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Performance of Cognitive Radio Networks with Finite Buffer Using Multiple Vacations and Exhaustive Service

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Abstract—In this paper, we analyze the performance of a cognitive radio network where the secondary transmitter, besides its own transmission, occasionally relays the primary signal. It is assumed that the secondary transmitter employs the exhaustive service mode to transmit the secondary signal and multiple vacations to relay the primary signal. When assisting the primary transmitter, we assume that the secondary transmitter utilizes the decode-and-forward protocol to process the primary signal and forwards it to the primary receiver. Furthermore, the secondary transmitter has a finite buffer, the arriving packets of the secondary network are modeled as a Poisson process, and all channels are subject to Nakagami- m fading. Modeling the system as an M/G/1/K queueing system with exhaustive service and multiple vacations, using an embedded Markov chain approach to analyze the system, we obtain several key queueing performance indicators, i.e., the channel utilization, blocking probability, mean number of packets, and mean serving time of a packet in the system. The derived formulas are then utilized to evaluate the performance of the considered system.

I. INTRODUCTION

Due to the growth of ubiquitous wireless communications, which inflicts increasing stress on the limited radio spectrum, cognitive radio technology has been developed as a means of alleviating the scarcity of radio spectrum [1]. In principle, there exist three significant schemes of the spectrum access, i.e., interweave, overlay, and underlay schemes [2]. In the interweave spectrum access, the secondary users opportunistically occupy the licensed spectrum when the primary user does not use its spectrum. This scheme requires the secondary users to sense the status of the primary channels to decide whether or not the channels are idle. In the overlay spectrum access scheme, secondary users can operate on the primary users' spectrum in concurrence with the primary users provided that the secondary network applies interference cancellation techniques to get rid of the interference to the primary network. With underlay spectrum access, a secondary user is allowed to access the primary radio spectrum at any time provided that the interference power at the primary receiver imposed by the secondary transmission is kept below a predefined threshold. The underlay spectrum access is endowed with several important advantages such as high spectrum utilization and simple implementation which has led to an increasing interest in this approach (see [3]–[6], and the references therein).

Although numerous recent studies have investigated outage probability or symbol error rate (SER) for cognitive radio networks (CRNs) (see [7], [8], and the references therein), there are few works evaluating queueing performance such as throughput, transmission delay, channel utilization, and blocking probability for CRNs. Specifically, modeled as an M/G/1 queueing system with infinite buffer, Poisson arrival process, and deterministic departure process, the system performance of a spectrum sharing system has been addressed in [9]. In addition, in [10], considering the Poisson arrival process and deterministic departure process, the performance of an opportunistic spectrum access system, which is modeled as an M/D/1 infinite queue, was investigated. Taking the time-out transmission and infinite queueing buffer into account, the studies of [9], [10] have investigated the queueing performance of a CRN. Recently, assuming a finite buffer at the secondary transmitter, the queueing performance of a CRN, which is modeled as an M/G/1/K queueing system, has been assessed in [11].

In this paper, we analyze the queueing behavior of an underlay CRN where the secondary transmitter serves in two roles, transmitting its own signal and relaying the primary signal. The idea of letting the secondary user forward the primary signal has been shown as a means of providing significant spatial diversity for the primary user [12], [13]. In the considered system, we assume that the secondary transmitter is subject to the interference power threshold of the primary receiver. Also, the secondary transmitter is equipped with a finite buffer, its arriving packets follow a Poisson process, and all channels are modeled as Nakagami- m fading. Applying an embedded Markov chain, the secondary system is modeled as an M/G/1/K queueing system with multiple vacations and exhaustive service. Then, the blocking probability, channel utilization, mean serving time of packet in the system, and mean number of packets in the system are analyzed.

The rest of this paper is organized as follows. A description of the system and channel models are given in Section II. Section III presents the queueing performance analysis. Selected numerical examples and discussions are provided in Section IV. Finally, Section V concludes the paper.

Notation: The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable

Fig. 1. System model for primary relaying assisted by a secondary network with finite buffer using multiple vacations and exhaustive service.

(RV) X are denoted as $f_X(\cdot)$ and $F_X(\cdot)$, respectively. $\Gamma(n)$ denotes the gamma function [14, eq. (8.310.1)] and $\Gamma(n, x)$ is the incomplete gamma function [14, eq. (8.350.2)]. Finally, $\mathbb{E}[\cdot]$ is the expectation operator.

II. SYSTEM AND CHANNEL MODEL

We consider a single hop underlay CRN, comprising a secondary transmitter SU_{TX} and a secondary receiver SU_{RX} , coexisting with a primary transmitter PU_{TX} and a primary receiver PU_{RX} (see Fig. 1). Besides its own transmission, to improve the performance of the primary communication, the secondary transmitter SU_{TX} offers to relay the transmission of PU_{TX} as long as it has no data in the buffer to transmit. In particular, the secondary transmitter SU_{TX} starts a vacation to forward a signal of the primary transmission only if the buffer becomes empty, so-called exhaustive service. We assume that SU_{TX} utilizes the decode-and-forward (DF) protocol to relay the primary signal since it can cancel the noise at SU_{TX} . It continues being on vacation if no secondary packets exist at SU_{TX} at the end of each vacation. Until there is at least one packet waiting upon an ended vacation, SU_{TX} returns to transmit its own packets. This paradigm is referred to as the multiple vacation model. Usually, the primary transmission is performed through the direct transmission from PU_{TX} to PU_{RX} . However, when SU_{TX} goes on vacation, it informs the PU_{TX} being available to relay the primary signal. The works of [12], [13] have revealed that with the assistance of SU_{TX} in forwarding the primary signal, the performance of the primary transmission is significantly enhanced. Thus, we do not delve into analyzing the performance of the primary network any more. Instead, we focus on analyzing the performance of the secondary network. Assume that all the involved channels are modeled as Nakagami- m fading with fading severity parameter m which covers a wide range of other fading channels as special cases. Let s_0 be the transmit signal of the secondary network with average transmit power, $P_0 = \mathbb{E}[|s_0|^2]$. Constrained by the interference power threshold Q imposed by PU_{RX} and the transmit power limit, P_{max} , the average transmit power P_0 of the secondary transmitter is determined as

$$P_0 = \min \{P_{max}, Q/|h_2|^2\} \quad (1)$$

where h_2 is the channel coefficient of the link from SU_{TX} to PU_{RX} with channel mean power Ω_2 , and positive integer fading severity parameter m_2 . Denote n_0 as the additive white Gaussian noise (AWGN) at SU_{RX} with zero-mean and variance N_0 and h_0 as the channel coefficient of the link from SU_{TX} to SU_{RX} with channel mean power Ω_0 , and positive integer fading severity parameter m_0 . Then, the received signal at SU_{RX} is given by

$$y_0 = h_0 s_0 + n_0 \quad (2)$$

From (1) and (2), the instantaneous signal-to-noise ratio (SNR) of the secondary transmission can be expressed as

$$\gamma_0 = \min \{\gamma_{max}|h_0|^2, \gamma_Q|h_0|^2/|h_2|^2\} \quad (3)$$

where $\gamma_{max} = P_{max}/N_0$ and $\gamma_Q = Q/N_0$. When the secondary user SU_{TX} is on vacation to forward the primary signal, the received signal at PU_{RX} can be formulated as

$$y_2 = h_2 s_2 + n_2 \quad (4)$$

where s_2 is the decoded symbol for the primary transmission at SU_{TX} which is then forwarded with power $P_{max} = \mathbb{E}[|s_2|^2]$, and n_2 is the AWGN at PU_{RX} with zero-mean and variance N_0 . Accordingly, the instantaneous SNR of the vacation transmission can be written as

$$\gamma_2 = P_{max}|h_2|^2/N_0 \quad (5)$$

III. PERFORMANCE ANALYSIS OF THE SECONDARY NETWORK

In this section, we analyze the queueing performance of the secondary transmission provided that the secondary transmitter takes vacations for relaying the primary transmission whenever it has no data for its own transmission.

A. Statistical Distributions of Transmission and Vacation Times

Assume that the packets in the buffer of SU_{TX} are transmitted according to the first come first serve (FCFS) principle. The transmission rate of these packets is equal to the channel capacity of the secondary channel from SU_{TX} to SU_{RX} . Thus, a distribution of the transmission rate, which is expressed as a general RV, can be straightforwardly derived from the distribution of the packet transmission time T_0 over the channel from SU_{TX} to SU_{RX} . This distribution, in turn, depends on the distribution of the corresponding instantaneous SNR according to the Shannon capacity theorem as

$$T_0 = \frac{1}{C_0} = \frac{1}{b \log_2(1 + \gamma_0)} \text{ [secs/packet]} \quad (6)$$

where $b = B/L$, B is the channel bandwidth in Hz, and L is the packet length in bits. From (6), we can write the CDF of T_0 as

$$F_{T_0}(t) = 1 - F_{\gamma_0}(2^{1/(bt)} - 1) \quad (7)$$

Applying the order statistics theory along with the total probability theorem to (3), we can derive an expression for

the CDF of the instantaneous SNR γ_0 as

$$F_{\gamma_0}(\gamma) = 1 - \sum_{i=0}^{m_0-1} \frac{\alpha_0^i \gamma^i e^{-\alpha_0 \gamma / \gamma_{max}}}{i! \gamma_{max}^i} + \frac{\alpha_0^{m_0}}{\Gamma(m_0)} \sum_{i=0}^{m_2-1} \frac{\alpha_2^i \gamma_Q^i}{i!} \times \frac{\gamma^{m_0} \Gamma\left(m_0 + i, \frac{\gamma_Q \alpha_2 + \alpha_0 \gamma}{\gamma_{max}}\right)}{(\gamma_Q \alpha_2 + \alpha_0 \gamma)^{m_0+i}} \quad (8)$$

where $\alpha_0 = m_0/\Omega_0$ and $\alpha_2 = m_2/\Omega_2$. Substituting (8) into (7), it follows that

$$F_{T_0}(t) = \sum_{i=0}^{m_0-1} \frac{\alpha_0^i}{i! \gamma_{max}^i} \left[2^{1/(bt)} - 1\right]^i e^{-\alpha_0 [2^{1/(bt)} - 1] / \gamma_{max}} - \frac{\alpha_0^{m_0}}{\Gamma(m_0)} \sum_{i=0}^{m_2-1} \frac{\alpha_2^i \gamma_Q^i}{i!} \frac{[2^{1/(bt)} - 1]^{m_0}}{\{\gamma_Q \alpha_2 + \alpha_0 [2^{1/(bt)} - 1]\}^{m_0+i}} \times \Gamma\left(m_0 + i, \frac{\gamma_Q \alpha_2 + \alpha_0 [2^{1/(bt)} - 1]}{\gamma_{max}}\right) \quad (9)$$

In principle, packet transmission time can approach positive infinity when the channel quality becomes very poor. Practically, if the packet transmission time goes beyond a predefined threshold, say time-out t_o , the packet is treated as dropped and the transmission of the subsequent packet follows. The dropping probability that a packet is considered being timed out is determined as, $P_0^o = P[T_0 \geq t_o] = 1 - F_{T_0}(t_o)$. Therefore, we introduce a new RV of packet transmission time T_0^o capturing this transmission situation, namely, which is defined as

$$T_0^o = \begin{cases} T_0, & 0 < T_0 < t_o \\ t_o, & T_0 \geq t_o \end{cases} \quad (10)$$

Then, the PDF of packet transmission time T_0^o , when considering time-out, can be written as

$$f_{T_0^o}(t) = \begin{cases} f_{T_0}(t), & 0 < t < t_o \\ P_0^o, & t = t_o \\ 0, & t > t_o \end{cases} \quad (11)$$

Accordingly, the mean packet transmission time $\mathbb{E}[T_0^o]$ can be formulated as

$$\mathbb{E}[T_0^o] = \int_0^{t_o} t f_{T_0}(t) dt + t_o P_0^o = t_o - \int_0^{t_o} F_{T_0}(t) dt \quad (12)$$

Substituting (9) into (12), we obtain

$$\mathbb{E}[T_0^o] = t_o - \sum_{i=0}^{m_0-1} \frac{\alpha_0^i}{i! \gamma_{max}^i} \int_0^{t_o} \left[2^{1/(bt)} - 1\right]^i e^{-\frac{\alpha_0 [2^{1/(bt)} - 1]}{\gamma_{max}}} dt + \frac{\alpha_0^{m_0}}{\Gamma(m_0)} \sum_{i=0}^{m_2-1} \frac{\alpha_2^i \gamma_Q^i}{i!} \int_0^{t_o} \frac{[2^{1/(bt)} - 1]^{m_0}}{\{\gamma_Q \alpha_2 + \alpha_0 [2^{1/(bt)} - 1]\}^{m_0+i}} \times \Gamma\left(m_0 + i, \frac{\gamma_Q \alpha_2 + \alpha_0 [2^{1/(bt)} - 1]}{\gamma_{max}}\right) dt \quad (13)$$

By using a similar derivation for the CDF $F_{T_0}(t)$ and the mean packet transmission time $\mathbb{E}[T_0^o]$, respectively, we obtain the CDF of vacation time T_2 and the mean vacation time $\mathbb{E}[T_2^o]$ as

$$F_{T_2}(t) = \sum_{i=0}^{m_2-1} \frac{\alpha_2^i}{i! \gamma_{max}^i} \left[2^{1/(bt)} - 1\right]^i e^{-\frac{\alpha_2 [2^{1/(bt)} - 1]}{\gamma_{max}}} \quad (14)$$

$$\mathbb{E}[T_2^o] = t_o - \sum_{i=0}^{m_2-1} \frac{\alpha_2^i}{i! \gamma_{max}^i} \int_0^{t_o} \left[2^{1/(bt)} - 1\right]^i e^{-\frac{\alpha_2 [2^{1/(bt)} - 1]}{\gamma_{max}}} dt \quad (15)$$

B. Queueing Performance of the Secondary Network

In this section, we further evaluate the performance of the considered secondary network in terms of its queueing behavior. We assume that the maximum number of packets in the secondary system is K , including one packet being transmitted and $K - 1$ packets waiting in the buffer. Because of the finite buffer length, as long as this buffer is completely occupied, new arriving packets cannot join the system and have to leave it without being transmitted. Assume that secondary packets arriving at SU_{TX} can be modeled as a Poisson process with arrival rate λ . Also, these packets are transmitted with a rate equal to the channel capacity of the secondary channel from SU_{TX} to SU_{RX} . When being on vacation for relaying the primary signal, the primary packets are forwarded with a rate equal to the channel capacity of the link from SU_{TX} to PU_{RX} . Therefore, an M/G/1/K queueing system with multiple vacations and exhaustive service can be applied to model the network depicted in Fig. 1. The embedded Markov points of the underlying Markov chain are selected immediately after either a secondary transmission completion or a vacation transmission completion. In addition, each state of the embedded Markov chain at the n -th Markov point is identified as a combination of the number of packets, L_n , in SU_{TX} immediately after the n -th Markov point and the type of transmission completion ζ_n :

$$\zeta_n = \begin{cases} 0, & \text{primary relaying completion} \\ 1, & \text{secondary transmission completion} \end{cases} \quad (16)$$

Then, the states of the embedded Markov chain is denoted as $L_n^{(\zeta_n)}$. For the states that correspond to the embedded Markov points selected immediately after a primary transmission completion ($\zeta_n = 0$), L_n can take a value from 0 up to K . However, for the states that correspond to the embedded Markov points selected immediately after a secondary transmission completion ($\zeta_n = 1$), $L_n \in \{0, \dots, K - 1\}$. Let $\pi_k^{(0)}$ and $\pi_k^{(1)}$ be the probabilities that k packets exist in the buffer of SU_{TX} immediately after a primary transmission completion and immediately after a secondary transmission completion, respectively. Clearly, $\pi_k^{(0)}$ and $\pi_k^{(1)}$ are the steady-state probabilities of State $k^{(0)}$ and State $k^{(1)}$, respectively. As defined in [15, eq. (3.2)], $\pi_k^{(0)}$ and $\pi_k^{(1)}$ are, respectively,

calculated as

$$\pi_k^{(0)} = \lim_{n \rightarrow \infty} \Pr[L_n^{(\zeta_n)} = k^{(0)}] = \lim_{n \rightarrow \infty} \Pr[\zeta_n = 0, L_n = k] \quad \text{for } 0 \leq k \leq K \quad (17)$$

$$\pi_k^{(1)} = \lim_{n \rightarrow \infty} \Pr[L_n^{(\zeta_n)} = k^{(1)}] = \lim_{n \rightarrow \infty} \Pr[\zeta_n = 1, L_n = k] \quad \text{for } 0 \leq k \leq K-1 \quad (18)$$

The state transition probability $p_{j,k}^{(uv)}$ from State $j^{(u)}$ to State $k^{(v)}$ of the Markov chain is defined as

$$p_{j,k}^{(uv)} = \Pr[\zeta_{n+1} = v, L_{n+1} = k | \zeta_n = u, L_n = j] \quad (19)$$

In particular, these transition probabilities can be classified into the following cases:

- Case 1: The state transition probabilities from State $0^{(0)}$ to State $k^{(0)}$ and from State $0^{(1)}$ to State $k^{(0)}$, $0 \leq k \leq K-1$, i.e.,

$$p_{0,k}^{(00)} = p_{0,k}^{(10)} = f_k, \quad 0 \leq k \leq K-1 \quad (20)$$

where f_k represents the probability that k packets arrive at SU_{TX} during its vacation time for relaying the primary signal and f_k can be expressed as [15, eq. (3.6)]

$$f_k = \int_0^{t_o} \frac{(\lambda t)^k}{k!} e^{-\lambda t} f_{T_2}(t) dt + \frac{(\lambda t_o)^k}{k!} e^{-\lambda t_o} P_2^o = \frac{(\lambda t_o)^k}{k!} \times e^{-\lambda t_o} + \int_0^{t_o} F_{T_2}(t) \frac{\lambda e^{-\lambda t} ((\lambda t)^k - k(\lambda t)^{k-1})}{k!} dt \quad (21)$$

Substituting (14) into (21) gives

$$f_k = \frac{(\lambda t_o)^k}{k!} e^{-\lambda t_o} + \sum_{i=0}^{m_2-1} \frac{\alpha_2^i}{i! \gamma_{max}^i} \int_0^{t_o} [2^{1/(bt)} - 1]^i \times e^{-\frac{\alpha_2}{\gamma_{max}} [2^{1/(bt)} - 1]} \frac{\lambda e^{-\lambda t} ((\lambda t)^k - k(\lambda t)^{k-1})}{k!} dt \quad (22)$$

- Case 2: The state transition probabilities from State $0^{(0)}$ to State $K^{(0)}$ and from State $0^{(1)}$ to State $K^{(0)}$ are

$$p_{0,K}^{(00)} = p_{0,K}^{(10)} = 1 - \sum_{k=0}^{K-1} f_k \quad (23)$$

- Case 3: The state transition probabilities from State $j^{(0)}$ to State $k^{(1)}$ and from State $j^{(1)}$ to State $k^{(1)}$, $0 \leq k \leq K-2$, $1 \leq j \leq k+1$, are given by

$$p_{j,k}^{(01)} = p_{j,k}^{(11)} = a_{k-j+1}, \quad 0 \leq k \leq K-2, 1 \leq j \leq k+1 \quad (24)$$

where a_k is the probability that k packets arrive at SU_{TX} during a secondary transmission time, i.e.,

$$a_k = \int_0^{t_o} \frac{(\lambda t)^k}{k!} e^{-\lambda t} f_{T_0}(t) dt + \frac{(\lambda t_o)^k}{k!} e^{-\lambda t_o} P_0^o = \frac{(\lambda t_o)^k e^{-\lambda t_o}}{k!} + \int_0^{t_o} F_{T_0}(t) \frac{\lambda e^{-\lambda t} ((\lambda t)^k - k(\lambda t)^{k-1})}{k!} dt \quad (25)$$

Substituting (9) in (25), we have

$$a_k = \frac{(\lambda t_o)^k}{k!} e^{-\lambda t_o} + \sum_{i=0}^{m_0-1} \frac{\alpha_0^i}{i! \gamma_{max}^i} \int_0^{t_o} [2^{1/(bt)} - 1]^i \times e^{-\frac{\alpha_0}{\gamma_{max}} [2^{1/(bt)} - 1]} \frac{\lambda e^{-\lambda t} ((\lambda t)^k - k(\lambda t)^{k-1})}{k!} dt - \frac{\alpha_0^{m_0}}{\Gamma(m_0)} \sum_{i=0}^{m_2-1} \frac{\alpha_2^i \gamma_Q^i}{i!} \int_0^{t_o} \frac{\Gamma\left(m_0+i, \frac{\gamma_Q \alpha_2}{\gamma_{max}} + \frac{\alpha_0 [2^{1/(bt)} - 1]}{\gamma_{max}}\right)}{(\gamma_Q \alpha_2 + \alpha_0 [2^{1/(bt)} - 1])^{m_0+i}} \times [2^{1/(bt)} - 1]^{m_0} \frac{\lambda e^{-\lambda t} ((\lambda t)^k - k(\lambda t)^{k-1})}{k!} dt \quad (26)$$

- Case 4: The state transition probabilities from State $j^{(0)}$ to State $(K-1)^{(1)}$, from State $j^{(1)}$ to State $(K-1)^{(1)}$, and from State $K^{(0)}$ to State $(K-1)^{(1)}$, $1 \leq j \leq K-1$, are

$$p_{j,K-1}^{(01)} = p_{j,K-1}^{(11)} = \sum_{k=K-j}^{\infty} a_k = 1 - \sum_{k=0}^{K-j-1} a_k, \quad 1 \leq j \leq K-1$$

$$p_{K,K-1}^{(01)} = 1 \quad (27)$$

Therefore, the set of limiting state probability equations and the probability conservation equation are given by

$$\pi_k^{(0)} = \pi_0^{(0)} p_{0,k}^{(00)} + \pi_0^{(1)} p_{0,k}^{(10)} = (\pi_0^{(0)} + \pi_0^{(1)}) f_k, \quad 0 \leq k \leq K-1 \quad (28)$$

$$\pi_K^{(0)} = \pi_0^{(0)} p_{0,K}^{(00)} + \pi_0^{(1)} p_{0,K}^{(10)} = (\pi_0^{(0)} + \pi_0^{(1)}) (1 - \sum_{k=0}^{K-1} f_k) \quad (29)$$

$$\pi_k^{(1)} = \sum_{j=1}^{k+1} [\pi_j^{(0)} p_{j,k}^{(01)} + \pi_j^{(1)} p_{j,k}^{(11)}] = \sum_{j=1}^{k+1} (\pi_j^{(0)} + \pi_j^{(1)}) a_{k-j+1}, \quad 0 \leq k \leq K-2 \quad (30)$$

$$\pi_{K-1}^{(1)} = \sum_{j=1}^{K-1} [\pi_j^{(0)} p_{j,K-1}^{(01)} + \pi_j^{(1)} p_{j,K-1}^{(11)}] + \pi_K^{(0)} \quad (31)$$

$$\sum_{k=0}^K \pi_k^{(0)} + \sum_{k=0}^{K-1} \pi_k^{(1)} = 1 \quad (32)$$

In order to find $\pi_k^{(0)}$, $k \in (0, \dots, K)$, and $\pi_k^{(1)}$, $k \in (0, \dots, K-1)$, we only need $2K+1$ independent equations. From (28), (29), (30), and (32), we can express the limiting state probability equations in matrix form as

$$\mathbf{a} = \pi \mathbf{P} \quad (33)$$

where $\mathbf{a}$ is a $1 \times (2K+1)$ vector, defined as

$$\mathbf{a} = (0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 1) \quad (34)$$

The limiting state probabilities are stacked into the vector π as

$$\pi = (\pi_0^{(0)} \pi_1^{(0)} \dots \pi_K^{(0)} \pi_0^{(1)} \pi_1^{(1)} \dots \pi_{K-1}^{(1)}) \quad (35)$$

$$\mathbf{P} = \begin{pmatrix} f_0 - 1 & f_1 & f_2 & \dots & f_{K-2} & f_{K-1} & 1 - \sum_{k=0}^{K-1} f_k & 0 & 0 & 0 & \dots & 0 & 0 & 1 \\ 0 & -1 & 0 & \dots & 0 & 0 & 0 & a_0 & a_1 & a_2 & \dots & a_{K-3} & a_{K-2} & 1 \\ 0 & 0 & -1 & \dots & \vdots & \vdots & \vdots & 0 & a_0 & a_1 & \dots & a_{K-4} & a_{K-3} & 1 \\ \vdots & \vdots & 0 & \ddots & 0 & 0 & 0 & \vdots & 0 & a_0 & \ddots & \vdots & \vdots & 1 \\ 0 & 0 & \vdots & \dots & -1 & 0 & 0 & 0 & \vdots & 0 & \dots & a_0 & a_1 & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 & 0 & 0 & 0 & \vdots & \dots & 0 & a_0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 & -1 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \\ f_0 & f_1 & f_2 & \dots & f_{K-2} & f_{K-1} & 1 - \sum_{k=0}^{K-1} f_k & -1 & 0 & 0 & \dots & 0 & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & a_0 & a_1 - 1 & a_2 & \dots & a_{K-3} & a_{K-2} & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & a_0 & a_1 - 1 & \dots & a_{K-4} & a_{K-3} & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 & a_0 & \ddots & \vdots & \vdots & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \vdots & 0 & \dots & a_1 - 1 & \vdots & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \vdots & \dots & a_0 & a_1 - 1 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & a_0 & 1 \end{pmatrix} \quad (36)$$

Eventually, $\mathbf{P}$ is an $(2K + 1) \times (2K + 1)$ matrix determined as (36). As a result, the linear equation system in (33) gives the solution to the limiting state probabilities as

$$\boldsymbol{\pi} = \mathbf{a}\mathbf{P}^{-1} \quad (37)$$

C. Channel Utilization

The channel utilization ρ' , defined as the long-run fraction of time that the secondary channel is utilized, is given by [15, eq. (3.12a)]

$$\rho' = \frac{(1 - \pi_0^{(0)} - \pi_0^{(1)})\mathbb{E}[T_0^o]}{(\pi_0^{(0)} + \pi_0^{(1)})\mathbb{E}[T_2^o] + (1 - \pi_0^{(0)} - \pi_0^{(1)})\mathbb{E}[T_0^o]} \quad (38)$$

D. Blocking Probability

Blocking probability is defined as the probability that an arriving packet is not accepted to the system when it is totally full. By definition, the blocking probability for the considered system is given by [15, eq. (1.20b)]

$$P_B = 1 - \frac{\rho'}{\rho} \quad (39)$$

where the offered load ρ is calculated as

$$\rho = \lambda \mathbb{E}[T_0^o] \quad (40)$$

Substituting (38) and (40) into (39), we get

$$P_B = 1 - \frac{1}{\lambda} \frac{(1 - \pi_0^{(0)} - \pi_0^{(1)})}{(\pi_0^{(0)} + \pi_0^{(1)})\mathbb{E}[T_2^o] + (1 - \pi_0^{(0)} - \pi_0^{(1)})\mathbb{E}[T_0^o]} \quad (41)$$

E. Mean Number of Packets in the System

Mean number of packets in the secondary system is expressed as [15, eq. (3.31)]

$$\mathbb{E}[L] = \frac{\sum_{k=1}^{K-1} k \pi_k^{(1)}}{\lambda[(\pi_0^{(0)} + \pi_0^{(1)})\mathbb{E}[T_2^o] + (1 - \pi_0^{(0)} - \pi_0^{(1)})\mathbb{E}[T_0^o]]} + KP_B \quad (42)$$

F. Mean Serving Time of a Packet in the System

Mean serving time of a packet in the secondary system is defined as the sum of the waiting time of a packet in the buffer of SU_{TX} and its transmission time. For the considered system, the mean time in the system can be computed as [15, eq. (3.69a)]

$$\mathbb{E}[T] = \frac{\mathbb{E}[T_0^o] \sum_{k=1}^{K-1} k \pi_k^{(1)}}{\lambda \rho'[(\pi_0^{(0)} + \pi_0^{(1)})\mathbb{E}[T_2^o] + (1 - \pi_0^{(0)} - \pi_0^{(1)})\mathbb{E}[T_0^o]]} + \frac{K(\rho - \rho')}{\lambda \rho'} \quad (43)$$

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical examples to illustrate the queueing performance of a secondary system wherein the secondary transmitter assists the primary transmission by using multiple vacations and exhaustive service. In all scenarios, the time-out threshold, bandwidth, and the number of bits per packet are, respectively, selected as $t_o = 100$ ms, $B = 1$ MHz, and $N = 4096$. In addition, we select the average SNR as $\gamma_{\max} = P_{\max}/N_0 = 15$ dB. Assume that all the channel mean powers are attenuated according to the exponential decaying model wherein the path-loss exponent is chosen as $\nu = 4$ representing a highly shadowed urban area. The normalized distances from SU_{TX} to SU_{RX} and from SU_{TX} to PU_{RX} are $d_0 = 2$ and $d_2 = 4$. In addition, the fading severity parameters

Fig. 2. Blocking probability against interference power-to-noise ratio Q/N_0 for various buffer lengths K .

Fig. 3. Channel utilization against interference power-to-noise ratio Q/N_0 for various buffer lengths K .

of the links from SU_{TX} to SU_{RX} and from SU_{TX} to PU_{RX} are $m_0 = 2$ and $m_2 = 2$.

Fig. 2 shows the blocking probability against interference power-to-noise ratio Q/N_0 for various buffer lengths K . As can be observed from this figure, when the buffer length becomes large, the blocking probability decreases. This implies that arriving packets have higher probability to be accepted to the system for transmission. Furthermore, at low value of Q/N_0 , the blocking probability decreases with the increasing of Q/N_0 . However, when the tolerable interference power-to-noise ratio increases beyond a threshold, the blocking probability remains constant. This effect is attributed to the fact that in the high interference power-to-noise ratio regime, the transmit power of SU_{TX} is limited to P_{max} , according to (1).

Fig. 3 plots channel utilization against interference power-

Fig. 4. Mean number of packets in the system against interference power-to-noise ratio Q/N_0 for various buffer lengths K .

Fig. 5. Mean time in the system against interference power-to-noise ratio Q/N_0 for various buffer lengths K .

to-noise ratio Q/N_0 for various buffer lengths K . As expected, the channel utilization increases according to the increase of the buffer length. Accordingly, when the buffer length becomes large there are more arriving packets to be accepted to the system and, therefore, more packets will be transmitted. When the interference power-to-noise ratio Q/N_0 becomes sufficiently large, the channel utilization becomes saturated. This effect is because in the high regime of the interference power-to-noise ratio, the transmit power of SU_{TX} is equal to the transmit power limit P_{max} that leads to the saturated channel utilization.

Fig. 4 shows the mean number of packets in the system against interference power-to-noise ratio Q/N_0 for various buffer lengths K . As can be seen from Fig. 4, the mean number of packets in the system becomes large when the buffer length increases. In addition, at high values of Q/N_0 , the transmit

power of SU_{TX} is only constrained by the transmit power limit P_{max} , such that there exists a floor of the mean number of packets in the system.

Fig. 5 shows the mean time in the system against interference power-to-noise ratio Q/N_0 for various buffer lengths K . For the low regime of Q/N_0 , i.e., from -10dB to 0dB , we can see that increasing the buffer length K results in an increase of the mean time in the system. This is because when the buffer length increases, the mean number of packets in the system also increases. As a result, the mean time that a packet spends in the system will increase. However, for higher values of Q/N_0 , as the buffer length K varies, the mean time in the system changes slightly. This is because the transmission rate becomes very large such that the secondary transmitter can serve all the waiting packets immediately. Thus, the effect of the buffer length K on the packet waiting time in the buffer becomes insignificant.

V. CONCLUSIONS

We have analyzed the performance of a secondary cognitive system over Nakagami- m fading where the secondary transmitter is on vacation to forward the primary signal. We have considered the scenario where the secondary transmitter adopts multiple vacations and exhaustive service. Also, the secondary transmitter has a finite buffer and its arriving packets are modeled as a Poisson process. Utilizing an embedded Markov chain to analyze the corresponding M/G/1/K queueing system with multiple vacations and exhaustive service, we have derived several key queueing performance indicators for the examined network. In particular, blocking probability, channel utilization, mean number of packets in the system, and mean time in the system of the secondary network are evaluated. Finally, selected numerical examples are presented to investigate the impact of various network parameters on the performance of the examined secondary network.

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