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Effect of primary network on performance of spectrum sharing AF relaying

T.Q. Duong, V.N.Q. Bao, H. Tran, G.C. Alexandropoulos and H.-J. Zepernick

Most of the research in spectrum sharing has neglected the effect of interference from primary users. In this reported work, the performance of spectrum sharing amplify-and-forward relay networks under interference-limited environment, where the interference induced by the transmission of primary networks is taken into account, is investigated. In particular, a closed-form expression tight lower bound of outage probability is derived. To reveal additional insights into the effect of primary networks on the diversity and array gains, an asymptotic expression is also obtained.

Introduction: Spectrum sharing relay networks have recently attracted much attention for providing higher reliability over direct transmission under scarce and limited spectrum conditions [1–4]. Specifically, the performance of decode-and-forward (DF) relay networks in spectrum sharing environments has been reported [1–3]. Recently, we have investigated the outage probability (OP) for spectrum sharing networks with amplify-and-forward (AF) relaying [4]. It has been shown in [1–4] that utilising DF/AF relaying significantly enhances system performance in such constrained transmission power conditions. However, most of the previous works have neglected the effect of the primary transmitter (PU-Tx), which significantly deteriorates the performance of the secondary network. In this Letter, to evaluate this interference effect, we derive a closed-form expression for OP and further calculate an asymptotic expression. We show that under fixed interference from primary networks, the diversity order remains unchanged and the loss only occurs in the array gain, which is theoretically quantified. However, when the interference is linearly proportional to the signal-to-noise ratio (SNR) of the secondary network, the system is severely affected, leading to an irreducible error floor of OP.

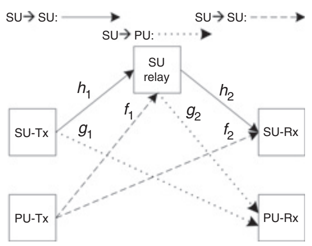

Fig. 1 *System model for spectrum sharing AF relay network considering interference from PU-Tx*

System model and outage probability analysis: Consider an underlay cognitive network where a secondary transmitter (SU-Tx) communicates with a secondary receiver (SU-Rx) through the assistance of a secondary relay (SU-relay) in co-existence with a primary network, as shown in Fig. 1. The transmit powers at the SU-Tx and the SU-relay are constrained so that their transmission will not cause any harmful interference to the PU-Rx, which is defined by the maximum tolerable interference power I_p. In the first hop, the SU-Tx transmits its signal, s, to the SU-relay under the power constraint that $P_s = \frac{I_p}{|g_1|^2}$, where g_1 is the channel coefficient for the link SU-Tx → PU-Rx. The received signal at the SU-relay, y_r, impaired by the transmission of the PU-Tx, is given by $y_r = \sqrt{P_s}h_1 s + \sqrt{P_I}f_1 x_1 + n_r$, where h_1 is the channel coefficient for the link SU-Tx → SU-relay, P_I is the average transmit power at the PU-Tx, x_1 is the transmitted signal of the PU-Tx in the first time slot, and n_r is additive white Gaussian noise (AWGN) at the SU-relay. Without loss of generality, we assume that $\mathbb{E}\{|s|^2\} = \mathbb{E}\{|x_1|^2\} = 1$, where $\mathbb{E}\{\cdot\}$ is the expectation. Then, the SU-relay amplifies y_r with an amplifying gain G and transmits the resulting signal to the SU-Rx with the average power $P_R = \frac{I_p}{|g_2|^2}$, where g_2 is the channel coefficient for the link SU-relay → PU-Rx. Owing to the concurrent transmission of the PU-Tx, the received signal at the SU-Rx can be written as $y_d = \sqrt{P_s}Gh_2h_1 s + Gh_2 n_r + Gh_2\sqrt{P_I}f_1x_1 + n_d + \sqrt{P_I}f_2x_2$, where h_2 and f_1 are the channel coefficients for the links SU-relay → SU-Rx and PU-Tx → SU-Rx, respectively, x_2 is the transmitted signal of the PU-Tx with $\mathbb{E}\{|x_2|^2\} = 1$, and n_d is AWGN at the SU-Rx. In this work, we consider non-identical Rayleigh fading in which all the fading channel coefficients h_1, h_2, g_1, g_2, f_1, f_2 are complex Gaussian distributed with zero mean and variances Ω_{h_1}, Ω_{h_2}, Ω_{g_1}, Ω_{g_2}, Ω_{f_1}, Ω_{f_2}, respectively, and AWGN components n_r, n_d have the same variance of N_0. The signal-to-interference ratio at SU-Tx is obtained as

$$\gamma_{AF} = \frac{\frac{\bar{\gamma}|h_1|^2}{|g_1|^2\bar{\gamma}_I|f_1|^2}\frac{\bar{\gamma}|h_2|^2}{|g_2|^2\bar{\gamma}_I|f_2|^2}}{\frac{\bar{\gamma}|h_1|^2}{|g_1|^2\bar{\gamma}_I|f_1|^2}+\frac{\bar{\gamma}|h_2|^2}{|g_2|^2\bar{\gamma}_I|f_2|^2}+1} \tag{1}$$

where $\bar{\gamma} = \frac{I_p}{N_0}$ and (1) is obtained by considering the interference-limited environment, i.e. $\bar{\gamma}_I = \frac{P_I}{N_0}$. To start our analysis, let us introduce an upper bound for γ_{AF} given in (1) as $\gamma_{AF} \le \gamma_{AF_{up}} = \min\,(\gamma_1,\ \gamma_2)$ with $\gamma_1 = \frac{\bar{\gamma}|h_1|^2}{|g_1|^2\bar{\gamma}_I|f_1|^2}$ and $\gamma_2 = \frac{\bar{\gamma}|h_2|^2}{|g_2|^2\bar{\gamma}_I|f_2|^2}$ To obtain the OP, we need to derive the CDF of $U = \frac{X}{YZ}$ where X, Y, and Z are exponentially distributed random variables with parameters λ_x, λ_y, and λ_z, respectively. It is easy to see that the CDF of U can be obtained as $F_U\,(u) = \int_0^\infty \int_0^\infty F_X\,(uyz)\ f_Y\,(y)\ f_Z\,(z)\ dydz$. Here, the CDF and probability density function (PDF) of $W \in \{X,\ Y,\ Z\}$ are written as $F_W\,(w) = 1 - e^{-\lambda_w}w$ and $f_W\,(w) = \lambda_w e^{-\lambda_w}w$ for $\lambda w \in \{\lambda_x,\ \lambda_y,\ \lambda_z\}$. After some simple calculations, the CDF of U can be easily derived as $F_U(u) = 1 - \frac{\lambda_y\lambda_z}{\lambda_x u}exp\left(\frac{\lambda_y\lambda_z}{\lambda_x u}\right)\Gamma\left(0,\ \frac{\lambda_y\lambda_z}{\lambda_x u}\right)$, where $\Gamma\ (\cdot,\ \cdot)$ is the incomplete gamma function [5, equation (8.350.2)]. As a result, the CDF of $\gamma_{AF_{up}}$, i.e. $F_{\gamma_{AF_{up}}}\,(\gamma) = 1 - [1 - F_{\gamma_1}\,(\gamma)]\ [1 - F_{\gamma_2}\,(\gamma)]$, can be written as

$$\begin{aligned} F_{\gamma_{AF_{up}}}(\gamma) = 1\,&\frac{\bar{\gamma}\Omega_{h_1}\Omega_{h_2}}{\bar{\gamma}_I^2\Omega_{g_1}\Omega_{f_1}\Omega_{g_2}\Omega_{f_2}\gamma^2}e^{\left(\frac{\bar{\gamma}\Omega_{h_1}}{\bar{\gamma}_I\Omega_{g_1}\Omega_{f_1}\gamma}\right)} \\ &\times e^{\left(\frac{\bar{\gamma}\Omega_{h_2}}{\bar{\gamma}_I\Omega_{g_2}\Omega_{f_2}\gamma}\right)}\Gamma\left(0,\ \frac{\bar{\gamma}\Omega_{h_1}}{\bar{\gamma}_I\Omega_{g_1}\Omega_{f_1}\gamma}\right) \\ &\times \Gamma\left(0,\ \frac{\bar{\gamma}\Omega_{h_2}}{\bar{\gamma}_I\Omega_{g_2}\Omega_{f_2}\gamma}\right) \end{aligned} \tag{2}$$

The lower bound for OP, P_{out}, can be immediately obtained from (2) utilising the fact that $P_{out} = F_{\gamma_{AF_{up}}}(\gamma\text{th})$, where γ_{th} is an outage threshold. The asymptotic representation of $\Gamma(\alpha,\ x)$ for large value of $|x|$ can be given by [5, equation (8.357.1)] $\Gamma(0, x) = x^{-1}e^{-x}\Big[\sum_{m=0}^{M-1}\frac{(-1)^m m!}{x^m} + \mathcal{O}(|x|^{-M})\Big]$, $M = 1, 2, \ldots, \infty$. By substituting this result into (2) and neglecting small terms, we obtain

$$P_{out} \overset{\bar{\gamma}\to\infty}{\simeq} \left(\frac{\Omega_{g_1}\Omega_{f_1}}{\Omega_{h_1}} + \frac{\Omega_{g_2}\Omega_{f_2}}{\Omega_{h_2}}\right)\frac{\bar{\gamma}I\,\gamma\text{th}}{\bar{\gamma}} \tag{3}$$

For comparison, we also derive an asymptotic expression for the case of neglecting the effect of the PU-Tx in [4], i.e. in the absence of $\bar{\gamma}_I$, Ω_{f_1}, and Ω_{f_2}. The lower bound for OP is shown as (detailed proof is omitted here due to space limitation) $P_{out} = 1 - \left(1 + \frac{\Omega_{g_1}}{\Omega_{h_1}\bar{\gamma}}\gamma_{th}\right)^{-1}\left(1 + \frac{\Omega_{g_2}}{\Omega_{h_2}\bar{\gamma}}\gamma_{th}\right)^{-1}$. Then, applying the McLaurin series expansion for $(1 + ax)^{-1} = \Sigma_{k=0}^{\infty}(-1)^k a^k x^k$, after some manipulations and ignoring small terms, the asymptotic OP of the system in [4] is shown as

$$P_{out} \overset{\bar{\gamma}\to\infty}{\simeq} \left(\frac{\Omega_{g_1}}{\Omega_{h_1}} + \frac{\Omega_{g_2}}{\Omega_{h_2}}\right)\frac{\gamma_{th}}{\bar{\gamma}} \tag{4}$$

From (3), i.e. in the presence of the PU-Tx, and (4), i.e. in the absence of the PU-Tx, we observe that under a fixed $\bar{\gamma}_I$, the two systems have the same diversity order. However, the array gain is reduced by an amount of $\mathcal{G}_\infty = 10\log_{10}\left(\frac{(\Omega_{g_1}\Omega_{f_1}\Omega_{h_2} + \Omega_{g_2}\Omega_{f_2}\Omega_{h_1})\bar{\gamma}_I}{\Omega_{g_1}\Omega_{h_2} + \Omega_{g_2}\Omega_{h_1}}\right)$. When the inference from the PU-Tx, $\bar{\gamma}_I$, is linearly proportional to the average SNR, i.e. $\bar{\gamma}_I = \rho\bar{\gamma}$ where ρ is a positive constant, the OP in (2) becomes $P_{out} \overset{\bar{\gamma}\to\infty,\,\bar{\gamma}_I=\rho\bar{\gamma}}{\simeq} \rho\left(\frac{\Omega_{g_1}\Omega_{f_1}}{\Omega_{h_1}} + \frac{\Omega_{g_2}\Omega_{f_2}}{\Omega_{h_2}}\right)\gamma_{th}$, which is independent of $\bar{\gamma}$. This causes an error floor in the OP for the whole SNR range yielding zero diversity order.

Numerical results: Similarly as in [4], a linear network topology is assumed here where the SU-Tx, the SU-relay, and the SU-Rx are located at co-ordinates (0,0), and (1,0), respectively. The average channel power for the link between node A and B, Ω_0, is inversely proportional to the distance from A to B, d_0, i.e. $\Omega_0 = \frac{1}{d_0^4}$ for a shadowed urban cellular radio, where A, B $\in$ {SU-Tx, SU-relay, SU-Rx, PU-Tx, PU-Rx}. The outage threshold γ_{th} is set to 3 dB for all examples. Fig. 2 displays the OP performance for PU-Rx(0.5,0.5) and $\bar{\gamma}_I = 2$ dB. Here, we consider three different scenarios where the location of the PU-Tx is set to (0.7, 0.7), (0.8, 0.8), and (0.9, 0.9). As expected, the performance increases when the PU-Tx moves away from the secondary network, i.e. (0.7, 0.7) $\rightarrow$ (0.8, 0.8) $\rightarrow$ (0.9, 0.9). The analysis matches very well with the simulation and the asymptotic result tightly converges to the exact value, which validates the proposed analysis. To understand the impact of the PU-Tx on the system performance better, Fig. 3 shows OP for different values of the interference power $\bar{\gamma}_I$. In the case of $\bar{\gamma}_I$ being independent of the average SNR $\bar{\gamma}$, i.e. $\bar{\gamma}_I = 2$, 4 6 dB, increasing $\bar{\gamma}_I$ degrades the array gain but not the diversity gain. The PU-Tx has a major impact on the secondary network since the performance loss of more than 10 dB is observed in the case of the interference of $\bar{\gamma}_I = 2$ dB compared to the scenario without the PU-Tx. More severely, as $\bar{\gamma}_I = 0.1\bar{\gamma}$ and $\bar{\gamma}_I = 0.5\bar{\gamma}$, the performance is significantly reduced owing to the error floor for the considered SNR range.

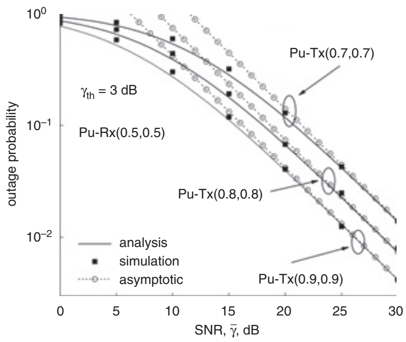

Fig. 2 *Performance comparison for different positions of PU-Tx*

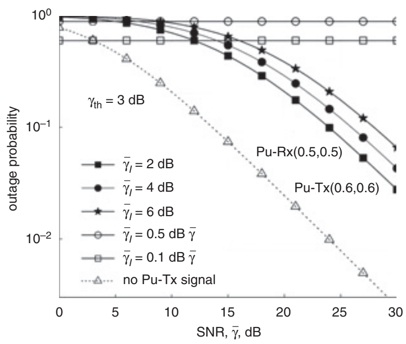

Fig. 3 *Performance comparison for different average powers from PU-Tx* $\bar{\gamma}_I$

Conclusion: The effect of the primary network on spectrum sharing AF relaying has been investigated in this Letter. Closed-form and asymptotic expressions for OP have been derived for non-identical Rayleigh fading channels. It has been shown that under a fixed interference from the primary network, the diversity order of the secondary network is not affected but only the array gain. However, when the interference power is dependent on the average SNR of the secondary network, it is infeasible to operate the secondary system as an irreducible error floor exists for the whole SNR regime.

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