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***Abstract*—In this paper, we study the performance for the primary and secondary transmissions in cognitive radio networks where the amplify-and-forward (AF) secondary relay helps to transmit the signals for both the primary and secondary transmitters over independent Nakagami-m fading. First, we derive exact closed-form expressions for outage probability and symbol error rate (SER) of the primary network. Then, we derive an exact closed-form expression for outage probability and a closed-form expression of a tight upper bound for SER of the secondary network. Furthermore, we also make a comparison for the performance of the primary system with and without the help of the secondary relay. Finally, we show a good agreement between analytical results and Monte-Carlo simulations.**

I. Introduction

The concept of cognitive relay transmission has been of great interest in the research community thanks to its advantageous features such as efficient utilization of frequency spectrum, reliable transmission, and radio coverage (see [1]–[5] for some recent studies, and the references therein). In particular, a brief overview about techniques for cooperative transmission in a cognitive radio network (CRN) was presented in [1], [2]. Based on the manner that secondary users access the licensed spectrum, there exist overlay cognitive and underlay cognitive spectrum sharing. Under the overlay paradigm, the secondary user is only allowed to use the licensed spectrum of the primary user when this spectrum is not occupied. To opportunistically utilize the licensed frequency, the secondary users adopt a specific spectrum sensing mechanism to decide whether the licensed spectrum is idle. On the contrary, under the underlay scheme, the primary and secondary users may access the same spectrum simultaneously provided that the interference incurred by the secondary transmission at the primary receiver remains below a pre-defined threshold. Consequently, the secondary transmitters must control its transmit power to meet the interference constraint at the primary receiver even if the primary transmitter is idle. For example, the studies of [3] have presented several spectrum sensing techniques for an overlay CRN. Differently, [4] has proposed a distributed power allocation strategy for underlay cognitive multiple-relay network to guarantee the interference contraint.

The two fundamental relaying protocols in conventional relay networks, namely amplify-and-forward (AF) and decode-and-forward (DF), have been also considered in cognitive relay systems. That is, in the works of [6], [7], outage performance of a cognitive AF network with single and multiple relay(s) has been addressed, respectively. In addition, outage probability of a cognitive DF relay network has been quantified in [5], [8]. Nevertheless, all of the aforementioned works have studied the system performance in the context that the relays only assist the transmission of the secondary network rather than the primary user. Thanks to the broadcast nature of wireless communications, theoretically, a secondary relay can receive signals from any neighbor transmitter, including both primary and secondary transmitters. Assisting both the primary and secondary transmissions, the secondary relay can be more helpful in the sense that the reliability of the primary receiver is enhanced. In addition, the above relay assisted infrastructure becomes more efficient in utilizing spectrum because of its high performance. To achieve this benefit, [9] investigates outage probability (OP) for a DF relay transmission in underlay cognitive scheme where a relay retransmits signals for both the primary transmission and the secondary transmission.

In this paper, we study the performance of the primary and secondary transmission in an CRN. We focuses on the scenario that the secondary relay operates in AF mode and is responsible to forward both the primary and secondary signals. In particular, closed-form expressions for the OP and symbol error rate (SER) of both the primary and secondary transmissions are derived to quantify and study the effect of the network parameters on the system performance. Numerical examples are conducted to validate the presented analysis.

The structure of this paper is organized as follows. Section II describes the system model and related definitions. Section III analyzes the OP and SER for both the primary and secondary transmissions. Section IV provides numerical results, discussion and evaluation about the results. Finally, conclusions are presented in Section V.

Notation: The following notations are used in this paper. A vector and a matrix are denoted by bold lower and upper case letters, respectively. Further, $f_X(\cdot)$ and $F_X(\cdot)$, respectively, stand for the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X. Additionally, expectation operator is denoted by $\mathbb{E}\{\cdot\}$ and $\mathcal{CN}(0, N_0)$ represents an additive white Gaussian noise (AWGN) RV with zero mean and variance N_0. We use $\Gamma(n)$ as the gamma function defined in [10, eq. 8.310.1] and $\Gamma(n, x)$ as the incomplete gamma function defined in [10, eq. 8.350.2]. Moreover, the nth order modified Bessel function of the second kind in [10, eq. (8.432.1)] and the Whittaker function in [10, eq. (9.222)] are denoted as $\mathcal{K}_n(\cdot)$ and $\mathcal{W}_{\lambda,\mu}(x)$, respectively. Finally, $U(a, b; x)$ is the confluent hypergeometric function [10, eq. (9.211.4)] and ${}_2F_1(a, b; c; x)$ denotes the Gauss hypergeometric function defined in [10, eq. (9.111)].

II. System and Channel Model

We consider a CRN consisting of a primary transmitter, $\mathrm{PU_{TX}}$, a primary receiver, $\mathrm{PU_{RX}}$, a secondary transmitter, $\mathrm{SU_{TX}}$, a secondary relay, $\mathrm{SU_R}$, and a secondary receiver, $\mathrm{SU_{RX}}$

operating according to the underlay scheme as shown in Fig. 1. We assume SU_{TX} is located far from PU_{RX} and its average transmit power per symbol is controlled based on the average channel gain from SU_{TX} to PU_{RX} to meet the interference constraint at PU_{RX}. Furthermore, due to the long distance and shadowing between SU_{TX} and SU_{RX}, the direct communication from SU_{TX} to SU_{RX} is not applicable. Therefore, the secondary network uses a relay SU_R to forward the signals. The secondary network co-exists with the primary network utilizing the frequency band licensed to the primary user by applying underlay spectrum sharing. On the other hand, the secondary relay will support both networks in forwarding their signals. Supposing that all channels are modeled as Nakagami-m fading with fading severity parameter m and all terminals operate in half-duplex mode.

In the first time slot, both PU_{TX} and SU_{TX} simultaneously broadcast their signals, namely x_s and x_p with average transmit power P_p and P_s, respectively. Hence, the received signals y_r at SU_R and y_{PD} at PU_{RX} are, respectively, given by

$$y_r = h_1 x_s + h_3 x_p + n_r \tag{1}$$

$$y_{PD} = h_5 x_p + n_p \tag{2}$$

where h_1, h_3, h_5 are the channel coefficients of the links $SU_{TX} \to SU_R$, $PU_{TX} \to SU_R$, and $PU_{TX} \to PU_{RX}$, respectively. In addition, n_r, n_p are the additive white Gaussian noise (AWGN) with zero mean and variance N_0 at SU_R and PU_{RX}, respectively. Note that $h_3 x_p$ now becomes the interference component for the secondary transmission, and $h_1 x_s$ is the interference to the primary transmission. These components are assumed to be large as compared to the noise at SU_R such that this noise in (1) can be neglected.

In the second time slot, SU_R amplifies the received signal, y_r, with amplifying gain G and then forwards the resulting signal. Therefore, the received signals at PU_{RX} and SU_{RX} are, respectively, expressed as

$$y_{PR} = G h_4 h_1 x_s + G h_4 h_3 x_p + n_p \tag{3}$$

$$y_S = G h_2 h_1 x_s + G h_2 h_3 x_p + n_s \tag{4}$$

where h_2 and h_4 are the channel coefficients of the links $SU_R \to SU_{RX}$ and $SU_R \to PU_{RX}$, respectively. Moreover, n_s is the AWGN at SU_{RX} with zero mean and variance N_0.

The amplifying gain G is selected to meet the interference power constraint at PU_{RX}. That is, the interference from the secondary transmission, imposed on PU_{RX}, must be limited to a predefined threshold Q, i.e. $\mathbb{E}\{(G h_1 x_s)^2\} = Q/h_4^2$ or

$$G^2 = \frac{Q}{P_s h_1^2 h_4^2} \tag{5}$$

As a result, the instantaneous signal-to-interference plus noise ratio (SINR) at SU_{RX}, γ_S, is represented as

$$\gamma_s = \frac{X_1 X_2}{a X_2 X_3 + b X_1 X_4} \tag{6}$$

Similarly, the instantaneous SINR of the relaying link, γ_{PR}, and the instantaneous signal-to-noise ratio (SNR) of the direct link, γ_{PD}, at PU_{RX} are, respectively, given by

$$\gamma_{PR} = \frac{X_3}{c X_1} \tag{7}$$

$$\gamma_{PD} = d X_5 \tag{8}$$

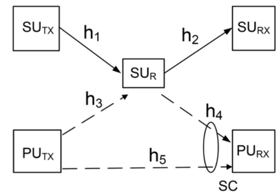

Fig. 1. System model for cognitive AF relay networks.

where $a = P_p/P_s$, $b = N_0/Q$, $c = (Q + N_0) P_s/(Q P_p)$, $d = P_p/N_0$ and $X_l = h_l^2$ with $l \in \{1, \ldots, 5\}$. It is assumed that PU_{RX} adopts selection combining (SC) to process the received signals. Therefore, its instantaneous end-to-end SNR at PU_{RX}, γ_P, is written as $\gamma_P = \max(\gamma_{PR}, \gamma_{PD})$. Based on the order statistics theory, the CDF of γ_P is expressed as

$$F_{\gamma_P}(\gamma) = F_{\gamma_{PR}}(\gamma) F_{\gamma_{PD}}(\gamma) \tag{9}$$

Let m_l and Ω_l be fading severity and channel mean power parameters of the l-th channel coefficient h_l, i.e., X_l follows gamma distribution with parameters $(m_l,\ \alpha_l^{-1})$, $\alpha_l = \frac{m_l}{\Omega_l}$, as

$$f_{X_l}(x_l) = \frac{\alpha_l^{m_l}}{\Gamma(m_l)} x_l^{m_l - 1} \exp(-\alpha_l x_l) \tag{10}$$

$$F_{X_l}(x_l) = 1 - \exp(-\alpha_l x_l) \sum_{p=0}^{m_1 - 1} \frac{\alpha_1^p x_1^p}{p!} \tag{11}$$

III. Performance of Primary Transmission

In this section, we present an exact closed-form expression for the CDF of the instantaneous SNR, γ_P, for the primary sub-system. Utilizing this outcome, we further derive exact closed-form expressions for the OP and the SER in the sequel.

From (7), the CDF of γ_{PR} is determined as

$$F_{\gamma_{PR}}(\gamma) = \int_0^{\infty} F_{X_3}(\gamma c x_1) f_{X_1}(x_1) dx_1 \tag{12}$$

Substituting (10) and (11) into (12), then applying [10, eq. (3.381.4)] to solve the remaining integral, $F_{\gamma_{PR}}(\gamma)$ is formulated as

$$F_{\gamma_{PR}}(\gamma) = 1 - \sum_{p=0}^{m_3 - 1} \frac{1}{p!} \frac{\Gamma(m_1 + p)}{\Gamma(m_1)} \frac{\alpha_1^{m_1} \alpha_3^p c^p \gamma^p}{(\alpha_3 \gamma c + \alpha_1)^{(m_1 + p)}} \tag{13}$$

From (8), the CDF of γ_{PD} is given by

$$F_{\gamma_{PD}}(\gamma) = F_{X_5}\left(\frac{\gamma}{d}\right) = 1 - \exp\left(-\frac{\alpha_5 \gamma}{d}\right) \sum_{q=0}^{m_5 - 1} \frac{\alpha_5^q \gamma^q}{d^q q!} \tag{14}$$

Substituting (13) and (14) into (9), the CDF of the end-to-end SNR γ_P is obtained as

$$\begin{aligned} F_{\gamma_P}(\gamma) = 1 &- \sum_{p=0}^{m_3 - 1} \frac{1}{p!} \frac{\Gamma(m_1 + p)}{\Gamma(m_1)} \frac{\alpha_1^{m_1} \alpha_3^p\ c^p\ \gamma^p}{(\alpha_3\ \gamma\ c + \alpha_1)^{(m_1 + p)}} \\ &- \exp\left(-\frac{\alpha_5 \gamma}{d}\right) \sum_{q=0}^{m_5 - 1} \frac{\alpha_5^q \gamma^q}{d^q q!} + \sum_{p=0}^{m_3 - 1} \frac{1}{p!} \frac{\Gamma(m_1 + p)}{\Gamma(m_1)} \\ &\times \frac{\alpha_1^{m_1} \alpha_3^p \gamma^p c^p}{(\alpha_3 \gamma c + \alpha_1)^{(m_1 + p)}} \exp\left(-\frac{\alpha_5 \gamma}{d}\right) \sum_{q=0}^{m_5 - 1} \frac{\alpha_5^q \gamma^q}{d^q q!} \end{aligned} \tag{15}$$

A. Outage Performance

Outage probability of the primary system, P_{out}^P, is defined as the probability that the instantaneous SNR, γ_P, falls below a predefined threshold, γ_{th}. This performance metric is found directly from the CDF of γ_P given in (15) as follows:

$$P_{out}^P = F_{\gamma_P}(\gamma_{th}) \tag{16}$$

B. Symbol Error Rate

In general, the SER of the primary system, P_E^P, can be given in terms of $F_{\gamma_P}(\gamma)$ as follows [11]:

$$P_E^P = \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \int_0^\infty F_{\gamma_P}(\gamma)\gamma^{-\frac{1}{2}} \exp(-b_1\gamma)d\gamma \tag{17}$$

where a_1 and b_1 are modulation parameters determined by the specific modulation scheme, e.g., $a_1 = 2$ and $b_1 = \sin(\frac{\pi}{M})^2$ for M-ary phase shift keying (M-PSK). Substituting (15) into (17), we rewrite the expression for the SER as

$$\begin{aligned} P_E^P &= \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \int_0^\infty \gamma^{-\frac{1}{2}} \exp(-b_1\gamma)d\gamma - \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \sum_{p=0}^{m_3-1} \frac{1}{p!} \\ &\times \frac{\Gamma(m_1+p)}{\Gamma(m_1)} \frac{\alpha_1^{m_1}}{\alpha_3^{m_1} c^{m_1}} \int_0^\infty \frac{\gamma^{p-\frac{1}{2}} \exp(-b_1\gamma)}{\left(\gamma + \frac{\alpha_1}{\alpha_3 c}\right)^{m_1+p}} d\gamma \\ &- \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \sum_{q=0}^{m_5-1} \frac{\alpha_5^q}{d^q q!} \int_0^\infty \exp\left(-\frac{(\alpha_5 + db_1)\gamma}{d}\right) \gamma^{q-\frac{1}{2}} d\gamma \\ &+ \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \sum_{p=0}^{m_3-1} \frac{1}{p!} \frac{\Gamma(m_1+p)}{\Gamma(m_1)} \sum_{q=0}^{m_5-1} \frac{\alpha_1^{m_1}}{q!\ \alpha_3^{m_1}} \frac{\alpha_5^q}{c^{m_1} d^q} \\ &\times \int_0^\infty \frac{\gamma^{p+q-\frac{1}{2}}}{\left(\gamma + \frac{\alpha_1}{\alpha_3 c}\right)^{(m_1+p)}} \exp\left(-\frac{(\alpha_5 + db_1)\gamma}{d}\right) d\gamma \end{aligned} \tag{18}$$

Applying [10, eq. (3.381.4)] to solve the first and third integrals, and [12, eq. (2.3.6.9)] to simplify the second and fourth integral of (18), we obtain

$$\begin{aligned} P_E^P &= \frac{a_1}{2} - \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \sum_{p=0}^{m_3-1} \frac{1}{p!} \frac{\Gamma(m_1+p)\Gamma\left(p+\frac{1}{2}\right)}{\Gamma(m_1)} \frac{\alpha_1^{\frac{1}{2}}}{\alpha_3^{\frac{1}{2}}} \\ &\times \frac{1}{c^{\frac{1}{2}}} U\left(p+\frac{1}{2}, \frac{3}{2} - m_1, b_1\frac{\alpha_1}{\alpha_3 c}\right) - \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \sum_{q=0}^{m_5-1} \frac{\alpha_5^q}{q!} \\ &\times d^{\frac{1}{2}} \frac{\Gamma\left(q+\frac{1}{2}\right)}{(\alpha_5 + db_1)^{q+\frac{1}{2}}} + \frac{a_1\sqrt{b_1}}{2\sqrt{\pi}} \frac{1}{p!} \frac{\Gamma(m_1+p)}{\Gamma(m_1)} \sum_{q=0}^{m_5-1} \frac{1}{q!} \\ &\times \frac{\alpha_5^q}{d^q} \frac{\alpha_1^{q+\frac{1}{2}}}{\alpha_3^{q+\frac{1}{2}} c^{q+\frac{1}{2}}} \Gamma\left(p+q+\frac{1}{2}\right) U\Bigg(p+q+\frac{1}{2}, p+q \\ &+ \frac{3}{2} - m_1 - p, \frac{\alpha_5\alpha_1 + db_1\alpha_1}{\alpha_3\ c\ d}\Bigg) \end{aligned} \tag{19}$$

IV. Performance of Secondary Transmission

In this section, an exact closed-form expression for the CDF of the instantaneous SNR γ_S is first derived to further quantify the outage performance of the secondary system. Moreover, the CDF of a tight upper bound on γ_S is also introduced to evaluate the SER of the secondary system.

A. Outage Probability

It can be seen from (6) that γ_S is expressed as a complicated function of multiple independent RVs, i.e., X_i with $i = 1, 2, 3, 4$; thus, the total probability theorem is utilized to obtain its CDF. Fixing the values of X_3 and X_4 as $X_3 = x_3$ and $X_4 = x_4$, we have the respective conditional CDF of γ_S, $F_{\gamma_S}(\gamma|x_3, x_4)$, as follows:

$$\begin{aligned} F_{\gamma_S}(\gamma|x_3, x_4) &= P\left\{\frac{X_1X_2}{aX_2x_3 + bX_1x_4} < \gamma\right\} \\ &= \underbrace{F_{X_2}(b\gamma x_4)}_{I(\gamma|x_4)} + \underbrace{\int_{b\gamma x_4}^\infty F_{X_1}\left(\frac{a\gamma x_2 x_3}{x_2 - b\gamma x_4}\right) f_{X_2}(x_2)dx_2}_{J(\gamma|x_3,x_4)} \end{aligned} \tag{20}$$

It can be seen that $I(\gamma)$ is simply achieved by averaging the conditional CDF $I(\gamma|x_4)$ over the PDF of X_4 as $I(\gamma) = \int_0^\infty I(\gamma|x_4)f_{X_4}(x_4)dx_4$. Using (10), (11) together with [10, eq. (3.381.4)], we have

$$I(\gamma) = 1 - \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{m_2-1} \frac{\alpha_2^q \gamma^q b^q}{q!} \frac{\Gamma(m_4+q)}{(\alpha_2\gamma b + \alpha_4)^{(m_4+q)}} \tag{21}$$

In addition, the expression for $J(\gamma|x_3, x_4)$ is rewritten as

$$\begin{aligned} &J(\gamma|x_3, x_4) \\ &= \int_0^\infty F_{X_1}\left(\gamma a x_3 + \frac{\gamma^2 abx_4x_3}{x_2}\right) f_{X_2}(x_2 + \gamma bx_4)dx_2 \end{aligned} \tag{22}$$

By substituting (10) and (11) into (22) and then utilizing the binomial theorem in [10, eq. (1.111)], i.e., $(a+b)^n = \sum_0^n C_k^n x^k y^{n-k}$ where $C_k^n = \frac{n!}{k!(n-k)!}$ is the binomial coefficient, as well as the result of [10, eq. (3.471.9)] to solve the remaining integral of (22), an analytic expression for $J(\gamma|x_3, x_4)$ is found as

$$\begin{aligned} J(\gamma|x_3, x_4) &= 1 - F_{X_2}(\gamma bx_4) - 2 \sum_{p=0}^{m_1-1} \sum_{q=0}^{p} \sum_{r=0}^{2m_2-1} \frac{C_q^p C_r^{m_2-1}}{p!\Gamma(m_2)} \\ &\times \alpha_1^{\frac{r+p+q+1}{2}} \alpha_2^{\frac{2m_2-r+p-q-1}{2}} a^{\frac{r+p+q+1}{2}} b^{\frac{2m_2+p-q-r-1}{2}} \gamma^{m_2+p} \\ &\times x_3^{\frac{r+p+q+1}{2}} x_4^{\frac{2m_2+p-q-r-1}{2}} \exp(-\alpha_1\gamma a\ x_3) \exp(-\alpha_2\gamma b\ x_4) \\ &\times \mathcal{K}_{r-p+q+1}\left(2\sqrt{\alpha_1\alpha_2\gamma^2 abx_3}\sqrt{x_4}\right) \end{aligned} \tag{23}$$

By integrating the product of $J(\gamma|x_3, x_4)$ and $f_{X_4}(x_4)$ over x_4, yields

$$J(\gamma|x_3) = \int_0^\infty J(\gamma|x_3, x_4)f_{X_4}(x_4)dx_4 \tag{24}$$

Substituting (23) and (10) in (24), then applying [10, eq. (3.381.4)] and [10, eq. (6.643.3)] to tabulate the first and second integrals, respectively, the expression for $J(\gamma|x_3)$ is represented as

$$J(\gamma|x_3) = \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{m_2-1} \frac{\alpha_2^q\ \gamma^q\ b^q}{q!} \frac{\Gamma(m_4+q)}{(\alpha_2\gamma\ b+\alpha_4)^{(m_4+q)}}$$
$$- \sum_{p=0}^{m_1-1} \sum_{q=0}^{p} \sum_{r=0}^{m_2-1} \frac{C_q^p C_r^{m_2-1} \Gamma(m_4+m_2+p-q-r-1)}{p!\Gamma(m_2)\Gamma(m_4)}$$
$$\times \Gamma(m_4+m_2)\ a^{\frac{r+p+q}{2}}\ b^{\frac{2m_2-r+p-q-2}{2}} \alpha_1^{\frac{r+p+q}{2}} \alpha_2^{\frac{2m_2-r+p-q-2}{2}}$$
$$\times \alpha_4^{m_4} \frac{\gamma^{m_2+p-1}\exp(-\alpha_1\gamma a x_3)}{(\alpha_4+\alpha_2\gamma b)^{\frac{2m_4+2m_2+p-q-r-2}{2}}} \exp\left(\frac{\alpha_1\alpha_2\gamma^2 abx_3}{2(\alpha_4+\alpha_2\gamma b)}\right)$$
$$\times x_3^{\frac{r+p+q}{2}} \mathcal{W}_{-\frac{2m_4+2m_2+p-q-r-2}{2}, \frac{r-p+q+1}{2}} \left(\frac{\alpha_1\ \alpha_2\ \gamma^2\ a\ b\ x_3}{\alpha_4+\alpha_2\gamma b}\right) \tag{25}$$

Now, $J(\gamma)$ is obtained by integrating the product $J(\gamma|x_3)f_{X_3}(x_3)$ over x_3 as

$$J(\gamma) = \int_0^\infty J(\gamma|x_3) f_{X_3}(x_3) dx_3 \tag{26}$$

Substituting (10) and (25) into (26) followed by utilizing [10, eq. (7.621.3)] to solve the integral (26), $J(\gamma)$ is now obtained as

$$J(\gamma) = \frac{\alpha_4^{m_4}}{\Gamma(m_4)} \sum_{q=0}^{m_2-1} \frac{\alpha_2^q b^q \gamma^q \Gamma(m_4+q)}{q!(\alpha_2\gamma b+\alpha_4)^{(m_4+q)}} - \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{p} C_q^p$$
$$\times \sum_{r=0}^{m_2-1} C_r^{m_2-1} \frac{\Gamma(m_4+m_2+p-q-r-1)\Gamma(m_4+m_2)}{\Gamma(m_2)\Gamma(m_4)\Gamma(m_3)}$$
$$\times \frac{\alpha_1^{r+q+1}\alpha_2^{m_2}\ \alpha_3^{m_3}\ \alpha_4^{m_4}\ a^{r+q+1}\ b^{m_2}\ \gamma^{m_2+r+q+1}}{(\alpha_4+\alpha_2\ \gamma\ b)^{m_4+m_2}(\alpha_3+\alpha_1\ \gamma\ a)^{m_3+r+q+1}} \Gamma(m_3+p)$$
$$\times \frac{\Gamma(m_3+r+q+1)}{\Gamma(m_3+m_4+m_2+p)} {}_2F_1\bigg(m_3+r+q+1, m_4+m_2, m_3$$
$$+m_4+m_2+p; \frac{\alpha_3\alpha_4+\alpha_2\alpha_3\gamma b+\alpha_1\alpha_4\gamma a}{\alpha_3\alpha_4+\alpha_2\alpha_3\gamma\ b+\alpha_1\alpha_4\gamma\ a+\alpha_1\alpha_2\gamma^2 ab}\bigg) \tag{27}$$

It is straightforward to see that the CDF of γ_S can be formulated by adding $I(\gamma)$ given in (21) and $J(\gamma)$ given in (27) together as (28) on the next page. As a result, the outage probability of the secondary network is obtained directly from (28) as $P_{out}^S = F_{\gamma_S}(\gamma_{th})$.

B. Symbol Error Rate

It is not feasible to evaluate the SER of the secondary transmission by using the CDF of γ_S, $F_{\gamma_S}(\gamma)$. Instead, here, a tightly bounded expression for $F_{\gamma_S}(\gamma)$ is adopted to quantify the SER performance as in [13, eq. (25)]. To be specific, the instantaneous SNR of the secondary transmission γ_S is upper bounded as $\gamma_S^U = \min(\gamma_1, \gamma_2)$ where $\gamma_1 = \frac{X_1}{aX_3}$ and $\gamma_2 = \frac{X_2}{bX_4}$. Hence, $F_{\gamma_S^U}(\gamma)$ is written as

$$F_{\gamma_S^U}(\gamma) = 1 - [1 - F_{\gamma_1}(\gamma)][1 - F_{\gamma_2}(\gamma)] \tag{29}$$

where $F_{\gamma_1}(\gamma) = \int_0^\infty F_{X_1}(\gamma a x_3) f_{X_3}(x_3) dx_3$ and $F_{\gamma_2}(\gamma) = \int_0^\infty F_{X_2}(\gamma b x_4) f_{X_4}(x_4) dx_4$. Utilizing (10) and (11) along with [10, eq. (7.621.3)] to simplify $F_{\gamma_i}(\gamma)$, $i = 1, 2$, yields

$$F_{\gamma_1}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \frac{\Gamma(m_3+p)}{\Gamma(m_3)} \frac{a^p \alpha_1^p \alpha_3^{m_3} \gamma^p}{(\alpha_1\gamma a+\alpha_3)^{(m_3+p)}} \tag{30}$$

$$F_{\gamma_2}(\gamma) = 1 - \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\Gamma(m_4+q)}{\Gamma(m_4)} \frac{b^q \alpha_2^q \alpha_4^{m_4} \gamma^q}{(\alpha_4+\alpha_2\gamma b)^{(m_4+q)}} \tag{31}$$

Substituting (30) and (31) into (29), $F_{\gamma_S^U}(\gamma)$ is given by

$$F_{\gamma_S^U}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \frac{\Gamma(m_3+p)}{\Gamma(m_3)} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\alpha_3^{m_3}\alpha_4^{N_4 m_4} a^{N_3 m_3}}{\alpha_1^{N_3 m_3}\alpha_2^{N_4 m_4}}$$
$$\times \frac{b^{N_4 m_4}\Gamma(m_4+q)}{\Gamma(m_4)} \frac{\gamma^{p+q}}{\left(\gamma+\frac{\alpha_3}{\alpha_1 a}\right)^{m_3+p}\left(\gamma+\frac{\alpha_4}{\alpha_2 b}\right)^{m_4+q}} \tag{32}$$

By replacing $F_{\gamma_S^U}(\gamma)$ for $F_{\gamma_P}(\gamma)$ and the modulation parameters $(a_2,\ b_2)$ of the secondary transmission for $(a_1,\ b_1)$ in (17), the SER for the secondary network is now written as

$$P_E^S = \frac{a_2}{2} - \frac{a_2\sqrt{b_2}}{2\sqrt{\pi}} \sum_{p=0}^{m_1-1} \frac{1}{p!} \frac{\Gamma(m_3+p)}{\Gamma(m_3)} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\alpha_3^{m_3}\ \alpha_4^{m_4}}{\alpha_1^{m_3}\alpha_2^{m_4}}$$
$$\times \frac{\Gamma(m_4+q)}{a^{m_3}b^{m_4}\Gamma(m_4)} \int_0^\infty \frac{\gamma^{p+q-\frac{1}{2}}\exp(-b_2\gamma)}{\left(\gamma+\frac{\alpha_3}{\alpha_1 a}\right)^{m_3+p}\left(\gamma+\frac{\alpha_4}{\alpha_2 b}\right)^{m_4+q}} d\gamma \tag{33}$$

Since the integral in (33) is not represented as a tabulable form, we apply the partial fraction [10, eq. (3.326.2)] for the integral expression and then utilizing [12, eq. (2.3.6.9)] to simplify the remaining integrals as follows:

$$P_E^S = \frac{a_2}{2} - \frac{a_2\sqrt{b_2}}{2\sqrt{\pi}} \sum_{p=0}^{m_1-1} \frac{1}{p!} \frac{\Gamma(m_3+p)}{\Gamma(m_3)} \sum_{q=0}^{m_2-1} \frac{1}{q!} \frac{\alpha_3^{m_3}\alpha_4^{m_4}}{\alpha_1^{m_3}\alpha_2^{m_4}}$$
$$\times \frac{\Gamma(m_4+q)\ \Gamma\left(p+q+\frac{1}{2}\right)}{a^{m_3}b^{m_4}\ \Gamma(m_4)} \Bigg[\sum_{i=1}^{m_3+p} \kappa_i \left(\frac{\alpha_3}{\alpha_1\ a}\right)^{p+q+\frac{1}{2}-i}$$
$$\times U\left(p+q+\frac{1}{2},\ p+q+\frac{3}{2}-i,\ b_2\frac{\alpha_3}{\alpha_1\ a}\right) + \sum_{j=1}^{m_4+q} \theta_j$$
$$\times \left(\frac{\alpha_4}{\alpha_2 b}\right)^{p+q+\frac{1}{2}-j} U\left(p+q+\frac{1}{2}, p+q+\frac{3}{2}-j, \frac{b_2\alpha_4}{\alpha_2 b}\right)\Bigg] \tag{34}$$

where

$$\kappa_i = \frac{1}{(m_3+p-i)!} \frac{d^{m_3+p-i}\left(\gamma+\frac{\alpha_4}{\alpha_2 b}\right)^{-m_4-q}}{d\gamma^{m_3+p-i}}\Bigg|_{\gamma=-\frac{\alpha_3}{\alpha_1 a}}$$

$$\theta_j = \frac{1}{(m_4+q-j)!} \frac{d^{m_4+q-j}\left(\gamma+\frac{\alpha_3}{\alpha_1 a}\right)^{-m_3-r}}{d\gamma^{m_4+q-j}}\Bigg|_{\gamma=-\frac{\alpha_4}{\alpha_2 b}}$$

V. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results are provided to verify the analysis and to illustrate the applicable prospect of the proposed scheme in improving system performance and enhancing frequency efficiency. Assuming that this system exits in a shadowed urban environment, so the path loss for an

$$F_{\gamma_s}(\gamma) = 1 - \sum_{p=0}^{m_1-1} \frac{1}{p!} \sum_{q=0}^{p} C_q^p \sum_{r=0}^{m_2-1} C_r^{m_2-1} \frac{\Gamma(m_4+m_2)\ \Gamma(m_4+m_2+p-q-r-1)}{\Gamma(m_2)\ \Gamma(m_4)\ \Gamma(m_3)} \alpha_1^{r+q+1}\ \alpha_2^{m_2}\ \alpha_3^{m_3}\ \alpha_4^{m_4}\ a^{r+q+1}\ b^{m_2}$$
$$\times \frac{\gamma^{m_2+r+q+1}}{(\alpha_4+\alpha_2\gamma\ b)^{m_4+m_2}\ (\alpha_3+\alpha_1\gamma\ a)^{N_3 m_3+r+q+1}} \frac{\Gamma(m_3+r+q+1)\ \Gamma(m_3+p)}{\Gamma(m_3+m_4+m_2+p)} {}_2F_1\Bigg(m_3+r+q+1, m_4+m_2, m_3+m_4$$
$$+ m_2 + p\ ;\ \frac{\alpha_3\alpha_4+\alpha_2\alpha_3\gamma b+\alpha_1\alpha_4\gamma a}{\alpha_3\alpha_4+\alpha_2\alpha_3\gamma b+\alpha_1\alpha_4\gamma a+\alpha_1\alpha_2\gamma^2 ab}\Bigg) \tag{28}$$

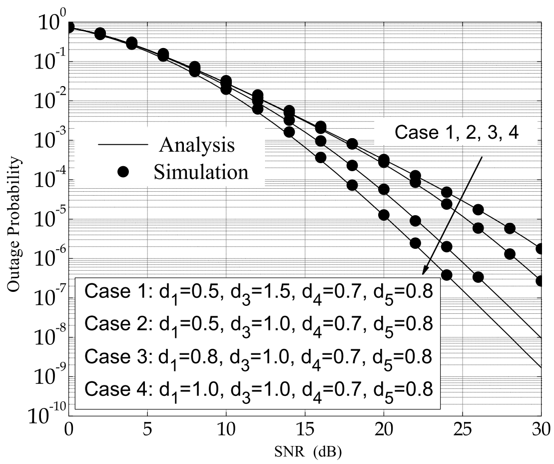

Fig. 2. Outage probability of the primary network with various distances of the links $SU_{TX} \to SU_R$ and $PU_{TX} \to SU_R$.

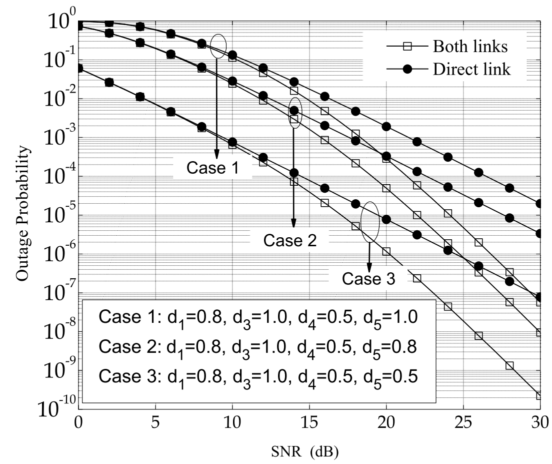

Fig. 4. A comparison of outage probability of the primary network with and without the help of the secondary relay.

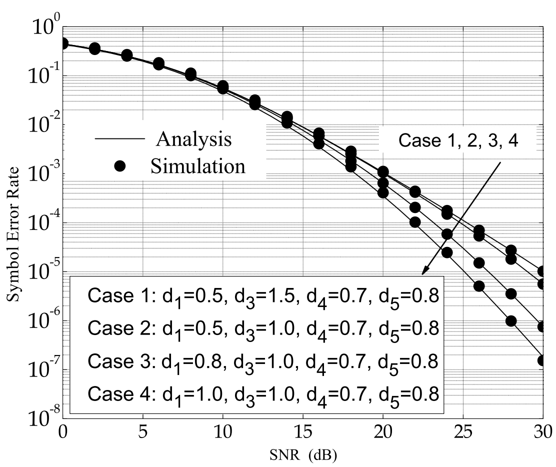

Fig. 3. Symbol error rate for 8-PSK of the primary network with various distances of the links $SU_{TX} \to SU_R$ and $PU_{TX} \to SU_R$.

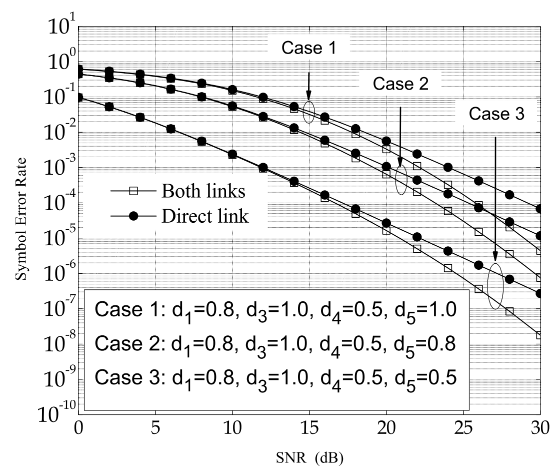

Fig. 5. A comparison of symbol error rate for 8-PSK of the primary network with and without the help of the secondary relay.

arbitrary transmission distance d may be modeled as $(\frac{d}{d_0})^{-n}$ with path loss exponent $n = 4$ where d_0 is the reference distance. For all considered scenarios, the fading severity parameters of all the links are set as $m_i = 2,\ i = 1, \cdots, 5$. Furthermore, the distances $SU_{TX} \to SU_R$, $SU_R \to SU_{RX}$, $PU_{TX} \to SU_R$, $SU_R \to PU_{RX}$, and $PU_{TX} \to PU_{RX}$ are normalized as d_1, d_2, d_3, d_4 and d_5, respectively. In this system model, the performance of the primary networks only depends on d_1, d_3, d_4, and d_5 and the performance of the secondary networks only depends on d_1, d_2, d_3 and d_4.

Fig. 2 and Fig. 3 depict the outage probability and SER of the primary network versus average SNR, P_p/N_0, for different distances $SU_{TX} \to SU_R$ and $PU_{TX} \to SU_R$ while the distances $PU_{TX} \to SU_R$ and $PU_{TX} \to PU_{RX}$ is kept constant. Further, P_s is set to 10 dBW and Q is set to 5 dBW. It can be seen from these figures that the outage probability and SER of the primary network are improved when d_1 increases. This performance enhancement can be attributed to the fact that when d_1 increases, there exits less interference from SU_{TX} to the primary network. Furthermore, when d_3 decreases from

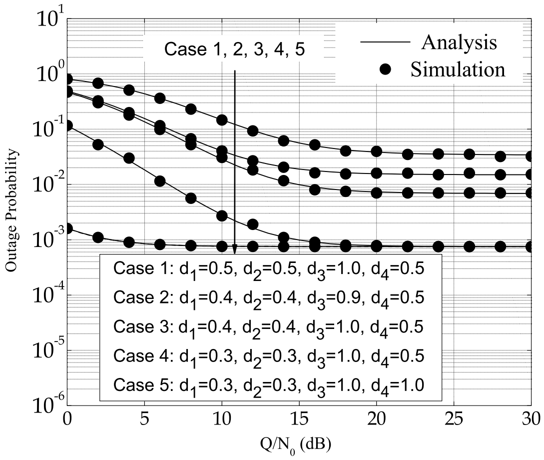

Fig. 6. Outage probability of the secondary network with various distances of the links $SU_{TX} \to SU_R$, $PU_{TX} \to SU_R$, and $SU_R \to SU_{RX}$.

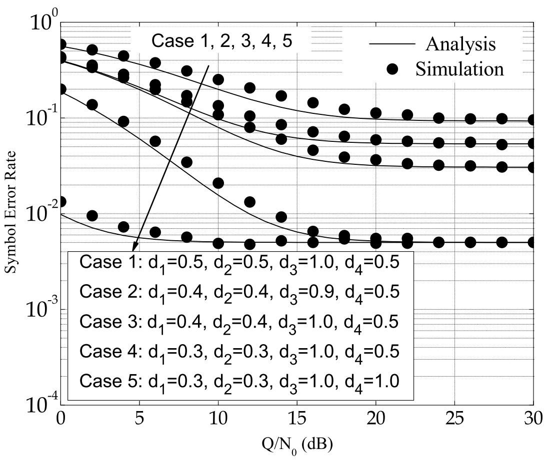

Fig. 7. Symbol error rate for 8-PSK of the secondary network with various distances of the links $SU_{TX} \to SU_R$, $PU_{TX} \to SU_R$, $SU_R \to SU_{RX}$.

$d_3 = 1.5$ in Case 1 to $d_3 = 1.0$ in Case 2, the attenuation of the primary signal in the link from PU_{TX} to SU_R will become less severe. As a consequence, the outage probability and SER performance is improved.

Fig. 4 and Fig. 5 present a comparison between the outage probability and SER of the primary transmission for various distances d_5 from SU_{TX} to SU_{RX} with and without the help of the secondary relay (only the direct communication is adopted for the primary transmission). As expected, the performance when combining both the direct link and relaying link always outperforms that of having only a direct link. Therefore, in our model, the primary network performance is enhanced with the help of the secondary relay.

Fig. 6 and Fig. 7 depict the outage probability and SER of the secondary transmission for various d_1, d_2, d_3 and d_4. The transmit power of the primary network and secondary network are set equally to $P_p = P_s = 10$ dBW. Given the same distances d_3 and d_4, when the distances from SU_{TX} to SU_R, d_1, and SU_R to SU_{RX}, d_2, decrease, the outage probability and SER of secondary transmission are decreased. This is because attenuation due to path loss increases with distance. Moreover, by comparing Case 2 and Case 3, having the same values of d_1, d_2 and d_4, the better performance is achieved in Case 3 with larger value of d_3 because of less interference from PU_{TX} to the secondary transmission.

Finally, when the distance from SU_R to PU_{RX}, d_4, is sufficiently large such as in Case 5 of Fig. 6 and Fig. 7, the outage probability and SER of the secondary transmission are almost invariant with the increase of the interference threshold Q. This is due to the fact that the signal from the secondary transmission is severely attenuated over the large distance d_4 from SU_R to SU_{RX}. Hence, the interference caused by the secondary transmission to PU_{RX} is negligible. As a result, for any interference threshold, Q, this interference is always smaller than Q.

VI. CONCLUSIONS

In this paper, we considered a system model for spectrum sharing where the AF cognitive relay assisted both the primary transmitter and the secondary transmitter in forwarding their signals. In particular, we derived exact closed-form expressions for the outage probability and SER of the primary transmission. Moreover, we also presented an exact closed-form expression for the outage probability and a tight approximation for the SER of the secondary transmission. Our works show that in this system not only secondary transmission benefits from using the primary networks' frequency but also the primary transmission gain in performance by utilizing the relay of the secondary system. Finally, our analytical results are in close agreement with Monte Carlo simulations, which validates the presented analysis.

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