On Nonlinear Parameter Estimation

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Abstract

The industrial demand on good dynamical simulation models is increasing. Since most structures show some form of nonlinear behavior, linear models are not good enough to predict the true dynamical behavior. Therefore nonlinear characterization, localization and parameter estimation becomes important issues when building simulation models. This paper presents identification techniques for nonlinear systems based on both random and harmonic excitation signals.

The identification technique based on random excitation builds on the well known reverse-path method developed by Julius S. Bendat. This method treats the nonlinearity as a feedback forcing term acting on an underlying linear system and the parameter estimation is performed in the frequency domain by using conventional MISO/MIMO techniques. Although this method provides a straightforward and systematic way of handling nonlinearities, it has been somewhat limited in use due to the complexity of creating uncorrelated inputs to the model. As is shown in this paper, the parameter estimation will not be improved with conditioned inputs and the nonlinear parameters and the underlying linear system can still be estimated with partially correlated inputs.

This paper will also describe a parameter estimation method to be used with harmonic input signals. By using the principle of harmonic balance and multi-harmonic balance it is possible to estimate an analytical frequency response function of the studied nonlinear system. This frequency response function can, in conjunction with measured nonlinear transfer functions, be used to estimate the nonlinearity present in the system. This method is also applicable on nonlinear systems with memory, e.g. systems with hysteresis effects.

The above mentioned methods are applied to multi-degree-of-freedom and single-degree-of-freedom systems with different types of nonlinearities. Also, techniques for locating nonlinearities are discussed.

1 Introduction

The overall aim with parameter estimation is to find suitable parameters to a mathematical model, based on measurements of the inputs and outputs of a system. The mathematical model can, for instance, be based on a beforehand known model structure. This is known as parametric modelling, which will be studied in this paper. Furthermore, the systems studied contain significant nonlinearities that must be included when creating the model, hence nonlinear parameter estimation.

Systems used in engineering can generally be separated into two distinct groups; linear or nonlinear. A system, H, is said to be linear if it fulfills the principle of superposition:

$$H\{c \cdot x\} = c \cdot H\{x\} \tag{1}$$

$$H\{x_1 + x_2\} = H\{x_1\} + H\{x_2\} \tag{2}$$

A system which does not satisfy these properties is defined as nonlinear. Additionally, nonlinear systems can be classified into different groups as described in [1]. In this paper, two different types of non-linearities will be discussed; zero-memory nonlinear systems and nonlinear systems with memory.

The next chapter will show how the frequency response functions can be used to identify and locate nonlinearities in a dynamic system. This will be followed by methods for parameter estimation with random noise signals in chapter 3, and next using sine excitation in chapter 4.

2 Nonlinear effects on the Frequency Response Functions

The frequency response functions can normally give sufficient information about a system. In structural dynamics it is often common to assume that there is an underlying linear system. By measuring the frequency response functions at different force amplitudes it is, in many cases, possible to identify the type of nonlinearity present as shown in Figure 2.1. For zero-memory nonlinear systems it is also possible to identify the type of nonlinearity with random noise signals as will be shown in Chapter 3.

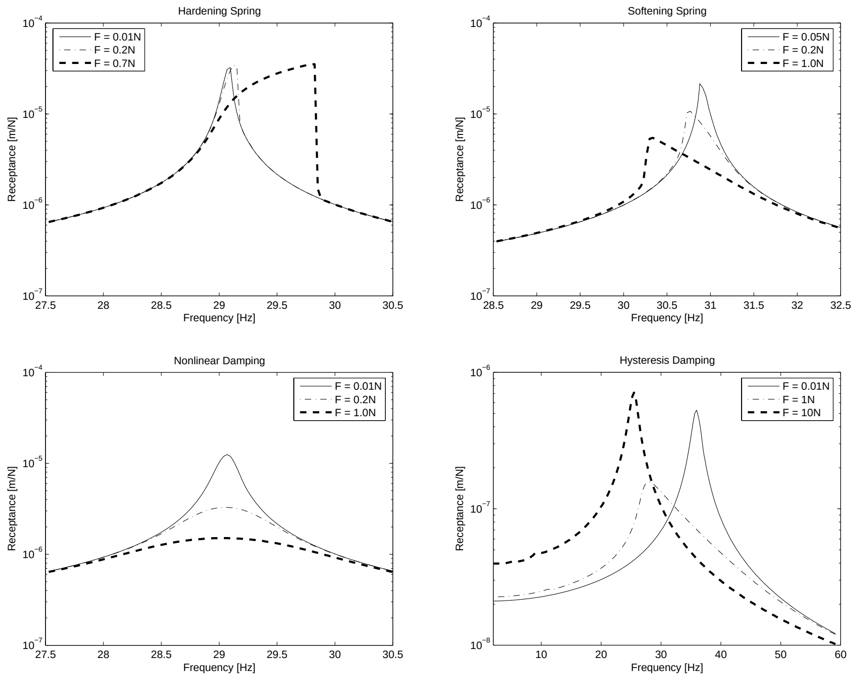

Figure 2.1: Frequency Response Functions measured with an upward sine sweep.

The nonlinear degree-of-freedoms can also be determined, from the frequency response functions, by using the fact that the nonlinearity usually only influence the system dynamics at high input amplitude. For example, consider a 2-dof system where a nonlinear element is connected from one dof to ground. Figure 2.2 shows the point-receptance measured at each degree-of-freedom.

Notice that, for H_{22}, the anti-resonance remains at the same position in frequency since the system is close to linear at this very low response amplitude. The same behavior can not be seen for H_{11} and thus, the nonlinear element must be connected to the second degree-of-freedom.

Hence, the frequency response function can be used to find the nonlinear degree of freedoms as well as identify the type of nonlinearity present. This information will be necessary in the following chapters, where a known model structure is assumed.

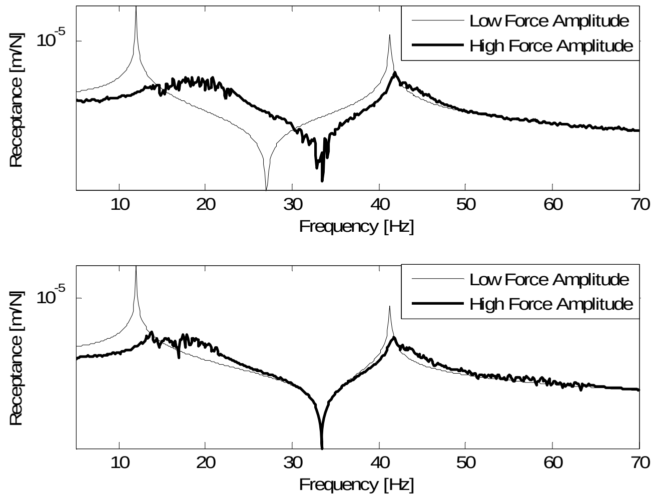

Figure 2.2: Simulated point receptance for a two-degree-of-freedom system with a nonlinear element connected to the second dof. H_{11} above and H_{22} below. Notice that the anti-resonance is not moving for H_{22} since the nonlinear element is connected to the second dof.

3 Parameter Estimation with Random Noise Signals

This chapter will study a method based on using random noise signals as initially developed by Bendat [1]. This method uses a least-square method in the frequency domain to estimate parameters and is suitable to identify zero-memory nonlinear systems. The multiple-input-multiple-output theory, used in the method, will be initially reviewed in section 3.1, followed by an application to nonlinear systems in section 3.2 and finally two simulated examples in section 3.3.

3.1 The MIMO Identification Technique

Consider a general linear system, with N input and M outputs, and noise on the measured outputs, as shown in Figure 3.1.

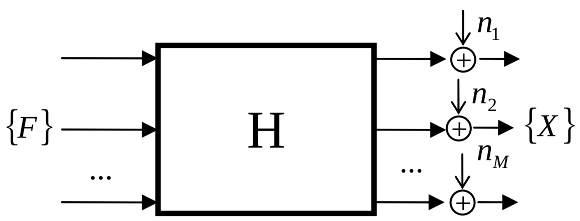

Figure 3.1: A Multiple-Input-Multiple-Output Model with noise on the outputs.

The *H1*-estimator for this system can be determined at each frequency as

$$\mathbf{H} = \mathbf{G}_{XF} \cdot \mathbf{G}_{FF}^{-1} \tag{3}$$

where $\mathbf{G}_{XF}$ is the cross-spectral density matrix with size $(N \times N)$, and $\mathbf{G}_{FF}$ is the auto-spectral density matrix with size $(M \times N)$.

Next, the coherence function is defined. Generally, the ordinary coherence function is defined as the linear relationship between any two signals.

For a single-input-single-output model the use of the coherence function is straightforward but for a multiple input case, as shown in Figure 3.1, several ordinary coherence functions can be formulated, for instance the ordinary coherence between input p and output q:

$$\gamma_{pq}^{2} = \frac{\left|\mathbf{G}_{X_p F_q}\right|^2}{\mathbf{G}_{F_q F_q} \cdot \mathbf{G}_{X_p X_p}} \tag{4}$$

Equation (4) yields a frequency dependent real-valued scalar equal to unity when perfect linear relationships exist between two spectra. However, for a MIMO case, the ordinary coherence function between an output spectrum and an input spectrum can be misleading. Due to the influence of other inputs, this coherence function can be much less than unity even though a perfect linear relationship exists between all inputs and outputs. For this reason the multiple coherence function is more useful in a multiple input case.

A multiple coherence function can be calculated for each output as

$$\gamma_{m}^{2} = \frac{\mathbf{G}_{XF} \cdot \mathbf{G}_{FF}^{-1} \cdot \mathbf{G}_{XF}^{H}}{\mathbf{G}_{XX}} \tag{5}$$

The size of $\mathbf{G}_{XF}$ will now be $(1 \times N)$ and $\mathbf{G}_{XX}$ will be scalar since only one output is selected. Thus, M number of multiple coherence functions can be calculated for the system shown in Figure 3.1.

3.2 Using MIMO identification For Nonlinear Systems

The MIMO identification technique can now be used to solve a nonlinear problem. Actually, there are several possible ways to formulate a nonlinear problem with MIMO-technique. For instance, consider duffing's equation:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x + p \cdot x^3 = f \tag{6}$$

In frequency domain this can be formulated as

$$B \cdot X + p \cdot \mathcal{F}\left(x^3\right) = F \tag{7}$$

where B is the impedance of the linear system and $\mathcal{F}(.)$ is the Fourier transform. Note that in Equation (7), p can be frequency dependent. Clearly, using the multiple input techniques from the previous section, this problem can be solved in, for instance, three different ways as shown in Figure 3.2 were $H = B^{-1}$.

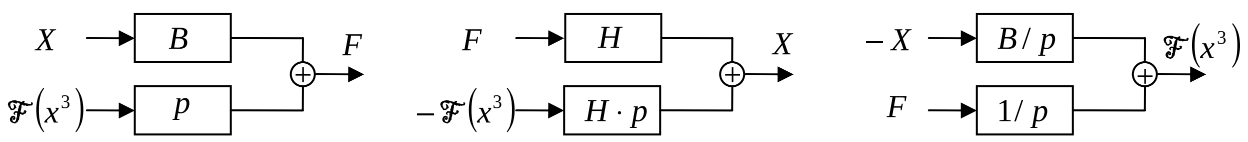

Figure 3.2: Three possible ways to solve Duffing's equation with multiple input techniques; Force as output (left), displacement as output (middle), nonlinear function as output (right)

In this paper, the applied force and the nonlinear feedback forces are used as input to the MIMO-model, since this formulation is easier to extend to a general nonlinear system as will be shown next. Various extensions to MDOF systems with several nonlinear elements have previously been done by, for instance, [2,3].

A general nonlinear system can be expressed as

$$\mathbf{M}\cdot\{\ddot{x}\}+\mathbf{C}\cdot\{\dot{x}\}+\mathbf{K}\cdot\{x\}=\{f\}-N[\{x\},\{\dot{x}\}] \tag{8}$$

where N is a set of nonlinear restoring forces depending on either x or $\dot{x}$. In more detail, the nonlinear operator can be written as

$$N[\{x\},\{\dot{x}\}]=\sum_{m=1}^{N_{EL}} p_m\cdot\{w_m\}\cdot g_m(\{x\},\{\dot{x}\}) \tag{9}$$

Thus, for each nonlinear component there is an unknown coefficient p_m, a position vector $\{w_m\}$ and a nonlinear function g_m. Typical nonlinear functions are shown in Equation (10)-(11). The position vector is here used in the formulation since the nonlinear function might act on the relative output between two degrees-of-freedom, as will be determined by the elements location, for instance:

$$g_m=\left(\{w_m\}^T\{x\}\right)^3 \qquad \text{(Cubic hardening spring)} \tag{10}$$

$$g_m=\left(\{w_m\}^T\{\dot{x}\}\right)^2 \qquad \text{(Quadratic damping)} \tag{11}$$

Equation (8) and (9) can be written in frequency domain as

$$\{X\}=\mathbf{H}\cdot\{F\}-\mathbf{H}\cdot\sum_{i=1}^{N_{EL}} p_m\cdot\{w_m\}\cdot\mathcal{F}\left(g_m(\{x\},\{\dot{x}\})\right) \tag{12}$$

From the expression above a new modified force vector can be defined which includes the nonlinear restoring forces as shown in Equation (13). This force vector is then used as input to a MIMO-model. To avoid confusion with the force vector used in the previous section this vector is notated as $\{R\}$.

$$\{R\}=\begin{Bmatrix}\{F\}\\ r_1\\ \vdots\\ r_m\end{Bmatrix} \tag{13}$$

In (13) each r_m is the Fourier-transform of a nonlinear function, i.e.

$$r_m=-\mathcal{F}\left(g_m(\{x\},\{\dot{x}\})\right) \tag{14}$$

As will be shown later, several nonlinear functions can be used for a single nonlinear element. For instance when several polynoms are used to estimate a gap- or clearance-nonlinearity.

Using this new modified force vector a set of transfer functions can be calculated using the equation for the *H1*-estimator as

$$\mathbf{H}_M=\mathbf{G}_{XR}\cdot\mathbf{G}_{RR}^{-1} \tag{15}$$

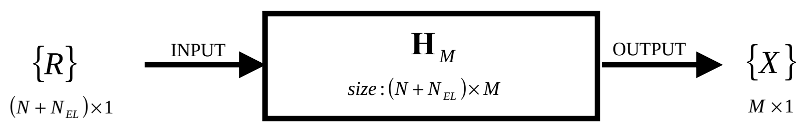

Figure 3.3: A force vector with applied force and the restoring nonlinear forces is used as input to a MIMO-model.

If the size of the matrices is defined as shown in Figure 3.3, the first column of the underlying linear system can be identified as the first column in $\mathbf{H}_M$. Thus, for a single force input the linear system can be extracted, $\{H_L\}$, and the nonlinear coefficients can, for each frequency, be calculated as

$$p_i = \frac{1}{\{w_i\}^T \cdot \{H_L\}} \cdot \mathbf{H}_M^{(r,i+1)}, \quad i \in \{1,2\ldots N_{EL}\} \tag{16}$$

$\mathbf{H}_M^{(r,i+1)}$ refer to a single element in matrix $\mathbf{H}_M$ at row r (force input) and column $i+1$.

Note that Equation (16) can easily be extended to a case where there is several force inputs. As can be seen in simulations, an applied force at each nonlinear dof will reduce the necessary simulation time considerably.

With the method described above, the column that corresponds to the force input will always be estimated in the linear system. Thus, modal parameter estimation can be performed to determine the complete linear system. If two forces are applied, two columns in the linear system will be estimated and so forth.

The force input and the nonlinear restoring forces may be partially correlated with each other and, after averaging, the matrix $\mathbf{G}_{RR}$ will be non-diagonal. This may introduce numerical instabilities when calculating the matrix inversion in Equation (15), particularly at the resonance. In general though, this error is very small and does not affect the result much.

The conditioning method, described in [1], where a set of uncorrelated inputs is created will not improve the parameter estimation in this case. For all simulated systems tested, it has not been possible to see any difference between using partially correlated inputs and the conditioned version with uncorrelated inputs. This is an important observation since it simplifies the calculations considerably.

The only advantage with using uncorrelated inputs is that the ordinary coherence can be calculated between the output and each input. These ordinary coherences will sum up to the multiple coherence and thus it is possible to see the significance of each nonlinear restoring force. This may be useful when searching for possible nonlinear functions and a method for calculating uncorrelated inputs is given below.

A new set of inputs can be defined by a transformation as

$$\{U\} = \mathbf{\Phi} \cdot \{R\} \tag{17}$$

Multiplying with the complex conjugate and averaging yields

$$[G_{UU}] = \mathbf{\Phi} \cdot \{R\} \cdot (\mathbf{\Phi} \cdot \{R\})^H = \mathbf{\Phi} \cdot \mathbf{G}_{RR} \cdot \mathbf{\Phi}^H \tag{18}$$

Thus, if the matrix Φ is defined as described in [1], G_{UU} will be a diagonal matrix.

Using this Φ-matrix the cross-correlation densities can also be calculated as

$$\mathbf{G}_{XU} = \mathbf{G}_{XR} \cdot \mathbf{\Phi}^H \tag{19}$$

With $\mathbf{G}_{UU}$ and $\mathbf{G}_{XU}$ available the ordinary coherences between a selected output and each input can be calculated with Equation (4). Using the notations in Figure 3.3 there will $N + N_{EL}$ ordinary coherences for each output.

3.3 Simulations

The method derived in section 3.2 is here demonstrated on two nonlinear systems. First, a single-degree-of-freedom with two nonlinear elements is used followed by a multi-degree-of-freedom system.

3.3.1 A SODF-system with a nonlinear spring and damper

The test system used is shown in Figure 3.4 and consists of an underlying linear system with a cubic hardening spring and a quadratic damping element connected. The time response is simulated from this system using digital filters. Figure 3.5 shows several frequency response functions calculated with different force amplitudes.

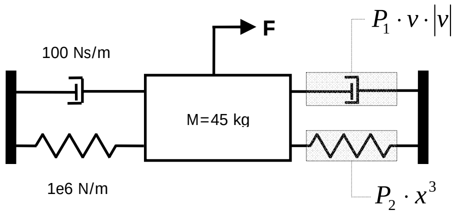

Figure 3.4: Test Case used in simulations. A single-degree of freedom system with two nonlinear elements. $P_1 = 2e5$ **and** $P_2 = 6e13$**.**

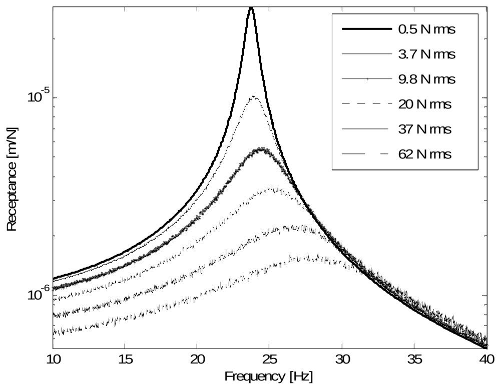

Figure 3.5: Several FRF's at different force amplitudes for the tested system.

With increasing force, the frequency response functions will decrease in amplitude as well as moving upward in frequency due to the presence of a damping nonlinearity and a cubic hardening spring. As mentioned in previous section it is possible to use several nonlinear functions as input and, with equation (17)-(19), calculate a new set of uncorrelated inputs and then studying the ordinary coherences. However, this is not straightforward as the order in which the functions are defined and the amplitude of the response signal will affect the result. In this case, testing different combinations of Equation (20) will show that the cubic spring element and the quadratic damping element are most significant.

$$\{R\} = \{F \quad -x^2 \quad -x^3 \quad -x \cdot sign(x) \quad -v \cdot |v| \quad -v^3 \quad -v^5\}^T \tag{20}$$

The position vector and the nonlinear functions are therefore defined as

$$w_{1,2} = \{1\} \qquad r_1 = -\mathcal{F}\left((x_1)^3 \right) \qquad r_2 = -\mathcal{F}\left((v_1 \cdot |v_1|)^3 \right) \tag{21}$$

Using Equation (13)-(16), the nonlinear analysis can be performed and the result is shown in Figure 3.6. The linear system can be identified and the calculated average value of the nonlinear coefficients differ no more than ± 0.01% from the theoretical value.

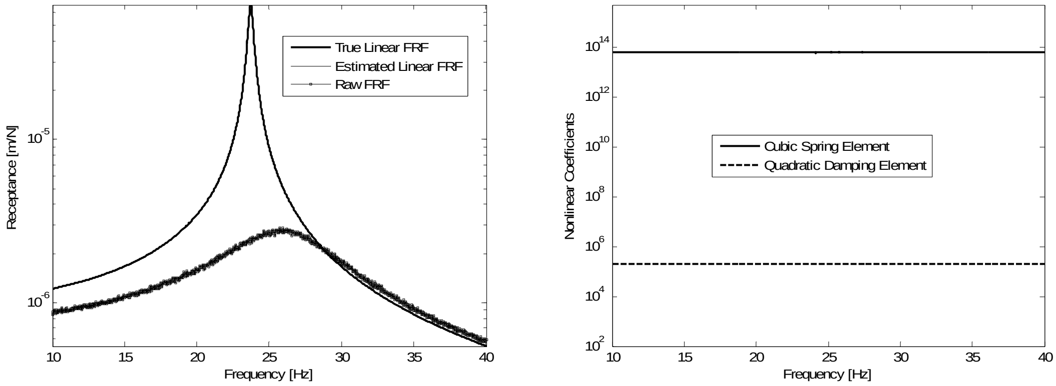

Figure 3.6: Estimate of the underlying linear system and the nonlinear coefficients.

3.3.2 A MDOF-system with a several nonlinear elements

The next example consists of a mdof system with a cubic- and a quadratic-spring as shown below.

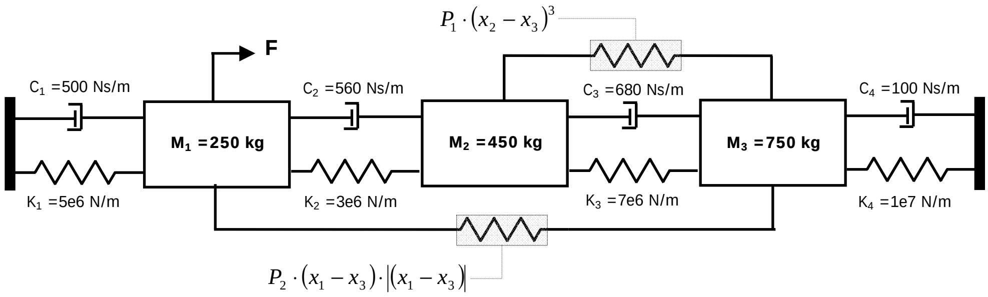

Figure 3.7: Test System used in Simulations, $P_1 = 4e16$ **and** $P_2 = 6e10$

Using the same procedure as in section 3.3.1, the positions vector and the nonlinear functions for each nonlinear element is defined as

$$w_1 = \{0 \quad -1 \quad 1\}^T, \; w_2 = \{-1 \quad 0 \quad 1\}^T \tag{22}$$

$$r_1 = -\mathscr{F}\left(\left(\{w_1\}^T \{x\}\right)^3 \right), \; r_2 = -\mathscr{F}\left(\{w_2\}^T \{x\} \cdot \left| \{w_2\}^T \{x\} \right| \right) \tag{23}$$

The first column in the linear system and the nonlinear coefficients can then be solved with Equation (13)-(16), and the result is shown in Figure 3.8.

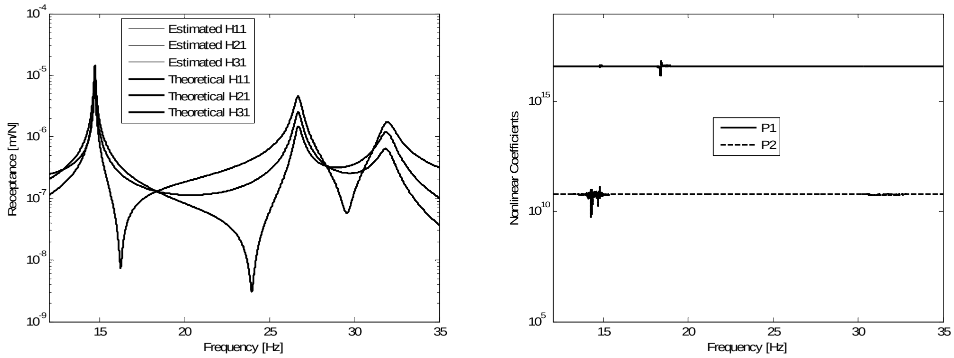

Figure 3.8: The first column in the linear system and the nonlinear coefficients.

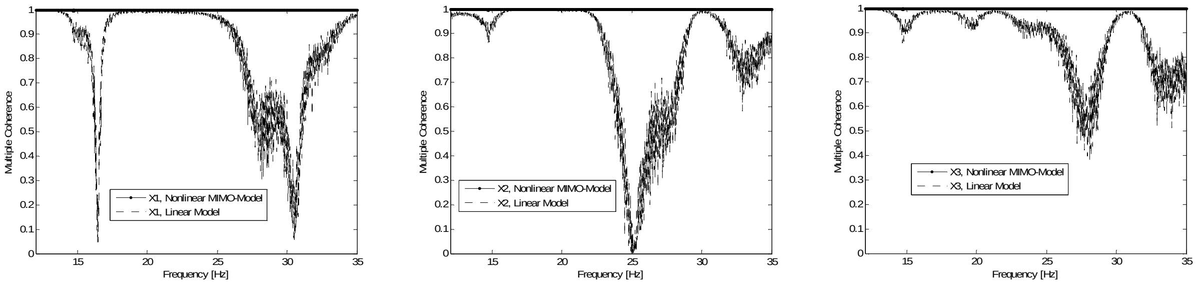

Figure 3.9: Multiple Coherence Functions

Furthermore, the multiple coherence can be calculated for each output which is then compared with the multiple coherence function calculated for a conventional linear analysis (Figure 3.9). Notice that the multiple coherence is above 0.99 for all frequencies with the nonlinear analysis.

4 Parameter Estimation With Harmonic Input

The difference between broadband and narrow band excitation signals is the amount of energy associated with every single frequency point. In the case of a broadband signal, the energy associated with each specific frequency point is low. The result is a frequency response function which appears linearised around the resonance frequency. However, if sinusoidal excitation is used it is possible to obtain a good estimate of the frequency response function for the studied nonlinear system. This is illustrated in figure 4.1.

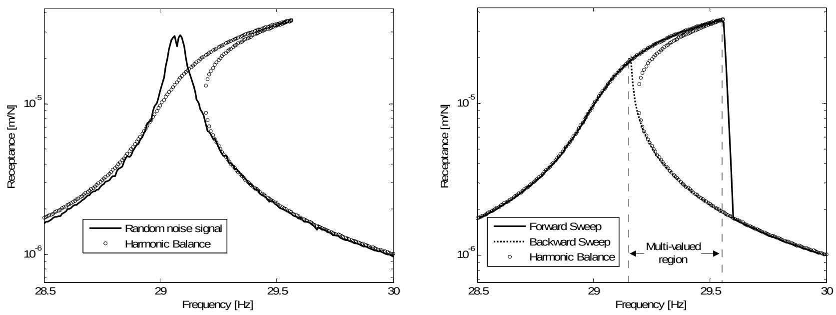

Figure 4.1: Transfer function of a nonlinear SDOF system obtained by random noise and sinusoidal excitation, compared with an analytical transfer function calculated by harmonic balance.

The frequency response function obtained by harmonic excitation can, in conjunction with an analytical frequency response function, be used to estimate the nonlinearity present in the system. In this chapter, the principle and method of a parameter estimation procedure based on harmonic input will be discussed. The method is then tested on a nonlinear stick-slip system in section 4.2.

4.1 Theoretical Background

By exciting a nonlinear system with either a low or a high force, depending on the type of nonlinearity, the frequency response function of the underlying linear system can be estimated. This frequency response function can be used as basis to calculate an analytical nonlinear frequency response function of the studied system. The basic principle of this will be described by using a nonlinear single degree of freedom system with an arbitrary nonlinearity, see Figure 4.2.

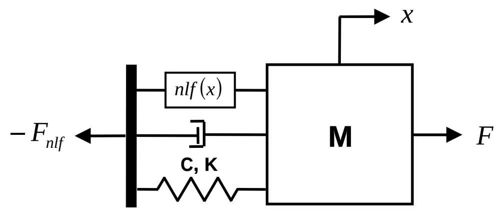

Figure 4.2: Single degree of freedom system with an arbitrary nonlinearity

The equation of motion for this system is:

$$m\ddot{x} + c\dot{x} + kx = F - F_{nlf} \tag{24}$$

F_{nlf} = displacement dependent force due to the nonlinearity.

$$F(t) = F_0 e^{j\omega t} \tag{25}$$

When the system above is excited with a pure sinusoidal force, according to Equation (25), the system response will contain higher harmonics due to the nonlinearity. This response can be expressed with a Fourier expansion.

$$x(t) = \sum_{k=1}^{\infty} x_k e^{jk\omega t} \tag{26}$$

Differentiation of this equation yields:

$$\dot{x}(t) = \sum_{k=1}^{\infty} jk\omega x_k e^{jk\omega t} \tag{27}$$

$$\ddot{x}(t) = -\sum_{k=1}^{\infty} k^2 \omega^2 x_k e^{jk\omega t} \tag{28}$$

The nonlinear force can be expressed in a similar fashion:

$$F_{nlf}(x) = \sum_{k=1}^{\infty} F_{nlk} e^{jk\omega t} \tag{29}$$

Putting Equation (25)-(29) into Equation (24) and considering for example three harmonics [1 3 5], the system of equation becomes:

$$\begin{aligned}
&-mx_1\omega^2 e^{j\omega t} + jcx_1\omega^2 e^{j\omega t} + kx_1 e^{j\omega t} - F_0 e^{j\omega t} + F_{nl1} e^{j\omega t} = 0 \\
&-9mx_3\omega^2 e^{j3\omega t} + 3jcx_3\omega^2 e^{j3\omega t} + kx_3 e^{j3\omega t} + F_{nl3} e^{j3\omega t} = 0 \\
&-25mx_5\omega^2 e^{j5\omega t} + 5jcx_5\omega^2 e^{j5\omega t} + kx_5 e^{j5\omega t} + F_{nl5} e^{j5\omega t} = 0
\end{aligned} \tag{30}$$

This system of equations can be rewritten in a more compact form:

$$\begin{aligned}
&(-m\omega^2 + jc\omega + k)x_1 - F_0 + F_{nl1} = 0 \\
&(-9m\omega^2 + 3jc\omega + k)x_3 + F_{nl3} = 0 \\
&(-25m\omega^2 + 5jc\omega + k)x_5 + F_{nl5} = 0
\end{aligned} \tag{31}$$

As is evident, the bracketed expressions contain the impedance for each specific harmonic. So, the equation can be written as:

$$\begin{aligned} &Z_1 x_1 - F_0 + F_{nl1} = 0 \\ &Z_3 x_3 + F_{nl3} = 0 \\ &Z_5 x_5 + F_{nl5} = 0 \end{aligned} \tag{32}$$

By using the frequency response function of the underlying linear system mentioned previously, the impedance of the system is estimated and an analytical frequency response function can be calculated using the method of harmonic balance. This presumes that the nonlinearity in the system is known. If the nonlinearity is not known, it can be estimated by fitting the analytical nonlinear frequency response function to a measured nonlinear frequency response function.

All nonlinear functions is defined by a set of parameters, the first step is to find out which type of nonlinearity is present in the current system and select a suitable model for this nonlinearity, some examples of how this can be done were described in chapter 2.

The parameters which define the nonlinear model can then be estimated by the method described in figure 4.3. The cost function used as error estimate is defined as

$$\varepsilon = \left\| H_{measured\_nonlinear} - H_{analytic\_nonlinear} \right\|_2 \tag{33}$$

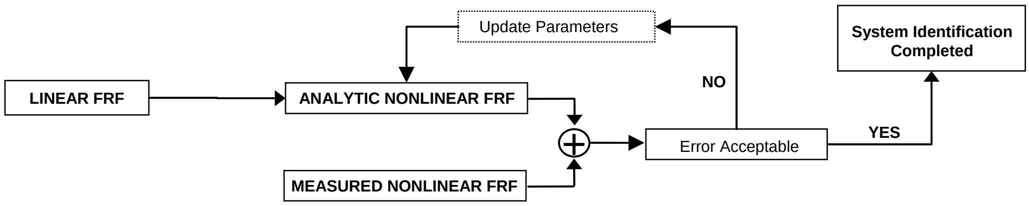

Figure 4.3: Flowchart of the parameter estimation procedure

One benefit of this method, compared to random data based parameter estimation procedures, is the ability to use any nonlinear function, e.g. bilinear functions or functions with memory, instead of being forced to use only polynomial estimates.

4.2 Simulations

The method described previously will be illustrated by an example where parameter estimation is done on a nonlinear system with memory, a stick slip system. The nonlinear system used as a reference, i.e. the system the simulated measurements are performed on, is shown in Figure 4.4.

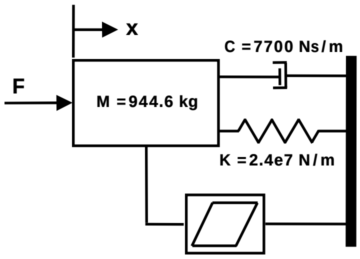

Figure 4.4: Nonlinear stick-slip system used as reference.

The nonlinearity in this system is defined by two parameters:

k_d - The stiffness during the stick-condition

F_d - The force level where the system starts to slip

Examples of frequency response functions obtained by swept-sine excitation at different force amplitudes are illustrated in Figure 4.5

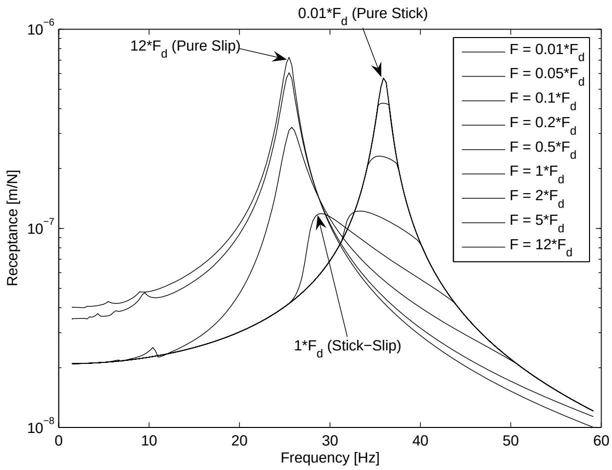

Figure 4.5: FRFs from different excitation amplitudes.

As shown in figure 4.5, the system has two different resonance frequencies. One at low levels of excitation and one at high levels, between these excitation levels the hysteretic effect in the system gives the best damping. At high levels of excitation the effects of the stick-slip nonlinearity is eliminated, due to the fact that the system is constantly in slip condition. Therefore the frequency response function at high levels of excitation can be used as an estimate of the underlying linear system and as input to the harmonic balance function.

Since the nonlinear function is supposed to be unknown a more general hysteresis function is used in the estimation. A system with this hysteresis function is shown in Figure 4.6.

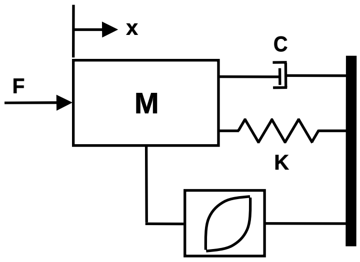

Figure 4.6: The assumed nonlinear system to be fitted to the reference system.

The virgin-curve of the hysteresis function in this system is governed by Equation (34).

$$F = \frac{k_d \cdot x}{\left(1+\left(\frac{k_d \cdot x}{F_d}\right)^N\right)^{\frac{1}{N}}} \tag{34}$$

Where:

k_d - The stiffness during stick-condition

F_d - The force level where the system starts to slip

N - Determines the curvature

The exponent N in Equation (34) makes it far more flexible than a normal stick-slip function and thereby more adaptable to an arbitrary hysteresis function.

Ten percent noise was added to the response of all time signals and the harmonic balance calculations were carried out using four harmonics [1 3 5 7]. The non-gradient search algorithm *fminsearch* in MATLAB® was used to find the desired parameters.

4.3 Results

The results from the simulation are displayed in table 4.1

	F_d	k_d	N
Reference Parameters	246.048 N	2.4e7 N/m	
Estimated Parameters	242.05 N	2.33e7 N/m	22.55

Table 4.1: Reference and estimated parameters.

Figure 4.7 shows a comparison between a reference hysteresis loop and the corresponding loop obtained by the estimated parameters of k_d, F_d and N.

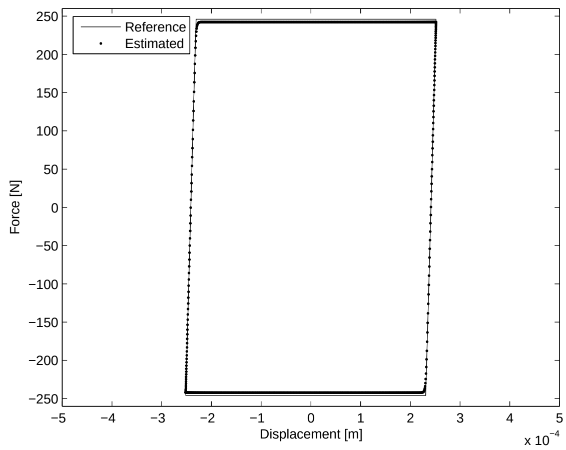

Figure 4.7: Comparison of the reference hysteresis loop and the loop obtained by the estimated parameters.

Figure 4.8 shows frequency response functions from sine-sweep measurements at different force amplitudes done on the estimated system and compared with the reference system.

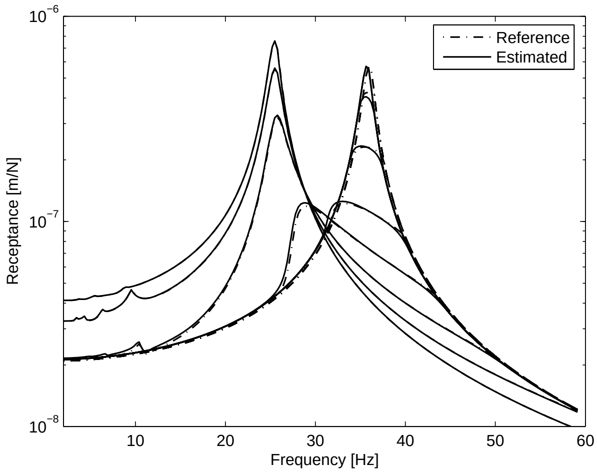

Figure 4.8: Comparison of frequency response functions from the reference and estimated system at different force amplitudes.

The match between the reference system and the estimated system is good but not perfect. Since the function used in the reference and estimated system are not the same and due to the fact that ten percent noise was added to all response signals, the result obtained looks promising. Also several different simulations have been carried out, not presented in this paper, both at different force amplitudes and with different initial guesses. They have all converged towards the same solution.

5. Discussion and Conclusion

Two methods of parameter estimation have been presented in this paper; with random noise and sinusoidal excitation. The method described in chapter 3 uses a least-square estimation in the frequency domain to find the parameters while the method in chapter 4 relies on the multi-harmonic-balance method and uses an optimizations routine to find suitable parameters.

A major benefit with using random noise signals is that conventional and already established methods for linear systems are used such as MIMO-technique. The method can also easily be extended to multiple-degree-of-freedoms with several nonlinear elements. However, with sine excitation and the harmonic-balance concept a wider range of nonlinear problems can be solved, including nonlinearities with memory as shown in section 4.2.

As demonstrated, both methods shows large potential in simulations and will therefore be further examined with experimental testing.

Acknowledgements

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