FUZZY AND ROUGH SET THEORY IN TREATMENT OF ELDERLY GASTRIC CANCER PATIENTS

Hang Zettervall

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To my family ……

谨以此书献给我的亲人 ……

Abstract

Fuzzy set theory was presented for the first time by Professor Lotfi A. Zadeh from Berkeley University in 1965 as his vision of research involved in the analysis of complex systems [Zadeh, 1965]. Fuzzy mathematics constitutes a new tool of dealing with imprecise or vague data. The definition of a fuzzy set is an extension of the classical Cantor set.

We are still accustomed to our traditional bases of reasoning being strict and precise. In conventional binary logic a statement can be true or false, and there is no place for even a little uncertainty in this judgment. By looking at sets, we can state that an element either belongs to a set or does not. We call these kinds of sets *crisp sets*. In practice we often experience those real situations that are represented by crisp sets as impossible to describe accurately. If we assign a truth-value of one to the element that is included in the set, and a truth-value comparable to zero to such an element that lies outside the set, then we will create the range of two-valued logic. This sort of logic assumes that precise symbols must be employed, and it is therefore not applicable to the real existence but only to an imagined existence [Zadeh, 1965; Zimmermann, 2001; Rakus-Andersson, 2007].

Lotfi Zadeh referred to the last hypothesis when he wrote: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until the threshold is reached beyond which precision and significance become almost mutually exclusive characteristics" [Zadeh, 1965].

If we consider the characteristic features of real world systems, we will conclude that real situations are very often uncertain or vague in a number of ways. If the information demanded by a system is lacking, the future state of such a system may not be known completely. This type of uncertainty has been handled by probability theories and statistics, and it is called stochastic uncertainty. The vagueness, concerning the description of the semantic meaning of the events, phenomena, or statements themselves, is called fuzziness [Zadeh, 1965].

Fuzziness can be found in many areas of daily life, especially in medicine. We look for the methods that help us to express the borders of such sets as "*young*", "*middle-aged*", "*old*", "*seldom*", "*rarely*", "*often*" and the like. Thus we introduce the fuzzy apparatus to extend a notion of the set under the circumstances of vagueness [Rakus-Andersson, 2007].

Since the introduction in 1965, fuzzy set theory has been frequently applied in a wide range of areas like, e.g., dynamic systems, militaries, medicine and other domains. One of the successful trials of technical adaptations of the theory is the construction of the smoothest subway developed in Japan.

Another theory, which copes with the problem of imprecision, is known as rough set theory [Pawlak, 1984, 1997, 2004; Małuszyński and Vitória, 2002, Skowron, 2001]. Rough set theory was proposed by Professor Zdzisław Pawlak in Warsaw in the 1980ties. Whereas imprecision is expressed in the category of a membership degree in fuzzy set theory, this is a matter of the set approximation in rough set theory. Due to the definition of a rough set formulated by means of the decision attribute value, two approximate sets of the rough set are determined. These contain sure and possible members of the universe considered, in which the rough set has been defined.

The objective of this study is to apply some classical methods of fuzzy set theory to medicine in order to estimate the survival length of gastric cancer patients and, by means of rough set classification, to verify the types of operations. These items will be discussed in conformity with the physicians' wishes to support results of statistical investigations. The current research is funded by the scientific grant obtained from Blekinge Research Board.

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Preface

For the first time I heard about "Fuzzy mathematics" in 2006. My first reaction was "What is fuzzy mathematics?" Mathematics has always been a subject consisting of precise information and how could it be fuzzy? To which areas can fuzzy mathematics be applied?"

With the curiosity I attended the course "Fuzzy logic" which was conducted by Professor Elisabeth Rakus-Andersson. After that I became more and more interested in the subject and decided to continue studying it.

Actually, there are many uncertain problems arising in our daily life. For instance, we often hear such statements as "It is very warm/cold today" on the TV and on the radio or other formulations like "She is tall/young/beautiful" and so on. These are simple examples of the uncertainty. We cannot define exactly how many degrees centigrade a warm/cold day should be characterized by, we cannot either decide how many centimeters a person should measure to be classified as tall, how many years old a person should be to be recognized as young or old and how she should look like to be beautiful. Thus, the sets of objects which fuzzy mathematics introduces have the vague and overlapping characteristics.

This thesis consists of five parts. In the introductory part some basic definitions and operations on fuzzy sets are presented to allow a reader to follow the next parts with understanding. Operations on continuous fuzzy numbers in different forms are discussed in part two. Since fuzzy numbers play the powerful role in determination of entry data in models further discussed, the closer study of their nature and representations has been made in the same chapter. By utilizing the knowledge about fuzzy numbers we further concentrate on the applications of fuzzy controllers in parts three and four to estimate the survival length of gastric cancer patients. Finally, we use the rough set classification to verify the decision of the operation types.

1 Introductory Items of Fuzzy Set Theory

Before discussing the essence of the medical project, tackling with the treatment of gastric cancer patients, we wish to introduce some basic formulations of fuzzy set theory.

1.1 Preliminaries of Fuzzy Set Theory

Definition 1.1: Fuzzy Set

If X is a finite universe set, $X = \{x_i, i = 1,...,n\}$, then a fuzzy set A in X is defined as a set of ordered pairs $A = \{(x_i, \mu_A(x_i)), i = 1,...,n\}$, in which the value of $\mu_A(x_i)$ is called the degree of membership of x_i in fuzzy set A and $\mu_A : X \to [0,1]$ is called the membership function mapping X into the unit interval [0,1] [Zadeh, 1965, 1973; Zimmermann, 2001; Rakus-Andersson, 2007; Buckley and Eslami, 2002; Carlsson and Fullér, 2001; Kaufmann and Gupta, 1991; Lowen, 1996; Mordeson et al., 2000; Pedrycz, 1995; Pedrycz and Gomide, 1998; Sadegh-Zadeh, 2000]. The support of the fuzzy set *A*, *S*(*A*) is defined as a non-fuzzy set of these elements, which have the membership degrees greater than zero.

There are four types of notations used for presenting a fuzzy set A.

1. A is formed as an ordered set of pairs, $A = \{(x_i, \mu_A(x_i)), i = 1,...,n\}$, where x_i denotes the element from the set X and $\mu_A(x_i)$ denotes the degree of the membership of x_i in the fuzzy set *A*.

Example 1.1

Let X = "integers between 1 and 10" = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then a fuzzy set A, "integers which are close to 5" can be defined as A = {(1, 0), (2, 0.3), (3, 0.5), (4, 0.7), (5, 1), (6, 0.9), (7, 0.8), (8, 0.6), (9, 0.3), (10, 0)}.

2. A corresponds to "the Zadeh notation" $A = \sum_{i=1}^{n} \mu_A(x_i)/x_i$. Elements with zero membership degrees are normally omitted. Note: the sigma-sign has just the symbolic function to connect the set elements.

Example 1.2

Let X = "integers between 1 and 10" = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
Hence, A = "close to 6" $= 0.3/2 + 0.5/3 + 0.7/4 + 0.9/5 + 1/6 + 0.9/7 + 0.7/8 + 0.3/9$.

3. *A* is characterized by an *n*-dimensional vector of membership degrees $A = (\mu_A(x_i), i = 1,...,n)$. Elements with zero membership degrees cannot be omitted.

Example 1.3

Let X = "integers between 1 and 10" = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
A is thus assisted by A = "close to 6" $=(0, 0.3, 0.5, 0.7, 0.9, 1, 0.9, 0.7, 0.3, 0)$.

4. *A* is listed as a collection of its elements and assigned to them degrees of membership in the form of $A = \left\{ \mu_A(x_i)/x_i,\ i = 1,\ldots,n \right\}$.

Example 1.4
If X = "integers between 1 and 10" = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} then
A = "close to 6" = $\left\{ 0/1,\ 0.3/2, 0.5/3, 0.7/4, 0.9/5, 1/6, 0.9/7, 0.7/8, 0.3/9, 0/10 \right\}$.

We now intend to demonstrate some useful definitions concerning fuzzy sets.

Definition 1.2
A fuzzy set *A* is said to be empty if all membership degrees assisting x_i, i = 1,…,n, are equal to zero in *X*. That is,

$$\mu_A(x_i) = 0,\ x_i \in X, i = 1,\ldots,n \Rightarrow A \text{ is empty.} \tag{1.1}$$

Definition 1.3
Two fuzzy sets *A* and *B* are equal, if and only if the membership degree in *A* is equal to the membership degree in *B* for each element $x_i \in X$. This property can be expressed as

$$A = B \Leftrightarrow \mu_A(x_i) = \mu_B(x_i),\ x_i \in X, i = 1,\ldots,n\ . \tag{1.2}$$

Definition 1.4
The complement of a fuzzy set *A*, denoted by A', is defined by the formula of A''s membership function

$$\mu_{A'}(x_i) = 1 - \mu_A(x_i),\ x_i \in X, i = 1,\ldots,n\ . \tag{1.3}$$

Definition 1.5
A fuzzy set *A* is said to be a subset of another fuzzy set *B* if and only if the membership degree in *A* is equal or less than the membership degree in *B* for each element $x_i \in X$, which is equivalent to

$$A \subseteq B \Leftrightarrow \mu_A(x_i) \le \mu_B(x_i),\ x_i \in X, i = 1,\ldots,n\ . \tag{1.4}$$

1.2 Basic Operations on Fuzzy Sets

To be able to perform some topological operations on fuzzy sets we determine their union, their intersection and their complements.

Definition 1.6: The topological (hard) union of two fuzzy sets *A* and *B*
Let *X* be a common universe for fuzzy sets *A* and *B*. If $A = \{(x_i, \mu_A(x_i)), i = 1,\ldots,n\}$ and $B = \{(x_i, \mu_B(x_i)), i = 1,\ldots,n\}$, then the union of *A* and *B* is denoted by $A \cup B$ and determined by a membership function

$$\mu_{A\cup B}(x_i) = \text{Max } [\mu_A(x_i), \mu_B(x_i)] = \mu_A(x_i) \vee \mu_B(x_i), \quad x_i \in X, i = 1,...,n . \tag{1.5}$$

Lemma 1.1
The union of fuzzy sets *A* and *B*, due to Def. 1.6 is the smallest fuzzy set which contains both *A* and *B*.

Proof: Suppose that $\mu_{A\cup B}(x_i) = \text{Max}[\mu_A(x_i), \mu_B(x_i)], \quad x_i \in X, i = 1,...,n$. Then,
Max $[\mu_A(x_i), \mu_B(x_i)] \geq \mu_A(x_i)$ and
Max $[\mu_A(x_i), \mu_B(x_i)] \geq \mu_B(x_i)$.
Let *D* be an arbitrary fuzzy set containing both *A* and *B*. This implies that
$\mu_D(x_i) \geq \mu_A(x_i)$ and $\mu_D(x_i) \geq \mu_B(x_i)$.
Hence, $\mu_D(x_i) \geq \mu_{A\cup B}(x_i), \quad x_i \in X, i = 1,...,n$.

Definition 1.7: The topological (hard) intersection of *A* and *B*
The intersection of two fuzzy sets *A* and *B*, $A = \{(x_i, \mu_A(x_i)), i = 1,...,n\}$ and $B = \{(x_i, \mu_B(x_i)), i = 1,...,n\}$, is recognized by the notation $A \cap B$ and expressed by the membership function

$$\mu_{A\cap B}(x_i) = \text{Min } [\mu_A(x_i), \mu_B(x_i)] = \mu_A(x_i) \wedge \mu_B(x_i), \quad x_i \in X, i = 1,...,n . \tag{1.6}$$

Lemma 1.2
The intersection of *A* and *B* is the largest fuzzy set, which is contained in both *A* and *B*.

Proof: By the definition of the intersection of *A* and *B* we get
$\mu_{A\cap B}(x_i) = \text{Min } [\mu_A(x_i), \mu_B(x_i)] = \mu_A(x_i) \wedge \mu_B(x_i), \quad x_i \in X, i = 1,...,n$.
It means that
Min $[\mu_A(x_i), \mu_B(x_i)] \leq \mu_A(x_i)$ and Min $[\mu_A(x_i), \mu_B(x_i)] \leq \mu_B(x_i)$.
Suppose that *M* is an arbitrary fuzzy set which is contained in both *A* and *B*. This implies that $\mu_A(x_i) \geq \mu_M(x_i)$ and $\mu_B(x_i) \geq \mu_M(x_i)$.
Hence, $\mu_{A\cap B}(x_i) \geq \mu_M(x_i), \quad x_i \in X, i = 1,...,n$.

Some of the basic identities, such as the commutative laws, the distributive laws and the associative laws, which hold for the topological union and intersection performed on the classical sets, can be also encountered in relations among fuzzy sets. We formulate and prove the following connections involving fuzzy sets.

Theorem 1.1
If *A* and *B* are two fuzzy sets then the commutative laws $A \cap B = B \cap A$ and $A \cup B = B \cup A$ hold.

Proof: $\mu_{A\cap B}(x_i) = \text{Min } [\mu_A(x_i), \mu_B(x_i)] = \text{Min } [\mu_B(x_i), \mu_A(x_i)] = \mu_{B\cap A}(x_i)$,

$x_i \in X, i = 1,...,n$. For

$$\mu_{A\cup B}(x_i) = \text{Max } [\mu_A(x_i), \mu_B(x_i)] = \text{Max } [\mu_B(x_i), \mu_A(x_i)] = \mu_{B\cup A}(x_i),$$

$$x_i \in X, i = 1,\ldots,n .$$

Theorem 1.2
For three fuzzy sets *A*, *B* and *C* we formulate the distributive laws
$C\cap(A\cup B)=(C\cap A)\cup(C\cap B)$ and $C\cup(A\cap B)=(C\cup A)\cap(C\cup B)$.

Proof: The existence of the first distributive law can be verified by means of the topological operation performance on membership degrees of the fuzzy sets *A, B* and *C*. Since the distributive laws are valid for the maximum and the minimum operators then

$$\mu_{C\cap(A\cup B)}(x_i) = \mu_C(x_i)\wedge\mu_{A\cup B}(x_i) = \mu_C(x_i)\wedge[\mu_A(x_i)\vee\mu_B(x_i)]$$
$$= [\mu_C(x_i)\wedge\mu_A(x_i)]\vee[\mu_C(x_i)\wedge\mu_B(x_i)] = \mu_{C\cap A}(x_i)\vee\mu_{C\cap B}(x_i) = \mu_{(C\cap A)\cup(C\cap B)}(x_i).$$

For the second distributive law

$$\mu_{C\cup(A\cap B)}(x_i) = \mu_C(x_i)\vee\mu_{A\cap B}(x_i) = \mu_C(x_i)\vee[\mu_A(x_i)\wedge\mu_B(x_i)]$$
$$= [\mu_C(x_i)\vee\mu_A(x_i)]\wedge[\mu_C(x_i)\vee\mu_B(x_i)] = \mu_{C\cup A}(x_i)\wedge\mu_{C\cup B}(x_i) = \mu_{(C\cup A)\cap(C\cup B)}(x_i).$$

A simple example confirms the validity of de Morgan's law $(A\cup B)' = A'\cap B'$ for topological union and intersection of two fuzzy sets.

Example 1.5
Let $X = \{1,2,3,4,5,6,7,8\}$. Define
$A =$"large integer in X"$= 0.25/5 + 0.5/6 + 0.8/7 + 1/8$ and
$B =$"small integer in X"$= 1/1 + 0.7/2 + 0.5/3 + 0.3/4 + 0.1/5$. Then
$A\cup B = 1/1 + 0.7/2 + 0.5/3 + 0.3/4 + 0.25/5 + 0.5/6 + 0.8/7 + 1/8$,
$(A\cup B)' = 0.3/2 + 0.5/3 + 0.7/4 + 0.75/5 + 0.5/6 + 0.2/7$,
$A' = 1/1 + 1/2 + 1/3 + 1/4 + 0.75/5 + 0.5/6 + 0.2/7$,
$B' = 0.3/2 + 0.5/3 + 0.7/4 + 0.9/5 + 1/6 + 1/7 + 1/8$ and
$A'\cap B' = 0.3/2 + 0.5/3 + 0.7/4 + 0.75/5 + 0.5/6 + 0.2/7$. Hence, $(A\cup B)' = A'\cap B'$.

We intend to formulate and to prove the existence of relations known as de Morgan's laws for the topological operations on fuzzy sets.

Theorem 1.3: De Morgan's laws:
For two fuzzy sets $A = \{(x_i, \mu_A(x_i)), i = 1,\ldots,n\}$ and $B = \{(x_i, \mu_B(x_i)), i = 1,\ldots,n\}$

$$\begin{cases} (A \cup B)' = A' \cap B' & (1.7) \\ (A \cap B)' = A' \cup B'. & (1.8) \end{cases}$$

Proof: We verify the first identity by means of the membership functions of x_i in A and B for each $x_i \in X$.

For

$$\mu_{(A \cup B)'}(x_i) = 1 - \mu_{A \cup B}(x_i) = 1 - \text{Max}[\mu_A(x_i), \mu_B(x_i)]$$
$$= 1 - [\mu_A(x_i) \vee \mu_B(x_i)] = [1 - \mu_A(x_i)] \wedge [1 - \mu_B(x_i)]$$
$$= \mu_{A'}(x_i) \wedge \mu_{B'}(x_i) = \text{Min}[\mu_{A'}(x_i), \mu_{B'}(x_i)] = \mu_{A' \cap B'}(x_i).$$

The second identity can also be verified by referring to the definitions of the topological operations. Hence

$$\mu_{(A \cap B)'}(x_i) = 1 - \mu_{A \cap B}(x_i) = 1 - \text{Min}[\mu_A(x_i), \mu_B(x_i)]$$
$$= 1 - [\mu_A(x_i) \wedge \mu_B(x_i)] = [1 - \mu_A(x_i)] \vee [1 - \mu_B(x_i)]$$
$$= \mu_{A'}(x_i) \vee \mu_{B'}(x_i) = \text{Max}[\mu_{A'}(x_i), \mu_{B'}(x_i)] = \mu_{A' \cup B'}(x_i).$$

If we determine other types of the intersection and the union for two fuzzy sets, e.g., the operations algebraically defined on membership degrees of A and B as $\mu_{A \cdot B}(x_i) = \mu_A(x_i) \cdot \mu_B(x_i)$ and $\mu_{A+B}(x_i) = \mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i) \cdot \mu_B(x_i)$, $x_i \in X$, $i = 1,\ldots,n$, then we can also prove the validity of de Morgan's laws.

Theorem 1.4

For two fuzzy sets $A = \{(x_i, \mu_A(x_i)), i = 1,...,n\}$ and $B = \{(x_i, \mu_B(x_i)), i = 1,...,n\}$ the law $(A + B)' = A' \cdot B'$ is referred to the operations of the algebraic (soft) intersection and the algebraic (soft) union yielded by the membership functions $\mu_{A \cdot B}(x_i) = \mu_A(x_i) \cdot \mu_B(x_i)$ and $\mu_{A+B}(x_i) = \mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i) \cdot \mu_B(x_i)$.

Proof: We perform the set operations on membership degrees assisting A and B. Thus,
$\mu_{(A+B)'}(x_i) = 1 - (\mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i)\mu_B(x_i)) = 1 - \mu_A(x_i) - \mu_B(x_i) + \mu_A(x_i)\mu_B(x_i) =$
$(1 - \mu_A(x_i))(1 - \mu_B(x_i)) = \mu_{A'}(x_i) \cdot \mu_{B'}(x_i) = \mu_{A' \cdot B'}(x_i)$.

Both the minimum operator and the multiplication operator, proposed in the intersection operations on membership degrees of fuzzy sets, fulfill the properties of the function called a t-norm. The t-norm satisfies the properties

1. $t(0,0) = 0;\ t(\mu_A(x_i),1) = t(1, \mu_A(x_i)) = \mu_A(x_i), x_i \in X$,
2. $t(\mu_A(x_i), \mu_B(x_i)) \le t(\mu_C(x_i), \mu_D(x_i))$ if $\mu_A(x_i) \le \mu_C(x_i)$ and $\mu_B(x_i) \le \mu_D(x_i)$,
3. $t(\mu_A(x_i), \mu_B(x_i)) = t(\mu_B(x_i), \mu_A(x_i))$,
4. $t(\mu_A(x_i), t(\mu_B(x_i), \mu_C(x))) = t(t(\mu_A(x_i), \mu_B(x_i)), \mu_C(x_i))$.

Generally, any function t, which has the features listed in points 1-4 can be adopted as an operator of intersection between two fuzzy sets.

To define different approaches to the union of two fuzzy sets the characteristics of the function s, named the s-norm, are stated as

1. $s(1,1) = 1;\ s(\mu_A(x_i),0) = s(0,\mu_A(x_i)) = \mu_A(x_i),\ x_i \in X$,
2. $s(\mu_A(x_i),\mu_B(x_i)) \le s(\mu_C(x_i),\mu_D(x_i))$
   if $\mu_A(x_i) \le \mu_C(x_i)$ and $\mu_B(x_i) \le \mu_D(x_i)$,
3. $s(\mu_A(x_i),\mu_B(x_i)) = s(\mu_B(x_i),\mu_A(x_i))$,
4. $s(\mu_A(x_i),s(\mu_B(x_i),\mu_C(x_i))) = s(s(\mu_A(x_i),\mu_B(x_i)),\mu_C(x_i))$.

It is easy to check that the maximum operator assisting the topological union as well as the definition of the algebraically created membership function for the algebraic union possess the properties 1-4 of the last connection.

All definitions and properties of fuzzy sets, discussed so far, can be transferred into the continuous universe of discourse $X = \{x\}$, in which a fuzzy set A is denoted as a collection $A = \sum_{x \in X} \mu_A(x)/x$.

1.3 The Concepts of *s*-class Functions and Fuzzy Numbers

Instead of predetermined discrete sets of membership degrees, corresponding to the set elements, continuous membership functions will be stretched over domains of fuzzy sets. The functions constitute the sets' restrictions in the universe X and, thus, they should take the values in the interval [0, 1]. One of the types of membership functions, commonly utilized in applications, is an s-class function defined below.

Definition 1.8

As the membership function of the fuzzy set A we demonstrate the s-class function $s(x, \alpha, \beta, \gamma)$ with the parameters α, β and γ that are included in the formula [Adlassnig, 1986; Buckley and Eslami, 2002; Kaufmann and Gupta, 1991; Novák and Perfilieva, 1999; Pal and Mitra, 2004; Rakus-Andersson, 2005]

$$y = \mu_A(x) = s(x,\alpha,\beta,\gamma) = \begin{cases} 0 & \text{for } x \le \alpha, \\ 2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2 & \text{for } \alpha < x \le \beta, \\ 1-2\left(\dfrac{x-\gamma}{\gamma-\alpha}\right)^2 & \text{for } \beta < x \le \gamma, \\ 1 & \text{for } x > \gamma, \end{cases} \tag{1.9}$$

where $\beta = \dfrac{\alpha+\gamma}{2}$.

Example 1.6
The function $s(x, 25, 37.5, 50)$ is plotted in Fig.1.1.

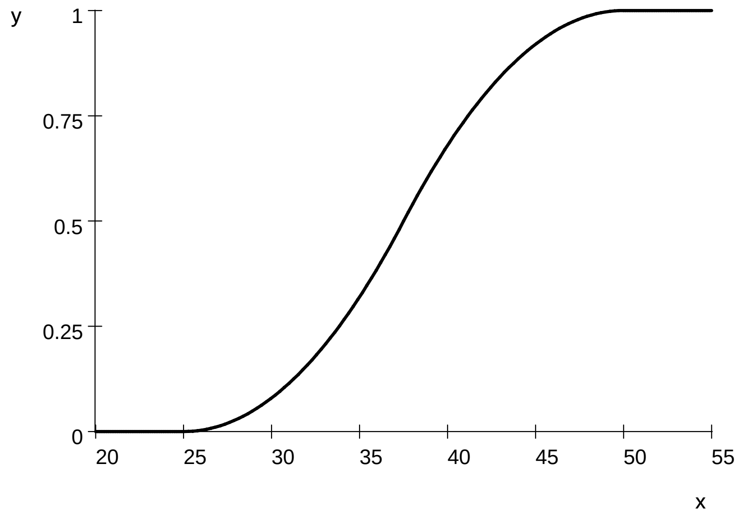

Figure 1.1: The function $s(x, 25, 37.5, 50)$

We also introduce another important definition of a non-fuzzy set assisting the fuzzy set A.

Definition 1.9

For a fuzzy continuous set $A = \{(x, \mu_A(x))\}$, $x \in X$, we determine a non-fuzzy set

$$A_\alpha = \{x : \mu_A(x) \geq \alpha\} \tag{1.10}$$

called the α-level of A.

Example 1.7
For A given by the membership function [Rakus-Andersson, 2007]

$$\mu_A(x) = \begin{cases} s(x,\ 30,40,50) & \text{for} \quad x \leq 50, \\ 1 - s(x,50,60,70) & \text{for} \quad x \geq 50, \end{cases}$$

we state, e.g., $A_{0.5} = [40, 60]$ in accordance with Fig. 1.2.

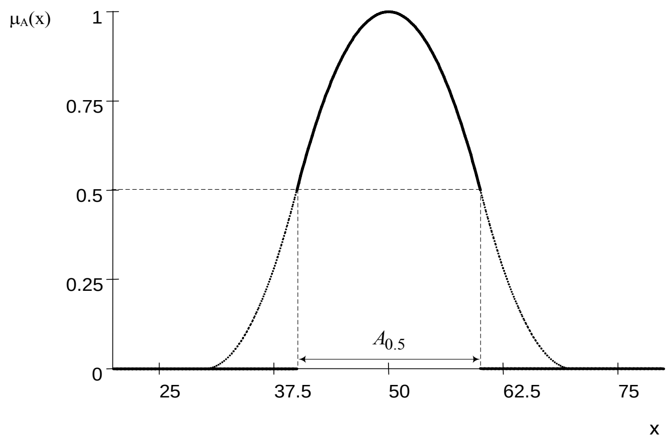

Figure 1.2: The 0.5-level of A

A particular sort of a fuzzy set is called a fuzzy number.

Definition 1.10: A definition of a continuous fuzzy number

A fuzzy number A of a universe $X = \{x\}$ is a normal fuzzy set possessing the ascending left function and the descending right function around this x in which the membership degree equals one. A fuzzy set is recognized as normal if at least one of its elements x has the degree of membership equal to 1.

We intend to discuss the character and the properties of continuous fuzzy numbers in the next chapter.

2. **Different Approaches to Operations on Continuous Fuzzy Numbers**

This chapter is particularly addressed to the readers as our own study in performing operations on continuous fuzzy numbers [Rakus-Andersson and Zettervall, 2008]. We discuss three different approaches to operations on fuzzy numbers to make comparisons of results in the aspect of their advantages and disadvantages.

The theory of fuzzy number arithmetic constitutes an exciting part of fuzzy set theory; therefore we have made a separate study of their nature. Fuzzy numbers will be utilized in the chapters devoted to medical applications in this dissertation and even they are planned to be crucial items of further investigations in developments of algorithms, which will be included in the doctor's thesis. By demonstrating different possibilities of making calculations on fuzzy numbers we intend to select the proper approach to the operations in the further parts of the actual and planned dissertations.

Fuzzy numbers, being normal fuzzy sets characterized by particularly designed membership functions, constitute important components of numerical operations involved in fuzzy calculus. The different approaches to the arithmetic over the space of fuzzy numbers have been suggested in many works [Buckley and Eslami, 2002; Chich-Hui Chiu and Wen-June Wang, 2002; Coupland and John, 2003; Dubois and Prade, 1978a; Dubois and Prade, 1978b; Dug Hun Hong, 2001; Dug Hun Hong and Hae Young Do, 1997; Divyendu Sinha, 1990; Filev and Yager, 1997; Goetschel and Voxman, 1986; Heilpern, 1997; Kacprzyk, 1986; Kechagias and Papadopoulos, 2007; Luke, 2006; Mendel, 2001; Pedrycz and Gomide, 1998].

We often adopt three methods of performing the operations on continuous fuzzy numbers dependently on their forms, namely, we recognize the *L-R* form, the interval form and the $\alpha - cut$ form.

2.1 Arithmetic Operations on Continuous Fuzzy Numbers in the L-R forms

Definition 2.1: Fuzzy numbers in the *L-R* representation

A continuous fuzzy number A can be represented by the *L-R* form if there exist reference functions *L* (for left) and *R* (for right) and scalars $\alpha > 0$ and $\beta > 0$, then the membership function $\mu_A(x)$ of A is defined as

$$y = \mu_A(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right) & \text{for} \quad m-\alpha \le x \le m, \\ \\ R\left(\dfrac{x-m}{\beta}\right) & \text{for} \quad m \le x \le m+\beta, \end{cases} \tag{2.1}$$

where *m* is called the mean value of A, $m \in \mathbf{R}$, α and β are called the left and the right spreads, respectively.

The fuzzy number A is represented by a triplet $A = (m, \alpha, \beta)_{LR}$, $m \in \mathbf{R}$. The support of A, $\mathrm{S}(A) = \{x \in X : \mu_A(x) > 0\}$ is thus stated as a set $S(A) = [m-\alpha, m+\beta]$. The functions $L(x)$ and $R(x)$ can have different definitions. One of them introduces a very comfortable appearance of $L(x) = R(x) = 1 - x$ created for linear branches of A*'s* membership functions. Due to this formula we restrict $\mu_A(x)$ in the form of

$$y=\mu_A(x)=\begin{cases} L\left(\dfrac{m-x}{\alpha}\right)=1-\dfrac{m-x}{\alpha} & \text{for} \quad m-\alpha \le x \le m, \\ R\left(\dfrac{x-m}{\beta}\right)=1-\dfrac{x-m}{\beta} & \text{for} \quad m \le x \le m+\beta. \end{cases} \tag{2.2}$$

Example 2.1
If $A = (6,3,2)_{LR}$, then

$$\mu_A(x)=\begin{cases} L\left(\dfrac{6-x}{3}\right)= \; 1-\left(\dfrac{6-x}{3}\right)= \; \dfrac{x-3}{3} & \text{for} \quad 3 \le x \le 6, \\ R\left(\dfrac{x-6}{2}\right)= \; 1-\left(\dfrac{x-6}{2}\right)= \; \dfrac{8-x}{2} & \text{for} \quad 6 \le x \le 8. \end{cases}$$

$A = (6,3,2)_{LR}$ has a triangular shape with the peak in 6. We plot the membership function of A in Fig. 2.1.

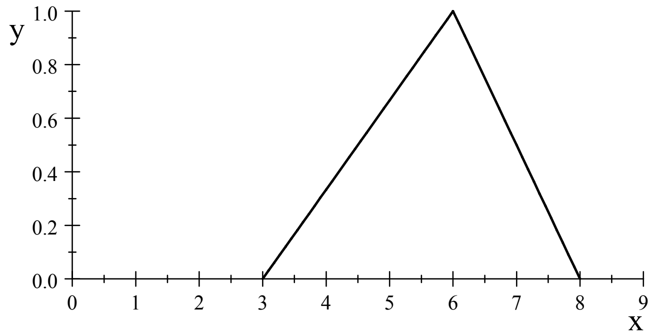

Figure 2.1. The membership function of $A=(6, 3, 2)_{LR}$

Let $A=(m_A,\alpha_A,\beta_A)_{LR}$ and $B=(m_B,\alpha_B,\beta_B)_{LR}$ be two continuous fuzzy numbers in the *L-R* form. We define the arithmetic operations on fuzzy numbers by the following way [Dubois and Prade, 1978a; Dubois and Prade, 1978b].

Addition of A_{LR} and B_{LR}

$$A+B=(m_A,\alpha_A,\beta_A)_{LR}+(m_B,\alpha_B,\beta_B)_{LR}=(m_A+m_B,\alpha_A+\alpha_B,\beta_A+\beta_B)_{LR}. \tag{2.3}$$

Example 2.2
If $A=(m_A,\alpha_A,\beta_A)_{LR}=(3,1,2)_{LR}$ and $B=(m_B,\alpha_B,\beta_B)_{LR}=(6,2,4)_{LR}$, then
$A+B=(3,1,2)_{LR}+(6,2,4)_{LR}=(9,3,6)_{LR}$.

After adopting (2.3) we plot the membership functions and the result in Fig. 2.2.

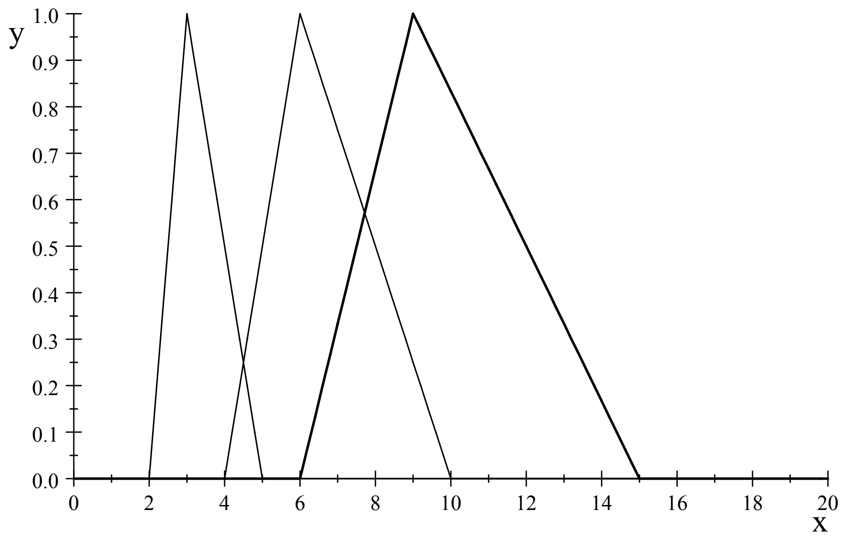

Figure 2.2. $A + B = (3,1,2)_{LR} + (6,2,4)_{LR}$

All membership functions are linear. If we wish to change the shape of the membership functions then we should design $L(x)$ and $R(x)$ in another form.

Subtraction of A_{LR} and B_{LR}

If we determine $-B$ as $-B = (-m_B, \beta_B, \alpha_B)_{LR}$, then

$$A - B = A + (-B) = (m_A, \alpha_A, \beta_A)_{LR} + (-m_B, \beta_B, \alpha_B)_{LR} = (m_A - m_B, \alpha_A + \beta_B, \beta_A + \alpha_B)_{LR}. \quad (2.4)$$

Example 2.3

For $A = (5,2,3)_{LR}$ and $B = (6,2,1)_{LR}$, $-B = (-6,1,2)_{LR}$, we get
$A - B = A + (-B) = (5,2,3)_{LR} + (-6,1,2)_{LR} = (5-6, 2+1, 3+2)_{LR} = (-1,3,5)_{LR}$.

Multiplication of A_{LR} and B_{LR} - Case 1

We differentiate results dependently on the relative values of m_A and m_B.

If $m_A > 0, m_B > 0$, then

$$A \cdot B = (m_A, \alpha_A, \beta_A)_{LR} \cdot (m_B, \alpha_B, \beta_B)_{LR} = (m_A \cdot m_B, m_A\alpha_B + m_B\alpha_A, m_A\beta_B + m_B\beta_A)_{LR}. \quad (2.5)$$

Example 2.4

For $A = (3,1,2)_{LR}$ and $B = (5,4,1)_{LR}$ we obtain a fuzzy number by multiplication
$A \cdot B = (3,1,2)_{LR} \cdot (5,4,1)_{LR} = (15,17,13)_{LR}$.

Multiplication of A_{LR} and B_{LR} - Case 2

If $m_A < 0, m_B > 0$, then we design

$$A \cdot B = (m_A, \alpha_A, \beta_A)_{LR} \cdot (m_B, \alpha_B, \beta_B)_{LR} = (m_A \cdot m_B, m_B\alpha_A - m_A\alpha_B, m_B\beta_A - m_A\beta_B)_{LR}. \quad (2.6)$$

Example 2.5

We state $A=(-3,2,1)_{LR}$ and $B=(4,3,2)_{LR}$ to expect a result
$A\cdot B=(-3,2,1)_{LR}\cdot(4,3,2)_{LR}=(-12,17,10)_{LR}$.

Multiplication of A_{LR} and B_{LR} - Case 3

In the case of $m_A<0, m_B<0$, the multiplication provides us with a fuzzy number

$$A\cdot B=(m_A,\alpha_A,\beta_A)_{LR}\cdot(m_B,\alpha_B,\beta_B)_{LR}=(m_A\cdot m_B,-m_A\beta_B-m_B\beta_A,-m_A\alpha_B-m_B\alpha_A)_{LR}. \tag{2.7}$$

Example 2.6

We accept $A=(-3,2,1)_{LR}$ and $B=(-1,3,2)_{LR}$ to compute
$A\cdot B=(-3,2,1)_{LR}\cdot(-1,3,2)_{LR}=(3,7,11)_{LR}$.

Let us notice that for $A=(m_A,\alpha_A,\beta_A)_{LR}$, the inverse number
$A^{-1}=\left(1/m_A, \alpha_A/m_A^2, \beta_A/m_A^2\right)_{LR}$.

Example 2.7

$A=(m_A,\alpha_A,\beta_A)_{LR}=(2,3,2)_{LR}\Rightarrow A^{-1}=\left(1/2, 3/4, 1/2\right)$.

Division of A_{LR} and B_{LR} - Case 1

We recognize the cases of different interpretations of results in conformity with the relative values of m_A and m_B as follows:

For $m_A>0, m_B>0$ we derive a formula

$$\begin{aligned}A:B&=(m_A,\alpha_A,\beta_A)_{LR}:(m_B,\alpha_B,\beta_B)_{LR}=\\&=\left(m_A:m_B,(m_A\beta_B+m_B\alpha_A):m_B^2,(m_A\alpha_B+m_B\beta_A):m_B^2\right)_{LR}.\end{aligned} \tag{2.8}$$

Example 2.8

The fuzzy numbers $A=(6,2,1)_{LR}$ and $B=(3,1,2)_{LR}$, inserted as arguments of the division operation, lead to a result

$$A:B=(6,2,1)_{LR}:(3,1,2)_{LR}=\left(6:3,(6\cdot2+3\cdot2):3^2,(6\cdot1+3\cdot1):3^2\right)_{LR}=(2,2,1)_{LR}.$$

Division of A_{LR} and B_{LR} - Case 2

We refer to $m_A>0, m_B<0$ to create a result

$$\begin{aligned}A:B&=(m_A,\alpha_A,\beta_A)_{LR}:(m_B,\alpha_B,\beta_B)_{LR}=\\&=\left(m_A:m_B,(m_A\beta_B-m_B\alpha_A):m_B^2,(m_A\alpha_B-m_B\beta_A):m_B^2\right)_{LR}.\end{aligned} \tag{2.9}$$

Example 2.9
We state $A=(2,3,1)_{LR}$ and $B=(-1,2,1)_{LR}$ to expect a result
$A:B=(2,3,1)_{LR}:(-1,2,1)_{LR}=(-2,5,5)_{LR}$.

Division of A_{LR} and B_{LR} - Case 3
In the case of $m_A<0, m_B<0$, the division provides us with a fuzzy number

$$\begin{aligned} A:B &= (m_A,\alpha_A,\beta_A)_{LR}:(m_B,\alpha_B,\beta_B)_{LR} \\ &= (m_A:m_B,(-m_A\alpha_B-m_B\beta_A):m_B^2,(-m_A\beta_B-m_B\alpha_A):m_B^2)_{LR}. \end{aligned} \tag{2.10}$$

Example 2.10
We accept $A=(-2,3,1)_{LR}$ and $B=(-1,2,3)_{LR}$ to compute
$A\cdot B=(-2,3,1)_{LR}\cdot(-1,2,3)_{LR}=(2,5,9)_{LR}$.

Let us notice that the formulas, derived for arithmetical operations on fuzzy numbers in the *L-R* form, provide us only with supports of results. To prognosticate shapes of the result membership functions in accordance with our expectations we have to model the reference functions *L*(*x*) and *R*(*x*) in advance.

2.2 Computations with Fuzzy Numbers in the Interval Form

The *L-R* form of a fuzzy number can be sometimes confusing when regarding the order of parameters listed as the mean, the left spread and the right spread. It seems to be natural to experience the fuzzy number to appear as a triplet: the left border of the support, the mean and the right border of the support. Let us suggest a concept of the fuzzy number in the interval form in conformity with the following definition.

Definition 2.2
If $A=(m_A,\alpha_A,\beta_A)_{LR}$, then the interval form of A can be written as $A_{\text{int}}=[a_A,m_A,b_A]$, where m_A is the mean value, a_A and b_A are the left and right borders respectively.
We determine $a_A=m_A-\alpha_A$ and $b_A=m_A+\beta_A$.

Example 2.11
Let $A=(6,3,2)_{LR}$. Then $A_{\text{int}}=[6-3,6,6+2]=[3,6,8]$.

Suppose that $A_{\text{int}}=[a_A,m_A,b_A]$ and $B_{\text{int}}=[a_B,m_B,b_B]$ are triangular fuzzy numbers in the interval form. We define the arithmetic operations on A_{int} and B_{int} by following the pattern below [Goetschel and Voxman, 1986; Buckley and Eslami, 2002].

Addition of A_{int} and B_{int}

$$[A+B]_{\text{int}}=[a_A+a_B,m_A+m_B,b_A+b_B]. \tag{2.11}$$

Example 2.12
Assume that $A=(3,2,1)_{LR} \equiv A_{\text{int}}=[3-2,3,3+1]=[1,3,4]$ and
$B=(4,1,2)_{LR} \equiv B_{\text{int}}=[4-1,4,4+2]=[3,4,6]$. The effect of the addition has been performed as
$[A+B]_{\text{int}}=[1+3,3+4,4+6]=[4,7,10] \equiv (7,3,3)_{LR}$.

Conclusion 2.1
If we calculate the addition $A+B$ in the *L-R* form, we will obtain
$A+B=(3+4,2+1,1+2)_{LR}=(7,3,3)_{LR}$, which confirms the full similarity of results when proving different approaches to addition of fuzzy numbers.

Subtraction of A_{int} and B_{int}

$$[A-B]_{\text{int}}=[a_A-b_B, m_A-m_B, b_A-a_B]. \tag{2.12}$$

Example 2.13
The setting of $A_{\text{int}}=[1,3,4] \equiv (3,2,1)_{LR}$ and $B_{\text{int}}=[3,6,8] \equiv (6,3,2)_{LR}$ in (2.12) yields

$[A-B]_{\text{int}}=[1-8,3-6,4-3]=[-7,-3,1] \equiv (-3,4,4)_{LR}$.

Conclusion 2.2
We execute the operation of subtraction for the same numbers $A_{\text{int}}=(3,2,1)_{LR}$,
$B_{\text{int}}=(6,3,2)_{LR}$ and $-B_{\text{int}}=(-6,2,3)_{LR}$ in the *L-R* representation to be furnished with a number

$A-B=A+(-B)=(3,2,1)_{LR}+(-6,2,3)_{LR}=(-3,4,4)_{LR}$.

This confirms the convergence of subtraction answers even if the computations procedures have been conducted according to different approaches. Further, we intend to compare the result reports for multiplication and division of two fuzzy numbers when performing the operations in the interval and *L-R* forms. Thus we ought to determine the rules of multiplying and dividing for two fuzzy numbers in the interval performance to be capable of making the mentioned confrontations.

Multiplication A_{int} and B_{int}
We suggest $A_{\text{int}}=[a_A, m_A, b_A]$ and $B_{\text{int}}=[a_B, m_B, b_B]$ as the data in the interval multiplication. Let $\varepsilon=\min(a_A a_B, a_A b_B, b_A a_B, b_A b_B)$ and $\phi=\max(a_A a_B, a_A b_B, b_A a_B, b_A b_B)$. We define [Buckley and Eslami, 2002]

$$[a_A, m_A, b_A]\cdot[a_B, m_B, b_B]=[\varepsilon, m_A\cdot m_B, \phi]. \tag{2.13}$$

Example 2.14
If $A=(3,1,2)_{LR} \equiv A_{\text{int}}=[2,3,5]$ and $B=(4,3,1)_{LR} \equiv B_{\text{int}}=[1,4,5]$, then, for

$\varepsilon=\min(2\cdot 1,2\cdot 5,5\cdot 1,5\cdot 5)=\min(2,10,5,25)=2$ and
$\phi=\max(2\cdot 1,2\cdot 5,5\cdot 1,5\cdot 5)=\max(2,10,5,25)=25$, we will perform the operation of multiplication to obtain a number

$[A\cdot B]_{\text{int}}=[2,3\cdot 4,25]=[2,12,25]\equiv(12,10,13)_{LR}$

Conclusion 3.3
We set $A=(3,2,1)_{LR}$ and $B=(4,3,1)_{LR}$, due to (2.5), in

$A\cdot B=(3,2,1)_{LR}\cdot(4,3,1)_{LR}=(12,13,11)_{LR}$

to discover the divergent answer of multiplication when comparing forms of exploiting the definitions created for the interval and the *L-R* fuzzy numbers.

Division of A_{int} and B_{int}
We introduce $[A:B]_{\text{int}}$ as $[A:B]_{\text{int}}=[a_A,m_A,b_A]\cdot[1/b_B,1/m_B,1/a_B]$.
If $\varepsilon=\min\left(a_A/b_B,a_A/a_B,b_A/b_B,b_A/a_B\right)$ and $\phi=\max\left(a_A/b_B,a_A/a_B,b_A/b_B,b_A/a_B\right)$, then

$$[A:B]_{\text{int}}=[a_A,m_A,b_A]\cdot\left[1/b_B,1/m_B,1/a_A\right]=\left[\varepsilon,m_A/m_B,\phi\right]. \tag{2.14}$$

Example 2.15
Suppose $A=(6,2,1)_{LR}$ and $B=(2,1,2)_{LR}$. We will calculate A:B in the interval and the *L-R* form by utilizing (2.14) and (2.8) respectively, to check if we can produce the same answer in both cases. We let $m_A>0$, $m_B>0$.

We take $A=(6,2,1)_{LR}\equiv A_{\text{int}}=[4,6,7]$ and $B=(2,1,2)_{LR}\equiv B_{\text{int}}=[1,2,4]$. We start with the calculation of $[A:B]_{\text{int}}=[4,6,7]\cdot[1/4,1/2,1/1]$. After evaluating of
$\varepsilon=\min\left(4/4,4/1,7/4,7/1\right)=1$ and $\phi=\max\left(4/4,4/1,7/4,7/1\right)=7$ we compute
$[A:B]_{\text{int}}=\left[\varepsilon,m_A/m_B,\phi\right]=\left[1,6/2,7\right]=[1,3,7]\equiv(3,2,4)_{LR}$.

We also calculate A:B in the *L-R* form as a number $(A:B)_{LR}$ for the same fuzzy numbers $A=(6,2,1)_{LR}$ and $B=(2,1,2)_{LR}$ to create the result
$(A:B)_{LR}=(6,2,1)_{LR}:(2,1,2)_{LR}=\left(6/2,(6\cdot 2+2\cdot 2)/2^2,(6\cdot 1+2\cdot 1)/2^2\right)_{LR}=(3,4,2)_{LR}$.

We get different division results when comparing the interval and the *L-R* forms.

Let us emphasize that even the conceptions of arithmetic operations performed on fuzzy numbers in the interval forms provide us with supports of results only. To attach the membership functions to these supports we should combine the point (*a,* 0) corresponding to the left border with the peak (*m,* 1) to design the left branch of the membership function as well as to tie (*m,* 1) to (*b,* 0) to produce the right part of the function. The shapes are styled due to the users´ expectations.

2.3 Arithmetic operations in the set of fuzzy numbers converted to $\alpha-cut$ forms

If we wish to find a membership curve fitted for results of arithmetical operations without making some special selection arrangements, then we should apply the operation formulas already discussed for interval forms of data, provided that the data is interpreted by means of $\alpha-cuts$. The concept of the $\alpha-cut$ set was introduced by Def. 1.9.

Each $\alpha-cut$ of *A* constitutes an interval stretched along the *x-axis*. The $\alpha-cut$ form of a fuzzy number *A*, based on the determination of A_α and denoted by $A(\alpha)$, is a non fuzzy set, dependent on α, defined as an interval $A[\alpha]=[a_1(\alpha),a_2(\alpha)]$, $0<\alpha<1$, where $a_1(\alpha),a_2(\alpha)$ are the left and the right reference functions [Buckley and Eslami, 2007; Luke, 2006].

Example 2.16

Let $A_{\text{int}}=[4,8,10]$. We will expand the general formula of $\alpha-cuts$ of *A*, $A[\alpha]=[a_1(\alpha),a_2(\alpha)]$ for $0<\alpha<1$. For $A_{\text{int}}=[4,8,10]$ we extract $a_A=4, m_A=8, b_A=10$.

Let us suppose that

$$\begin{aligned} a_1(\alpha)&=k_1\alpha+c_1,\\ a_2(\alpha)&=k_2\alpha+c_2, \end{aligned} \tag{2.15}$$

where k_1,k_2,c_1 and c_2 are constants.

Equations (2.15) must satisfy the following conditions.

For $\alpha=0\Rightarrow\begin{cases}a_1(\alpha)&=&a_A,\\a_2(\alpha)&=&b_A,\end{cases}$ and for $\alpha=1\Rightarrow\begin{cases}a_1(\alpha)&=&m_A,\\a_2(\alpha)&=&m_A.\end{cases}$

Value $\alpha=0\Rightarrow\begin{cases}a_1(\alpha)&=&a_A,\\a_2(\alpha)&=&b_A,\end{cases}$ leading to $\begin{cases}k_1\cdot0+c_1&=&4,\\k_2\cdot0+c_2&=&10,\end{cases}$ provides us with $c_1=4$, $c_2=10$.

If $\alpha=1$ we will expect that the system $\begin{cases}a_1(\alpha)&=&m_A\\a_2(\alpha)&=&m_A\end{cases}$ equivalent to $\begin{cases}k_1\cdot1+c_1&=&8\\k_2\cdot1+c_2&=&8\end{cases}$ reveals, after inserting $c_1=4$ and $c_2=10$, the rest of coefficients $k_1=4$ and $k_2=-2$.

Due to (2.15) we obtain $a_1(\alpha)=4+4\alpha$ and $a_2(\alpha)=10-2\alpha$.

Hence, $A[\alpha]=[a_1(\alpha),a_2(\alpha)]=[4+4\alpha, 10-2\alpha]$, $0<\alpha<1$.

The $\alpha-cuts$ of fuzzy numbers are always closed and bounded intervals for every $\alpha\in[0,1]$. Let us emerge two fuzzy numbers *A* and *B* in the $\alpha-cut$ forms as $A[\alpha]=[a_1(\alpha),a_2(\alpha)]$ and $B[\alpha]=[b_1(\alpha),b_2(\alpha)]$, $0<\alpha<1$. On the basis of the arithmetic principles valid for interval forms of fuzzy numbers we thus generate the operations on fuzzy numbers in $\alpha-cut$ forms in the following way:

Addition of *A*[*α*] and *B*[*α*]

$$A[\alpha]+B[\alpha]=[a_1(\alpha)+b_1(\alpha),\, a_2(\alpha)+b_2(\alpha)],\, 0<\alpha<1. \tag{2.16}$$

Example 2.17

Let $A_{\text{int}}=[1,3,4]$ and $B_{\text{int}}=[2,5,7]$ be two triangular fuzzy numbers. We wish to develop $A[\alpha]+B[\alpha],\ 0<\alpha<1$.

After converting the interval forms to the $\alpha-cut$ forms we will get $A_{\text{int}}=[1,3,4]\Rightarrow A[\alpha]=[1+2\alpha,\, 4-\alpha]$ and $B_{\text{int}}=[2,5,7]\Rightarrow B[\alpha]=[2+3\alpha,\, 7-2\alpha]$ to accomplish the addition as
$A[\alpha]+B[\alpha]=[a_1(\alpha)+b_1(\alpha),\, a_2(\alpha)+b_2(\alpha)]$ = $[(1+2\alpha)+(2+3\alpha),(4-\alpha)+(7-2\alpha)]$ = $[3+5\alpha,\, 11-3\alpha]$.

If $\alpha=0$, we obtain $S(A+B)=[3,11]$.
If $\alpha=1$, we obtain $m_{A+B}=8$.

Let us now demonstrate a method of employing the $\alpha-cut$ form of the addition result to generate its membership function consisting of two unknown branches.

Example 2.18

Since α constitutes the $\mu_{A+B}(x)$ value, $\alpha\in[0,1]$, then we set $x=3+5\alpha$ for $3\le x\le 8$ and $x=11-3\alpha$ for $8\le x\le 11$. We solve the equation with respect to α to obtain $\alpha=\dfrac{x-3}{5}$ and $\alpha=\dfrac{11-x}{3}$ i.e.,

$$\mu_{A+B}(x)=\begin{cases}\dfrac{x-3}{5} & \text{for} \quad 3\le x\le 8,\\[2ex] \dfrac{11-x}{5} & \text{for} \quad 8\le x\le 11.\end{cases}$$

Subtraction of *A*[*α*] and *B*[*α*]

$$(A-B)[\alpha]=[a_1(\alpha)-b_2(\alpha),\, a_2(\alpha)-b_1(\alpha)],\, 0<\alpha<1. \tag{2.17}$$

Example 2.19

Let $A_{\text{int}}=[1,3,4]$ and $B_{\text{int}}=[3,6,8]$. We concatenate them to get $(A-B)[\alpha],\ 0<\alpha<1$.
After interpreting $A_{\text{int}}=[1,3,4]$ as $A[\alpha]=[1+2\alpha,\, 4-\alpha]$ and $B_{\text{int}}=[3,6,8]$ as $B[\alpha]=[3+3\alpha,\, 8-2\alpha]$, we are provided with the result

$(A-B)[\alpha]=[(1+2\alpha)-(8-2\alpha),(4-\alpha)-(3+3\alpha)]=[-7+4\alpha,\, 1-4\alpha]$,
which possesses $S(A-B)=[-7,1]$ $(\alpha=0)$ and $m_{A-B}=-3$, $(\alpha=1)$.

By extracting α from the equations $x=-7+4\alpha$ and $x=1-4\alpha$ we expand the membership function of $(A-B)[\alpha]$ in the form

$$\mu_{A-B}(x)=\begin{cases}\dfrac{x+7}{4} & \text{for} \quad -7\le x\le -3,\\[2ex] \dfrac{1-x}{4} & \text{for} \quad -3\le x\le 1.\end{cases}$$

Multiplication of *A*[*α*] and *B*[*α*]

$$(A\cdot B)[\alpha]=[a_1(\alpha)\cdot b_1(\alpha),\, a_2(\alpha)\cdot b_2(\alpha)],\, 0<\alpha<1. \tag{2.18}$$

Example 2.20

For $A_{\text{int}}=[1,2,4]$ and $B_{\text{int}}=[3,5,7]$ we will find the $(A\cdot B)[\alpha]$, $0<\alpha<1$.

We first convert the proposed fuzzy numbers in the interval form to the $\alpha-cut$ forms as follows:

$A_{\text{int}}=[1,2,4]\Rightarrow A[\alpha]=[1+\alpha,\, 4-2\alpha]$ and $B_{\text{int}}=[3,5,7]\Rightarrow B[\alpha]=[3+2\alpha,\, 7-2\alpha]$ to estimate $(A\cdot B)[\alpha]=[(1+\alpha)\cdot(3+2\alpha), (4-2\alpha)\cdot(7-2\alpha)]=[2\alpha^2+5\alpha+3,\, 4\alpha^2-22\alpha+28]$ as the number with $S(A\cdot B)=[3,\, 28]$ and $m_{A\cdot B}=10$.

To find the membership functions over the intervals $[3,10]$ and $[10,28]$, let $2\alpha^2+5\alpha+3=x$ and $4\alpha^2-22\alpha+28=x$, respectively. After solving $2\alpha^2+5\alpha+3=x$ with respect to α we obtain two roots $\alpha_1=\mu_1(x)=\dfrac{-5+\sqrt{1+8x}}{4}$ and $\alpha_2=\mu_2(x)=\dfrac{-5-\sqrt{1+8x}}{4}$. To make a decision about which part should be adopted as a left branch of the $A\cdot B$`s membership function we suppose that the function value in the left border $x=3$ is equal to zero. Because of $\mu_1(3)=0$ contra $\mu_2(3)\neq 0$ we select μ_1 as the left part of the membership function of the product. We can even confirm the selection of μ_1 by analyzing another property of $\mu_1(10)=1$ typical of the mean of a fuzzy number versus $\mu_2(10)\neq 1$.

We repeat the same procedure for the equation $4\alpha^2-22\alpha+28=x$ to be furnished with the appropriate membership function $\mu(x)=\dfrac{22+\sqrt{36+16x}}{8}$ playing role of the right membership function of $A\cdot B$ over $x\in[10,28]$.

Finally, we submit the membership function of $A\cdot B$ as a split definition

$$\mu_{A\cdot B}(x)=\begin{cases}\dfrac{-5+\sqrt{1+8x}}{4} & \text{for} \quad 3\le x\le 10,\\[2ex] \dfrac{22-\sqrt{36+16x}}{8} & \text{for} \quad 10\le x\le 28,\end{cases}$$

with a graph drawn in Fig. 2.3.

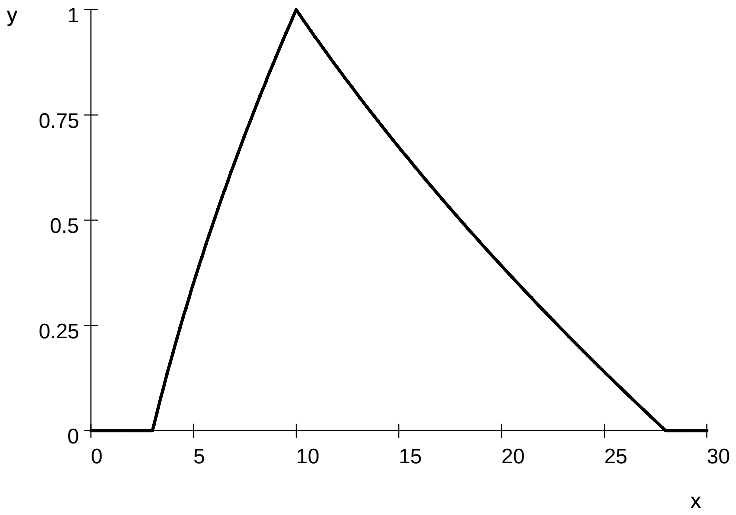

Figure 2.3: The multiplication of two fuzzy numbers in the α-cut forms

Division of $A[\alpha]$ and $B[\alpha]$

$$(A:B)[\alpha]=[a_1(\alpha),a_2(\alpha)]\cdot\left[1/b_2(\alpha),1/b_1(\alpha)\right]=\left[a_1(\alpha)/b_2(\alpha),a_2(\alpha)/b_1(\alpha)\right],\ 0<\alpha<1. \quad (2.19)$$

Example 2.21

Let $A[\alpha]=[\alpha,2-\alpha]$, $B[\alpha]=[1+\alpha,3-\alpha]$, $0\le\alpha\le1$. Hence

$(A:B)[\alpha]=[\alpha,2-\alpha]\cdot\left[1/(3-\alpha),1/(1+\alpha)\right]=\left[\alpha/(3-\alpha),(2-\alpha)/(1+\alpha)\right]$ with $S(A:B)=[0,2]$ and $m_{A:B}=\frac{1}{2}$. The left membership function, that constitutes the solution of $\frac{\alpha}{3-\alpha}=x$ with respect to α, is found as $\mu_{left}(x)=\frac{3x}{1+x}$, $0\le x\le\frac{1}{2}$. For $\frac{2-\alpha}{1+\alpha}=x$ we seek $\mu_{right}(x)=\frac{2-x}{1+x}$, $\frac{1}{2}\le x\le2$. Both functions are depicted in Fig. 2.4.

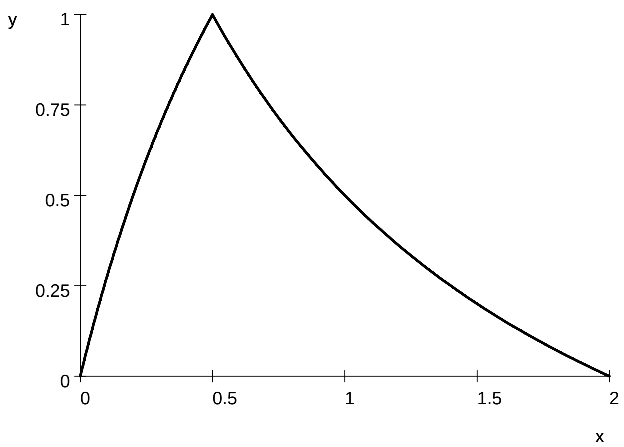

Figure 2.4: The division of A and B in α-cut forms

To demonstrate the utilization of the lastly introduced operations, let us apply them in the prediction of the survival length, expected as a fuzzy number *Y* and affected by the biological marker *CRP*-value *X* (*C*-active proteins). The *CRP* is also estimated as a fuzzy number and considered by a physician as the most crucial biological recognizer of the presence of gastric cancer. We thus seek an equation of the fuzzy regression line, which binds *X* and *Y*.

Example 2.22

Suppose that the physician has reported some data concerning the heightened *CRP*-value presence in a group of patients who died after suffering from gastric cancer. As the values of *CRP* for almost the same post-operative survivals do not differ very much from each other then we will decide to use fuzzy numbers in their mathematical representation. We sample the close *CRP*-values assisting the close survival periods in the connected groups to treat the medians of the respective groups as mean values of fuzzy numbers being their representatives. The groups' minimal values play roles of left borders of the numbers while the groups' maximal quantities will constitute the right borders in fuzzy numbers.

Suppose that we have stated "*CRP* close to 16" = $X_1 = [13,16,18]_{\text{int}} = [13+3\alpha,18-2\alpha]$ corresponding to "survival length close to 2 years" = $Y_1 = [1,2,3]_{\text{int}} = [1+\alpha,3-\alpha]$ and "*CRP* very close to 30" = $X_2 = [28,30,32]_{\text{int}} = [28+2\alpha,32-2\alpha]$ characteristic of "survival length close to 1 year" = $Y_2 = [0,1,2]_{\text{int}} = [\alpha,2-\alpha]$.

We want to find the equation $Y = AX + B$ which is particularly satisfied for the pairs (X_1,Y_1) and (X_2,Y_2).

By solving the system of equations

$$\begin{cases} AX_1 + B = Y_1 \\ AX_2 + B = Y_2 \end{cases}$$

with respect to A and B we evaluate $A = \dfrac{Y_2 - Y_1}{X_2 - X_1}$ and $B = Y_2 - \dfrac{Y_2 - Y_1}{X_2 - X_1} X_2$.

Thus $A = \dfrac{[\alpha,2-\alpha]-[1+\alpha,3-\alpha]}{[28+2\alpha,32-2\alpha]-[13+3\alpha,18-2\alpha]} = \left[\dfrac{-3+2\alpha}{19-5\alpha},\dfrac{1-2\alpha}{10+4\alpha}\right]$ and

$$B = [\alpha,2-\alpha]-\left[\frac{-3+2\alpha}{19-5\alpha},\frac{1-2\alpha}{10+4\alpha}\right][28+2\alpha,32-2\alpha] = \left[\frac{-32+76\alpha}{10+4\alpha},\frac{122-79\alpha+\alpha^2}{19-5\alpha}\right].$$

The regression line is built as

$$[Y_1[\alpha],Y_2[\alpha]] = \left[\frac{-3+2\alpha}{19-5\alpha},\frac{1-2\alpha}{10+4\alpha}\right][X_1[\alpha],X_2[\alpha]] + \left[\frac{-32+76\alpha}{10+4\alpha},\frac{122-79\alpha+\alpha^2}{19-5\alpha}\right]$$

For, e.g., $X = [8.9.10]_{\text{int}} = [8+\alpha,10-\alpha]$ we estimate the survival length as fuzzy number "close to 5 years" in spite of its borders and the mean, which are computed as fractions.

Even if the fuzzy regression constitutes a useful tool of inference we intend to test other methods to find more accurate evaluations of survival length. These can be affected by more independent variables than only one. In the next chapters we will prove the action of fuzzy controllers.

To sum up the current chapter we can mention that all operations on fuzzy numbers, independently of their forms, are grounded on the extension principle [Zimmermann, 2002; Dubois and Prade, 1978a, 1978b] that substantially affects obtained results.

The utilization of the $\alpha - cut$ forms of results facilitates a procedure of generating their membership functions fitted best for the results´ occurrences, which should be regarded as an advantage of the method promoting the $\alpha - cut$ forms involved in calculations.

3 The Mamdani Controller in Prediction of the Survival Length in Elderly Gastric Patients

3.1 The Introduction of a Control System

In the current chapter we will study the control actions to improve the estimates of the survival length. We wish to still treat the survival length as a dependent variable, which is affected by some biological parameters. Strict analytic formulas are the tools usually derived for determining the formal relationships between a sample of independent variables and a variable which they affect.

If we cannot formalize the function tying the independent and dependent variables then we will utilize some control actions. Apart from crisp version of control we often adopt its fuzzy variant developed by Mamdani and Assilian [Mamdani and Assilian, 1973]. Fuzzy control algorithm is furnished with softer mechanisms when comparing it to classical control, which constitutes its advantage in the processing of vague or imprecise information.

The algorithm is particularly adaptable to support medical systems, often handling uncertain premises and conclusions. From the medical point of view it would be desirable to prognosticate the survival length for patients suffering from gastric cancer. We thus formulate the objective of the chapter as the utilization of fuzzy control action for the purpose of making the survival prognoses [Zettervall, Rakus-Andersson and Forssell, 2011].

Fuzzy set theory allows us to describe complex systems by using our knowledge and experience in transparent English-like rules. It does not need complex mathematical equations and system modeling that governs the relation between inputs and outputs. Fuzzy rules are very appealing to use by non-experts also.

Expert-knowledge designs together with assumptions of fuzzy set theory have given rise to the creation of fuzzy control, see e.g., [Mamdani and Assilian, 1973; Sutton and Towill, 1985; Nguyen et al., 2002; Andrei, 2005; Al-Odienat and Al-Lawama, 2008]. Experience-based rules constitute the crucial part of fuzzy controllers, which have found many adherents to apply them in order to support solutions of complex systems not characterized by formally stated structures. Typical applications of fuzzy control have mostly concerned technical processes. The applications range from cameras to model cars and trains [Zimmermann, 2001].

Due to the possibility of making input and output variables verbally expressed, which constitutes an advantage in the process of preparing the imprecise data strings, fuzzy control has also been tested in medicine. The reduction of post-operative pain [Hernández et al., 1992] or the blood pressure regulation [Chin-Te Chen et al., 1996; Isaka and Sebald, 1989] constitute some examples of medical experiments accomplished by fuzzy controllers. We can learn about how adaptive mechanisms coupled with fuzzy controllers regulate the mean arterial pressure and how the fuzzy control solutions deprive a patient of the postoperative pain.

The evaluation of survival length was already accomplished by statistical methods.

In the first trials of survival approximation a survival curve from censored data was introduced [Kaplan and Meier, 1958]. The model was used in cancer patient examinations to estimate the length of living [Newland et al., 1994]. The Cox regression [Cox, 1972] of life length prediction was developed in such studies as logistic Cox regression [Sargent, 2001]. The statistics-based models predicting the survival were compared by Everitt and Rabe-

Hesketh [Everitt and Rabe-Hesketh, 2001] who found such model disadvantages as the lack of normal distribution or missing values among survival times.

A typical fuzzy control system normally consists of three parts. The first part refers to the fuzzification process of input and output variables. These are first linguistically differentiated in levels. The levels are listed as names, which are demonstrated by fuzzy numbers with assigned to them appropriate membership functions.

The second part consists of processing procedures. We rely on own experience when we prepare rules to link the linguistic terms of input variables to the control output state. The rules employed in the model are constructed as IF-THEN statements. On the basis of the rules verbally formulated and actual for individual data values we estimate their mathematical consequences expressed as a collection of fuzzy sets. The creation of a sampling of all consequence sets results in one final consequence fuzzy set and terminates the procedures of the second step of fuzzy control.

The last step of the fuzzy control system is to defuzzify the final fuzzy output set being the result of the second stage. We adopt the centre of gravity (COA) method to convert the fuzzy set into a crisp value corresponding to the initial crisp input data.

We thus intend to prove fuzzy control to make some prognoses concerning the survival length in patients whose disease has been diagnosed as gastric cancer. To make the conclusions reliable we select two clinical markers "*age*" and "*CRP-value*" due to the physicians' expertise. The choice of *CRP* and age, as representative markers of post-surgical survival in cancer diseases, has been made due to the latest investigations revealing associations of these indices with the progression of disease in many cancer types [Do Kyong-Kim et al., 2009; de Mello et al., 1983].

3.2 Fuzzification of Input and Output Variable Entries in Survival Length Estimation

Fuzzy control model is applied in research to some relationships between a collection of independent variables and the dependent of them variable when we cannot mathematically formalize the functional connection among them. We are expected to evaluate the survival length in patients with diagnosis "*gastric cancer*". The period of survival is affected by two biological parameters X = "*age*" and Y = "*CRP-value*", which are selected as the most essential markers of making the prognosis. We cannot formally derive a function, which relates the independent variables X = "*age*" and Y = "*CRP-value*" to the dependent variable Z = "*survival length*"; therefore we will adapt such fuzzy controller, which supports estimation of dependent values in spite of the lack of a formula concerning $z = f(x, y)$, $x \in X$, $y \in Y$, $z \in Z$.

All variables are now differentiated into levels, which are expressed by lists of terms. The terms from the lists are represented by fuzzy numbers, restricted by the parametric *s*-functions lying over the variable domains $[x_{\min}, x_{\max}]$, $[y_{\min}, y_{\max}]$ and $[z_{\min}, z_{\max}]$ respectively.

In conformity with the physician's suggestions we introduce five levels of X , Y and Z as the collections

X = "*age*" = { X_0 = "*very young*", X_1 = "*young*", X_2 = "*middle-aged*", X_3 = "*old*", X_4 = "*very old"*}

Y = "*CRP-value*" = { Y_0 = "*very low"*, Y_1 = "*low*", Y_2 = "*medium*", Y_3 = "*high*", Y_4 = "*very high*"}

and

Z = "*survival length*" = { Z_0 = "*very short*", Z_1 = "*short*", Z_2 = "*middle-long*", Z_3 = "*long*", Z_4 = "*very long*"}.

To obtain a family of membership functions of fuzzy numbers standing for the terms of the respective lists we will modify the parametric s-class functions [Zimmermann, 2001; Rakus-Andersson, 2007]. For X_i, i = 0,...,4, we design [Rakus-Andersson and Jain, 2009a; Rakus-Andersson, 2009b; Rakus-Andersson, Salomonsson and Zettervall, 2008; Rakus-Andersson, Salomonsson and Zettervall, 2008; Rakus-Andersson, Zettervall and Erman, 2010]

$$\mu_{X_i}(x) = \begin{cases} left\mu_{X_i}(x), \\ right\mu_{X_i}(x), \end{cases} \tag{3.1}$$

where

$$left\mu_{X_i}(x) = \begin{cases} 2\left(\frac{x-((x_{\min}-h_X)+h_X\cdot i)}{h_X}\right)^2 & \text{for} \quad (x_{\min}-h_X)+h_X\cdot i \le x \le \left(x_{\min}-\frac{h_X}{2}\right)+h_X\cdot i, \\ 1-2\left(\frac{x-(x_{\min}+h_X\cdot i)}{h_X}\right)^2 & \text{for} \quad \left(x_{\min}-\frac{h_X}{2}\right)+h_X\cdot i \le x \le (x_{\min})+h_X\cdot i, \end{cases} \tag{3.2}$$

and

$$right\mu_{X_i}(x) = \begin{cases} 1-2\left(\frac{x-(x_{\min}+h_X\cdot i)}{h_X}\right)^2 & \text{for} \quad (x_{\min})+h_X\cdot i \le x \le \left(x_{\min}+\frac{h_X}{2}\right)+h_X\cdot i, \\ 2\left(\frac{x-((x_{\min}+h_X)+h_X\cdot i)}{h_X}\right)^2 & \text{for} \quad \left(x_{\min}+\frac{h_X}{2}\right)+h_X\cdot i \le x \le (x_{\min}+h_X)+h_X\cdot i. \end{cases} \tag{3.3}$$

Formulas (3.2) and (3.3) depend on the minimal value $x_{\min}$, which starts the X-variable domain. The structures (3.2) and (3.3) are also affected by the value of a parameter h_X, which estimates the length between the beginnings of membership functions constructed for two adjacent terms of X. The h_X quantity is adjusted to the number of functions in the X-list and to the distance between the minimal and the maximal value of the X-variable domain.

The membership functions of Y_j, j = 0,…,4, constructed for the accommodated values of parameters h_Y and $Y_{\min}$ to the conditions of Y, are yielded by

$$\mu_{Y_j}(y) = \begin{cases} left\mu_{Y_j}(y), \\ right\mu_{Y_j}(y), \end{cases} \tag{3.4}$$

for

$$left\mu_{Y_j}(y) = \begin{cases} 2\left(\frac{y-((y_{\min}-h_Y)+h_Y\cdot j)}{h_Y}\right)^2 & \text{for} \quad (y_{\min}-h_Y)+h_Y\cdot j \le y \le \left(y_{\min}-\frac{h_y}{2}\right)+h_y\cdot j, \\ 1-2\left(\frac{y-(y_{\min}+h_Y\cdot j)}{h_Y}\right)^2 & \text{for} \quad \left(y_{\min}-\frac{h_Y}{2}\right)+h_Y\cdot j \le y \le (y_{\min})+h_Y\cdot j, \end{cases} \tag{3.5}$$

and

$$right\mu_{Y_j}(y) = \begin{cases} 1-2\left(\frac{y-(y_{\min}+h_Y\cdot j)}{h_Y}\right)^2 & \text{for} \quad (y_{\min})+h_Y\cdot j \le y \le \left(y_{\min}+\frac{h_Y}{2}\right)+h_Y\cdot j, \\ 2\left(\frac{y-((y_{\min}+h_Y)+h_Y\cdot j)}{h_Y}\right)^2 & \text{for} \quad \left(y_{\min}+\frac{h_Y}{2}\right)+h_Y\cdot j \le y \le (y_{\min}+h_Y)+h_Y\cdot j. \end{cases} \tag{3.6}$$

Finally, the Z_k's functions, $k = 0,\ldots,4$, are derived as

$$\mu_{Z_k}(z) = \begin{cases} left\mu_{Z_k}(z) \\ middle\mu_{Z_k}(z) \\ right\mu_{Z_k}(z) \end{cases} \tag{3.7}$$

with

$$left\mu_{Z_k}(z) = \begin{cases} 2\left(\frac{z-\left(z_{\min}-\frac{h_Z}{2}+h_Z\cdot k\right)}{\frac{h_Z}{2}}\right)^2 & \text{for} \quad z_{\min}-\frac{h_Z}{2}+h_Z\cdot k \le z \le z_{\min}-\frac{h_Z}{4}+h_Z\cdot k, \\ 1-2\left(\frac{z-\left(z_{\min}+h_Z\cdot k\right)}{\frac{h_Z}{2}}\right)^2 & \text{for} \quad z_{\min}-\frac{h_Z}{4}+h_Z\cdot k \le z \le z_{\min}+h_Z\cdot k, \end{cases} \tag{3.8}$$

the central part

$$middle\mu_{Z_k}(z) = 1 \quad \text{for} \quad z_{\min}+h_Z\cdot k \le z \le z_{\min}+\frac{h_Z}{2}+h_Z\cdot k, \tag{3.9}$$

and

$$right\mu_{Z_k}(z) = \begin{cases} 1-2\left(\frac{z-\left(z_{\min}+\frac{h_Z}{2}+h_Z\cdot k\right)}{\frac{h_Z}{2}}\right)^2 & \text{for} \quad z_{\min}+\frac{h_Z}{2}+h_Z\cdot k \le z \le z_{\min}+\frac{3h_Z}{4}+h_Z\cdot k, \\ 2\left(\frac{z-\left(z_{\min}+h_Z+h_Z\cdot k\right)}{\frac{h_Z}{2}}\right)^2 & \text{for} \quad z_{\min}+\frac{3h_Z}{4}+h_Z\cdot k \le z \le z_{\min}+h_Z+h_Z\cdot k. \end{cases} \tag{3.10}$$

The parameter h_Z allows designing five functions of fuzzy numbers from Z over [$z_{\min}$, $z_{\max}$].

We return to the variable X = "*age*", which is differentiated in five levels X = "*age*" = {X_0 = "*very young*", X_1 = "*young*", X_2 = "*middle-aged*", X_3 = "*old*", X_4 = "*very old"*} and restricted over the interval $[x_{\min}, x_{\max}]$ = [0, 100], to state $x_{\min} = 0$, $h_X = 25$ and $i = 0,...,4$. For the terms of "*age*" from the list above we will obtain by (3.2) and (3.3) a family of the membership functions sketched in Fig. 3.1. The h_X value is specified to be equal to 25 since we wish to make a design in which X_0 has its peak in (0, 1) and the peak of X_4 should be moved to (100, 1). On the other hand five symmetric functions of fuzzy numbers have to find their placements over [0, 100] which, together with previously made assumptions, initiates $h_X = 25$.

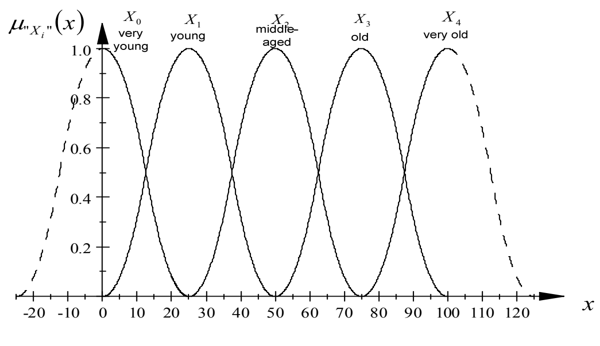

Figure 3.1: The membership functions for the "*age*"

By inserting new parameters of $y_{\min}$ and h_Y in (3.5) and (3.6) we generate the membership functions for "*CRP-value*". We have stated Y as Y = "*CRP-value*" = { Y_0 = "*very low*", Y_1 = "*low*", Y_2 = "*medium*", Y_3 = "*high*", Y_4 = "*very high*"} over the interval [0, 50]. The design of installing five functions with the peak of Y_0 in (0, 1) and the peak of Y_4 in (50, 1) demands the selection of $h_Y = 15$. We plot the Y_j functions in Fig. 3.2 by setting in turn the different values of j in (3.5) and (3.6), where j = 0,...,4.

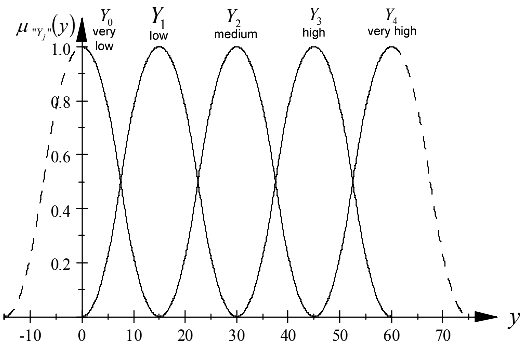

Figure 3.2: The membership functions for the "*CRP-value*"

The output variable Z = "*survival length*" = { Z_0 = "*very short*", Z_1 = "*short*", Z_2 = "*middle-long*", Z_3 = "*long*", Z_4 = "*very long*"} takes the values in the interval [0, 5]. We determine $h_Z = 1$ and set $k = 0,...,4$ in (3.8), (3.9) and (3.10) to initialize the functions depicted in Fig. 3.3.

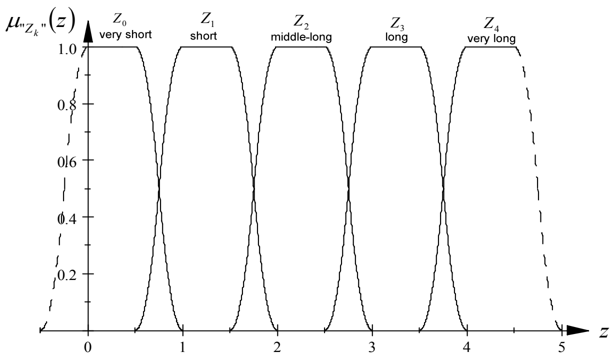

Figure 3.3: The membership functions for the "*survival length*"

We emphasize the importance of the parametric design of functions. Instead of implementing fifteen formulas of the similar nature we have sampled all functions in three generic groups. Any time we can involve the desired function in necessary computations by setting its number in the proper formula concerning *X*, *Y* or *Z*. Moreover, the mathematical scenario of membership functions is established in the formal and elegant designs, which can be segments of a computer program for the reason of their nature letting the creation of loops.

3.3 The Rule Based Processing Part of Surviving Length Model

After the fuzzification procedure we are able to create the rule bases, which link the states of the two input variables to the state of the output variable. We thus design a table in which the entries are filled with terms of "*survival length*". To express the states of the survival length as logically as possible, we have studied the behavior of variables on the basis of biological data samplings. The cells of the table are characterized by subintervals of domains of X and Y.

We first estimate the survival length median in the samplings of the data corresponding to considered cells. The median value was set as z in the membership functions of all sets listed in the Z-space. We select this fuzzy number Z_k as a representative of the cell, in which the membership degree of the median was largest. The technique of combining the human experience with data sets obtained for discrete samples to make conclusions for continuous samples is a modern branch of so-called "integration systems".

The estimations of survival length are collected in Table 3.1.

Table 3.1. Rule base of fuzzy controller estimating "*survival length*"

age X_i \ CRP-value Y_j	very low	low	medium	high	very high
very young					
young					
middle-age	middle-long				
old	middle-long	short	short	short	very short
very old	short	short	very short	very short	very short

Some entries in the table are empty, since the essential data was lacking for younger people. It rarely happens to find young patients with diagnosis "*gastric cancer*"; therefore we could not make any reliable conclusions concerning survival in this age group.

Suppose that we would like to make the survival prognosis for (x, y), $x \in X$, $y \in Y$ – with other words we want to evaluate $z = f(x, y)$ when assuming that the f-formula is not developed.

Furthermore, x belongs to the different fuzzy numbers X_i, i = 0,...,4, being the fuzzy subsets of X, with different membership degrees equalling $\mu_{X_i}(x)$. Element y associated to x is a member of the some fuzzy numbers Y_j, j = 0,...,4, constituting the fuzzy subsets of Y with the membership degrees $\mu_{Y_j}(y)$. By means of the IF-THEN statements grounded on the basis of Table 1, we can determine the contents of rules by attaching the pair of input variable levels to a level of the output variable according to:

$$\text{Rule } R_{(x,y):l} : \text{ If } x \text{ is } X_{i:l} \text{ and } y \text{ is } Y_{j:l}, \text{ then } z \text{ is } Z_{k:l}, \qquad (3.11)$$

where l is the rule number. The expressions $X_{i:l}$, $Y_{j:l}$ and $Z_{k:l}$ denote the fuzzy numbers X_i, Y_j and Z_k assisting rule number l, which has been found for actual x and y.

To evaluate the influence of the input variables on the output consequences we need an estimate $\alpha_{(x,y):l}$ found by performing the minimum operation

$$\alpha_{(x,y):l} = \min\left(\mu_{X_i:l}(x), \mu_{Y_j:l}(y)\right) \tag{3.12}$$

for each $X_{i:l}$ and $Y_{j:l}$ concerning the choice of (x,y).

We use $\alpha_{(x,y):l}$ and the minimum operator to determine consequences of all rules $R_{(x,\,y):l}$ for the output. Fuzzy sets $R_{(x,y):l}^{conseq}$, stated in the output space Z, will have the membership functions

$$\mu_{R_{(x,y):l}^{conseq}}(z) = \min\left(\alpha_{(x,y):l}, \mu_{Z_k:l}(z)\right). \tag{3.13}$$

In the last step of the algorithm we aggregate the consequence sets $R_{(x,y):l}^{conseq}$ in one common set $conseq_{(x,y)}$ allocated in Z over a continuous interval $[z_0, z_n]$. To derive the membership function of $conseq_{(x,y)}$ we prove the action of the maximum operator in the form of

$$\mu_{conseq_{(x,y)}}(z) = \max_l\left(\mu_{R_{(x,y):l}^{conseq}}(z)\right) \tag{3.14}$$

3.4 Defuzzification of the Output Variable

In order to assign a crisp value Z to the selected pair (x, y) we defuzzify the consequence fuzzy set (3.14) in Z. We will thus indicate the expected value of the survival length for a gastric cancer patient whose age x and *CRP*-value y have been examined.

One can find different kinds of defuzzification methods in literature (Zimmermann, 2001). We are appealed by properties of the centre of gravity method (COG) as a model of computing which is easy to perform and clearly interpretable. We expand COG as

$$z = f(x, y) = \frac{\int_{z_0}^{z_n} z \cdot \mu^{conseq}(z)dz}{\int_{z_0}^{z_n} \mu^{conseq}(z)dz} = \frac{\int_{z_0}^{z_1} z \cdot \mu^{conseq}(z)dz + \cdots + \int_{z_{n-1}}^{z_n} z \cdot \mu^{conseq}(z)dz}{\int_{z_0}^{z_1} \mu^{conseq}(z)dz + \cdots + \int_{z_{n-1}}^{z_n} \mu^{conseq}(z)dz} \tag{3.15}$$

with the inner borders $z_1,...,z_{n-1}$ being either z-coordinates of intersection points between adjacent branches of the $conseq_{(x,y)}$ membership function or characteristic support values of fuzzy numbers included in $conseq_{(x,y)}$.

3.5 The Survival Length Prognosis for a Selected Patient

Suppose that we examine a 77-year-old patient, whose *CRP*-value is 16. His diagnosis is determined by a physician as "*gastric cancer*". We wish to estimate theoretically the expected value of his survival length by proving the algorithm sketched in previous sections. The information is confidential and used only by the physician.

Let $x = 77$ and $y = 16$. Age 77 belongs to the fuzzy number X_3 = "*old*". Therefore, for i = 3, h_X = 0.25 and $x_{\min}$ = 0, we allocate x = 77 in the interval
$(x_{\min}) + h_X \cdot i \le x \le \left(x_{\min} + \frac{h_X}{2}\right) + h_X \cdot i \leftrightarrow 0 + 25 \cdot 3 \le x \le \left(0 + \frac{25}{2}\right) + 25 \cdot 3 \leftrightarrow 75 \le x \le 87.5$

with the membership degree

$$\mu_{X_3}(77) = \mu_{\text{"}old\text{"}}(77) = 1 - 2\left(\tfrac{77-(x_{\min}+h_X\cdot i)}{h_X}\right)^2 = 1 - 2\left(\tfrac{77-(0+25\cdot 3)}{0.25}\right)^2 = 0.9872\,.$$

The same x = 77 is a member of another fuzzy number X_4 = "*very old*" with the membership degree of $\mu_{X_4}(77) = \mu_{\text{"}very\ old\text{"}}(77) = 0.0128$. The *CRP*-value 16 belongs to the fuzzy number Y_1 = "*low*" with the membership degree $\mu_{Y_1}(16) = \mu_{\text{"}low\text{"}}(16) = 0.991$ and Y_2 = "*medium*" with the membership degree $\mu_{Y_2}(16) = \mu_{\text{"}medium\text{"}}(16) = 0.009$.

In accordance with (3.11) the rules which connect the states of the input variables to the output variable levels are established as:

$R_{(77,16):1}$: IF "*age*" is "*old*" and the "*CRP-value*" is "*low*", THEN "*survival length*" will be "*short*".
$R_{(77,16):2}$: IF "*age*" is "*old*" and "*CRP-value*" is "*medium*", THEN the survival length will be "*short*".
$R_{(77,16):3}$: IF "*age*" is "*very old*" and "*CRP-value*" is "*low*", THEN the survival length will be "*short*".
$R_{(77,16):4}$: IF "*age*" is "*very old*" and "*CRP-value*" is "*medium*", THEN the survival length will be "*very short*".

To evaluate the influences of the input variables on the output consequences due to (3.12), we estimate $\alpha_{(77,16):l}$, l = 1,...,4 as four quantities

$$\alpha_{(77,16):1} = \min\left(\mu_{X_3:1}(77), \mu_{Y_1:1}(16)\right) = \min\left(\mu_{\text{"}old\text{"}}(77), \mu_{\text{"}low\text{"}}(16)\right) = \min(0.9872, 0.991) = 0.9872,$$

$$\alpha_{(77,16):2} = \min\left(\mu_{X_3:2}(77), \mu_{Y_2:2}(16)\right) = \min\left(\mu_{\text{"}old\text{"}}(77), \mu_{\text{"}medium\text{"}}(16)\right) = \min(0.9872, 0.009) = 0.009,$$

$$\alpha_{(77,16):3} = \min\left(\mu_{X_4:3}(77), \mu_{Y_1:3}(16)\right) = \min\left(\mu_{\text{"}very\ old\text{"}}(77), \mu_{\text{"}low\text{"}}(16)\right) = \min(0.0128, 0.991) = 0.0128$$

and

$$\alpha_{(77,16):4} = \min\left(\mu_{X_4:4}(77), \mu_{Y_2:4}(16)\right) = \min\left(\mu_{\text{"}very\ old\text{"}}(77), \mu_{\text{"}medium\text{"}}(16)\right) = \min(0.0128, 0.009) = 0.009.$$

In conformity with formula (3.13) we obtain the fuzzy subsets of the consequences.
Set $R_{(77,16):1}{}^{conseq}$ has a membership function

$$\mu_{\text{"}R_{(77,16):1}\text{"}}{}^{conseq}(z) = \min\left(\alpha_{(77,16):1}, \mu_{Z_1:1}(z)\right) = \min\left(0.9872, \mu_{Z_1:1}(z)\right) = \min\left(0.9872, \mu_{\text{"}short\text{"}}(z)\right)$$

given by Fig. 3.4.

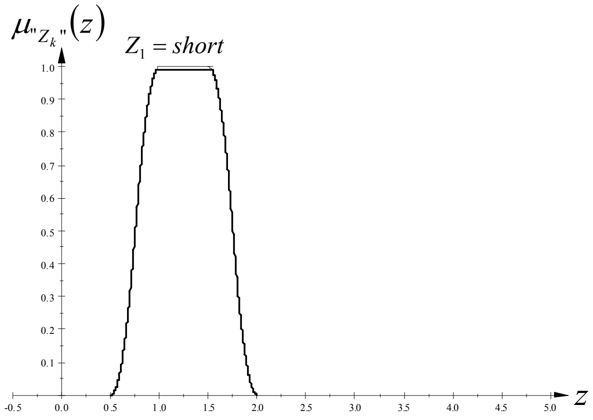

Figure 3.4: The fuzzy subset of consequence constructed due to $R_{(77,16):1}$

The next set $R_{(77,16):2}{}^{conseq}$ is characterized by a constraint

$$\mu_{"R_{(77,16):2}"}^{conseq}(z) = \min\left(\alpha_{(77,16):2}, \mu_{"Z_1:2"}(z)\right) = \min\left(0.009, \mu_{"Z_1:2"}(z)\right) = \min\left(0.009, \mu_{"short"}(z)\right)$$

drawn in Fig. 3.5.

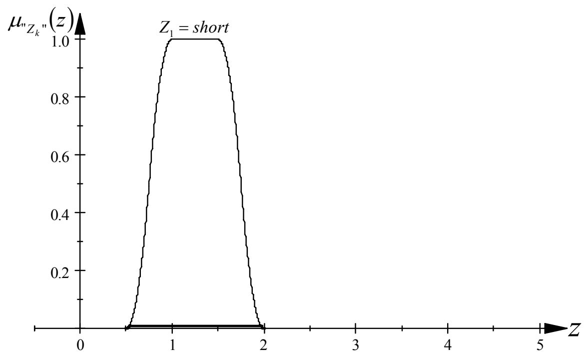

Figure 3.5: The fuzzy subset of consequence constructed due to $R_{(77,16):2}$

The third set $R_{(77,16):3}{}^{conseq}$ possesses a function

$$\mu_{"R_{(77,16):3}"}^{conseq}(z) = \min\left(\alpha_{(77,16):3}, \mu_{Z_1:3}(z)\right) = \min\left(0.009, \mu_{Z_1:3}(z)\right) = \min\left(0.0128, \mu_{"short"}(z)\right)$$

whose graph is revealed in Fig. 3.6.

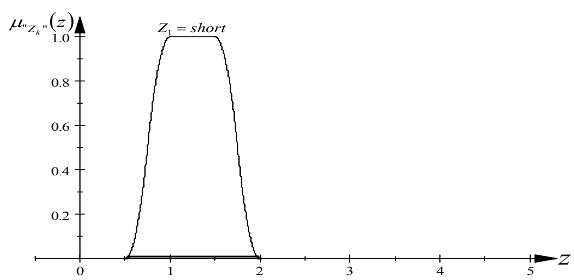

Figure 3.6: The fuzzy subset of consequence constructed for $R_{(77,16):3}$

The last set $R_{(77,16):4}^{conseq}$ is restricted by a function

$$\mu_{"R_{(77,16):4}"}^{conseq}(z) = \min\left(\alpha_{(77,16):4}, \mu_{Z_0:4}(z)\right) = \min\left(0.009, \mu_{Z_0:4"}(z)\right) = \min\left(0.009, \mu_{"very\ short"}(z)\right)$$

seen in Fig. 3.7.

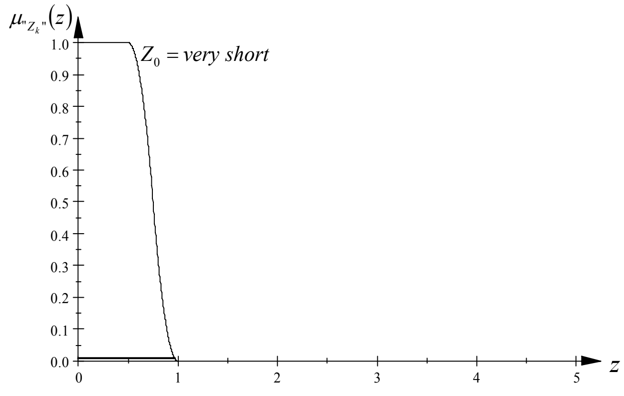

Figure 3.7: The fuzzy subset of consequence constructed in accord to $R_{(77,16):4}$

When applying formula (3.14) we concatenate all $\mu_{"R_{(x,y):l}"}^{conseq}(z)$, l = 1,2,3,4, in order to determine a common consequence of rules (3.11) fitted for the pair (77, 16). The fuzzy subset of the universe Z will be thus yielded by its membership function

$$\mu_{conseq_{(77,16)}}(z) = \max_{1 \le l \le 4}\left(\mu_{R_{(77,16):l}}^{conseq}(z)\right).$$

The fuzzy set $conseq_{(77,16)}$ is aggregated in Fig. 3.8.

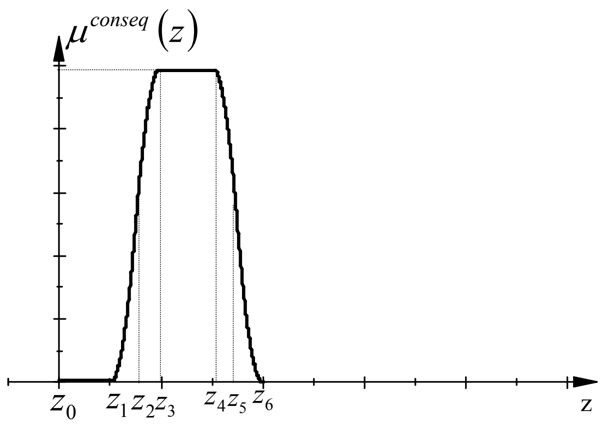

Figure 3.8: The consequence set $conseq_{(77,16)}$ in Z

Formula (3.15) constitutes a basis of an estimation of the survival length expected when assuming "*age*" = 77 and "*CRP-value*" = 16. Over interval $[z_0, z_6]$ = [0, 2], which contains characteristic points z_0 = 0, z_1 = 0.533, z_2 = 0.75, z_3 = 0.96, z_4 = 1.54, z_5 = 1.75 and z_6 = 2, we compute the z-prognosis

$$z = f(77,16)\frac{\int_{0}^{0.5335} 0.009z\,dz + \cdots + \int_{1.75}^{2} 2\left(\frac{z-2}{0.5}\right)^2 z\,dz}{\int_{0}^{0.5335} 0.009\,dz + \cdots + \int_{1.75}^{2} 2\left(\frac{z-2}{0.5}\right)^2 dz} = 1.05$$

For the patient who is 77 years old and has the *CRP*-value equal to 16, the theoretical estimated survival length is about 1 year. The result converges with the physician's own judgment made on the basis of his medical reports. For each pair (x,y) we can arrange new computations due to the fuzzy control algorithm to estimate the patient's period of surviving in the case of suffering from gastric cancer.

The fuzzy Mamdani control system is a powerful method, which mostly is applied to technologies controlling complex processes by means of human experience. In this work we have proved that the expected values of patients' survival lengths can be estimated even if the mathematical formalization involving independent and dependent variables is unknown. For each x and each y belonging to continuous spaces X and Y respectively, we are able to repeat the control algorithm in order to cover the space of pairs over the Cartesian product of X and Y with a continuous surface. This will constitute a part of our future work.

In the next chapter we wish to confirm the magnitude of survival length approximation by testing the Sugeno controller.

4. Verification of Survival Length Results by Means of Sugeno Controller

In the rules, constituting the crucial part of control processing, the levels of the independent variables have been tied to a selected level representing the dependent parameter. All levels have been further replaced by fuzzy numbers. The operations recommended by the Mamdani controller have been performed on membership functions of these fuzzy representatives of levels.

To shorten the action of the processing part in the Mamdani controller, Sugeno [Sugeno, 1985; Sugeno and Nishida, 1985] proposed another approach to the creation of rules, in which the dependent variable level will be determined by a functional connection of independent variables.

4.1 Adaptation of the Processing Part of the Fuzzy Controller to Sugeno-made Assumptions

We still wish to evaluate the survival length in gastric cancer patients due to information about their age and *CRP*-value. In this new version of a fuzzy controller, called the Sugeno controller, we preserve the former results of the fuzzification of independent variables, i.e., we still keep alive the levels of variables X = *"age"* and Y = *"CRP-value"* with assisting membership functions (3.1), (3.2) and (3.3) for X, as well as (3.4), (3.5) and (3.6) for Y.

The dependent variable Z = "*survival length*" is not differentiated into levels anymore. Instead, for each combination of X- and Y-levels we derive a linear two-dimensional function of the general shape $f(x,y) = ax+by+c$. This procedure can only work in the case of possessing some data points (x,y), which come from the examinations carried out on patients belonging to the desired combinations of levels. We support our experience by engaging discrete point sets to predict the information, which can be obtained for continuous intervals of X and Y.

Example 4.1

The triples ("*age*", "*CRP-value*", "*survival length*") = (x,y,z) belong to the set {(77, 18, 0.5),…,(81, 21, 0.9)}, in which x-values correspond to level X_3 = "*old*" and y-values are typical of Y_1 = "*low*". The dependent variable $z = f(x,y)$ has taken values between 0.1 and 0.8. The data is withdrawn from the patients' reports. Thus, for the couple of levels X_3 and Y_1 we find the functional dependency $z = f(x,y) = 0.13057x + 3.4256 \cdot 10^{-3} y - 9.4426$ shown in Fig. 4.1

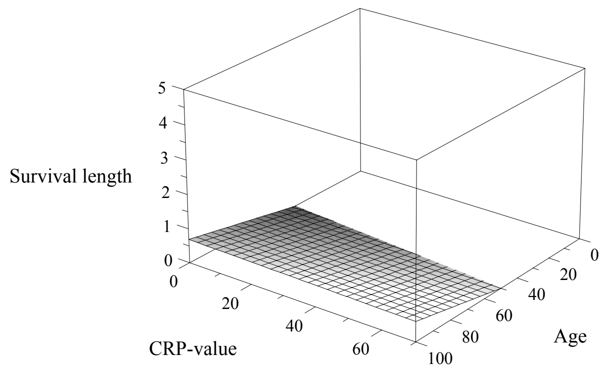

Figure 4.1: The example of the functional dependency between independent and dependent variables

In the IF…THEN… rule (3.11) the Z-level indicates the character of dependency between levels of X and Y. In the Sugeno IF…THEN… rule the function $z = f(x,y)$ is not assimilated to any level of variable Z from Chapter 3. We formulate a new pattern of (3.11) as

$$R_{(x,y):l}: \text{IF } x \text{ is } X_{i:l} \text{ and } y \text{ is } Y_{j:l}, \text{ THEN } z_{(x,y):l} \text{ is } z_k = f_k(x,y) \quad (4.1)$$

where l is the rule number, $X_{i:l}$ and $Y_{j:l}$ represent the fuzzy numbers X_i and Y_j, which are associated with the rule number l for the actual pair (x,y).

The formula

$$z_{(x,y):l} = z_k = f_k(x,y) = a_k x + b_k y + c_k$$

is a control function found for levels $X_{i:l}$ and $Y_{j:l}$. The quantities a_k, b_k and c_k are constants. The control functions have been constructed by the Maple computer program.

As we want to find the functional evaluations for all possible connections of levels, selected for independent variables due to Table 3.1, then the number of functions will equal 11. Hence, the index k, $k = 1,\ldots,11$, constitutes the function number with accordance to the next rule base table, introduced as Table 4.1.

Table 4.1 The functional rule base table for combinations of X- and Y-levels in estimations of survival length

age X_i \ CRP-value Y_j	very low	low	medium	high	very high
very young					
young					
middle-age	z_1				
old	z_2	z_3	z_4	z_5	z_6
very old	z_7	z_8	z_9	z_{10}	z_{11}

As before, we have left some empty cells in the table because of the lack of data for younger patients with diagnosis "gastric cancer".

Instead of the sophisticated procedure of looking for the final consequence set, characteristic of the Mamdani controller, we adopt the control function [Sugeno, 1985; Zimmermann, 2001]

$$z = f^{Sugeno}(x,y) = \frac{\sum_l \alpha_{(x,y):l} \cdot \left(z_{(x,y):l} = f_k(x,y)\right)}{\sum_l \alpha_{(x,y):l}}, k = 1,\ldots,11, \quad (4.2)$$

which directly delivers a crisp control value of the output variable "*survival length*".

To obtain the value of $\alpha_{(x,y):l}$, we need to perform the minimum operation for the membership degrees of $\mu_{X_i:l}(x)$ and $\mu_{Y_j:l}(y)$ according to (3.12).

4.2 Applications of the Sugeno Fuzzy Controller to Estimation of the Survival Length in Gastric Cancer Patients

We return to the case of the patient already presented in 3.5. By testing the Sugeno controller let us now evaluate the expected value of the survival length for a 77-year-old patient, whose *CRP*-value is 16.

We put $x = 77$ and $y = 16$. We have previously stated that age 77 belongs to the fuzzy number X_3 = "*old*" with membership degree $\mu_{X_3}(77) = 0.9872$.

Element $x = 77$ also is a member of X_4 = "*very old*" with the membership $\mu_{X_4}(77) = 0.0128.$

The *CRP*-value $y = 16$ is found in fuzzy number Y_1 = "*low*", where its membership equals $\mu_{Y_1}(16) = 0.991.$ The same $y = 16$ takes place in Y_2 = "*medium*", but the membership degree is determined as $\mu_{Y_2}(16) = 0.009.$

The rules IF…THEN, which associate the input variables with the output variable are determined according to formula (4.1) and Table 4.1 as:

$R_{(77,16):1}$: IF x is X_3 and y is Y_1, THEN

$$z_{(77,16):1} = z_3 = f_3(x,y) = -4.3948 \cdot 10^{-2} x - 9.6411 \cdot 10^{-3} y + 4.3938,$$

$R_{(77,16):2}$: IF x is X_3 and y is Y_2, THEN

$$z_{(77,16):2} = z_4 = f_4(x,y) = -1.0077 \cdot 10^{-2} x - 2.7872 \cdot 10^{-2} y + 2.0658,$$

$R_{(77,16):3}$: IF x is X_4 and y is Y_1 THEN

$$z_{(77,16):3} = z_8 = f_8(x,y) = -6.2637 \cdot 10^{-2} x + 0.26035 y + 4.1361$$

and

$R_{(77,16):4}$: IF x is X_4 and y is Y_2 THEN

$$z_{(77,16):4} = z_9 = f_9(x,y) = -9.7739 \cdot 10^{-3} x + 3.3194 \cdot 10^{-4} y + 1.1162.$$

To estimate the value of $\alpha_{(x,y):l}$, we perform the minimum operation on each pair of values $\mu_{X_i:l}(x)$ and $\mu_{Y_j:l}(y)$. We refer to previously known results

$$\alpha_{(77,16):1} = \min\left(\mu_{X_3:1}(77), \mu_{Y_1:1}(16)\right) = 0.9872,$$

$$\alpha_{(77,16):2} = \min\left(\mu_{X_3:2}(77), \mu_{Y_2:2}(16)\right) = 0.009,$$

$$\alpha_{(77,16):3} = \min\left(\mu_{X_4:3}(77), \mu_{Y_1:3}(16)\right) = 0.0128$$

and

$$\alpha_{(77,16):4} = \min\left(\mu_{X_4:4}(77), \mu_{Y_2:4}(16)\right) = 0.009.$$

After substituting the values of $\alpha_{(x,y):l}, l = 1,...,4$ and corresponding to them $z_{(x,y):l} = f_k(x,y)$, k = 3,4,8 and 9, in (4.2) we get

$$z = f^{Sugeno}(77,16) =$$

$$\frac{\alpha_{(77,16):1} \cdot f_3(77,16) + \alpha_{(77,16):2} \cdot f_4(77,16) + \alpha_{(77,16):3} \cdot f_8(77,16) + \alpha_{(77,16):4} \cdot f_9(77,16)}{\alpha_{(77,16):1} + \alpha_{(77,16):2} + \alpha_{(77,16):3} + \alpha_{(77,16):4}} =$$

$$= \frac{0.9872 \cdot 0.8555 + 0.009 \cdot 0.84392 + \cdots + 0.009 \cdot 0.36892}{0.9872 + 0.0009 + \cdots + 0.009} \approx 0.88.$$

When comparing the results yielded by two controllers we make the conclusion about their convergence to the approximated survival value about one year. The deviation between quantities, related to survival length of the 77-year-old patient with *CRP* equaling 16, can be an effect of using the planar surface instead of an irregular one in the approximation of point sets in the Sugeno controller. We wish to formulate the following conclusion summing up the comparison of both controllers.

Conclusion 4.1
The algorithm of the Mamdani controller demands a large number of operations in the processing phase, but we can always construct logical rules IF…THEN… based on fuzzy numbers. Even if the data from point sets is lacking it is still possible to make a trial of designing membership functions for all levels of variables by relying on the human expertise.
The Sugeno controller does not need so many operations in the processing stage. Nevertheless, its use is impossible in practice when we cannot be furnished with discrete data sets to accomplish the design of functions f_k. The choice of the method is thus dependent on the access to data.

The controllers encounter results coming from statistical experiments, and they do not need special assumptions like normal distributions of the dependent variables.

In the future experiments we want to construct the computer program to cover the rectangle $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$ with a surface, which immediately allows to read off the desired value of z for the pair (x, y). In that way we will solve the problem of the continuous evaluation of survival length as it has been recommended by our co-operating physicians.

In the end we emphasize that, unlike the traditional control methods, fuzzy control is the methodology, which deals with many real-life problems successfully. As the conventional control methods often are based on advanced mathematical models, such as differential equations sometimes impossible to solve, the method of fuzzy control is much more convenient to apply.

5. Rough Set Classification in the Operation Type Selections accomplished for Gastric Cancer Patients

There are about 20 different operation types recommended by surgeons in recovery from gastric cancer. How to choose the most suitable operation type for a cancer patient is an important decision to make for a physician. The main idea of the current study part focuses on verifications of the primary operation types for gastric patients by means of rough set approach. The method provides physicians with a theoretical secondary operation prognosis.

5.1 Foundations of Rough Set Theory

For a physician, who judges the operation possibility due to some values of clinical markers, it is important to know if a cancer patient needs to be operated or not. If an operation is necessary, then what kind of surgery is the best alternative? To verify the initial decision and provide a theoretical secondary opinion may be necessary in the decision-making.

Suppose that U is a finite universe set. Each element of U is associated with some information values. Subsets of U consisting of elements which have the identical information values are defined as elementary sets [Pawlak, 1984, 1997, 2004; Pawlak et al., 1995; Małuszyński and Vitória, 2002, Skowron, 2001; Pal and Mitra, 2004; Rakus-Andersson, 2007, 2009c]. The union of elementary sets constitutes a definable set [Pawlak, 1984, 1997]. The definable set cannot be expressed exactly. However, it can be approximated by a lower- and an upper- approximation.

In rough set theory, a large amount of information is usually collected in a table, whose rows are labelled by objects, columns are labelled by different attributes and entries of the table are labelled by attribute values. Such kinds of tables are known as information systems, data tables, decision tables or information tables [Pawlak, 1984, 1997]. An example of the information table is given in Table 5.1.

Example 5.1

Table 5.1 The example of the information table

Objects in U	Conditional attributes $C=\{a_1,a_2,a_3\}$			Decision attribute D
	a_1	a_2	a_3	
x_1	vl	o	F	Type 2
x_2	m	vo	M	Type 11
x_3	vh	m	M	Type 11
x_4	vl	o	F	Type 8
x_5	m	y	F	Type 8
x_6	m	vo	M	Type 5

The data table above describes an example of the preliminary decision making in the case of different operation types suggested for sex gastric cancer patients. The objects $x_1, x_2,..., x_6$ represent sex patients. The condition attributes a_1, a_2 and a_3 stand for such essential factors deciding of cancer diagnosis as the *CRP*-value, age and sex respectively, and the decision attributes, “the type of operation”, are stated by “type 2”, “type 11”,…, and so on. Furthermore, the conditional attribute values are assigned by linguistic lists a_1 = *CRP*-value = {very low = vl, medium = m, very high = vh}, a_2 = age = {young = y, middle-aged = m, old = o, very old = vo} and a_3 = sex = {Female = F, Male = M}.

Normally, an information table consists of 4-tuple $S = (U, A, V, f)$ [Pang et al., 2007; Feng et al., 2005], where $U = \{x_1, x_2, ..., x_m\}$ is a nonempty finite set called the universe, A is also a nonempty finite set composed of attributes. Furthermore, $A = C \cup D$, in which $C = \{a_1, a_2, \cdots, a_i\}$ is a finite set of condition attributes and $D = \{d_1, d_2, \cdots, d_j\}$ is a finite set of decision attributes. $V = \{v_1, v_2, \cdots, v_k\}$ is a set of attributes values and finally $f: U \times A \to V$ is called the information function such that $f(x,a) \in V$ for each $x \in U$, $a \in A$. The information function refers to a value of x at an attribute a and the value usually is represented by a linguistic term.

5.2. Indiscernibility relation

The indiscernibility relation is an important concept in rough set theory. Suppose that we have an information system $S = \{U, A, V, f\}$. *C* is a subset of the attribute set *A*, $C \subseteq A$. A binary relation for indiscernibility on the subset *C* is defined as follows:

$$IND(C) = \{(x_p, x_q) : f(x_p, a) = f(x_q, a)\}, \text{ where } (x_p, x_q) \in U \times U, \forall a \in C \qquad (5.1)$$

The indiscernibility relation is an equivalence relation since it is reflexive, symmetric and transitive. The proof of all properties is accomplished below.

(i) Reflexivity
Suppose that $(x_p, x_p) \in IND(C)$ then, according to the definition of the relation, we get $f(x_p, a) = f(x_p, a)$, which is equivalent to $f(x_p, a) = f(x_p, a)$. Hence, $(x_p, x_p) \in IND(C)$ after inverting the order of x_p in each pair (x_p, x_p).

(ii) Symmetry
Let $(x_p, x_q) \in IND(C)$. The definition of the indiscernibility relation provides us with $f(x_p, a) = f(x_q, a) \Leftrightarrow f(x_q, a) = f(x_p, a)$. This means that $(x_q, x_p) \in IND(C)$, which confirms the symmetric property of the relation.

(iii) Transtivity
Assume $(x_p, x_q) \in IND(C)$, $(x_q, x_r) \in IND(C)$ and $x_p, x_q, x_r \in U$. If $f(x_p, a) = f(x_q, a)$ and $f(x_q, a) = f(x_r, a)$, then $f(x_p, a) = f(x_r, a)$. Thus $(x_p, x_r) \in IND(C)$.

As the equivalence relation, *IND*(*C*) has equivalence classes defined by

$$[x_p]_{IND(C)} = \{x_q \in U : f(x_p, x_q) \in IND(C)\}, \; p, q = 1, \cdots, m, \; \forall a \in C.$$

Example 5.2

Suppose $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $C = \{a_1, a_2, a_3\} = \{CRP - value, age, sex\} \subseteq A$, With respect to these three condition attributes, the universe set U can be divided into 4 parts, which are represented by $\{x_1, x_4\}$, $\{x_2, x_6\}$, $\{x_3\}$ and $\{x_5\}$. From Table 5.1, we see that x_1 and x_4 have the same condition attributes values $f(x_1, a) = f(x_4, a) = \{vl, o, w\}$, x_2 and x_6 also are characterized by other condition attributes values $f(x_2, a) = f(x_6, a) = \{m, vo, m\}$. Therefore, x_1 is indiscernible to x_4 and x_2 is indiscernible to x_6. Normally, the equivalence class which includes elements x_p is denoted by $[x_p]_{IND(C)}$, where $IND(C)$ refers to "the indiscernibility relation with respect to C". From table 5.1, we obtain the following equivalence classes

$[x_1]_{IND(C)} = [x_4]_{IND(C)} = \{x_1, x_4\}$,

$[x_2]_{IND(C)} = [x_6]_{IND(C)} = \{x_2, x_6\}$,

$[x_3]_{IND(C)} = \{x_3\}$

and

$[x_5]_{IND(C)} = \{x_5\}$.

5.3 Lower and Upper Approximation

The creation of lower and upper approximations are two basic system operations on sets in the rough set theory. Suppose that $S = (U, A, V, f)$ is an information system, $C \subseteq A$ is a subset of attributes, and $X \subseteq U$ is a subset of the universe set created with respect to decision attribute d_l, $l = 1,...,j$. The set

$$B_{-}(X) = \{ x_p \in U : [x_p]_{IND(C)} \subseteq X \}, p = 1,...,m, \tag{5.2}$$

is called the B-lower approximation of X, whereas

$$B^{-}(X) = \{ x_p \in U : [x_p]_{IND(C)} \cap X \neq \phi \}, p = 1,...,m, \tag{5.3}$$

is named the B-upper approximation of X.

The elements of the B-lower approximation are considered as sure members of X, while the elements of the B-upper approximation are treated as X's possible members.

If we look at the elements x_1 and x_4 in Table 5.1, we will find that these elements have the same values of the condition attributes, but the values of the decision attribute are different. The elements x_2 and x_6 show the same tendency. These kinds of elements are called inconsistent. In this situation, rough set theory provides a method to deal with this kind of conflicts. The method is called the creation of the lower and the upper approximation of X.

Example 5.3

We use Table 5.1 again to explain the definitions of the lower and upper approximations in details. The universe set U consists of sex patients $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, moreover set C is a subset of the attributes, $C = \{a_1, a_2, a_3\} = \{CRP - value, age, sex\} \subseteq A$. Suppose that operation type is defined as "type 8". The set $X = \{x_4, x_5\}$ contains patients who are assigned to the primary decision "type 8". By the definition of the indiscernibility relation of C, we obtain the equivalence classes as $[x_1]_{IND(C)} = [x_4]_{IND(C)} = \{x_1, x_4\}$, $[x_2]_{IND(C)} = [x_6]_{IND(C)} = \{x_2, x_6\}$, $[x_3]_{IND(C)} = \{x_3\}$ and $[x_5]_{IND(C)} = \{x_5\}$.

Due to (5.2), the lower approximation is computed as follows.
For x_1, the equivalence class is determined as $[x_1]_{IND(C)} = \{x_1, x_4\}$. Since $\{x_1, x_4\} \not\subset \{x_4, x_5\}$ then $x_1 \notin B\_(X)$. We prove the same operation for the next element of U until we find $B\_(X) = \{x_5\}$.

The upper approximation is decided with compliance with (5.3). For x_1 the equivalence class $[x_1]_{IND(C)} = \{x_1, x_4\}$ has the nonempty intersection with $X = \{x_4, x_5\}$ stated as the set $\{x_4\}$, which includes x_1 to the B-upper approximation. We repeat the same procedure for each x_p, $p = 1,...,m$, until we select the contents of $B^{-}(X) = \{x_1, x_4, x_5\}$. The inclusion relation $B\_(X) \subset X \subset B^{-}(X)$ holds. The lower approximation refers to "certainty", while the upper approximation refers to "possibility". This means that, e.g., the decision referring to patient x_5 and concerning the operation type 8 is sure, while patients who possibly should be operated by surgery type 8 are x_1, x_4 or even x_5.

5.4 The Concept of a Rough Set

After having introduced the information about the lower and the upper approximations, it is the high time to formalize the concept of a rough set. A crisp set consisting of indistinguishable elements with respect to some attributes is called a rough set. A rough set is normally associated with a lower and an upper approximation [Pawlak, 1984, 1997, 2004].

In order to evaluate the degree of certainty for x_p to be included in set X we compute the degree of certainty

$$\mu_{d_l}(x_p) = \frac{card\ \{X \cap [x_p]_{IND(C)}\}}{card\ \{[x_p]_{IND(C)}\}}, l = 1,..., j.. \tag{5.4}$$

The formula (5.4) determines the truth value of element x_p to be a member of X, grounded due to decision value d_l, $l = 1,...,j$. The sure elements placed in the lower approximation take the certainty value equal one. The expression $card\ \{\cdots\}$ denotes a number of objects belonging to a crisp set. It happens that the same object x_p reveals differentiated certainty degrees due to different decision values. We extract the optimal decision for the object in compliance with its highest certainty degree when comparing the object's all degrees obtained for diverse decision values.

5.5. Application of Rough Set Classification in Verification of Operation Type

The rough classification has been adopted by us to answer the second question posed by the physicians, namely, the selection of the most promising operation type, when deciding it by recognizing the patient's clinical conditions.

We have based our investigation on the clinical data concerning 156 patients with the diagnosis gastric cancer. The data was obtained from Associate Professor H. Forsell, employed at the Blekinge Research Competence Centre. Due to the physician's suggestions, we have chosen three parameters (*CRP*-value, age and sex) as the crucial condition attributes in the information system, since they play an essential role in making the operation decision.

The decision attribute *D* is characterized by 12 different operation codes, JAH00, JAH01, JCC96 and others. For the reason of simplicity, we rename them as 1, 2, 3, …, 12.

The information is collected in Table 5.2.

Table 5.2 The information system for gastric cancer patients

Objects in *U*	Conditional attributes $C=\{a_1,a_2,a_3\}$			Decision attribute *D*
	a_1	a_2	a_3	
x_1	l	vo	F	9
x_2	l	o	M	6
x_3	vl	m	M	8
x_4	vl	o	M	4
...	...	...	...	...
x_{86}	vl	o	F	2
x_{87}	vl	vo	F	11
x_{88}	l	m	M	11
...	...	...	...	...
x_{156}	m	vo	F	1

The values of the condition attributes are defined by the linguistic terms:
CRP-value = {very low = l, low = l, medium = m, high = h, very high = vh},
Age = {young = y, middle-age = m, old = o, very old = vo},
Sex = {female = f, male = m}

The indiscernibility relation consists of pairs of elements, which have the same information with respect to condition attributes. The relation is listed as the set of pairs
IND(*C*)=
$$\{(x_1,x_1),(x_2,x_2),\cdots,(x_{156},x_{156}),(x_1,x_{28}),(x_{28},x_1),\cdots,(x_6,x_{21}),(x_{21},x_6),\cdots,(x_{156},x_{145}),(x_{145},x_{156})\}.$$

The equivalence classes
$$[x_p]_{IND(C)} = \{\, x_q \in U : f(x_p,x_q) \in IND(C)\},\ p,q = 1,\cdots,156,\ \forall a \in C$$
are discerned as sets

$[x_1]=[x_{27}]=[x_{28}]=[x_{46}]=[x_{138}]=\{x_1,x_{27},x_{28},x_{46},x_{138}\},$

$[x_2]=[x_{12}]=[x_{26}]=[x_{32}]=[x_{61}]=[x_{93}]=[x_{112}]=\{x_2,x_{12},x_{26},x_{32},x_{61},x_{93},x_{112}\},$

$[x_3]=[x_{78}]=[x_{81}]=\{x_3,x_{78},x_{81}\},$

$[x_4]=[x_5]=[x_7]=\cdots=[x_{130}]=[x_{132}]=\{x_4,x_5,x_7,\cdots,x_{130},x_{132}\},$

$[x_5]=[x_4]=[x_7]=\cdots=[x_{130}]=[x_{132}]=\{x_5,x_4,x_7,\cdots,x_{130},x_{132}\},$

$[x_6]=[x_{21}]=[x_{77}]=\{x_6,x_{21},x_{77}\},$

$[x_7]=[x_4]=[x_5]=\cdots=[x_{130}]=[x_{132}]=\{x_7,x_4,x_5,\cdots,x_{130},x_{132}\}$

$[x_8]=\{x_8\}$

$[x_9]=[x_{19}]=[x_{25}]=\cdots=[x_{146}]=[x_{148}]=\{x_9,x_{19},x_{25},\cdots,x_{146},x_{148}\}$

$[x_{10}]=\{x_{10}\}$

$[x_{11}]=[x_{34}]=[x_{35}]=\cdots=[x_{134}]=[x_{149}]=\{x_{11},x_{34},x_{35},\cdots,x_{134},x_{149}\}$

$[x_{13}] = \{x_{13}\}$
$[x_{14}] = \{x_{14}\}$
$[x_{15}] = [x_{41}] = [x_{64}] = \cdots [x_{153}] = \{x_{15}, x_{41}, x_{64}, \cdots, x_{153}\}$
$[x_{16}] = [x_4] = [x_5] = [x_7] = \cdots = [x_{130}] = [x_{132}] = \{x_{16}, x_4, x_5, x_7, \cdots, x_{130}, x_{132}\}$
$[x_{17}] = [x_{18}] = \cdots = [x_{136}] = [x_{150}] = \{x_{17}, x_{18}, \cdots, x_{136}, x_{150}\}$
$[x_{18}] = [x_{17}] = \cdots = [x_{136}] = [x_{150}] = \{x_{17}, x_{18}, \cdots, x_{136}, x_{150}\}$
$[x_{19}] = [x_9] = [x_{25}] = \cdots = [x_{146}] = [x_{148}] = \{x_9, x_{19}, x_{25}, \cdots, x_{146}, x_{148}\}$
$[x_{20}] = [x_4] = [x_5] = \cdots = [x_{130}] = [x_{132}] = \{x_4, x_5, x_{20}, \cdots, x_{130}, x_{132}\}$
$[x_{21}] = [x_6] = [x_{77}] = \{x_{21}, x_6, x_{77}\}$
$[x_{22}] = [x_{24}] = [x_{49}] = \cdots = [x_{102}] = [x_{121}] = [x_{143}] = \{x_{22}, x_{24}, x_{49}, \cdots, x_{102}, x_{121}, x_{143}\}$
$[x_{23}] = [x_{47}] = [x_{57}] = [x_{63}] = [x_{125}] = \{x_{23}, x_{47}, x_{57}, x_{63}, x_{125}\}$
$[x_{29}] = \{x_{29}\}$
$[x_{30}] = \{x_{30}\}$
$[x_{36}] = [x_{37}] = [x_{51}] = [x_{58}] = [x_{144}] = \{x_{36}, x_{37}, x_{51}, x_{58}, x_{144}\}$
$[x_{38}] = [x_{73}] = [x_{89}] = \cdots = [x_{140}] = [x_{147}] = \{x_{38}, x_{73}, x_{89}, \cdots, x_{140}, x_{147}\}$
$[x_{39}] = [x_{139}] = \{x_{39}, x_{139}\}$
$[x_{40}] = [x_{92}] = [x_{108}] = [x_{155}] = \{x_{40}, x_{92}, x_{108}, x_{155}\}$
$[x_{43}] = [x_{52}] = [x_{75}] = [x_{152}] = \{x_{43}, x_{52}, x_{75}, x_{152}\}$
$[x_{48}] = [x_{84}] = \cdots = [x_{142}] = [x_{154}] = \{x_{48}, x_{84}, \cdots, x_{142}, x_{154}\}$
$[x_{66}] = [x_{133}] = [x_{151}] = \{x_{66}, x_{133}, x_{151}\}$
$[x_{80}] = [x_{88}] = [x_{99}] = \{x_{80}, x_{88}, x_{99}\}$
$[x_{85}] = \{x_{85}\}$
$[x_{87}] = [x_{114}] = \{x_{87}, x_{114}\}$
$[x_{98}] = [x_{107}] = [x_{131}] = \{x_{98}, x_{107}, x_{131}\}$
$[x_{116}] = [x_{126}] = \{x_{116}, x_{126}\}$
$[x_{117}] = \{x_{117}\}$
$[x_{122}] = [x_{127}] = [x_{145}] = [x_{156}] = \{x_{122}, x_{127}, x_{145}, x_{156}\}$

We demonstrate an example in decision-making about the operation type for a gastric patient by rough set classification. We randomly choose the patient, x_{29}, and verify the initial operation type. According to the physician's suggestion, the patient should have the operation type 7. We define $X_{d_7} = \{x_{25}, x_{29}\}$, which consists of all patients who have operation type 7 assigned in the last column. The lower approximation of X_{d_7} is grounded as $B_{d_7}\_(X) = \{x_{29}\}$ and the upper approximation is determined by $B_{d_7}^{-}(X) = \{x_9, x_{19}, x_{29}, x_{59}, x_{74}, \cdots, x_{148}\}$. $B_{d_7}\_(X) \subseteq X_{d_7} \subseteq B_{d_7}^{-}(X)$. The degree of certainty of operation type 7 for the patient x_{29} is computed by: $\mu_{d_7}(x_{29}) = \frac{|X_{d_7} \cap [x_{29}]_{IND(C)}|}{|[x_{29}]_{IND(C)}|} = \frac{1}{1} = 1$. The certainty degree of the operation type 7 for the patient 29 is 1, which can be assimilated with the 100% of security feeling to make this decision.

Sometimes it happens that to one patient several degrees of certainty are computed according to different constructions of *X* sets. In this situation, we choose the largest value of these degrees to obtain the optimal result. The operation decision, to each the largest certainty degree is assigned will be treated as the most recommended. To demonstrate this action we show another example. We also randomly choose patient x_{89} from the universe set *U*. The preliminary operation type of x_{89} is 11, d_{11}. But another patient x_{147}, who has been marked with the operation type 10, belongs to the same equivalence class in which x_{89} already exists. It coincides that x_{89} can have another operation type than d_{11}. The set $X_{d_{11}}$, determined by decision d_{11} is presented by
$X_{d_{11}} = \{x_{10}, x_{23}, x_{27}, \cdots, x_{63}, x_{69}, x_{73}, ..., x_{89}, ..., x_{144}, x_{150}, x_{155}\}$.

Its lower approximation is decided as $B_{d_{11}}\_(X) = \phi$ and the upper approximation is designed as $B_{d_{11}}^{-}(X) = \{x_{10}, x_{23}, x_{27}, ..., x_{57}, x_{63}, ..., , x_{89}, ..., x_{125}, \cdots, x_{155}\}$.
$B_{d_{11}}\_(X) \subseteq X_{d_{11}} \subseteq B_{d_{11}}^{-}(X)$. The degree of certainty of operation type 11 for the patient x_{89} is computed by: $\mu_{d_{11}}(x_{89}) = \frac{|X_{d_{11}} \cap [x_{89}]_{IND(C)}|}{|[x_{89}]_{IND(C)}|} = \frac{7}{8} = 87.5\%$ and the degree of certainty of operation type 10 for the patient x_{89} is computed by: $\mu_{d_{10}}(x_{89}) = \frac{|X_{d_{10}} \cap [x_{89}]_{IND(C)}|}{|[x_{89}]_{IND(C)}|} = \frac{1}{8} = 12.5\%$.

For patient x_{89}, there exist two different degrees of certainty to make a decision concerning the operation choice. We choose the larger degree, $\mu_{d_{11}}(x_{89}) = 87.5\%$ assisting d_{11}. The operation type d_{11} for x_{89} is much more suitable than d_{10}.

Rough set theory was proposed by a Polish scientist Zdzisław Pawlak in the early 1980s. It has proved to be a very efficient tool in dealing with vague and uncertain information. In particular, the applications of rough set theory have been crucial contributions in the domain of technical diagnostics, finance and conflict analysis.

Medical diagnosis is also an important practical area for rough set theory to obtain relevant information. Many countries such as Japan and China have done many research works in this area. A research group from Shimane University of Japan has utilized rough set theory for diagnosis of the antibiotic allergy reaction and got reliable results [Pang et al., 2007]. The result shows that there are good chances for the development of rough set theory in medical fields.

We have utilized the theory to give the answer to the question of the secondary opinion referring the operation type choice. In the future research a computer program is desired to simplify the computations. We plan to introduce the technique of reducts, which should make the choice of the operation type more general by reducing the number of different equivalence classes. By utilizing other algorithms of computational intelligence like, e.g., immunological tests we wish to find the operation type by segregation of self (pro-operative) regions against non-self (anti-operative).

6. Concluding Remarks

The project titled “Fuzzy Sets, Rough Sets and Fuzzy Statistics in Treatment of Gastric Cancer Patients” has given rise to the current study. The idea of cooperation between mathematicians and physicians in Blekinge Region in Sweden has resulted in gaining the grant from Blekinge Research Board, which is used for research devoted to mathematical applications to medicine. Since the scientific profile of the medical consultant, Associate Professor Henrik Forssell has been stretched over the stomach cancer diseases then we have formulated the objectives of the project as mathematical complements of his queries.

One of them has been the estimation of the patient survival length when making it dependent on two biological markers “age” and “the CRP-value”. We have engaged fuzzy regression to evaluate the length of living as a fuzzy number, which could be verbalized. To obtain a crisp value of the survival length, in spite of an unknown relation between the collection of independent variables and the variable dependent of them, we have proved the action of two fuzzy controllers of the Mamdani and the Sugeno type. Luckily, we have been provided with data concerning some samples of patients. This has simplified the processing part of controllers, in which the information collected for the discrete samplings has supported the knowledge-based reasoning when creating the logical rules IF... THEN.... The results of the controllers’ actions have converged to the expected survival value, which confirms the reliability of the mathematical experiment. In the process of variable fuzzification own parametric designs of membership functions have been used. The formulas of membership functions are already adapted to be included as segments of future computer programs. By the way, we should emphasize that the common parametric formula, which produces a family of functions instead of several distinct designs, constitutes an elegant and formal mathematical solution.

The second problem has touched a choice of the most appropriate operation type. To choose the surgery proposal, we have tested the model of rough set classification. For patients who found their places in the sets of lower classification the selection of operation type has been a clear-cut decision since the certainty value of the decision choice has equalled one. For other patients who have been assigned to several decisions the selection of the most reliable operation type has been accomplished by comparing magnitudes of certainty degrees corresponding to different decisions. The largest degree has pointed out the optimal decision.

Finally, we want to emphasize the usefulness of that interdisciplinary cooperation between mathematicians and physicians, which has awoken useful questions and brought novel solutions to them. The solutions can be confronted with statistical results and, even more, we can use the imprecise models of fuzzy and rough ser theories more generally as they do not require special assumptions like the probability distribution or a sample size that are necessary in statistical developments.

7 Future Prognoses

We treat the research, done in the phase discussed in the thesis, as the first part of our investigations and we still intend to work on the other solutions to the questions posed.

In the domain of controllers we need computer programs to test the large samples of patients to implement dense surfaces of points instead of singletons. It would be desirable to prove controllers with the larger number of independent variables than two.

Another computer program is needed to simplify the computing procedure in rough set classification. A possibility to introduce some reducts of biological markers is also considered in order to diminish a number of equivalence classes. This should make the decision choice simpler since we expect that the certainty grade one would be assigned to more patients.

We can alternatively prove other algorithms of computational intelligence, like immunological and evolutionary, to improve the quality of effects.

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ABSTRACT

Fuzzy set theory was presented for the first time by Professor Lotfi A. Zadeh from Berkeley University in 1965.

In conventional binary logic a statement can be true or false, and there is no place for even a little uncertainty in this judgment. An element either belongs to a set or does not. We call these kinds of sets crisp sets. In practice we often experience those real situations that are represented by crisp sets as impossible to describe accurately. A two-valued logic assumes that precise symbols must be employed, and it is therefore not applicable to the real existence.

If the information demanded by a system is lacking, the future state of such a system may not be known completely. One of the instruments used to handle the vagueness in the real-world situations is fuzzy set theory, which has been frequently applied in a wide range of areas like, e.g., dynamic systems, militaries, medicine and other domains.

Another theory, which copes with the problem of imprecision, is known as rough set theory. It was proposed by Professor Zdzisław Pawlak in Warsaw in the 1980ties. Whereas imprecision is expressed in the category of a membership degree in fuzzy set theory, this is a matter of the set approximation in rough set theory. Due to the definition of a rough set formulated by means of the decision attribute value, two approximate sets of the rough set are determined. These contain sure and possible members of the universe considered, in which the rough set has been defined.

One of the objectives of this study is to apply some classical methods of fuzzy set theory to medicine in order to estimate the survival length of gastric cancer patients. We have decided to test the action of fuzzy controllers of the Mamdani and Sugeno type. Two clinical markers, playing roles of the independent variables, have been included in the algorithm as the base information assisting the survival prognosis. Since the model results have been convergent to the expected experimental values then we will intend to make some extensions of the model concerning the larger number of independent variables.

We have also utilized rough set classification, to verify the types of operations. These items are discussed in the thesis in conformity with the physicians' wishes to support results of statistical investigations. The current research is funded by the scientific grant obtained from Blekinge Research Board.

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