A NEW SUBBAND WEIGHT TRANSFORM FOR DELAYLESS SUBBAND ADAPTIVE FILTERING STRUCTURES

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ABSTRACT

In applications like echo cancellation and speech enhancement, where there is need to track changes continuously, adaptive filtering is usually used. Long adaptive filters gives problems like slow convergence and high complexity. Subband adaptive filtering has been introduced to overcome these problems. The filter banks used in subband adaptive filtering introduce large delays. In order to compensate for the delays, delayless subband adaptive filtering is introduced. Delayless subband adaptive filtering is used in both open loop and closed loop configuration, where the subband filters are transformed to a fullband filter using a weight transform. This paper proposes a new subband weight transform based on the filter banks that are used. We investigate the performance of the weight transform using Monte Carlo simulations of a system identification situation. Different adaptive algorithms are used to compare the weight transform to previously proposed weight transforms.

1. INTRODUCTION

Adaptive filtering is widely used in applications like echo cancellation and speech enhancement. Fullband adaptive filters are afflicted with problems as high computational complexity and slow convergence. Computational complexity can be reduced using frequency domain filtering [1]. Another way to reduce computational complexity and to improve convergence speed is to use adaptive filters in subbands, i.e. subband adaptive filtering. A drawback with subband adaptive filtering is that delays, proportional to the number of subbands, are introduced by the filter banks. In [2], the delayless subband adaptive filtering structure has been introduced to overcome the problem with large delays. One important aspect with delayless subband adaptive filtering is the transformation of the subband filter weights to a fullband filter. Previously proposed weight transforms [2] [3], are all based on DFT transforms and properly stacking of subband filter weights to a fullband filter. The proposed weight transform is depending of the filter banks used. By using a linear transformation matrix, the subband filters are transformed to a corresponding fullband filter. In order to evaluate the performance of the new weight transform, compared to previously proposed weight transforms, two different simulations are carried out. Monte Carlo simulations of a system identification situation, show the performance for the different weight transforms. An echo cancellation situation, using a conference room impulse response, shows the adaptive behavior for the different weight transforms.

In Section 2, the delayless subband adaptive filtering is described. More details about the uniform DFT-filter banks are de-

scribed in Section 3, and in Section 4 the different weight transforms are described. The simulations and the results can be found in Section 5 and 6, respectively, and finally Section 7 concludes this paper.

2. DELAYLESS SUBBAND ADAPTIVE FILTERING

In the delayless subband adaptive filtering structures, the adaptive filter weights in the subbands are transformed to a corresponding fullband filter. The signal filtering is done using the fullband filter while adaptive filters are adapted in the subbands. The additional signal path delays, introduced by the filter banks in ordinary subband filtering, are eliminated by the delayless subband adaptive filtering structure.

Fig. 1. Open loop delayless subband adaptive filter structure with subband weight transform and fullband filtering. Subband error signals are fed back to the adaptive filters in the subbands.

Fig. 2. Closed loop delayless subband adaptive filter structure with subband weight transform and fullband filtering. A fullband error signal is fed back to the adaptive control algorithm through an analysis filter bank.

In the open loop configuration, illustrated in Fig. 1, local errors are calculated for each subband and used in the adaptive control algorithm. Even if the fullband filtering is delayless, the adaptive control algorithm is working with an input signal that is delayed due to the analysis filter bank. In the closed loop configuration, illustrated in Fig. 2, a fullband error signal is calculated and fed into the analysis filter bank in order to create the local errors. This introduces yet another delay to the adaptive control algorithm. Influences on performance, caused by this extra delay, is beyond the scope of this paper.

3. UNIFORM DFT-MODULATED FILTER BANKS

An uniform DFT-modulated filter bank consists of an analysis filter bank and a synthesis filter bank, see Fig. 3. In an uniform DFT-

Fig. 3. Subband filtering structure. Signal $X(z)$ is input to the analysis filter bank A , which gives the subband signals $X_m(z)$. The subband signals are then filtered by the subband filters $P_m(z)$. The subband output signals $Y_m(z)$ are fed into the synthesis filter bank S , which gives the output signal $Y(z)$.

modulated filter bank, where all subbands have the same bandwidth, the analysis filters and the synthesis filters are modulated versions of prototype filters, $H(z)$ and $G(z)$, according to

$$h_m(n) = h(n)W_M^{-mn} \iff H_m(z) = H(zW_M^m) \quad (1)$$

$$g_m(n) = g(n)W_M^{-mn} \iff G_m(z) = G(zW_M^m), \quad (2)$$

where $W_M = e^{-j2\pi/M}$, M equals the number of subbands and the subband index is denoted by m . The input signal $X(z)$ is fed into the analysis filters, according to Fig. 3. The branch signals, $V_m(z)$, are filtered versions of the input signal according to

$$V_m = H_m(z)X(z) = H(zW_M^m)X(z). \quad (3)$$

The signal at each branch is then decimated with a factor D . The analysis filter output signals $X_m(z)$, are given by

$$\begin{aligned} X_m(z) &= \frac{1}{D} \sum_{d=0}^{D-1} V_m(z^{1/D}W_D^d) \\ &= \frac{1}{D} \sum_{d=0}^{D-1} H_m(z^{1/D}W_D^d)X(z^{1/D}W_D^d), \end{aligned} \quad (4)$$

where $W_D = e^{-j2\pi/D}$ and the aliasing term index is denoted by d . The filtering is performed by subband filters $P_m(z)$, one for

each subband m , see Fig. 3. The output signals $Y_m(z)$, are given by

$$Y_m(z) = P_m(z)X_m(z). \quad (5)$$

The synthesis filters in Fig. 3, creates the output signal $Y(z)$ from the subband signals $Y_m(z)$. The interpolated signals $U_m(z)$ are given by

$$\begin{aligned} U_m(z) &= Y_m(z^D) \\ &= \frac{1}{D} \sum_{d=0}^{D-1} P_m(z^D)H_m(zW_D^d)X(zW_D^d). \end{aligned} \quad (6)$$

The output signal, $Y(z)$, is reconstructed by filtering each branch with a corresponding synthesis filter $G_m(z)$, whereafter the signals are added according to

$$Y(z) = \sum_{m=0}^{M-1} G_m(z)U_m(z). \quad (7)$$

Inserting Eq. (6) into Eq. (7), gives the output signal expressed in terms of the input signal, according to

$$Y(z) = \frac{1}{D} \sum_{m=0}^{M-1} \sum_{d=0}^{D-1} G_m(z)P_m(z^D)H_m(zW_D^d)X(zW_D^d). \quad (8)$$

4. SUBBAND FILTER WEIGHT TRANSFORM

The purpose of the subband filter weight transform is to transform a set of M subband filters $p_m(n)$ of length N , into a corresponding fullband filter $p(n)$ of length L_p . In the following three subsections we describe the previously proposed weight transforms in [2] [3], and the last subsection the proposed linear weight transform.

4.1. DFT-1 Stacking Method

The DFT-1 stacking weight transform is proposed in [2]. A set of N DFT coefficients, P_m , are calculated from each subband filter weights, $p_m(n)$, using a N point DFT. By properly stacking the DFT coefficients into a L_p long array and calculating a L_p point inverse DFT, the fullband impulse response, $p(n)$, is obtained.

The procedure is as follows. For $m = 0$,

$$P(l) = P_0(l) \quad \text{for } 0 \leq l \leq \frac{N}{4} - 1. \quad (9)$$

For $m = 1$ to $M - 1$ and if m is odd,

$$P(l + (m-1)\frac{N}{2}) = P_m(l) \quad \text{for } \frac{N}{4} \leq l \leq \frac{3N}{4} - 1, \quad (10)$$

and if m is even, then

$$P(l + (m-2)\frac{N}{2}) = P_m(l) \quad \text{for } \frac{3N}{4} \leq l \leq N - 1 \quad (11)$$

and

$$P(l + m\frac{N}{2}) = P_m(l) \quad \text{for } 0 \leq l \leq \frac{N}{4} - 1. \quad (12)$$

The last $\frac{N}{4}$ points of $\mathbf{P}$ are obtained by

$$P(l + (M-2)\frac{N}{2}) = P_0(l) \quad \text{for } \frac{3N}{4} \leq l \leq N - 1. \quad (13)$$

The fullband impulse response $\mathbf{p}$ is the inverse DFT of $\mathbf{P}$.

4.2. DFT-2 Stacking Method

The DFT-2 stacking weight transform is proposed in [3]. The method is similar to the DFT-1 stacking method. Instead of a N point DFT, the subband filter weights are zero padded into a length of $2N$ and then transformed into the frequency domain using a $2N$ point DFT. The DFT coefficients are then properly stacked into a $2L_p$ long array. The fullband impulse response, $p(n)$ are obtained by truncating the last L_p points of a $2L_p$ inverse DFT.

The procedure is as follows. The DFT of $[\mathbf{p}_m, \mathbf{0}]$, where $\mathbf{0}$ is a $(1 \times N)$ zero padding vector, is denoted by $\mathbf{P}'_m$. For $m = 0$,

$$P'(l) = P'_0(l) \quad \text{for } 0 \leq l \leq \frac{N}{2} - 1. \quad (14)$$

For $m = 1$ to $M - 1$ and if m is odd,

$$P'(l + (m - 1)N) = P'_m(l) \quad \text{for } \frac{N}{2} \leq l \leq \frac{3N}{2} - 1, \quad (15)$$

and if m is even, then

$$P'(l + (m - 2)N) = P'_m(l) \quad \text{for } \frac{3N}{2} \leq l \leq 2N - 1 \quad (16)$$

and

$$P'(l + mN) = P'_m(l) \quad \text{for } 0 \leq l \leq \frac{N}{2} - 1. \quad (17)$$

The last $\frac{N}{2}$ points of $\mathbf{P}'$ are obtained by

$$P'(l + (M - 2)N) = P'_0(l) \quad \text{for } \frac{3N}{2} \leq l \leq 2N - 1. \quad (18)$$

The fullband impulse response $\mathbf{p}$ is the first L_p points of a $2L_p$ point inverse DFT of $\mathbf{P}'$.

4.3. DFT-FIR Weight Transform

The DFT-FIR weight transform is proposed in [3]. The fullband impulse response, $p(n)$, is obtained by using the subband filter weights as input signals to the synthesis filter. The transform is given, corresponding to Eq. (7), by the following expression

$$P(z) = \sum_{m=0}^{M-1} G_m(z) P_m(z^D) z^{L/2}, \quad (19)$$

where the delay of the synthesis filters is $\frac{L}{2}$. This delay is compensated by multiplying with $z^{L/2}$. The fullband impulse response, $p(n)$, is given by

$$p(n) = \sum_{m=0}^{M-1} g_m(n) * p_{m\uparrow D}(n) * \delta(n + \frac{L}{2}), \quad (20)$$

where $*$ denotes convolution.

4.4. Linear Weight Transform

This paper proposes a linear subband filter weight transform for delayless subband adaptive filtering. The weight transform is depending on the number of subbands M , the decimation factor D , the analysis filters $h_m(n)$ and the synthesis filters $g_m(n)$.

The weight transform is based on the expression of the reconstructed output signal $Y(z)$ in terms of the analysis filters $H_m(z)$,

the subband filters $P_m(z)$ and the synthesis filters $G_m(z)$, see Eq. (8).

If the nonlinearities, i.e. in-band and output aliasing terms in the filter bank structure are neglectable, the subband filter structure in Fig. 3 can be approximated by a fullband finite impulse response (FIR) filter $p(n)$. A design method for filter banks with low levels of aliasing is presented in [4]. The aspects of filter bank design in relation to weight transform has been evaluated in [5].

The expression for the output signal $Y(z)$ can be transformed to the time domain according to

$$y(n) = \frac{1}{D} \sum_{d=0}^{D-1} \sum_{m=0}^{M-1} g_m(n) * p_{m\uparrow D}(n) * h_{m,d}(n) * x_d(n), \quad (21)$$

where the discrete time index is denoted by n , the modulated input signal is defined as

$$x_d(n) = x(n) W_D^{-nd} \quad (22)$$

and the modulated analysis filters are defined as

$$h_{m,d}(n) = h(n) W_D^{-nd} W_M^{-nm}. \quad (23)$$

The impulse response $p_{m\uparrow D}(n)$ is an interpolated version of the m -th subband filter impulse response with interpolation factor D . When the aliasing terms in the output signal (the terms for $d = 1, \dots, D-1$) have much lower power levels compared to the non-aliasing terms (the terms for $d = 0$), the output signal can be approximated by

$$y(n) \approx \left[\frac{1}{D} \sum_{m=0}^{M-1} g_m(n) * p_{m\uparrow D}(n) * h_{m,0}(n) \right] * x(n). \quad (24)$$

In order to compensate for the total group-delay of τ_{A+S} samples inflicted by the analysis filter bank and the synthesis filter bank, the fullband filter impulse response is delay compensated by convolution with $\delta(n + \tau_{A+S})$. The delayless fullband filter impulse response $p(n)$ is defined as

$$p(n) = \begin{cases} \frac{1}{D} \sum_{m=0}^{M-1} g_m(n) * p_{m\uparrow D}(n) * h_{m,0}(n) * \delta(n + \tau_{A+S}) & n \geq 0 \\ 0 & n < 0 \end{cases} \quad (25)$$

The output signal $\hat{y}(n)$ of the fullband filter, in Fig. 1 and 2, is

$$\hat{y}(n) = p(n) * x(n) = \mathbf{p}^T \mathbf{x}(n) \approx y(n + \tau_{A+S}), \quad (26)$$

where the fullband filter coefficient vector is defined as $\mathbf{p} = [p(0), \dots, p(ND - 1)]^T$ and the input data buffer is defined as $\mathbf{x}(n) = [x(n), \dots, x(n - ND + 1)]^T$. The length of the subband filters is N , the length of the analysis and synthesis filters is L and $(\cdot)^T$ denotes transpose.

The fullband filter coefficient vector $\mathbf{p}$ is obtained by a linear transform of the subband weights $\mathbf{p}_m$, according to

$$\mathbf{p} = \mathbf{Q}[\mathbf{p}_0^T, \dots, \mathbf{p}_{M-1}^T]^T, \quad (27)$$

where $\mathbf{p}_m = [p_m(0), \dots, p_m(N - 1)]^T$. The transformation matrix $\mathbf{Q}$ is defined as

$$\mathbf{Q} = \frac{1}{D} [\mathbf{Q}_0, \dots, \mathbf{Q}_{M-1}], \quad (28)$$

where $\mathbf{Q}_m = \mathbf{S}\mathbf{H}_m\mathbf{G}_m\mathbf{J}_D$. The matrix $\mathbf{S}$ is a delay compensation matrix according to

$$\mathbf{S} = \begin{pmatrix} \mathbf{0}_I & \mathbf{I} & \mathbf{0}_{II} \end{pmatrix}_{(ND+2L-2)}^{(ND \times (ND+2L-2))} \quad (29)$$

where $\mathbf{0}_I$ is a $(ND \times \tau_{A+S})$ zero matrix, $\mathbf{I}$ is a $(ND \times ND)$ identity matrix and $\mathbf{0}_{II}$ is a $(ND \times (2L - \tau_{A+S} - 2))$ zero matrix. The matrix $\mathbf{H}_m$ is a convolution matrix, based on the analysis filter coefficients, according to

$$\mathbf{H}_m = \begin{pmatrix} h_m(0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ h_m(L-1) & & \ddots & 0 \\ 0 & \ddots & & h_m(0) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_m(L-1) \end{pmatrix}_{(ND+2L-2) \times (ND+L-1)} \quad (30)$$

The matrix $\mathbf{G}_m$ is a convolution matrix based on the synthesis filter coefficients, according to

$$\mathbf{G}_m = \begin{pmatrix} g_m(0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ g_m(L-1) & & \ddots & 0 \\ 0 & \ddots & & g_m(0) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & g_m(L-1) \end{pmatrix}_{(ND+L-1) \times ND} \quad (31)$$

The matrix $\mathbf{J}_D$ is an interpolation matrix, according to

$$\mathbf{J}_D = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ \{D-1 \text{ Rows with zeros}\} & & & & \\ 0 & 1 & 0 & \cdots & 0 \\ \{D-1 \text{ Rows with zeros}\} & & & & \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \end{pmatrix}_{ND \times N} \quad (32)$$

5. SIMULATIONS

Monte Carlo simulations of delayless subband adaptive filtering, with open loop configuration, applied in system identification, with Least Squares estimation of the subband filter weights, show the importance of incorporating the filter banks in the weight transform.

The misalignment $\mathcal{M}$ is used as performance measure and is defined as

$$\mathcal{M} = 10 \log_{10} \left(\frac{\sum_n |\tilde{p}(n) - p(n)|^2}{\sum_n |\tilde{p}(n)|^2} \right), \quad (33)$$

where $\tilde{p}$ is the impulse response to be identified. The misalignment measure describes how much the transformed fullband filter deviates from the system impulse response $\tilde{p}$.

For each iteration in the Monte Carlo simulations, a new system impulse response has been randomly generated. Least Squares optimal weights of the subband filters are transformed to a fullband filter using the four different weight transforms. The four different fullband filters are compared using the misalignment measure in Eq. (33).

The performance regarding convergence speed, in a real echo cancellation situation, is evaluated using a Normalized Least Mean Squares (NLMS) and a Recursive Least Squares (RLS) adaptive algorithms [6] in the subbands. The NLMS adaptive algorithm is evaluated in both open loop and closed loop configuration and the RLS adaptive algorithm only in open loop configuration. A impulse response measured from a real conference room is used as echo path, see Fig. 4. White noise is used as excitation signal

Fig. 4. Measured conference room impulse response.

of the room impulse response. The convergence behavior of the delayless subband adaptive filter, using the four different weight transforms, are compared to a fullband NLMS.

In all simulations, the following filter bank parameters are used. The number of subbands $M = 256$, the decimation factor $D = \frac{M}{2}$, the subband filter length $N = 2$, the fullband filter length $L = 512$ and the total delay for analysis and synthesis filter banks $\tau_{A+S} = 512$. In the Monte Carlo simulations, the number of iterations is 5000. The system impulse response, that is to be identified, is a 256 tap FIR filter. In the adaptive algorithms the following parameters are used. The step size for the LMS algorithm, $\beta = 0.005$, and the exponential weighting factor for the RLS algorithm, $\lambda = 0.995$.

6. SIMULATION RESULTS

6.1. Monte Carlo Simulations

The misalignment histogram in Fig. 5, with the misalignment distribution plotted against the misalignment error, shows the difference in performance for the four different weight transforms. Clearly, for this parameter setting, the linear weight transform outperforms the previously proposed weight transforms.

6.2. Open Loop NLMS

Fig. 6 shows the misalignment, $\mathcal{M}$, as a function of discrete time. The convergence rate for the linear weight transform is comparable to existing transforms.

Fig. 5. Misalignment distributions obtained from the Monte Carlo simulations.

6.3. Open Loop RLS

Fig. 7 shows the misalignment, $\mathcal{M}$, as a function of discrete time. The convergence rate for the linear weight transform is comparable to existing transforms.

6.4. Closed Loop NLMS

Fig. 8 shows the misalignment, $\mathcal{M}$, as a function of discrete time. The convergence rate for the linear weight transform is comparable to existing transforms.

7. CONCLUSIONS

In this paper, we proposed a new subband weight transform based on the filter bank that is used. We investigated the performance of the weight transform using Monte Carlo simulations of a system identification situation. Different adaptive algorithms are evaluated in a real echo cancellation situation, in order to compare the convergence behavior of the linear weight transform to previously proposed weight transforms.

Fig. 6. Convergence behavior for an open loop NLMS delayless subband adaptive filter, using different weight transforms.

Fig. 7. Convergence behavior for an open loop RLS delayless subband adaptive filter, using different weight transforms.

Fig. 8. Convergence behavior for a closed loop NLMS delayless subband adaptive filter, using different weight transforms.

8. REFERENCES

- [1] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice-Hall, 1993.
- [2] D. R. Morgan and J. C. Thi, "A Delayless Subband Adaptive Filter Architecture," *IEEE Transactions on Signal Processing*, pp. 1819–1830, Aug. 1995.
- [3] J. Huo, S. Nordholm, and Z. Zang, "New Weight Transform Schemes for Delayless Adaptive Filtering," in *Proceedings of IEEE Global Telecommunications Conference*, 2001, pp. 197–201.
- [4] J. M. de Haan, N. Grbić, I. Claesson, and S. Nordholm, "Design of Oversampled Uniform DFT Filter Banks with Delay Specification using Quadratic Optimization," in *Proceedings of ICASSP*, May 2001, pp. 3633–3636.
- [5] J.M. de Haan, L.-O. Larson, and I. Claesson, "Filter Bank Design for Delayless Subband Adaptive Filtering Structures with Subband Weight Transformation," *submitted to IEEE Digital Signal Processing Workshop*, Oct. 2002.
- [6] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 2002.