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INITIAL INVESTIGATIONS CONCERNING MODELLING OF SOUND PROPAGATION IN DUCTS WITH ANC BY MEANS OF TWO-PORT THEORY AND FEM

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Today mechanical ventilation systems are installed in many buildings to handle the ventilation. Such ventilation systems constitute a potential source of unwanted background noise in the buildings. In order to reduce the noise propagating through the ducts passive silencers are normally installed as a part of the ventilation system. However, the passive silencers are relatively ineffective in the low frequency range. A solution can be to use a combination of active noise control (ANC) and passive techniques where the ANC system extends the noise attenuation to include the low frequency noise.
The construction of the ventilation systems can vary substantially between different buildings and may contain many different duct parts in different compositions etc. Hence, it may not be trivial to find a cost- and performance efficient installation of an ANC system in a ventilation system, e.g. to find a proper installation position of it and suitable passive silencers to combine it with, finding out if one or several ANC systems should be used, and so on. In order to make the design of a complete ventilation system (including one or several ANC systems) cost- and time efficient, an adequate mathematical model of the duct is required. The purpose of the model is to describe sound propagation in the duct. Such model can be built e.g. based on the two-port theory or by finite element method. In this paper initial investigations concerning modelling of standard duct parts were performed. Simulations were carried out to find parameters such as Noise Reduction etc., using both plane wave two-port theory and finite element modelling. The results of the simulations were compared with measurement results.

1. Introduction

The installation design when applying an ANC system to e.g. a ventilation system is important in order to utilize the full noise attenuation potential of the ANC system [1]. Such design may include installations to reduce the influence of turbulent noise in the microphone signals due to the airflow in the duct [1, 2, 3], but it may also include the installation position of the ANC system and what type of passive silencer to combine it with etc. For example, the acoustic feedback between the secondary source and the reference microphone in a feedforward ANC system may result in performance reduction and stability problems of the control system. It has been found that efficient acoustic feedback cancellation can be achieved by using a proper passive silencer with the proper placement [4]. Further, using passive silencers may result in less pronounced standing wave patterns in the duct resulting in an increased performance of the ANC system and a less complex forward path to estimate [4, 5]. Since the construction of ventilation systems can vary substantially between different buildings, finding the proper placement of the ANC system and the passive silencers, the proper passive silencers, etc. may not be trivial. To avoid tedious and costly experimental investigations it can be desirable to use a mathematical model in order to describe the sound propagation inside duct systems of different configurations. Such model can be built e.g. based on the two-port theory or by finite element method (FEM). It can be also of interest to simulate the effect of applying an ANC system. A similar simulation of active suppression of boring bar vibrations in a lathe application using FEM has for example been reported in [6]. Such simulations could be useful to predict the positions where an ANC system produce the highest attenuation.

One way of describing sound propagation in ducts in the plane wave propagation region, is by using two-port theory [7, 8]. A two-port can be seen as a linear system with an input and an output, where the states at the input or output can be described by two state variables in a state vector. By choosing different state variables in the state vectors, the two-port can be obtained in different forms, such as the transfer matrix form, the mobility matrix form, and the scattering matrix form [9]. In this paper the transfer matrix method is used. The system is considered to be composed of several duct elements connected in cascade. The transfer matrix for the complete system is generated by successively multiplying the transfer matrices of all included duct elements.

Another way to describe the sound propagation is by using the finite element method (FEM), where a model consisting of discrete finite elements is used to approximate the geometry of the duct system [10]. Compared to two-port theory, FEM has several advantages. For example sound propagation above the plane wave propagation region can be modeled. Further, it has no limitation with respect to geometry of the duct elements whereas no closed form expressions generally exist for the transfer matrices of complex duct elements [8].

The performance of an acoustic filter is generally measured using one or several of the parameters: Insertion loss (IL), transmission loss (TL), or noise reduction (NR) [8]. Insertion loss is the difference between the acoustic power radiated without-, and with the filter [8]. Transmission loss is the difference between the acoustic power incident on the filter and that transmitted downstream when using an anechoic termination [8]. Noise reduction is the difference between sound pressure levels at two arbitrarily selected points, e.g. upstream and downstream of the filter [8]. The three different measures have advantages and disadvantages, even though IL is the one that represents the true performance of the filter for the user [8]. However, NR is the easiest to measure since it does not require knowledge of source impedance and does not need anechoic termination. Therefore it has become a commonly used measure for experimental verification of calculated transmission behavior [8]. Further, a feedforward ANC system adapts to filter the primary noise signal as a part of the acoustic path between the reference-and error microphones does. This can be seen as that the ANC system performs a system identification of a part of the acoustic path between the reference-and error microphones. Thus, of the three measures IL, TL, and NR, NR in some sense is the closest of what the ANC sytem uses to perform the system identification.

With the future goal of being able to model different complex duct systems including one or several ANC systems, in this paper initial investigations concerning modelling of four different duct systems assembled based on standard simple duct parts were performed. Simulations were carried out to find the NR, using both two-port theory and FEM. The results of the simulations were compared with measurement results. The results show relatively good correlation between the estimates produced using the different methods in the frequency range up to the cut-on frequency of the first higher order mode.

2. Materials and Methods

2.1 Experimental Setup

Noise reduction of four duct elements with circular cross-section, here considered as acoustic elements, was estimated experimentally as well as using two-port theory and FE modelling in four different setups. The considered duct elements were: a linear duct element (i.e. a straight duct), a sudden area expansion and a sudden area contraction between two linear duct elements, and a complex duct element consisting of these three elements. Noise reduction of an acoustic element is defined as follows [7]:

$$NR = 10\log_{10}\left|\frac{p_1^2}{p_2^2}\right| \tag{1}$$

where p_1 and p_2 are sound pressure levels at the two sides of the element.

The experiments were conducted using the following instrumentation: two microphones G.R.A.S. 40AE supported by two pre-amplifiers G.R.A.S. 26CA, loudspeaker JBL high power 400W GTX series, signal analyzer HP35670A.

The schematic illustrations of the test arrangements for noise reduction estimation carried out in a duct system are shown in the Figs. 1-4. A duct system here is a system that consists of a loudspeaker enclosed into the chain of considered duct elements and terminated by a concrete block. The part of the duct system between the two microphones denoted "mic1" and "mic2" is considered here as the acoustic element. However, it should be noted that the noise reduction between the microphones will be dependent on the complete duct system, not just the duct element itself [7]. A normally distributed random noise excitation signal in the frequency range 0 - 800 Hz is generated by the signal analyzer and transmitted by the enclosed loudspeaker, coupled with the duct system input cross-section. The microphone denoted "mic1" is placed closest to the loudspeaker at the position considered to be at the input cross-section of the acoustic element. The distance between "mic1" and the output cross-section of the complete duct system is denoted by $x1$. The microphone denoted "mic2" is placed at the position considered to be at the output cross-section of the acoustic element. The distance between microphone "mic2" and the output cross-section of the complete duct system is denoted $x2$. The values of the distances $x1$ and $x2$ for each setup are given in the figure captions of the setups schematic illustrations, see Figs. 1-4.

The lengths of the tested duct elements are $L1 = 1.72\ m, L2 = 1.02\ m, L3 = 0.15\ m$ and diameters are $d1 = 0.315\ m, d2 = 0.5\ m$. The output of the complete duct system is closed by the 0.17 m thick concrete block in the four experimental setups. The microphones signals are recorded and processed by the signal analyzer and the average of 50 power spectral density magnitudes of each microphone is acquired. The frequency resolution in the power spectral densities estimates was set to 0.5 Hz.

The sound pressure at the the microphone "mic1" position is estimated by power spectral density, denoted by $\hat{G}_{11}^{PSD}(f)$ and the sound pressure at the the microphone "mic2" position is estimated by power spectral density, denoted by $\hat{G}_{22}^{PSD}(f)$. Noise reduction of the duct element is estimated experimentally as given by Eq. 2.

2.2 Estimation of Noise Reduction

Noise reduction of a duct element can be estimated based on the power spectral densities estimates of the sound pressure obtained at two different positions:

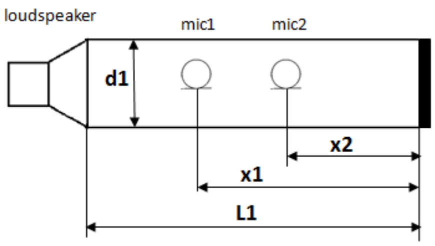

Figure 1. Setup 1 for NR estimation of a linear duct element, $x1 = 0.88\ m, x2 = 0.30\ m$.

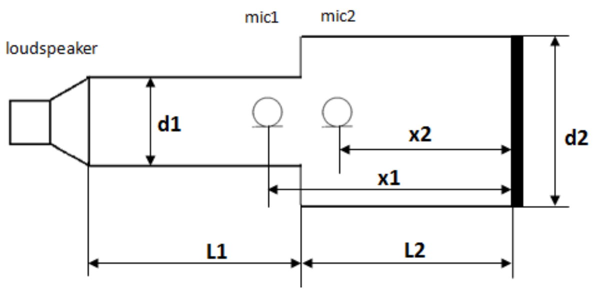

Figure 2. Setup 2 for NR estimation of a sudden area expansion enclosed by two linear duct elements, $x1 = 1.06\ m, x2 = 0.98\ m$.

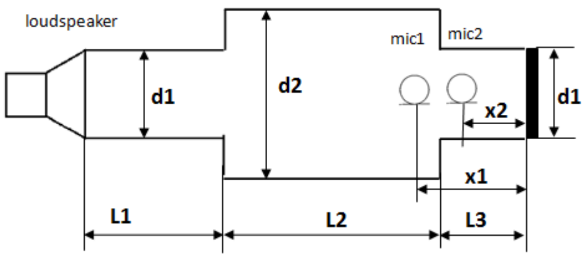

Figure 3. Setup 3 for NR estimation of a sudden area contraction enclosed by two linear duct elements, $x1 = 0.18\ m, x2 = 0.10\ m$.

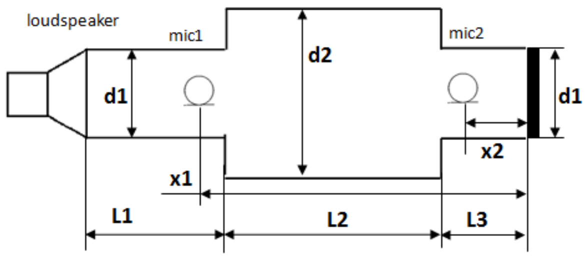

Figure 4. Setup 4 for NR estimation of a complex duct element, $x1 = 1.21\ m, x2 = 0.10\ m$.

$$NR(f) = 10\log_{10}\left(\frac{\hat{G}_{11}^{PSD}(f)}{\hat{G}_{22}^{PSD}(f)}\right). \qquad (2)$$

where the power spectral densities of the sound pressure signals at the position 1, $\hat{G}_{11}^{PSD}(f)$, and the position 2, $\hat{G}_{22}^{PSD}(f)$, can be obtained using e.g. Whelch's method [11].

2.3 Two-Port Theory and Transfer Matrices

The simplest mathematical description of acoustic wave propagation in ducts can be done using the concept of two-ports established originally in electrodynamics [12]. In the case of acoustic two-ports, sound pressure p and mass velocity v compose the vector of the state variables at each port, see Fig. 5. The state variables at the port 1, $[p_1, v_1]^T$, are related to the state variables at the port 2,

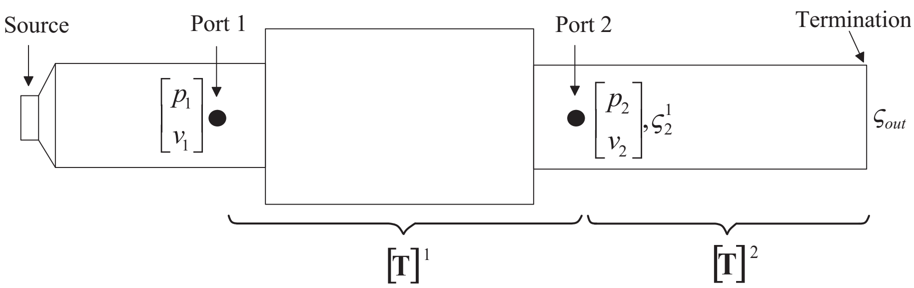

Figure 5. A scheme of a general duct representing two-port theory.

$[p_2, v_2]^T$, linearly versus transfer matrix $[\mathbf{T}]^1$, see Eq. 3 [8]. Acoustic two-ports can only be used to describe acoustic field in ducts within the plane wave limit.

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T^1_{11} & T^1_{12} \\ T^1_{21} & T^1_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \tag{3}$$

The transfer matrices of fundamental acoustic elements such as ducts, sudden area discontinuities etc., are reported in [8, 12]. An arbitrary shaped duct composed of simple acoustic elements may be modeled using two-port theory. Its transfer matrix is obtained as a matrix product of transfer matrices for included simple acoustic elements.

If the transfer matrix $[\mathbf{T}]^1$ of an arbitrary shaped duct element is known, the noise reduction, i.e. the difference between the sound pressure levels on the two sides of the duct element is calculated as follows [7]:

$$NR_{TM} = 10log_{10}((T^1_{11} + T^1_{12}/\zeta^1_2)^2). \tag{4}$$

where T^1_{11} and T^1_{12} are frequency dependent element of the transfer matrix of an arbitrary shaped duct element and ζ^1_2 is the output impedance for this duct element. The noise reduction can be calculated between any two positions inside the duct. The output impedance ζ^1_2 is calculated based on the output impedance for the duct ζ_{out} and transmission matrix between the position 2 and termination of the duct $[\mathbf{T}]^2$ [7]:

$$\zeta^1_2 = (T^2_{12}\zeta_{out} + T^2_{12})/(T^2_{21}\zeta_{out} + T^2_{22}). \tag{5}$$

In correspondence to the experimental investigations and the FE model simulations (discussed further), the two-port theory was used to model a duct terminated by a rigid wall. In this case the output impedance of the duct is $\zeta_{out} = \infty$ [7].

2.4 Finite Element Modelling

In general case of a duct with complex geometry (e.g., containing bends or parts with varying cross-sectional area), the closed form expression for the transfer matrices of its elements are not available. The elements of such transfer matrices may either be calculated by numerical integration of the wave equation [8] or be estimated experimentally [12].
One way to describe the sound propagation in an arbitrary shaped duct is by utilizing its finite element model. In finite element modelling the acoustic medium of the duct, a spatial model, assembled with discrete finite elements connected via the endpoints called nodes that approximate the actual spatial geometry of the duct, is produced. The sound pressures at each node are considered as degrees of freedom [10]. A spatial solution is assumed for each finite element and approximated by a low-order polynomial known as a shape function.

The finite element models of the simple duct elements were built using commercial finite element software MSC.MARC. The acoustic modes of the duct can be calculated based on equation of dynamic equilibrium, i.e.

$$[\mathbf{M}]\{\ddot{\mathbf{p}}(t)\} + [\mathbf{K}]\{\mathbf{p}(t)\} = \{\mathbf{0}\}, \tag{6}$$

where $[\mathbf{M}]$ is the $N \times N$ acoustic mass matrix of the duct, $[\mathbf{K}]$ is the $N \times N$ acoustic stiffness matrix of the duct, and $\{\mathbf{p}(t)\}$ is the space and time dependent $N \times 1$ sound pressure vector. Here N is the number of degrees of freedom of the finite element model [10]. The modal analysis was conducted by using the Lanczos iterative method in the of MSC.MARC software [13]. The transient response of the duct excited by a point source was also calculated by means of the modal superposition method based on the results previously obtained from the modal analysis of the duct [13].

2.4.1 "3-D" FE Models of Simple Duct Elements

As a basic finite element a tetrahedron was chosen, as the most convenient element to describe the geometry of an arbitrary shaped duct element. Three finite element models corresponding to the four experimental setup were developed (since geometries of the complete duct in the setups 3 and 4 are identical see Figs. 3-4). The FE models are shown in Fig. 6.

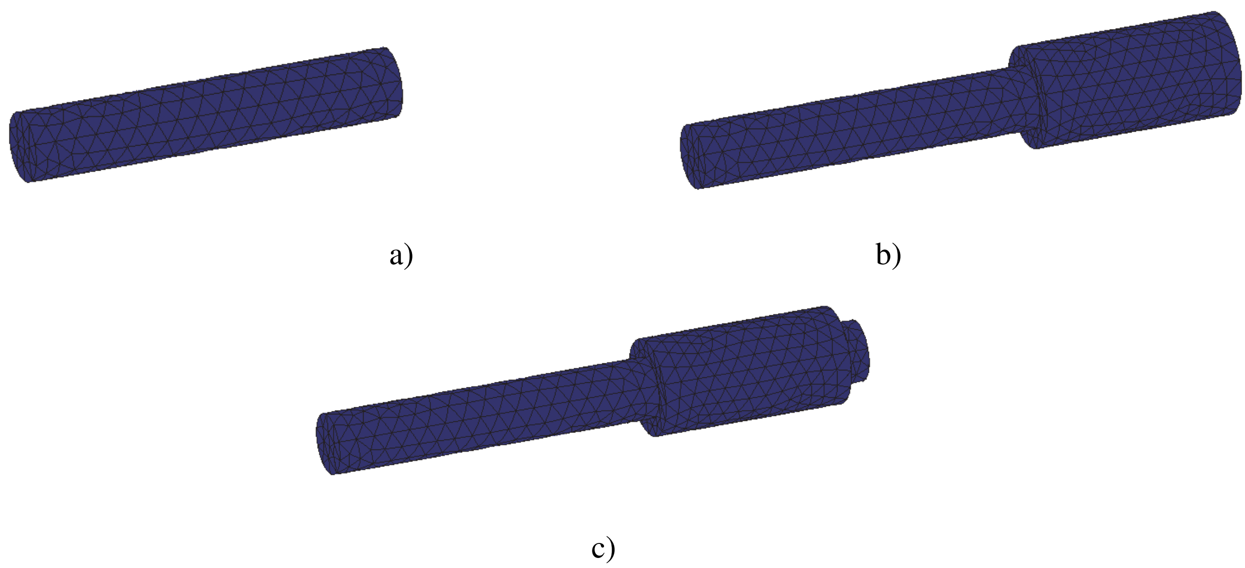

Figure 6. The "3-D" finite element model of a) a linear circular cross-section duct element, b) a sudden area expansion enclosed by two linear duct elements and c) a complex duct element.

2.4.2 Boundary Conditions

Since it was not possible to model sound propagation through the open end of the duct using available software, the acoustic medium of the duct was modeled as a cavity with rigid reflecting boundaries. Therefore as a boundary condition the pressure gradient at the nodes of the FE model that are located at the boundary is set to zero. This was the reason of ducts termination with a concrete block during experimental investigations as well as making corresponding assumptions for the transfer matrix method.

The noise reduction between any two positions 1 and 2 in the duct can be estimated based on the results of the transient simulation of its finite element model according to Eq. 2. A normally distributed random noise was with a flat spectrum and RMS value of 1 $[Pa]$ was applied as a point source to the nodes of FE models of ducts corresponding to the location of the loudspeakers in the respective experimental setups. The power spectral densities estimates were produced based on 10 averages, with the frequency resolution of 0.5 Hz.

3. Results

Noise reduction of four circular cross-section duct elements, estimated by experiments, FEM and transfer matrix theory are presented in this section. In Figs. 7-10 the solid black line represents the results using transfer matrix theory, the dashed line the results using FEM, and the solid grey line the experimental measurement results. Noise reduction of the linear duct element (see Fig. 1) is shown in the Fig. 7. Noise reduction of the sudden area expansion enclosed between two linear duct elements (see Fig. 2) is shown in the Fig. 8. Noise reduction of the sudden area contraction enclosed between two linear duct elements (see Fig. 3) is shown in the Fig. 9. Noise reduction of the complex duct element consisting of sudden area expansion, linear duct elements and sudden area contraction (see Fig. 4) is shown in the Fig. 10.

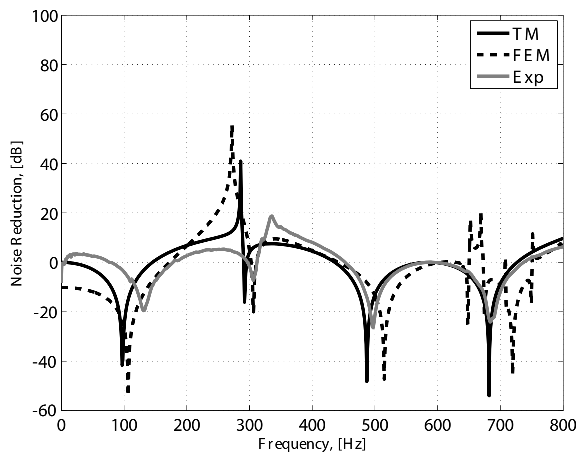

Figure 7. Noise reduction of the linear duct element.

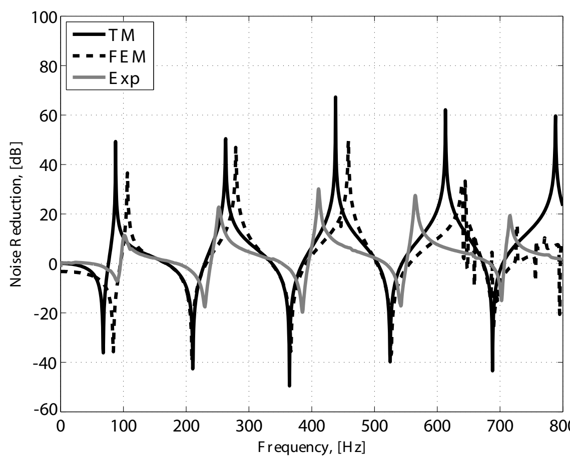

Figure 8. Noise reduction of the sudden area expansion enclosed between two linear duct elements.

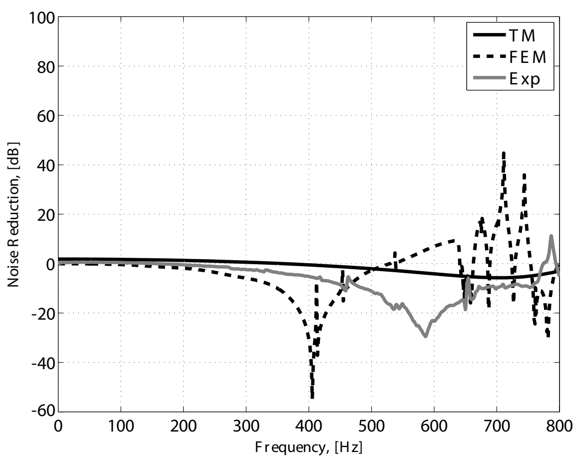

Figure 9. Noise reduction of the sudden area contraction enclosed between two linear duct elements.

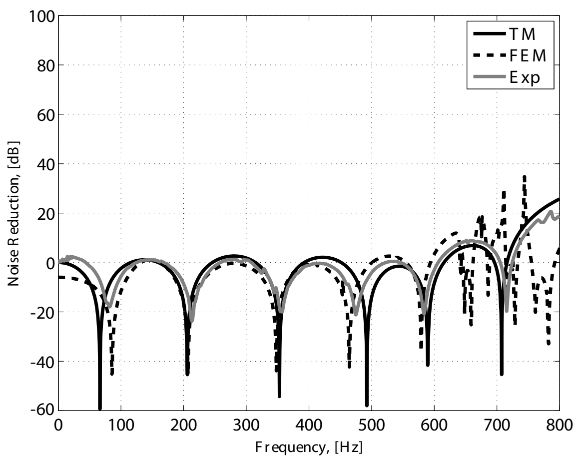

Figure 10. Noise reduction of the complex duct element consisting of sudden area expansion, linear duct elements and sudden area contraction.

4. Conclusions

Noise reduction performance of four duct elements with different geometry was estimated experimentally, using finite element modeling and transfer matrix theory. The results presented in Figs. 7-10 show fairly good correlation between the estimates produced using different methods in the frequency range up to the cut-on frequency of the first higher-order mode which is around 640 Hz for setup 1, 407 Hz for setup 2, and 412 Hz for setups 3 and 4. However, disturbances can be observed in noise reduction estimates produced for all setups based on the corresponding finite element models in the higher frequency range above the plane wave limit. This is due to the fact that higher order modes were considered in transient response simulation using the finite element models. Since the closed form expression for an arbitrary shaped duct element is often not available, finite element modeling seems to be a fairly accurate and convenient method to predict the sound propagation in ducts.

5. Acknowledgment

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