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Quantification of Packet Delay Variation through the Coefficient of Throughput Variation

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ABSTRACT

Packet delay variation plays an important role in network performance degradation and affects the user-perceptual quality, especially in case of real-time services such as video streaming, VoIP etc. Lightweight methods for detecting network performance issues are desirable as compared to task-intensive measurement and analysis. On this background, this paper discusses the applicability of the Coefficient of Throughput Variation (CoTV) to quickly and reliably detect bottleneck behavior between two arbitrary points in a network. The CoTV can be used as a Reduced Reference Metric. It is relatively simple to calculate, compare and interpret, yet powerful to provide control feedback when facing changes in network performance. In this paper, we demonstrate above mentioned properties of the CoTV through a formula relating it to the sample interval of the throughput and the variability of the delay. The latter is shown to be detectable on time scales that are significantly larger than that of the variability itself. This observation is a key enabler for reducing the load on the device that performs the analysis due to the possibility to use large sample intervals. We also describe how difference and ratio of CoTV at outlet and inlet can be used to identify network performance issues.

Categories and Subject Descriptors

C.2.3 [Computer Communication Networks]: Network Operations

General Terms

Measurement, Performance Analysis, Traffic Shaper

Keywords

Packet Delay Variation, Throughput, Coefficient of Throughput Variation, Network Measurement Traffic Shaping

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IWCMC'10, Jun 28- July 02, 2010, Caen, France

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1. INTRODUCTION

We are moving into a period where more people will use mobile handsets, or small handled devices, to access services via networks (this includes mobile, wireless and wired networks). The performance of the services will be crucial to ensure that the users remain loyal to the service, and that the user group grows. However, it is a challenge to monitor the individual service perception for a particular user, in a centralized way [1]. It becomes even more challenging if the user is located on a mobile or handled device that is connected to a mobile or wireless network. To this end we are investigating lightweight measurement techniques and analysis methods to enable individual service monitoring.

From earlier work [2], we are aware of that the comparison of traffic characteristics at inlet and outlet of a network provides information of the nature and wildness of a potential bottleneck in-between. In this paper we use the *Coefficient of Throughput Variation* (CoTV) as indicator for performance problems. At this point, we focus on packet delay variation instead of data loss as in [2]. Because for certain applications packet delay variation has a severe impact on the user perception as compared to occasionally lost packets. As we want to leave a minimal footprint on the monitored system, we will focus on time scales that are relatively high as compared to the disturbances inflicted by the network. However, the time scale may not be so big that we miss important information. We will also describe the benefit of using comparative analysis based on the CoTV at the inlet and outlet of a network to highlight the bottleneck in between. The results are underlined by experiments.

The Coefficient of Variation (CoV) is used in many areas, here we just mention some papers that are related to our work. In some cases, the CoTV is used explicitly. In [3] the authors simulate TCP behaviour over wireless wide area networks. Their results hint that a satisfactory TCP behaviour implies a CoTV of not more than 10%. Similarly [4] use CoTV as the performance metric for measuring stability of the long-term sending rate, when analysing the performance of two different TCP congestion control mechanisms, MulTCP [5] and PA-MulTCP [4]. The CoV of the application level throughput is used in [6] to measure the relative spread in per-user goodput, i.e the rate at which the destination application receives information. This work is done to evaluate the TCP rate control algorithm. In [7] the CoV of the service time for a packet is used to explain the behaviour of mean re-sequencing delay and the variability of the service time distribution. To analyze application

performance, [8] reports that for Skype via UMTS network, a growth of the CoTV by 10% implies a decrease in the Perceptual Evaluation of Speech Quality (PESQ) value from about 2.5 to 2.0, which shows an interesting parallel to the above-cited result [3].

The outline of this paper is as follows: In Section 2 we discuss definitions and method. This is followed by an analytical approach to derive the CoTV from delay variations with known distribution in section 3. After this we provide a use case; done in a lab (section 4) using a network emulator and artificial traffic, with the goal to establish a relationship between CoTV and delay variation and to show CoTV's capability to detect and quantify changes in the delay. This is followed by conclusions and future work in section 5.

2. DEFINITIONS AND METHOD

Let the bit rate R_i in interval i be calculated as the number of bits that have arrived in sample interval i , divided by the sample interval duration ΔT .

$$R_i = \frac{b_{i,-} + \sum_{k=1}^N b_k + b_{i,+}}{\Delta T} \quad (1)$$

And let R_i^P specify the bit rate at a point of reference (location) specified by P . Using this, the average bit rate is over a window of n intervals calculated as

$$m_{R,j}^P = \frac{1}{n} \sum_{i=j-n+1}^j R_i^P, \quad j \geq n. \quad (2)$$

where n defines the window size. The standard deviation is calculated as:

$$s_{R,j}^P = \sqrt{\frac{1}{n-1} \sum_{i=j-n+1}^j (R_i^P - m_{R,j}^P)^2}, \quad j \geq n. \quad (3)$$

Based on the average and standard deviation, we can now define the CoTV as

$$c_{V,j}^P = \frac{s_{R,j}^P}{m_{R,j}^P}. \quad (4)$$

As we are interested in steady state analysis, we let n span the whole session, i.e. $c_V^P = c_{V,n}^P$.

Furthermore, we will also investigate two different ways of relating the input and output coefficient of variation to each other. The first is the *CoTV difference*

$$\Delta c_{V,j} = c_{V,j}^{\text{out}} - c_{V,j}^{\text{in}}, \quad (5)$$

the second is the *CoTV ratio*

$$\gamma_{V,j} = \frac{c_{V,j}^{\text{out}}}{c_{V,j}^{\text{in}}}, \quad c_{V,j}^{\text{in}} > 0. \quad (6)$$

3. THEORY

Given that we have a bit rate stream that is injected into the network. In the network the stream is subjected to a uniform delay distribution $D \pm \Delta D$. Furthermore, we assume the following:

- $\Delta T \geq \Delta D$, as we are particularly interested in the scaling behaviour of the CoTV for large sample intervals ΔT ;
- $n \rightarrow \infty$, i.e. steady state;

- $R_i^{\text{in}} \equiv 0$, i.e. a constant rate at the inlet of the shaper, which means that c_V^{in} is undefined.

The latter implies that with a nominal inter-packet time of ΔT_p , we observe a nominal number of

$$N = \frac{\Delta T}{\Delta T_p} \quad (7)$$

packets per interval at the inlet. For the resulting coefficient of throughput variation, we obtain

$$\hat{c}_V^{\text{out}} = \sqrt{\frac{\frac{\Delta D}{\Delta T} (1 - \frac{\Delta D}{2\Delta T})}{N}}, \quad \frac{\Delta D}{\Delta T} \leq 1. \quad (8)$$

A sketch of the proof of Equation (8) is found in the appendix.

We now turn our attention to the scaling behaviour of (8) for large sample intervals, i.e. $\Delta T \gg \Delta D$. If the inter-packet time remained unchanged, a growth of ΔT by a certain factor implies a growth of N by the same factor according to (7). This observation leads to

$$\hat{c}_V^{\text{out}} \sim \frac{1}{\Delta T}, \quad \Delta T \gg \Delta D > 0. \quad (9)$$

Interestingly enough, Equation (9) communicates that as soon there is a delay variation on a constant-rate stream ($\Delta D > 0$), the consequences are seen in the coefficient of throughput variation on *any* larger time scale, no matter how large it is. However, the impact of the – uniformly distributed – ΔD onto the CoTV vanishes with $O(\frac{1}{\Delta T})$ so that, in practice, it might become unobservable at very large time scales.

On the other hand, given the number of nominal packets to be observed in an interval N and the actually measured coefficient of throughput variation at the outlet of a potential bottleneck $\hat{c}_V^{\text{out}}$, we can actually estimate the variable component of the delay as

$$\Delta \hat{D} = \left(1 - \sqrt{1 - 2N(\hat{c}_V^{\text{out}})^2}\right) \Delta T. \quad (10)$$

In other words, throughput estimations on large time scales can help to get an idea of (equivalent) delay variations that act on much smaller time scales.

In the following section we will use this analysis method for the use case of a traffic shaper.

4. TRAFFIC SHAPER

Traffic shapers [9, 10, 11] can be used to emulate networks with regards to throughput, delay and loss. Here we use the network emulator NetEm [10] to study the effect of jitter on the CoTV. Into such an emulated network, we inject test traffic at (approximately) constant rate $1/\Delta T_p$. The shaper imposes varying delay for the traffic flows on a similar path. By default NetEm uses the uniform distribution for delay variation, hence further expressed as $D \pm \Delta D$ [10]. We introduced a constant delay $D = 100 \text{ ms} > \Delta D$ in order to avoid that $D - \Delta D$ becomes negative and that the uniform distribution becomes truncated.

4.1 Experiment

The experiment setup is shown in Figure 1. The sender (S) sends 10 000 UDP datagrams with constant size at a rate of 1000 packets/s via the shaper to the destination (D). Furthermore the network is a 100BaseT-FD network, and the

Figure 1: Experiment Setup

shaper acts as a bridge. The cables that connect the sender and destination to the shaper are tapped using wiretaps. From these taps, we connect the cables in such a way that all traffic going from sender to destination will be sent to one measurement point (MP), in this case MP1, while the traffic going in the other direction will be sent to MP2. This allows us to get reliable time synchronization for the packets [12, 13]. The measurement setup uses the Distributed Passive Measurement Infrastructure (DPMI) [14] to control and manage the measurements. In this case the MPs were equipped with DAG3.5 cards [15]. The data collected by the MPs are captured by a consumer and stored to file for off-line analysis.

For the evaluation we used four different network settings. We focused on emulating, delay and varying delay, henceforth described by $D \pm \Delta D$. The settings used were $(100 \pm \{0, 10, 20, 40\})$ ms. For each setting we obtained results from 32 runs, sufficient for calculating confidence intervals. To minimize uncertainties in the bit rate values, we used a tool that can handle fractional bits [13].

4.2 Comparison between Inlet and Outlet

Figures 2 and 3 show the CoTV at the inlet and outlet of the shaper on the y -axis versus the time interval ΔT on the x -axis. Table 1 shows the numerical results of estimated and calculated CoTV for $\Delta T \geq 10$ ms, where the 3rd and 4th column, $\hat{c}_V^{\text{in}}$ and $\hat{c}_V^{\text{out}}$ are the estimated CoTV at inlet and outlet of the shaper with confidence intervals, and in 5th column γ_V is defined by (6). In the last column, $\hat{c}_V^{\text{out}}$ is the calculated outlet from (8).

In Figure 2 the delay was set to 100 ms. We see that the inlet and outlet CoTV are not identical on all time scales. Apparently, system and shaper enqueueing and dequeuing latency do affect the CoTV. But for $\Delta T \geq 50$ ms both CoTVs are almost identical. As ΔT decreases, the outlet CoTV grows faster than the inlet CoTV. On the other hand, the absolute values of CoTV for $\Delta T = 1$ ms are still quite small. In Table 1, for $\Delta D = 0$, the values in the last column γ_V are approximately equal to one, which emphasises that the values of inlet and outlet CoTV are nearly equal for all ΔT .

We are turning our attention to Figure 3, where the shaper now applies a varying delay of (100 ± 10) ms. We notice that the CoTV for the outlet requires a different scale (ten time larger), as compared to the fixed delay in Figure 2, whereas CoTV values at the inlet are similar to the $\Delta D = 0$ case. The delay variation of $\Delta D = 10$ ms has perceptible impact on the CoTV for outlet on almost all time scales below 1 s. As ΔT is increasing, the gap between CoTV of outlet and inlet is decreasing, but both remain distinct from each other. While looking at Table 1, we also see that $\hat{c}_V^{\text{out}}$ is increasing with a decrease in ΔT , and γ_V is greater than one for $\Delta D = 10$ ms for all time scales. Figure 4 exhibits

a similar behavior to that shown in Figure 3, except that the variable delay was (100 ± 40) ms and that a significant difference between $\hat{c}_V^{\text{in}}$ and $\hat{c}_V^{\text{out}}$ occurs for larger values of ΔT .

Figure 5 compares the outlet CoTVs for $\Delta D \in \{0, 10, 20, 40\}$ ms. All outlet CoTVs for $\Delta D > 0$ demonstrate comparable behavior; they follow a similar path (as if they were shifted versions of each other) and increase as ΔT decreases. From $\Delta T = 3$ ms to $\Delta T = 0.7$ s, the curves are disjoint. Considering for example $\Delta T = 100$ ms and Table 1, the value of the outlet CoTV 0.027 grows from 0.027 via 0.037 to 0.053 for $\Delta D = \{10, 20, 40\}$ ms, respectively. Not surprisingly more jitter implies a growth of the outlet CoTV.

Table 1: Estimated CoTV at inlet and outlet of the shaper with calculated CoTV for outlet of shaper

ΔD [ms]	ΔT [ms]	$\hat{c}_V^{\text{in}}$	$\hat{c}_V^{\text{out}}$	γ_V	$\hat{c}_V^{\text{out}}$
0	1000	0.002±0.0002	0.002±0.0002	0.970	0.000
	500	0.002±0.0002	0.002±0.0003	0.982	0.000
	200	0.004±0.0003	0.004±0.0004	0.985	0.000
	100	0.006±0.0005	0.006±0.0005	1.003	0.000
	40	0.013±0.0007	0.012±0.0005	0.981	0.000
	20	0.017±0.0008	0.019±0.0007	1.124	0.000
	10	0.02±0.0001	0.028±0.0007	1.373	0.000
10	1000	0.001±0.0001	0.003±0.0003	1.911	0.003
	500	0.002±0.0002	0.005±0.0003	2.379	0.006
	200	0.004±0.0003	0.014±0.0007	3.418	0.016
	100	0.006±0.0004	0.027±0.0008	4.678	0.031
	40	0.012±0.0005	0.064±0.0015	5.239	0.074
	20	0.016±0.0006	0.131±0.0019	8.052	0.137
	10	0.02±0.0006	0.243±0.0019	12.269	0.224
20	1000	0.001±0.0002	0.004±0.0004	2.865	0.004
	500	0.002±0.0002	0.008±0.0005	3.832	0.009
	200	0.004±0.0003	0.019±0.0007	5.186	0.022
	100	0.005±0.0004	0.037±0.0011	7.057	0.042
	40	0.012±0.0004	0.093±0.0014	7.926	0.097
	20	0.016±0.0006	0.171±0.0020	10.861	0.158
	10	0.019±0.0006	0.278±0.0020	14.828	N/A
40	1000	0.002±0.0002	0.006±0.0006	3.616	0.006
	500	0.002±0.0002	0.011±0.0011	5.015	0.012
	200	0.004±0.0003	0.026±0.0012	6.763	0.030
	100	0.006±0.0005	0.053±0.0017	9.576	0.057
	40	0.012±0.0005	0.125±0.0022	10.251	0.112
	20	0.016±0.0008	0.201±0.0021	12.244	N/A
	10	0.02±0.0008	0.301±0.0021	15.107	N/A

4.3 Validation of the Theoretical Results

We now turn our attention on the validation of Equation 8. In Table 1, $\hat{c}_V^{\text{in}}$ is almost constant for any timescale and naturally for all ΔD , while $\hat{c}_V^{\text{out}}$ is varying. We observe some minor deviations between $\hat{c}_V^{\text{out}}$ and $\hat{c}_V^{\text{out}}$ that in many cases make the estimated value lie just outside of the corresponding confidence intervals. However, when performing this comparison, it has to be realised that the theoretical value $\hat{c}_V^{\text{out}}$ assumes that the inlet CoTV should be zero, which is not the case in practice. Actually, the CoTV difference Δc_V in combination with the corresponding confidence intervals capture $\hat{c}_V^{\text{out}}$ in most cases.

Given these circumstances, the degree of the matching between measured value $\hat{c}_V^{\text{out}}$ and predicted value $\hat{c}_V^{\text{out}}$ might be regarded as satisfactory. We also notice the scaling behaviour of both analytically and experimentally determined CoTVs. The transition from $\Delta T = 100$ ms to $\Delta T = 1$ s implies factors of roughly ten as predicted by Equation 9.

Figure 2: CoTV at inlet and outlet of the shaper for different ΔT , shaper delay (100 ± 0) ms.

Figure 3: CoTV at inlet and outlet of the shaper for different ΔT , shaper delay (100 ± 10) ms.

Figure 4: CoTV at inlet and outlet of the shaper for different ΔT , shaper delay (100 ± 40) ms.

4.4 The Role of the CoTV Ratio

The CoTV ratio γ_V is approximately equal to one in case of $\Delta D = 0$, as expected. In all other cases, γ_V is significantly greater than one and rising as ΔD increases and ΔT decreases. In other words, the CoTV ratio reveals the existence of significant jitter $\Delta D > 0$ on a time scale ΔT independently of ΔD .

One more interesting finding from Table 1 is that if ΔD gets equal to ΔT , we obtain typically a CoTV ratio $\gamma_V \geq 10$. In order to investigate the latter observation in more detail,

Table 2: Estimated CoTV at inlet and outlet of the shaper with calculated CoTV for outlet of shaper

PDU Size 256 Bytes					
$D \pm \Delta D$ ms	ΔT ms	$\hat{c}_V^{\text{in}}$	$\hat{c}_V^{\text{out}}$	Δc_V	γ_V
100±10	10	0.020±0.001	0.243±0.002	0.223	12.269
100±20	20	0.016±0.001	0.171±0.002	0.155	10.861
100±40	40	0.012±0.000	0.125±0.002	0.113	10.251
PDU Size 512 Bytes					
$D \pm \Delta D$ ms	ΔT ms	$\hat{c}_V^{\text{in}}$	$\hat{c}_V^{\text{out}}$	Δc_V	γ_V
100±10	10	0.026±0.019	0.247±0.01	0.221	9.521
100±20	20	0.013±0.001	0.173±0.002	0.160	13.455
100±40	40	0.011±0.001	0.126±0.002	0.116	11.945
PDU Size 1024 Bytes					
$D \pm \Delta D$ ms	ΔT ms	$\hat{c}_V^{\text{in}}$	$\hat{c}_V^{\text{out}}$	Δc_V	γ_V
100±10	10	0.015±0.002	0.236±0.003	0.221	15.856
100±20	20	0.010±0.001	0.172±0.002	0.162	17.076
100±40	40	0.008±0.001	0.124±0.002	0.117	15.823
PDU Size 1470 Bytes					
$D \pm \Delta D$ ms	ΔT ms	$\hat{c}_V^{\text{in}}$	$\hat{c}_V^{\text{out}}$	Δc_V	γ_V
100±10	10	0.012±0.002	0.236±0.002	0.224	18.979
100±20	20	0.010±0.001	0.170±0.002	0.160	17.522
100±40	40	0.008±0.001	0.124±0.003	0.117	16.524

we consider different PDU sizes [256 B, 512 B, 1024 B, and 1470 B]. The corresponding results are reported in Table 2. We can see that the difference Δc_V for different PDU sizes at $\Delta D = \Delta T$ is approximately equal for the different packet sizes, and that the ratio $\gamma_V \geq 10$ (except of one value of 9.521). However, the CoTV value ratio is increasing with the increase of the PDU size, as for bigger PDUs, the bitrate at inlet become smoother, cf. also Table 2.

In practice, we can employ the most recent observations to detect performance issues as follows: Consider a given value of $\hat{c}_V^{\text{in}}(\Delta T)$. On the same time scale, $\hat{c}_V^{\text{out}}(\Delta T)$ can be measured, and $\gamma_V(\Delta T)$ can be calculated. If $\gamma_V(\Delta T) > 10$, we can deduce the existence of uniformly distributed *equivalent delay jitter* of $\Delta D = \Delta T$. The real jitter distribution might look differently, but at least, an indication of the order of its magnitude is provided.

5. CONCLUSIONS AND OUTLOOK

In this paper we investigated to which extent the Coefficient of Throughput Variation (CoTV) can be used for detecting and quantifying performance problems. In particular, we are interested in practical methods that allow for estimating the bottleneck parameters using lightweight techniques, involving both the measurement and the analysis. Here, we particularly focused on variable delay as the

Figure 5: TV at inlet and outlet of the shaper for different ΔT , shaper delay ($100 \pm \{0, 10, 20, 40\}$) ms.

disturbance introduced by the network.

Based on a model involving uniformly distributed delays, we derived an analytical solution for the CoTV at the outlet of a network, given a non-varying traffic intensity at the inlet. Amongst others, this solution revealed the scaling behaviour of the CoTV for large sample intervals, which is asymptotically governed by the reciprocal value of the duration of the sample interval. This means not only that delays are visible on time scales far beyond their own time scale, but that vice versa the measurement of CoTV behind a bottleneck allows for estimating the underlying delay according to the chosen model. From comparisons to experimental results using a shaper that introduced uniformly distributed packet delay variation, we found evidence that the analytic expression captures the impact of that delay jitter onto the CoTV. Furthermore, the ratio of the CoTV at sender and receiver gave us a clear indicator about network disturbance in terms of equivalent uniformly distributed jitter. It was found that a CoTV ratio of ten or more revealed the existence of equivalent jitter in the same order of magnitude than that given by the observation interval. Thus the CoTV is a Reduced Reference Metric revealing jitter problems.

Future work will address how to extend the method and develop an analytical model that can work with any type of service and jitter. We will also investigate CoTV thresholds for different services and networks and the practical use of the method for discovering jitter issues under dynamic network conditions.

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Appendix

Sketch of the proof for Equation 8:

- Assume $\Delta D \leq \Delta T$ and independency of the delays.
- $\Pr \{\text{arbitraty packet arrives within }]D, D + \Delta T]\} = 1 - \frac{\Delta D}{2\Delta T}$.
- $\Pi_i = \Pr \{i \text{ out of } N \text{ packets } \in]D, D + \Delta T]\} = \frac{N!}{(N-i)! i!} \left(1 - \frac{\Delta D}{2\Delta T}\right)^i \left(\frac{\Delta D}{2\Delta T}\right)^{N-i}$.
- $\Pi_j = \Pr \{j \text{ out of } N \text{ packets from outside }]D, D + \Delta T]\} = \frac{N!}{(N-j)! j!} \left(\frac{\Delta D}{2\Delta T}\right)^j \left(1 - \frac{\Delta D}{2\Delta T}\right)^{N-j}$.
- $(c_V^{\text{out}})^2 = \sum_{k=0}^{2N} \left(\frac{k}{N} - 1\right)^2 \Pi_k$ with $\Pi_k = \Pr \{k \text{ packets } \in]D, D + \Delta T]\} = \sum_{(i,j): i+j=k} \Pi_i \Pi_j$.
- Simplify the expression.

The proof has so far been carried out analytically for $N = 2$ with aid of Mathematica, but a general N cannot be treated that way, as it appears in two sums.