

Minimization of Regret versus Unequal Multi-objective Fuzzy Decision Process in a Choice of Optimal Medicines

Elisabeth Rakus-Andersson

Blekinge Institute of Technology,
S-37179 Karlskrona, Sweden
Elisabeth.Andersson@bth.se

Abstract

Some researches have already developed fuzzy decision making theoretical algorithms, but there are only a few medical fields of their applications. We thus make a trial of adopting two decision-making algorithms based on unequal objectives and minimization of regret to extract the optimal medicine from a collection of drugs recommended to a patient. A choice of the most efficacious remedy is based on clinical symptoms typical of the considered morbid unit.

Keywords: Fuzzy decision making, weighted objectives, eigen values, payoff matrix, regret matrix.

1 Introduction

Theoretical fuzzy decision-making models, mostly described in [2, 5, 12, 13, 15, 16], give rise to successfully accomplished technical applications. However, there are not so many medical applications to decision-making proposals, especially in the domain of pharmacy matters.

Some auxiliary methods coming from Fuzzy Set Theory have been already applied to make attempts of solutions in appreciation of drug efficiency. In own research work [4, 9] we have tested fuzzy eigen set techniques [11] to evaluate the optimal levels of one drug influence on several symptoms characteristic of a disease.

We often experience that there can occur such a morbid process in which the symptoms do not disappear after the treatment when treating patients with one medicine only. The medication improves too high or too low level of a quantitative symptom but the symptom still indicates the presence of the disease. We sometimes experience some problems to choose the medicine which acts best, since it can happen that most of drugs affect the same symptoms while they do not improve the others.

By employing fuzzy decision models based on utility theory [3, 5], we have facilitated the choice of this drug, which affects most of the symptoms in the highest degree [6, 7]. We have even considered how to extract the best medicine in the circumstances when some decision-makers have different opinions about the priority of the tested drugs [8].

The models, which we intend to use in making the best choice of medicine have as a theoretical basis Yager's conceptions of fuzzy decision making. We first apply fuzzy sets to multi-objective decision making with particular emphasis on degrees of importance attached to different objectives [12, 13]. As a comparative decision-making model we discuss a method founded on regret minimization [15, 16]. In the paper we build the contents of decisive pharmacological objectives in Section 2 to extend their conclusive power by adding importance weights in Section 3. The regret minimization method is formalized and practised in Section 4.

The appropriate communication between interdisciplinary researchers makes their cooperation much easier. Therefore we have added a large part of text, which throws some light on the problem of a connection between semantic expressions and numbers.

2 Construction of Objectives

We introduce the notions of a space of states $X = \{x_1, \dots, x_m\}$ and a decision space (a space of alternatives) $A = \{a_1, \dots, a_n\}$. We consider a decision model in which n alternatives $a_1, \dots, a_n \in A$ act as drugs used to treat patients who suffer from a disease. The medicines should influence m states $x_1, \dots, x_m \in X$, which are identified with m symptoms typical of the morbid unit under consideration.

The drugs-decisions constitute n elements in supports of fuzzy sets K_k , $k = 1, \dots, m, m+1, m+2$, determined as some criteria-objectives, which restrict the set A [13]. Hence, we can treat each set K_k as a fuzzy subset of A , i.e., $K_k : A \rightarrow [0, 1]$, $k = 1, \dots, m+2$.

After preparing the criteria-objectives K_k we are ready to make a fuzzy decision, which is affected by all of them.

The fuzzy decision D , taking into account K_1 and K_2 and ... and K_{m+2} , is made in accordance with the minimum decision rule [2, 12, 13]

$$D = K_1 \cap \dots \cap K_m \cap \dots \cap K_{m+2}. \quad (1)$$

This provides us with the function

$$\mu_D(a_i) = \min(\mu_{K_1}(a_i), \dots, \mu_{K_m}(a_i), \dots, \mu_{K_{m+2}}(a_i)) \quad (2)$$

for each $a_i \in A$.

The optimal drug-decision is accepted as this a_i , $i = 1, \dots, n$, which has the maximal value of the membership degree in D according to

$$\mu_D(a_{\text{optimal}}) = \max_{1 \leq i \leq n} (\mu_D(a_i)). \quad (3)$$

Firstly we intend to discuss a fuzzy decision making model supported by [13]. We present a solution in steps, which are assisted by appro-

priate examples added in order to explain the decision-making procedure successively.

In the model of accepting the most optimal medicine a_i , $i = 1, \dots, n$, we assume that the first m restriction sets K_j , $j = 1, \dots, m$, are defined by

$$K_j = \text{"influence of } a_1, \dots, a_n \text{ on symptom } x_j \text{"} = a_1 \text{'s effect concerning } x_j/a_1 + \dots + a_n \text{'s effect concerning } x_j/a_n. \quad (4)$$

In spite of drug effectiveness, which definitely is the most important factor in the appreciation of drug action, we can introduce other substantial elements assisting drug decision-making like side effects of medicines or their prices. We thus form the next fuzzy set

$$K_{m+1} = \text{"side effects of } a_1, \dots, a_n \text{ supporting the decision positively"} = 1 - \text{side effects of } a_1/a_1 + \dots + 1 - \text{side effects of } a_n/a_n \quad (5)$$

in which a physician estimates the strength of all side effects of the drugs. The side effects of drugs a_i , $i = 1, \dots, n$, are rather unfavorable occurrences; therefore their lack in a_i should be emphasized by the larger membership value assigned to a_i as an indication of a safe medicine consumption. For the purpose of enlarging membership values of these medicines that have not extensive side effects we use the complement operation 1-estimation of side effects.

The last constraint

$$K_{m+2} = \text{"estimation of price availability for } a_1, \dots, a_n \text{"} = \text{price availability of } a_1/a_1 + \dots + \text{price availability of } a_n/a_n \quad (6)$$

is added in order to enlarge a number of decisive indications.

Not all symptoms retreat after the cure has been carried out. One can only sometimes soothe their negative effects by, for example, the lowering of an excessive level of the indicator, the relief of pain, and the like.

Let us find a practical way of determining effectiveness of drugs as mathematical expressions, which should take place in the first m objectives. To simplify the symbols we assume

that each symptom $x_j \in X$, where X is a space of symptoms (states), is understood as the result of the treatment of the symptom after the cure with the drugs a_1, a_2, \dots, a_n has been carried out.

On the basis of earlier experiments, the physician knows how to define in words the curative drug efficiency in the case of considered symptoms. In accordance with his advice we suggest a list of terms, which introduces a linguistic variable named “the curative drug effectiveness concerning a symptom” = $\{R_1 = \text{none}, R_2 = \text{almost none}, R_3 = \text{very little}, R_4 = \text{little}, R_5 = \text{rather little}, R_6 = \text{medium}, R_7 = \text{rather large}, R_8 = \text{large}, R_9 = \text{very large}, R_{10} = \text{almost complete}, R_{11} = \text{complete}\}$ [1]. Each notion from the list of terms is the name of a fuzzy set. Assume that all sets are defined in the space $Z = [0, 100]$, which is suitable as a reference set to measure the number of patients who have been affected by a medicine in the grade corresponding to each name.

Let us propose the membership functions for the fuzzy sets from the list, called “the curative drug effectiveness concerning a symptom” by applying as constrains simple linear functions [6, 7, 8, 9]

$$L(z, \alpha, \beta) = \begin{cases} 0 & \text{for } z \leq \alpha \\ \frac{z - \alpha}{\beta - \alpha} & \text{for } \alpha < z \leq \beta \\ 1 & \text{for } z > \beta \end{cases} \quad (7)$$

and

$$\pi(z, \alpha, \gamma, \beta) = \begin{cases} 0 & \text{for } z \leq \alpha \\ L(z, \alpha, \gamma) & \text{for } \alpha < z \leq \gamma \\ 1 - L(z, \gamma, \beta) & \text{for } \gamma < z \leq \beta \\ 0 & \text{for } z > \beta \end{cases} \quad (8)$$

in which z is an independent variable belonging to $[0, 100]$, and α, β, γ are some parameters.

Let us define

$$\mu_{R_t}(z) = \begin{cases} 1 - L(z, \alpha_t, \beta_t) & \text{for } t = 1, 2, 3, 4, 5 \\ L(z, \alpha_t, \beta_t) & \text{for } t = 7, 8, 9, 10, 11 \end{cases} \quad (9)$$

and

$$\mu_{R_6}(z) = \pi(z, \alpha_6, \gamma, \beta_6), \quad (10)$$

where $z \in Z = [0, 100]$, while $\alpha_t, \beta_t, \gamma$ are the borders for supports of the fuzzy sets $R_1 - R_{11}$ and they also constitute some numbers from the interval $[0, 100]$.

We should decide the values of the boundary parameters $\alpha_t, \beta_t, \gamma$ in order to construct constrains for the fuzzy sets that represent the terms of the mentioned list “the curative drug effectiveness concerning a symptom”.

Example 1

We suggest the following functions that can be approved as the membership functions of terms composing the contents of the effectiveness list

$$\begin{aligned} \mu_{R_1}(z) &= \mu_{\text{none}}(z) = 1 - L(z, 0, 20), \\ \mu_{R_2}(z) &= \mu_{\text{almost none}}(z) = 1 - L(z, 10, 30), \\ \mu_{R_3}(z) &= \mu_{\text{very little}}(z) = 1 - L(z, 20, 40), \\ \mu_{R_4}(z) &= \mu_{\text{little}}(z) = 1 - L(z, 30, 50), \\ \mu_{R_5}(z) &= \mu_{\text{rather little}}(z) = 1 - L(z, 40, 60), \\ \mu_{R_6}(z) &= \mu_{\text{medium}}(z) = \pi(z, 30, 50, 70), \\ \mu_{R_7}(z) &= \mu_{\text{rather large}}(z) = L(z, 40, 60), \\ \mu_{R_8}(z) &= \mu_{\text{large}}(z) = L(z, 50, 70), \\ \mu_{R_9}(z) &= \mu_{\text{very large}}(z) = L(z, 60, 80), \\ \mu_{R_{10}}(z) &= \mu_{\text{almost complete}}(z) = L(z, 70, 90), \\ \mu_{R_{11}}(z) &= \mu_{\text{complete}}(z) = L(z, 80, 100) \end{aligned}$$

for $z \in [0, 100]$.

The parameters α_t, β_t and γ in (9) and (10) have been proposed in conformity with the physician’s suggestion.

To each effectiveness, expressed as a continuous fuzzy set, we would like to assign only one value.

Example 2

To decide the adequate $z \in [0, 100]$, which represent the effectiveness terms from the introduced list we take as z , in compliance with (9), α_t for $t = 1, 2, 3, 4, 5$, and β_t for $t = 7, 8, 9, 10, 11$, respectively γ from (10). By finding the values of α_t, β_t and γ in Ex. 1, we decide z -values from $[0, 100]$, which stand for the representatives of the following expressions: $z_{\text{none}} = 0$, $z_{\text{almost none}} = 10$, $z_{\text{very little}} = 20$, $z_{\text{little}} = 30$, $z_{\text{rather little}} = 40$, $z_{\text{medium}} = 50$, $z_{\text{rather large}} = 60$, $z_{\text{large}} = 70$, $z_{\text{very large}} = 80$, $z_{\text{almost complete}} = 90$, $z_{\text{complete}} = 100$. These

z -values are elements of the support of a new fuzzy set “effectiveness” whose membership function is expressed over the interval $[0, 100]$ by $\mu^{\text{effectiveness}}(z) = L(z, 0, 100)$. For the representatives of z , sorted above, we finally compute membership values $\mu^{\text{effectiveness}}(z)$, which replace the terms of effectiveness decided by the physician in the drug choice matter. We summarize the obtained results in Table 1.

To state a connection between a_i (medicine) and the effectiveness of the retreat of x_j (symptom) the physician uses the word from the list “the curative drug effectiveness concerning a symptom” and this word is “translated” into the associated value, taking place in Table 1.

The associations between average words and quantities replacing these semantic structures allow us to “compute with words”.

Table 1: The representatives of effectiveness

| Effectiveness | Representing z -value | $\mu(z)$ |
|-----------------|-------------------------|----------|
| none | 0 | 0 |
| almost none | 10 | 0.1 |
| very little | 20 | 0.2 |
| little | 30 | 0.3 |
| rather little | 40 | 0.4 |
| medium | 50 | 0.5 |
| rather large | 60 | 0.6 |
| large | 70 | 0.7 |
| very large | 80 | 0.8 |
| almost complete | 90 | 0.9 |
| complete | 100 | 1 |

Let us now give the example of a medical task to be solved.

Example 3

We have obtained the clinical data, which concerns the diagnosis “*coronary heart disease*”. We consider the most substantial symptoms $x_1 =$ “*pain in chest*”, $x_2 =$ “*changes in EKG*” and $x_3 =$ “*increased level of LDL-cholesterol*”. The recommended medicines that can improve the patient’s state are listed as $a_1 =$ *nitroglycerin*, $a_2 =$ *beta-adrenergic blockade*, $a_3 =$ *acetylsalicylic acid (aspirin)* and $a_4 =$ *statine LDL-reductor*.

The physician has judged the relationship between efficiency of drugs and retreat of symptoms. We express the connections in Table 2.

Table 2: The relationship between medicine action and retreat of symptom

| Drug action \Symptoms | x_1 | x_2 | x_3 |
|-----------------------|----------|------------|-------------|
| a_1 | complete | very large | almost none |
| a_2 | medium | medium | little |
| a_3 | little | little | very little |
| a_4 | little | little | very large |

We take into account Eq. (4) to construct the sets K_j , $j = 1, 2, 3$, by applying the contents of Table 2 and the last column of Table 1. Hence,

$$K_1 = \text{"influence of } a_1, a_2, a_3, a_4 \text{ on } x_1 \text{"} = \\ \text{complete}/_{a_1} + \text{medium}/_{a_2} + \text{little}/_{a_3} + \text{little}/_{a_4} = \\ 1/a_1 + 0.5/a_2 + 0.3/a_3 + 0.3/a_4,$$

$$K_2 = \text{"influence of } a_1, a_2, a_3, a_4 \text{ on } x_2 \text{"} = \\ \text{very large}/_{a_1} + \text{medium}/_{a_2} + \text{little}/_{a_3} + \text{little}/_{a_4} = \\ 0.8/a_1 + 0.5/a_2 + 0.3/a_3 + 0.3/a_4$$

and

$$K_3 = \text{"influence of } a_1, a_2, a_3, a_4 \text{ on } x_3 \text{"} = \\ \text{almost none}/_{a_1} + \text{little}/_{a_2} + \text{very little}/_{a_3} + \text{very large}/_{a_4} = \\ 0.1/a_1 + 0.3/a_2 + 0.2/a_3 + 0.8/a_4.$$

The physician has evaluated side effects of the drugs in the set K_4 by assimilating the words from the first column of Table 1. We have already mentioned that the side effects of a_i , $i = 1, \dots, 4$, are negative appearances and their lack in a_i , e.g., “*side effects of } a_i*” = “*almost none*”, should be expressed by the larger membership value assigned to a_i . We thus adopt the complement operation 1–estimation of side effects. The set K_4 is established in accordance with (5) as

$$\begin{aligned}
K_4 &= \text{"side effects of } a_1, a_2, a_3, a_4 \\
&\text{ supporting the decision positively"} = \\
&1\text{-very little}/a_1 + 1\text{-little}/a_2 + 1\text{-rather large}/a_3 + 1\text{-very little}/a_4 = \\
&1-0.2/a_1 + 1-10.3/a_2 + 1-0.6/a_3 + 1-0.2/a_4 = \\
&0.8/a_1 + 0.7/a_2 + 0.4/a_3 + 0.8/a_4.
\end{aligned}$$

The prices of all medicines are not at the least inconvenient for patients to purchase them. If we note that the large value of a membership degree corresponds to a rather cheap and available medicine we can state the set K_5 by examining (6) as

$$\begin{aligned}
K_5 &= \text{"price availability for } a_1, a_2, a_3, a_4 \text{"} = \\
&0.8/a_1 + 0.8/a_2 + 0.9/a_3 + 0.8/a_4.
\end{aligned}$$

All objectives are created now and we could have put them in Eqs (1) and (2) to emerge the optimal decision. As the influence of medicines on considered symptoms has a conclusive character in making the right decision then we, in addition to all tools discussed already, want to add a factor of importance, which should lift up the meaning of some objectives in the decision action.

3 Importance of Objectives

If we can associate with each fuzzy objective K_k , $k = 1, \dots, m, m+1, m+2$, a non negative number, which indicates its power or importance in the decision according to the rule: the higher the number the more important criterion K_k , then we could raise each fuzzy criterion set to this power before combining to form D . If we regard $w_1, \dots, w_m, \dots, w_{m+2}$ as powers-weights of $K_1, \dots, K_m, \dots, K_{m+2}$ then we will modify (1) as a richer and more extended decision

$$D = K_1^{w_1} \cap \dots \cap K_m^{w_m} \cap \dots \cap K_{m+2}^{w_{m+2}} \quad (11)$$

in which the membership degree of each $a_i \in A$ is determined as

$$\begin{aligned}
\mu_D(a_i) &= \min((\mu_{K_1}(a_i))^{w_1}, \dots, \\
&(\mu_{K_m}(a_i))^{w_m}, \dots, (\mu_{K_{m+2}}(a_i))^{w_{m+2}}).
\end{aligned} \quad (12)$$

A procedure for obtaining a ratio scale of importance for a group of $m+2$ elements was developed by Saaty in [10].

Assume that we have $m+2$ objectives and we want to construct a scale, rating these objectives as to their importance with respect to the decision. We ask a decision maker to compare the objectives in paired comparison. If we compare objective k with objective l , we assign the values b_{kl} and b_{lk} as follows:

- (1) $b_{lk} = \frac{1}{b_{kl}}$,
- (2) If objective k is more important than objective l then b_{kl} gets assigned a number according to the following scheme

| b_{kl} | Intensity of importance of K_k over K_l |
|----------|---|
| 1 | Equal |
| 3 | Weak |
| 5 | Strong |
| 7 | Demonstrated |
| 9 | Absolute. |

If objective l is more important than objective k , then we will assign the value of b_{lk} .

Having obtained the above judgments a $(m+2) \times (m+2)$ square matrix $B = (b_{kl})_{k,l=1}^{m+2}$ is constructed in the drug decision problem sketched above.

Example 4

The physical status of a patient is subjectively better if the pain disappears, which means that a physician tries to release the patient from the symptom $x_1 = \text{"pain in chest"}$. The next priority is assigned to $x_2 = \text{"changes in EKG"}$ and finally, we concentrate our attention on getting rid of $x_3 = \text{"increased level of LDL cholesterol"}$.

These remarks are helpful when constructing a content of the matrix B as

$$B = \begin{matrix} & \begin{matrix} K_1 & K_2 & K_3 & K_4 & K_5 \end{matrix} \\ \begin{matrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \end{matrix} & \begin{bmatrix} 1 & 3 & 5 & 7 & 7 \\ \frac{1}{3} & 1 & 3 & 7 & 7 \\ \frac{1}{5} & \frac{1}{3} & 1 & 7 & 7 \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & 1 & 3 \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{3} & 1 \end{bmatrix} \end{matrix}.$$

The matrix B constitutes a crucial part in the procedure of determining the degrees of importance $w_1, \dots, w_m, \dots, w_{m+2}$, which affect the decision set D in the substantial way (in accord with (12)). The weights are decided as components of the eigen vector, which corresponds to the largest in magnitude eigen value of the matrix B .

Example 5

To get the largest eigen value of B we consider an equation

$$\det(B - \lambda I) = 135\lambda^5 - 675\lambda^4 - 2172\lambda^2 - 1440\lambda - 320 = 0$$

in which I is a unit matrix of the same type $(m+2) \times (m+2)$ and λ denotes B 's eigen value. B has only one real eigen value $\lambda = 5.5805$. The associated eigen vector $V = (0.83215, 0.46393, 0.26609, 0.08575, 0.055586)$ is composed of coordinates that are interpreted as the weights sought for $K_k, k = 1, \dots, 5$. The computations have been performed by the package Maple 9.0.

The sets $K_k, k = 1, \dots, 5$, already found in Ex. 3, are now completed by introducing their grades of importance.

We update the objectives as the sets

$$K_1 = 1^{0.83215}/a_1 + 0.5^{0.83215}/a_2 + 0.3^{0.83215}/a_3 + 0.3^{0.83215}/a_4,$$

$$K_2 = 0.8^{0.46393}/a_1 + 0.5^{0.46393}/a_2 + 0.3^{0.46393}/a_3 + 0.3^{0.46393}/a_4,$$

$$K_3 = 0.1^{0.26609}/a_1 + 0.3^{0.26609}/a_2 + 0.2^{0.26609}/a_3 + 0.8^{0.26609}/a_4,$$

$$K_4 = 0.8^{0.08575}/a_1 + 0.7^{0.08575}/a_2 + 0.4^{0.08575}/a_3 + 0.8^{0.08575}/a_4$$

and

$$K_5 = 0.8^{0.055586}/a_1 + 0.8^{0.055586}/a_2 + 0.9^{0.055586}/a_3 + 0.8^{0.055586}/a_4.$$

The final decision D is obtained, according to the recommended Eqs (11) and (12), as a fuzzy set

$$D = 0.5418/a_1 + 0.5616/a_2 + 0.3671/a_3 + 0.3671/a_4.$$

We conclude that the curative power of considered medicines is ranked in the order $a_2 \approx a_1 \succ a_4 = a_3$ provided that the symbol $a_i \succ a_j$ is assigned to the statement “ a_i acts better than a_j ”. We have considered not only effectiveness of drugs regarding their action on symptoms but also the priority of symptoms, which should disappear for the reason of their harm influence on the patient's physical and psychical condition.

4 Minimization of regret

The action of the minimum operation in the final decision formula has provided us with a very cautious prognosis referring to the drug hierarchy. Some high values of degrees, which reflect a positive effect of medicine impact on considered symptoms, have no chance of influencing finally computed decision values. We can even say that the minimum operation acts as a filter for high values by depriving them of their decisive power.

We try to obtain clearer results by applying another fuzzy decision-making technique known as a minimization of regret [15, 16]. Let us prepare a new medical apparatus by reorganizing the sets previously introduced. We preserve a decision space of alternatives $A = \{a_1, \dots, a_n\}$ but we complement a space of states as $X = \{x_1, x_2, \dots, x_m, x_{m+1}, x_{m+2}\}$. In X there are symbols possessing the following meanings: x_1 – the 1st symptom, \dots , x_m – the m^{th} symptom, x_{m+1} – medicine side effects, x_{m+2} – medicine price availability. We form a basic payoff matrix

$$C = \begin{matrix} & x_1 & \dots & x_k & \dots & x_{m+2} \\ \begin{matrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{matrix} & \left[\begin{array}{cccccc} & & & & & \\ & & & & & \\ & & & & & \\ & & & c_{ik} & & \\ & & & & & \\ & & & & & \end{array} \right] & (13) \end{matrix}$$

where c_{ik} is the payoff to a decision maker if he connects a_i to $x_k, i = 1, \dots, n, k = 1, \dots, m+2$.

In a continuation of the proposed approach to a choice of the optimal medicine we first obtain a regret matrix R whose components r_{ik} indicate the decision-maker's regret in selecting alternative a_i when the state of X is x_k . We then calculate the maximal regret for each alternative.

A procedure of selecting an optimal a_i should follow the steps listed below:

1. For each x_k calculate $C_k = \max_{1 \leq i \leq n} c_{ik}$,
2. For each pair a_i and x_k calculate $r_{ik} = C_k - c_{ik}$,
3. Suppose that matrix B from Section 3 consists of b_{kl} , which now reflect the importance scale when comparing states x_k and x_l , $k, l = 1, \dots, m + 2$. The coordinates of this eigen vector that assists the largest in magnitude eigen value of B still constitute weights w_1, \dots, w_{m+2} assigned to states x_1, \dots, x_{m+2} stated in X . The weights are involved in the computations of estimates $RT_i = w_1 r_{i1} + \dots + w_{m+2} r_{i,m+2}$ for each a_i . It can be proved that the formulas derived for calculations of RT_i satisfy the conditions of OWA operators [14].
4. Select a_{i^*} , such that $RT_{i^*} = \min_{1 \leq i \leq n} RT_i$.

The values r_{ik} constitute the entries of the matrix R called the regret matrix. We shall refer to C_k as the horizon under x_k .

Example 6

The sets K_1 – K_5 found in Ex. 3 are now utilized as columns of the matrix C , determined by a table

$$C = \begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \left[\begin{array}{ccccc} 1^* & 0.8^* & 0.1 & 0.8^* & 0.8 \\ 0.5 & 0.5 & 0.3 & 0.7 & 0.8 \\ 0.3 & 0.3 & 0.2 & 0.4 & 0.9^* \\ 0.3 & 0.3 & 0.8^* & 0.8^* & 0.8 \end{array} \right] \end{matrix}$$

in which “*” points to the largest element in each column as recommended by 1.

The regret matrix R is determined in the form of the next table

$$R = \begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \left[\begin{array}{ccccc} 0 & 0 & 0.7 & 0 & 0.1 \\ 0.5 & 0.3 & 0.5 & 0.1 & 0.1 \\ 0.7 & 0.5 & 0.6 & 0.4 & 0 \\ 0.7 & 0.5 & 0 & 0 & 0.1 \end{array} \right] \end{matrix}$$

For $w_1 \approx 0.83$, $w_2 \approx 0.46$, $w_3 \approx 0.27$, $w_4 \approx 0.09$, $w_5 \approx 0.05$ (Ex. 5) the values of RT_i , $i = 1, \dots, 4$, are appreciated as

$$RT_1 = 0.83 \cdot 0 + 0.46 \cdot 0 + 0.27 \cdot 0.7 + 0.09 \cdot 0 + 0.05 \cdot 0.1 = 0.1895$$

$$RT_2 = 0.6894, RT_3 = 1.009, RT_4 = 0.816.$$

Finally, we decide the hierarchical order of drugs with respect to their curative abilities. We state them in a sequence $a_1 \succ a_2 \succ a_4 \succ a_3$, which totally confirms the results obtained by the technique of unequal objectives. Moreover, we notice that the last decision is very clearly interpretable and easy to make without special doubts. This emphasizes an advantage of applying the OWA weighted operations, which prevent a loss of substantial information.

5 Conclusions

We have adapted Yager’s mathematical fuzzy decision models in the process of extracting the best medicine from the collection of proposed remedies. The basis of investigations has been mostly restricted to a judgment of medicine influence on clinical symptoms, which accompany the disease. By employing the factors of importance associated with decisive objectives we could strengthen their crucial power as well.

As a result of fuzzy decision models involving unequal objectives and minimal regret we have obtained the hierarchical order of curative abilities of the recommended drugs. After the consultation with the experienced physician we can confirm the medical reliability of decisive effects. Nevertheless, we should admit that the decision, made by applying the model with unequal objectives, is very cautious because of the minimum operation used in the final decision set. The pure action of minimum, used to high effectiveness indicators, has deprived them of their positive decisive power. On the contrary, the results, brought by the application of regret minimization combined with using of OWA mean operators, have given us the opportunity of a reliable and undoubted medicine choice. It is a result of the simultaneous engagement of all effectiveness quantities in mean decision-making values involved in the regret model. In order to further improve the quality of a decision

it would be recommended to test Dempster-Shafer belief structures in decision-making [16].

Finally, we hope that the medical application of fuzzy decision-making, proved in the context of appreciation of drug efficiency, contributes an interesting applicable insertion in the fuzzy set theory environment.

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