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The capability of time- and frequency-domain algorithms for bistatic SAR processing

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ABSTRACT

The paper presents a study of the capability of time- and frequency-domain algorithms for bistatic SAR processing. Two typical algorithms, Bistatic Fast Backprojection (BiFBP) and Bistatic Range Doppler (BiRDA), which are both available for general bistatic geometry, are selected as the examples of time- and frequency-domain algorithms in this study. Their capability is evaluated based on some criteria such as processing time required by the algorithms to reconstruct SAR images from bistatic SAR data and the quality assessments of those SAR images.

Keywords: Bistatic SAR, algorithm, time-domain, frequency-domain, BiFBP, BiRDA, LORA, CARABAS-II

1. INTRODUCTION

Bistatic SAR refers to SAR systems whose transmitter and receiver(s) are separated. This separation allows bistatic SAR having some advantages over monostatic SAR. For example, the ability to avoid Radio Frequency Interference (RFI) of bistatic SAR is higher than that of monostatic SAR illuminating the same ground scene. Processing SAR data normally requires much effort and may only be performed at ground stations. However, this is not a problem for bistatic SAR due to the flexibility in deploying receivers of bistatic SAR. With the multiple deployed receivers, a target can be observed at different angles with bistatic SAR. This enhances the classification of that target. From a system designer's point of view, the design of bistatic SAR is more flexible than of monostatic SAR and the cost to build a bistatic SAR system can therefore be optimized. Based on the relative position of transmitter with respect to receiver, we can categorize bistatic SAR in three main geometries: quasi-monostatic, azimuth-invariant and general. For quasi-monostatic geometry, transmitter and receiver are mounted on a single platform whereas transmitter and receiver are separated in two different platforms for azimuth-invariant and general geometries. However, in azimuth-invariant geometry, the relative position of transmitter with respect to receiver is a constant. The study presented in this paper focuses on the general bistatic geometry due to its generality.

Bistatic SAR processing is based on SAR algorithms. Similar to the monostatic case, the bistatic SAR algorithms can be divided in two groups: time- and frequency-domain. The time-domain algorithms refer to the backprojection algorithms such as Bistatic Global Backprojection (BiGBP), Bistatic Fast Backprojection (BiFBP), and Bistatic Fast Factorized Backprojection (BiFFBP),^{1,2} The processing of these algorithms is performed only in time-domain whereas other algorithms such as Bistatic Range Doppler (BiRDA),³ Bistatic Chirp Scaling (BiCSA)⁴ and Bistatic Range Migration (BiRMA)⁵ mainly process the SAR data in frequency-domain. In the monostatic case, the frequency-domain algorithms generally have the advantage of processing time over time-domain algorithms. However, the frequency-domain algorithms also shows the limitations for bandwidth, motion error compensation, approximations, real-time processing and so forth which do not appear in the time-domain algorithms. Such limitations may constrain the usage of the frequency-domain algorithms.

In practice, an algorithm can work well with a SAR system but might not work with others due to the limitations of that algorithm. Another algorithm can be available for all SAR systems but it might require extremely long processing time. This is due to the fact that the algorithms have usually been developed for a specific SAR system. The choice of an algorithm among a number of algorithms is therefore seen as a practical issue in bistatic SAR processing. Bases for this choice can be operating frequency range, requirements of motion

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error, ratio of aperture length to minimum range and processing time and thus determine the capability of algorithms.

The aim of this study is to investigate the capability of time- and frequency-domain algorithms for bistatic SAR processing. Two algorithms, BiFBP and BiRDA, are selected as the examples of time- and frequency-domain algorithms for the investigations. These algorithms are chosen for the following reasons. Firstly, both algorithms are supposed to be available for general bistatic geometry. Secondly, the processing time of these two algorithms are supposed to be intermediate. BiRDA runs faster than BiRMA with highly computational interpolation but slower than BiCSA which does not require any interpolation. Similarly, the processing required by BiFBP is shown to be less than BiGBP but more than BiFFBP. Finally, the phase errors generated by these two algorithms are considered to be intermediate. This can be explained by the number of approximations in BiFBP and BiRDA and the validity of these approximations in comparison to other algorithms. To investigate the capability, the algorithms are examined with two different sets of system parameters. One is LORA,⁶ which can be configured as a narrow-band (NB) system, and the other CARABAS-II,⁷ an ultrawide-band and -beam (UWB) system.

The rest of the paper is organized as follows. Section 2 and Section 3 present the frequency- and time-domain algorithms in general and BiFBP and BiRDA in particular, respectively. The capability of the selected algorithms is examined in Section 4. Section 5 provides the conclusions.

2. TIME-DOMAIN ALGORITHMS

The backprojection algorithms are generally interpreted by a linear and direct transformation process from radar echoes into a complex SAR image. GBP was developed in 1980's and has been used for bistatic SAR processing recently.¹ The differences of BiGBP in comparison to GBP are that the ground plane instead of the slant-range plane is utilized for SAR scene reconstruction¹ and that the spherical projection in GBP replaced by the ellipsoidal projection in BiGBP. Let's consider a bistatic SAR system with the geometry given in Fig. 1. The time dependent bistatic range calculated for a single target (x_0, y_0) in the ground scene is given by

$$R_0(t) = \sqrt{R_{T,0}^2 + (v_T t)^2 - 2\cos\theta_{T,0}R_{T,0}v_T t} + \sqrt{R_{R,0}^2 + (v_R t)^2 - 2\cos\theta_{R,0}R_{R,0}v_R t} \quad (1)$$

where v_T and v_R are speeds of the transmitting and receiving platforms, respectively, $R_{T,0}$ and $R_{R,0}$ are initial half-way ranges with respect to the transmitter and receiver, and $\theta_{T,0}$ and $\theta_{R,0}$ are angles formed by platform velocities and initial steering vectors. If $g(t, \tau)$ is the range-compressed radar echo where t and τ indicate azimuth-time (slow-time) and range time (fast-time), respectively, then the superposition of the radar echo to reconstruct the SAR scene is expressed by the integral

$$h(x_0, y_0) = \int_{-\frac{t_i}{2}}^{+\frac{t_i}{2}} g\left(t, \frac{R_0(t)}{c}\right) dt \quad (2)$$

where t_i is the integration time and c is the speed of propagation.

Processing SAR data in a subaperture and subimage basis is carried out in BiFBP.² The full SAR scene to be reconstructed is segmented into K subimages whereas the complete transmitting and receiving apertures are split into L subapertures. The reconstruction of the imaged scene is performed in two stages: beamforming and backprojection. In the beamforming stage, all radar echo $g(t, \tau)$ belonging to the l -th subaperture are first shifted in range time τ with respect to the center of the k -th subimage, then superposed to form the k -th beam. In the next stage, an image sample belonging to the k -th subimage is then backprojected by the corresponding sample of the k -th beam formed in the previous stage. This procedure is repeated for K subimages and then L subapertures. The full reconstructed SAR scene is finally retrieved by a coherent combination of all subimages. The mathematical expression of BiFBP can be written by

$$h(x_0, y_0) = \bigcup_{k=1}^K \sum_{l=1}^L \int_{t_l - \frac{t_s}{2}}^{t_l + \frac{t_s}{2}} g\left(t, \frac{R_{l,k} - R_{l,k}^c + R_0^c(t)}{c}\right) dt \quad (3)$$

Figure 1. Bistatic SAR geometry.

where t_l is the time corresponding to the center of the l -th subaperture, t_s is the integration time along a subaperture,

$$R_{l,k}(t) = \sqrt{R_{T,k}^2 + (v_T t)^2 - 2\cos\theta_{T,k}R_{T,k}v_T t} + \sqrt{R_{R,k}^2 + (v_R t)^2 - 2\cos\theta_{R,k}R_{R,k}v_R t} \quad (4)$$

$$R_{l,k}^c = \sqrt{R_{T,k}^2 + (v_T t_l)^2 - 2\cos\theta_{T,k}R_{T,k}v_T t_l} + \sqrt{R_{R,k}^2 + (v_R t_l)^2 - 2\cos\theta_{R,k}R_{R,k}v_R t_l} \quad (5)$$

and

$$R_0^c = \sqrt{R_{T,0}^2 + (v_T t_l)^2 - 2\cos\theta_{T,0}R_{T,0}v_T t_l} + \sqrt{R_{R,0}^2 + (v_R t_l)^2 - 2\cos\theta_{R,0}R_{R,0}v_R t_l} \quad (6)$$

If we compare (2) and (3), one can see that BiFBP is only an approximation version of BiGBP. Hence, $R_0(t)$ in (2) is approximated by R_0^c in (3) whereas also in (3), $R_{l,k}^c$ is approximated by $R_{l,k}(t)$. To be sure that these approximations are valid, subaperture length and subimage size in BiFBP must be selected so that the far field condition is fulfilled. A combination of the approximations results in $R_{l,k}(t) - R_{l,k}^c + R_0^c \approx R_0(t)$. Processing SAR data in a subaperture and subimage basis allows reducing the processing time.

In BiFFBP,³ processing SAR data in a subaperture and subimage basis is still utilized. The difference between BiFBP and BiFFBP is only the number of beamforming stages. Hence, there is more than one beamforming stage in BiFFBP. A number of beams formed in the previous beamforming stage are used to form a new beam in the next beamforming stage. This procedure is repeated until the final beamforming stage. The backprojection stages in BiFBP and BiFFBP are identical. Processing SAR data in a subaperture and subimage basis and in multiple beamforming stages help us to reduce further the processing time.

3. FREQUENCY-DOMAIN ALGORITHMS

The frequency-domain algorithms are also called the FFT-based algorithms as they are based on Fast Fourier Transform (FFT). The first frequency-domain algorithm and also the first SAR imaging algorithm based on computer processing is RDA and was first introduced in 1970's. However, extending RDA to the bistatic case is successful very recently. The prerequisite for an extension of the frequency-domain algorithms in general and RDA in particular to the bistatic case is a two-dimensional spectrum for the bistatic case. BiRDA, BiRMA and BiCSA for the general bistatic geometry are based on the two-dimensional spectrum, i.e. the two-dimensional FFT of the radar echoes, which was derived in⁸ using the stationary phase principle and the method of series reversion. Let's consider again the bistatic SAR system with the geometry given in Fig. 1. According to,⁸ the two-dimensional spectrum of such as SAR system under certain circumstances can be shown to be

$$G(f_t, f_\tau) \approx W_{az} \left(f_t + (f_\tau + f_c) \frac{k_1}{c} \right) W_r(f_\tau)$$

$$\begin{aligned} & \times \exp \left\{ -i2\pi (f_\tau + f_c) \frac{R_{T,0} + R_{R,0}}{c} - i\pi \frac{f_\tau^2}{\kappa} + i2\pi \frac{c}{4k_2 (f_\tau + f_c)} \left[f_t + (f_\tau + f_c) \frac{k_1}{c} \right]^2 \right\} \\ & \times \exp \left\{ i2\pi \frac{k_3 c^2}{8k_2^3 (f_\tau + f_c)^2} \left[f_t + (f_\tau + f_c) \frac{k_1}{c} \right]^4 + i2\pi \frac{(9k_3^2 - 4k_2 k_4) c^3}{64k_2^5 (f_\tau + f_c)^3} \left[f_t + (f_\tau + f_c) \frac{k_1}{c} \right]^5 \right\} \end{aligned} \quad (7)$$

where $W_{az}(\cdot)$ and $W_r(\cdot)$ are the shape of the Doppler spectrum and the spectral shape of the transmitted pulse, respectively. Azimuth and range frequencies are represented by f_t and f_τ whereas f_c and κ denote the center frequency and chirp rate. The terms $k_1 \cdots k_4$, which are functions of v_T , v_R , $R_{T,0}$, $R_{R,0}$, $\theta_{T,0}$ and $\theta_{R,0}$, are clearly defined in.⁸

In BiRDA, the phase term of two-dimensional spectrum (11) is analyzed by a sum of the range modulation, azimuth modulation, the range cell migration, the range/azimuth coupling and residual phases. The range modulation and the azimuth/range coupling are removed in two-dimensional frequency-domain by the functions

$$\phi_1(f_t, f_\tau) = \exp \left\{ i\pi \frac{f_\tau^2}{\kappa} \right\} \quad (8)$$

and

$$\begin{aligned} \phi_2(f_t, f_\tau) = & \exp \left\{ -i2\pi \frac{c}{4k_2 f_c} \left(\frac{f_\tau^2}{f_c^2} - \frac{f_\tau^3}{f_c^3} \right) f_t^2 - i2\pi \frac{k_3 c}{8k_2^3 f_c} \left[3k_1 \left(\frac{f_\tau^2}{f_c^2} - \frac{f_\tau^3}{f_c^3} \right) f_t^2 + \frac{c}{f_c} \left(3\frac{f_\tau^2}{f_c^2} - 4\frac{f_\tau^3}{f_c^3} \right) f_t^3 \right] \right\} \\ & \times \exp \left\{ -i2\pi \frac{(9k_3^2 - 4k_2 k_4) c}{64k_2^5 f_c} \left[6k_1^2 \left(\frac{f_\tau^2}{f_c^2} - \frac{f_\tau^3}{f_c^3} \right) f_t^2 + \frac{4k_1 c}{f_c} \left(3\frac{f_\tau^2}{f_c^2} - 4\frac{f_\tau^3}{f_c^3} \right) f_t^3 + \frac{c^2}{f_c^2} \left(6\frac{f_\tau^2}{f_c^2} - 10\frac{f_\tau^3}{f_c^3} \right) f_t^4 \right] \right\} \end{aligned} \quad (9)$$

After removing the range modulation and the azimuth/range coupling, the two-dimensional spectrum is transformed to the range-Doppler domain by one-dimensional Inverse Fast Fourier Transform (IFFT) in azimuth for range cell migration correction (RCMC), azimuth compression and residual phase removal. The range cell migration is corrected by an interpolation, e.g. nearest neighbor, based on the azimuth frequency dependent time delay

$$\tau_0(f_t) = \frac{-1}{4k_2} \left(\frac{k_1^2}{c} - \frac{c}{f_c^2} f_t^2 \right) - \frac{k_3}{8k_2^3} \left(\frac{k_1^3}{c} - \frac{3k_1 c}{f_c^2} f_t^2 - \frac{2c^2}{f_c^3} f_t^3 \right) - (9k_3^2 - 4k_2 k_4) \left(\frac{k_1^4}{c} - \frac{6k_1^2 c}{f_c^2} f_t^2 - \frac{8k_1 c^2}{f_c^3} f_t^3 - \frac{3c^3}{f_c^4} f_t^4 \right) \quad (10)$$

The azimuth compression and the residual phase removal are handled by the functions

$$\begin{aligned} \phi_3(f_t, \tau) = & \exp \left\{ -i2\pi \frac{1}{4k_2} \left(2k_1 f_t + \frac{c}{f_c} f_t^2 \right) - i2\pi \frac{k_3}{8k_2^3} \left(3k_1^2 f_t + \frac{3k_1 c}{f_c} f_t^2 + \frac{c^2}{f_c^2} f_t^3 \right) \right\} \\ & \times \exp \left\{ -i2\pi \frac{9k_3^2 - 4k_2 k_4}{64k_2^5} \left(4k_1^3 f_t + \frac{6k_1^2 c}{f_c} f_t^2 + \frac{4k_1 c^2}{f_c^2} f_t^3 + \frac{c^3}{f_c^3} f_t^4 \right) \right\} \end{aligned} \quad (11)$$

and

$$\phi_4(f_t, \tau) = \exp \left\{ -i2\pi f_c \left[\frac{(R_{T,0} + R_{R,0})}{c} + \frac{k_1^2}{4k_2 c} + \frac{k_1^3 k_3}{8k_2^3 c} + \frac{(9k_3^2 - 4k_2 k_4) k_1^4 c}{64k_2^5 c} \right] \right\} \quad (12)$$

After the azimuth compression and the residual phase removal, the reconstructed SAR scene is achieved by an one-dimensional IFFT in azimuth for the signal in the range-Doppler domain.

BiCSA⁵ is quite similar to BiRDA. The difference between BiRDA and BiCSA is only that RCMC is performed in two parts, i.e. a differential and a bulk part. Hence, the equalization of the migration at different ranges is first carried out in the range-Doppler domain whereas the bulk RCMC is then handled in two-dimensional frequency domain. The interpolation is therefore avoided. For BiRMA,⁶ all processing including the Stolt interpolation are handled in two-dimensional frequency domain. The reconstructed SAR scene is obtained with a two-dimensional IFFT for the processed signal.

Table 1. Parameters of the simulated NB SAR system.

Parameter	Transmitter	Receiver
Operating frequency range	307 MHz–333 MHz	
Platform speed	130 m/s	125 m/s
Flight altitude	3000 m	5000 m
PRF	1300 Hz	
Aperture step	0.1 m	
Number of aperture positions	6000	
Minimum range	≈ 12000 m	≈ 12000 m
Direction of movement	-15°	$\approx 15.6^\circ$

4. CAPABILITY OF BIFBP AND BIRDA

The capability of BiFBP and BiRDA is investigated in this part. As previously mentioned, an algorithm can work well with a SAR system but might not work with others. Hence, to examine the capability of BiFBP and BiRDA, these algorithms are first applied to process different bistatic SAR data sets and the results given by these algorithms are then evaluated. For a SAR system in general and a bistatic SAR system in particular, the most crucial parameters are the operating frequency and the synthetic aperture length. Based on those parameters, SAR systems can be categorized into two main groups: NB SAR and UWB SAR. In the investigations of the capability of BiFBP and BiRDA, we therefore select two different sets of parameters. One is of LORA⁷ configured for ground moving target indication (GMTI) purposes as a NB SAR system, and the other is of CARABAS-II,⁸ a UWB SAR system.

4.1 Simulated LORA data set

Table 1 provides the main LORA's parameters. These parameters are used simulate the transmitter with the given operating frequency range and PRF, the passive receiver and the movement of the platforms. The flight tracks of the simulated platforms are assumed to be linear. It is also assumed that there is no motion error which allows us to exclude the motion error compensation in processing. At the initial coordinates in a Cartesian coordinate system, two platforms are separated by (1000 m, 1000 m, 2000 m). The illuminated ground scene is simulated by a number of point-like scatterers equally separated. One of them is located at the center of the simulated ground scene. The image of point-like scatterers normally skews with a certain angle corresponding to bistatic geometry. To remove this skew, we can define a suitable image coordinates before processing for BiFBP or use k_1 to adjust the 2-D spectrum of SAR images for BiRDA. In the simulations, we define the coordinates of the transmitting and receiving aperture positions so that the central points of the transmitting and receiving aperture positions should form a line which parallel to the x -axis of the attached Cartesian coordinate system. This also allows us to avoid the skew of the image of the point-like scatterers without losing the generality.

For the time-domain algorithms including BiFBP, a reconstructed area of the ground scene can arbitrarily be selected. The selection of sampling in SAR scene reconstruction is also flexible. However, such flexibilities are not available for the frequency-domain algorithms in general and BiRDA in particular. For the frequency-domain algorithms, the number of samples of a reconstructed area is fixed and defined by the number of aperture position, i.e. 6000 samples, and the number of range sample, i.e. depending on the fast-time (range-time) axis. The number of range samples will be 800 based on the minimum and maximum range-times, which depend on the coordinates of the simulated point-like scatterers, and the sampling time. In the monostatic case, aperture step and sampling time will directly decide the sampling in SAR scene reconstruction. However, in the bistatic case, our investigations show that the sampling depends on not only the aperture step and the sampling time but also bistatic geometry namely the speeds of the platforms, the angle formed by the flight tracks, the flight altitudes and so forth.

Figure 2. Bistatic SAR image reconstructed with the simulated LORA data set using BiRDA.

Figure 3. Bistatic SAR image reconstructed with the simulated LORA data set using BiFBP.

Fig. 2 shows the SAR image of the ground scene reconstructed with the simulated LORA data set using BiRDA. The nearest neighbor method is used for interpolation with the oversampling rate of three (3). As observed, the point-like scatterers in the ground scene are focused in correct coordinates in the x direction but not totally correct in the y direction. This is supposed to be caused by approximations in BiRDA. The point-like scatterers also have different focusing. The closer to the center of the ground scene the scatterers are, the better the scatterers have. This can be explained by the reference range using in BiRDA is associated with the center of the ground scene. The strong sidelobe in the x and y direction can be observed as the result of the narrow synthetic angle (less than 3°) or the short synthetic aperture.

Based on Fig. 2, we can find the sampling in the SAR scene reconstruction. The sampling in y is shown to be approximately 3.0 m which is larger than the sampling calculated with the sampling time in the monostatic case, i.e. 2.9 m. In spite of the incorrect positions of the scatterers in the SAR image, we can still find the approximate sampling in x of about 0.1 m. This value is reasonable since the speeds of the platforms are quite similar.

For the SAR scene reconstruction with BiFBP, the samplings in x and y are selected by $0.1 \text{ m} \times 3 \text{ m}$. The reconstructed area is set to identical to the previous one, i.e. $600 \text{ m} \times 2400 \text{ m}$. The number of samples of the reconstructed area will also be 6000×800 samples. The subaperture length and subimage area for BiFBP can be estimated with the phase error equation published in.¹ In the SAR scene reconstruction, the subaperture length

Table 2. Parameters of the simulated UWB SAR system.

Parameter	Transmitter	Receiver
Operating frequency range	20 MHz–90 MHz	
Platform speed	125 m/s	130 m/s
Flight altitude	5000 m	3000 m
PRF	137 Hz	
Aperture step	0.9375 m	
Number of aperture positions	20480	
Minimum range	≈ 12000 m	≈ 12000 m
Direction of movement	$\approx 15.6^\circ$	-15°

is chosen by 60 m for the transmitter side (600×0.1 m). This means that 600 aperture positions will form one subaperture in the transmitter side and the same number is also applied to the subaperture in the receiver side. The reasonable subimage area will therefore be $60 \text{ m} \times 60 \text{ m}$ or 600×20 samples. Totally, we have 10 subapertures and 10×40 subimages. This selection ensures that the phase error caused by BiFBP is smaller than $\pi/8$. The data is upsampled with the over sampling rate of three (3) and the data interpolation is the nearest neighbor method in the beamforming.

Fig. 3 shows the SAR image of the ground scene reconstructed with the simulated LORA data set using BiFBP. As shown in the figure, all the point-like scatterers in the ground scene are well focused to point targets and in the correct coordinates.

The simulation results show that, both BiFBP and BiRDA works with the simulated LORA data set with the parameters given in Table 1. The targets in the ground scene can be seen to be focused to the point targets and in the approximately correct coordinates. However, the performance of BiFBP is shown to be better than BiRDA if the quality assessments such as focusing and accuracy are taken into account.

4.2 Simulated CARABAS-II data set

The main CARABAS-II's parameters is provided in Table 2. The given operating frequency range and PRF are changed while the motion parameters of transmitter and receiver are exchanged. The assumptions of the flight tracks, motion error, and initial coordinates are similar to the previous simulation. The same ground scene with a number of point-like scatterers is illuminated again by the new simulated bistatic system. The number of range samples will increase to 1820 since the sampling time reduces in this simulation.

The SAR scene reconstruction using BiRDA faces some difficulties. The first one is the dimension of the data matrix is 20480×1820 samples. Processing for such a data matrix demands powerful computers with a large amount of memory. To simplify this task, we only process a part of the data matrix with the dimension of 5120×1820 samples. The focusing will therefore be decreased in comparison to the case where the full aperture is used.

Fig. 4 shows a part of the full SAR image of the ground scene, i.e. $600 \text{ m} \times 2400 \text{ m}$ instead of $4600 \text{ m} \times 2400 \text{ m}$, reconstructed with a part of the simulated CARABAS-II data set using BiRDA. The interpolation method and the oversampling rate for the data are selected the same as the previous processing. As shown in Fig. 4, the point-like scatterers in the ground scene are either not well focused or strongly smeared. The scatterer in the center of the SAR scene has the best focusing due to the reference range associated with the center of the ground scene in BiRDA. The focusing problem is supposed to originate from the initial approximation in the derivation of two-dimensional spectrum in.⁸ Hence, in the derivation of two-dimensional spectrum, it is assumed that the aperture is small in comparison to the minimum range which allows the following approximation using Taylor series

$$R_0(t) \approx R_{T,0} + R_{R,0} + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4 \quad (13)$$

Figure 4. A part of bistatic SAR image reconstructed with the simulated CARABAS-II data set using BiRDA.

Figure 5. A part of bistatic SAR image reconstructed with the simulated CARABAS-II data set using BiFBP.

For a long aperture, e.g. larger than the minimum range in this example, the Taylor series (13) will not converge. In the SAR scene reconstruction, we processed only a part of the data matrix, the aperture length is still smaller than the minimum range and the Taylor approximation is converge. However, we need to consider the higher order terms in the Taylor expansion (13). This makes the two-dimensional spectrum become very complicated and the phase functions in BiRDA may not therefore be easy to be derived. This can be seen as the second difficulty of using BiRDA for bistatic SAR data processing.

Fig. 4 also shows that the best focused point-like scatterer is still associated with a very high sidelobe level and a widely spreading of the sidelobes. This effect originates from the approximation for the inverse range frequency in the derivation of BiRDA. According to,³ in order to decompose the phase of the two-dimensional spectrum into the components of range modulation, azimuth modulation, range cell migration, range/azimuth coupling and residual, the inverse range frequency is approximated by

$$\frac{1}{f_{\tau} + f_c} \approx \frac{1}{f_c} \left[1 - \frac{f_{\tau}}{f_c} + \left(\frac{f_{\tau}}{f_c} \right)^2 \right] \quad (14)$$

However, this approximation is only valid if and only if $f_c > f_{\tau}$ and the Taylor series quickly converges if $f_c \gg f_{\tau}$. For CARABAS-II, the fractional bandwidth, i.e. the ratio of the bandwidth to the center frequency, is

even larger than one or in other words some range frequency components are larger than center frequency. The approximation (14) is therefore only valid with some range frequency components. This can be considered as the third difficulty of using BiRDA for bistatic SAR data processing.

Based on Fig. 2, we can find the sampling in the SAR scene reconstruction. In the x direction, the sampling is shown to be approximately 0.9174 m while it is about 1.3 m in the y direction. These samplings are used for the SAR scene reconstruction with BiFBP. Other settings for BiRDA are also applied to BiFBP. Hence, instead of using full aperture, we use only 5120 aperture positions to reconstruct SAR scene. The number of samples are also selected by 5120×1820 or in other words the reconstructed area is chosen by $4600 \text{ m} \times 2400 \text{ m}$. The subaperture length and subimage area for BiFBP are also estimated with the phase error equation published in.¹ Thus, the subaperture length is chosen by 60 m ($64 \times 0.9375 \text{ m}$), i.e. 64 aperture positions will form one subaperture in the transmitter side. The same number of subaperture positions is applied to the subaperture in the receiver side. The corresponding subimage area will be less than $60 \text{ m} \times 60 \text{ m}$. For convenience, we select 64×40 samples. Thus, we have 80 subapertures and 80×46 subimages. The over sampling rate for the data interpolation is also three (3) and the nearest neighbor method is utilized.

A part of the SAR image of the ground scene reconstructed with a part of the simulated CARABAS-II data set using BiFBP is given in Fig. 5. We can see that all the point-like scatterers in the ground scene are very well focused to point targets and in the correct coordinates. The sidelobes of the point-targets are significantly suppressed in comparison to the previous result.

This simulation hence points out that only BiFBP works well with the simulated CARABAS-II data set with the parameters given in Table 2. The simulated data set cannot be processed correctly with BiRDA due to the dimension of the data matrix, the long synthetic aperture, and the large fractional bandwidth.

4.3 Processing time

Processing time is always the crucial advantage of the frequency-domain algorithms in comparison to the time-domain algorithms. The following analysis shows how many times BiRDA runs faster than BiFBP. One way to implement this analysis is to estimate the number of operations required by these two algorithms and then compare them together. Since the flexibilities in the SAR reconstructed area and the image sample are not available for BiRDA, we assume that the SAR reconstructed area and the image sample of BiFBP is the same as the ones of BiRDA and given by $N_x \times N_y$ where N_x is also the number of aperture positions.

For BiRDA, we assume that all phase functions are available. The number of operations required by the algorithm corresponds to

- Two-dimensional FFT: $(N_x N_y) \log_2 (N_x N_y)$
- Range modulation & coupling removal: $2(N_x N_y)$
- One-dimensional IFFT in range: $(N_x N_y) \log_2 (N_y)$
- Interpolation: $(N_x N_y) + (N_x N_y) \log_2 (N_y) + (N_x \lambda N_y) \log_2 (\lambda N_y)$ where λ is the data over sampling rate (for FFT/IFFT upsampling and nearest neighbor).
- Azimuth compression & residual removal: $2(N_x N_y)$
- One-dimensional IFFT in azimuth: $(N_x N_y) \log_2 (N_x)$

The number of operations required by BiRDA can thus be estimated by

$$\Sigma_R = (N_x \times N_y) \left[2\log_2 (N_x \times N_y) + \lambda \log_2 (\lambda) + (\lambda + 1) \log_2 (N_y) + 5 \right] \quad (15)$$

The computational cost of BiFBP depends not only on N_x and N_y but also on the selections of subaperture length and subimage area. If the algorithm processes the radar echoes on a basis of L subapertures and K subimages, the needed number of operations is counted for

- Interpolation: $(N_x N_y) \log_2(N_y) + (N_x \lambda N_y) \log_2(\lambda N_y)$ where λ is the data over sampling rate (for FFT/IFFT upsampling and nearest neighbor).
- Beamforming: $N_x K \left(\sqrt{2} \lambda N_y / \sqrt{K} \right)$
- Backprojection: $L N_x N_y$

We can calculate the number of operations required by BiFBP by

$$\Sigma_F = (N_x \times N_y) \left[\lambda \log_2(\lambda) + (\lambda + 1) \log_2(N_y) + \lambda \sqrt{2K} + L \right] \quad (16)$$

The ratio of the processing time demanded by BiFBP to the one of BiRDA will simply be given by

$$\Gamma = \frac{\lambda \log_2(\lambda) + (\lambda + 1) \log_2(N_y) + \lambda \sqrt{2K} + L}{2 \log_2(N_x \times N_y) + \lambda \log_2(\lambda) + (\lambda + 1) \log_2(N_y) + 5} \quad (17)$$

Based on (17), we can estimate the value of Γ for the first test with the simulated LORA data set. Hence, for $K = 400$, $L = 10$, $\lambda = 3$ and $N_x \times N_y = 6000 \times 800$, the value will be $\Gamma \approx 1.5$. This ratio is quite matched to the ratio of the processing times in reality $\Gamma = 135 \text{ s} / 85 \text{ s} \approx 1.6$.

Although the processing time of BiFBP is longer than the one of BiRDA, we can reduce the processing time of BiFBP easily by reducing reconstructed area, increasing image sample, and selecting small values for K and L with higher phase error. For example, if we decrease N_y to 1500 for only BiFBP since there is no information outside the area $(-300 : 300, -750 : 750)$ and at the same time we increase the image sample in azimuth from 0.1 to 2 for BiFBP since this value is still much smaller than the azimuth resolution obtained with the simulated bistatic LORA system, the ratio will be reduced to $\Gamma \approx 1.2$. However, the possibility to reduce the processing time is not available for BiRDA.

5. CONCLUSION

The capability of BiFBP and BiRDA which are the typical time- and frequency-domain algorithms for bistatic SAR processing are investigated in this paper. The examined algorithms have been tested with two different SAR data sets. One data set is simulated according to LORA's parameters and the other CARABAS-II's parameters. The simulation results indicate that both BiFBP and BiRDA can work with the simulated LORA data set, i.e. NB SAR data set. BiRDA faces different challenges such as dimension of data matrix, long synthetic aperture and high signal fractional bandwidth when it is used to process the simulated CARABAS-II data set, i.e. UWB SAR data set, while BiFBP does not. The processing time of BiRDA is analytically and experimentally shown to be shorter than the one of BiFBP. Thanks to the flexibility of the time-domain algorithm in general and BiFBP in particular, there are different ways to shorten the processing time.

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