The Choquet and Sugeno Integrals as Measures of Total Effectiveness of Medicines

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Abstract. The concepts of the Choquet and Sugeno integrals, based on a fuzzy measure, can be adopted as useful tools in estimation of the total effectiveness of a drug when appreciating its positive influence on a collection of symptoms typical of a considered diagnosis. The expected effectiveness of the medicine is evaluated by a physician as a verbal expression for each distinct symptom. By converting the words at first to fuzzy sets and then numbers we can regard the effectiveness structures as measures in the Choquet and Sugeno problem formulations. After comparing the quantities of total effectiveness among medicines, expressed as the values of the Choquet or Sugeno integrals, we accomplish the selection of the most efficacious drug.

Keywords: Fuzzy utilities, weights of importance, fuzzy decision-making model, Choquet integral, Sugeno integral.

1 Introduction

Theoretical fuzzy decision-making models, mostly described in [8, 9], give rise to successfully accomplished technical applications. However, there are not so many medical applications to decision-making proposals, especially they are lacking in the domain of pharmacy matters. In own research works [5, 6] we have already made attempts of appreciation of drug efficiency. We thus begin the paper with recalling and modernizing the mentioned model of the drug selection in Section 2. Since we need to develop the conception of effectiveness as a fuzzy measure then we will thoroughly discuss the connection between a symptom and a medicine in Section 3. In Section 4 we introduce the item of the Choquet integral seen from the point of view concentrating attention on the utility measure of drugs. After obtaining satisfactory results we continue discussing the effective role of integrals by extending – in Section 5 – the measurable effect of effectiveness on the subject of the Sugeno integral. The authors should mention that the approach to the use of integrals as computation tools of utility constitutes their original and genuine insertion in fuzzy decision-making.

2 The General Outline of a Drug Decision-making Model

We introduce the notions of a space of states $X = \{x_1,...,x_m\}$ and a decision space (a space of alternatives) $A = \{a_1,...,a_n\}$. We consider a decision model in which n alternatives $a_1,...,a_n \in A$ act as drugs used to treat patients who suffer from a disease. The medicines should influence m states $x_1,...,x_m \in X$, which are identified with m symptoms typical of the morbid unit considered.

If a rational decision maker makes a decision $a_i \in A$, $i = 1, 2, \ldots, n$, concerning states-results $x_j \in X$, $j = 1, 2, ..., m$, then the problem is reduced to the consideration of the ordered triplet (X, A, U), where X is a set of states-results, A – a set of decisions and U – the utility matrix [5, 6, 8, 9]

$$U = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_m \end{array} \\ \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \vdots & & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nm} \end{bmatrix} \end{array} \tag{1}$$

in which each element u_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, is a representative value belonging to [0, 1] for the fuzzy utility following from the decision a_i with the result x_j.

The theoretical model with the triplet (X, A, U) can find its practical application in the processes of choosing an optimal drug from a sample of tested medicines [5, 6].

Let us further associate with each state-symptom x_j, $j = 1, \ldots, m$, a non negative number that indicates its power or importance in decision making in accordance with the rule: the higher the number is, the greater significance of symptom x_j will be expected, when regarding its harmful impact on the patient's condition. If we assign w_1, w_2, …, w_m as powers-weights to x_1, x_2, …, x_m, $w_j \in W$, $j = 1, 2, \ldots, m$, where W is a space of weights, then we will modify (1) as the weighted matrix

$$U_W = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_m \end{array} \\ \begin{bmatrix} w_1 \cdot u_{11} & w_2 \cdot u_{12} & \cdots & w_m \cdot u_{1m} \\ w_1 \cdot u_{21} & w_2 \cdot u_{22} & \cdots & w_m \cdot u_{2m} \\ \vdots & & \ddots & \vdots \\ w_1 \cdot u_{n1} & w_2 \cdot u_{n2} & \cdots & w_m \cdot u_{nm} \end{bmatrix} \end{array}. \tag{2}$$

In compliance with data entries determined in (2), the common decisive power of a_i is approximated by the quantity $U_W(a_i)$ defined as an OWA operation [8, 9]

$$U_W(a_i) = \sum_{j=1}^{m} w_j \cdot u_{ij} \ . \tag{3}$$

As a final optimal decision a^* we select this a_i that satisfies

$$U_W(a^*) = \max_{1 \le i \le n} U_W(a_i), \tag{4}$$

i.e., we pick out the decision-drug possessing the highest utility grade with respect to symptoms cured. The distinct utility u_{ij} is comprehended to be the ability of the symptom retreat after medication. In other words, we define utility u_{ij} of a_i taken to x_j as effectiveness of drug a_i observed in the case of x_j.

3 The Decisive Role of Effectiveness in the Final Decision

Let us find a way of determining effectiveness of drugs as mathematical expressions that should take place in the matrix U_W. On the basis of earlier experiments, the physician defines in words the curative drug efficiency with respect to considered symptoms. He suggests a list of terms that introduce a linguistic variable named "drug effectiveness concerning symptom" = {R_1 = *none*, R_2 = *almost none*, R_3 = *very little*, R_4 = *little*, R_5 = *rather little*, R_6 = *medium*, R_7 = *rather large*, R_8 = *large*, R_9 = *very large*, R_{10} = *almost complete*, R_{11} = *complete*} [5, 6]. Each notion from the list of terms is the name of a fuzzy set. Assume that all sets are defined in the space $Z = [0,100]$, suitable as a reference set for supports of R_1-R_{11}.

We propose constrains for the fuzzy sets R_1-R_{11} by applying linear functions [5, 6]

$$L(z,\alpha,\beta) = \begin{cases} 0 & \text{for} \quad z \le \alpha \\ \dfrac{z-\alpha}{\beta-\alpha} & \text{for} \quad \alpha < z \le \beta \\ 1 & \text{for} \quad z > \beta \end{cases} \tag{5}$$

and

$$\pi(z,\alpha,\gamma,\beta) = \begin{cases} 0 & \text{for} \quad z \le \alpha \\ L(z,\alpha,\gamma) & \text{for} \quad \alpha < z \le \gamma \\ 1 - L(z,\gamma,\beta) & \text{for} \quad \gamma < z \le \beta \\ 0 & \text{for} \quad z > \beta \end{cases} \tag{6}$$

where z is an independent variable from [0, 100], whereas α, β, γ are parameters.

Let us now define

$$\mu_{R_t}(z) = \begin{cases} 1 - L(z,\alpha_t,\beta_t) & \text{for} \quad t = 1,2,3,4,5 \\ L(z,\alpha_t,\beta_t) & \text{for} \quad t = 7,8,9,10,11 \end{cases} \tag{7}$$

and

$$\mu_{R_6}(z) = \pi(z, \alpha_6, \gamma, \beta_6) \quad (8)$$

in which α_t, β_t, γ are the borders for supports of the fuzzy sets R_1-R_{11}.

We decide the values of the boundary parameters α_t, β_t, γ in Ex. 1 below.

Example 1

Figure 1 collects the graphs of fuzzy sets R_1-R_{11} that can be approved as the terms composing the contents of the effectiveness list.

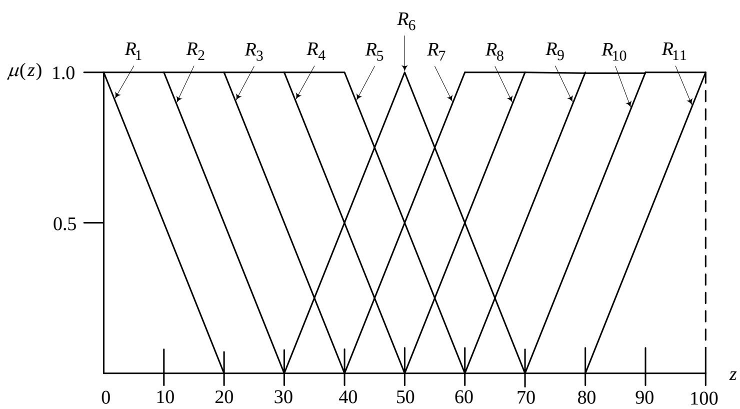

Fig. 1. The fuzzy sets R_1-R_{11}

To each effectiveness, expanded as a continuous fuzzy set, we would like to assign only one value.

Example 2

To find the adequate $z \in [0, 100]$ representing the effectiveness terms R_1-R_{11} we adopt as z the values α_t for $t = 1, 2, 3, 4, 5$, and β_t for $t = 7, 8, 9, 10, 11$ in compliance with (7), respectively γ due to (8). We simply read off the values of α_t, β_t and γ from Fig. 1 in order not to introduce evident calculations. These z-values are elements of the support of a new fuzzy set "*effectiveness*" whose membership function is expressed over the interval [0, 100] by $\mu_{\text{"effectiveness"}}(z) = L(z, 0, 100)$. For the z-representatives of R_1-R_{11}, we finally compute membership values $\mu_{\text{"effectiveness"}}(z)$, which replace the terms of effectiveness-utility as quantities u_{ij}. We summarize the obtained results in Table 1.

The next problem to put into discussion is a procedure of obtaining an importance ratio scale for a group of m symptoms [7]. Assume that for m states-symptoms we wish to construct a scale, rating them as to their importance for making the decision. We compare the symptoms in pairs with respect to their harmful impact on the patient's health. If we confront symptom j with symptom l, then we can assign the values b_{jl} and b_{lj} to the pair (x_j, x_l) as follows, $j, l = 1, 2, \ldots, m$:

Table 1: The representatives of effectiveness

Effectiveness	Representing z-value for effectiveness	$\mu(z) = u_{ij}$
none	0	0
almost none	10	0.1
very little	20	0.2
little	30	0.3
rather little	40	0.4
medium	50	0.5
rather large	60	0.6
large	70	0.7
very large	80	0.8
almost complete	90	0.9
Complete	100	1

(1) $b_{lj} = \frac{1}{b_{jl}}$,

(2) If symptom j is more important than symptom l then b_{jl} gets assigned one of the numbers 1, 3, 5, 7 or 9 due to the difference of importance being *equal*, *week*, *strong*, *demonstrated* or *absolute*, respectively. If symptom l is more important than symptom j, we will assign the value of b_{lj}.

Having obtained the above judgments an $m \times m$ matrix $B = (b_{jl})_{j,l=1}^{m}$ is constructed. The weights $w_1, w_2, \ldots, w_m \in W$ are decided as components of the eigen vector corresponding to the largest in magnitude eigen value of the matrix B. We normalize the weights w_j by dividing them all by the largest weight $w_{largest}$. We suggest this simple operation to keep all w_j within interval [0, 1] that now replaces W.

Let us denote the normalized weights by $\hat{w}_j = \frac{w_j}{w_{largest}}$. Afterwards we reorder $\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_m$ to generate an arrangement of the normalized weights as the ascending sequence $\hat{w}_1^a, \hat{w}_2^a, \ldots, \hat{w}_m^a$ satisfying the condition $0 \le \hat{w}_1^a \le \hat{w}_2^a \le \ldots \le \hat{w}_m^a = 1$. The symptoms x_j follow the new replacement of associated weights. In order to avoid too many designation signs let us name the ordered and normalized weights $\omega_j = \hat{w}_j^a$ and attached to them symptoms $\chi_j \in X$, $j = 1, 2, \ldots, m$. The matrix U_W is accommodated to new assumptions as $U_{[0,1]}$ given by

$$U_{[0,1]} = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} \chi_1 & \chi_2 & \cdots & \chi_m \end{array} \\ \begin{bmatrix} \omega_1 \cdot u_{11} & \omega_2 \cdot u_{12} & \cdots & \omega_m \cdot u_{1m} \\ \omega_1 \cdot u_{21} & \omega_2 \cdot u_{22} & \cdots & \omega_m \cdot u_{2m} \\ \vdots & & \ddots & \vdots \\ \omega_1 \cdot u_{n1} & \omega_2 \cdot u_{n2} & \cdots & \omega_m \cdot u_{nm} \end{bmatrix} \end{array}. \quad (9)$$

The formula (3) has been replaced by

$$U_{[0,1]}(a_i) = \sum_{j=1}^{m} \omega_j \cdot u_{ij} \qquad (10)$$

in regard to the new order of weights.

After the theoretical accomplishments of proofs yielding methods of stating crucial decision elements u_{ij} (effectiveness of assimilating a_i with x_j or, rather, with χ_j) and ω_j, let us show the results of the procedure that selects an optimal medicine.

Example 3

The following clinical data concerns the diagnosis "*coronary heart disease*". We consider the most substantial symptoms x_1 = "*pain in chest*", x_2 = "*changes in ECG*" and x_3 = "*increased level of LDL-cholesterol*". The medicines improving the patient's state are recommended as a_1 = *nitroglycerin*, a_2 = *beta-adrenergic blockade* and a_3 = *statine LDL-reductor*.

The physician has judged the relationship among efficiency of the drugs and retreat of the symptoms. We express the connections in Table 2.

Table 2: The relationship between medicine action and retreat of symptom

$a_i \backslash x_j$	x_1	x_2	x_3
a_1	complete, $u_{11} = 1$	very large, $u_{12} = 0.8$	almost none, $u_{13} = 0.1$
a_2	medium, $u_{21} = 0.5$	rather large, $u_{22} = 0.6$	little, $u_{23} = 0.3$
a_3	little, $u_{31} = 0.3$	little, $u_{32} = 0.3$	very large, $u_{33} = 0.8$

Further, we conclude that the physical status of a patient is subjectively better if the symptom x_1 = "*pain in chest*" disappears. The next priority is assigned to x_2 = "*changes in ECG*" and lastly, we concentrate our attention on getting rid of x_3 = "*increased level of LDL cholesterol*". We thus construct B as

$$B = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 5 \\ \frac{1}{3} & 1 & 3 \\ \frac{1}{5} & \frac{1}{3} & 1 \end{bmatrix} \end{array} .$$

The largest eigen value of B has the associated eigen vector $V = (0.93295, 0.30787, 0.18659)$. V is composed of coordinates that are interpreted as the weights w_1, w_2, w_3 sought for x_1, x_2, x_3. We normalize and rearrange the weights to obtain $\omega_1 = 0.2$ attached to $\chi_1 = x_3$, $\omega_2 = 0.33$ connected to $\chi_2 = x_2$ and $\omega_3 = 1$ as the power of $\chi_3 = x_1$.

Due to (10) we approximate the utilities $U_{[0,1]}(a_i)$ of medicines a_i, $i = 1, 2, 3$, as

$U_{[0,1]}(a_1) = 0.2 \cdot 0.1 + 0.33 \cdot 0.8 + 1 \cdot 1 = 1.284$,
$U_{[0,1]}(a_2) = 0.2 \cdot 0.3 + 0.33 \cdot 0.6 + 1 \cdot 0.5 = 0.758$,
$U_{[0,1]}(a_3) = 0.2 \cdot 0.8 + 0.33 \cdot 0.3 + 1 \cdot 0.3 = 0.559$.

After placing the utilities of drugs in the decreasing order (see (4)) we establish the hierarchy of medicines as $a_1 \succ a_2 \succ a_3$, when supposing that the notion $a_i \succ a_k$ stands for "a_i acts better than a_k" with respect to all involved symptoms, i, k = 1, 2, 3.

4 The Choquet Integral as Total Effectiveness

The normalization and the rearrangement of weights have been made in the intention of proving that formula (10) can be interpreted as a rule corresponding to the Choquet integral calculation [1, 2, 3, 4].

We know that the symptoms $\chi_1,...,\chi_m \in X$ act as objects in X. To them let us assign the measures $m(\{\chi_j|a_i\})$ = u_{ij}, where the symbols $\chi_j|a_i$ reflect the association between symptom χ_j and medicine a_i, j = 1, 2, …, m, i = 1, 2, …, n. The values $m(\{\chi_j|a_i\})$ are listed in the last column of Table 1.

The weights ω_j are set as the range values $f(\chi_j)$ of a function $f : X \to W = [0, 1]$.

By considering the latest suggestions we define the total utility of a_i gathered for all symptoms χ_1, χ_2, …, χ_m as the Choquet integral

$$U^{Ch}_{[0,1]}(a_i) = \int_{X=\{\chi_1,\chi_2,...,\chi_m\}} f(\chi_j)dm(\chi_j|a_i) \tag{11}$$

with respect to the measures $m(\{\chi_j|a_i\})$.

To find a precise calculus formula of integral (11) we study Fig. 2, made for three symptoms χ_1, χ_2, χ_3. This associates to (11) the following equation

$$\begin{aligned} U^{Ch}_{[0,1]}(a_i) &= \int_{X=\{\chi_1,\chi_2,\chi_3\}} f(\chi_j)dm(\chi_j|a_i) = (\omega_1 - \omega_0)\cdot m\{\chi_j|a_i : f(\chi_j) \ge \omega_1\} \\ &+ (\omega_2 - \omega_1)\cdot m\{\chi_j|a_i : f(\chi_j) \ge \omega_2\} + (\omega_3 - \omega_2)\cdot m\{\chi_j|a_i : f(\chi_j) \ge \omega_3\}, \end{aligned} \tag{12}$$

that practically explains how to understand the Choquet integral arithmetic. The measures of sets consisting of elements $\chi_j|a_i$, defined by properties $f(\chi_j) \ge \omega_1$, $f(\chi_j) \ge \omega_2$ and $f(\chi_j) \ge \omega_3$, are estimated as sums of utilities corresponding to respective $\chi_j|a_i$ fulfilling conditions above.

The general formula of the Choquet integral is revealed in the form

$$U^{Ch}_{[0,1]}(a_i) = \int_{X=\{\chi_1,\chi_2,...,\chi_m\}} f(\chi_j)dm(\chi_j|a_i) = \sum_{j=1}^{m} (\omega_j - \omega_{j-1})\cdot m\{\chi_l|a_i : f(\chi_l) \ge \omega_j\} \tag{13}$$

for $\omega_0 = 0$, l = 1, 2, …, m, i = 1, 2, …, n.

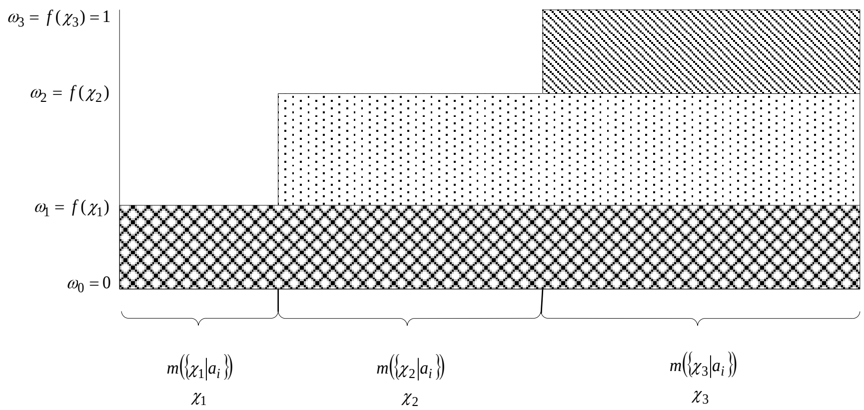

Fig. 2. The Choquet integral in evaluation of a_i's total curative effect

Let us recall that *m* is a utility measure with values in {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1} (the set of u_{ij}-values standing for effectiveness) defined for symptoms after treating them by medicines. If the utility is *none* then its measure will be equal to zero. For the total utility *complete* we reserve the measuring quantity of one. The physician can decide the common utility of medicine for two symptoms being less then the sum of utilities for distinct symptoms, e.g., the effectiveness of a_2 for "*pain in chest* and *changes in ECG*" together is judged as 0.5 while the separate measures of effectiveness emerge 0.6 and 0.5 (see Table 2). The last remark reveals the non-additive property of the effectiveness measure. Without any formal proofs made for confirmation of effectiveness as a fuzzy measure, we intend to use it in Choquet integrals constructed for the sample of medicines to approximate their curative effects.

In the next example we compute the entire effectiveness of medicines from Ex. 3 to compare the results obtained there.

Example 4

Let us involve formula (12) together with Fig. 2 to estimate

$$U_{[0,1]}^{Ch}(a_1) = \int_{X=\{\chi_1,\chi_2,\chi_3\}} f(\chi_j)dm(\chi_j|a_1) = (0.2-0)\cdot m\{\chi_j|a_1 : f(\chi_j) \geq 0.2\}$$
$$+(0.33-0.2)\cdot m\{\chi_j|a_1 : f(\chi_j) \geq 0.33\} + (1-0.33)\cdot m\{\chi_j|a_1 : f(\chi_j) \geq 1\}$$
$$= 0.2\cdot m\{\chi_1|a_1, \chi_2|a_1, \chi_3|a_1\} + 0.13\cdot m\{\chi_2|a_1, \chi_3|a_1\} + 0.67\cdot m\{\chi_3|a_1\}$$
$$= 0.2\cdot(0.1+0.8+1) + 0.13\cdot(0.8+1) + 0.67\cdot 1 = 1.284,$$

$$U_{[0,1]}^{Ch}(a_2) = 0.2\cdot(0.3+0.6+0.5) + 0.13\cdot(0.6+0.5) + 0.67\cdot 0.5 = 0.758$$

and

$U^{Ch}_{[0,1]}(a_3) = 0.2 \cdot (0.8 + 0.3 + 0.3) + 0.13 \cdot (0.3 + 0.3) + 0.67 \cdot 0.3 = 0.559$.

The results are identical with calculations obtained in Ex. 3, which confirms the proper interpretation of the Choquet integral in the drug ranking $a_1 \succ a_2 \succ a_3$.

5 The Sugeno Integral in Hierarchical Drug Order

To be able to introduce the Sugeno-like integral in the calculations leading to the choice of an optimal medicine, we normalize the measures $m\{\chi_l | a_i : f(\chi_l) \geq \omega_j\}$ from (13), j, l = 1, 2, …, m, when dividing them all by the largest value in the sequence. This operation provides us with the quantities $\hat{m}\{\chi_l | a_i : f(\chi_l) \geq \omega_j\}$ belonging to [0, 1].

As the next estimate of a_i's entire utility we propose a formula [1, 2, 3, 4]

$$U^{S}_{[0,1]}(a_i) = \int_{X=\{\chi_1,\chi_2,\ldots,\chi_m\}} f(\chi_j)dm(\chi_j | a_i) = \max_{1 \leq j \leq m}\left(\min\left(\omega_j, \hat{m}\{\chi_l | a_i : f(\chi_l) \geq \omega_j\}\right)\right) \qquad (14)$$

for j, l = 1, 2, …, m, i = 1, 2, …, n.

Example 5

The measures $m\{\chi_l | a_1 : f(\chi_l) \geq 0.2\} = 1.9$, $m\{\chi_l | a_1 : f(\chi_l) \geq 0.33\} = 1.8$ and $m\{\chi_l | a_1 : f(\chi_l) \geq 1\} = 1$ found for a_1 in Ex. 4 are now divided by the largest value of m equal to 1.9 to generate their normalized versions $\hat{m}\{\chi_l | a_1 : f(\chi_l) \geq 0.2\} = 1$, $\hat{m}\{\chi_l | a_1 : f(\chi_l) \geq 0.33\} = 0.947$ and $\hat{m}\{\chi_l | a_1 : f(\chi_l) \geq 1\} = 0.526$. In the scenario of (14) we estimate the utility of a_1 as

$$U^{S}_{[0,1]}(a_1) = \int_{X=\{\chi_1,\chi_2,\chi_3\}} f(\chi_j)dm(\chi_j | a_1) = \max(\min(0.2,1), \min(0.33,0.947), \min(1,0.526))$$
$$= 0.526.$$

For a_2 we get the utility value

$$U^{S}_{[0,1]}(a_2) = \max(\min(0.2,1), \min(0.33,0.786), \min(1,0.357)) = 0.357,$$

while a_3 possesses the affection grade on symptoms from X approximated as

$$U^{S}_{[0,1]}(a_3) = \max(\min(0.2,1), \min(0.33,0.428), \min(1,0.214)) = 0.33.$$

Even the application of the Sugeno integral provides us with the same hierarchy ladder of medicines upgraded in the order $a_1 \succ a_2 \succ a_3$. We should mention that the utility values in the last computations are comparable to the "ideal" utility equal to one that can be reached in the state of absolute absence of all symptoms.

6 Conclusions

As a primary method of fuzzy decision-making we have adopted Yager's model in the process of extraction of the best medicine from the collection of proposed remedies. The basis of investigations has been mostly restricted to a judgment of medicine influence on clinical symptoms accompanying the disease. We have also employed the indices of the symptoms' importance to emphasize the essence of additional factors in the final decision. By interpreting the utilities of drugs as measures the authors have furnished such tools as the Choquet and Sugeno integrals to rearrange the conception of the classical fuzzy decision-making, which constitutes an original contribution in decision model. The adaptation of the Choquet integral has provided us with the same results like these ones obtained by applying of the classical method. Even the adjustment of the Sugeno integral to the medication problem has brought the effects totally confirming the medicine order previously determined.

References

1. Grabisch, M., Murofushi, T., Sugeno, M., Kacprzyk, J.: Fuzzy Measures and Integrals. Theory and Applications. Physica Verlag, Berlin Heidelberg (2000)
2. Mirofushi, T., Sugeno, M.: An Interpretation of Fuzzy Measures and the Choquet Integral as an Integral with Respect to a Fuzzy Measure. Fuzzy Sets and Systems 29 (1989) 201-227
3. Mirofushi, T., Sugeno, M.: Fuzzy Measures and Fuzzy Integrals. In: Grabisch, M., Murofushi, T., Sugeno, M., Kacprzyk, J. (eds.): Fuzzy Measures and Integrals. Theory and Applications. Physica Verlag, Berlin Heidelberg (2000) 3-34
4. Sugeno, M.: Fuzzy Measures and Fuzzy Integrals - a Survey. In: Gupta, M. M., Saridis, G. N., Gaines, B. R. (eds.): Fuzzy Automata and Decision Processes, New York North-Holland (1977) 89-102
5. Rakus-Andersson, E.: A Choice of Optimal Medicines by Fuzzy Decision Making Including Unequal Objectives. Issues in the Representation and Processing of Uncertain and Imprecise Information, EXIT – The publishing House of The Polish Academy of Sciences (2005) 307-321
6. Rakus-Andersson, E.: Minimization of Regret versus Unequal Multi-objective Fuzzy Decision Process in a Choice of Optimal Medicines. Proceedings of the XIth International Conference IPMU 2006 – Information Processing and Management of Uncertainty in Knowledge-based Systems, vol. 2, Edition EDK, Paris-France (2006) 1181-1189
7. Saaty, T. L.: Exploring the Interface between Hierarchies, Multiplied Objectives and Fuzzy Sets. Fuzzy Sets and Systems 1 (1978) 57-68
8. Yager, R. R.: Fuzzy Decision-making Including Unequal Objectives. Fuzzy Sets and Systems 1 (1978) 87-95
9. Yager, R. R.: Decision-making Using Minimization of Regret. International Journal of Approximate Reasoning 36 (2004) 109-128