Control Algorithm For Sine Excitation On Nonlinear Systems

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NOMENCLATURE

A	Estimate of Jacobian
f	Frequency, $[Hz]$
F_M	Measured Force Vector with sine and cosine components for each harmonic
F_V	Difference between measured and desired Force vector
J	Jacobian including partial derivates
k	Number of harmonics to be controlled
n	Iteration counter
r_n	Sine and cosine components in response (force) signal
t	Time, $[\sec]$
v_n	Sine and cosine components in input (volt) signal
V_N	Input voltage vector with sine and cosine components for each harmonic

ABSTRACT

When using electrodynamic vibration exciters to excite structures, the actual force applied to the structure under test is the reaction force between the exciter and the structure. The magnitude and phase of the reaction force is dependent upon the characteristics of the structure and exciter. Therefore the quality of the reaction force i.e. the force applied on the structure depends on the relationship between the exciter and structure under test.

Looking at the signal from the force transducer when exciting a structure with a sine wave, the signal will appear harmonically distorted within the regions of the resonance frequencies. This phenomenon is easily observed when performing tests on lightly damped structures. The harmonic distortion is a result of nonlinearities produced by the shaker when undergoing large amplitude vibrations, at resonances.

When dealing with non-linear structures, it is of great importance to be able to keep a constant force level as well as a non-distorted sine wave in order to get reliable results within the regions of the resonance frequencies. This paper presents theoretical methods that can be used to create a non-distorted sinusoidal excitation signal with constant force level.

1. INTRODUCTION

When performing measurements on nonlinear structures a stepped-sine excitation is a favorable method. Unlike random noise signals, a sine-excitation can be controlled to desired force amplitude as well as providing a better physical understanding of the actual problem.

Most of the theory developed for nonlinear systems [1] relies on the fact that the structure under test is excited with a pure sine wave. However, if the structure has a strong nonlinear behavior, the response signal will contain higher harmonics or sub-harmonics. This will also give distortion in the force signal since the force applied is the reaction force between the exciter and the structure.

This problem will be further amplified since the shaker in itself shows a nonlinear behavior at larger displacements. This is due to the fact that the coil is moving in the non-linear parts of the magnetic field. Distortion in the force signal can therefore be observed even when exciting linear systems, particularly weakly damped structures with large vibrations at resonance frequencies.

A principal simulation model used when developing the control algorithm is shown in Figure 1.1. A pure sine signal is sent as a voltage signal through a nonlinear system and gives a distorted signal with higher harmonics as output. The system consists of an unknown nonlinear relationship depending on the shaker, the interaction between the shaker and the structure and the nonlinearity in the structure. Results from simulations with different types of nonlinearity for system in Figure 1.1 will be shown in chapter 5, where the black box is described as a lumped mass-model with added non-linear elements.

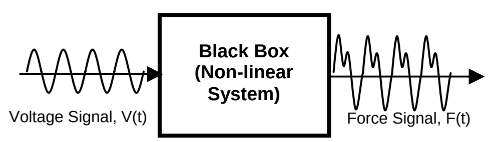

Figure 1.1. A black box used in simulations. A harmonic voltage signal gives a distorted force signal containing higher harmonics from an unknown non-linear system.

2. DEVELOPMENT OF A CONTROL ALGORITM

The essential concept used, based on methods developed by Bucher, Ewins et al [1,2,3], is to send a voltage signal including higher harmonics or sub-harmonics into the system. The phase and amplitude of the added signal content should be designed to decrease the distortion in the force signal. However, there is an unknown and nonlinear relationship between the input signal and the output force signal. Therefore an iterative procedure is required to find the correct voltage signal. This chapter describes the development of this procedure.

A general expression for the actual force measured can be written as a sum of sine and cosine components including k number of harmonics i.e.

$$F(t) = \sum_{n=1}^{k} \left(r_{2n-1} \cdot \sin(2\pi \cdot n \cdot f \cdot t) + r_{2n} \cdot \cos(2\pi \cdot n \cdot f \cdot t) \right) \qquad (2.1)$$

where f is the fundamental harmonic. For each harmonic there is a sine and cosine component to describe the actual amplitude and phase.

Alternatively, equation (2.1) can be written in matrix form for each time sample as:

$$\begin{bmatrix} \sin(2\pi \cdot f \cdot t_0) & \cos(2\pi \cdot f \cdot t_0) & \cdot & \cos(2\pi \cdot k \cdot f \cdot t_0) \\ \sin(2\pi \cdot f \cdot t_1) & \cos(2\pi \cdot f \cdot t_1) & \cdot & \cos(2\pi \cdot k \cdot f \cdot t_1) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \sin(2\pi \cdot f \cdot t_T) & \cos(2\pi \cdot f \cdot t_T) & \cdot & \cos(2\pi \cdot k \cdot f \cdot t_T) \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ r_{2k} \end{bmatrix} = \begin{bmatrix} F(t_0) \\ F(t_2) \\ \cdot \\ \cdot \\ F(t_T) \end{bmatrix} \quad (2.2)$$

From equation (2.2) it is clear that the force vector $\{r_1\ r_2\ r_3\ ...\ r_{2k}\}$ can be determined with a least-square-estimate using measurement data from time t_0 to t_T.

The actual amplitude and phase at the fundamental harmonic can be calculated as

$$Z_0 = \sqrt{r_1^2 + r_2^2} \quad (2.3)$$

$$\psi_0 = \tan^{-1}(r_2 / r_1) \quad (2.4)$$

Using (4.3) and (4.4) we can define

$$\{F_M\} = \{Z_0\ , \psi_0\ , r_3\ , r_4\ ...\ , r_{2k}\} \quad (2.5)$$

$$\{F_D\} = \{Z_d\ , \psi_d\ , 0\ , 0\ , 0\ ...\ \} \quad (2.6)$$

Thus, $\{F_M\}$ is the force vector we actually measure, and $\{F_D\}$ is the desired force vector when tuning is completed.

Similarly as in equation (2.1), the voltage signal with added higher harmonics can be defined as

$$V(t) = \sum_{n=1}^{k} \left(v_{2n-1} \cdot \sin(2\pi \cdot n \cdot f \cdot t) + v_{2n} \cdot \cos(2\pi \cdot n \cdot f \cdot t) \right) \quad (2.7)$$

$$\{V_N\} = \{v_1\ , v_2\ , v_2\ , ...,\ , v_{2k}\} \quad (2.8)$$

$\{V_N\}$ is vector containing all sine and cosine coefficients for the voltage signal. Here it is assumed that the voltage signal should initially contain the same harmonic components as observed in the force signal. Consequently, if we want to control k number of harmonics (including the fundamental), there are $2k$ numbers of unknowns.

The required voltage signal cannot be identified by simply studying the force signal since there is an unknown non-linear relationship between $\{V_N\}$ and $\{F_M\}$. Instead the problem is dealt with as a system of non-linear equations. Solving this type of problems can presents some major difficulties. Normally there are several roots and the selected algorithm can be unable to find a solution if the initial guess is too far away.

The method selected in this work is based on a multidimensional Newton-Raphson scheme [4, 5]. Unlike more general optimization routines tested, such as Powell and Nelder-Mead, this method has quadratic convergence. Unfortunately the method requires the derivates to actually be calculated which can present some difficulties when dealing with measurement data.

The Newton-Raphson method also has relatively large sensitivity against starting values. However, this may not become a large problem in real measurements. Since the force signal is most distorted at resonances, it is possible to start in an area where the number of harmonics are low and then slowly step towards the resonance.

During the frequency sweep the previous set of input voltage, which gave a non-distorted force signal, is always used as initial guess.

Next, the Newton-Raphson procedure and its application to our problem are described. A general $n\ x\ n$ nonlinear problem can be formulated as

$$\begin{Bmatrix} g_1(x_1, x_2, ..., x_N) \\ g_2(x_1, x_2, ..., x_N) \\ \vdots \\ g_N(x_1, x_2, ..., x_N) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{2.9}$$

where $\{g_1, g_2, ..., g_N\}$ are the functions to minimize. For our case the functions will be the difference between measured force vector and desired force vector. In vector form this can be written as

$$\{F_V\} = \{F_M\} - \{F_D\} \tag{2.10}$$

Each of the functions in (2.10) are then assumed to be dependent on the input voltage vector described in Equation (4.8) so that

$$\{F_V(v_1, v_2, ..., v_N)\} = \{0\} \tag{2.11}$$

Once an initial guess is provided on $\{V_N\}^n$ we would now like to determine $\{V_N\}^{n+1}$ so that it is a step towards the right solution i.e.

$$\{F_V\}^{n+1} < \{F_V\}^n \tag{2.12}$$

In order to fulfill (2.12) we must calculate the partial derivates between $\{F_V\}$ and $\{V_N\}$.

$$\begin{Bmatrix} \Delta F_V^1 \\ \Delta F_V^2 \\ \Delta F_V^3 \\ \Delta F_V^4 \\ \vdots \\ \Delta F_V^{2k} \end{Bmatrix} = \begin{bmatrix} j_{11} & j_{12} & j_{13} & j_{14} & \cdot & j_{1(2k)} \\ j_{21} & j_{22} & j_{23} & j_{24} & \cdot & j_{2(2k)} \\ j_{31} & j_{32} & j_{33} & j_{34} & \cdot & j_{3(2k)} \\ j_{41} & j_{42} & j_{43} & j_{44} & \cdot & j_{4(2k)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & j_{5(2k)} \\ j_{(2k)1} & j_{(2k)2} & j_{(2k)4} & j_{(2k)4} & \cdot & j_{6(2k)} \end{bmatrix} \cdot \begin{Bmatrix} \Delta v_1 \\ \Delta v_2 \\ \Delta v_3 \\ \Delta v_4 \\ \cdot \\ \Delta v_{2k} \end{Bmatrix} \tag{2.13}$$

Equation (2.13) is actually a linearization and since we have a nonlinear problem this will only be valid for small Δ. It is therefore essential that the initial guess on $\{V_N\}$ is sufficiently close to the root. Furthermore, the functions must be relatively smooth without large discontinuities.

All j_{nm} components in Equation (2.13) are a shorter notation for the actual partial derivate and the complete matrix is defined as the systems jacobian, i.e

$$[J_{nm}] = \frac{\partial F_V^n}{\partial v_m} \tag{2.14}$$

The partial derivates can be determined with a finite difference approach so that

$$[J_{nm}] = \frac{1}{2h} \cdot \left(\Delta F_V^n(v_m + h) - \Delta F_V^n(v_m - h) \right) \tag{2.15}$$

where h is the step length used to estimate the derivate.

By using an appropriate value for h and change the m :th value in the voltage vector, we can observe all the differences between the measured force vector and the desired force vector. As a result, the complete m :th column in $[J]$ can be determined from two function evaluations i.e measurements. If *k* numbers of harmonics are controlled we have $2k$ unknown and thus $4k$ number of measurements are required to determine the complete Jacobian.

From Equation (2.13) a new guess on the input voltage vector can be formulated as

$$\{V_N\}^{n+1} = \{V_N\}^n - [J]^{-1} \cdot \{F_V\} \tag{2.16}$$

If the initial $\{V_N\}$ was sufficiently close to the root, $\{V_N\}^{n+1}$ should be one step closer to the correct solution. From here a new jacobian can be calculated and a new step is taken. Rather than attempting to find the exact solution, the quantity

$$F_{crit} = \sum_{n=1}^{2k} \{F_V\}^n \tag{2.17}$$

is calculated and a global criteria on F_{crit} is used for when the new solution should be accepted.

Once (2.17) is fulfilled the force signal is said to be close to a non-distorted sine. The response at selected points on the structure can therefore be saved for FRF-calculation. Then the desired fundamental frequency on the force signal, Equation (2.1), can be incremented and the previous $\{V_N\}$ is used as initial guess. If new harmonic components become relevant in the force signal they are added to the controlled set of frequencies and new iterations are performed.

3. BROYDENS METHOD

The Newton-Raphson technique described in chapter 2 is useful for solving small systems of equations but is not very practical when the number of unknowns becomes too large. As previously described k number of unknowns yields 4 k number of measurement to determine the jacobian. Therefore, a single iteration where four harmonics are controlled can take up to 200-300 seconds (depending on measurement time required to find a steady-state solution).

To deal with this problem and increase the speed of the control algorithm several quasi-newton methods have been tested. This class of methods avoids the direct calculation of partial derivates. In this work Broydens method [4, 5] is tested where a secant method is used to approximate the partial derivates.

In a one-dimensional case the secant method replaces the derivate as

$$g'(x) = \frac{g(x^n) - g(x^{n-1})}{x^n - x^{n-1}} \tag{3.1}$$

$$g'(x)\cdot\left(x^n - x^{n-1}\right) \approx \left(g(x^n) - g(x^{n-1})\right) \tag{3.2}$$

Extended to higher dimensions and to our application we can define

$$J\left(\{V_N\}^n\right)\cdot\left(\{V_N\}^n - \{V_N\}^{n-1}\right) \approx F_V\left(\{V_N\}^n\right) - F_V\left(\{V_N\}^{n-1}\right) \tag{3.3}$$

Broyden then uses an approximation to the jacobian matrix

$$A\left(\{V_N\}^n\right) \approx J\left(\{V_N\}^n\right) \tag{3.4}$$

which after two successive iterations can be updated with Equation (3.3). To achieve an updating formula for $[A]$, as suggested by Broyden [4,5], the following vectors are defined

$$\{y_k\} = \{F_V\}^n - \{F_V\}^{n-1} \tag{3.5}$$

and

$$\{s_k\} = \{V_N\}^n - \{V_N\}^{n-1} \tag{3.6}$$

Using Equation (3.5) and (3.6), the estimate of the jacobian can be updated with

$$A^n = A^{n-1} + \frac{y_k - A^{n-1}s_k}{\left\|s_k\right\|_2^2}\left(s_k\right)^T \tag{3.7}$$

Where $\left\|.\right\|_2$ denotes the euclidean norm.

Equation (3.7) requires the two initial values on the voltage vector. In practice the first jacobian can be calculated and then using Equation (2.16) to determine $\{V_N\}^1$. Alternatively a scalar multiple of the unit matrix can be us as initial guess for the jacobian.

Once the first two values on the voltage vector are given, Equation (3.7) are used and

$$\{V_I\}^{n+1} = \{V_I\}^n - \left[A^n\right]^{-1}\cdot\{F_V\} \tag{3.8}$$

until Equation (2.17) is fulfilled.

4. EXTENDED FORCE CONTROL ALGORITM

Since the Jacobian (calculated as in chapter 2 or estimated with Broyden) can contain a lot of disturbance for measurement data it is necessary to have a built in strategy to keep the solution-process stable. A global strategy method is therefore used. The idea is to not always take the full step, instead equation (3.8) is rewritten as

$$\{V_I\}^{n+1} = \{V_I\}^n + u\cdot\left[A^k\right]^{-1}\cdot\{F_V\} \tag{4.1}$$

As suggested by [1, 4] we find a value u that minimize

$$G_V = \frac{1}{2} F_V^T \cdot F_V \tag{4.2}$$

To find the exact minimum may require several function evaluations (measurements) so instead selected values between $\{0.1, ..., 1\}$ is tested and using interpolation it is possible to find a satisfactory value. This will help the algorithm to converge even when the starting values are less good or the Jacobian contains noise. To achieve a good starting value the multidimensional solver does not start until the amplitude on the fundamental harmonic is correct as suggested in [1, 2, 6].

The complete force algorithm from chapter 2, 3 and 4 is illustrated with a flow chart in Figure 4.1. After the complete voltage vector $\{V_N\}$ is defined we can choose to either use Broydens method or Newton-Raphsons method as indicated in the flow chart.

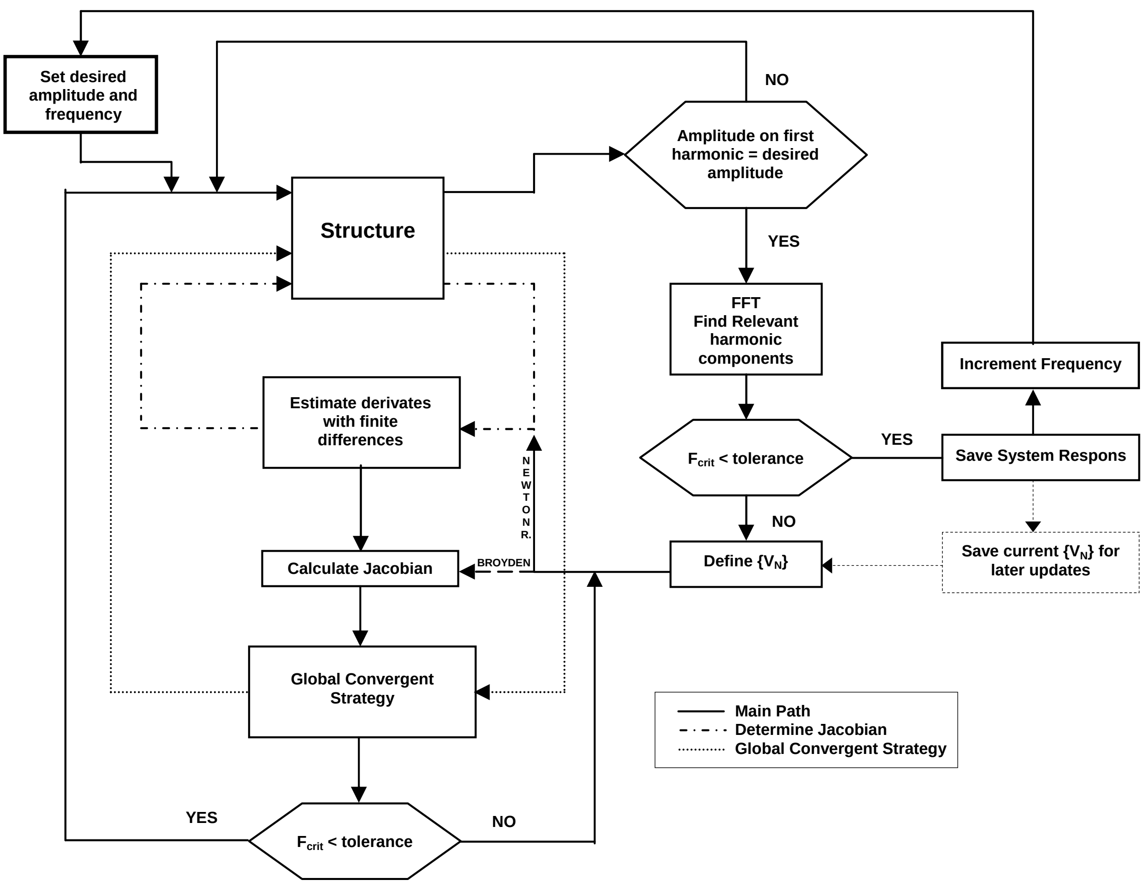

Fig 4.1. Flow chart of Force control algorithm. A voltage signal is sent to the amplifier and the resulting force signal can be measured with a force sensor. The control algorithm the updates the voltage signal to achieve a non-distorted force signal.

5. SIMULATIONS AND RESULTS

A simulation model is created as explained in chapter 1. The shaker, the interaction between the shaker and the structure and the nonlinearity in the structure is here regarded as a black box and modeled as lumped mass model.

This is a simplification since the true model between input voltage and output force is a non-linear electro-mechanic system. However, for simulations this provides a good way to actually test the algorithm. The values selected for m, c, k and P has no real physical meaning but are merely estimated from transfer functions calculated between voltage/force. All calculations are performed in MATLAB® [7].

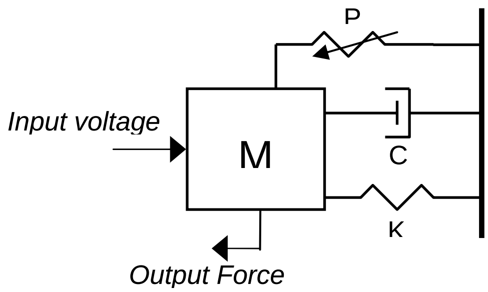

Figure 5.1. Simulation model used to test the algorithm. The values used are $m = 0.024\ kg$, $C = 1.5\ Ns/m$, $K = 1.6 \cdot 10^6\ N/m$ and $P = 1.6 \cdot 10^8 \cdot x^3\ N/m$

The desired amplitude on the output signal is selected and the input signal is first adjusted to give an output signal with correct fundamental harmonic. Then, using the Newton-Raphson procedure, an input signal with higher harmonics is created as described in chapter 2.

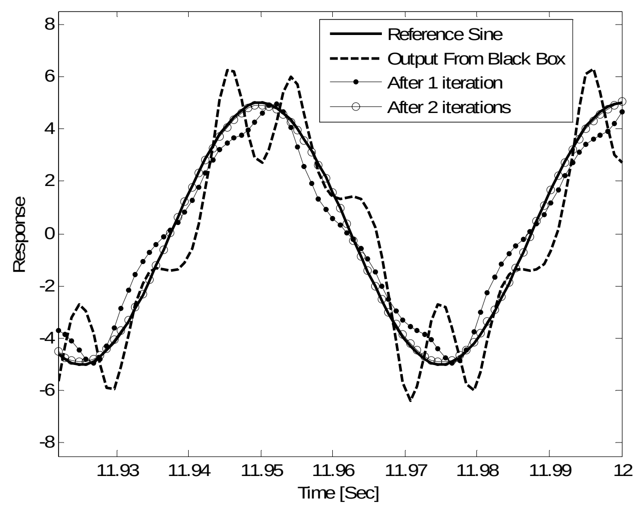

Figure 5.2 Simulation results where the algorithm is used to control the output signal from the non-linear system in Figure 5.1. In this case 40 simulated measurements were required to correct the signal.

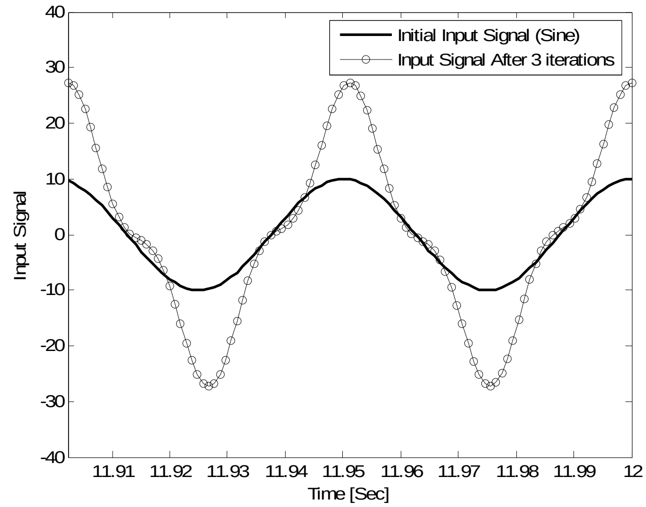

Figure 5.3 The higher harmonics in the input signal can be seen after 3 iterations.

The output from the system contains a significant third and fifth harmonic component as shown in Figure 5.2. After two iterations the difference between the output signal and the reference signal is said to be acceptable and the algorithm can continue with a new frequency step. The initial input signal and the input signal after two iterations is shown in Figure 5.3. The actual amplitude on the final signal is larger to compensate for the loss to higher harmonics. Additionally, a slight change on phase of the main harmonic can observed as well as a significant third harmonic in the final input signal.

Next, the modified control algorithm as described in chapter 3 is used to control the input signal. The first jacobian is here calculated and used as initial guess for Broydens method. The results can be studied in Figure 5.4. As observed this method requires an additional iteration before the solution is accepted since Broydens method converges slower. However the amount of function evaluations (measurements) saved is still significant.

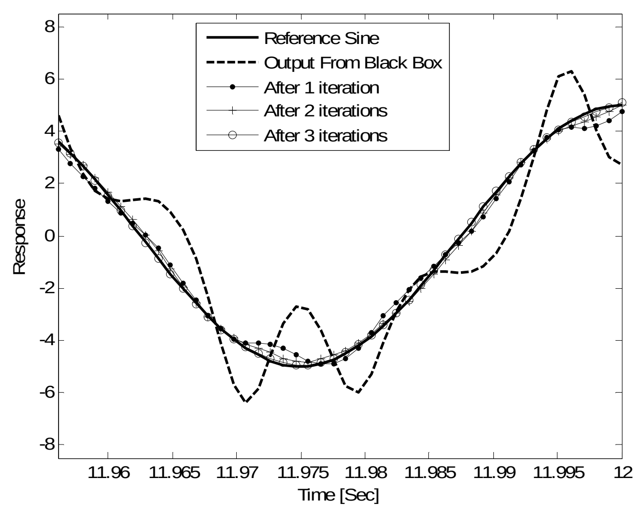

Figure 5.4 Broydens method for the same system requires 28 simulated measurements to find the correct voltage signal.

6. CONCLUSIONS AND FURTHER WORK

A non-distorted force signal can be difficult to obtain when measuring non-linear structures or weak linear structures at resonances and thus a control algorithm is necessary. This work has presented theoretical methods that can be used to reduce the distortion in the force signal. This can be achieved by creating a multiharmonic voltage signal with phase and amplitude designed to decrease all higher harmonics or sub-harmonics in the force signal. A control algorithm based on Newton-Raphsons method have been applied i.e. an iterative process is used to actually find the correct voltage signal. Further extensions have been made on the control algorithm by using Broydens method to estimate the derivates.

A complete control system is built in MATLAB® where simulations have been performed on several arbitrary non-linear black box systems as described in chapter 1 and chapter 5. The result from simulations shows that the control algorithm can obtain a non-distorted output signal even when there is a significant nonlinearity. The potential of Broydens method have been observed in simulations and further experiments is required to determine whether this method can be used in real measurements.

Additional work in the future will also be to create an improved electro-mechanic model of the shaker including the nonlinearities. From here a better understanding of the actual phenomena can be gained and the control algorithm can be further improved.

7. REFERENCES

1. Ferreira V. Janito, Dynamic Response Analysis of structures with non-linear components, Phd Thesis, Imperial College, London, 1998
2. Ewins, D.J and Perinpanayagam S., Testing Technique for identifying and Characterizing Non-Linear Behaviour of and Aero-Engine Structure, 12th International Conference on Experimental Mechanics, 2004
3. Bucher I, Exact Adjustment of Dynamic Forces in Presence of Non-linear Feedback and Singularity – Theory and Algorithm, Journal of Sound and Vibration, 218(1), 1998
4. Press H William, Teukolsky A. Saul, Vetterling T. William and Flannery P. Brian, Numerical Recipes in C - The art of Scientific Computing Second Edition, ISBN 0-521-43108-5, Cambridge University, 1992
5. Lindfield George and Penny John, Numerical Methods using Matlab ,Second Edition, Aston University, 1999
6. Rossmann Stephane, Development of Force Controlled Modal Testing on Rotor Supported by Magnetic Bearings, Master Thesis at Imperial College, London, 1999
7. MATLAB Computing Software v 7.0 (R14), The MathWorks, Inc. 2005