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Impact of Primary Networks on the Performance of Secondary Networks

Hung Tran, Maarig Aregawi Hagos, Marshed Mohamed, and Hans-Jürgen Zepernick

Blekinge Institute of Technology
SE-371 41 Karlskrona, Sweden
E-mail: htr@bth.se

Abstract—In this paper, we investigate the performance of a single-input and multiple-output cognitive radio network over Rayleigh fading. In particular, we assume that secondary transmitter (SU-Tx) and primary transmitter (PU-Tx) are equipped with a single antenna while secondary receiver (SU-Rx) and primary receiver (PU-Rx) have multiple antennas. Additionally, the SU-Tx transmit power is subject to outage constraint of the primary network and peak transmit power of the secondary network. Given these settings, an adaptive transmit power allocation policy for the SU-Tx, a closed-form expression for outage probability, and an approximation for ergodic capacity are obtained. These formulas will be used to examine the impact of the PU-Tx transmit power, the number of antennas at receivers, and channel mean powers on the performance of the secondary network. More importantly, our results reveal that the SU-Tx using the power allocation policy can obtain optimal performance.

Index Terms—Cognitive Radio Networks, Outage Constraint, Outage Probability, Ergodic Capacity.

I. INTRODUCTION

The rapid growth of wireless services has led to radio resources getting exhausted. However, measurement campaigns have shown that almost all allocated spectrum is under-utilized at a specific geographical area, time, and frequency [1], [2]. This implies that the shortage of the radio spectrum is mainly due to inefficient usage rather than real scarcity. In order to overcome this problem, cognitive radio networks (CRNs) have been proposed as a promising solution [3].

In a CRN, secondary users (SUs) are allowed to access the licensed radio frequency band of primary users (PUs) as long as the quality of service (QoS) of the primary network is assured. According to [4], CRNs are classified into three main techniques, spectrum overlay, interweave, and spectrum underlay. In spectrum overlay, the SU can transmit simultaneously with the PU, given that the SU knows channel state information (CSI), codebook, and messages of PUs to keep the transmission rate of the primary network stable. In the interweave approach, the secondary transmitter (SU-Tx) can access the licensed frequency if and only if it is not occupied by the PU. In contrast to interweave and overlay, the SU in spectrum underlay may cause limited interference to the PU as long as this interference is kept below a predefined level.

Recently, the multiple antennas technique has been considered to improve the performance of underlay networks. Accordingly, many works have studied the impact of multiple

antenna terminals on the system performance under various interference constraints of primary networks, e.g. [5]–[11] and the references therein. More particularly, in [5], the maximum sum-rate of a single-input multiple-output (SIMO) CRN, where the SU-Tx is subject to the peak transmit power and interference constraint of the PU, has been analyzed. Regarding the capacity of spectrum underlay CRN, fundamental capacity limits for a multiple-input multiple-output (MIMO) system has been investigated in [6]. In [7], the mean uplink capacity of a MIMO CRN under average interference power constraint has been derived. The results reported in [7], have shown the impact of the number of PUs and SUs on system performance. In [12], the performance of a SIMO CRN, in which the SU-Tx is subject to the combined constraint of average interference power and peak interference power of the PU, has been investigated for Nakagami- m fading. Following this study, an optimal power allocation, beamforming weights, and outage probability of a secondary network have been derived. Later, in [11], a MIMO CRN with transmit antenna selection has been studied, and the system performance in terms of ergodic capacity and outage capacity has been analyzed. More recently, in [13], a pre-coding scheme has been introduced for a MIMO CRN in order to minimize interference to the PU and maximize the sum capacity. Our most recent works, reported in [14], have examined the impact of the number of antennas, and distances between SU and PU on the performance of a SIMO CRN. The results have shown that subject to the peak interference power constraint, the system performance degrades significantly as the number of antennas of the primary receiver (PU-Rx) increases. However, the impact of interference from the primary transmitter (PU-Tx) to the secondary receiver (SU-Rx) on the performance of the secondary network has not been investigated.

In this paper, we analyze the performance of a SIMO CRN in which the impact of interference from the primary network on the system performance of the secondary network is studied. In particular, we assume that the PU-Tx and SU-Tx are equipped with a single antenna while the PU-Rx and SU-Rx are equipped with multiple antennas. Additionally, the SU-Tx transmit power is subject to the outage constraint of the PU as well as the peak transmit power constraint of the SU. We further assume that both PU-Rx and SU-Rx use selection combining (SC) to process the received signal, and all channels undergo Rayleigh fading. The contributions of this paper are

summarized as follows:

- An adaptive transmit power allocation policy for the SU-Tx is derived. Utilizing this policy, the cumulative distribution function (CDF) and probability density function (PDF) of the signal-to-interference-plus-noise ratio (SINR) are also obtained.
- A closed-form expression for the outage probability and an approximate expression for the ergodic capacity are obtained. These formulas will be used to examine the impact of the PU-Tx transmit power, the number of antennas at the receivers, and channel mean powers on the performance of the secondary network.
- The results indicate that the performance of the secondary network is improved significantly as the number of antennas of the PU-Rx or SU-Rx increases.

The remainder of this paper is organized as follows. In Section II, the system model and assumptions are introduced. In Section III, the performance analysis in terms of outage probability and ergodic capacity are provided. Numerical results are presented in Section IV. Finally, conclusions are given in Section V.

II. SYSTEM MODEL

Let us consider a spectrum underlay network as shown in Fig. 1 in which the SU shares the same frequency band with the PU. The SU-Tx and PU-Tx are equipped with a single antenna while the SU-Rx and PU-Rx have N and M antennas, respectively. Both SU-Rx and PU-Rx use SC to process the received signal. For mathematical modeling, we denote the instantaneous channel power gains of the SU-Tx→SU-Rx and PU-Tx→PU-Rx links by g_i and h_j , $i = 1, 2, \dots, N$, $j = 1, 2, \dots, M$, respectively. The instantaneous channel power gains of the PU-Tx→SU-Rx and SU-Tx→PU-Rx interference links are denoted, respectively, by f_i and β_j . We assume that all channels undergo Rayleigh fading and system bandwidth is normalized to one. Accordingly, the instantaneous channel power gains, h_j , g_i , β_j , and f_i are independent exponentially distributed random variables (RVs) with channel mean powers denoted by Ω_h , Ω_g , Ω_β , and Ω_f , respectively.

It is noted that the PU allows the SU to access its licensed frequency band as long as the PU QoS is not compromised by the interference from the SU. In the considered system model, the QoS of the PU is assured if the outage probability is kept below a predefined constraint. This condition may be formulated as

$$P_{\text{out}}^p = \Pr \{ \gamma_{sc}^p \leq \gamma_{th} \} \leq \varepsilon \quad (1)$$

where $\gamma_{th} = 2^{r_p} - 1$, r_p and ε are outage transmission rate and outage constraint of the primary network, respectively. The symbol γ_{sc}^p represents the instantaneous SINR at the PU-Rx defined as

$$\gamma_{sc}^p = \max_{j=1,2,\dots,M} \left\{ \frac{P_p h_j}{\beta_j P_s + N_0} \right\} \quad (2)$$

where P_p , N_0 , and P_s are average transmit power of the PU-Tx, average noise power, and instantaneous transmit power of

Fig. 1. System model of a CRN in which SU-Tx and PU-Tx have a single antenna while the SU-Rx and PU-Rx are equipped with N and M antennas, respectively.

the SU-Tx, respectively. It is important that the SU-Tx transmit power must be controlled such that the condition given in (1) is satisfied.

Moreover, the transmit power is usually limited in practice. Thus, the SU-Tx transmit power is subject to an additional constraint as

$$P_s \leq P_{pk} \quad (3)$$

where P_{pk} is the peak transmit power of the SU-Tx. In the next section, the combined constraint given in (1) and (3) will be used to derive the adaptive transmit power allocation policy for the SU-Tx.

III. PERFORMANCE ANALYSIS

In this section, we derive the transmit power allocation policy, outage probability, and approximate expression for the ergodic capacity. For this purpose, let us commence with the following lemma.

Lemma 1: Assume X_j and Z_j , $j = 1, 2, \dots, L$ are independent exponentially distributed RVs with mean Ω_x and Ω_z respectively, and a_1, a_2, a_3 are positive constants. A random variable Y is defined as

$$Y = \max_{j=1,2,\dots,M} \left\{ \frac{a_1 X_j}{a_2 Z_j + a_3} \right\} \quad (4)$$

with CDF and PDF, respectively, as

$$F_Y(y) = \left[1 - \frac{\exp(-yC)}{yD + 1} \right]^L \quad (5)$$

$$f_Y(y) = L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \exp(-yC(k+1)) \times \left(\frac{C}{(1+Dy)^{k+1}} + \frac{D}{(1+Dy)^{k+2}} \right) \quad (6)$$

where $C = \frac{a_3}{a_1 \Omega_x}$ and $D = \frac{a_2 \Omega_z}{a_1 \Omega_x}$.

Proof: See Appendix A. ■

A. Transmit Power Allocation Policy of the SU-Tx

Substituting (1) into (2), we can derive a closed-form expression for the outage probability of the primary network by using (5) in *Lemma 1* as

$$\begin{aligned} P_{\text{out}}^p &= \Pr \left\{ \max_{j=1,2,\dots,M} \left\{ \frac{P_p h_j}{\beta_j P_s + N_0} \right\} \leq \gamma_{th} \right\} \\ &= \left[1 - \frac{P_p \Omega_h}{\gamma_{th} P_s \Omega_\beta + P_p \Omega_h} \exp \left(-\frac{N_0 \gamma_{th}}{P_p \Omega_h} \right) \right]^M \leq \varepsilon \end{aligned} \quad (7)$$

From (7), the maximum transmit power of the SU-Tx under the outage constraint can be obtained after some manipulations as

$$P_s = \frac{P_p \Omega_h}{\gamma_{th} \Omega_\beta} \left[\frac{1}{1 - \sqrt[M]{\varepsilon}} \exp \left(-\frac{N_0 \gamma_{th}}{P_p \Omega_h} \right) - 1 \right]^M \quad (8)$$

Moreover, combining (8) with (3) yields the adaptive transmit power allocation policy for the SU-Tx as

$$\mathcal{P} = \min \left\{ P_{pk}, \frac{P_p \Omega_h}{\gamma_{th} \Omega_\beta} \left[\frac{\exp \left(-\frac{N_0 \gamma_{th}}{P_p \Omega_h} \right)}{1 - \sqrt[M]{\varepsilon}} - 1 \right]^M \right\} \quad (9)$$

By dividing both side of (9) by N_0 , we can rewrite (9) in terms of transmit signal-to-noise ratio (SNR) as

$$\gamma_{ST} = \min \left\{ \gamma_{pk}, \frac{\gamma_{PT} \Omega_h}{\gamma_{th} \Omega_\beta} \left[\frac{\exp \left(-\frac{\gamma_{th}}{\gamma_{PT} \Omega_h} \right)}{1 - \sqrt[M]{\varepsilon}} - 1 \right]^M \right\} \quad (10)$$

where $\gamma_{PT} = P_p/N_0$ and $\gamma_{pk} = P_{pk}/N_0$ denote the PU-Tx transmit SNR and the SU-Tx peak transmit SNR, respectively.

B. CDF and PDF of the SINR at the SU-Rx

Now, the SU-Tx uses the power allocation policy given in (9) to transmit the signal to SU-Rx. Accordingly, the SINR at each antenna branch of the SU-Rx is expressed as

$$\gamma_i^s = \frac{\mathcal{P} g_i}{P_p f_i + N_0} \quad (11)$$

As the SU-Rx uses SC, the SINR at the SU-Rx is formulated as

$$\gamma_{sc}^s = \max_{i=1,2,\dots,N} \left\{ \frac{\mathcal{P} g_i}{P_p f_i + N_0} \right\} \quad (12)$$

Consequently, the CDF and PDF of γ_{sc}^s can be obtained by applying *Lemma 1* as

$$\begin{aligned} F_{\gamma_{sc}^s}(\gamma) &= \Pr \left\{ \max_{i=1,2,\dots,N} \left\{ \frac{\mathcal{P} g_i}{P_p f_i + N_0} \right\} \leq \gamma \right\} \\ &= \left[1 - \frac{\exp(-G\gamma)}{(1+A\gamma)} \right]^N \end{aligned} \quad (13)$$

$$\begin{aligned} f_{\gamma_{sc}^s}(\gamma) &= N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \exp(-\gamma G(k+1)) \\ &\quad \times \left(\frac{G}{(1+A\gamma)^{k+1}} + \frac{A}{(1+A\gamma)^{k+2}} \right) \end{aligned} \quad (14)$$

where $A = \frac{\gamma_{PT} \Omega_f}{\gamma_{ST} \Omega_g}$, $G = \frac{1}{\gamma_{ST} \Omega_g}$.

C. Outage Probability of the Secondary Network

Outage probability of the secondary network is defined as the probability that the instantaneous SINR is dropped below a given threshold. Using (13), the outage probability can be easily obtained as

$$\begin{aligned} P_{\text{out}}^s &= \Pr \{ \gamma_{sc}^s \leq \beta_{th} \} = F_{\gamma_{sc}^s}(\beta_{th}) \\ &= \left[1 - \frac{\exp(-G\beta_{th})}{(1+A\beta_{th})} \right]^N \end{aligned} \quad (15)$$

where $\beta_{th} = 2^{r_s} - 1$ and r_s is the outage transmission rate of the secondary network.

D. Ergodic Capacity of the Secondary Network

Ergodic capacity is defined as the maximum long-term achievable rate over all channel state information and can be expressed as

$$\begin{aligned} C_{er} &= \int_0^\infty \log_2(1+\gamma) f_{\gamma_{sc}^s}(\gamma) d\gamma = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \\ &\quad \times \left[\int_0^\infty \frac{G \exp(-\gamma G(k+1)) \log_2(1+\gamma)}{(1+A\gamma)^{k+1}} d\gamma \right. \\ &\quad \left. + \int_0^\infty \frac{A \log_2(1+\gamma) \exp(-\gamma G(k+1))}{(1+A\gamma)^{k+2}} d\gamma \right] \end{aligned} \quad (16)$$

To the best of our knowledge, a closed-form expression for the ergodic capacity (16) is not available. However, we can derive an approximate ergodic capacity to examine the system performance by applying the well-known Taylor series expansion of the logarithm function as

$$\begin{aligned} \ln(1+\gamma) &\approx \ln(1+\bar{\gamma}) + \frac{\gamma-\bar{\gamma}}{1+\bar{\gamma}} - \frac{(\gamma-\bar{\gamma})^2}{2(1+\bar{\gamma})^2} \\ &\quad + o[(\gamma-\bar{\gamma})^2] \end{aligned} \quad (17)$$

where $o[\gamma^n]$ is defined as a polynomial function with power equal or greater than n and $\bar{\gamma} = \mathbb{E}[\gamma] = \int_0^\infty \gamma f_{\gamma_{sc}^s}(\gamma) d\gamma$. Then, the ergodic capacity can be approximated by using (17) as

$$C_{er} = \mathbb{E}[\log_2(1+\gamma)] \approx \log_2(e) \left[\ln(1+\bar{\gamma}) - \frac{\mathbb{E}[\gamma^2] - \bar{\gamma}^2}{2(1+\bar{\gamma})^2} \right] \quad (18)$$

Clearly, the approximate ergodic capacity (18) can be obtained if the mean of SINR $\bar{\gamma}$ and mean square SINR $\mathbb{E}[\gamma^2]$ are found. By definition, the first and second moment of SINR can be written, respectively, as

$$\mathbb{E}[\gamma] = \int_0^\infty \gamma f_{\gamma_{sc}^s}(\gamma) d\gamma = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k (I_0 + I_1) \quad (19)$$

$$\mathbb{E}[\gamma^2] = \int_0^\infty \gamma^2 f_{\gamma_{sc}^s}(\gamma) d\gamma = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k (I_2 + I_3) \quad (20)$$

where

$$I_0 = G \int_0^\infty \frac{\gamma \exp(-\gamma G(k+1))}{(1+A\gamma)^{k+1}} d\gamma \quad (21)$$

$$I_1 = A \int_0^\infty \frac{\gamma \exp(-\gamma G(k+1))}{(1+A\gamma)^{k+2}} d\gamma \quad (22)$$

$$I_2 = G \int_0^\infty \frac{\gamma^2 \exp(-\gamma G(k+1))}{(1+A\gamma)^{k+1}} d\gamma \quad (23)$$

$$I_3 = A \int_0^\infty \frac{\gamma^2 \exp(-\gamma G(k+1))}{(1+A\gamma)^{k+2}} d\gamma \quad (24)$$

In order to solve the integrals I_0 , I_1 , I_2 , and I_3 , let us consider the following lemma.

Lemma 2: Given $1 \leq n \leq 2$, $a > 0$, $b > 0$, and $m > 0$, we have

$$I = \int_0^\infty \frac{t^n \exp(-bt)}{(1+at)^m} dt = \frac{\Gamma(1+n)}{a^{n+1}} \Phi(1+n, 2-m+n, \frac{b}{a}) \quad (25)$$

where $\Phi(\cdot, \cdot, \cdot)$ and $\Gamma(\cdot)$ are the confluent hypergeometric function [15, eq. (9.211.4)] and gamma function [15, eq. (8.310.1)], respectively.

Proof: See Appendix B. ■

Applying *Lemma 2* for the integrals given in (21), (22), (23), and (24), yields

$$I_0 = \frac{G}{A^2} \Phi\left(2, 2-k, \frac{G(k+1)}{A}\right) \quad (26)$$

$$I_1 = \frac{1}{A} \Phi\left(2, 1-k, \frac{G(k+1)}{A}\right) \quad (27)$$

$$I_2 = \frac{2G}{A^3} \Phi\left(3, 3-k, \frac{G(k+1)}{A}\right) \quad (28)$$

$$I_3 = \frac{3}{A^2} \Phi\left(3, 2-k, \frac{G(k+1)}{A}\right) \quad (29)$$

Substituting (26) and (27) into (19), we find an exact expression for the first moment of the SINR as

$$\bar{\gamma} = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \left[\frac{G}{A^2} \Phi\left(2, 2-k, \frac{G(k+1)}{A}\right) + \frac{1}{A} \Phi\left(2, 1-k, \frac{G(k+1)}{A}\right) \right] \quad (30)$$

Similarly, an exact expression for the second moment of the SINR is obtained by substituting (28) and (29) into (20) as

$$\mathbb{E}[\gamma^2] = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \left[\frac{2G}{A^3} \Phi\left(3, 3-k, \frac{G(k+1)}{A}\right) + \frac{3}{A^2} \Phi\left(3, 2-k, \frac{G(k+1)}{A}\right) \right] \quad (31)$$

Eventually, an approximate expression for the ergodic capacity is established by substituting (30) and (31) into (18).

Fig. 2. SU-Tx transmit SNR versus PU-Tx transmit SNR, γ_{PT} , with different channel mean powers.

IV. NUMERICAL RESULTS

In this section, numerical results for the outage probability and ergodic capacity are presented. We set the outage transmission rate of the primary network $r_p = 0.4$ bit/s, outage transmission rate of the secondary network $r_s = 0.1$ bit/s, outage constraint of the primary network $\epsilon = 0.01$, and the SU-Tx peak transmit SNR $\gamma_{pk} = 4$ dB.

Fig. 2 plots the SU-Tx transmit SNR as a function of the PU-Tx transmit SNR. We can see that as the PU-Tx transmit SNR increases, the SU-Tx transmit SNR increases also. However, as the PU-Tx transmit SNR increases further, the SU-Tx transmit SNR is saturated at the SU-Tx peak transmit SNR, $\gamma_{pk} = 4$ dB.

Fig. 3 shows the outage probability as a function of PU-Tx transmit SNR. We can see that the outage probability firstly decreases to a minimum value, and then increases as the PU-Tx transmit SNR, γ_{PT} , increases. This is thought to be due to the fact that the SU-Tx transmit SNR is controlled following the policy given in (10). Thus, an increasing PU-Tx transmit SNR γ_{PT} leads to an increase of the SU-Tx transmit SNR γ_{PT} (see Fig. 2). Accordingly, the SU-Rx SINR is increased, i.e., the outage probability decreases. However, as the PU-Tx transmit SNR increases beyond a certain value, e.g., $\gamma_{PT} > 5$ dB, the SU-Tx cannot adapt to higher transmit SNR of the PU-Tx due to the limitation of peak transmit SNR, $\gamma_{pk} = 4$ dB (see Fig. 2). In this context, if the PU-Tx transmit SNR increases further, the SU-Rx will suffer strong interference from the PU-Tx. As a result, the system performance decreases. Additionally, as expected, the system performance can be improved as the channel mean powers of the SU-Tx $\rightarrow$ PU-Rx and PU-Tx $\rightarrow$ SU-Rx interference links decrease from $\Omega_f = \Omega_\beta = 2$ to $\Omega_f = 1$ or $\Omega_\beta = 1$.

Fig. 4 illustrates that the ergodic capacity increases to a maximum value, and then decreases as the PU-Tx transmit SNR, γ_{PT} , increases. Clearly, the approximation tightly matches with the simulation. These results are also in line with the observations for the outage probability depicted in Fig. 3. This is because of the same reasons explained above. Hence, an increase of the PU-Tx transmit SNR firstly leads to an

Fig. 3. Outage probability versus the PU-Tx transmit SNR, γ_{PT} , with different channel mean powers.

Fig. 4. Ergodic capacity versus the PU-Tx transmit SNR, γ_{PT} , with different channel mean powers.

increasing SU-Tx transmit SNR, i.e., the SU-Tx transmission rate is increased. On the other hand, as the SU-Tx transmit SNR is restricted due to the peak transmit SNR γ_{pk} , it is not able to adapt its transmit SNR following a further increase of the PU-Tx transmit SNR. As a result, the PU-Tx transmit SNR causes strong interference to the SU-Rx, i.e., the SU transmission rate is decreased.

Fig. 5 and Fig. 6 show the impact of the number of antennas on the system performance in terms of outage probability and ergodic capacity. As expected, the system performance can be improved as the number of antennas N or M increases. This is due to the fact that the diversity of received signals at the SU-Rx and PU-Rx is increased as the number of antennas increases. Accordingly, the SU-Rx and PU-Rx have higher probability to decode the received signals from the SU-Tx and PU-Tx. Interestingly, in contrast to the results reported in [14],

Fig. 5. Outage probability versus the PU-Tx transmit SNR, γ_{PT} , with different number of antennas.

Fig. 6. Ergodic capacity versus PU-Tx transmit SNR, γ_{PT} , with different number of antennas.

the results presented in this paper indicates that increasing the number of antennas of the PU-Rx does not degrade the performance of the secondary network. This is because of three reasons: 1) In [14], the impact of the communication links and interference of the primary network on the secondary network have not been included. 2) The SC of the PU-Rx has not been considered. 3) The SU-Tx in [14] is only subject to the peak interference power constraint of the PU-Rx. In other words, the considered system model has more advantage than the one reported in [14] as it reveals conditions for optimal performance. However, the SU-Tx requires more information from the primary network to be able to improve the system performance.

V. CONCLUSIONS

In this paper, we have examined the performance of a SIMO CRN based on the outage constraint of the primary network and the peak transmit power constraint of the secondary network. An adaptive transmit power allocation policy for the SU-Tx, a closed-form expression for the outage probability, and an approximation for the ergodic capacity have been obtained.

APPENDIX

A. Proof for the Lemma 1

According to the probability definition, we can define the CDF of Y in (4) as

$$F_Y(y) = \Pr \left\{ \max_{j=1,2,\dots,L} \left\{ \frac{a_1 X_j}{a_2 Z_j + a_3} \right\} \leq y \right\} \quad (32)$$

Because the RVs X_j and Z_j are independent, $F_Y(y)$ in (32) can be rewritten as

$$F_Y(y) = \prod_{j=1}^L \int_0^\infty \underbrace{\Pr \left\{ \frac{a_1 X_j}{a_2 z + a_3} \leq y \right\}}_K f_{Z_j}(z) dz \quad (33)$$

Additionally, Z_j and X_j are exponentially distributed RVs with mean Ω_z and Ω_x . Thus, the PDF of Z_j and K are expressed, respectively, as

$$f_{Z_j}(z) = \frac{1}{\Omega_z} \exp \left(-\frac{z}{\Omega_z} \right) \quad (34)$$

$$K = 1 - \exp \left(-\frac{y(a_2 z + a_3)}{a_1 \Omega_x} \right) \quad (35)$$

Substituting (34) and (35) into (33), and after some manipulations yields the CDF of Y as

$$\begin{aligned} F_Y(y) &= \prod_{j=1}^L \int_0^\infty \left[1 - \exp \left(-\frac{y(a_2 z + a_3)}{a_1 \Omega_x} \right) \right] \frac{\exp \left(-\frac{z}{\Omega_z} \right)}{\Omega_z} dz \\ &= \left[1 - \frac{a_1 \Omega_x}{a_2 \Omega_z y + a_1 \Omega_x} \exp \left(-\frac{a_3 y}{a_1 \Omega_x} \right) \right]^L \\ &= \left[1 - \frac{\exp(-yC)}{yD + 1} \right]^L \end{aligned} \quad (36)$$

where $C = \frac{a_3}{a_1 \Omega_x}$ and $D = \frac{a_2 \Omega_z}{a_1 \Omega_x}$.

Differentiating (36) with respect to y and then applying the binomial expansion yields the PDF of Y as

$$\begin{aligned} f_Y(y) &= L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \exp(-yC(k+1)) \\ &\quad \times \left(\frac{C}{(1+Dy)^{k+1}} + \frac{D}{(1+Dy)^{k+2}} \right) \end{aligned} \quad (37)$$

B. Proof for the Lemma 2

By setting $x = at$ and applying an exchange of variables in the integral (25) yields

$$I = \frac{1}{a^{n+1}} \int_0^\infty \frac{x^n}{(1+x)^m} \exp \left(-\frac{b}{a} x \right) dx$$

Moreover, using [15, eq. (9.211.4)], an exact expression for I is obtained as

$$I = \frac{\Gamma(1+n)}{a^{n+1}} \Phi \left(1+n, 2-m+n, \frac{b}{a} \right) \quad (38)$$

where $\Phi(\cdot, \cdot, \cdot)$ and $\Gamma(\cdot)$ are the confluent hypergeometric function [15, eq. (9.211.4)] and gamma function [15, eq. (8.310.1)], respectively.

REFERENCES

- [1] "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," Federal Communications Commission (FCC), Tech. Rep. 03-108, Mar. 2005.
- [2] "Cognitive radio technology - A study for Ofcom," 2010. [Online]. Available: <http://www.ofcom.org.uk/research/>
- [3] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Personal Commun. Mag.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.
- [4] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [5] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 38–51, Jan. 2008.
- [6] S. Sridharan and S. Vishwanath, "On the capacity of a class of MIMO cognitive radios," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 103–117, Feb. 2008.
- [7] X. Hong, C.-X. Wang, H.-H. Chen, and J. Thompson, "Performance analysis of cognitive radio networks with average interference power constraints," in *Proc. IEEE International Conference on Communications*, Beijing, China, May 2008, pp. 3578–3582.
- [8] H. Dhillon and R. Buehrer, "Cognitive MIMO radio: Incorporating dynamic spectrum access in multiuser MIMO network," in *Proc. IEEE Global Telecommunications Conference*, Hawaii, U.S.A., Dec. 2009, pp. 1–6.
- [9] M. Sarkar, T. Ratnarajah, M. Sellathurai, and C. Cowan, "Outage performance of SIMO multiple access cognitive radio channel," in *Proc. International Workshop on Cognitive Wireless Systems*, New Delhi, India, Dec. 2010, pp. 1–5.
- [10] F. Negro, I. Ghauri, and D. Slock, "Beamforming for the underlay cognitive MISO interference channel via UL-DL duality," in *Proc. International Conference on Cognitive Radio Oriented Wireless Networks Communications*, Cannes, France, Jun. 2010, pp. 1–5.
- [11] Asaduzzaman and H. Kong, "Ergodic and outage capacity of interference temperature-limited cognitive radio multi-input multi-output channel," *IET Commun.*, vol. 5, no. 5, pp. 652–659, May 2011.
- [12] Y. Yang, X. Liu, Q. Wu, and J. Wang, "Outage capacity analysis for SIMO Nakagami-m fading channel in spectrum sharing environment," in *Proc. IEEE International Conference on Information Theory and Information Security*, Beijing, China, Dec. 2010, pp. 1058–1063.
- [13] M. Jung, K. Hwang, and S. Choi, "Interference minimization approach to precoding scheme in MIMO-based cognitive radio networks," *IEEE Commun. Lett.*, vol. 15, no. 8, pp. 789–791, Aug. 2011.
- [14] H. Tran, H.-J. Zepernick, and H. Phan, "Impact of the number of antennas and distances among users on cognitive radio networks," in *Proc. International Conference on Advanced Technologies for Communications*, Hanoi, Vietnam, Oct. 2012, pp. 1–5.
- [15] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Elsevier, 2007.