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A Module Based Active Noise Control System for Ventilation Systems, Part I: Influence of Measurement Noise on the Performance and Convergence of the Filtered-x LMS Algorithm

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Low noise level is an essential feature when installing ventilation systems today. To achieve attenuation over a broad frequency range, the passive silencers traditionally used to attenuate ventilation noise can be combined with an active noise control (ANC) system. To insure reliable operation and desirable levels of attenuation when applying ANC to duct noise, it is highly important to be able to suppress the contamination of the microphone signals due to the turbulent pressure fluctuations, which arise as the microphones are exposed to the airflow in the duct. This paper is the first in a series of two regarding the problem of turbulence-induced noise originating from the airflow inside the ducts. Part I is concerned with theoretical and experimental investigations of the influence of the turbulence-induced noise on the adaptive algorithm in the ANC system. Part II is concerned with the design and the investigations of microphone installations for turbulence suppression and the results concerning the performance of an ANC system with the different microphone installations are presented. Some of the results were obtained at an acoustic laboratory according to an ISO-standard. The attenuation achieved with ANC was approximately 15-25 dB between 50-315 Hz, even for airflow speeds up to 20 m/s.

1. INTRODUCTION

Low-frequency noise can have negative effects on human well-being, is often annoying, and can also affect our ability to perform different tasks—for example, when working.[1] Ventilation systems constitute a well-known source of low-frequency noise in environments like, schools, factories, hospitals, office buildings etc, as well as in our homes. As awareness of the negative effects of low-frequency noise on human well-being has increased and so too has the requirement for quieter ventilation installations.

A technique which has proven to be an effective way to attenuate low-frequency noise is active noise control (ANC).[2–4] The basic idea of active noise control is to let a secondary source generate a secondary sound field, which destructively interferes with the undesired primary sound field. A single-channel feedforward adaptive control system[2,3] used to attenuate ventilation noise generally consists of two microphones, one loudspeaker, and a control unit. Even though the control unit relies on adaptive digital signal processing, it is of highest importance that the physical arrangement is designed such to insure reliable operation and desirable levels of attenuation.[3,5]

Placing the microphones in airflow will result in noise contamination of the microphone signals, since they will each contain a signal component induced by the turbulent pressure fluctuations, which arise when the diaphragm of the microphones are exposed to the airflow in the duct. A high level of turbulence-induced noise in the reference- and error microphones will lead to a decreased performance of the ANC system.[2,3]

Two papers are presented, which analyze the influence of the turbulence induced noise on the algorithm used in the controller, as well as the design and investigations of different microphone installations for the reduction of the turbulent noise when applying ANC to ducts. In Part I (the present paper), the influence of the turbulent noise on the algorithm is analyzed both theoretically and experimentally. The results show that a high level of turbulent noise present at the reference microphone, as compared to the level of acoustic noise, will affect the optimal filter weight solution and therefore lower the ability of the ANC system to cancel the acoustic noise. It is also shown that measurement noise at the reference sensor will lower the maximum step size and, hence, the maximum convergence rate. Further, the results show that a high level of turbulent noise at the error sensor, as compared to the level of acoustic noise, will not affect the filter weights in mean, but will increase the convergence time of the algorithm. In Part II,[6] different microphone installations for reducing the turbulent noise are investigated. Further, comparative results concerning the performance of an ANC system using the different microphone installations are presented. Some of the results were obtained in an acoustic laboratory according to an ISO-standard.

2. THE ACTIVE NOISE CONTROL SYSTEM

In this work it was desirable to apply ANC in the frequency range up to 400 Hz, which is in the plane-wave propagation region for the ducts used. Hence, a single-channel ANC system could be used instead of a multiple channel ANC system, which would have to be used if ANC was to be applied above the plane wave propagation region.[7,8] The ANC system used was a single-channel feedforward adaptive control system based on the time-domain leaky filtered-x LMS (FxLMS) algorithm[2,3,9] given by Eq. (1),

$$\begin{aligned} y(n) &= \mathbf{w}^T(n)\,\mathbf{x}(n) \\ e(n) &= d(n) + y_F(n) \\ \mathbf{w}(n+1) &= v\mathbf{w}(n) - 2\mu\mathbf{x}_{\hat{F}}(n)\,e(n), \end{aligned} \tag{1}$$

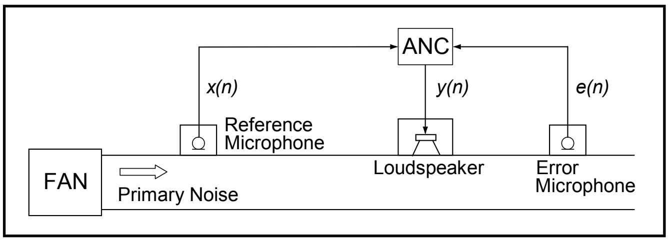

Figure 1. Single-channel feedforward ANC system applied to a duct.

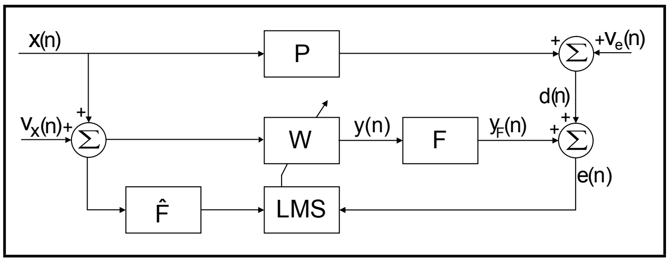

Figure 2. Block diagram of the filtered-x LMS algorithm with measurement noise at the reference- and error sensors, $v_x(n)$ and $v_e(n)$, respectively.

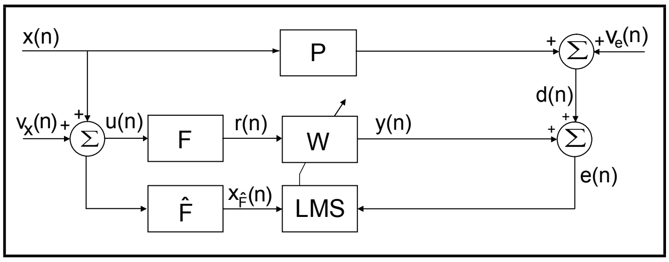

Figure 3. Rearranged version of Fig. 2.

where $y(n)$ is the output from the adaptive filter, $\mathbf{w}(n)$ is the adaptive filter weight vector, $d(n)$ is the desired signal, $y_F(n)$ is the output after the forward path, $\mathbf{x}_{\hat{F}}(n)$ is a filtered reference signal vector produced by filtering the reference microphone signal $x(n)$ with an estimate, $\hat{F}$, of the forward path F between the loudspeaker input to the error microphone output, and $e(n)$ is the error microphone signal. The leaky factor and the step size are given by v and μ, respectively. A simplified schematic illustration of the ANC system installed in a duct is shown in Fig. 1.

2.1. Influence of Measurement Noise on the ANC System Performance

In reality, the performance of the ANC system is affected by the measurement noise present in the microphone signals. The measurement noise can be electrical but, in this application, it is likely that the noise due to the airflow around the microphones have more impact on the performance of the controller. A convergence analysis has been performed for one case with the measurement noise present at the reference sensor and not at the error sensor, and for one case with the measurement noise present at the error sensor and not at the reference sensor. Also, a frequency domain analysis of the influence of measurement noise on the ANC system performance with the measurement noise present at both the reference sensor and the error sensor has been performed. In the convergence analysis with the measurement noise present only at the reference sensor, it was analyzed if the adaptive filter weights in mean converge to the optimal filter weight solution. The analysis performed with the measurement noise present only at the error sensor concerns convergence in the mean-square sense—i.e., the influence of the measurement noise at the error sensor on the variance of the adaptive filter weights around the optimal filter weight solution. The theory in this section applies to all types of uncorrelated measurement noise although, here, it is the noise generated due to the airflow around the microphones at which it is aimed. Further, in the analysis in this section, no acoustic feedback between the output of the adaptive filter and the reference microphone is considered. The ANC system used for the practical experiments was based on the leaky filtered-x LMS algorithm and, in the experiments, the leaky factor was chosen to be very close to one. In the different analyzes of the influence of measurement noise on the ANC system's performance, the leaky factor was assumed to equal 1. Thus, the analyses were carried out for the filtered-x LMS algorithm. A block diagram of the filtered-x LMS algorithm, including the measurement noise on the reference- and error sensors is illustrated in Fig. 2, where P is the primary path between the reference and error microphones.

If the adaptive filter is assumed to be changing slowly compared to the time constant for the impulse response of the forward path, the order of the adaptive filter and the forward path in Fig. 2 may change,[3] and, hence, Fig. 2 can be rearranged to Fig. 3. This assumption was made in the following convergence analyses, which were thus performed on the system illustrated in Fig. 3.

2.1.1. Convergence Analysis with Measurement Noise at the Reference Sensor

In this analysis it is assumed that the reference signal $x(n)$, desired signal $d(n)$, and the measurement noise $v_x(n)$ are weakly stationary stochastic processes, and the measurement noise $v_x(n)$ is uncorrelated with both $x(n)$ and $d(n)$. Also, no measurement noise is present at the error sensor, i.e., $v_e(n) = 0$. Further, it is assumed that the estimate of the forward path equals the true forward path, $\hat{F} = F$. Under these assumptions, the weight adjustment equation for the filtered-x LMS algorithm can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - 2\mu\left(\mathbf{x}_F(n) + \mathbf{v}_F(n)\right)e(n), \tag{2}$$

where $\mathbf{x}_F(n)$ is a vector containing the acoustic noise component, $x(n)$, of the reference microphone signal filtered with the forward path F, and $\mathbf{v}_F(n)$ is a vector containing the measurement noise component, $v_x(n)$, of the reference microphone signal filtered with the forward path F.

Further, the error signal $e(n)$ can be written as

$$e(n) = \left(\mathbf{x}_F^T(n) + \mathbf{v}_F^T(n)\right)\mathbf{w}(n) + d(n). \tag{3}$$

Thus, the update equation may be expressed as

$$\begin{aligned}\mathbf{w}(n+1) = \mathbf{w}(n) - 2\mu\left(\mathbf{x}_F(n) + \mathbf{v}_F(n)\right)\\ \left(\left(\mathbf{x}_F^T(n) + \mathbf{v}_F^T(n)\right)\mathbf{w}(n) + d(n)\right).\end{aligned} \tag{4}$$

Taking the expected value of both sides of the equation and assuming that $\mathbf{w}(n)$ is independent of $\mathbf{x}_F(n)$ as well as $\mathbf{v}_F(n)$,[10] and that $\mathbf{x}_F(n)$ and $\mathbf{v}_F(n)$, as well as $\mathbf{v}_F(n)$ and $d(n)$, are uncorrelated results in

$$E\left[\mathbf{w}(n+1)\right] = \left(\mathbf{I} - 2\mu\left(\mathbf{R}_F + \mathbf{V}_F\right)\right)E\left[\mathbf{w}(n)\right] - 2\mu\mathbf{p}_F, \tag{5}$$

where $\mathbf{R}_F = E\left[\mathbf{x}_F(n)\mathbf{x}_F^T(n)\right]$ and $\mathbf{V}_F = E\left[\mathbf{v}_F(n)\mathbf{v}_F^T(n)\right]$ are symmetric autocorrelation matrices and $\mathbf{p}_F = E\left[\mathbf{x}_F(n)d(n)\right]$ is the cross-correlation vector.

Further, assuming that $E[\mathbf{w}(n+1)] = E[\mathbf{w}(n)]$ as $n \to \infty$ yields that the limit for expectation of the coefficient vector is

$$\lim_{n\to\infty} E[\mathbf{w}(n)] = -(\mathbf{R}_F + \mathbf{V}_F)^{-1}\mathbf{p}_F. \tag{6}$$

If no measurement noise is present at the reference sensor, and uncorrelated white noise with mean-square value σ^2 is added to the *filtered* reference signal $\mathbf{x}_F(n)$, Eq. (6) becomes $\lim_{n\to\infty} E[\mathbf{w}(n)] = -\left(\mathbf{R}_F + \sigma^2\mathbf{I}\right)^{-1}\mathbf{p}_F$, where $\mathbf{I}$ is an identity matrix.[10,11] An equivalent result would be produced for the leaky filtered-x LMS algorithm without any measurement noise, and with a leaky factor equal to $v = 1 - 2\mu\sigma^2$, see Eq. (1).[10,11] Equation (6) is the optimal filter weight solution with measurement noise at the reference sensor, which the algorithm approaches in the mean as $n \to \infty$. Comparing this with the optimal solution approached without measurement noise at the reference sensor, $\lim_{n\to\infty} E[\mathbf{w}(n)] = -\mathbf{R}_F^{-1}\mathbf{p}_F$, it can be seen that the measurement noise at the reference sensor will cause the adaptive filter to converge in the mean toward a solution that is biased compared to the Wiener-solution for the case of no measurement noise affecting the reference sensor.[10] Hence, the measurement noise at the reference sensor will lower the ability of the ANC system to attenuate the acoustic noise. To investigate the influence of $v_x(n)$ on the convergence rate of the algorithm, the following analysis is performed.

The difference between the expected value of the filter weights and the filter weight vectors optimal Wiener-solution can be defined as[11]

$$\tilde{\mathbf{w}}(n) = E[\mathbf{w}(n)] - \mathbf{w}_{\text{opt}}. \tag{7}$$

In this case, with measurement noise present at the reference sensor, the optimal weight vector is given by $\mathbf{w}_{\text{opt}} = -(\mathbf{R}_F + \mathbf{V}_F)^{-1}\mathbf{p}_F$. Thus, $\mathbf{p}_F = -(\mathbf{R}_F + \mathbf{V}_F)\mathbf{w}_{\text{opt}}$. Substitution into Eq. (5), subtracting $\mathbf{w}_{\text{opt}}$ from both sides of the resulting equation and using Eq. (7) yields

$$\tilde{\mathbf{w}}(n+1) = (\mathbf{I} - 2\mu(\mathbf{R}_F + \mathbf{V}_F))\tilde{\mathbf{w}}(n). \tag{8}$$

Defining the sum of the two symmetric autocorrelation matrices $\mathbf{R}_F$ and $\mathbf{V}_F$ as the symmetric matrix $\mathbf{U}_F = \mathbf{R}_F + \mathbf{V}_F$, Eq. (8) can be written as

$$\tilde{\mathbf{w}}(n+1) = (\mathbf{I} - 2\mu\mathbf{U}_F)\tilde{\mathbf{w}}(n). \tag{9}$$

The symmetric matrix $\mathbf{U}_F$ can be factorized as[10,11]

$$\mathbf{U}_F = \mathbf{Q}_U\mathbf{\Lambda}_U\mathbf{Q}_U^T, \tag{10}$$

where $\mathbf{Q}_U$ is a unitary matrix that satisfies $\mathbf{Q}_U^T\mathbf{Q}_U = \mathbf{I}$ and contains the eigenvectors to $\mathbf{U}_F$. Further, $\mathbf{\Lambda}_U$ is a diagonal matrix containing the eigenvalues of $\mathbf{U}_F$. Thus, Eq. (9) can be written as

$$\tilde{\mathbf{w}}(n+1) = \left(\mathbf{I} - 2\mu\mathbf{Q}_U\mathbf{\Lambda}_U\mathbf{Q}_U^T\right)\tilde{\mathbf{w}}(n). \tag{11}$$

Multiplying both sides with $\mathbf{Q}_U^T$ from the left and defining

$$\tilde{\mathbf{w}}'(n) = \mathbf{Q}_U^T\tilde{\mathbf{w}}(n) \tag{12}$$

yields

$$\tilde{\mathbf{w}}'(n+1) = (\mathbf{I} - 2\mu\mathbf{\Lambda}_U)\tilde{\mathbf{w}}'(n), \tag{13}$$

where element i of $\tilde{\mathbf{w}}'(n+1)$ is given by

$$\tilde{w}_i'(n+1) = (1 - 2\mu\lambda_{U_i})\tilde{w}_i'(n). \tag{14}$$

Thus, $\tilde{w}_i'(n)$ will approach zero so that the algorithm in mean will converge to the optimal solution, given that $|1 - 2\mu\lambda_{U_i}| < 1$, yielding that[11]

$$0 < \mu < \frac{1}{\lambda_{U_i}}, \tag{15}$$

which should be valid for all i. Since the largest eigenvalue, $\lambda_{U\max}$, of $\mathbf{U}_F$ gives the most stringent condition of convergence, the algorithm will converge provided that

$$0 < \mu < \frac{1}{\lambda_{U\max}}. \tag{16}$$

A stronger condition is obtained using the fact that the eigenvalues of the symmetric correlation matrix $\mathbf{U_F}$ are $\lambda_{U_i} \geq 0$, so that the largest eigenvalue is upper bounded by the trace of $\mathbf{U}_F$, which gives the more restrictive bound on the step size:[10]

$$0 < \mu < \frac{1}{\text{trace}[\mathbf{U}_F]}, \tag{17}$$

where

$$\text{trace}[\mathbf{U}_F] = L_w\left(\sigma_{x_F}^2 + \sigma_{v_{x_F}}^2\right). \tag{18}$$

In Eq. (18) $\sigma_{x_F}^2$ and $\sigma_{v_{x_F}}^2$ is the power of the acoustic part of the reference signal filtered with the forward path, and the power of the measurement noise at the reference sensor filtered with the forward path, respectively. Further, L_w is the length of the adaptive FIR-filter. Thus, the bound on the step size with measurement noise at the reference sensor is given by

$$0 < \mu < \frac{1}{L_w\left(\sigma_{x_F}^2 + \sigma_{v_{x_F}}^2\right)}. \tag{19}$$

In comparing this with the bound on the step size without measurement noise at the reference sensor, $0 < \mu < 1/L_w\sigma_{x_F}^2$, it can be seen that the maximum step size for the case of measurement noise at the reference sensor is likely to be reduced, resulting in a decreased maximum convergence rate.

2.1.2. Convergence Analysis with Measurement Noise at the Error Sensor

When measurement noise uncorrelated with the reference signal $x(n)$ and the desired signal $d(n)$ is present at the error sensor of the ANC system using the filtered-x LMS algorithm, it can easily be shown that this will not affect the optimal Wiener filter solution (assuming infinite numerical precision), i.e., in this case, the optimal filter solution is identical to the case of no measurement noise present at the error sensor. However, measurement noise present at the error sensor may affect the statistical properties of the adaptive filter's weight vector and the time it takes to converge in the mean-square sense.

Gradient-based algorithms update the filter weights by taking a step of size μ in the negative direction of the gradient of a cost function. In the filtered-x LMS algorithm the gradient is instantaneously estimated at each iteration, which produces noise in the gradient estimate.[3] However, measurement noise at the error sensor may also cause an increased level of gradient noise. The estimate of the gradient vector in filtered-x

LMS algorithm may be written as a sum of the true gradient at time n, $\nabla\xi(n)$, and a gradient noise vector $\mathbf{n}(n)$, as[10]

$$\nabla\hat{\xi}(n) = \nabla\xi(n) + \mathbf{n}(n). \quad (20)$$

The direction of the gradient estimate in the filtered-x LMS algorithm will not instantaneously equal the direction of the actual gradient, however, in mean, it will.[12] Based on the steepest descent algorithm, which minimizes the cost function $\xi(n) = E\left[e^2(n)\right]$ instead of the cost function $\hat{\xi}(n) = e^2(n)$ as the filtered-x LMS algorithm minimizes, the true gradient vector is given by $\nabla\xi(n) = 2(\mathbf{R}_F\mathbf{w}(n) + \mathbf{p}_F)$ where $\mathbf{R}_F = E\left[\mathbf{x}_F(n)\mathbf{x}_F^T(n)\right]$ and $\mathbf{p}_F = E[d(n)\mathbf{x}_F(n)]$.[10] The optimal filter weight solution is given by $\mathbf{w}_{\text{opt}} = -\mathbf{R}_F^{-1}\mathbf{p}_F$. In this analysis it is assumed that the reference signal $x(n)$, desired signal $d(n)$, and the measurement noise are weakly stationary stochastic processes, and that the measurement noise $v_e(n)$ is uncorrelated with both $x(n)$ and $d(n)$. Also, no measurement noise is present at the reference sensor, i.e., $v_x(n) = 0$. Further, it is assumed that the estimate of the forward path equals the true forward path $\hat{F} = F$. The gradient estimate in Eq. (20) can now be rewritten as

$$\nabla\hat{\xi}(n) = 2\mathbf{R}_F(\mathbf{w}(n) - \mathbf{w}_{\text{opt}}) + \mathbf{n}(n). \quad (21)$$

Consider the filter weight update equation given by $\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\nabla\xi(n)$. Subtracting the optimal Wiener solution $\mathbf{w}_{\text{opt}}$ from both sides of the equation, and substituting the gradient estimate with Eq. (21) yields a recursive equation describing the error in the adaptive filter weights. Defining a weight error vector according to $\tilde{\mathbf{w}}(n) = \mathbf{w}(n) - \mathbf{w}_{\text{opt}}$, the equation is given by

$$\tilde{\mathbf{w}}(n+1) = (\mathbf{I} - 2\mu\mathbf{R}_F)\tilde{\mathbf{w}}(n) - \mu\mathbf{n}(n). \quad (22)$$

To decouple Eq. (22), a decomposition of the autocorrelation matrix $\mathbf{R}_F$ is used and is given by

$$\mathbf{R}_F = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^T, \quad (23)$$

where the columns of matrix $\mathbf{Q}$ consist of the eigenvectors to $\mathbf{R}_F$ and satisfies $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$, and $\boldsymbol{\Lambda}$ is a diagonal matrix whose elements consist of the eigenvalues to $\mathbf{R}_F$. Defining rotated vectors as

$$\begin{aligned}\tilde{\mathbf{w}}'(n) &= \mathbf{Q}^T\tilde{\mathbf{w}}(n)\\ \mathbf{n}'(n) &= \mathbf{Q}^T\mathbf{n}(n)\end{aligned} \quad (24)$$

yields a decoupled form of Eq. (22), where the equation for element i is given by

$$\tilde{w}_i'(n+1) = (1 - 2\mu\lambda_i)\tilde{w}_i'(n) - \mu n_i'(n). \quad (25)$$

The mean squared value of $\tilde{w}_i'$ is denoted as $\rho_i(n) = E\left[\tilde{w}_i'^2(n)\right]$, and the mean value of $\tilde{w}_i'$ is zero, $E\left[\tilde{w}_i'(n)\right] = 0$. Squaring both sides and taking expectations of Eq. (25), assuming that $E\left[\tilde{w}_i'(n)n_i'(n)\right] = 0$, yields

$$\rho_i(n+1) = (1 - 2\mu\lambda_i)^2\rho_i(n) + \mu^2E\left[n_i'^2(n)\right]. \quad (26)$$

To find an expression for $E\left[n_i'^2(n)\right]$ it is assumed that the algorithm has converged to a condition where the weight vector $\mathbf{w}(n)$ is near the optimal weight solution $\mathbf{w}_{\text{opt}}$. Then, the true gradient vector $\nabla\xi(n)$ will be approximately zero. Hence, the gradient estimate will then equal the gradient estimation noise vector, $\nabla\hat{\xi}(n) = \mathbf{n}(n)$; see Eq. (20). The gradient estimate for the filtered-x LMS algorithm is given by $\nabla\hat{\xi}(n) = 2\mathbf{x}_F(n)e(n)$, which for this condition with $\nabla\xi(n) = 0$, is equal to the gradient estimation noise vector $\mathbf{n}(n)$.[10]

With measurement noise, $v_e(n)$, present at the error sensor, the error sensor signal is given by $e(n) + v_e(n)$. The gradient estimate, or gradient noise when $\nabla\xi(n) = 0$, is thus given by

$$\nabla\hat{\xi}(n) = \mathbf{n}(n) = 2\mathbf{x}_F(n)(e(n) + v_e(n)), \quad (27)$$

where $e(n)$ and $v_e(n)$ are assumed to be uncorrelated. The term $2\mathbf{x}_F(n)v_e(n)$ due to the measurement noise in the error signal becomes a disturbance of the gradient estimate, as it is not present without measurement noise in the error signal. Based on the assumption that $v_e(n)$ and $\mathbf{x}_F(n)$ are uncorrelated, the weights on the average will not be affected since the influence of the disturbance term will average to zero. However, the instantaneous weights with measurement noise in the error signal will not equal the instantaneous weights without measurement noise in the error signal. Thus, this can be seen as a disturbance that misleads the convergence of the algorithm, resulting in a degraded performance of the ANC system.

Assuming that the weight vector $\mathbf{w}(n)$ remains in the vicinity of its optimal solution $\mathbf{w}_{\text{opt}}$ results in the error signal $e(n)$ and the filtered input signal $\mathbf{x}_F(n)$ not correlating with each other.[10] Also assume that $v_e(n)$ is uncorrelated with $e(n)$ as well as with $\mathbf{x}_F(n)$. The covariance in the mean square of the gradient estimate when $\nabla\xi(n) = 0$ is given by

$$\begin{aligned}E\left[\mathbf{n}(n)\mathbf{n}^T(n)\right] &= 4E\left[e^2(n)\right]E\left[\mathbf{x}_F(n)\mathbf{x}_F^T(n)\right] + \\ &+4E\left[v_e^2(n)\right]E\left[\mathbf{x}_F(n)\mathbf{x}_F^T(n)\right] = 4\left(\xi_{\min} + \sigma_{v_e}^2\right)\mathbf{R}_F,\end{aligned} \quad (28)$$

where $\xi_{\min}$ is the minimum mean-square error defined as the mean-square error obtained when using the optimal filter weights and there is no measurement noise at the error sensor, and $\sigma_{v_e}^2$ is the power of the measurement noise at the error sensor.[10] Equation (26) is based on the gradient noise vector $\mathbf{n}'(n)$ in the eigenvector coordinate system. The corresponding equation for Eq. (28) based on $\mathbf{n}'(n)$ is given by

$$\begin{aligned}E\left[\mathbf{n}'(n)\mathbf{n}'^T(n)\right] &= \mathbf{Q}^TE\left[\mathbf{n}(n)\mathbf{n}^T(n)\right]\mathbf{Q} = \\ &= 4\left(\xi_{\min} + \sigma_{v_e}^2\right)\mathbf{Q}^T\mathbf{R}_F\mathbf{Q} = 4\left(\xi_{\min} + \sigma_{v_e}^2\right)\boldsymbol{\Lambda}.\end{aligned} \quad (29)$$

This matrix is diagonal and element i is given by

$$E\left[n_i'^2(n)\right] = 4\left(\xi_{\min} + \sigma_{v_e}^2\right)\lambda_i. \quad (30)$$

Assume that the expression for $E\left[n_i'^2(n)\right]$ is valid for all n. Accordingly, combining Eq. (30) with Eq. (26) and applying repeated substitutions results in the solution

$$\begin{aligned}\rho_i(n) &= (1 - 2\mu\lambda_i)^{2n}\rho_i(0) + \\ &+4\mu^2\sum_{j=0}^{n-1}(1 - 2\mu\lambda_i)^{2j}\left(\xi_{\min} + \sigma_{v_e}^2\right)\lambda_i.\end{aligned} \quad (31)$$

If the step size μ is properly chosen for convergence of the weight update equation, $0 < \mu < \frac{1}{\lambda_i}$, the term due to $\rho_i(0)$ decays to zero as time n increases. Accordingly, Eq. (31) is now given by

$$\begin{aligned}\lim_{n\to\infty}\rho_i(n) &= 4\mu^2\left(\xi_{\min} + \sigma_{v_e}^2\right)\lambda_i\left[\frac{1}{1-(1-2\mu\lambda_i)^2}\right] = \\ &= \frac{\mu\left(\xi_{\min}+\sigma_{v_e}^2\right)}{1-\mu\lambda_i}.\end{aligned} \quad (32)$$

Thus, an *increased* power of the measurement noise, $\sigma_{v_e}^2$, will result in an *increased* variance of the weight error vector. A decreased variance can be obtained by decreasing the step size μ at the expense of an increased convergence time.

Variations of the adaptive weights around its optimal solution in a steady state will lead to variations of the instantaneous mean-square error around the minimum mean-square error. The average of these increases in mean-square error is denoted as excess mean-square error, defined as[10]

$$\xi_{ex} = E\left[\hat{\xi}(n) - \xi_{\min}\right], \tag{33}$$

where $\xi_{\min}$ is the minimum mean-square error defined as the mean-square error obtained when using the optimal filter weights and there is no measurement noise at the error sensor.[10] An approximation of the excess mean-square error can, for the filtered-x LMS algorithm, be found from[10]

$$\xi_{ex} = E\left[(\mathbf{w}(n) - \mathbf{w}_{\text{opt}})^T \mathbf{R}_F (\mathbf{w}(n) - \mathbf{w}_{\text{opt}})\right]. \tag{34}$$

By examining Eq. (33) and Eq. (34), it follows that deviations of $\mathbf{w}(n)$ from the optimal weights $\mathbf{w}_{\text{opt}}$, increase the excess mean-square error, i.e., the mean-square error exceeds $\xi_{\min}$. Hence, random weight variations around the optimal solution cause an increase in mean-square error, and result in excess mean-square error even without any measurement noise in the reference signal $x(n)$, and error signal $e(n)$.[10]

Rewriting Eq. (34) results in the excess mean-square error possibly being found from $\xi_{ex} = E\left[\tilde{\mathbf{w}}^T(n)\mathbf{R}_F\tilde{\mathbf{w}}(n)\right] = E\left[\tilde{\mathbf{w}}'^T(n)\mathbf{\Lambda}\tilde{\mathbf{w}}'(n)\right]$,[10] or as

$$\xi_{ex} = \sum_{i=0}^{L_w-1} \lambda_i E\left[\tilde{w}_i'^2(n)\right] = \sum_{i=0}^{L_w-1} \lambda_i \rho_i(n), \tag{35}$$

where L_w is the length of the adaptive filter. Substituting $\rho_i(n)$ with the result in Eq. (32) gives

$$\xi_{ex} = \sum_{i=0}^{L_w-1} \lambda_i \mu \left(\xi_{\min} + \sigma_{v_e}^2\right) \frac{1}{1-\mu\lambda_i}. \tag{36}$$

For small step sizes, μ, it can be assumed that $\mu\lambda_i << 1$ in Eq. (36). Hence, Eq. (36) may be approximated as

$$\begin{aligned}\xi_{ex} &\approx \sum_{i=0}^{L_w-1} \lambda_i \mu \left(\xi_{\min} + \sigma_{v_e}^2\right) = \\ &= \mu\left(\xi_{\min} + \sigma_{v_e}^2\right) \sum_{i=0}^{L_w-1} \lambda_i = \mu\left(\xi_{\min} + \sigma_{v_e}^2\right) \text{tr}\left[\mathbf{R}_F\right].\end{aligned} \tag{37}$$

This equation shows that the excess mean-square error is directly proportional to the step size μ. With a larger value of the step size, a reduced steady-state performance is obtained with higher excess mean-square error. On the other hand, a large value of the step size results in faster convergence. Accordingly, there is a design trade off between the excess mean-square error and the speed of convergence. The measurement noise, v_e, at the error sensor also has an impact on the excess mean-square error. The larger the level of measurement noise present at the error sensor is, the higher the excess mean-square error. However, to reduce the impact of the measurement noise, a decreased value of the step size can be used. Further, Eq. (37) yields that the excess mean-square error for the filtered-x LMS algorithm *without* measurement noise at the error sensor ($\sigma_{v_e}^2 = 0$) may approximately be written as[10]

$$\xi_{ex} \approx \mu\xi_{\min}\text{tr}\left[\mathbf{R}_F\right] = \mu\xi_{\min} L_w \sigma_{x_F}^2. \tag{38}$$

That is, the excess mean-square error will depend on the step size, the minimum mean-square error, the number of weights in the adaptive filter, and the power of the filtered reference signal. An approximation of the excess mean-square error *with* measurement noise in the error signal may, according to Eq. (37), be written as

$$\xi_{ex_{v_e}} \approx \mu_{v_e}\left(\xi_{\min} + \sigma_{v_e}^2\right) L_w \sigma_{x_F}^2, \tag{39}$$

where μ_{v_e} denotes the step size with measurement noise present at the error sensor. Assume that the same excess mean-square error is wanted with measurement noise present at the error sensor as without. Using Eq. (38) and Eq. (39) gives

$$\xi_{ex_{v_e}} = \xi_{ex} \Leftrightarrow \mu_{v_e}\left(\xi_{\min} + \sigma_{v_e}^2\right) L_w \sigma_{x_F}^2 = \mu\xi_{\min} L_w \sigma_{x_F}^2. \tag{40}$$

Solving for the step size with measurement noise at the error sensor, μ_{v_e}, yields

$$\mu_{v_e} = \frac{\xi_{\min}}{\xi_{\min} + \sigma_{v_e}^2}\mu. \tag{41}$$

Accordingly, to achieve an excess mean-square error with the same level with measurement noise present at the error sensor as without, the step size needs to be decreased with a factor of the power of the measurement noise, $\sigma_{v_e}^2$. This will also affect the convergence time. If, for example $\mu_{v_e} = \mu/10$, it would take 10 times as long for the algorithm to converge with measurement noise at the error sensor as without measurement noise.

2.1.3. Frequency Domain Analysis of the Influence of Measurement Noise on the ANC System Performance

Assuming that the system's transfer paths and the adaptive filter in Fig. 3 are linear and time invariant, and also that the adaptive FIR-filter is double-sided, infinite and non-causal; an analysis of this system is carried out in the frequency domain.

The unconstrained optimal filter in the frequency domain for the system illustrated in Fig. 3 is given by[9]

$$W_{\text{opt}}(f) = -\frac{S_{dr}(f)}{S_{rr}(f)}, \tag{42}$$

where $S_{dr}(f)$ is the two-sided, cross-spectral density between the input to the adaptive filter $r(n)$ and the desired signal $d(n)$, and $S_{rr}(f)$ is the two-sided power spectral density of $r(n)$.

By examining Fig. 3, and assuming that the measurement noise $v_x(n)$ and $v_e(n)$ are uncorrelated with each other as well as with as with the acoustic signals, it follows that the power spectral density for the reference signal is given by

$$S_{rr}(f) = F^*(f)F(f)\left(S_{xx}(f) + S_{v_xv_x}(f)\right), \tag{43}$$

and that

$$S_{dr}(f) = F^*(f)P(f)S_{xx}(f), \tag{44}$$

where $S_{v_xv_x}(f)$ is the two-sided power spectral density of $v_x(n)$. Substitution of Eqs. (43) and (44) into Eq. (42) yields

$$W_{\text{opt}}(f) = -\frac{P(f)S_{xx}(f)}{F(f)\left(S_{xx}(f) + S_{v_xv_x}(f)\right)}, \tag{45}$$

which is the optimal unconstrained transfer function for the system illustrated in Fig. 3. The expression of Eq. (45) can be further examined in terms of the signal-to-noise ratio (SNR) at the reference sensor, which can be defined as[9]

$$SNR_x(f) = \frac{S_{xx}(f)}{S_{v_x v_x}(f)}. \tag{46}$$

In terms of the SNR defined in Eq. (46), the solution for the optimal filter response can be written as

$$\begin{aligned} W_{\text{opt}}(f) &= -\frac{SNR_x(f)}{1+SNR_x(f)}\frac{P(f)}{F(f)} = \\ &= -\frac{1}{\left(\frac{1}{SNR_x(f)}+1\right)}\frac{P(f)}{F(f)}. \end{aligned} \tag{47}$$

Equations (45) and (47) show that the optimal solution is not affected by the noise associated with the error sensor, but will be affected by the noise associated with the reference sensor, which will decrease the optimum weight and, hence, lower the ability to cancel the acoustic noise. However, from practical experience it is known that it takes longer time for the adaptive algorithm to converge if there is measurement noise at the error signal. This is also shown in the previous analysis concerning the convergence analysis with measurement noise at the error sensor, in section 2.1.2.

An estimate of the frequency-dependent attenuation achieved by the ANC system is given by[2,3]

$$A(f) = -10\log_{10}\left(1-\gamma_{du}^2(f)\right) \quad [\text{dB}], \tag{48}$$

where $\gamma_{du}^2(f)$ is the ordinary coherence function given by

$$\begin{aligned} \gamma_{du}^2(f) &= \frac{|S_{du}(f)|^2}{S_{uu}(f)S_{dd}(f)} = \\ &= \frac{|P(f)S_{xx}(f)|^2}{(S_{xx}(f)+S_{v_x v_x}(f))\left(|P(f)|^2 S_{xx}(f)+S_{v_e v_e}(f)\right)}, \end{aligned} \tag{49}$$

where $0 \leq \gamma_{du}^2(f) \leq 1$. Hence, the equation for the theoretical attenuation from the coherence can be written as

$$A(f) = -10\log_{10} \left(1-\frac{|P(f)S_{xx}(f)|^2}{(S_{xx}(f)+S_{v_x v_x}(f))\left(|P(f)|^2 S_{xx}(f)+S_{v_e v_e}(f)\right)}\right). \tag{50}$$

Equation (50) indicates a decreased level of achievable attenuation with measurement noise present at the reference sensor, which is in agreement with Eqs. (45) and (47) for the optimal solution of the adaptive filter $W_{\text{opt}}(f)$. However, Eq. (50) also indicates that measurement noise present only at the error sensor would decrease the achievable attenuation. Equations (45) and (47) do not indicate any impact on $W_{\text{opt}}(f)$ for measurement noise at the error sensor. Accordingly, it is important to investigate where the measurement noise is present when investigating the possibility of applying ANC to a certain system, using the coherence function. For example, if measurement noise is present at the error sensor and not at the reference sensor, and the coherence is measured between the outputs of the reference sensor, and the error sensor Eq. (48) would indicate a low level of attenuation, which is not the case according to Eq. (45). If the actual attenuation is then measured at the error sensor, it would be low since the attenuation of the acoustic noise would be masked by the measurement noise (depending on the SNR of the error microphone signal). Therefore, it is also important to use an evaluation microphone that is, if possible, not affected by measurement noise, and with a different position than the error microphone.

2.1.4. Experimental Investigations of the Influence of Measurement Noise on the ANC System Performance

The results obtained in the previous section indicate that measurement noise present at the reference sensor and not at the error sensor will affect the optimal weight vector solution. The obtained results also indicate that measurement noise present at the error sensor will affect the variance of the adaptive weights, resulting in a design trade-off between convergence rate and excess mean-square error. Further, it is shown in the previous section that the use of the coherence function to investigate the level of achievable attenuation should be treated with some care, since it would indicate low levels of attenuation with measurement noise present at the error sensor, which is not necessarily the case. These results have also been experimentally investigated. This was done by applying ANC in a duct system for three cases: 1) One case in which no measurement noise was present at the microphones, 2) one in which measurement noise was present only at the reference microphone, 3) and one in which measurement noise was present only at the error microphone. The noise source was a standard axial fan. The measurement noise was flow-induced, by placing the reference microphone and/or the error microphone in the direct airflow inside the duct, and by using an airflow speed of 6.7 m/s. Thus, with a microphone placed inside the duct, it contains both turbulent noise and acoustic noise. When a microphone was considered measurement-noise free, it was placed inside an outer microphone box based on T-duct. These microphone boxes are described in Part II.[6] In the current experimental setup, a standard axial ventilation fan was used as a noise source. The ventilation fan generated both the acoustic noise and the airflow in the ventilation system. In other words, the level of the generated acoustic noise and the airflow speed were not individually adjustable. Therefore, it was not possible to separate the acoustic noise generated by the fan and the turbulent noise generated by the airflow in the duct. Hence, the SNR (acoustic noise to turbulence noise ratio) of the microphone signals in the experiments could not be found and thus be reported. The attenuation achieved by the ANC system was then measured both in the error microphone and in an evaluation microphone positioned outside the duct at the duct outlet. The coherence between the error- and reference microphones in absence of control was also measured. The results from the experimental investigation are illustrated here. The weights of the adaptive filter were saved after running the ANC system for twice as long with measurement noise present at the reference sensor and at the error sensors as compared to when no measurement noise was present on either sensor. Also, the attenuation achieved by the ANC system was measured after running the ANC system for twice as long with measurement noise present at the reference sensor and at the error sensor as compared to the case without measurement noise present on either sensor. The step size with measurement noise at the error sensor was $1/10$ of the step size used in the case with no measurement noise present on either sensor. Several different values of the step size in the case with measurement noise present only at the reference sensor were used, including reduced values, compared to the case without any measurement noise on either sensor. However, this did not result in higher attenuation produced by the ANC system than illustrated in Fig. 4. It can be noted that in the theoretical analysis, no modeling error of the forward path is considered. In Fig. 4 (a), which

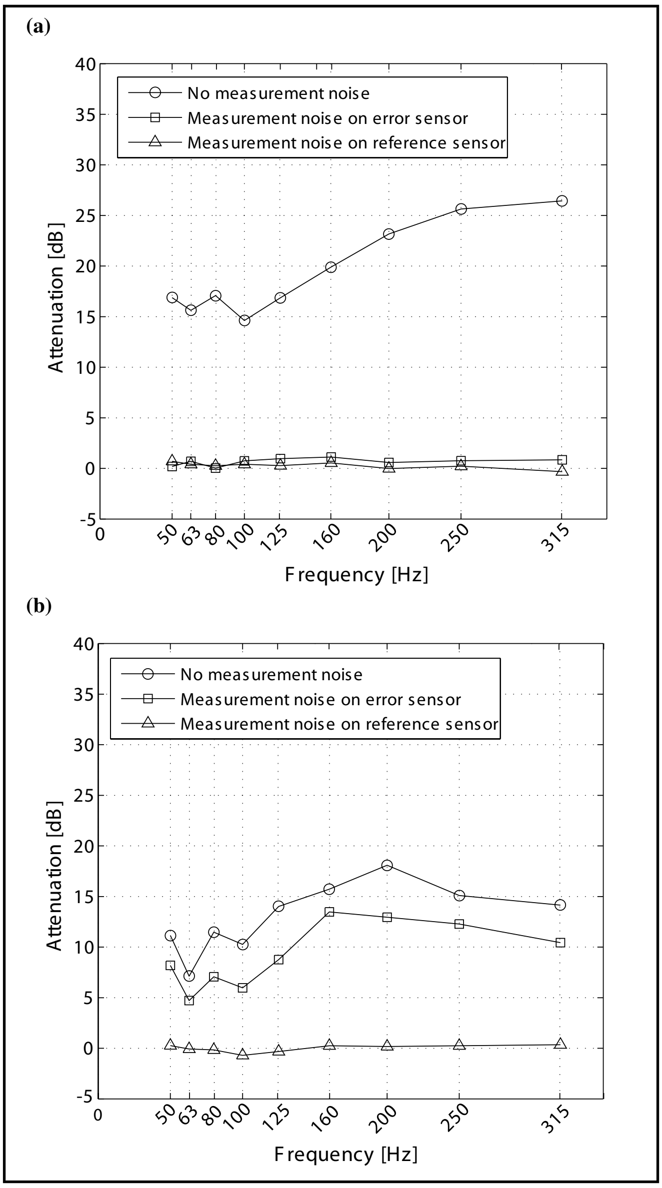

Figure 4. One-third octave spectrum of the attenuation achieved with the ANC system in (a) measured in the error microphone and in (b) measured in the evaluation microphone at the duct outlet. For both (a) and (b), the circles indicates no measurement noise present, squares indicate measurement noise present only at the error microphone, and triangles indicate measurement noise present only at the reference microphone. The measurements were performed after running the ANC system for twice as long for the two cases with measurement noise present as compared to cases when measurement noise was not present. The measurement noise was flow-induced by using an airflow of 6.7 m/s.

shows the level of attenuation at the error microphone, it can be seen that the level of attenuation measured at the error microphone is decreased both when measurement noise is present at the reference sensor and the error sensor. However, the level of attenuation of the acoustic noise is masked by the measurement noise in the error microphone. This can also be seen from Fig. 4 (b), which shows the level of attenuation in the evaluation microphone. Here, it can be seen that the acoustic noise is not attenuated with measurement noise at the reference sensor, but it is in fact attenuated with measurement noise at the error sensor even if it is not to the degree as when no measurement noise is present at either of the microphones.

In Fig. 5, the power spectral density of the error microphone signal with the ANC system off and with the ANC system turned on is illustrated. Here the reference and error microphones were placed in the microphone boxes based on T-ducts.

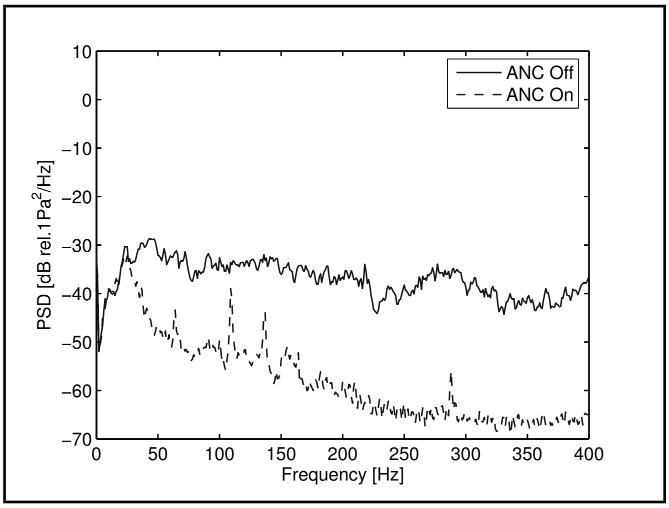

Figure 5. Power spectral density (PSD) of the error microphone signal with the reference and error microphones placed in microphone boxes based on T-duct. Solid line shows the ANC system off, and dashed line shows the ANC system on.

Thus, Fig. 5 corresponds to the upper curve with circles in Fig 4 (a). In Fig. 6 (a) the weights of the adaptive filter for the three different cases are illustrated. Observe that this figure is zoomed in to better show the differences: the actual filter lengths were 256 weights. From this figure, it can be seen that with measurement noise at the reference sensor the filter weights are smaller as compared to when measurement noise is not present, which is in agreement with Eq. (45). They are also somewhat smaller with measurement noise present at the error sensor. The results in Fig. 4 (b) and Fig. 6 (a) thus show that for this case, the step size should be decreased even more with measurement noise at the error sensor to achieve the same weights and level of attenuation as in the case when measurement noise is not present on either sensor.

Finally, Fig. 6 (b) illustrates the coherence between the error- and reference sensor in the absence of control, for the three different cases. Here, it can be seen that with measurement noise at the reference sensor as well as at the error sensor the coherence is very low even though the acoustic noise is attenuated a great deal with measurement noise present at the error sensor. Accordingly, it has also been shown experimentally that Eq. (48) should be used with some care.

3. CONCLUSIONS

It has been shown both theoretically and experimentally that the performance of an ANC system based on the filtered-x algorithm and applied to duct noise can be significantly decreased by the noise generated in the microphone signals due to the airflow present in ducts. The obtained results from the theoretical analysis show that measurement noise at the reference sensor will affect the optimal filter weight solution, resulting in a lowered ability of the ANC system to cancel the acoustic noise. It is also shown that measurement noise at the reference sensor will lower the maximum step size, resulting in a decreased maximum convergence rate of the algorithm. The theoretical analysis also shows that measurement noise present at the error sensor will affect the variance of the adaptive weights, resulting in a design trade-off between convergence rate and excess mean-square error. Further, it is shown that using the coherence function as a theoretical measure of the ability of the ANC system to cancel the acoustic noise should be treated with some care, since it would indicate low levels of attenua-

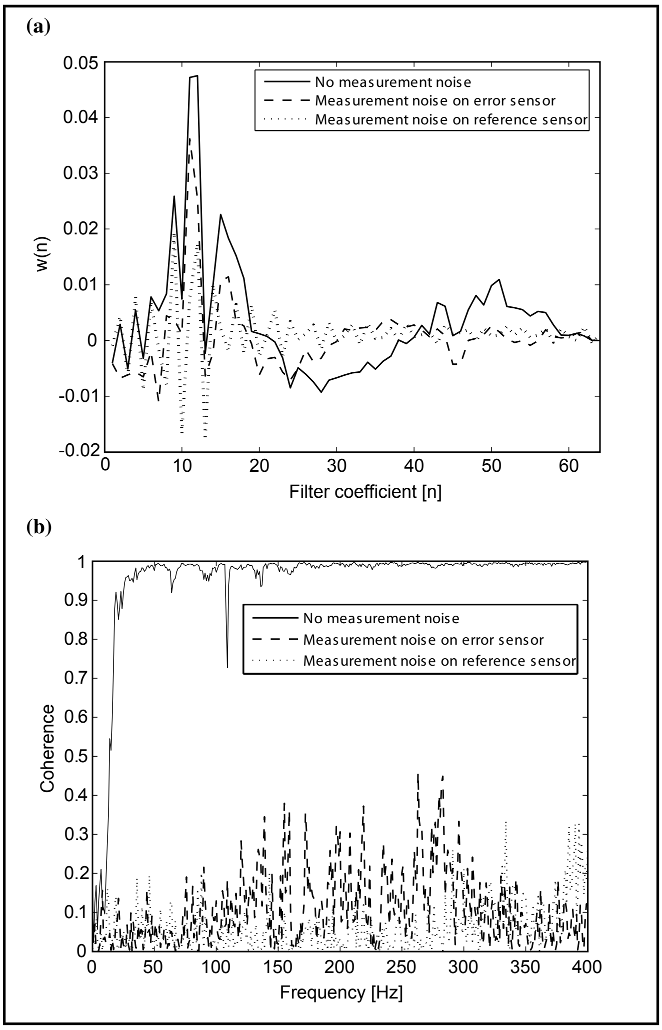

Figure 6. In (a), the weights of the adaptive filter after convergence for (solid line) no measurement noise present, (dashed line) measurement noise present at the error microphone, and (dotted line) measurement noise present at the reference microphone.The filter weights were saved after running the ANC system for twice as long for the two cases with measurement noise present as compared to case when measurement noise was not present. In (b), the coherence between the outputs of the error- and reference sensor in absence of control, for (solid line) no measurement noise present, (dashed line) measurement noise present at the error microphone, and (dotted line) measurement noise present at the reference microphone.

tion with measurement noise present at the error sensor, which is not necessarily the case.

The experimental investigations of measurement noise influence on the ANC system confirm the theoretical results. It is shown experimentally that measurement noise at the reference sensor affects the adaptive weights and lowers the attenuation of the acoustic noise produced by the ANC system as compared to when measurement noise is not present on either sensor (see Figs. 4 and 6 (a)). The results also indicate that measurement noise at the error sensor somewhat lowers the adaptive weights and the achievable attenuation as compared to when measurement noise is not present on either sensor (see Figs. 4 and 6 (a)). For the case described here, the results were obtained after running the ANC system for twice as long with measurement noise present as without, and a further decreased step size could result in an increased attenuation of the acoustic noise with measurement noise present at the error sensor. Further, the experimental results illustrate that the coherence is low with measurement noise present at the error sensor even if the acoustic noise is attenuated a great deal at the evaluation microphone (see Figs. 4 and 6 (b)). Thus, the common way of using the coherence function as a measure of the achievable attenuation should be treated with some care and, if possible, it should be investigated at which of the sensors the measurement noise is present. Further, since the attenuation is masked by the measurement noise at the error sensor even though, relatively high at the evaluation sensor, the results illustrate the importance of using an evaluation sensor which is not affected by measurement noise, i.e., in this case, a sensor positioned away from the direct airflow.

Both the theoretical as well as the experimental results obtained indicate that a high level of measurement noise compared to the level of acoustic noise at the reference microphone can result in very low levels of attenuation, if eny at all, achieved by the ANC system. Furthermore, high levels of measurement noise present at the error microphone are likely to degrade the performance of the ANC system. Hence, when applying ANC in ducts, it is emphasized that it is important for the performance of the ANC system to substantially reduce the turbulence-induced noise generated by the airflow around the microphones. Different microphone installations with this purpose, and the performance of an ANC system when using them, are investigated in Part II of this article series.[6]

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