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Citation for the published paper:

Title:

Author:

Journal:

Year:

Vol:

Issue:

Pagination:

URL/DOI to the paper:

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On Synthetic Aperture Radar Azimuth and Range Resolution Equations

This paper discusses spatial resolutions for ultrawideband-ultrawidebeam (UWB) synthetic aperture radar (SAR) in comparison to narrowband-narrowbeam (NB) SAR. The study shows that in the intensity interval from -6 dB to 0 dB, the behavior of the impulse response function in NB SAR imaging (IRF-NSAR)—sinc function—and the impulse response function in UWB SAR imaging (IRF-USAR) in azimuth and range are similar. This similarity is utilized in a derivation of new spatial resolution equations for UWB SAR based on -3 dB width or half power beamwidth (HPBW). Signal fractional bandwidth and associated integration angle are shown to affect the behavior of IRF-USAR. The effects to HPBW are described by the so-called HPBW narrowing/broadening factors.

I. INTRODUCTION

Spatial resolutions are considered as the most significant parameter for image quality measurements since it describes the distance between two objects on the ground at which the objects appear distinct and separate in a synthetic aperture radar (SAR) image. Spatial resolution equations are used to estimate the azimuth and range resolutions which can be achieved by a specific SAR system. Such information is important from a SAR system designer's point of view. The azimuth resolution equation given in [1] and range resolution equation in [2] are supposed to aim at narrowband-narrowbeam (NB) side-looking SAR without a squint angle. These resolution equations do not consider the coupling between azimuth and range frequencies. They may therefore be invalid for ultrawideband-ultrawidebeam (UWB) SAR. Taking system frequency dependence into account, a new spatial resolution equation is introduced in [3]. However, this equation only provides the information of the area resolution, not azimuth and range resolutions which may be of interest.

The assessment of spatial azimuth and range resolutions is based on the -3 dB width of the impulse response function in SAR imaging

Fig. 1. An example of 2-D FT of NB SAR image of point-like scatterer marked by light gray color.

(IRF-SAR) or the half power beamwidth (HPBW), i.e., the distances between the points with intensities of 3 dB below the peak intensity. For the availability of the impulse response function in NB SAR imaging (IRF-NSAR)—the sinc function, the derivation of the azimuth and range resolution equations for NB SAR seems to be simple. However, the complicated mathematical representation of the impulse response function in UWB SAR imaging (IRF-USAR) [4] creates difficulties for the derivation of the azimuth and range resolution equations for UWB SAR.

The objective of this paper is to discuss spatial resolution equations for UWB SAR in relation to the ones for NB SAR. This study is built on IRF-USAR which was previously published in [4]. On account of the similar behavior of the IRF-NSAR and IRF-USAR in the intensity interval from -6 dB to 0 dB, the spatial resolution equations for UWB SAR are built on the ones for NB SAR with the consideration of the coupling between azimuth and range frequencies through the so-called HPBW narrowing/broadening factors. The numerical results of these factors are also provided and summarized in graph.

The paper is organized as follows. In Section II, the spatial resolution equations for NB SAR are derived based on IRF-NSAR. The relations to the commonly known SAR resolution equations are also examined. Section III investigates the similarity and difference between IRF-NSAR and IRF-USAR. New spatial resolution equations for UWB SAR are then introduced. Section IV provides the conclusion.

II. SPATIAL RESOLUTION EQUATIONS FOR NB SAR

As illustrated in Fig. 1, for NB SAR where both the integration angle and the fractional bandwidth are very small, the following approximations are valid for the two-dimensional (2-D) Fourier transform (FT) of an NB SAR image of a point-like scatterer $H(k_x, k_y)$ marked by the light gray color

$$k_{y,\min} \sin \frac{\phi_0}{2} \approx k_c \sin \frac{\phi_0}{2} \approx k_{y,\max} \sin \frac{\phi_0}{2} \quad (1)$$

$$k_{y,\max} - k_{y,\min} \approx (k_{y,\max} - k_{y,\min}) \cos \frac{\phi_0}{2} \quad (2)$$

Manuscript received November 12, 2010; revised March 18, 2011; released for publication June 10, 2011.

IEEE Log No. T-AES/48/2/943852.

Refereeing of this contribution was handled by W. Blanding.

This work was supported by the KK-Foundation.

0018-9251/12/\$26.00 © 2012 IEEE

where k_x and k_y are the azimuth and the range wavenumber. The subscripts c, max, and min denote the terms of center, maximum, and minimum, respectively. The SAR image coordinates x and y indicate azimuth and range while ϕ_0 refers to the integration angle. The approximations in (1) and (2) are also interpreted as follows: a rectangular and flat approximation can be applied to the 2-D FT of an NB SAR image of a point-like scatterer. As shown in [4], IRF-NSAR $h(x, y)$ can be achieved from a 2-D inverse FT (IFT) of this approximation as

$$h(x, y) \approx \text{sinc} \left[k_c \sin \left(\frac{\phi_0}{2} \right) x \right] \text{sinc} \left(\frac{k_{y, \max} - k_{y, \min}}{2} y \right). \quad (3)$$

According to definitions of SAR image quality assessments proposed in [5]–[8], the spatial resolution, integrated sidelobe ratio (ISLR), and peak sidelobe ratio (PSLR) are based on a point-like scatterer approximation in a SAR image which can be a corner reflector deployed in measurement campaigns. The assessment of spatial azimuth and range resolutions is based on the -3 dB width or the HBPW of the mainlobe, i.e., the distances between the points with intensities of 3 dB below the peak intensity. From the function (3), the spatial resolutions for NB SAR can be estimated by solving the set of equations

$$\text{sinc} \left(k_c \frac{\Delta_x}{2} \sin \frac{\phi_0}{2} \right) = -3 \text{ dB} \quad (4)$$

$$\text{sinc} \left(\frac{k_{y, \max} - k_{y, \min}}{2} \frac{\Delta_y}{2} \right) = -3 \text{ dB}. \quad (5)$$

The azimuth and range resolutions are then given by

$$\Delta_{N, x} = \frac{0.2211 \lambda_c}{\sin \left(\frac{\phi_0}{2} \right)} \approx \frac{\lambda_c}{4 \sin \left(\frac{\phi_0}{2} \right)} \quad (6)$$

$$\Delta_{N, y} = \frac{0.4422 \lambda_c}{B_r} \approx \frac{c}{2B} \quad (7)$$

where λ_c is the center wavelength and B_r is the fractional bandwidth, defined by the ratio of the radar signal bandwidth B to the center frequency f_c . The approximated version of the azimuth resolution equation (6) is similar to the one given in [1] which is derived from the diffraction limit of the synthetic aperture. Similarly, the range resolution equation given in [2] has exactly the same expression as the approximated version of the range resolution equation (7). It should be mentioned here that the range resolution equation proposed in [2] is a general equation and used widely in the radar field. This equation is based on the minimum delay between two adjacent radar echoes so that they can be distinguished after pulse compression. For the cases where the

Fig. 2. Example of 2-D FT of UWB SAR image of point-like scatterer $H(k_x, k_y)$ marked by dark gray color. Light gray area associates with NB approximation. Using NB approximation marked by light gray color for $H(k_x, k_y)$ marked by dark gray color is unreasonable.

integration angle is very small, i.e., $\sin(\phi_0/2) \approx \phi_0/2$, (6) can be further simplified as

$$\Delta_{N, x} \approx \frac{\lambda_c}{2\phi_0}. \quad (8)$$

III. SPATIAL RESOLUTION EQUATIONS FOR UWB SAR

Let us look at the 2-D FT of a UWB SAR image of a point-like scatterer $H(k_x, k_y)$, which is marked by the dark gray area given in Fig. 2. As illustrated, using NB approximation for $H(k_x, k_y)$ in this case is unreasonable. As a consequence, the approximations (1) and (2) should not be applied to UWB SAR systems. Equation (3) is not a good approximation for IRF-USAR. Also, the spatial resolution equations (6) and (7) or (8), which are derived from (3), may result in large errors when used for UWB SAR. However, from a geometric point of view, (7) may still be valid for the cases of large fractional bandwidths and not very wide integration angles.

For UWB SAR systems with large fractional bandwidth and wide integration angle, the coupling between azimuth and range frequencies must be taken into account. This coupling can be represented by the relationship

$$\omega = \frac{c}{2} \sqrt{k_x^2 + k_y^2} = \frac{c}{2} \kappa \quad (9)$$

where ω is the radar signal frequency and κ represents a transformation of the wavenumbers k_x and k_y from a Cartesian coordinate system to a polar coordinate system. Considering the coupling between azimuth and range frequencies, the lower bound of the area resolution for UWB SAR is suggested in [3] but not the separate azimuth and range resolutions.

A. IRF-USAR

As shown in the previous section, the spatial resolution equations for NB SAR were derived successfully from the IRF-NSAR. Similarly, the

derivation of spatial resolution equations for UWB SAR can be based on an IRF-USAR. In other words, the possibility to develop spatial resolution equations for UWB SAR depends somewhat on the availability of an IRF-USAR. Using an assumption of a flat 2-D FT of a SAR image of a point-like scatterer and without the NB approximation, i.e., considering the coupling between azimuth and range frequencies, an IRF-USAR in polar coordinates (ρ, φ) is introduced in [4] as

$$h(\rho, \varphi, B_r, \phi_0) = \frac{e^{-i\varphi}}{\rho} \left[\phi_0 \sum_{n=-\infty}^{+\infty} \frac{i^n h_{1,n-1}(\rho, B_r)}{e^{i(n-1)\varphi}} \text{sinc}\left(n \frac{\phi_0}{2}\right) \right] + \frac{e^{-i\varphi}}{\rho} h_2(\rho, \varphi, B_r, \phi_0). \quad (10)$$

Equation (10), which is a function of fractional bandwidth B_r and integration angle ϕ_0 , has been proved in [4] to be valid for both NB and UWB SAR. This means that IRF-USAR with small fractional bandwidth and narrow integration angle is almost identical to IRF-NSAR with the corresponding values. This function is used here as a basis for derivation of spatial resolution equations for UWB SAR.

B. A Comparison Between IRF-NSAR and -USAR

In the derivation of (10), κ in (9) has been normalized with respect to the center wavenumber k_c . To compare between IRF-NSAR and -USAR, a similar normalization can be applied to IRF-NSAR. The normalized form of (3) can be shown to be

$$h_n(x, y) \approx \text{sinc}\left[\sin\left(\frac{\phi_0}{2}\right)x\right] \text{sinc}\left(\frac{B_r}{2}y\right). \quad (11)$$

The comparison will focus only on the mainlobe area since the resolution information is extracted from the mainlobe of IRF-SAR. Since the azimuth and range resolutions are of interest, the 2-D plots of IRF-USAR are informative enough to be used in the comparison. For IRF-NSAR, the IRF-SAR in azimuth and range are given by $\text{sinc}[\sin(\phi_0/2)x]$ and $\text{sinc}[(B_r/2)y]$ while $h(\rho, 90^\circ, B_r, \phi_0)$ and $h(\rho, 0^\circ, B_r, \phi_0)$ result in the IRF-USAR in azimuth and range, respectively. By rescaling the azimuth and range axes with the inverse of $\sin(\phi_0/2)$ and $B_r/2$, the IRF-NSAR and -USAR can be plotted in the same figure. The rescaling also facilitates a comparison between IRF-NSAR and -USAR associated with different SAR systems. The fractional bandwidths and the integration angles associated with the antenna beamwidth (B_r, ϕ_0) used in the comparison are summarized in Table I. Due to the rescaling of the azimuth and range axes, the plots of IRF-NSAR with these parameters would be identical. For this reason, only one of the pairs

Fig. 3. Plots of IRF-NSAR and -USAR with different values of (B_r, ϕ_0) . Azimuth and range axes are rescaled with inverse of $\sin(\phi_0/2)$ and $B_r/2$, respectively. (a) Mainlobe in azimuth. $\text{sinc}[\sin(\phi_0/2)x]$ and $h(x, 90^\circ, B_r, \phi_0)$. (b) Mainlobe in range. $\text{sinc}[(B_r/2)y]$ and $h(y, 0^\circ, B_r, \phi_0)$.

TABLE I
SAR System Parameters

	System 1	System 2	System 3
Fractional bandwidth B_r	0.1	0.35	1.1
Associated integration angle ϕ_0	10°	35°	110°

(B_r, ϕ_0) needs to be used to generate IRF-NSAR in this comparison.

Fig. 3 plots the mainlobes of IRF-USAR given by (10) and IRF-NSAR given by (11). In this plot, the intensity of an IRF-SAR is normalized with respect to its peak and then converted to the dB scale. As observed from Fig. 3, IRF-NSAR and -USAR are almost identical at small fractional bandwidth and narrow integration angle, e.g. $(B_r, \phi_0) = (0.1, 10^\circ)$. At large fractional bandwidths and wide integration angles, the behavior of IRF-USAR is much more complicated than the behavior of a sinc function. This can be explained by the azimuth-range frequency coupling which has been taken into account in IRF-USAR.

For comparison purposes, we can normalize the azimuth and range axes with respect to the HPBW of the IRF-SAR and then plot them in the same figure. The 2-D plots of the IRF-NSAR and -USAR with this normalization are given in Fig. 4 where the normalized azimuth and range are denoted by x_n and y_n . As shown, the behavior of IRF-NSAR and -USAR are very similar down to -6 dB. In other words, the mainlobe in the intensity interval from -6 dB to 0 dB may still be approximated as a sinc

Fig. 4. Mainlobes in azimuth and range with different values of (B_r, ϕ_0) . The azimuth and range normalized with respect to the -3 dB beamwidths of IRF-SAR. (a) Mainlobe in azimuth. $\text{sinc}[\sin(\phi_0/2)x_n]$ and $h(x_n, 90^\circ, B_r, \phi_0)$, (b) Mainlobe in range. $\text{sinc}[(B_r/2)y_n]$ and $h(y_n, 0^\circ, B_r, \phi_0)$.

curve with the corresponding intensity interval. Hence, the function (3) may therefore be used to estimate the spatial resolutions in UWb SAR. However, the difference between IRF-NSAR and -USAR at the -3 dB beamwidth must be taken into account in the estimations. This difference is described by an HPBW narrowing/broadening factor which is introduced in the next section.

C. HPBW Narrowing/Broadening Factors

Let us consider a SAR system utilizing a fractional bandwidth B_r and an integration angle ϕ_0 . If the estimations of HPBW using IRF-NSAR are given by $\Delta_{N,x}(B_r, \phi_0)$ and $\Delta_{N,y}(B_r, \phi_0)$, and if $\Delta_{U,x}(B_r, \phi_0)$ and $\Delta_{U,y}(B_r, \phi_0)$ indicate the same estimations, however using IRF-USAR, then the HPBW narrowing/broadening factors in azimuth ϵ_x and in range ϵ_y will be defined by

$$\epsilon_x = \frac{\Delta_{U,x}(B_r, \phi_0)}{\Delta_{N,x}(B_r, \phi_0)} \quad (12)$$

$$\epsilon_y = \frac{\Delta_{U,y}(B_r, \phi_0)}{\Delta_{N,y}(B_r, \phi_0)} \quad (13)$$

respectively. To investigate HPBW narrowing/broadening factors associated with different SAR systems, IRF-NSAR (3) and IRF-USAR (10) in the intensity interval $[-3, 0]$ dB are again plotted in Fig. 5 in which the same axial scaling used in Fig. 3, i.e., scaling the axes with the inverse of $\sin(\phi_0/2)$ and $B_r/2$, is used. IRF-NSAR is therefore independent of fractional bandwidth B_r and integration angle ϕ_0 . In other words, $\Delta_{N,x}(B_r, \phi_0)$ and $\Delta_{N,y}(B_r, \phi_0)$ are

Fig. 5. Investigation into HPBW narrowing/broadening factors with different values of (B_r, ϕ_0) . Azimuth and range are rescaled so that $\Delta_{N,x}/2 = 1$ and $\Delta_{N,y}/2 = 1$. (a) Mainlobe in azimuth. $\text{sinc}[\sin(\phi_0/2)x]$ and $h(x, 90^\circ, B_r, \phi_0)$, (b) Mainlobe in range. $\text{sinc}[(B_r/2)y]$ and $h(y, 0^\circ, B_r, \phi_0)$.

a constant. The azimuth and range axes are then rescaled with a factor of 0.72 so that $\Delta_{N,x}/2 = 1$ and $\Delta_{N,y}/2 = 1$. In this case, the horizontal axes at intensity of -3 dB give, directly, the values of ϵ_x and ϵ_y with the different (B_r, ϕ_0) .

Some comments on the HPBW narrowing/broadening factors may be given as follows. The HPBW narrowing/broadening factors depend on the fractional bandwidth and integration angle of SAR systems. However, the deviation of these factors from one can be observed clearly when fractional bandwidth and integration angle are very large, e.g. $(B_r, \phi_0) = (1.1, 110^\circ)$. For SAR systems utilizing a small fractional bandwidth and a narrow integration angle, e.g. $(B_r, \phi_0) = (0.1, 10^\circ)$, these HPBW narrowing/broadening factors are very close to one. Even at a larger fractional bandwidth and a wider integration angle, e.g. $(B_r, \phi_0) = (0.35, 35^\circ)$, we may still approximate ϵ_x and ϵ_y to one.

D. Azimuth and Range Resolution Equations

The azimuth and range resolution equations can be achieved by solving the set of equations

$$h\left(\frac{\Delta_x}{2}, 90^\circ, B_r, \phi_0\right) = -3 \text{ dB} \quad (14)$$

$$h\left(\frac{\Delta_y}{2}, 0^\circ, B_r, \phi_0\right) = -3 \text{ dB} \quad (15)$$

where $h(\rho, \varphi, B_r, \phi_0)$ is given by (10). This set of equations is not easy to solve analytically. The possibility to find the roots in a numerical way is also constrained due to the existence of two variables

B_r and ϕ_0 in the equation. Thus, we propose to build the spatial resolution equations for UWB SAR upon (6) and (7). IRF-USAR is therefore approximated to IRF-NSAR in the intensity interval from -6 dB to 0 dB by the consideration of the HPBW narrowing/broadening factors. The azimuth and range resolution equations for UWB SAR are proposed as follows

$$\Delta_{U,x} = \epsilon_x \Delta_{N,x} = \epsilon_x \frac{0.2211\lambda_c}{\sin\left(\frac{\phi_0}{2}\right)} \quad (16)$$

$$\Delta_{U,r} = \epsilon_y \Delta_{N,y} = \epsilon_y \frac{0.4422c}{B}. \quad (17)$$

The HPBW narrowing/broadening factors ϵ_x and ϵ_y in (16) and (17) represent the effects of fractional bandwidth and the integration angle to the azimuth and range resolutions or, in other words, the coupling between azimuth and range frequencies.

For any SAR system, its fractional bandwidth, which is the ratio of the signal bandwidth to the center frequency, can only be in the interval $(0, 2]$. If the flight track of a SAR system is a straight line, the associated integration angle of this SAR system cannot be beyond the interval $[0^\circ, 180^\circ)$. These intervals are considered in the numerical calculations of ϵ_x and ϵ_y in (16) and (17). The values of these factors associated with different SAR systems are summarized in Figs. 6 and 7. As shown, for small signal fractional bandwidth and narrow integration angle, the factors ϵ_x and ϵ_y can be approximated to one. In this case, (6) and (7) are identical to (16) and (17). The effect of the fractional bandwidth to the azimuth resolution is not as strong as the effect of the integration angle to the range resolution. A SAR system utilizing small fractional bandwidth can obtain very high range resolution by increasing the integration angle. For example, the range resolution of a SAR system with $B_r = 0.1$ can be enhanced two times when the system integration angle increases from $\phi_0 = 10^\circ$ to $\phi_0 = 70^\circ$.

The integration angle ϕ_0 , where equal azimuth and range resolutions are reached, is estimated by equalizing (16) and (17) as

$$\bar{\phi}_0 = 2 \arcsin\left(\frac{\epsilon_x B_r}{\epsilon_r}\right). \quad (18)$$

E. Validation

The validation of (16) and (17) is based on the parameters of the SAR system 3 given in Table I. The SAR scene illuminated by this system is simulated only with one point-like scatterer. Using simulated data will avoid undesired effects such as impacts of antenna pattern, clutter backscattering, noise, and local reflection which may affect the resolutions of the system. The azimuth and range resolutions achieved

Fig. 6. Numerical results of HPBW narrowing/broadening factor in azimuth ϵ_x .

Fig. 7. Numerical results of HPBW narrowing/broadening factor in range ϵ_y .

with this system in the simulation are $\Delta_{x,\text{ref}} \approx 1.36$ m and $\Delta_{y,\text{ref}} \approx 2.49$ m.

If the spatial resolutions are estimated by the approximated versions of (6) and (7), the results will be $\Delta_{N,x} \approx 1.75$ m and $\Delta_{N,y} \approx 2.61$ m. These values are quite different from $\Delta_{x,\text{ref}}$ and $\Delta_{y,\text{ref}}$. The differential resolution (DRES) introduced in [8] should be used to evaluate the accuracy of the resolution equations. In this case, DRES directly indicates the estimation errors. The errors are calculated as $\Delta_{dx} \approx 28.7\%$ and $\Delta_{dy} \approx 4.8\%$. The former can be seen to be significant.

Based on Figs. 6 and 7, we can determine the HPBW broadening/narrowing factors to be $\epsilon_{x,-3} \approx 0.825$ and $\epsilon_{y,-3} \approx 1.085$ (marked by black points on the figures). With these factors, the azimuth and range resolutions retrieved from (16) and (17) are $\Delta_{N,x} \approx 1.28$ m and $\Delta_{N,y} \approx 2.50$ m, respectively.

The estimation errors are reduced to $\Delta_{dx} \approx 5.8\%$ and $\Delta_{dy} \approx 0.4\%$. These errors are quite small and originated from the flat spectrum assumption.

IV. CONCLUSION

In this paper we discuss the spatial resolution equations for NB and UWB SAR. The proposed spatial resolution equations can be seen as extended versions of the commonly known spatial resolution equations for NB SAR. The comparison between IRF-NSAR and -USAR and the HPBW narrowing/broadening factors, which must be considered in the equations for UWB SAR, can be seen to be the basis for this proposal. The HPBW narrowing/broadening factors are shown to be dependent on the fractional bandwidth and the integration angle of SAR systems. The numerical results of HPBW narrowing/broadening factors are also provided in this study.

ACKNOWLEDGMENT

The authors would like to thank the Swedish Defence Research Agency, Saab Bofors Dynamics, Saab Electronic Defence Systems, and RUAG Space for their cooperation.

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