Non-contact nonlinear acoustic damage localization in plates.

Part 2: Localized resonance through dynamically trapped modes. C.M. Hedberg (author to contact), and

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Abstract

This work is the second part of three that presents new tools to be used for damage localization in plates by nonlinear acoustical methods. It introduces an important in-plane localization technique, which is based on the existence of resonant spatially localized wave fields. The wave from the transducer is acting as a dynamic influence on the plate surface, making the waves reflect in a non-ideal way. The non-ideal reflections make the modes underneath the transducer have different resonant frequencies than the modes beside the insonified area.

They appear both for contact and non-contact sources. In the nonlinear damage localization application, the trapped mode wave field interacts with another signal at lower frequency. This results in sidebands around the high frequency whose amplitudes are related to the amount of damage underneath the transducer.

I. INTRODUCTION

This work is the second part of three presenting new tools that are useful for damage localization in plates by nonlinear acoustical methods for contact and non-contact sources. In the first part, non-contact air wave fields in a plate-air-plate system were investigated [1]. This paper treats the in-plane localization in the plate (in x - y), while the third will deal with the depth localization (in z), see Figure 1.

The trapped local mode approach in this paper is much more important for the *nonlinear* acoustic methods treated because they use lower frequencies than the linear ultrasonic methods.

Resonant waves are used in the fields of both linear and nonlinear nondestructive acoustic testing [2]. Examples of the resonant methods

are Resonant Ultrasound Spectroscopy [3], Nonlinear Wave Modulations Spectroscopy [4], Resonant Frequency Shift [5] and Slow Dynamics [6]. Most of the mentioned methods are nonlinear. The linear ultrasound use wavelengths that are smaller or on the same length scale as the damage. Thus the linear ultrasound waves often are in the shape of directional beams. They are by their nature local as they spread less and are more attenuated. On the other hand, with the nonlinear methods, be several orders smaller than the wavelength. The low frequencies used by the nonlinear methods are prone to spread all over the test objects because of the long wavelengths and low attenuation. Therefore, these waves are not local and it is difficult to obtain localization by the ordinary nonlinear acoustic methods. In this work we are looking at plates with the in-plane dimensions x and y being much larger than the thickness z , see Figure 1. At the same time the thickness may not be too small ($z_0 > z_{min}$). Plate structures exist for example in buildings, airplanes, tubes, containers, ships, and cars.

FIGURE 1. The non-contact source above a plate.

There are three reasons why a local wave can be necessary in nonlinear nondestructive testing: 1) When the object is very large, the complete object can not be examined all at once, examples are buildings or ships. 2) When closely lying nonlinear regions create disturbances. 3) When an image of the position and extent of damage is needed.

One common nonlinear methods in use to detect damage is the Non-linear Wave Modulation Spectroscopy (NWMS) [7], [8], [9], [10], [11], [12]. Like the other nonlinear methods, it is based on the fact that material with cracks has much greater material nonlinearity than one with no

cracks. Therefore, by measuring the material nonlinearity, an indication of the damage level is obtained. The general approach to do this can be understood by considering propagation of a one-dimensional wave with two frequencies inside a material

$$u(x = 0, \tau) = u_0(\tau) = a_L \sin(\Omega_0 \tau) + a_H \sin(\omega_0 \tau) . \quad (1)$$

The parameter $\tau = t - x/c_0$ is the retarded time, where t is time, x is distance from source, and c_0 is the sound velocity. We assume that the angular frequencies fulfill $\omega_0 \gg \Omega_0$. The nonlinear interaction between the waves (here expressed in particle velocity u) will be obtained by investigating the solution (2) (see e.g. [13] [14]) for a small distance x away from the source:

$$u = u_0(\tau + \frac{\epsilon}{c_0^2} u x) , \quad (2)$$

where ϵ is a material nonlinearity parameter. The signal (1) inserted into the solution (2) gives us an approximate solution at $z = \epsilon x/c_0^2$:

$$u(z, \tau) = a_L \sin(\Omega_0[\tau + z(a_L \sin(\Omega_0 \tau) + a_H \sin(\omega_0 \tau))]) + a_H \sin(\omega_0[\tau + z(a_L \sin(\Omega_0 \tau) + a_H \sin(\omega_0 \tau))]) \quad (3)$$

With the MacLaurin expansion in z , $f(z) \sim f(0) + f'(0) \cdot z + \dots$, we get

$$\begin{aligned} u(z, \tau) = & a_L \sin(\Omega_0 \tau) + \underbrace{a_L \cos(\Omega_0 \tau) \Omega_0 \cdot z \cdot a_L \sin(\Omega_0 \tau)}_I \\ & + \underbrace{a_L \cos(\Omega_0 \tau) \Omega_0 \cdot z \cdot a_H \sin(\omega_0 \tau)}_{II} + a_H \sin(\omega_0 \tau) \\ & + \underbrace{a_H \cos(\omega_0 \tau) \omega_0 \cdot z \cdot a_L \sin(\Omega_0 \tau)}_{III} + \underbrace{a_H \cos(\omega_0 \tau) \omega_0 \cdot z \cdot a_H \sin(\omega_0 \tau)}_{IV} . \quad (4) \end{aligned}$$

The first and fourth terms are the linear source frequency terms. The more interesting ones are the nonlinear interaction terms I, II, III and IV (which are the second, third, fifth and sixth terms respectively).

The product with mixed frequencies in term II is

$$\cos(\Omega_0 \tau) \sin(\omega_0 \tau) = [\sin((\omega_0 + \Omega_0) \tau) + \sin((\omega_0 - \Omega_0) \tau)]/2 . \quad (5)$$

The product with mixed frequencies in term III is

$$\sin(\Omega_0 \tau) \cos(\omega_0 \tau) = [\sin((\omega_0 + \Omega_0) \tau) - \sin((\omega_0 - \Omega_0) \tau)]/2 . \quad (6)$$

Both terms II and III from equation (4) contribute to the sidebands at frequencies $\omega_0 + \Omega_0$ and $\omega_0 - \Omega_0$

$$\begin{aligned} u_{SB} = & 2a_L a_H z \{ \Omega_0 [\sin((\omega_0 + \Omega_0) \tau) + \sin((\omega_0 - \Omega_0) \tau)]/2 \\ & + \omega_0 [\sin((\omega_0 + \Omega_0) \tau) - \sin((\omega_0 - \Omega_0) \tau)]/2 \} \\ = & a_L a_H z \{ (\omega_0 + \Omega_0) \sin((\omega_0 + \Omega_0) \tau) - (\omega_0 - \Omega_0) \sin((\omega_0 - \Omega_0) \tau) \} \quad (7) \end{aligned}$$

There is an amplitude difference between the sidebands. The sideband with lower frequency is smaller than the upper by the factor $(\omega_0 - \Omega)/(\omega_0 + \Omega)$.

No sidebands exist when there are no cracks, because high levels of material nonlinearity is directly connected to cracks. The sideband amplitudes naturally increase with the amplitude of the low frequencies in the damage region. With a constant low frequency amplitude, the sideband amplitudes will increase (or decrease) along with the transducer high frequency amplitude and along the low frequency amplitude. Experiments that show this are described in reference [15]. The type of curve that is obtained from equation (4) is shown in Figure 2.

FIGURE 2. Frequency response spectrum after nonlinear propagation over the distance $x = 0.1$ meter of two initial frequencies $\omega_0 = 15000$ Hz and $\Omega_0 = 2000$ Hz. The nonlinearity parameter $\epsilon = 500$.

As one is searching for the nonlinear interaction between these two signals, it is enough to localize one of the wave fields, either $a_L \sin(\Omega_0 \tau)$ or $a_H \sin(\omega_0 \tau)$. It is easier to limit the geometric extent of the high frequency (HF) than the low frequency (LF).

For a free plate, there does not exist any localized trapped modes through the thickness. They do not exist in the free plates and do not appear when making eigen value finite element simulations. They exist only when an acoustic source fulfills the open resonance conditions for the plate thickness [1]. Therefore, in experiments they are found when the insonified surface area on the plate is large enough, while at the same time the frequency which gives resonance through the thickness is used. These kinds of localized vibrations, or trapped modes, may in general appear as a result of thickness variation [16],[17], bending of the domain [18], and perturbations of the elastic modulus [19]. A local change of the

boundary conditions as a possible reason for trapped modes is mentioned by Förster [19], but has not been thoroughly treated to our knowledge. In our case it is a *dynamic* local change in the boundary condition. The excitation can come from either a contact transducer or a non-contact air wave. The local property of such an open resonator wave field was noted in another publication [20] - while its physical explanation was not given. In the present article the explanation for the local resonant field will be presented, as well as measurements of both contact and non-contact localizations in plates.

II. CONCEPTS OF LOCAL ACOUSTIC OPEN RESONATOR WAVE FIELD

When placing a transducer on a plate, there can exist two kinds of wave fields which will limit the thickness resonance wave field, see Figure 3 top (Contact). The first way (type A) has a resonant wave being reflected by the transducer's far side. The transducer acts as an integral part of the medium. The time of propagation of the wave is the sum of the propagation time through the plate and the transducer. A local wave field appears because the mode for the system with transducer and plate has a frequency which is different from the wave field in the plate by itself. This resonant wave (type C) has soft reflections from both the top and bottom surfaces.

FIGURE 3. Plate with transducers. Top: contact transducer. Bottom: non-contact air transducer.

The second kind of wave field (type B) is where the interface between the transducer and plate acts as reflector. The wave coming in from the transducer to the interface affects the boundary pressure values and the acoustic impedances given by the normal material values can not be used.

The incoming wave affects the plate surface *boundary condition*. This mode propagates only through the plate, so it has a propagation time close to the propagation through the plate only, and has a higher frequency than type A. This kind of local wave field may appear only under conditions of resonance when the conditions for open resonator is fulfilled for the plate surface wave field (under the transducer). In short, the diameter of the wave field (the transducer) must be large enough, and must increase with increase in plate thickness and in wavelength λ [1].

The ideal estimated frequency of the mode A (completely through the sample and transducer) is $f_{Ai} = 1/(2a/c_s + 2b/c_t)$, where a is thickness and c_s is sound velocity of sample, and b is thickness and c_t is sound velocity of piezoelectric transducer. The "ideal" frequency of mode B, not accounting for the wave field influence, is simply $f_{Bi} = c_s/a = f_C$, but the real frequency of mode B contains a strong non-ideal influence. The non-ideal influence is expected to depend strongly on frequency, like the way any other mass-spring component shows a frequency dependent response. For practical purposes, the details about the exact influence of any specific non-ideal properties of the transducer-plate system are not important. *It will be enough to have the knowledge that this phenomenon exists, and therefore will create conditions for trapped mode local wave fields.*

For the non-contact case, Figure 3 bottom, the resonant modes of types B and C still exist, but not type A. The pressure field from the air on the plate surface under the transducer still causes the boundary condition to be non-ideal, making the mode type B have a different frequency from type C.

III. ONE DIMENSIONAL WAVE FIELD MEASUREMENTS

The simultaneous existence of mode types A and B in a one-dimensional system is evident from acoustics of layered media, see e.g. [21],[22]. The concepts are clear from a measurement indicating the influence of the transducer. The types A and B modes are shown for a steel bar of length $a = 0.119$ and diameter 15 mm where a transmitting piezoelectric transducer with diameter 30 mm and thickness 2.5 mm was attached to one

end. It was driven by an Agilent 33220A signal generator.

The fundamental frequency responses (for a steel bar length of $a \approx \lambda/2$) were found at $f_{qA} = 1/T_A = 21100$ Hz, and at $f_{qB} = 21300 = 1/T_s$ Hz, see Figure 4. The peak at 21 300 Hz is from the reflection against the interface between steel and transducer. The ideal frequency shift with an added length of $b = 2.5$ mm transducer material with sound velocity $c_T = 4030$ m/s is 135 Hz. The measured value is 200 Hz. The influence of the transducer is non-ideal, meaning that the reflection is neither perfectly soft or perfectly hard.

FIGURE 4. The peaks from the mode of type A (at 21 300 Hz) and from the type B (at 21 100 Hz) for a steel bar with a PZT attached to one end.

If we assume a perfect hard interface (ideal mode type B), making the reflections hard (immovable) on the transducer side and soft (pressure release) on the free plate surface we can evaluate the modes. For the resonant wave with one hard reflection and one soft, the relation between plate thickness a and wavelength λ is $a = \lambda/4 + n\lambda/2$, $n = 0, 1, 2, \dots$

For a plate, the regions outside the transducer area with two soft reflections the thickness resonance criterion is $a = n\lambda/2$, $n = 1, 2, 3, \dots$. For a given frequency, both these criteria can not be fulfilled - when resonance exist under the transducer it does not exist on the sides. This is the reason why the wave field is local when open resonance exist under the transducer.

In a one-dimensional system, like slender bars, the resonance of a layer of type B (see Figure 3) is seen as evident and exist for specific frequencies. For these kinds of systems the resonance which includes the wave passing through the transducer (type A) also always exist. What is not evident,

is that resonance of type B in a two-dimensional system must fulfill the open resonance criteria to exist. The resonance mode type C (soft-soft) is found outside the region with the incoming wave field.

IV. CONTACT LOCAL WAVE FIELD

The measurements with a 30 mm diameter transducer glued to a 5 mm thick, 800 mm wide and 1100 mm long plate of plexiglass is a good example of the resonance localization type A. The mode includes the transducer itself which has thickness $b = 2.5$ mm. The frequency estimate is $n \cdot \frac{1}{2} \cdot 1/(0.005/2700 + 0.0025/4030) = n \cdot 202236$ Hz. This gives the following estimates of those resonant mode frequencies: 202, 405, 607, 809, 1011 kHz.

The wave field was measured with a pressure sensor, PCB 132M14 with a diameter of 3.3 mm, which was attached to the plate by a medium hard wax. It was moved along lines away from the center of the transmitting transducer, see Figure 1. In the measurement a suitable frequency was 1034 kHz (to be compared to the theoretical value 1011 kHz), and the amplitude is shown as function of radial position r in Figure 5. Both the same side as the transducer (top) and the opposite side (bottom) were measured. For this frequency the wave field is very local with most energy inside the 15 mm transducer radius.

FIGURE 5. Wave amplitude for 5 mm plexiglass plate opposite ("Bottom"), and beside ("Top") the 30 mm diameter contact transducer with the frequency 1034.40 kHz. Horizontal axis: distance from center in mm.

V. NON-CONTACT LOCAL WAVE FIELD

It may be easy to claim that attachment of a structure on a plate is what makes the local wave field even of the type B, and that the interface conditions are a result of this only. But then, the similar effect would not exist for a non-contact transducer. That this happens will be shown in this section. The claim that the wave field is local because it is close to the transducer will be disproved by showing a wave field where the amplitude is near zero under the transducer, while there exist resonance on the sides.

Non-contact measurements were made on a 71.6 mm thick carbonite-divinyl-carbonite laminate plate. The non-contact transducer consisting of a 115 mm diameter plexi-glass plate with seven 30 mm PZT transducers glued onto it was placed at a height of 9.5 mm above the laminate plate. The surface of the transducer was flat because a distributed field on the plate surface was preferred [1]. Frequency sweeps were performed where the amplitude was measured by two PZT sensors. One sensor was attached by medium hard wax to the plate underside directly beneath the transducer device, and the other at 30 cm from the first one, see Figure 6.

FIGURE 6. Frequency sweeps of a laminate excited by an non-contact 115 mm transducer. The measurements were made directly under the transducer ("r=0 cm"), and at 30 cm distance.

There exist frequencies where the wave amplitude in the center ($r = 0$) is more or less zero, which happens at the anti-resonances. The pressure at $r = 30$ cm has relatively large values, approximately 1 kPa at frequency 249 kHz, while at the center, they are more or less zero.

The plate thickness mode type B from the contact transducer measurement is found for the non-contact excitation case too, see the bottom picture in Figure 3. The transducer excites the air, which in turn excites the plate. The air wave field will act, for the plate thickness modes, equivalently to the contact transducer field on the plate. This *non-contact* mode type B has the same origin in the non-ideal boundary conditions as the *contact* mode type B. Beside the transducer the boundary condition is soft and beneath the transducer the boundary condition is non-ideal. This means that there is a thickness resonance under the transducer whose eigen frequency will not be the frequency for the thickness resonance on the sides. It shows the existence of the anti-resonant modes of the air gap-plate system, and that the resonances for the air gap-plate system are not the same as for the plate by itself. Even though the plate is surrounded by air, the air interface under the transducer *does not act as a perfect soft boundary*. It acts as a partly hard surface.

The amplitude as a function of the radial distance from the center is shown for frequency 242.5 kHz in Figure 7. The high amplitude wave field is being concentrated almost fully within the radius of the non-contact transducer (57.5 mm).

FIGURE 7. Wave amplitude in laminate plate (thickness 71.6 mm), measured at the opposite side of the transducer, as function of radial distance from the center of the 115 mm diameter non-contact transducer (air-gap = 9.5 mm) at frequency 242.5 kHz. Horizontal axis: distance from center in mm.

It must be the incoming wave that makes the insonified region having different boundary condition from the rest of the plate surface, because the *only* difference between the resonant system conditions of the insonified region and the rest, is the insonification itself.

VI. IN-PLANE LOCALIZATION APPLICATION

The main application in mind is nonlinear acoustical Non-Destructive Testing. To see how this works, we assume a plate with a damaged region at a position x , see Figures 1 and 8. By moving a non-contact transducer device above the plate surface along the x -axis, the local field, seen in Figure 7, will move with it.

FIGURE 8. The damage localization test with the non-contact high frequency source, the low frequency shaker, and the PZT sensor.

The plate is a composite laminate plate made by carbonite-divinyl-carbonite layers of 71.6 mm thickness (a different one from the one in section V.). The material and nature of the plate is not affecting the results to any significant degree as the nonlinear methods work for more or less all types of solid materials. The transducer is moved along a straight line in steps of 1 cm. To create the low frequency components a shaker was attached to the plate and driven to create chirp signals with constant excitation force, exciting several of the plate's resonance frequencies. Simultaneously the high frequency wave was introduced by the non-contact transducer driven at fixed frequencies by an Agilent 33220A signal generator and a Krohn-Hite 7500 Wide-Band Amplifier. The response was measured using a PZT sensor at a fixed position which had a diameter of 10 mm and a thickness of 1 mm. The fixed position was chosen randomly as the method works for almost any positions. Moving the sensor with the

transducer kept fixed, the response amplitude would increase slightly, but for limited sizes of plates the nonlinear response variations from damage will in most cases be much bigger. The frequency spectrum was produced and stored with a LeCroy Lt262. Five different high frequencies [240.5 242.5 244.5 246.5 248.5] kHz were sent separately and the amplitude of the sidebands was averaged as described by Kazakov *et al.* [23]. The amplitude of the sidebands were used to quantify the nonlinearity. In Figure 9 the resulting sideband amplitude is plotted over the scanned x-direction.

FIGURE 9. Noncontact transducer scanning of plate with damage localized around ± 5 cm beside the origin.

It has the same shape as the transducer wave field. The artificially introduced damage has the center located at $x=0$ and has a 10 cm length in the x-direction. The result shows that damage start to give response 4 cm before and stop 5 cm after the middle point. It is clear from this graph that using an open resonator, the nonlinear wave modulation technique is possible to use for localization of defects in this laminate. Noticeable is the fair resolution of the damage localization compared to the physical extent of the resonator. Only the high amplitudes at small radius in Figure 7 have enough strength to create a large nonlinear response. The resolution in position x of the damage is much finer than the transducer diameter that is 115 mm. The damage region can in this case be mapped with an accuracy of around 1 cm. The size of small individual cracks on millimeter-scale or smaller is not possible with this type of set-up.

Damage position and extent can be determined, and by doing a non-linear acoustical C-scan an image can be presented.

VII. CONCLUSIONS

This paper presents measurements of localized resonant wave fields caused by dynamically trapped modes in plates. These are of applied interest for nonlinear acoustical non-destructive testing. The influence of the incoming wave field on the reflections from the insonified plate surface makes the modes under the transducer and the modes beside the transducer have different resonant frequencies. The wave field modes under the transducers are non-ideal, having different resonant frequencies from the plate regions beside the exciting wave field on the plate surface. The local wave field under the transducer is useful for the damage localization in nonlinear acoustics, where the relatively low frequencies that are used generally spread throughout the object. The explanation is the dynamic change of boundary impedance through the active wave field influence. The same behavior is found both for the contact and the non-contact measurements. In order to get the local resonant wave field, the insonified region on the plate surface must fulfill the criteria for an open resonator.

The anti-local wave field was indicated, where the wave field under the transducer had negligible amplitude due to a local anti-resonance condition, while there existed large amplitude resonant waves beside the transducer region. This proves that the local wave field does not exist only because of proximity to the transducer. The trapped mode wave field was used as the high frequency in a Nonlinear Wave Modulation Spectroscopy measurement exemplifying the localization ability of the nonlinear acoustical techniques.

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FIGURE CAPTIONS

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