 to 0.418 and N_6 to 0.5. The right fuzzy sets are identified by the following numbers: N_7 by 0.582, N_8 by 0.726, N_9 by 0.844, N_{10} by 0.958 and N_{11} by 0.9946.

Example 3

After considering therapies a_i and symptoms X_j in the radiation cystitis, the judgments of verbal utilities $U_{ij} = N_q$ for pairs (a_i, X_j), $i = 1,\dots,10$, $j = 1,\dots,5$, $q = 1,\dots,11$, are accomplished by physicians. We convert N_q into numbers like shown in Table 1. For the reason of sparse data reports, collected for the radiation cystitis, mostly the professional experience of physicians is involved in predicting the utility evaluations [12].

5 The Choquet Integral as the Total Utility of a Therapy

The quantities of medical estimates N_q, taking place in Table 1, are interpreted as utilities U_{ij} of the pairs (a_i, X_j).

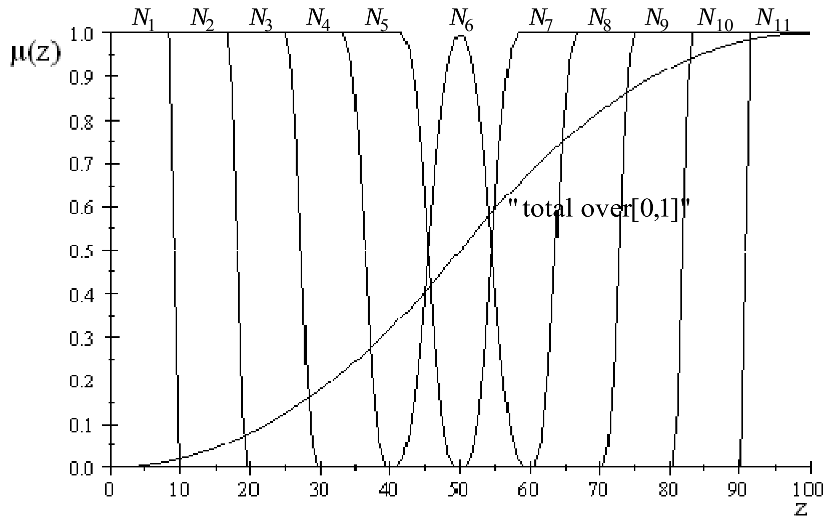

Fig. 1. The fuzzy sets N_1–N_{11} and “total over [0, 100]”

Table 1. Utilities $U_{ij} = N_q$ of pairs (a_i, X_j)

$a_i \backslash x_j$	X_1	X_2	X_3	X_4	X_5
a_1	N_7=0.582	N_3=0.156	N_5=0.418	N_3=0.156	N_1=0.0054
a_2	N_8=0.726	N_2=0.042	N_3=0.156	N_2=0.042	N_1=0.0054
a_3	N_2=0.042	N_3=0.156	N_2=0.042	N_3=0.156	N_3=0.156
a_4	N_5=0.418	N_4=0.274	N_4=0.274	N_6=0.5	N_6=0.5
a_5	N_5=0.418	N_3=0.156	N_5=0.418	N_1=0.0054	N_1=0.0054
a_6	N_{10}=0.958	N_4=0.274	N_3=0.156	N_2=0.042	N_1=0.0054
a_7	N_6=0.5	N_3=0.156	N_{10}=0.958	N_8=0.726	N_9=0.844
a_8	N_{10}=0.958	N_{11}=0.9946	N_{11}=0.9946	N_{11}=0.9946	N_{11}=0.9946
a_9	N_7=0.582	N_3=0.156	N_2=0.042	N_4=0.274	N_3=0.156
a_{10}	N_7=0.582	N_6=0.5	N_7=0.582	N_6=0.5	N_4=0.274

We remember that symptoms $X_1,...,X_m \in X$ act as objects in X. To them let us assign the measures $M(X_j|a_i) = U_{ij}$, where symbols $X_j|a_i$ reflect the association between symptom X_j and medicine a_i, $i = 1,\ldots,n$, $j = 1,\ldots,m$.

The weights W_j are set as the range values $f(X_j)$ of a function $f : X \to = [0, 1]$.
In this context formula (4) gets a new shape of

$$U_{a_i} = \sum_{j=1}^{m} U_{ij} \cdot W_j = \sum_{j=1}^{m} M\left(X_j | a_i \right) \cdot W_j \qquad (9)$$

The second part of formula (9) is comparable to the area of a figure, composed of rectangles. These possess one side equal to $M\left(X_j|a_i\right)$ and the other side measured as the W_j value.

Example 4

Let us estimate the influence of therapy a_1 on symptoms X_j, $j = 1,\ldots,5$, characteristic of the radiation cystitis. The total utility of a_1 is measured as the surface of the pattern sketched in Fig. 2.

Hence,

$U_{a_1} = \sum_{j=1}^{5} M\left(X_j \lceil a_1\right) \cdot W_j = 0.582 \cdot 0.33 + 0.156 \cdot 0.264 + 0.418 \cdot 0.198 + 0.156 \cdot 0.132 + 0.0054 \cdot 0.066 = 0.337.$

The same result will be reached after converting the contents of Fig. 2 to the figure drawn in Fig. 3.

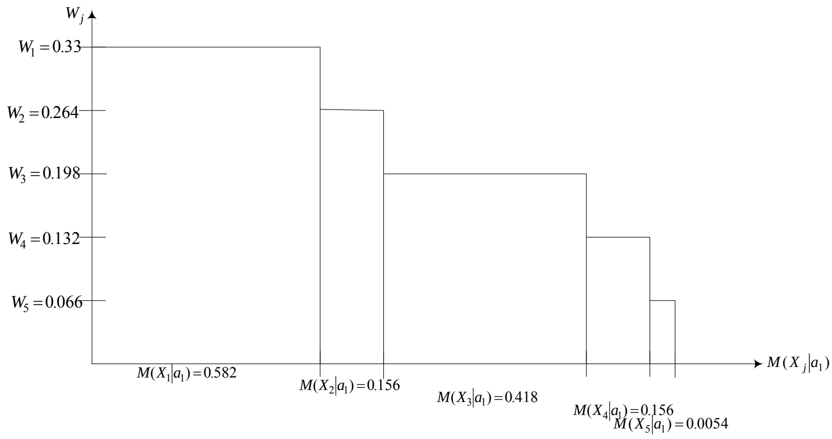

Fig. 2. The graph of $U_{a_1} = \sum_{j=1}^{5} M\left(X_j \lceil a_1\right) \cdot W_j$

The area of the pattern, sketched in Fig. 3, corresponds to

$U_{a_1} = 0.582(0.33 - 0.264) + (0.582 + 0.156)(0.264 - 0.198) + (0.582 + 0.156 + 0.418)(0.198 - 0.132) + (0.582 + 0.156 + 0.418 + 0.156)(0.132 - 0.066) + (0.582 + 0.156 + 0.418 + 0.156 + 0.0054)(0.066 - 0) = 0.337.$

The last result in Ex. 4 is fully compatible with

$$U_{a_1} = \\ (W_1 - W_2) \cdot M\left(X_{1\leq s\leq j}\middle|a_1 : f\left(X_{1\leq s\leq j}\right) \geq W_1\right)_{j=1} + \\ (W_2 - W_3) \cdot M\left(X_{1\leq s\leq j}\middle|a_1 : f\left(X_{1\leq s\leq j}\right) \geq W_2\right)_{j=2} + \\ (W_3 - W_4) \cdot M\left(X_{1\leq s\leq j}\middle|a_1 : f\left(X_{1\leq s\leq j}\right) \geq W_3\right)_{j=3} + \quad (10)$$

$(W_4 - W_5) \cdot M(X_{1 \le s \le j} | a_1: f(X_{1 \le s \le j}) \ge W_4)_{j=4} +$
$(W_5 - W_6) \cdot M(X_{1 \le s \le j} | a_1: f(X_{1 \le s \le j}) \ge W_5)_{j=5}$
$= \sum_{j=1}^{5} (W_j - W_{j+1}) \cdot M(X_{1 \le s \le j} | a_1) : f(X_{1 \le s \le j}) \ge W_j$

provided that $W_6 = 0$ and $M(X_{1 \le s \le j} | a_1) = \sum_{s=1}^{j} M(X_s | a_1)$.

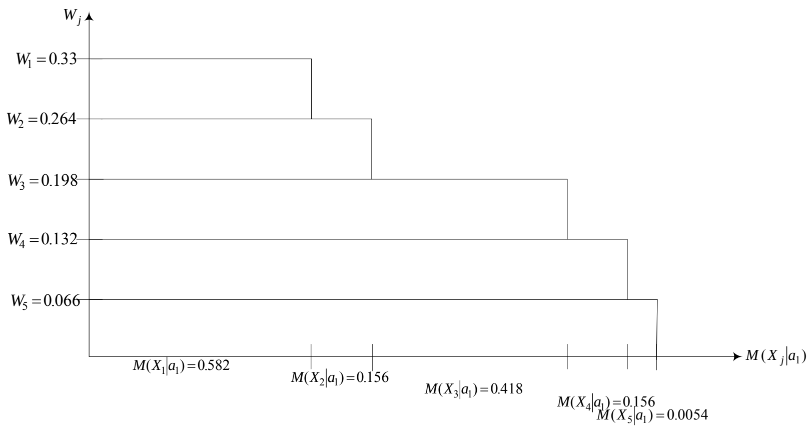

Fig. 3. The pattern of computing U_{a_1} after converting Fig. 2

Formula (10), on the other hand, can be formalized as

$$\sum_{j=1}^{5} (W_j - W_{j+1}) \cdot M(X_{1 \le s \le j} | a_1) : f(X_{1 \le s \le j}) \ge W_j = \int_{X_j \in \{X_1, X_2, X_3, X_4, X_5\}} f(X_j) dM(X_j | a_1) \quad (11)$$

and, after the confrontation with [3, 5, 13], is identified as the Choquet integral over $X = \{X_1, X_2, X_3, X_4, X_5\}$. In general, we are able to perform the computations of total utilities of a_i, $i = 1,\ldots,n$, by [7]

$$U_{a_i} = \int_{X_j \in X} f(X_j) dM((X_j | a_i) = \sum_{j=1}^{m} (W_j - W_{j+1}) \cdot M(X_{1 \le s \le j} | a_1) : f(X_{1 \le s \le j}) \ge W_j \quad (12)$$

We perform (12) to estimate: U_{a_2} =0.287, U_{a_3} = 0.094, U_{a_4}= 0.366, U_{a_5} = 0.263, U_{a_6}= 0.425, U_{a_7}= 0.547, U_{a_8} = 0.973, U_{a_9} = 0.288, and $U_{a_{10}}$= 0.523.

As the most efficacious therapy, a_i assists the largest U_{a_i} value, i = 1,…,10. We will thus set the therapies in the sequence $a_8 > a_7 > a_{10} > a_6 > a_4 > a_1 > a_9 > a_2 > a_5 > a_3$, when assuming that "$>$" means "$a_i$ shows the stronger power in receding symptoms than a_c, i, c = 1,…,n".

6 Conclusions

The basis of investigations has been mostly restricted to a judgment of the therapy influence on clinical symptoms characterizing radiation cystitis. In the classical model of fuzzy decision making [14–15] we have also employed the indices of the symptoms' importance to emphasize the essence of the symptom priority to recede in the final decision. By interpreting the utilities of treatments as measures we have furnished total utility estimates with such tools as the Choquet integrals to extend the model of the classical fuzzy decision-making. This complement to decision making constitutes an original contribution as it has been impossible to find the similar effects in literature apart from our own [7]. The method of constructing families of membership functions, depending only of the length of a reference set and the number of constraints, reveals another own idea of the authors.

The results of the decision model, obtained in this paper, converge to the final decision from [12]. These seem to be reasonable from the clinicians' point of view and, with some exceptions, they match results of the algorithm proposed by Martinez-Rodriques [4]. Nevertheless, we would strongly emphasize that the symptoms, their intensities and treatment efficacies are mostly based on the personal experience and obviously can vary among the centers. We also note that the treatment of radiation cystitis is most often multimodal when combining various methods. The final scale of therapy priorities from Section 5 should absolutely not be regarded as a guideline for future prognoses of treatments but the model itself, with dynamic input categories, seems to be a very valuable tool helping to determine the appropriate treatment path.

To sum up we would like to state that computational intelligence methods can constitute perfect bridges between the expert judgments and real evidence based medicine in case of diseases that lack data from the good quality clinical trials.

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