ABSTRACT

Vibration in internal turning is a problem in the manufacturing industry. Vibrations appear under the excitation applied by the material deformation process during the machining of a workpiece. In order for a lathe to perform an internal turning or boring operation, for example, in a pre-drilled hole in a workpiece, it is generally required that the boring bar should be long and slender; therefore extra sensitive to vibrations. These vibrations will affect the result of machining, in particular the surface finish, also the tool life may be reduced. As a result of tool vibration, severe acoustic noise frequently occurs in the working environment.

This thesis comprises three parts and the first part presents a method for active control of boring bar vibration. This method consists of an active boring bar controlled by, for example, an analog controller. The focus lies on the analog controller and the advantages that may be obtained from working in the analog domain. The controller is a lead-lag compensator with digitally controlled parameters, such as gain and phase. However, signals remain in the analog domain. In addition, the analog controller is compared with a digital adaptive controller and it is found that both controllers yield an attenuation of the vibration by up to 50 dB.

The second part of this thesis concerns the dynamic properties of a clamped boring bar used by the industry. In order to design a robust controller for a certain system, knowledge about the system’s dynamic properties is required. On the workshop floor, a boring bar is dismounted and remounted, and reconfiguration of boring bars will alter the dynamic properties of the clamped boring bar. The dynamic properties of a standard boring bar and an active boring bar for a number of possible clamping conditions, as well as for a linearized clamping have been investigated based on an experimental approach. Also simple Euler-Bernoulli modeling of clamped boring bars incorporating simple non-rigid models of the boring bar clamping are investigated. Initial simulations of nonlinear SDOF systems have been carried out: one with a signed squared stiffness and one with a cubic stiffness. The purpose of these simulations was to identify a nonlinearity that introduces a similar behavior in the SDOF system dynamics as the nonlinear behavior observed in the dynamic properties of a clamped boring bar.

The third and final part of this thesis focuses on vibration analysis methods in engineering education. A signal analyzer (which is a commonly used instrument in signal processing and vibration analysis) was made accessible via the Internet. Assignments were developed for students to learn and practice vibration analysis on real signals from a real setup of a relevant structure: a clamped boring bar. Whilst the experimental setup was fixed, the instrument and sensor configuration nonetheless enable a variety of experiment, for example: excitation signal analysis, spectrum analysis and experimental modal analysis.
Active control of vibration and analysis of dynamic properties concerning machine tools

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Abstract

Vibration in internal turning is a problem in the manufacturing industry. Vibrations appear under the excitation applied by the material deformation process during the machining of a workpiece. In order for a lathe to perform an internal turning or boring operation, for example, in a pre-drilled hole in a workpiece, it is generally required that the boring bar should be long and slender; therefore extra sensitive to vibrations. These vibrations will affect the result of machining, in particular the surface finish, also the tool life may be reduced. As a result of tool vibration, severe acoustic noise frequently occurs in the working environment.

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a real setup of a relevant structure; a clamped boring bar. Whilst the experimental setup was fixed, the instrument and sensor configuration nonetheless enable a variety of experiment, for example: excitation signal analysis, spectrum analysis and experimental modal analysis.
Preface

This licentiate thesis summarizes my work at the Department of Signal Processing at Blekinge Institute of Technology. The thesis is comprised of three parts:

Part

I On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry.

II Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions.

III Vibration Analysis of Mechanical Structures over the Internet Integrated into Engineering Education.
Acknowledgments

Firstly, I would like to express my sincere gratitude to Professor Ingvar Claesson for giving me the opportunity to begin a PhD candidacy. Special thanks to my supervisor and friend Lars Håkansson, for his encouragement, his profound knowledge within the field of both applied signal processing and mechanical engineering, improvements of this thesis, and many fruitful discussions. I would also like to thank former and present colleagues at the department of Signal Processing for all their help and fun, especially my dear friend and colleague Benny Sällberg for all his support and his constructive ideas, Tatiana for all her help. Also gratitude to Ingvar Gustavsson for his clear-sightedness and advice. Finally, I would like thank my wife Lisa for her love, support and patience throughout the years and for being the person she is.

Henrik Åkesson
Ronneby, 12th March 2007
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### Part

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Vibration concerns the repetitive motion of an object or objects relative to a stationary frame referred to as the equilibrium of the vibration. Vibrations may be measured in terms of displacement, velocity or acceleration. Vibrations exist everywhere and may have a great impact on the surrounding environment. One general phenomenon of vibration is the "self-oscillation" or resonance [1], meaning that a system exposed to even a weak force, which excites a resonance, may result in a substantial vibration level that eventually results in damage to or failure of the system. Thus, it is of great importance in engineering design to consider the properties of the system from a vibration point of view, referred to as the dynamic properties of the system. In Fig. 1 an example of the simplest vibrating system, a single degree of freedom system, is presented in conjunction with a diagram corresponding to the displacement \( x(t) \) of the mass.

Figure 1: A single degree of freedom system, with natural frequency \( f_0 \) and damping ratio \( \zeta \), and its oscillation behavior for an initial displacement \( A \).

Vibration is a frequent problem in the manufacturing industry, particular in the workshop, where metal cutting operations such as external and internal turning, boring, milling etc., take place. Vibrations affect the surface finish of the workpiece, the tool life and the noise level in the working environment. In order to increase productivity and tool life and improve the tolerance of machined workpieces, it is necessary to develop/utilize methods
which increase the stability and restrain the tool vibration in metal cutting. In many cases, the tooling structure may be considered a bottleneck with regard to the achievable accuracy imposed by static deflections and the cutting regimes, as well as surface finish due to forced and self-excited vibrations [2,3]. Long-overhang cantilever tooling (e.g. boring bars) is often the critical component of the tooling structure [2,3]. Fig. 2 presents a typical configuration for internal turning using a boring bar with long overhang, in which a tube is clamped at one side in the chuck by the jaws and the boring bar is clamped in the clamping house.

Figure 2: Typical configuration for internal turning, illustrating the long boring bar overhang required to turn deep holes.

There are several possible sources of boring bar vibrations: transient excitations due to rapid movements or the engagement phase of cutting; periodic excitation related to the residual rotor mass unbalance in the spindle-chuck-workpiece system; random excitation from the material deformation process, etc. [4]. The two most widely used theories for explaining self-excited chatter or tool vibration are the regenerative effect and the mode coupling effect [5–8]. These theories generally are explained based on the dynamic interaction of the cutting process and the machine tool structure, as the basic causes of chatter [5–8]. During cutting, the cutting force $F_r(t)$ is generated between the tool and the workpiece, see Fig. 3 b). The cutting force applied by the material deformation process during turning will strain the tool-boring bar structure and may introduce a relative displacement of the tool and the workpiece, changing
the tool and workpiece engagement. This relation between cutting force and tool displacement is commonly described by the feedback system in Fig. 3 a). The causes of instability are generally considered to derive from mechanisms providing energy, the regenerative effect and the mode coupling effect. The regenerative effect is considered to be the most frequent cause of instability and chatter, and may appear when the tool is removing an undulation on the workpiece surface that was cut on the previous revolution of the workpiece. Fig. 3 b) illustrates this scenario, where $h_0(t)$ is the desired cutting depth or chip thickness, $h(t)$ the actual chip thickness, $y(t)$ is the displacement of the tool at time $t$ and $y(t-T)$ is the displacement of the tool at the preceding revolution of the workpiece.

![Figure 3: a) Block diagram describing the cutting process-machine tool structure feedback system and b) the principle for regenerative chatter.](image)

However, the classical self-excited chatter models fails to explain some types of vibration which might exist in the system [9]. For instance, one model considers the material deformation process as a broadband vibration excitation of the boring bar. This model is the "self-excited vibration with white noise excitation" [9]. The irregularities of the workpiece surface, chemical composition, inhomogeneities, microstructure and spatial stochastic variation
of the hardening [10] results in a cutting force which may be considered as a stochastic process [11, 14]. Fig. 4 illustrates the material structure of one common work material, chromium molybdenum nickel steel SS 2541-03 (AISI 3239).

![Material Structure](image)

Figure 4: The material structure of chromium molybdenum nickel steel SS 2541-03 (AISI 3239).

A number of methods have been proposed to reduce harmful tool vibration. Three of these methods are as follows:

- "trial and error" - the operator tries to adapt the cutting data in an iterative fashion;
- passive control - constructional enhancement of the dynamic stiffness - can be achieved by increasing the structural damping and/or stiffness of the boring bar;
- active control - selective increase of the dynamic stiffness of a fundamental boring bar’s natural frequency.
The "trial and error" approach requires continuous supervision and control of the machining process by a skilled operator. Passive control is frequently tuned to increase the dynamic stiffness at a certain eigenfrequency, (for example, that of a particular boring bar) and thus is an inflexible solution [16,17]. Active control, such as based on an adaptive feedback controller and a boring bar with integrated actuator and vibration sensor, can be easily adapted to various configuration. Thus, active control provides a more flexible, and therefore preferable, solution. Active control of a boring bar can be implemented using either a digital or an analog approach. A digital controller based on a feedback filtered-x LMS algorithm [18] results in substantial attenuation of vibrations, and exhibit stable behavior. However, an analog controller with the corresponding vibration attenuation performance is required in order to avoid unnecessary delay in control authority and eventual tool failure in the engagement phase of the tool. A digital controller always introduces delay associated with controller processing time, A/D-and D/A- conversion processes and anti-aliasing and reconstruction filtering. Other benefits may include, low complexity, reduced cost and flexible bandwidth.

Both the analog and digital domain may be utilized for the implementation of feedback controllers [19–21]. Implementation of an active control solution requires a modification of the boring bar structure, thus a change of the bar's dynamic properties is likely. Modifications should be carried out with care to avoid undesired problems, such as making the boring bar too flexible, or moving boring bar resonance frequencies so that they coincide with other structural resonance frequencies of the machine tool system.

**PART I - On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry**

The application of active control of boring bar vibration in industry requires reliable, robust adaptive feedback controllers or manually tuned feedback controllers, that are simple to adjust on the workshop floor by the lathe operator. This part of the thesis presents the development of a simple adjustable robust analog controller, based on a digitally controlled, analog design that is suitable for the control of boring bar vibration in industry. Fig. 5 illustrates a block diagram of a feedback control system.

Initially, a digitally controlled, analog manually adjustable lead compen-
sator was developed. This manually adjustable controller approach was further developed to provide controller responses appropriate for the active control of boring bar vibration. The controller relies on a lead-lag compensator and enables manual tuning by the lathe operator. The emphasis has been on designing a controller that enables simple adjustment of its gain and phase to provide robust control appropriate for industry application.

Furthermore, this part of the thesis features a comparative evaluation of the two analog controllers and a digital controller based on the feedback filtered-x LMS algorithm [20,22] for performance and robustness in the active control of boring bar vibration. This evaluation includes a number of different dynamic properties of the boring bar, produced using a set of different clamping conditions likely to occur in industry. Both the developed analog controller and the adaptive digital controller manage to reduce the boring bar vibration level by up to approximately 50 dB.

PART II - Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions

Successful implementation of active control, such as the method presented in Part I, requires substantial knowledge concerning the dynamic properties of the tooling system. Furthermore, the interface between the boring bar and the lathe (i.e. clamping house) has a significant influence on the dynamic properties of the clamped boring bar. Part II of this thesis presents dynamic properties of boring bars for different clamping conditions, based on exper-
imental and analytical results. The different cases reflect on the variation that may be introduced in the clamping conditions of a boring bar when the operator mounts the boring bar in the clamping house and tightens the clamp screws, as illustrated in Fig. 6. Thus, this section focuses on those dynamic properties of a boring bar which arise due to different clamping conditions of the boring bar introduced by a clamping house (commonly used in the manufacturing industry). Also, this section briefly addresses the nonlinear behavior frequently observed in the dynamic properties of a clamped boring bar.

Figure 6: A lathe operator, mounting a boring bar in a clamping house using a standard wrench, accomplishing an arbitrary tightening torque. The boring bar may be expected to exhibit different properties when clamped or mounted in the clamping house by different operators and from time to time.

PART III - Vibration Analysis of Mechanical Structures over the Internet Integrated into Engineering Education

The final part of this thesis presents a pedagogical view related to experimental vibration analysis. Since experimental vibration analysis is the, or one of the most, important tools for analyzing the dynamic properties of mechanical structures, a contribution to enhance this field within engineering education
is provided.

Information derived from experimental vibration analysis is used in the development of products to obtain a required dynamic behavior or, for instance, to classify vibration problems in different public, industrial environments, etc. In order to carry out such experiments efficiently, experience of different analysis methods is of great importance. As is the case in any field, reliable results are usually obtained by those with a high degree of experience conducting experiments.

Blekinge Institute of Technology (BTH), Sweden, provides the opportunity for engineering students to remotely access practical and theoretical knowledge advancement in experimental vibration analysis via the Internet. This combination of practical and theoretical experience offered by BTH is highly attractive to industry. Fig. 7 illustrates the experimental setup and the transfer of the front panel of the instrument to the screen of the client.

Figure 7: An overview of the remote experimental vibration laboratory.

This section presents the remote vibration laboratory and how remote experimental vibration analysis has been integrated into engineering education as a complement to ordinary lectures and experiments in traditional laboratories.

References

Introduction


Part I

On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry
Part I is submitted as:

On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry

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Abstract

Vibration in internal turning is a problem in the manufacturing industry. This problem may, however, be reduced by utilizing active control based on active boring bars with an embedded actuator and sensor, and a suitable feedback controller. A digital adaptive controller may not be a sufficient solution to the problem. Due to the inherent delay in a digital adaptive controller, the delay of control authority in the engagement phase of the cutting edge (the abrupt change from no cutting to cutting) may result in tool failure. The controller delay may be reduced, for example, with an analog approach. A robust and simple adjustable analog controller suitable for industry application has been developed. The emphasis has been on designing a controller that enables simple adjustment of its gain and phase to provide robust control appropriate for the industry application. It relies on a lead-lag compensator and enables manual tuning by the lathe operator. The performance and robustness of the developed controller has been investigated and compared with an adaptive digital controller based on the feedback filtered-x algorithm. In addition, this paper takes into account those variations in boring bar dynamics which are likely to occur in industry; for example, when the boring bars is clamped in a lathe. Both the analog and the digital controller manage to reduce the boring bar vibration level by up to approximately 50 dB.
1 Introduction

Degrading vibrations in metal cutting e.g. turning, milling, boring and grinding are a common problem in the manufacturing industry. In particular, vibration in internal turning operations is a pronounced problem. To obtain required tolerances of the workpiece shape, and adequate tool-life, the influence of vibration in the process of machining a workpiece must be kept to a minimum. This necessitates extra care being taken in production planning and preparation. Vibration problems in internal turning have a considerable influence on important factors such as productivity, production costs, working environment, etc. Generally, the tooling structure -the interface between the cutting tool or insert and the machine tool- is the weakest link in a machining system [1,2]. In many cases, the tooling structure is the bottleneck concerning the achievable accuracy imposed by static deflections and the cutting regimes as well as the surface finish due to forced and self-excited vibrations. Long-overhang cantilever tooling, e.g. boring bars, is often the critical component of the tooling structure [1,2]. In internal turning (or when the metal cutting process is carried out in pre-drilled holes or holes in cast etc.) the dimensions of the workpiece hole will generally determine the length and limit the diameter or cross-sectional size of the boring bar. As a result, boring bars are frequently long and slender - long-overhang cantilever tooling- and thus sensitive to excitation forces introduced by the material deformation process in the turning operation [1–3].

A number of experimental studies have been carried out on the mechanism that results in the vibrations during the turning operation [4–8] and on the dynamic properties of tool holder systems and boring bars [8–12]. As early as 1946 Arnold [4] determined the principles of the traditional theory of chatter in simple machine-tool systems.

The vibration problems in internal turning can be addressed using both passive and active methods [1, 2, 13]. Common methods used to increase dynamic stiffness of cantilever tooling involve making them (in high Young’s modulus) non-ductile materials, such as sintered tungsten carbide and machinable sintered tungsten, and/or utilizing passive Tuned Vibration Absorbers (TVA) [1,2,14]. These passive methods are known to enhance the dynamic stiffness and stability (chatter-resistance) of long cutting tools and thus, enable the allowable overhang to be increased [1,2,14]. The passive methods offer solutions with a fix enhancement of the dynamic stiffness frequently tuned for a narrow frequency range comprising a certain bending mode frequency that
On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry

in some cases may be manually adjusted [1, 2, 14]. On the other hand, the active control of tool vibration enables a flexible solution that selectively increases the dynamic stiffness at the actual frequency of the dominating bending modes, until the level of the chatter component in the feedback signal is negligible [2, 13, 15]. In the sixties, control strategies concerning the reduction of tool vibration turning was reported by Comstock et al. [16]. This concerned the active control of the position of the cutting tool relative to the workpiece in the cutting depth direction [16]. The controller was further developed by Mitchell and Harrison [17]. In the seventies Inamura and Sata [18] introduced strategies concerning the control of cutting data with respect to the stability of machining. A more recent approach was reported by Tewani et al. [19, 20] concerning active dynamic absorbers in boring bars controlled by a digital state feedback controller. Browning et al. [21] reported an active clamp for boring bars controlled by a feedback version of the filtered-x LMS algorithm. Claesson and Håkansson [15] controlled tool vibration by using the feedback filtered-x LMS algorithm to control tool shank vibration in the cutting speed direction without applying the traditional regenerative chatter theory.

Two important constraints concerning the active control of tool vibration involve the difficult environment in a lathe and industry demands. It is necessary to protect the actuator and sensors from the metal chips and cutting fluid. Also, the active control system should be applicable to a general lathe. Pettersson et al. [22] reported an adaptive active feedback control system based on a tool holder shank with embedded actuators and vibration sensors. This control strategy was later applied to boring bars by Pettersson et al. [23]. Åkesson et al. [24] reported successful application of active adaptive control of boring bar vibration in industry using an active boring bar with embedded actuators and vibration sensors.

Long-overhang boring bars frequently form the most critical part of the tooling structure and the vibrations of a boring bar are often directly related to its low-order bending modes [8, 11, 23]. Therefore, the intention behind the active control of boring bar vibration is to selectively increase the dynamic stiffness at the actual frequency of the dominating bending mode. This is achieved by controlling the boring bar response via an actuator embedded in the boring bar [2, 13, 23, 24]. The actuator is steered by a controller in such way that the secondary vibration introduced by means of the actuator interferes destructively with the boring bar tool vibration excited by the material deformation process [25]. However, in this active control application, the controller is constrained to the feedback approach [2, 25]. The controller feedback
signal is measured by an accelerometer attached to the boring bar, close to the cutting tool.

During the process of machining a workpiece in a lathe, the boundary conditions applied by the workpiece on the cutting tool may exhibit large and abrupt variation; particularly in the engagement phase between the cutting tool and workpiece. The abrupt change of load applied by the workpiece on the tool - which always occurs in the engagement phase - may result in tool vibration. However, when utilizing active adaptive digital control of tool vibration, the problem of tool failure in the engagement phase may remain. For instance, the time required for the tuning of an adaptive digital controller’s response, the inherent delay in, controller processing time, A/D and D/A-conversion processes, and analog anti-aliasing and reconstruction filtering, might impede an active adaptive digital control system to produce control authority sufficiently fast to avoid tool failure.

The application of active control of boring bar vibration in industry requires reliable, robust adaptive controllers or manually tuned controllers that are simple to adjust on the shop floor by the lathe operator. Both the analog and digital domain may be utilized for the implementation of feedback controllers [26–28]. The revolution and development of cost-effective digital computers, such as the digital signal processor (DSP) have resulted in embedding of digital logic in automatic control applications. One reason for this may involve the simplicity of implementing various controller algorithms on the same hardware. This implementation means that hardware and software design are almost independent of each other, thereby reducing overall development time and costs significantly. Furthermore, having the algorithm implemented in software allows easy and accurate parameter adjustment, thus it might be easier and more convenient to tune a digital controller. Also, the simplicity of adding adaptivity, nonlinear functions, estimated system models etc. allow the application more sophisticated and efficient controllers [29,30]. These advantages, however, do not come for free. Digital controllers have an inherent delay associated with the controller processing time, A/D- and D/A- conversion processes, and analog anti-aliasing and reconstruction filtering, and digital solutions introduce quantization errors [26, 27]. Delay and quantization error may, however, be reduced by using high speed DSPs and longer word length. Such preventive measures are, on the other hand, likely to increase the total power consumption, as well as the total price per controller unit. To enable low controller delay without using an uneconomically high sampling rate, an analog controller approach may be suitable [26,27].
This article focuses on the development of a simple adjustable robust analog controller, based on digitally controlled analog design, that is suitable for the control of boring bar vibration in industry. Initially, a digitally controlled analog manually adjustable lead compensator was developed. This manually adjustable controller approach was further developed to provide controller responses appropriate for the active control of boring bar vibration. A manually adjustable bandpass lead-lag compensator was developed. Gain and phase of the controller response may be independently adjustable on the two developed analogue controller prototypes. Also, the performance and robustness using the two analog controllers was evaluated and compared with a digital adaptive controller based on the feedback filtered-x LMS-algorithm [25,27] in the active control of boring bar vibration.

2 Materials and Methods

2.1 Experimental setup

Experiments concerning active control of tool vibration have been carried out in a Mazak SUPER QUICK TURN - 250M CNC turning center. The CNC lathe has 18.5 kW spindle power and a maximal machining diameter of 300 mm, 1005 mm between the centers, a maximal spindle speed of 400 revolutions per minute (r.p.m.) and a flexible turret with a tool capacity of 12 tools. The CNC lathe is presented by the photo in Fig. 1 a) and Fig. 1 b) shows the room in the lathe in which the machining is carried out. In this photo (Fig. 1 b)), the turret configured with a boring bar clamped in a clamping house, and a workpiece clamped in the chuck are observable.

A coordinate system was defined: z was in the feed direction, y in the reversed cutting speed direction and x in the cutting depth direction (see upper left corner of Fig. 1 b)). The cutting tool insert is attached to a WIDAX S40T PDUNR15 boring bar which, in turn, is mounted in a clamping house attached with bolts to the turret. The turret may be controlled to move in the cutting depth direction, i.e. x-direction, and in the feed direction, i.e. z-direction, as well as to rotate about the z-axis for tool change. The turret is supported by a slide mounted on the lathe bed. Another important component is the spindle-chuck system which holds the workpiece. All components together have a total weight of approximately 5650 kg.
2.1.1 Work material - Cutting data - Tool Geometry

The cutting experiments used the work material chromium molybdenum nickel steel SS 2541-03 (AISI 3239). The material deformation process of this material during turning excites the boring bar with a narrow bandwidth and has a susceptibility to induce severe boring bar vibration levels [8, 11], resulting in poor surface finish, tool breakage and severe acoustic noise levels. The workpiece used in the cutting experiments had a diameter of 225 mm and a length of 230 mm. To enable supervision of the metal-cutting process during continuous turning and to perform experiments and measurements successfully without destroying any sensitive and expensive equipment, the cutting operation was performed externally, see Fig. 1 b). An active boring bar was firmly clamped in a clamping house rigidly attached to the lathe turret. Only one side of the workpiece shaft’s end was firmly clamped into the chuck of the lathe, see Fig. 1 b). As cutting tool, a standard 55° diagonal insert with geometry DNMG 150608-SL and carbide grade TN7015 for medium roughing was used. After a preliminary set of trials, suitable cutting data was selected: Cutting speed \( v = 60 \text{ m/min} \), Depth of cut \( a = 1.5 \text{ mm} \) and Feed \( s = 0.2 \text{ mm/rev} \). This cutting data set was selected to cause significant tool vibrations and an observable deterioration of the workpiece surface, as well as severe acoustic noise.
2.1.2 Measurement Equipment and Setup

A block diagram of the experimental setup for the active control of boring bar vibrations is presented in Fig. 2.

![Block Diagram of Experimental Setup](image)

Figure 2: A block diagram describing the experimental setup for the active vibration control system.

The control experiments used an active boring bar, described in detail in Section 2.2. The active boring bar was equipped with an accelerometer and an embedded piezoceramic stack actuator. The actuator was powered with an actuator amplifier, custom designed for capacitive loads, and the accelerometer was connected to a charge amplifier. A floating point signal processor and Successive Approximation Register (SAR) AD- and DA- converters were used; they introduce a significantly smaller delay as compared to the sigma-delta type. Two commercial signal conditioning filters with adjustable pre- and post-amplifiers, as well as adjustable filter characteristics and cut-off frequencies were utilized in the control experiments. The software I-DEAS and a VXI Mainframe E8408A with two 16-channel 51.2kSa/s cards were used for data collection. The data was recorded as time data and Matlab was used to analyze, for example, the time records of the boring bar vibrations.
2.2 Active Boring Bar

The active boring bar used in this experiment is based on the standard WIDAX S40T PDUNR15 boring bar with an accelerometer and an embedded piezoceramic stack actuator, see Fig. 3. The accelerometer was mounted 25 mm from the tool tip to measure the vibrations in the cutting speed direction (y-). This position was selected as close as possible to the tool tip, but at a sufficient distance to avoid damage to the accelerometer from metal chips during the material removal process. The actuator was embedded into a milled space in the longitudinal direction (z-direction), below the centerline of the boring bar. By embedding accelerometers and piezoceramic stack actuators in conventional boring bars, a solution for the introduction of control force to the boring bar with physical features and properties that fit the general lathe application may be obtained.

![Figure 3: The active boring bar.](image)

2.2.1 Piezoelectric Stack Actuator

Piezoelectric stack actuators consist of a stack of $\eta$ piezo ceramic plates separated by thin metallic electrodes. When a voltage $V(t)$ is applied over the piezo ceramic plates they will attempt to expand their thickness. The simultaneous expansion of the $\eta$ plates or layers of the stack will, in the unloaded case, result in a free expansion $\Delta L_a(t)$. A piezoelectric stack actuator is illustrated in Fig. 4. If the piezoelectric stack actuator is assumed to have a negligible hysteresis, the free expansion of an unloaded piezoelectric stack actuator may be expressed by [31]:

$$\Delta L_a(t) = \eta d_{33} V(t)$$  \hspace{1cm} (1)
where $d_{33}$ is the strain coefficient parallel with polarization for the actuator expressed in m/V. The actuator equivalent spring constant is given by [31]

$$k_a = E_a A_a / L_a$$

were $A_a$ is the cross-sectional area of the actuator, $E_a$ is the Young’s elastic modulus of the actuator material and $L_a$ is the original unloaded and unexcited length of the actuator.

### 2.2.2 Active Boring Bar - Simple Model

A Euler-Bernoulli beam [8,11] may be used as a simple model to illustrate the structural dynamic properties of a boring bar. Most realistic structural systems are characterized by the ability to support transverse shear and exhibit internal stiffness. The Euler-Bernoulli beam model assumes that the deflection of the centerline is small and only transverse. While this theory assumes the presence of a transverse shear force, it neglects any shear deformation. Also, the rotary inertia is neglected by this model [32,33].

The used sign convention for displacements forces and moments is as follows; forces and displacements in the positive directions of the coordinate system axes are positive and a moment about a coordinate system axis that is in the counter clockwise direction if viewed from the end of the coordinate system axis towards the origin is considered to be positive.

The Euler-Bernoulli differential equation describing the transversal motion in the y direction of the boring bar may be written as [31,32,34].

$$\rho A(z) \frac{\partial^2 u(z,t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left[ EI(z) \frac{\partial^2 u(z,t)}{\partial z^2} \right] = \frac{\partial m_e(z,t)}{\partial z}$$

(3)
where \( \rho \) is the density of the boring bar, \( A(z) \) the cross-sectional area, \( u(z,t) \) the deflection in \( y \) direction, \( E \) the Young’s elastic modulus, \( I(z) \) the cross-sectional area moment of inertia and \( m_e(z,t) \) the space- and time-dependent external moment load per unit length. From the theory of mechanics of material, the beam sustains a bending moment, \( M(z,t) \), which is related to the beam deflection, \( u(z,t) \). This relation is given by \([33]\)

\[
M(z,t) = -EI(z) \frac{\partial^2 u(z,t)}{\partial z^2}.
\]  

(4)

In this context, it is assumed that both the cross-sectional area \( A(z) \) and the flexural stiffness \( EI(z) \) are constant along the boring bar. In other words, it is assumed that the geometric features of the tool end of the boring bar are negligible to the overall bar dynamics. The boundary conditions of the boring bar depend on the suspension of the boring bar ends, and (in this case) a clamped-free model is suggested [8,11,33].

To obtain the forced response of the beam, the method of eigenfunction expansion can be applied [35]. That is, the time-domain dynamic response of the boring bar can be expressed as:

\[
u(z,t) = \sum_{r=1}^{\infty} \psi_r(z)y_r(t)
\]  

(5)

where \( \psi_r(z), 1 \leq r \) are the normal modes determined by the boundary conditions for the beam and the normalization of the eigenfunctions or mode shapes, and \( y_r(t) \) is the modal displacement. Each modal displacement \( y_r(t) \) is determined by a convolution integral or mechanical filter, named Duhamel’s integral [36];

\[
y_r(t) = \frac{1}{m_r2\pi f_r} \int_0^t f_r(\tau) \sin(2\pi f_r(t - \tau)) d\tau
\]  

(6)

where \( m_r \) is the modal mass, \( f_r \) the undamped system’s eigenfrequency, \( f_{load,r}(t) \) the generalized load for mode \( r \). In the case of an external moment load, the generalized load for mode \( r \), \( f_{load,r}(t) \) is given by:

\[
f_{load,r}(t) = \int_0^L \psi_r(z) \frac{\partial m_e(z,t)}{\partial z} dz
\]  

(7)
Figure 5: The configuration of the piezoceramic stack actuator in the active boring bar.

were $L$ is the length of the beam, i.e. the length of the boring bar overhang.

Fig. 5 illustrates a boring bar with a piezoelectric stack actuator embedded in a milled space in the underside of the boring bar. If a voltage is applied over the piezoceramic stack actuator it will attempt to expand its length in the $z$-direction, but will be restrained elastically by the boring bar. Thus, the actuator will apply loads on the boring bar, in the actuator - boring bar interfaces, $f_{a1}(t)$ and $f_{a2}(t)$ - in parallel with the $z$-axis. These loads are asymmetrically distributed in the $y$-direction of the boring bar i.e. the boring bar will both bend and stretch. As a result of the applied loads the boring bar actuator interfaces will respond with displacements $z_1(t)$ and $z_2(t)$ in the $z$-direction. Assuming that the Fourier transform of the responses, $Z_1(f)$ and $Z_2(f)$, and of the loads, $F_{a1}(f)$ and $F_{a2}(f)$, (where $f$ is frequency) exists, then the point receptance at respective actuator interface of the active boring bar may be written

$$H_1(f) = -\frac{Z_1(f)}{F_{a1}(f)}$$

respectively:

$$H_2(f) = \frac{Z_2(f)}{F_{a2}(f)}$$

Assuming that the actuator operates well below its resonance frequency, thus neglecting inertial effects of the actuator, the force the actuator exerts on the boring bar, $F_a(f) = -F_{a1}(f) = F_{a2}(f)$, may approximately be related to the constraint expansion or motion of the actuator, the relative displacement
$Z(f) = Z_2(f) - Z_1(f), \text{ according to } [31]:$

$$F_a(f) = \frac{E_a(\Delta L_a(f) - Z(f))}{L_a} A_a$$ (10)

where $\Delta L_a(f)$ is the Fourier transform of the free expansion of an unloaded piezoelectric stack actuator, see Eq. 1. If the point receptance at the respective actuator end are summed to form the receptance $H_B(f) = H_1(f) + H_2(f)$, the relative displacement $Z(f)$ may be expressed as:

$$Z(f) = H_B(f)F_a(f)$$ (11)

Using Newton’s second law, combining Eq. 10 and Eq. 11, and the Fourier transform of the expression for free expansion of an unloaded piezoelectric stack actuator in Eq. 1 yields an expression for the actuator force applied on the boring bar as a function of the actuator voltage $V(f)$ [31]:

$$F_a(f) = \frac{k_a \eta d_{33} V(f)}{1 + k_a H_B(f)} = \frac{k_a \eta d_{33}}{1 + k_a H_B(f)} V(f) = H_{f_v}(f)V(f)$$ (12)

where $H_{f_v}(f)$ is the electro-mechanic frequency function between input actuator voltage $V(f)$ and output actuator force $F_a(f)$.

If the boring bar is subject to pure bending a plane, the natural surface, will exist in the boring bar where there is no strain [37]. Assume that the natural surface of the boring bar (in the section where the actuator is situated) coincides with the z-axis, and the y-axis is normal to the natural surface. Then, the distance between the actuator - boring bar interface center and the natural surface of the active boring bar in the y-direction is $\alpha$ as shown in Fig. 5. Thus, the external moment per unit length applied on the boring bar by the actuator force $F_a(f)$ may be approximated as:

$$m_e(z, f) = \alpha F_a(f)(\delta(z - z_1) - \delta(z - z_2)))$$ (13)

where $\delta(z)$ is the Dirac delta function, $z_1$ and $z_2$ are the z-coordinates for the actuator - boring bar interfaces introduced by the DC-voltage offset required for dynamic operation of the actuator, and $z_2 - z_1 \approx L_a$ as $\max(\Delta L_a(t))/L_a \leq 0.2\%$ [38] Expressing the Fourier transform of Eq. 7 with Eq. 13 and integrate yields the following expression for the generalized load of mode $r$:

$$F_{load,r}(f) = \alpha F_a(f)(\psi'_r(z_2) - \psi'_r(z_1))$$ (14)
If $z_1 = 0$, one actuator - boring bar interfaces is at the clamped end of the boring bar, the generalized load of mode $r$ is given by:

$$F_{\text{load},r}(f) = \alpha F_a(f) \psi'_r(z_2)$$  \hspace{1cm} (15)

Thus, the frequency-domain dynamic response of the boring bar in the $y$ direction may be expressed as:

$$u(z,f) = \sum_{r=1}^{\infty} \psi_r(z) H_r(f) \alpha H_{fv}(f) V(f) \psi'_r(z_2)$$  \hspace{1cm} (16)

where $H_r(f)$ is the frequency response function for mode $r$. i.e. the Fourier transform of $1/(m e^{2\pi f r}) \sin(2\pi f r t)$.

### 2.3 System Identification

The design of feedback controllers usually rely on detailed knowledge of the dynamic properties of the system to be controlled, e.g. a dynamic model of the system to be controlled [39, 40]. Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [41, 42].

A non-parametric estimate of the power spectral density $P_{xx}(f)$, $f$ is frequency for a signal $x(t)$ may for example be estimated using the the Welch spectrum estimator [42], given by:

$$\hat{P}_{xx}(f_k) = \frac{1}{LNUF_s} \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} x_l(n) w(n) e^{-j2\pi nk/N}$$

$$f_k = \frac{k}{N} F_s$$  \hspace{1cm} (17)

where $k = 0, \ldots, N - 1$, $L$ the number of periodograms, $N$ the length of the data segments used to produce the periodograms, $x_l(n)$ the sampled signal in segment $l$, $F_s$ the sampling frequency, $w(n)$ the used data window and where

$$U = \frac{1}{N} \sum_{n=0}^{N-1} w^2(n)$$

is the window dependent effective analysis bandwidth normalization factor.

Thus, the input signal $x(t)$ and output signal $y(t)$ of a single-input–single-output system (SISO system) are simultaneously measured and the sampled
signals $y(n)$ and $x(n)$ are recorded. By using, for example, the Welch spectrum estimator [42] the cross-power spectral density $\hat{P}_{xy}(f_k)$ between the input signal $x(n)$ and the output signal $y(n)$ and the power spectral density $\hat{P}_{xx}(f_k)$ for the input signal $x(n)$ may be produced [11,41]. A least-squares estimate of a frequency response function between the input signal $x(n)$ and the output signal $y(n)$ may be produced according to [41,42]:

$$\hat{H}_{yx}(f_k) = \frac{\hat{P}_{yx}(f_k)}{\hat{P}_{xx}(f_k)}$$

(18)

The random error in frequency response function estimates for the amplitude function [41] are

$$\varepsilon_r(|\hat{H}_{yx}(f_k)|) \approx \sqrt{1 - \frac{\gamma^2_{yx}(f_k)}{\gamma^2_{yx}(f_k)2L_e}}$$

(19)

and for the phase function [41]

$$\varepsilon_r(|\hat{\Theta}_{yx}(f_k)|) \approx \arcsin\left(\varepsilon_r(|\hat{H}_{yx}(f_k)|)\right)$$

(20)

where $\gamma^2_{xy}(f_k)$ is the coherence function between the input signal $x(n)$ and the output signal $y(n)$ and $L_e$ is the equivalent number of uncorrelated periodograms used in the average to produce the spectrum estimate and is given by [42]:

$$L_e = \frac{L}{1 + 2 \sum_{q=1}^{L-2} \frac{L - q}{L} \rho(q)}$$

(21)

where

$$\rho(q) = \frac{\left(\sum_{n=0}^{N-1} w(n)w(n + qD)\right)^2}{\left(\sum_{n=0}^{N-1} w^2(n)\right)^2}$$

(22)

In other words, information regarding the quality of a frequency response function estimate is related to the coherence function between the input signal and the output signal. The coherence function is a frequency-dependent function which assumes values between 0 and 1. The interpretation of the coherence function depends on the nature of the signals. Coherence values
close to unity, in the case of random signals, indicate that the output signal may, to a large extent, be explained linearly from the input signal \[11, 41\]. The coherence function estimate may be produced as \[41\]

\[
\hat{\gamma}^2_{yx}(f_k) = \frac{|\hat{P}_{xy}(f_k)|^2}{\hat{P}_{xx}(f_k)\hat{P}_{yy}(f_k)}
\]  

(23)

where \(\hat{P}_{xy}(f_k)\) is the cross power spectral density estimate between input \(x\) and output \(y\), \(\hat{P}_{xx}(f_k)\) and \(\hat{P}_{yy}(f_k)\) are the power spectral density estimates of \(x\) and \(y\) respectively. The coherence function satisfies the condition \(0 \leq \hat{\gamma}^2_{xy}(f_k) \leq 1\). The normalized random error of a coherence function estimate is \[41, 42\]

\[
\varepsilon_r [\hat{\gamma}^2_{yx}(f_k)] \approx \sqrt{2} \left[1 - \hat{\gamma}^2_{yx}(f_k)\right] \left|\hat{\gamma}_{yx}(f_k)\right| \sqrt{L_c}
\]  

(24)

where \(L_c\) is defined by Eq. 21.

Table 1 gives the spectral density estimation parameters used in the production of forward path frequency function estimates, with various clamping conditions for the boring bar. Also, the spectral density estimation parameters used in the production of forward path frequency function estimates during continuous metal cutting can be found in Table 2. In Table 3, the spectral density estimation parameters used in the production of frequency function estimates for the controller responses are given, and in Table 4 the spectrum estimation parameters used for the production of power spectral density estimates for boring bar vibration with and without active control are presented.

2.4 Controllers

The application of active control of boring bar vibration in industry requires reliable robust adaptive feedback control or manually tuned feedback control which is simple to adjust at the shop floor by the lathe operator. A controller suitable for active control of boring bar vibrations might be implemented using different approaches. However, the abrupt changes that occur in the turning operations, i.e. in the engagement phase, suggest that an analog control approach might be suitable with respect to, for example, controller delay and thus delay in control authority. A simple controller approach is given by the proportional-integral-derivative (PID) controller which is the most widely...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Burst Random</td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>8192 Hz</td>
</tr>
<tr>
<td>Block Length $N$</td>
<td>16384</td>
</tr>
<tr>
<td>Frequency Resolution $\Delta f$</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>160</td>
</tr>
<tr>
<td>Burst Length</td>
<td>90%</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>0-4000 Hz</td>
</tr>
<tr>
<td>Window $w(n)$</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Overlap</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 1: Spectral density estimation parameters used in the production of forward path frequency function estimates with various clamping conditions of the boring bar.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>True Random</td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block Length $N$</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency Resolution $\Delta f$</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>265-635</td>
</tr>
<tr>
<td>Burst Length</td>
<td>-</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>-</td>
</tr>
<tr>
<td>Window $w(n)$</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 2: Spectral density estimation parameters used in the production of forward path frequency function estimates during continuous machining.
On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>True Random</td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block Length $N$</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency Resolution $\Delta f$</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>160</td>
</tr>
<tr>
<td>Burst Length</td>
<td>-</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>-</td>
</tr>
<tr>
<td>Window $w(n)$</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 3: Spectral density estimation parameters used in the production of frequency response function estimates for the controller responses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Cutting process</td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block Length $N$</td>
<td>10240</td>
</tr>
<tr>
<td>Frequency Resolution $\Delta f$</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>100</td>
</tr>
<tr>
<td>Burst Length</td>
<td>-</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>-</td>
</tr>
<tr>
<td>Window $w(n)$</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 4: Spectral density estimation parameters used for the production of boring bar vibration spectra, with and without active control.
used controller. It is estimated that over 90% of the industrial controllers are still implemented based on the PID algorithm [28,30,43]. The PID controller was first described by Albert Callender and Allan Stevenson in 1936, later patented in 1939 [44], and 1942 Ziegler and Nichols [45] published a paper in which they developed a set of mathematical rules for selecting the parameters associated with the controller. However, analog versions of the PD controller or the PID controller are problematic, as the derivative part (D) will cause the magnitude of the controller response to grow with increasing frequency [28]. Simple and effective compensators, or controllers that may be implemented in the analog domain are the lag and lead compensators that approximate the PI controller and the PD controller respectively. Moreover, the PID controller may be approximated in the analog domain by combining a lag compensator with a lead compensator [28].

Feedback control usually concerns the manipulation of the output quantity from a plant under control to follow some prescribed reference signal or command signal, thus servo-control systems [26,28]. A typical single channel feedback control system is presented in Fig. 6 where \( r(t) \) is the reference input signal or command signal to the system, \( y(t) \) is the output signal from the controller \( W \), \( C \) is the plant or forward path and \( e(t) \) is the error signal, i.e. the output signal from the control system, produced by the sum of the disturbance signal \( d(t) \) and the output signal \( y_c(t) \) from the plant. However, active control systems are generally designed for disturbance rejection [26] and this is also an adequate approach for the active control of tool vibration in metal-cutting [2]. A block diagram of the active boring bar vibration feedback control system, a feedback control system for disturbance rejection, is presented in Fig. 7. Thus, the feedback control system for disturbance rejection lacks a reference input signal. As in the servo-control case, \( W \) is the controller, \( y(t) \) controller output signal or control signal, \( C \) is the forward path or control path, \( y_c(t) \) is the forward path output vibration (secondary vibration), \( d(t) \) is the undesired
tool vibration or disturbance excited by the chip formation process during machining. If the forward path and the controller are linear time invariant stable systems, their dynamic properties may be described by the frequency response functions $C(f)$ and $W(f)$ respectively. Then, the feedback control systems' open-loop frequency function can be written as

$$H_{ol}(f) = W(f)C(f)$$

(25)

and the closed-loop frequency function for the servo-control system is given by

$$H_{cls}(f) = \frac{W(f)C(f)}{1+W(f)C(f)}$$

(26)

while the closed-loop frequency function for the active boring bar vibration feedback control system is given by

$$H_{cl}(f) = \frac{1}{1+W(f)C(f)}$$

(27)

### 2.4.1 Controller performance and robustness

Principally, there are two important aspects of the behavior of feedback controllers, their performance and their robustness; that is, the ability of the controller to reject disturbance and to remain stable under varying conditions [39]. Hence, good performance in feedback control requires high loop gain while robust stability requires lower loop gain. Usually, the discussion concerning the performance and robust stability of feedback controllers is based on the sensitivity function $S(f) = 1/(1+W(f)C(f))$ and the complementary function $T(f) = W(f)C(f)/(1+W(f)C(f))$ [39,40]. The sensitivity function $S(f)$ gives a measure on tracking performance and disturbance reduction of a feedback control system. It is also the sensitivity of the closed
loop response to a plant perturbation \[39,40\]. In the determination of the stability properties of the system, the complementary function \( T(f) \) has a vital role. It also governs the performance of the control system regarding the reduction of noise from the sensor detecting the error \( e(t) \). Generally, the design of controllers rely on a model of the system to be controlled or the so called plant. However, a model of a physical system is an approximation of the true dynamics of the system and it is therefore likely to affect the performance of the control system \[39, 40\]. To incorporate the plant uncertainty in the design procedure of a controller, a model of the plant uncertainty is usually included. A common way to model the plant uncertainty is with a multiplicative perturbation, yielding a frequency function model of the forward path as \[39, 40, 46\];

\[
C_{\text{actual}}(f) = C_{\text{nominal}}(f)(1 + \Delta C(f))
\]  

(28)

where \( \Delta C(f) \) is an unstructured perturbation given by;

\[
\Delta C(f) = \frac{C_{\text{actual}}(f)}{C_{\text{nominal}}(f)} - 1
\]  

(29)

and \( C_{\text{nominal}}(f) \) is a nominal plant model of the forward path. Thus, assuming that a control system design based on the nominal plant model results in a theoretically stable control system, then the denominator \( 1 + W(f)C_{\text{nominal}}(f) \) has no zeros in the right half complex plane. However, the actual control system’s frequency response function will have the denominator \( 1 + W(f)C_{\text{actual}}(f) \). If

\[
|1 + W(f)C_{\text{nominal}}(f)| > |W(f)C_{\text{nominal}}(f)\Delta C(f)|, \forall f
\]  

(30)

then \( 1 + W(f)C_{\text{nominal}}(f) \) and \( 1 + W(f)C_{\text{actual}}(f) \) will have the same number of zeros in the right half complex plane \[26, 39, 47\]. In other words, if the inequality in Eq. 30 is fulfilled, the actual control system is stable. Hence, for stable control the unstructured perturbation is upper limited as

\[
|\Delta C(f)| < \frac{1}{|T_{\text{nominal}}(f)|}, \forall f
\]  

(31)

If \( \Delta C(f) \) is bounded as

\[
|\Delta C(f)| < \beta(f), \forall f
\]  

(32)

The condition for robust stability of a control system is given by \[26\]

\[
\beta(f) < \frac{1}{|T_{\text{nominal}}(f)|}, \forall f
\]  

(33)
The "robustness" of a feedback control system generally refers to its sensitivity in stability and performance to variations in the dynamics of the forward path (plant dynamics). A controller that performs well for substantial variations in the dynamics of the forward path is said to be robust [39]. The performance and robustness of an active feedback control system may also be visualized by a polar plot of its open-loop frequency response function in a Nyquist diagram [26, 39, 47]. If the closed loop system is to be stable, the polar plot of the open loop frequency response for the feedback control system $W(f)C(f)$ must not enclose the polar coordinate (-1, 0) in the Nyquist diagram. The larger the distance between the polar plot and the (-1, 0) point, the more robust the feedback control system becomes, with respect to variation in forward path response.

### 2.4.2 Compensators

In the context of traditional feedback control, the purpose of the introduction of a lead compensator to a feedback control system is to enhance its transient response properties, i.e. to lower the rise time and to decrease the transient overshoot of the control system [28, 47]. Thus, generally a lead compensator increases a control system's bandwidth [28, 47]. The purpose of a lead controller is to advance the phase of the open loop frequency response $W(f)C(f)$ for a feedback control system, usually by adding maximal positive phase shift in the frequency range where the loop gain $|W(f)C(f)|$ the magnitude of the open loop frequency response- equals 0 dB, i.e. at the crossover frequency [28, 47]. This will increase the phase margin and generally increase the bandwidth of a feedback control system [28, 47].

A lag compensator or controller is generally used to provide an improved steady-state accuracy of a feedback control system [28, 47]. By utilizing a lag compensator, the low frequency loop gain of the feedback control system may be increased as the phase-lag filter attenuates the high frequency gain. In this way, the gain margin of the open loop frequency response for the feedback control system can be improved, and the phase shift added by the compensation filter can be minimized [26, 28, 47].

The characteristic equation or frequency function for a lead compensator may be written as [48]:

$$W_{\text{Lead}}(f) = K_{\text{lead}} \frac{1}{\alpha_{\text{lead}} f + z_{\text{lead}}} = K_{\text{lead}} \frac{\tau_{\text{lead}} f + 1}{\alpha_{\text{lead}} \tau_{\text{lead}} f + 1}$$ (34)
Where $-z_{\text{lead}}, z_{\text{lead}} > 0$ and $z_{\text{lead}} \in \mathbb{R}$, is the compensator zero, $-p_{\text{lead}}, p_{\text{lead}} > 0$ and $p_{\text{lead}} \in \mathbb{R}$, is the compensator pole, $\alpha_{\text{lead}} = z_{\text{lead}}/p_{\text{lead}} < 1$ is the inverse lead ratio for a lead compensator, $K_{\text{lead}}$ is the compensator gain and $\tau_{\text{lead}} = 1/z_{\text{lead}}$. The lead compensator has similar properties to the PD-controller, however, towards high frequencies the PD-controller approaches an infinite gain and a positive phase shift of $90^\circ$, while the lead compensator approaches a gain of $K_{\text{lead}}/\alpha_{\text{lead}}$ and zero phase shift [28,47]. A large phase margin does not, however, necessarily guarantee that the control system is robust [28,47]. For instance, the Nyquist plot (or polar plot) of the open-loop frequency function $W(f)C(f)$ of a feedback control system with substantial phase margin may have a small distance to $(-1,0)$ on the Nyquist diagram, see Fig. 8.

$$\Im [W(f)C(f)]$$

$$\Re [W(f)C(f)]$$

Figure 8: A Nyquist plot example of the open-loop response $W(f)C(f)$ for a control system which is not robust but has a large phase margin.

The characteristic equation or frequency function for a lag compensator is fairly similar to the lead compensator characteristic equation and may be expressed as [48]:

$$W_{\text{Lag}}(f) = K_{\text{lag}} \frac{1}{\alpha_{\text{lag}}} \frac{j2\pi f + z_{\text{lag}}}{j2\pi f + p_{\text{lag}}} = K_{\text{lag}} \frac{\tau_{\text{lag}} j2\pi f + 1}{\alpha_{\text{lag}}\tau_{\text{lag}} j2\pi f + 1}$$  \hspace{1cm} (35)

Here $-z_{\text{lag}}, z_{\text{lag}} > 0$ and $z_{\text{lag}} \in \mathbb{R}$, is the compensator zero, $-p_{\text{lag}}, p_{\text{lag}} > 0$ and $p_{\text{lag}} \in \mathbb{R}$, is the compensator pole, $\alpha_{\text{lag}} = z_{\text{lag}}/p_{\text{lag}} > 1$ is the inverse lag ratio for a lag compensator, $K_{\text{lag}}$ is the compensator gain and $\tau_{\text{lag}} = 1/z_{\text{lag}}$. 
Thus, the lag compensator pole and zero placing on the negative real axis in the complex plane is reversed as compared to the lead compensator pole and zero placing. The lag compensator has similar properties to the PI-controller, however, towards zero frequency the PI-controller approaches an infinite gain and a negative phase shift of $-90^\circ$ while the lag compensator approaches a gain of $K_{\text{lag}}$ and zero phase shift [28,47].

If it is desired to simultaneously improve the transient response properties and steady-state accuracy of a feedback control system, thus improving bandwidth and low-frequency gain of the system, a lead-lag compensator may be utilized [28,47].

The lead-lag compensator is obtained by connecting a lead compensator and a lag compensator in series, thus by combining Eqs. 34 and 35. This yields the characteristic function for the lead-lag compensator:

$$W_{\text{LeadLag}}(f) = K_{\text{leadlag}} \frac{1}{\alpha_{\text{lead}} j \omega f + z_{\text{lead}}} \frac{1}{\alpha_{\text{lag}} j \omega f + p_{\text{lag}}}$$

Thus, the lag compensator may be adjusted to provide a suitable low frequency loop gain of the feedback control system. Subsequently, the lead compensator may be adjusted to provide an additional positive phase shift in the frequency range, where the loop gain equals 0 dB, i.e. at the crossover frequency [28,47].

### 2.4.3 Digitally Controlled Analog Controller

The intention is to develop an analog controller with response properties that can be easily adjusted manually, without necessitating the replacement of discrete components, i.e. resistors and capacitances. If a lead compensator is considered, its frequency response function is given by Eq. 34 and the selected values for the parameters $\alpha_{\text{lead}}$, $K_{\text{lead}}$ and $\tau_{\text{lead}}$ [28,47]. A lead compensator may be designed according to the circuit diagram shown in Fig. 9, where $R_{d,2}$ and $R_{d,f}$ are adjustable resistors, $R_{d,1}$ is a suitable fixed resistor and $C_d$ is a suitable fixed capacitor. The compensator gain is related to the resistors by

$$K_{\text{lead}} = \frac{R_{d,f}}{R_{d,2} + R_{d,1}}$$

and

$$\tau_{\text{lead}} = C_d R_{d,2}$$
and finally the inverse lead ratio is produced as
\[ \alpha_{\text{lead}} = \frac{R_{d,1}}{R_{d,2} + R_{d,1}} \] (39)

If \( R_{d,2} \) and \( R_{d,f} \) are implemented by digitally controlled potentiometers the phase and gain may be adjusted independently at one selectable frequency, in discrete steps, to successively increase or decrease phase and gain respectively. For instance, by using two knobs for the compensator tuning (one for phase adjustment and one for gain adjustment) a function of the two knob angles may be produced according to:
\[ [a, g] = AG_{\text{lead}}(\text{gain knob angle, phase knob angle}) \] (40)

This function produces integers \( a \in \{1, 2, \ldots, L_a\} \) and \( g \in \{1, 2, \ldots, L_g\} \), selecting the appropriate analog compensator frequency response function in the set of \( L_a \times L_g \) different analog compensator frequency response functions:
\[ W_{\text{lead; } a, g}(f) = K_{a, g} \frac{\tau_{a, g} j 2\pi f + 1}{\alpha_{a, g} \tau_{a, g} j \omega + 1}, \quad a \in \{1, 2, \ldots, L_a\} \text{ and } g \in \{1, 2, \ldots, L_g\} \] (41)

The micro-controller realized the \( AG(\text{gain knob angle, phase knob angle}) \) function, by controlling the adjustable resistors \( R_{d,2} \) and \( R_{d,f} \), (the so called digital potentiometers that have a digital control interface and an analog signal path). Such a micro-controller will allow the implementation of an analog lead-circuit which enabling orthogonal adjustment of the phase function and magnitude function at one selectable frequency of the compensator response. A photo of such a lead compensator circuit is shown in Fig. 10.
Figure 10: The circuit board realizing the lead compensator with gain and phase separately adjustable at one selectable frequency, implemented using standard components.

If the frequency for orthogonal adjustment of the phase function and magnitude function is set to 500 Hz, the magnitude and phase functions of the frequency response function realized by this circuit may be adjusted with the phase knob according to the 3-D plots in Fig. 11 a) and Fig. 11 b) respectively. The gain knob only adjusts the level of the magnitude function surface and has no influence on its shape or the shape of the phase function surface.

A lag compensator with orthogonal adjustment of the phase function and the magnitude function of the compensator response may be designed similar to the analog lead compensator with orthogonal adjustment of the phase function and the magnitude function at one selectable frequency of the compensator response. Thus, a lead-lag compensator with orthogonal adjustment of the phase function and the magnitude function at one selectable frequency of its response may be realized by connecting the adjustable lead compensator in series with the adjustable lag compensator. A lag compensator may be designed according to the circuit diagram shown in Fig. 12. Where $R_{g,1}$ and $R_{g,f}$ are adjustable resistors and $R_{g,2}$ is a suitable fixed resistor and $C_g$ is a suitable fixed capacitor. The frequency response function for the lag compensator is given by Eq. 35 and the selected values for the parameters $\alpha_{lag}$, $K_{lag}$ and $\tau_{lag}$ [28,47]. The compensator gain is related to the resistors by

$$K_{lag} = \frac{R_{g,f}}{R_{g,1} + R_{g,3}}$$  (42)
Figure 11: a) Magnitude function and b) phase function of lead compensator frequency response function as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.

Figure 12: Circuit diagram of a lag compensator.
and
\[ \tau_{\text{lag}} = C_g R_{g,2} \]  
(43)

and finally the inverse lag ratio is produced as
\[ \alpha_{\text{lag}} = \frac{R_{g,1} R_{g,2} + R_{g,1} R_{g,3} + R_{g,2} R_{g,3}}{R_{g,1} R_{g,2} + R_{g,2} R_{g,3}} \]  
(44)

In much the same way as the orthogonally adjustable lead compensator, a micro-controller realizes an \( AG_{\text{lag}}(\text{gain knob angle, phase knob angle}) \) function suitable for steering the digital potentiometers implementing the adjustable resistors \( R_{g,1} \) and \( R_{g,f} \) for the lag compensator. If the frequency for orthogonal adjustment of the phase function and magnitude function is selected to 500 Hz, the magnitude and phase functions of the frequency response function realized by this circuit may be adjusted with the phase knob, according to the 3-D plots in Fig. 13 a) and Fig. 13 b) respectively. Observe that the gain knob only adjusts the level of the magnitude function surface and has no influence on its shape or the shape of the phase function surface.

By connecting the adjustable lead compensator in series with the adjustable lag compensator, a lead-lag compensator with orthogonal adjustment of the phase function and the magnitude function at one selectable frequency of the response may be realized. The implemented lead-lag compensator circuit is shown in Fig. 14.
The function relating the gain knob angle and the phase knob angle for the selection of the appropriate compensator frequency response function is, however, replaced with $AG_{\text{lead-lag}}(\text{gain knob angle}, \text{phase knob angle})$ appropriate for the lead-lag compensator. If the frequency for orthogonal adjustment of the phase function and magnitude function is set to 500 $Hz$, the magnitude and phase functions of the lead-lag compensator may be adjusted with the phase knob e.g. according to the 3-D plots in Fig. 15 a) and Fig. 15 b) respectively. Also, the gain knob only adjusts the level of the lead-lag compensator’s magnitude function surface and has no influence on its shape or the shape of the phase function surface.

Also, to utilize the capacity of the actuator amplifier, to limit the active control frequency range, a suitable high-pass filter, followed by a suitable low-pass filter was connected in series with the lead-lag compensator. The block diagram of the obtained analog band-pass lead-lag controller is shown in Fig. 16.

Selecting the frequency for orthogonal adjustment of the phase function and magnitude function to 500 $Hz$, this controller’s frequency response magnitude and phase functions are adjustable with the phase knob, according to the 3-D plots in Fig. 17 a) and Fig. 17 b) respectively. As for previous adjustable compensators, the gain knob only adjusts the level of the magnitude function surface and has no influence on its shape or the shape of the phase function surface.

The implemented analog lead-lag controller consists of several blocks:
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Figure 15: a) Magnitude function and b) phase function of lead-lag compensator frequency response function, as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.

Figure 16: Block diagram of the implemented lead-lag circuit.
Figure 17: a) Magnitude function and b) phase function of band-pass lead-lag compensator frequency response function, as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.

2.4.4 Feedback Filtered-x LMS algorithm

The feedback filtered-x LMS algorithm [25, 27, 49] is an adaptive digital feedback controller suitable for narrow-band applications. This algorithm is based on the method of steepest descent and the objective of the control is to minimize the disturbance signal or desired signal in the mean square sense [25, 27, 49]. A block diagram of the feedback filtered-x LMS algorithm is shown in Fig. 18.

The feedback filtered-x LMS algorithm is defined by the following equations:

\[
\begin{align*}
y(n) &= w^T(n)x(n) \quad (45) \\
e(n) &= d(n) + y_C(n) \quad (46) \\
w(n + 1) &= w(n) - \mu x_C(n)e(n) \quad (47)
\end{align*}
\]

\[
x_C(n) = \left[ \sum_{i=0}^{I-1} \hat{c}_i x(n - i), \ldots, \sum_{i=0}^{I-1} \hat{c}_i x(n - i - M + 1) \right]^T \quad (48)
\]
where $\mu$ is the adaptation step size, $x_C(n)$ is the filtered reference signal vector, which usually is produced by filtering the reference signal $x(n)$ with an $I$-coefficients FIR-filter estimate, $\hat{c}_i$, $i \in 0, 1, \ldots, I - 1$, of the forward path. Furthermore, $w(n)$ is the adaptive FIR filter coefficient vector, $y(n)$ is the output signal from the adaptive FIR filter, $e(n)$ is the error signal, $y_C(n)$ the secondary vibration (the output signal from the forward path), $\hat{C}$ is an estimate of the forward path and $d(n)$ is the primary disturbance. The reference signal vector $x(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$ is related to the error signal as [25,27,49]:

$$x(n) = e(n-1)$$  \hspace{1cm} (49)

This relation is introduced by the fact that the adaptive digital filter is used in a feedback control system. In order to select a step length $\mu$ to enable the feedback filtered-x LMS algorithm to converge, the following inequality may be used [25,27,49]:

$$0 < \mu < \frac{2}{E[x_C^2(n)](M + \delta)}$$  \hspace{1cm} (50)

where $\delta$ is the overall delay in the forward path, $M$ is the length of the adaptive FIR filter and $E[x_C^2(n)]$ is the mean square value of the filtered reference signal to the algorithm. The forward path, or control path, is estimated prior to active control by, for example, a second adaptive FIR filter steered by the LMS algorithm.

By introducing a leakage factor $\gamma$ in the feedback filtered-x LMS-algorithm, the "memory" of the adaptive algorithm is reduced, thereby reducing the
energy in the response of the adaptive FIR filter and also the energy in the control signal to the forward path. This causes the loop gain of the control system to be reduced and thereby increases the distance between the polar plot of the open loop frequency response and the point (-1,0) [26, 27]. The leaky version of the feedback filtered-x LMS algorithm is obtained by replacing the coefficient adjustment algorithm in Eq. 47 for the feedback filtered-x LMS algorithm with [25,27]

\[ w(n+1) = \gamma w(n) - \mu x(n) e(n) \]  

(51)

where \( \gamma \) is the leakage coefficient \( 0 < \gamma < 1 \), usually selected close to unity.

3 Results

3.1 The Forward Path

The system to be controlled, \( C \), consists of the forward (or control) path in the CNC lathe and is comprised of several parts: a signal conditioning filter, an actuator amplifier, an actuator, the structural path between the force applied by actuator on the boring bar and the boring bar response (measured by an accelerometer mounted close to the tool-tip).

In order to clamp the boring bar, it is first inserted into the cylindrical space of the clamping house. It is then clamped by means of four/six clamping screws; two/three on the tool side and two/three on the opposite side of the boring bar. The two standard versions of the clamping house are distinguishable only by the fact that one supports four clamping screws whilst the other support six. It is obvious that the boundary conditions applied by the four-screw version of the clamping house will differ from the boundary conditions applied by the sex-screw version of the clamping house. Also, to enable the boring bar to be inserted in the clamping house, the diameter of the clamping house’s cylindrical clamping space is slightly larger than the diameter of the boring bar. Thus, the exact spatial position of the clamped boring bar end in the clamping space of the clamping house is difficult to pinpoint. Furthermore, the tightening torque of the clamping screws, i.e. the clamping force, is likely to vary between the screws, each time the boring bar is clamped and each screw tightened. Thus, each time the boring bar is clamped it is likely that the clamped boring bar will have different dynamic properties.

Forward path frequency function estimates were produced when the boring bar was not in contact with the workpiece, i.e. off-line. The forward path was
estimated off-line for 10 different possible clamping conditions with respect to the tightening torque of the screws and the spatial position of the boring bar in the clamping space of the clamping house. Two spatial positions within clamping space were selected. The first was that in which the upper side of the boring bar’s end (the tool side) was clamped in contact with the upper section of the clamping space surface. The second was that in which the opposite, underside of the boring bar’s end was clamped in contact with the lower section of the clamping space surface. For each of these two spatial position configurations, five various tightening torques were used. The different off-line clamping conditions are presented by Table 5.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Torque [Nm]</th>
<th>Clamping Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC₁</td>
<td>10</td>
<td>Upper side</td>
</tr>
<tr>
<td>BC₂</td>
<td>20</td>
<td>Upper side</td>
</tr>
<tr>
<td>BC₃</td>
<td>30</td>
<td>Upper side</td>
</tr>
<tr>
<td>BC₄</td>
<td>40</td>
<td>Upper side</td>
</tr>
<tr>
<td>BC₅</td>
<td>50</td>
<td>Upper side</td>
</tr>
<tr>
<td>BC₆</td>
<td>10</td>
<td>Under side</td>
</tr>
<tr>
<td>BC₇</td>
<td>20</td>
<td>Under side</td>
</tr>
<tr>
<td>BC₈</td>
<td>30</td>
<td>Under side</td>
</tr>
<tr>
<td>BC₉</td>
<td>40</td>
<td>Under side</td>
</tr>
<tr>
<td>BC₁₀</td>
<td>50</td>
<td>Under side</td>
</tr>
</tbody>
</table>

Table 5: The 10 different clamping conditions of the boring bar used for the production of forward path estimates when the boring bar is not in contact with the workpiece (off-line). Four clamping screws were used, two on the tool side and two on the opposite side.

The spectrum estimation parameters and identification signal used in the production of off-line frequency function estimates are given in Table 1. Fig. 19 shows frequency function estimates of the forward path for the five different clamping screw tightening torques. The tool side, upper side, of the boring bar end was clamped in contact with the upper part of the clamping space surface. These forward path frequency function estimates are shown in the frequency range of the fundamental resonance frequency of the boring bar in Fig. 20.

For the other spatial position of the bar end, in the clamping house, the
Figure 19: Frequency function estimates of the forward path, for the five different tightening torques of the clamping screws, when the tool side, upper side, of the boring bar end is clamped in contact with the upper part of the clamping space surface.
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Figure 20: Frequency function estimates of the forward path in the frequency range of the fundamental resonance frequency of the boring bar, for the five different tightening torques of the clamping screws, when the tool side, upper side, of the boring bar end is clamped in contact with the upper part of the clamping space surface.
frequency function estimates for the forward path for the five different tightening torques are shown in Fig. 21. These forward path frequency function estimates are shown in the frequency range of the fundamental resonance frequency of the boring bar in Fig. 22.

As opposed to a situation in which the boring bar is not in contact with the workpiece, contact with the workpiece during a continuous cutting operation will cause the boundary conditions on the cutting tool to change [8,11]. Hence, the dynamic properties of the forward path will be different when the boring bar is not in contact with the workpiece and during continuous turning. Also, different cutting data and work material are likely to affect the dynamic properties of the forward path during continuous turning [8]. Estimates of the forward path may be produced during continuous turning, on-line, by using a suitable identification or excitation signal which is measured simultaneously with the forward path response or output. However, the forward path response during such circumstances will be based both on the identification signal excitation, and the material deformation process excitation of the forward path during continuous turning. High boring bar vibration levels excited by the material deformation process will have a degrading effect on the estimation and will influence the extent to which the measured forward path response may be linearly explained from the identification signal. Also, non-linear dynamic behavior of the boring bar may contribute to the forward path response [8,11].

Forward path frequency function estimates were produced during continuous turning for a variety of different cutting data. The clamping conditions with respect to the tightening torque of the clamping screws and the spatial position of the boring bar in the clamping space of the clamping house were fixed and are given by clamping condition $BC_{10}$ in Table 5. The spectrum estimation parameters and identification signal used in the production of on-line frequency function estimates are given in Table 2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Cutting Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cutting Depth</td>
</tr>
<tr>
<td>On-line 1</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>On-line 2</td>
<td>1 mm</td>
</tr>
</tbody>
</table>

Table 6: Cutting data used for forward path frequency response function estimation during continuous turning (on-line).
Figure 21: Frequency function estimates of the forward path, for the five different tightening torques of the clamping screws, when the underside, of the boring bar end, is clamped in contact with the lower part of the clamping space surface.
Figure 22: Frequency function estimates of the forward path in the frequency range of the fundamental resonance frequency of the boring bar, for the five different tightening torques clamping screws, when the underside, of the boring bar end is clamped in contact with the lower part of the clamping space surface.
Fig. 23 presents two different forward path frequency function estimates produced during continuous turning (on-line) with different cutting data (see Table 6). This diagram also presents a forward path frequency function estimate produced when the boring bar is not in contact with the workpiece (off-line). The off-line frequency function estimate was produced using the spectrum estimation parameters and identification signal according to Table 1.

The coherence functions corresponding to both the on-line forward path frequency response function estimates and the off-line estimates are shown in Fig. 24 a). Estimates of the random error for the on-line and off-line frequency response function estimates in Fig. 23 are shown in Fig. 24 b).

### 3.2 Active Boring Bar Vibration Control Results

The cutting experiments utilized three different feedback controllers in the active control of boring bar vibration: first, an analog manually adjustable controller based on lead compensation. Secondly, a manually adjustable analog stand-alone controller, based on a lead-lag compensation and finally an adaptive digital controller based on the feedback filtered-x LMS algorithm. To illustrate the results of the active control of boring bar vibration using the three different controllers, power spectral densities of boring bar vibration with and without active vibration control are presented in the same diagram. The spectrum estimation parameters used in the production of boring bar vibration power spectral density estimates are shown in Table 4.

Initially, a simple analog manually adjustable lead compensator, based on digitally controlled analog design was developed. The adjustable lead compensator was tuned manually and it was possible to provide an attenuation of the boring bar vibration level by up to approximately 35 dB, see Fig. 25. However, using the manually adjustable lead compensator in the active control of boring bar vibration frequently resulted in stability problems.

By using the manually adjustable band-pass lead-lag controller in the active control of boring bar vibration, the vibration level was reduced by up to approximately 50 dB after a simple manual tuning of the controller (see Fig. 26). Furthermore, in numerous cutting experiments, the active control of boring bar vibration based on the band-pass lead-lag controller after initial manual tuning has provided stable control with significant vibration attenuation.

By utilizing the feedback filtered-x LMS algorithm as controller in the
Figure 23: Frequency function estimates of the forward path during a continuous cutting operation (on-line) and when the boring bar is not in contact with the workpiece (off-line). The on-line estimation of the forward path was produced using workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015. On-line 1; feed rate $s=0.2\text{mm}/\text{rev}$, cutting depth $a=1.2\text{mm}$, cutting speed $v=80\text{m}/\text{min}$ and On-line 2; feed rate $s=0.2\text{mm}/\text{rev}$, cutting depth $a=1\text{mm}$, cutting speed $v=150\text{m}/\text{min}$. 
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Figure 24: a) Coherence function estimates between input and output signal of the forward path during continuous cutting (on-line) and when the boring bar is not in contact with the workpiece (off-line). b) Estimate of the random error for the on-line and off-line frequency response function estimates.
Figure 25: a) Power spectral densities of boring bar vibration in the cutting speed direction with active control using the adjustable lead compensator (solid line) and without active control (dashed line). b) The corresponding spectra zoomed in to the three first resonance peaks. Workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015, feed rate $s=0.24$ mm/rev, cutting depth $a=2$ mm, cutting speed $v=60$ m/min.
Figure 26: a) Power spectral densities of boring bar vibration in the cutting speed direction with active control using the adjustable lead-lag controller (solid line) and without active control (dashed line). b) The corresponding spectra zoomed in to the three first resonance peaks. Workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015, feed rate $s=0.24$ mm/rev, cutting depth $a=2$ mm, cutting speed $v=60$ m/min.
active control of boring bar vibration, the adaptive controller will tune the adaptive FIR filter to de-correlate the error signal with the filtered reference signal vector (i.e. previous error signal samples filtered by the FIR filter forward path estimate). Therefore, high loop gain is provided in the frequency range of the resonance frequency that dominates the boring bar vibration. Frequently, at high boring bar vibration levels, the feedback filtered-x LMS algorithm in the active control of boring bar vibration yields an attenuation of the vibration by more than 50 dB at the dominating resonance frequency, see Fig. 27. However, feedback filtered-x LMS algorithm requires leakage to provide stable and robust control [25, 49] and the cost for improved robustness is a somewhat reduced vibration attenuation performance, as illustrated in Fig. 28.

3.3 Stability and Robustness of the Controllers

The stability of a feedback control system requires that its open loop frequency response $H_{ol}(f)$ does not violate the closed loop stability requirements, i.e. the Nyquist stability criterion [26, 39, 47]. A closed loop system is said to be stable if the polar plot of the open loop frequency response $H_{ol}(f)$ for the feedback control system does not enclose the (-1, 0) point in the Nyquist diagram. The greater the shortest distance between the polar plot and the (-1, 0) point, the more robust the feedback control system is with respect to variation in forward path or plant response and controller response. The system fulfills the conditions for robust stability [26, 39, 47] if there is no phase function present which (in combination with maximal magnitude of the possible plant uncertainties at each frequency) can result in a feedback control system open loop frequency response that encloses the (-1,0) point in the Nyquist diagram.

An estimate of the open loop frequency function for a feedback control system may be produced based on the controller frequency response function and the forward path frequency response function. The analog controller frequency response function was estimated after manual tuning for active control of boring bar vibration. In the case of the adaptive digital controller, the controller frequency response function was estimated after convergence with the step size set to zero. All the controllers were estimated with the spectrum estimation parameters and the identification signal shown in Table 3. The open loop frequency responses for the active boring bar vibration control system were produced for the manually adjustable lead compensator and the off-line forward path frequency function estimate, for each of the 10 different
Figure 27: a) Power spectral densities of boring bar vibration in the cutting speed direction (solid line) with, and (dashed line) without active control based on the feedback filtered-x LMS algorithm, number of adaptive filter coefficients M=20, step size $\mu = -0.5$, sampling frequency of the DSP $F_s=8$ kHz. b) the corresponding spectra zoomed in to the three first resonance peaks. Workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015, feed rate $s=0.24$ mm/rev, cutting depth $a=2$ mm, cutting speed $v=60$ m/min.
Figure 28: a) Power spectral densities of boring bar vibration in the cutting speed direction (solid line) with, and (dashed line) without active control based on the leaky feedback filtered-x LMS algorithm number of adaptive filter coefficients M=20, leaky coefficient $\gamma = 0.9999$ step size $\mu = -0.5$, sampling frequency of the DSP $Fs=8$ kHz. b) the corresponding spectra zoomed in to the three first resonance peaks. Workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015, feed rate $s=0.24$ mm/rev, cutting depth $a=2$ mm, cutting speed $v=60$ m/min.
clamping conditions of the boring bar (see Table 5). Also, open loop frequency responses for the manually adjustable lead compensator and the two different on-line forward path frequency function estimates were produced. Observe that, in order to facilitate interpretation of the Nyquist diagrams, the number of the open loop frequency response functions plotted in the same diagram were limited to four. These open loop frequency response functions were selected in order to avoid redundancy in the Nyquist diagrams, and to form the open loop frequency response functions. The forward path frequency response function estimates corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$ were selected (see Table 5). The Nyquist diagram in Fig. 29 shows polar plots of the selected open loop frequency responses. The corresponding Bode plot is shown in Fig. 30.

Observe, in Fig. 29, that the polar plots of the open loop frequency responses for the feedback control system based on the manually adjustable lead compensator are close to, or enclose, the (-1, 0) point in the Nyquist diagram.
Figure 30: Open loop frequency response function estimates for a boring bar vibration control system, based on a manually adjustable lead compensator for the four different forward path frequency response function estimates corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$. 
Fig. 30 demonstrates it can be seen that open loop frequency responses for the boring bar vibration control system based on the manually adjustable lead compensator provide substantial loop gain at resonance frequencies above 2000 Hz. Open loop frequency responses for the boring bar vibration control system were produced based on the manually adjustable band-pass lead-lag compensator and the off-line forward path frequency function estimate, for each of the ten different boring bar clamping conditions (see Table 5). Also, open loop frequency responses were produced for the manually adjustable lead compensator and the two different on-line forward path frequency function estimates. The selected open loop frequency functions for the active boring bar control system based on band-pass lead-lag compensator are shown in the Nyquist diagram in Fig. 31. These estimates were based on the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$, which represent the variance of the forward path. The corresponding Bode plot of the open loop frequency response functions for the active boring bar control system are shown in Fig. 30.

![Nyquist diagram](image)

Figure 31: Nyquist diagram for a boring bar vibration control system, based on a manually adjustable band-pass lead-lag compensator for the four different forward path frequency response function estimates corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$.

Observe, the distance between the polar plots of the open loop frequency response function estimates for the boring bar vibration control system based
Figure 32: Open loop frequency response function estimates, for a boring bar vibration control system, based on a manually adjustable band-pass lead-lag compensator for the four different forward path frequency response function estimates corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$.
on the manually adjustable band-pass lead-lag compensator and the \((-1, 0)\) point in the Nyquist diagram in Fig. 31. In addition, the Bode plot (see Fig. 30) demonstrates low loop gain above 1000 Hz.

The adaptive digital control of boring bar vibration was carried out with and without a leakage factor in the feedback filtered-x LMS algorithm. The Nyquist diagram in Fig. 33 shows the polar plots of the four open loop frequency responses, based on the feedback filtered-x LMS algorithm. The four open loop frequency functions for the active boring bar control system based on the feedback filtered-x LMS algorithm are shown in the Nyquist diagram in Fig. 33. The corresponding Bode plots of the open loop frequency response functions based on the feedback filtered-x LMS algorithm are shown in Fig. 34. The feedback filtered-x LMS algorithm will automatically tune the adaptive FIR filter to de-correlate the error signal with the filtered reference signal vector, i.e., previous error signal samples filtered by the FIR filter forward path estimate. Thus, a high loop gain will be provided in the frequency range...
Figure 34: Open loop frequency response function estimates for a boring bar vibration control system, based on the feedback filtered-x LMS algorithm for the four different forward path frequency response function estimates, corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$. Number of adaptive filter coefficients $M=20$, step size $\mu = -0.5$, sampling frequency of the DSP $Fs=8$ kHz.
Figure 35: Nyquist diagram for a boring bar vibration control system, based on the leaky feedback filtered-x LMS algorithm for the four different forward path frequency response function estimates, corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$. Number of adaptive filter coefficients $M=20$, leaky coefficient $\gamma = 0.999$ step size $\mu = -0.5$, sampling frequency of the DSP $Fs=8$ kHz.

of the resonance frequency that dominates the boring bar vibration (see the Bode plot in Fig. 34). Finally, the open loop frequency responses produced for the adaptive digital controller, based on the leaky feedback filtered-x LMS algorithm, are plotted in the Nyquist diagram in Fig. 35; the corresponding magnitude and phase functions are shown in Fig. 36.

If, for example, the forward path frequency function estimate corresponding to the clamping condition $BC_{10}$ is selected as a nominal plant or forward path model of the forward path, then an estimate of the upper bound $\beta(f)$ for the multiplicative perturbation modeling the forward path uncertainty may be produced based on Eq. 29 using all the other forward path frequency function estimates (not the forward path frequency function estimate corresponding to the clamping condition $BC_{10}$). In Fig. 37 the magnitude of the inverse of the nominal complementary function for each of the controllers is plotted in the same diagram as the estimated upper bound $\hat{\beta}(f)$ for the multiplicative perturbation.
Figure 36: Open loop frequency response function estimates for a boring bar vibration control system, based on the leaky feedback filtered-x LMS algorithm for the four different forward path frequency response function estimates, corresponding to the clamping conditions; $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$. Number of adaptive filter coefficients $M=20$, leaky coefficient $\gamma = 0.999$ step size $\mu = -0.5$, sampling frequency of the DSP $Fs=8$ kHz.
4 Discussion and Conclusions

The above results demonstrate that boring bar vibration in internal turning may be reduced by utilizing active control based on active boring bars with embedded actuator and sensor, and a suitable feedback controller such as an analog manually adjustable band-pass lead-lag controller and an adaptive digital controller based on the feedback filtered-x LMS algorithm. It has been established that for each time the boring bar is clamped, it is likely that the clamped boring bar will have different dynamic properties (see Figs. 19, 20, 21 and 22). Also, the dynamic properties of the clamped boring bar will differ between these instances when the boring bar is not in contact with the workpiece and during continuous turning (see Fig. 23). In addition, different cutting data and work materials will likely affect the dynamic properties of the clamped boring bar during continuous turning [8]. Thus, the forward path in the active boring bar vibration system may display significant variations in its dynamic properties. Hence, a robust controller that performs well for substantial variations in the dynamics of the forward path is required for the active control of boring bar vibration.

Figure 37: Magnitude of the inverse of the nominal complementary function for each of the controllers and the estimated upper bound for the multiplicative perturbation.
The development of a simple adjustable analog controller, based on digitally controlled analog design, started initially with a lead compensator. However, the vibration to be controlled is related to the fundamental bending modes of the boring bar and not its higher order modes [8, 11]. Thus, high loop gain (provided by a controller above the fundamental bending modes eigenfrequencies) will likely be an issue concerning the stability and robustness of the active boring bar vibration control system. However, if the manually adjustable lead control is utilized for active boring bar vibration control, it will (when stable) perform a broad-band attenuation of the tool-vibration. Therefore, the vibration level is reduced by over 35 dB at 460 Hz and harmonics of the 460 Hz boring bar resonance frequency are attenuated (see Fig. 25). Polar plots of the open loop frequency responses (for the feedback control system based on the manually adjustable lead compensator) which approach or enclose the (-1, 0) point of the Nyquist diagram also demonstrate problems with robustness and stability (see Fig. 29). According to the Bode plots (Fig. 30), the manually adjustable lead compensators provides (as expected) a substantial loop gain at the boring bar resonance frequencies above 2000 Hz.

Thus, to provide high boring bar vibration attenuation, high loop gain is only required in the frequency range of the boring bar’s fundamental bending modes eigenfrequencies. Basically, a manually adjustable controller should be able to provide adjustable band-pass gain and adjustable phase enabling to control the forward path in the frequency range of the boring bar’s fundamental bending modes eigenfrequencies. This will produce adequate anti-vibration, canceling the original vibration excited by the material deformation process.

The feedback filtered-x LMS algorithm has successfully been used in both active control tool holder shank vibration in external turning [25, 49] and active control of boring bar vibration. Occasionally, however, in the active control of boring bar vibration based on feedback filtered-x LMS algorithm occasionally, the abrupt change of load applied on the tool by the workpiece may result in tool failure. This generally occurs during the engagement phase. A further shortcoming regarding a digital adaptive controller based on the feedback filtered-x LMS algorithm involves the delay in the control authority during the tool’s engagement phase. The adaptive digital feedback control based on feedback filtered-x LMS algorithm (with and without leakage) performs a broad-band attenuation of tool-vibration, and is able to reduce vibration levels by over 50 dB at 460 Hz, as well as to attenuate the harmonics of the 460
Hz boring bar resonance frequency (see Figs. 27 and 28). However, slightly lower vibration attenuation might be observed for the leaky feedback filtered-x LMS algorithm (see Fig. 28). The introduction of a leakage factor or a "forgetting factor" in the feedback filtered-x LMS algorithm's recursive coefficient adjustment algorithm will induce bias in the coefficient vector and may thereby cause a somewhat reduced attenuation of the boring bar vibration.

The active boring bar vibration control system based on the manually adjustable band-pass lead-lag control performs a broad-band attenuation of tool-vibration, and is also able to reduce the vibration level by over 50 dB at 460 Hz, as well as to attenuate the harmonics of the 460 Hz boring bar resonance frequency (see Fig. 26). Thus, the band-pass lead-lag controller provides attenuation performance comparable to that of the adaptive controller; by tuning the adaptive FIR filter to decorrelate the error signal with the filtered reference signal vector. It thereby provides high loop gain in the frequency range of the resonance frequency that dominates the boring bar vibration (see Figs. 34 and 36). By comparing the loop gains provided by the adaptive digital controller with the loop gains provided by the band-pass lead-lag controller (see Fig. 30) it follows that the analog lead-lag controller is tuned to provide high loop gain over a broader frequency range as compared to the loop gain resulting from the adaptive digital control. On the other hand, by examining the Nyquist plots for the open loop frequency response functions concerning the band-pass lead-lag controller (see Fig. 31) and the feedback filtered-x LMS controller (see Fig. 33) it follows that the shortest distance between the polar plots and the (-1, 0) point is greater for the analog controller as compared with the feedback filtered-x LMS controller. While the adaptive digital controller based on the leaky feedback filtered-x LMS algorithm results in the greatest shortest distance between the polar plots and the (-1, 0) point (see Fig. 35). Generally, by introducing leakage in the feedback filtered-x LMS-algorithm the "memory" of the adaptive algorithm is reduced. This reduces the energy in the response of the adaptive FIR filter, as well as the energy in the control signal to the forward path or plant. Thus the introduction of leakage factor in the feedback filtered-x LMS-algorithm will provide a restraining influence on the loop gain of the control system. In this context it should, however, be noted that the objective of the adaptive digital controllers, at each time instant \( n \), is basically to iteratively adjust the controller response to minimize the error signal power or boring bar vibration power.

If robust stability is considered (see Fig. 37) it follows that the active bor-
ing bar vibration control system based on the leaky feedback filtered-x LMS controller is the only system that fulfills the conditions for robust stability. However, the active boring bar vibration control system based on the band-pass lead-lag controller, tuned initially, remained stable during the course of numerous trials with varied clamping conditions and cutting data.

The dynamic response of the boring bar frequently exhibits nonlinear properties and, as a consequence, several of the dominating fundamental resonance frequencies’ harmonics may be observable in the boring bar response [8]. Thus, by utilizing active boring bar vibration control, a linearization of the boring bar vibration might be a probable explanation for the reduction of the harmonics of the dominating fundamental resonance frequency during active control [25].

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References


PART II

Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions
Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions

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Abstract

The boring bar is one of the most widely used type of tool holders in metal cutting operations. The turning process subjects the tool to vibrations, and cutting in deep workpiece cavities is likely to result in high vibration levels. The consequences of such vibration levels generally results in: reduced tool life, poor surface finishing and disturbing sound. Internal turning frequently requires a long and slender boring bar in order to machine inside a cavity, and the vibrations generally become highly correlated with one of the fundamental bending modes of the boring bar. Different methods can be applied to reduce the vibrations, the implementation of the most efficient and stable methods require in depth knowledge concerning the dynamic properties of the tooling system. Furthermore, the interface between the boring bar and the clamping house has a significant influence on the dynamic properties of the clamped boring bar. This report focuses on the dynamic properties of a boring bar that arise under different clamping conditions of the boring bar and are introduced by a clamping house (commonly used in the manufacturing industry). The dynamic properties of a bor-
Part II

ing bar (for different cases of boundary condition of the boring bar) are presented partly analytically but also experimentally.

1 Introduction

In industry where metal cutting operations such as turning, milling, boring and grinding take place, degrading vibrations are a common problem. In internal turning operations vibration is a pronounced problem, as long and slender boring bars are usually required to perform the internal machining of workpieces. Tool vibration in internal turning frequently has a degrading influence on surface quality, tool life and production efficiency, whilst also resulting in severe environmental issues such as high noise levels. By applying, for example, an active control scheme, these vibrations can be significantly reduced, with the result of improved workpiece surface finish and increased tool life [1]. In order to successfully implement such a scheme, the dynamic properties of system (boring bar - clamping structure) be known, as must the nature of the disturbing vibrations.

A number of experimental studies have been carried out on mechanisms explaining tool vibration during turning operations [2–4] and on the dynamic properties of boring bars [5–7]. In 1946 the principles of the traditional theory of chatter in simple machine-tool systems were worked out by Arnold [8] based on experiments carried out on a rigid lathe, using a stiff workpiece but a flexible tool. In this way he was able to investigate chatter under controlled conditions. Later in 1965 Tobias [4] presented further investigations of the chatter phenomena, involving, for example, the chip-thickness variation and the phase lag of the undulation of the surface. Also, in the same year, Meritt et al. [2] discussed the stability of structures with n-degrees of freedom, assuming no dynamics in the cutting process; they also proposed a simple stability criterion. Parker et al. [9] investigated the stability behavior of a slender boring bar by representing it with a two-degree-of-freedom mass-spring-damper system and experimenting with regenerative cutting. They also investigated how the behavior of the vibration was affected by coupling between modes, by using different cutting speeds, feed rates and angles of the boring bar head relative to the two planes of vibrations. Pandit et al. [10] developed a procedure for modeling chatter from time-series by including unknown factors of random disturbances present in the cutting process, they formulated self-excited random vibrations with white noise as a forcing function. Kato et al. [11] investigated regenerative chatter vibration due to deflection of the workpiece,
and introduced a differential equation describing chatter vibration based on experimental data. Furthermore, various analytical models/analysis methods relating to the boring bar/or cutting process have been continuously developed, assuming various conditions. For example, Zhang et al. [5] who’s model is derived from a two-degree-of-freedom model of a clamped boring bar and four cutting force components. In addition, Rao et al. [6] includes variation of chip cross-sectional area in their model, whilst Kuster et al. [12] developed a computer simulation based on a three-dimensional model of regenerative chatter. Walter et al. [13] developed a model of the chuck-workpiece connection when the workpiece is considered to be weak, using Finite Element Method (FEM) and experimental studies of a ring shaped weak workpiece; this model focused on the influence from clamping forces when using jaw chucks. A time series approach was used by Andrén et al. [7] to investigate boring bar chatter and the results were compared with an analytical Euler-Bernoulli model. Also, Euler-Bernoulli beam modeling, experimental modal analysis and operating deflection shape analysis were used by Andrén et al. [14] to investigate the dynamic properties of a clamped boring bar. Results obtained demonstrate observable differences concerning the fundamental bending modes. They found that that the bending motion of the first two resonance frequencies is, to a large extent, in the direction of cutting speed. Scheuer et al. [15] investigated the dynamic properties of a boring bar, based on experimental modal analysis under different clamping conditions. Two different clamping houses were used: one clamping from two sides with clamp screws and one circular clamping sleeve; clamping along the circular surface of the boring bar. Results indicate that both the eigenfrequencies and the directions of the fundamental bending modes vary for different clamping pressures; in particular, for the circular clamping sleeve.

The problem of boring bar vibration can be addressed using conventional methods, such as redesigning the machine tool system, implementing tuned passive damping or implementing active control [16,17].

However, the order of stability improvement achieved usually correlates to the quality and extent of knowledge of the dynamic properties of the tooling structure -the interface between the cutting tool, or insert, and the machine tool. Boring bar vibrations are usually directly related to the lower order bending modes and the dynamic properties of a boring bar installed in a lathe are directly influenced by the boundary conditions, i.e. the clamping of the bar [14]. Following the literature review, it appears that little work has been done on the clamping properties’ influence on the dynamic properties of a clamped boring bar. Thus, it is of significance to investigate the clamping
properties’ influence on the dynamic properties of the clamped boring bar in order to gain further understanding of the dynamic behavior of clamped boring bars in the metal cutting process. This report focuses on the variation in the dynamic properties of a clamped boring bar imposed by controlled discrete variations in the clamping conditions produced by a standard clamping house of the variety commonly used in industry today. The clamping house has a circular cavity that the boring bar fits easily into; the clamping is then carried out by means of screws on the tool side and on the opposite side of the boring bar. To investigate the influence of clamping properties on the dynamic properties of a clamped standard boring bar, experimental modal analysis have been conducted both for a clamped standard boring bar and a clamped active boring bar under different clamping conditions. Also, analytical Euler-Bernoulli beam models incorporating clamping flexibility through the use of transverse springs and rotational springs have been investigated for the modeling of a clamped boring bar. Finally, some simulations of nonlinear models have also been studied for observed nonlinear behavior of the clamped boring bar.

2 Materials and Methods

Experimental modal analysis has been carried out on different boring bars for various clamping conditions in order to investigate the changes of the clamped boring bar’s dynamic properties.

2.1 Experimental Setup

The experimental setup and subsequent measurements were carried out in a Mazak SUPER QUICK TURN - 250M CNC turning center. The CNC lathe has 18.5 kW spindle power and a maximal machining diameter of 300 mm, with 1005 mm between the centers, a maximal spindle speed of 400 revolutions per minute (r.p.m.) and a flexible turret with a tool capacity of 12 tools. The lathe is presented by the photo in Fig. 1.

Initially, a right-hand cartesian coordinate system was defined: z in the feed direction, y in the reversed cutting speed direction and x in the direction of cutting depth, see Fig. 1 b) (upper left corner). Subsequently a sign convention was defined for use throughout the report. The coordinate system and sign convention are based on the right-hand definition where the directions of displacements and forces in positive directions of the coordinate axes are considered positive. Moreover, moment about an axis in the clockwise
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Figure 1: a) Mazak SUPER QUICK TURN - 250M CNC lathe and b) the room in the lathe where machining is carried out.

direction (when viewing from the origin in the positive direction of the axis) is considered positive, see Fig. 2.

Figure 2: Right hand definition of the cartesian coordinate system and the sign convention, where direction shown by the arrows defines the positive direction of displacement, force and moment.

The boring bars were positioned in the operational position, mounted in a clamping house attached to a turret with screws, during all measurements. The turret may be controlled to move in the cutting depth direction, x-direction, and in the feed direction, z-direction, as well as to rotate about the z-axis for tool change. The turret, etc. is supported by a slide which in turn, is mounted onto the lathe bed. Even though the turret is a movable component, it is relative rigid, rendering the dynamic properties of the boring
bars observable.

2.1.1 Measurement Equipment and Setup

The following equipment was used in the experimental setup:

- 12 Brüel & Kjær 333A32 accelerometer
- 2 Brüel & Kjær 8001 Impedance Head
- 1 Brüel & Kjær NEXUS 2 channel conditioning amplifier 2692.
- OSC audio power amplifier, USA 850
- Ling Dynamic Systems shaker v201
- Gearing & Watson Electronics shaker v4
- A custom designed amplifier for capacitive loads.
- Active boring bar with embedded piezo ceramic actuator.
- Hewlett Packard 54601B Oscilloscope.
- Hewlett Packard 35670A Signal Analyzer.
- Hewlett Packard VXI Mainframe E8408A.
- Hewlett Packard E1432A 4-16 Channel 51.2 kSa/s Digitizer
- PC with I-DEAS 10 NX Series

Twelve accelerometers and two cement studs for the impedance heads were attached onto the boring bars with X60 glue (a cold hardener two component glue). The sensors were evenly distributed along the centerline, on the underside and on the back-side of the boring bar; six accelerometers and one stud on the respective side (see Figs. 3 and 4). To excite all the lower order bending modes, two shakers were attached via stinger rods to the impedance heads, one in the cutting speed direction (y-) and the other in the cutting depth direction (x-) see Fig. 3. The shakers were positioned relatively close to the cutting tool.

2.1.2 Boring Bars

Two different boring bars were used in the experimental setup in order to be able to analyze the changes of the eigenfrequencies and the mode-shapes in different cases. The first boring bar used in the modal analysis was a standard "non-modified" boring bar, WIDAX S40T PDUNR15F3 D6G, presented in Fig. 5. The second boring bar used in this experiment was an active boring bar, based on the standard WIDAX S40T PDUNR15 boring bar, with
Figure 3: The experimental setup, in a) two shakers suspended from the ceiling are observable as well as a workpiece and the turret. b) shows a closeup of the sensors and the shaker configuration on the boring bar.

an accelerometer and an embedded piezo-stack actuator, see Fig. 6. The accelerometer was mounted 25 mm from the tool tip to measure the vibrations in the cutting speed direction (y-). This position was as close as possible to the tool tip, but at a sufficient distance to prevent metal-chips from the material removal process from damaging the accelerometer. The actuator was embedded into a milled space in the longitudinal direction (z-direction), below the centerline of the boring bar. By embedding accelerometers and piezo stack actuators in conventional boring bars, a solution was obtained for the introduction of control force to the boring bar with physical features and properties that fit the general lathe application. Assuming a constant cross-section along the boring bar, neglecting the head, the dimensions from Fig. 5 b) result in a cross-sectional area $A$ and a moment of inertia $I_x$, $I_y$ presented by Table 1. The standard WIDAX S40T PDUNR15 boring bar is manufactured in the material 30CrNiMo8, (AISI 4330) which is a heat treatable steel alloy (for high strength), see Table 2 for material properties.
Figure 4: Drawings of the boring bar including clamp screws, cement studs and sensors. The sensors are attached along the underside and the backside of the boring bar. The threaded holes denoted $B_1$, $B_2$ and $B_1$ are screw positions for clamping the boring bar from top and bottom. The dimensions are in mm, where $l_1 = 10.7$, $l_2 = 18$, $l_3 = 101$, $l_4 = 250$, $l_5 = 17$, $l_6 = 100$, $l_7 = 18.5$, $l_8 = 25$ and $l_9 = 35$.

Figure 5: a) Top-view of the standard boring bar "WIDAX S40T PDUNR15F3 D6G, b) the cross section of the boring bar where $C_C$ is the center of the circle and $M_C$ is the mass center of the boring bar."
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Figure 6: The active boring bar with an accelerometer close to the tool tip and an embedded piezo-stack actuator in a milled space below the centerline.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$1.19330295 \cdot 10^{-3}$</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$I_x$</td>
<td>$1.13858895 \cdot 10^{-7}$</td>
<td>$m^4$</td>
</tr>
<tr>
<td>$I_y$</td>
<td>$1.13787004 \cdot 10^{-7}$</td>
<td>$m^4$</td>
</tr>
</tbody>
</table>

Table 1: Cross-sectional properties of the boring bar, illustrated in Fig. 5 b), where $A$ is the cross-sectional area and $I_x$, $I_y$ are the moments of inertia around the denoted axis.

<table>
<thead>
<tr>
<th>Material composition besides Fe in percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>0.26-0.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>205 GPa</td>
</tr>
</tbody>
</table>

Table 2: Composition and properties of the material 30CrNiMo8.
2.1.3 Clamping Houses

The clamping house is a basic 8437-0 40mm Mazak holder, presented in Fig. 7 a) and b), and clamps the tool holder by means of either four or six screws: two/three from the top and two/three from bottom. The basic holder itself is mounted onto the turret with four screws.

In addition to the screws, the clamping house also features a guide matching a track on the turret; this guide positions the clamping house along the z-axis on the xy plane, whilst the guide pin positions the clamping house on the z-axis, see Fig. 7 a). The clamping house has a default thread size of M8.

A second clamping house of the same model was rethreaded to the thread size M10. Furthermore, a third clamping house, also of the same type, was used in the construction of a so-called "linearized" clamping of the boring bar.

2.1.4 Clamping Conditions

A number of different setups were considered using different boring bars described in section 2.1.2 in conjunction with the different clamping houses. In
the first setup, the reference boring bar was clamped using four M8 class 8.8 screws. The screws were tightened first from the top and then from the underside.

The recommended tightening torque for this class is 26.6Nm, however, evaluations of the screws revealed that threads remained intact and screws did not break for a tightening torque of 30Nm.

The second setup involved the same five torques as for the previous setup, but four screws of size M10 class 8.8, which were, again, tightened first from the top and then from the underside. As only four clamp screws were used, the clamping house center screw positions were not used.

The third setup involved the use of six screws of size M10, with the reference bar and same torques as previous. The use of six screws involved the use of all clamping house center screw positions. Setup one and two were then repeated, using the active boring bar.

In order to accomplish a linearized clamping condition, the standard clamping was modified. A boring bar WIDAX S40T PDUNR15F3 D6G, the same model as the standard boring bar, was used together with three steel wedges produced of the material SS 1650 (AISI 1148). The steel wedges were glued with epoxy on the flat surfaces of the boring bar along the clamping length of the bar end. The steel wedges were shaped geometrically to render a circular cross section on the boring bar along its clamped end. After the epoxy was set; the boring bar end with circular cross section was pressed into the clamping house and glued to it with epoxy to make the clamping rigid, see Fig 8.

![Figure 8: The linearized boring bar-clamping house setup.](image-url)
2.2 Experimental Modal Analysis

The primary goal of experimental modal analysis is to identify the dynamic properties of the system under examination, the modal parameters; i.e. determine the natural frequencies, mode shapes, and damping ratios from experimental vibration measurements. The procedure of modal analysis may be divided into two parts: the acquisition of data and the parameter estimation or parameter identification from these data, also known as curve fitting [18]. These procedures are often referred to as a discipline of art since the process of acquiring good data and performing accurate parameter identification is an iterative process, based on various assumptions along the way [18].

2.3 Spectral Properties

Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [19].

A non-parametric estimate of the power spectral density \( P_{xx}(f) \), where \( f \) is frequency, for a signal \( x(t) \) may be estimated using the Welch spectrum estimator [20], given by:

\[
\hat{P}_{xx}(f_k) = \frac{1}{LNF_s} \sum_{l=0}^{L-1} \left| \sum_{n=0}^{N-1} w(n)x_l(n)e^{-j2\pi nk/N} \right|^2, \quad f_k = \frac{k}{N}F_s
\]

(1)

where \( k = 0, \ldots, N - 1 \), \( L \) is the number of periodograms, \( N \) is the length of the data segments used to produce the periodograms, \( x_l(n) \) is the sampled signal in segment \( t \), \( F_s \) is the sampling frequency.

Thus, for each input signal \( x(t) \) and output signal \( y(t) \), a single-input-single-output system (SISO system) is simultaneously measured and the sampled signal \( y(n) \) and \( x(n) \) are recorded. By using, for example, the Welch spectrum estimator [20], the cross-power spectral density \( \hat{P}_{yx}(f_k) \) between the input signal \( x(n) \) and the output signal \( y(n) \) and the power spectral density \( \hat{P}_{xx}(f_k) \) for the input signal \( x(n) \) may be produced [14,19].

A least-squares estimate of a frequency response function between the input signal \( x(n) \) and the output signal \( y(n) \) may be produced according to [19]:

\[
\hat{H}(f_k) = \frac{\hat{P}_{yx}(f_k)}{\hat{P}_{xx}(f_k)}
\]

(2)
and the coherence function as [19]

$$\hat{\gamma}_{yx}(f_k) = \frac{\hat{P}_{yx}(f_k)\hat{P}_{yy}(f_k)}{\hat{P}_{xx}(f_k)\hat{P}_{yy}(f_k)}$$  \hspace{1cm} (3)$$

In the case of a multiple-input-multiple-output system (MIMO system) with $P$ number of responses and $Q$ number of references, an estimate of the cross spectrum matrix $[\hat{P}_{xx}(f_k)]$ between all the inputs is produced as

$$[\hat{P}_{xx}(f_k)] = \begin{bmatrix}
\hat{P}_{1,1}(f_k) & \hat{P}_{1,2}(f_k) & \cdots & \hat{P}_{1,Q}(f_k) \\
\hat{P}_{2,1}(f_k) & \hat{P}_{2,2}(f_k) & \cdots & \hat{P}_{2,Q}(f_k) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{P}_{Q,1}(f_k) & \hat{P}_{Q,2}(f_k) & \cdots & \hat{P}_{Q,Q}(f_k)
\end{bmatrix}$$  \hspace{1cm} (4)$$

where the diagonal elements are power spectral densities for the respective input signal. Also a cross spectrum matrix $[\hat{P}_{yx}(f_k)]$ between all the inputs and outputs may be estimated as

$$[\hat{P}_{yx}(f_k)] = \begin{bmatrix}
\hat{P}_{y1,1}(f_k) & \hat{P}_{y1,2}(f_k) & \cdots & \hat{P}_{y1,Q}(f_k) \\
\hat{P}_{y2,1}(f_k) & \hat{P}_{y2,2}(f_k) & \cdots & \hat{P}_{y2,Q}(f_k) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{P}_{yQ,1}(f_k) & \hat{P}_{yQ,2}(f_k) & \cdots & \hat{P}_{yQ,Q}(f_k)
\end{bmatrix}$$  \hspace{1cm} (5)$$

The least-square estimate for the (SISO) system in Eq. 2, can be rewritten for the (MIMO system) yielding the estimate of the system matrix $[\hat{H}(f_k)]$ as [19]

$$[\hat{H}(f_k)] = [\hat{P}_{yx}(f_k)][\hat{P}_{xx}(f_k)]^{-1}$$  \hspace{1cm} (6)$$

In the case of a multiple inputs, case, the multiple coherence is of interest as a quality of the measurements. The multiple coherence function is defined by the ratio of that part of the spectrum which can be expressed as a linear function of the inputs to the total output spectrum (including extraneous noise), and which is an extension of the ordinary coherence function from the SISO case. If the inputs are uncorrelated, the multiple coherence $\hat{\gamma}^2_{yp,x}(f)$ for the response in point $p$ is given by [19]

$$\hat{\gamma}^2_{yp,x}(f) = \hat{\gamma}^2_{yp,x_1}(f) + \hat{\gamma}^2_{yp,x_2}(f) + \cdots + \hat{\gamma}^2_{yp,x_Q}(f)$$  \hspace{1cm} (7)$$
where $Q$ is the number of inputs and ":" denotes "linear dependent on". However, usually there is some correlations between the inputs, then the multiple coherence is given by [19]

$$
\gamma_{yp:x}^2(f) = 1 - (1 - \gamma_{yp:x_1}^2(f))(1 - \gamma_{yp:x_2}^2(f)) \cdots (1 - \gamma_{yp:x_Q(x_{(Q-1)})}^2(f))
$$

where ":" denotes "independent off" and "!" denotes factorial. The multiple coherence may also be expressed using an expanded spectral matrix $[P_{yp,x}(f)]$, who’s determinant is a measure of extraneous on the output and is written as

$$
[P_{yp,x}(f)] = 
\begin{bmatrix}
P_{yp,yp}(f) & P_{yp,x_1}(f) & \cdots & P_{yp,x_Q}(f) \\
P_{x_1,yp}(f) & P_{x_1,x_1}(f) & \cdots & P_{x_1,x_Q}(f) \\
\vdots & \vdots & \ddots & \vdots \\
P_{x_Q,yp}(f) & P_{x_Q,x_1}(f) & \cdots & P_{x_Q,x_Q}(f)
\end{bmatrix}
$$

Based on the extended spectral matrix the multiple coherence may be expressed

$$
\gamma_{yp:,x}^2(f) = 1 - \left( \frac{\left| [P_{yp,x}(f)] \right|}{P_{yp,yp}(f) \left| [P_{xx}(f)] \right|} \right)
$$

The normalized random error in frequency response function estimates for the amplitude function [19] is approximately given by

$$
\varepsilon_r(|\hat{H}_{yx}(f_k)|) \approx \frac{\sqrt{1 - \gamma_{yp,x}(Q-1)}(f_k)}}{\sqrt{\gamma_{yp,x}(Q-1)}(f_k)2(L_e + 1 - Q)}
$$

for the phase function it is approximately given by [19]

$$
\varepsilon_r(|\hat{\Theta}_{yx}(f_k)|) \approx \arcsin \left( \varepsilon_r(|\hat{H}_{yx}(f_k)|) \right)
$$

Finally, an estimate of the normalized random error for the multiple coherence function is given by [19]

$$
\varepsilon_r(\hat{\gamma}_{yp:,x}^2(f_k)) \approx \frac{\sqrt{2(1 - \hat{\gamma}_{yp:,x}^2(f_k))}}{\sqrt{\hat{\gamma}_{yp:,x}^2(f_k)(L_e + 1 - Q)}}
$$

where $L_e$ is the number of uncorrelated periodograms [19, 20] used in the average to produce the spectrum estimate.
2.3.1 Parameter Estimation

There are several different methods for identification of the modal parameters [18, 21]. There are two basic curve fitting methods: curve fitting in frequency domain using measured Frequency Response Function (FRF) data and a parametric model of the FRF; or curve fitting towards the measured Impulse Response Function (IRF) data using a parametrical model of the IRF [18]. Many methods use both domains, depending on which parameter is estimated [18]. A parametric model of an FRF, \( H(f) \), expressed as the receptance between the reference point, input signal \( q \), and the response, output signal in point \( p \) of a structure, may be written as [18],

\[
H_{pq}(f) = \sum_{r=1}^{N} \frac{A_{pqr}}{j2\pi f - \lambda_r} + \frac{A_{pqr}^*}{j2\pi f - \lambda_r^*}
\]  

(14)

where \( r \) is the mode number, \( N \) the number of modes used in the model, \( A_{pqr} \) the residue belonging to mode \( r \) between reference point \( q \) and response \( p \) and \( \lambda_r \) is the pole belonging to mode \( r \). The parametric model of the IRF, input force to output displacement impulse response may be expressed as

\[
h_{pq}(t) = \sum_{r=1}^{N} A_{pqr} e^{\lambda_r t} + A_{pqr}^* e^{\lambda_r^* t}
\]  

(15)

Due to the fact that two sources (references) were used during data acquisition, a method capable of handling multi-references is required. One such method is the Polyreference least square complex exponential method developed by Vold [22,23]. This method is defined for identification of MIMO-systems with the purpose of obtaining a global least squares estimates of the modal parameters. The estimated system matrix \( \hat{H}(f) \) is of size \( P \times Q \), where \( P \) is the number of responses and \( Q \) the number of references, and is written as

\[
[\hat{H}(f)] = \begin{bmatrix}
\hat{H}_{11}(f) & \hat{H}_{12}(f) & \cdots & \hat{H}_{1Q}(f) \\
\hat{H}_{21}(f) & \hat{H}_{22}(f) & \cdots & \hat{H}_{2Q}(f) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{H}_{P1}(f) & \hat{H}_{P2}(f) & \cdots & \hat{H}_{PQ}(f)
\end{bmatrix}
\]  

(16)

The procedure of modal parameter estimation starts by determining the model order of the system under analysis. This can be done with the aid of
a Mode Indicator Function (MIF) and a stability diagram [21]. The function used was the multivariate MIF and is expressed as [24]

$$\min_{\|\{F(f)\}\|_2=1} (\{F(f)\}^T [H_{\Re}(f)]^T [H_{\Re}(f)] \{F(f)\}) = \lambda(f)$$ (17)

which yields a value $0 \leq \lambda(f) \leq 1$, where $[F(f)]$ is a force vector, $[H_{\Re}(f)]$ and $[H_{\Im}(f)]$ is the real part and imaginary part, respectively, of the system matrix $[H(f)]$ and $^T$ is the transpose operator. This minimization problem can be reformulated into an eigenvalue problem as [21]

$$[H_{\Re}(f)]^T [H_{\Re}(f)] \{F(f)\} = ([H_{\Re}(f)]^T [H_{\Re}(f)] + [H_{\Im}(f)]^T [H_{\Im}(f)]) \{F(f)\} \lambda(f)$$ (18)

where the smallest eigenvalue $\lambda(f)$ corresponds to the minimization problem in Eq. 17. Eq. 18 forms an eigenvalue problem of size $Q \times Q$, thus the problem yields the same number of solutions for each frequency as the number of sources. Plotting all solutions, repeated roots will be detected if the references excited those modes.

A stability diagram is constructed using estimates of systems poles and modal participation factors as a function of model order [21]. As the model order is increased, more and more modal frequencies are estimated but, hopefully, the estimates of the physical modal parameters will stabilize as the correct model order is found. From empirical evaluation of the stability diagram, the physical "true" poles seem to asymptotically go to the true values, whereas computational (nonphysical) poles which arise due to leakage, low signal to noise ratio (SNR), frequency shift etc, appear more unstructured [21]. Using the stability diagram with the multivariate MIF overlayed, stable poles which appear to have physical correspondence are selected. Along with the poles and a driving point, real or complex residues are estimated. Mode shapes were estimated using the frequency polyreference method [25].

As quality assessment of the estimated parameters the FRF's were synthesized using the estimated parameters and overlayed with the estimated FRF's. Furthermore the Modal Assurance Criterion (MAC) [18] defined by Eq. 19.

$$MAC_{kl} = \frac{(\{\psi\}_k^H \{\psi\}_l)^2}{\{\psi\}_k^H \{\psi\}_k \{\psi\}_l^H \{\psi\}_l^H}$$ (19)

was used as a measure of correlation between mode shape $\{\psi\}_k$ belonging to mode $k$, and mode shape $\{\psi\}_l$ belonging to mode $l$, where $^H$ is the Hermitian transpose operator.
2.3.2 Excitation Signal

For the experimental modal analysis, burst random was used as the excitation signal. Based on initial experiments concerning suitable burst length and frequency resolution (data segment time or data block length time), a burst length of 90% of the data block length time was selected, see Table 3. Coherence function estimates and magnitude functions of frequency response function estimates were utilized for the selection of burst length and frequency resolution. Basically, the frequency resolution was tuned to provide high overall coherence in the analysis bandwidth and the burst length was tuned to provide high coherence at resonance frequencies. In other words, the time of the dead period of the burst random signal was set so as to be sufficiently long enough to enable the structural response to decay in order to render influences from leakage negligible. The data block length was set so as to maintain a sufficient signal to noise ratio.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Burst Random</td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block Length $N$</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency Resolution $\Delta f$</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>200</td>
</tr>
<tr>
<td>Window</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Overlap</td>
<td>0%</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>0-4000 Hz</td>
</tr>
<tr>
<td>Burst Length</td>
<td>90%</td>
</tr>
</tbody>
</table>

Table 3: Spectral density estimation parameters.

Furthermore, four different excitation levels with the proportion \(\{1, 2, 3, 4\}\) were applied for each of the boundary conditions of the boring bars. By using a number of different excitation levels and carrying out system identification for each of the excitation levels, differences between the estimates of the system may indicate nonlinear behavior of the system and might provide information concerning the structure of the nonlinearity or the nonlinearities involved.
2.4 Analytical Models of the Boring Bars

The boring bar has a cross section $A(z)$ and a length of $l$. Also associated with the beam is a flexural (bending) stiffness $EI(z)$, where $E$ is Young’s elastic modulus for the beam and $I(x)$ is the cross-sectional area moment of inertia about the "z axis." From mechanic theory, the beam sustains a bending moment $M(z,t)$, which is related to the beam deflection, or bending deformation $u(z,t)$, by the conservation of momentum. This can be derived as follows [26]: when the beam deflects, see Fig. 9, the longitudinal displacement

\[
\frac{d\theta(z,t)}{dz} \delta z
\]

Figure 9: Deflection model of a beam undergoing pure bending.

\[\theta(z,y,t)\] will cause a strain (illustrated by the arrow-field in the figure) due to the rotation of the cross-sectional plane which is parallel with $s$ in Fig. 9. Hooke’s law states that the uniaxial stress $\sigma$ (or axial force per unit sectional area) applied to a bar in the $z$ direction is proportional to the strain $\varepsilon$ (or elongation per unit length) within the elastic limit according to

\[E = \frac{\sigma}{\varepsilon}\]  \hspace{1cm} (20)

Assuming that the cross-section-plane remains flat after deformation; then the relation of strain $\varepsilon$ and stress $\sigma$ is given by

\[\varepsilon_{zz}(z,y,t) = \frac{\partial \theta(z,y,t)}{\partial z} = -y \frac{\partial^2 u(z,t)}{\partial z^2}\] \hspace{1cm} (21)

\[\sigma_{zz}(z,y,t) = -yE \frac{\partial^2 u(z,t)}{\partial z^2}\] \hspace{1cm} (22)
The total moment about an axis parallel with the x-axis in the mid-section, where strain is zero, will cause to stress become

\[ M_x(z, t) = \int \int_{y(z) x(z)} \sigma_{zz}(x, y, t) y \, dx \, dy \]  

(23)

using the relation in Eq. 22, we get

\[ M_x(z, t) = -E \frac{\partial^2 u(z, t)}{\partial z^2} \int \int_{y(z) x(z)} y^2 \, dx \, dy \]  

(24)

where

\[ I_x(z) = \int \int_{y(z) x(z)} y^2 \, dx \, dy \]  

(25)

and we get

\[ M_x(z, t) = -EI_x(z) \frac{\partial^2 u(z, t)}{\partial z^2} \]  

(26)

A model of bending vibration may be derived by examining the force diagram of an infinitesimal element of the beam [27] as indicated in Fig. 10. Assuming the deformation is small enough so that the shear deformation is much smaller than displacement \( u(z, t) \) (i.e., so that the sides of the element \( dz \) do not bend), a summation of forces in the \( y \) direction yields

\[ V(z, t) + \frac{\partial V(z, t)}{\partial z} \, dz - V(z, t) + f(z, t) \, dz = \rho A(z) \, dz \frac{\partial^2 u(z, t)}{\partial t^2} \]  

(27)
Here $V(z,t)$ is the shear force at the left end of the element $dz$, $V(z,t) + \frac{\partial V(z,t)}{\partial z}dz$ is the shear force at the right end of the element $dz$, $f(z,t)$ is the total external force applied to the element per unit length, and the term on the right side of the equality sign is the inertial force of the element. The assumption of small shear deformation used in the force balance is true if length divided by the smallest radius of the beam is less than 10 (i.e., for long slender beams). Next, the moments acting on the element $dz$ about the $x$ axis through point $Q$ are summed. This yields

$$M_x(z,t) - \left[ M_x(z,t) + \frac{\partial M_x(z,t)}{\partial z}dz \right] + \left[ V(z,t) + \frac{\partial V(z,t)}{\partial z}dz \right] dz + \left[ f(z,t)dz \right] (dz)^2 = 0$$

(28)

Here, the left-hand side of the equation is zero since the rotary inertia of element $dz$ is assumed to be negligible, simplifying the expression yields

$$\left[ V(z,t) - \frac{\partial M_x(z,t)}{\partial z} \right] dz + \left[ \frac{\partial V(z,t)}{\partial z} + \frac{f(z,t)}{2} \right] (dz)^2 = 0$$

(29)

Since $dz$ is assumed to be very small, $(dz)^2$ is assumed to be almost zero, thus the moment expression becomes

$$V(z,t) = \frac{\partial M_x(z,t)}{\partial z}$$

(30)

This states that the shear force is proportional to the spatial change in the bending moment. Substitution of this expression for the shear force into Eq. 27 yields

$$\frac{\partial^2}{\partial z^2} M_x(z,t) dz + f(z,t)dz = \rho A(z)dz \frac{\partial^2 u(z,t)}{\partial t^2}$$

(31)

Further substitution of Eq. 26 into Eq. 31 and dividing by $dz$ yields

$$\rho A(z)dz \frac{\partial^2 u(z,t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left[ EI(z) \frac{\partial^2 u(z,t)}{\partial z^2} \right] = f(z,t)dz$$

(32)

Eq. 32 is often referred to as the Euler-Bernoulli beam equation. The assumptions regarding the beam, used in formulating this model are:
Analysis of Dynamic Properties of Boring Bars
Concerning Different Clamping Conditions

- Uniform along its span, or length, and slender (diameter to length ratio > 10).
- Composed of a linear, homogenous, isotropic elastic material, without axial loads.
- Plane section remains plane.
- The plane of symmetry of the beam is also the plane of vibration, so rotation and translation are decoupled.
- Rotary inertia and shear deformation can be neglected.

Assuming that the cross-sectional area is constant \( A(z) = A \), the beam equation can be rewritten as

\[
\frac{\partial^2 u(z, t)}{\partial t^2} + c^2 \frac{\partial^4 u(z, t)}{\partial z^4} = 0, \quad c = \sqrt{\frac{EI}{\rho A}}
\]  

(33)

The solution for Eq. 33 is subjected to four boundary conditions and two initial conditions, however, in order to calculate the resonance frequencies and mode shapes we only need the boundary conditions. A separation-of-variables solution of the form \( u(z, t) = u(z)u(t) \) is assumed, thus the equation of motion to yields

\[
c^2 \frac{\partial^4 u(z)}{\partial z^4} \frac{1}{u(z)} = -\frac{\partial^2 u(t)}{\partial t^2} \frac{1}{u(t)} = (2\pi f)^2
\]  

(34)

The spatial equation results from rearranging Eq. 34, which yields

\[
\frac{\partial^4 u(z)}{\partial z^4} - \left( \frac{2\pi f}{c} \right)^2 u(z) = 0
\]  

(35)

By defining

\[
\beta^4 = \left( \frac{2\pi f}{c} \right)^2 = \frac{\rho A(2\pi f)^2}{EI}
\]  

(36)

the general solution of Eq. 35 can be written as [27]

\[
u(z) = a_1 \cos \beta z + b_1 \sin \beta z + c_1 \cosh \beta z + d_1 \sinh \beta z
\]  

(37)

where \( a_1, b_1, c_1 \) and \( d_1 \) are constants of integration determined by the boundary conditions. The general solution produces infinity solution for \( \beta \) (an infinite number of resonance frequencies and mode shapes), we denote each
solution as $\beta_r$, belonging to mode $r$. The equation for calculating the natural frequency is [27]

$$f_r = \frac{\beta_r^2}{2\pi} \sqrt{\frac{EI}{\rho A}}$$

(38)

The boundary conditions required in order to solve the spatial equation from the separation-of-variables solution of Eq. 33 are obtained by examining the deflection $u(z, t)$, the slope of the deflection $\partial u(z, t)/\partial z$, the bending moment $EI\partial^2 u(z, t)/\partial z^2$ and the shear force $EI\partial^3 u(z, t)/\partial z^3$ at all boundaries. From examining these boundary conditions, four equations should be found and may be written in matrix form as:

$$[C] \{a\} = \{0\}$$

(39)

where $\{a\} = \{ a_1 \ b_1 \ c_1 \ d_1 \}^T$ is the vector with unknown constants of integration and $[C]$ the coefficient matrix determined from the boundary conditions. By equating the determinant of the coefficient matrix to zero, the characteristic equation and the eigenfrequencies may be found [27]. For each eigenfrequency $f_r$, three of the four unknown constants of integration can be found or expressed in terms of the fourth. This is sufficient in order determine the mode shape $\{\Psi_r\}$. The fourth constant is found using the initial conditions and determines the participation of each mode in temporal solution. However, since we only consider the dynamic properties of the system is considered, the mode shape $\{\Psi_r\}$ is normalized.

### 2.4.1 Multi-span beam

The previous discussion concerned a beam with constant cross-section properties and boundary conditions at each end. In order to apply Euler-Bernoulli modeling to more complex beam structures with boundary conditions along the beam at discrete points and/or beam segments with different properties, the beam may be divided into several sub-beams, also referred to as a multi-span beam [28]. Each sub-beam will have the same general solution as the single span beam in Eq. 37, thus the mode shape for each sub-beam may be expressed as

$$u_j(z) = a_j \cos \beta(z - z_j) + b_j \sin \beta(z - z_j) +
+ c_j \cosh \beta(z - z_j) + d_j \sinh \beta(z - z_j)$$

(40)
where \( j \) is the sub-beam number, \( J \) the number of sub-beams and \( z_j \) the local coordinate offset. The local coordinate is expressed as

\[
z_j = \sum_{k=1}^{j-1} l_k
\]

where \( l_k \) is the length of section \( 1 \leq k \leq J \). The equation system will now consist of 4 times \( J \) coupled equations. Thus the coefficient matrix \([C]\) will have the size \( 4J \times 4J \) and the vector \( \{a\} \) the size \( 4J \times 1 \). The eigenfrequencies and mode shapes are found in the same way as for the single span beam, i.e. by first finding the solutions to the characteristic equation and then the corresponding eigenvectors.

### 2.4.2 Linearized Model

The simplest and most straightforward model of a boring bar is the Euler-Bernoulli model, which consists of a homogenous single span beam with constant cross-sectional area \( A(z) = A \) and constant cross-sectional moment of inertia \( I(z) = I \). The beam has four boundary conditions, two at each end. One end is clamped and the other is free, see Fig. 11.

![Figure 11: Model of a Clamped - Free beam, where \( E \) is the elasticity modulus (Young’s coefficient), \( \rho \) the density, \( A \) the cross-sectional area, \( I \) the moment of inertia and \( l \) the length of the beam.](image)

The clamped side of the beam will be fixated, thus the displacement and the slope of the displacement in this point \( z = 0 \) will equal zero and the two first boundary conditions become

\[
\left. u(z,t) \right|_{z=0} = 0
\]

\[
\left. \frac{\partial u(z,t)}{\partial z} \right|_{z=0} = 0
\]

The other end is free, thus no bending moment or shear force constrains the beam at the coordinate \( z = l \) when the beam vibrates, this yielding the other
two boundary condition as

\[
EI \frac{\partial^2 u(z,t)}{\partial z^2} \bigg|_{z=l} = 0 \tag{44}
\]

\[
EI \frac{\partial^3 u(z,t)}{\partial z^3} \bigg|_{z=l} = 0 \tag{45}
\]

The general solution for \( u(z) \) is then combined with the boundary condition, which yields

\[
u(z) \bigg|_{z=0} = a_1 + c_1 = 0 \tag{46}
\]

\[
\frac{du(z)}{dz} \bigg|_{z=0} = b_1 + d_1 = 0 \tag{47}
\]

\[
EI \frac{d^2 u(z)}{dz^2} \bigg|_{z=l} = EI \left( -a_1 \beta^2 \cos \beta l + b_1 \beta^2 \sin \beta l + c_1 \beta^2 \cosh \beta l + d_1 \beta^2 \sinh \beta l \right) = 0 \tag{48}
\]

\[
EI \frac{d^3 u(z)}{dz^3} \bigg|_{z=l} = EI \left( a_1 \beta^3 \sin \beta l - b_1 \beta^3 \cos \beta l + c_1 \beta^3 \sinh \beta l + d_1 \beta^3 \cosh \beta l \right) = 0 \tag{49}
\]

and in matrix form

\[
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
-\beta^2 \cos \beta l & -\beta^2 \sin \beta l & \beta^2 \cosh \beta l & \beta^2 \sinh \beta l \\
\beta^3 \sin \beta l & -\beta^3 \cos \beta l & \beta^3 \sinh \beta l & \beta^3 \cosh \beta l
\end{bmatrix}
\begin{bmatrix}
a_1 \\
b_1 \\
c_1 \\
d_1
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} \tag{50}
\]

Setting the determinant of the coefficient matrix equal to zero, i.e. \( \det([C]) = 0 \), yields the characteristic equation as

\[
2\beta^5 \cos \beta l \cosh \beta l + \beta^5 \cos^2 \beta l + \beta^5 \sin^2 \beta l + \beta^5 \cosh^2 \beta l - \beta^5 \sinh^2 \beta l = 0 \tag{51}
\]

since \( \cos^2 x + \sin^2 x = 1 \) and \( \cosh^2 x - \sinh^2 x = 1 \). Eq. 51 can be simplified into

\[
\cos \beta l \cosh \beta l + 1 = 0 \tag{52}
\]

and \( \beta_r \) is determined by finding the roots of this equation.

### 2.4.3 Multi-span Boring bar with Elastic Foundation

The boring bar was clamped with either two screws on the top and two on the underside or three screws on the top and three on the underside. In addition,
two different bolts were used: M8 and M10. If we consider the clamping house to be a rigid body, and the screws to be rigid in the transverse direction, a number of boundary conditions are yielded, i.e. approximated as pinned. The pinned boundary condition assumes an infinitely stiff spring in the transverse direction but no rotational stiffness. Letting the screws assume more realistic properties as deformable bodies will yield "elastic supports" [29] as a boundary condition, instead of the pinned condition. The elastic support can be seen as two springs in one point, with one spring in the transverse direction; thus, transverse stiffness resistance and one rotational spring exhibits rotational stiffness resistance. The configurations of the "elastic support" condition are presented in Fig. 12.

Figure 12: a) A model of a three span beam with elastic support, b) a model of a four span beam with elastic support, where \( E \) is the elasticity modulus (Young's coefficient), \( \rho \) the density, \( A \) the cross-sectional area, \( I \) the moment of inertia, \( k_T \) the transverse spring coefficient, \( k_R \) the rotational spring coefficient the length of the different spans in mm are \( l_1 = 35 \), \( l_2 = 50 \), \( l_3 = 215 \) and \( l_4 = 25 \).

Two types of boundary conditions may be categorized from the models presented in Fig. 12, where \( \text{z}_{\text{pos}} \) denotes the position of the boundary condi-
One is the "free" boundary condition, previously expressed as

\[
EI \left. \frac{\partial^2 u(z,t)}{\partial z^2} \right|_{z=z_{\text{pos}}} = 0 \tag{53}
\]
\[
EI \left. \frac{\partial^3 u(z,t)}{\partial z^3} \right|_{z=z_{\text{pos}}} = 0 \tag{54}
\]

where there is no bending or shear forces present. The other boundary conditions derive from the "elastic support" condition and may be expressed as [29]

\[
EI \left. \frac{\partial^2 u(z,t)}{\partial z^2} \right|_{z=z_{\text{pos}}} = -k_R \left. \frac{\partial u(z,t)}{\partial z} \right|_{z=z_{\text{pos}}} \tag{55}
\]
\[
EI \left. \frac{\partial^3 u(z,t)}{\partial z^3} \right|_{z=z_{\text{pos}}} = k_T u(z,t) \tag{56}
\]

where the transverse spring produces a transverse force proportional to the displacement, and the rotational spring produces a bending moment proportional to the slope. However, if we let the rotational spring coefficient equal zero \( k_R = 0 \), and the transverse spring coefficient go to infinity \( k_T = \infty \), we will have a third boundary condition termed "pinned". The pinned boundary condition can be expressed as

\[
\left. u(z,t) \right|_{z=z_{\text{pos}}} = 0 \tag{57}
\]
\[
\left. EI \frac{\partial^2 u(z,t)}{\partial z^2} \right|_{z=z_{\text{pos}}} = 0 \tag{58}
\]

The coefficient matrix may now be formulated in order to find the characteristic equations for the different models. The three span model will yield a 12x12 coefficient matrix and the four span model will yield a 16x16 coefficient matrix. Calculating the determinate of these matrices by hand is fairly time-consuming and the roots of the characteristic equation are often not possible to express explicitly [30,31]. The solutions produced using these models were found using Matlab and by numerically finding the roots for the characteristic equations. The boundary conditions for the four models, (three span model with rotational and transverse springs; three span model without rotational spring and infinitely stiff transverse spring; four span model with rotational and transverse springs and four span model without rotational spring and infinitely stiff transverse spring) are presented in appendix A.
2.4.4 Screws - Elastic Foundation

The boring bar was clamped using screws. Since the screws are not rigid bodies they may, for example, be modeled as flexible bodies using springs. A model based on a transverse spring and a rotational spring was assumed. Thus, the two different stiffness coefficients may be calculated using very simplified models of what is going on whilst clamping the boring bar. Hence, one of the coefficients corresponds to a transverse spring and the second coefficient corresponds to a rotational spring. The screws’ end surfaces, which are in contact with the boring bar, apply pressure to the bar. This screw pressure on the boring bar is related to the screws’ tightening torque.

The screws used to clamp the boring bar were of type MC6S norm "DIN 912, ISO 4762", presented in Fig. 13. The screws are zinc-plated, steel socket, head cap screws with the strength class 8.8, with a tensile yield strength of $R_{p02} = 660 MPa$. Two various sizes were used: first M8 and then M10; see Table 4 for dimensions.

The stiffness constants of the screws were calculated by modeling the screws as a beam rigidly clamped at one end, and free at the other. The beam was considered to be homogenous, having a constant cross-sectional area $A$, a constant cross-sectional moment of inertia $I$ and a length of $l$. When a screw is threaded in the clamping house and is clamping the boring bar, a part of the screw’s tip will not be in contact with the clamping house; thus yielding both transverse, lateral and bending elasticity. This is due to the fact that the inside of the clamping house is circular with a radius of 40 mm plus tolerance and the boring bar has a thickness of 37 mm, plus tolerance.
Table 4: Dimensions of the ISO 4762/DIN 912 socket head cap screw

where the boring bar is clamped. Two different lengths of the screw beam models were selected. The shortest length represents rigid clamping of the screws within the clamping house and the longer length represents the case of flexible clamping of the screw by the clamping house thread.

The screw clamping model is presented in Fig. 14, where a) shows the clamping configuration, b) illustrates the beam model of transverse vibrations and the transverse spring coefficient, and c) illustrates the beam model of the rotational spring coefficient.

The spring coefficients from a beam with one end fixed and the other end subjected to axial (vertical) loading and bending moment are the transverse spring constant $k_T$ and rotational spring constant $k_R$, respectively. These constants are calculated from beam bending theory \[27,32\] as

$$k_T = \frac{EA}{l} \quad (59)$$

$$k_R = \frac{EI}{l} \quad (60)$$

where $E$ is the elasticities modulus, cross-sectional area $A$, cross-sectional moment of inertia $I$, and $l$ the length of the beam model. These spring constants are used in the elastic support models.

Other important factors which affect clamping conditions include the coupling properties between the screws and the boring bar, and coupling properties between the screws and the clamping house. The force applied by the
Figure 14: a) Sketch illustrating screw clamping of the boring bar, via the clamping house, b) the transverse stiffness model, and c) the rotational stiffness model

clamp screws in the axial direction of the screws on the boring bar surface may be related to the tightening torque $M$ of the screws, and is identical to the prestressing force $F_p$. The tightening torque can be divided into two parts, one moment $M_T$ (arising due to friction in the contact surfaces between the threads and the geometric relation from the prestressing force) and a second moment, $M_C$ (due to the friction in the contact surface between the screw and boring bar). The moment due to the threads may be expressed as [33]

$$M_T = F_p r_m \tan(\varphi + \varepsilon)$$  \hspace{1cm} (61)

where $r_m$ is the average radius of the screw equal to $d/2$ in Table 4, $\varepsilon$ is the angle of friction force component and $\varphi$ is the pitch angle. The moment $M_C$, due to the friction between the boring bar and the screw, may be expressed as

$$M_C = F_p \mu_c r_c$$  \hspace{1cm} (62)

where $\mu_c$ is friction coefficient between the surfaces, and $r_c$ is the average radius of the contact surface [33], with regard to a contact surface with an inner and an outer circle. Then the average radius of contact equals

$$r_c = \frac{r_m}{2}$$  \hspace{1cm} (63)

thus, the total moment $M$ equals [33]

$$M = M_T + M_C = F_p r_m \tan(\varphi + \varepsilon) + F_p \mu_c r_c$$  \hspace{1cm} (64)
Thus, the expression for the force $F_p$ (which the screw is enacts upon the boring bar when applying the moment $M$) may be expressed as

$$F_p = \frac{M}{r_m \tan(\varphi + \varepsilon) + \mu_c r_c} \quad (65)$$

The pitch angle $\varphi$ is calculated from the geometry of the screw as

$$\tan \varphi = \frac{P}{\pi d_m} \quad (66)$$

where $P$ is the pitch and $d_m$ is the average diameter of the screw, equal to $d_2$ in Table 4. The angle of the friction force component $\varepsilon$ is related to the pitch angle $\varphi$ and friction in the thread as

$$\tan \varepsilon = \frac{\mu_T}{\cos \frac{\beta}{2}} \quad (67)$$

where $\mu_T$ is the friction coefficient between the surfaces of the inner and outer threads, (i.e. the threads in the clamping house and the threads of the screws) and $\beta$ is the profile angle, equal to $60^\circ$ from the ISO-standard presented in Fig. 13.

### 2.4.5 Spring Coefficients and Clamping Forces

Materials and properties are required in order to model the screws as the transversal spring and the rotational spring (both previously presented). The screws’ material is steel (zinc-plated). Whilst the exact type of steel is not specified, this is in general not relevant. Types of steel vary for different screw suppliers; usually, more important is the tensile yield strength. However, according to the supplier of the screws used in this experiment, the elasticity modulus $E$ is approximately $200 \cdot 10^9$ N/m$^2$. The dimensions are specified by the standard and are presented in Table 4. The tensile stress area $A$, according to the standard ISO 724, is given by

$$A = \frac{\pi}{4} \left( \frac{d_2 + d_1 - H}{2} \right)^2 \quad (68)$$

and the moment of inertia $I$ was calculated as

$$I = \frac{\pi}{4} \left( \frac{d_2 + d_1 - H}{4} \right)^4 \quad (69)$$
where \( d_1, d_2 \) and \( H \) are given in Table 4. The length \( l \) is the length of the "overhang" of the screw; an example was used in which the overhang length was considered to be 1.5 mm. This length was selected, because the circular inner diameter of the clamping space of the clamping hose is 40 mm and the height of the boring bar is 37 mm, thus resulting in a space from each side of the boring bar to the circular clamping house boundary of 1.5 mm. Furthermore, since the boring bar is clamped with screws from both the upper-side and the underside at the same positions along the \( z \)-axis, each axial spring coefficient includes the longitudinal stiffness from two screws, i.e. two springs in parallel.

The calculated spring coefficients and the spring parameters used in the spring models are presented in Table 5 together with the dimensions and elasticity modulus.

Table 5: The longitudinal and rotational spring coefficients and the spring parameters used in the spring models.

<table>
<thead>
<tr>
<th>Size</th>
<th>( A ) [m(^2)]</th>
<th>( I ) [m(^4)]</th>
<th>( E ) [N/m(^2)]</th>
<th>( l ) [m]</th>
<th>( k_T ) [N/m]</th>
<th>( k_R ) [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M8</td>
<td>( 3.661 \cdot 10^{-5} )</td>
<td>( 1.067 \cdot 10^{-19} )</td>
<td>( 200 \cdot 10^9 )</td>
<td>( 1.5 \cdot 10^{-3} )</td>
<td>( 4.881 \cdot 10^4 )</td>
<td>( 1.422 \cdot 10^4 )</td>
</tr>
<tr>
<td>M10</td>
<td>( 5.799 \cdot 10^{-5} )</td>
<td>( 2.676 \cdot 10^{-19} )</td>
<td>( 200 \cdot 10^9 )</td>
<td>( 1.5 \cdot 10^{-3} )</td>
<td>( 7.732 \cdot 10^4 )</td>
<td>( 3.568 \cdot 10^4 )</td>
</tr>
</tbody>
</table>

Estimates of the forces applied by the clamp screws on the boring bar were calculated using the torque-force relation presented in section 2.4.4, and are presented in Table 6. The screw force was calculated using the dimension previously presented in Table 4, assuming identical friction coefficients for the contact surface between the boring bar and the screws, and also the contact surface between the threads in the clamping house and the screw threads, i.e. \( \mu_c = 0.14 \) and \( \mu_T = 0.14 \).

Table 6: The clamp screw torque-force relation based on the presented clamp screw model.
2.5 Nonlinear Model

A linear model may not always be sufficient to explain different results from the experimental data. Nonlinearities may be caused by several different factors. A common source of nonlinearity is the contact problem, in which elements of the system come into contact with the surrounding environment due to "large" displacement, which, in turn, create a new set of boundary conditions. Another form of the contact problem is friction in joints, or sliding surfaces. This problem also involves large forces and/or deformation which may cause the properties of the material to behave in a nonlinear manner.

All of these nonlinearities are likely to exist to some extent in the boring bar - clamping house system, perhaps some of them more than others. The question is if, or to what extent they influence the dynamic behavior of the boring bar and how they can be determined (if relevant). In these simulations, nonlinearities regarding stiffness was examined, a so-called "softening spring" [18,34,35]. This nonlinearity was investigated as empirical data has shown that the fundamental boring bar resonance frequencies display a tendency to move towards lower frequencies as a result of increasing excitation force level.

The softening spring may be modeled in two different ways, yielding different properties with respect to the displacement. The first of these models yields a force proportional to a nonlinear stiffness coefficient, multiplied by the displacement, squared with sign. The equation describing a SDOF system with this type of a softening spring nonlinearity is given by [36]

\[ m \frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) - k_s x|\frac{d}{dt}x| = f(t) \]  \hspace{1cm} (70)

where \( m, c \) and \( k \) are the mass, damping and stiffness coefficients of the underlying linear system, \( x(t) \) - the displacement, \( f(t) \) - the force, and \( k_s \) the nonlinear stiffness coefficient. The second model yields a force proportional to a nonlinear stiffness coefficient multiplied by the displacement cubed, and inserted into the equation of motion describing a SDOF system results in [36]

\[ m \frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) - k_c x^3(t) = f(t) \]  \hspace{1cm} (71)

where \( k_c \) is the nonlinear stiffness coefficient.

In order to see if any of the proposed nonlinearities may explain the different results from the experimental modal analysis, a number of different simulations were carried out using different parameters.
2.5.1 Nonlinear Synthesis

There are different ways of simulating linear and nonlinear systems: Probably the most common method to solve ordinary differential equations (ODE) is the Runge-Kutta method, implemented in Matlab [37]. Another method is the digital filter method [38].

There are multiple advantages with using ODE solvers: firstly, they are rather straightforward to use and secondly they are well known. The disadvantage, however, is that they are relatively time consuming if large amounts of data are involved. The filter method, on the other hand, is significantly faster than the ODE solvers [38]. But is not as well documented as the ODE solvers with regarding to, for example, accuracy and the ability to handle nonlinear systems [39]. However, for linear systems, the limitations of the filter method are known [40] and depends on the sampling frequency and the transformation method used to convert the continuous time parameters to discrete time parameters [40].

2.5.2 Ordinary Differential Equation Methods

The simulation method used for simulating the nonlinear system is based on explicit Runge-Kutta of order (4,5) formula, the Dormand-Prince pair [37,41], referred to as ode45 in Matlab. The ode45 method combines a fourth order method and a fifth order method, both of which are similar to the classical fourth order Runge-Kutta [41, 42]. The numerical technique solves ordinary differential equations of the form

\[
\frac{dx(t)}{dt} = f(x(t), t), \quad x(t_0) = x_0 \tag{72}
\]

The Runge-Kutta 4th order method is based on the following expressions

\[
x(t + \Delta t) = x(t) + \left( a_1 k_1(x(t), t) + a_2 k_2(x(t), t) + a_3 k_3(x(t), t) + a_4 k_4(x(t), t) \right) \Delta t \tag{73}
\]

where \( \Delta t \) is the step size, \( \{a_1, \cdots, a_4\} \) and \( \{k_1(x(t), t), \cdots, k_4(x(t), t)\} \) are constants and functions respectively, determined based on the first five terms of the Taylor series [42]:

\[
x(t + \Delta t) = x(t) + f(x(t), t) \Delta t + \frac{1}{2!} f'(x(t), t) \Delta t^2 + \frac{1}{3!} f''(x(t), t) \Delta t^3 + \frac{1}{4!} f'''(x(t), t) \Delta t^4 \tag{74}
\]
Rewriting Eq. 74 into Eq. 73 yields [42]

\[ x(t + \Delta t) = x(t) + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \Delta t \]  

(75)

where

\[ k_1(x(t), t) = f(x(t), t) \]  

(76)

\[ k_2(x(t), t) = f \left( x(t) + \frac{\Delta t}{2} k_1, t + \frac{\Delta t}{2} \right) \]  

(77)

\[ k_3(x(t), t) = f \left( x(t) + \frac{\Delta t}{2} k_2, t + \frac{\Delta t}{2} \right) \]  

(78)

\[ k_4(x(t), t) = f \left( x(t) + \Delta t k_3, t + \Delta t \right) \]  

(79)

The Runge-Kutta method only solves first order differential equations, thus this requires that the second order differential equations in Eqs. 70 and 71 are rewritten to coupled first order differential equations as in Eqs. 82 and 83.

The nonlinear models simulated with the differential equation solvers were based on the softening spring using the quadratic model in Eq. 80, and the cubed model in Eq. 81

\[ g_q(x(t)) = k_s x| x(t) \]  

(80)

\[ g_c(x(t)) = k_c x^3(t) \]  

(81)

where \( g_q(x(t)) \) and \( g_c(x(t)) \) replaces \( g(x_1(t)) \) in Eq. 83 for respective model.

\[ \frac{dx_1(t)}{dt} = x_2(t) \]  

(82)

\[ m \frac{dx_2(t)}{dt} = -cx_2(t) - kx_1(t) + g(x_1(t)) + f(t) \]  

(83)

where \( x(t) \) is the response of the system, and \( f(t) \) is the driving force.

### 2.5.3 Filter Method

The filter method is a time-discrete method for extracting digital filter coefficients from the analog system using an appropriate transformation method. Thus, the differential equation is transformed into a difference equation, represented by a digital filter [38]. The continuous time filtering may be expressed
in terms of a convolution integral as [43]

\[ x(t) = \int_{\tau=-\infty}^{\infty} h(\tau)f(t-\tau)d\tau \tag{84} \]

where \( x(t) \) is the response or output signal and \( f(t) \) is the input to the system with impulse response \( h(t) \). The corresponding filtering procedure in the discrete time domain is given by

\[ x(n) = \sum_{k=-\infty}^{\infty} h(k)f(n-k) \tag{85} \]

where, again, \( x(n) \) is the response and \( f(n) \) is the input to the system with the impulse response \( h(n) \), but in discrete time domain.

The transformation may be performed by first dividing the total system (multiple degree of freedom) into subsystems using the modal superposition theorem and transforming each subsystems parameter into the filter coefficients. The frequency response function for a dynamic system may be expressed in terms of modal superposition as [18]

\[ H(f) = \sum_{r=1}^{R} \frac{A_r}{j2\pi f - \lambda_r} + \frac{A_r^*}{j2\pi f - \lambda_r^*}, \tag{86} \]

where \( R \) is the number of modes, \( A_r \) is the system’s residues belonging to mode \( r \), and \( \lambda_r \) is the pole belonging to mode \( r \). The poles and residues may be extracted from a lumped parameter system, a Finite Element Model, distributed parameter system, or estimated from experimental modal analysis [27]. Another approach is to directly express the system as in Eq. 87 and transform the analog filter coefficients into digital filter coefficients. The frequency function for an analog filter can be expressed as [43]

\[ H(s) = \frac{D(s)}{C(s)} = \frac{d_0 + d_1s + \ldots + d_Ms^M}{1 + c_1s + \ldots + c_Ks^K} \tag{87} \]

where \( M \) is the order of the polynomial \( D(s) \) in the numerator, and \( K \) is the order of the polynomial \( C(s) \) in the denominator. Transforming the analog filter yields a digital filter whose \( z \)-transform may be expressed as [43]

\[ H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1z^{-1} + \ldots + b_Mz^{-M}}{1 + a_1z^{-1} + \ldots + a_Kz^{-K}} \tag{88} \]
where $M$ is the order of the polynomial $B(z)$ in the numerator, and $K$ is the order of the polynomial $A(z)$ in the denominator. In the discrete time domain, the difference equation describing the filter may be written as [43]

$$x(n) = \sum_{m=0}^{M} b_m f(n - m) - \sum_{k=1}^{K} a_k x(n - k)$$  \hspace{1cm} (89)$$

One of the most common transformation methods is the so-called "impulse invariant" method, which allows the digital signal to represent the analog signal by an impulse at sampled intervals, i.e. $x(t) \rightarrow Tx(nT)$, where $T$ is sampling period [44]. Other methods include the step invariant, ramp invariant, centered step invariant, cubic spline invariant and Lagrange method; each with different properties [40]. The ramp invariant method was used in my simulations. This transform method introduces zero error at DC, low error at Nyquist frequency and low phase distortion [40]. However, the ramp invariant method introduces a large error at a resonance frequency than, for example, the impulse, step and centered step invariant methods [40]. However, the error introduced in the area of a resonance frequency only becomes large as the resonance frequency approaches the Nyquist frequency. Therefore, an over-sampling of 20 times the highest resonance frequency was used in the simulations.

The filter method for simulation of nonlinear systems is performed by using the digital filter coefficient for the linear system, and finding the solutions for the nonlinear difference equation. A nonlinear, single degree of freedom system may be described by differential equation

$$m \frac{d^2 x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) + g(x(t)) = f(t)$$  \hspace{1cm} (90)$$

the corresponding equation in time discrete domain may be written using a t difference equation as [38]

$$x(n) = \sum_{m=0}^{M} b_m (x(n - m) - g(x(n - m))) - \sum_{k=1}^{K} a_k f(n - k)$$  \hspace{1cm} (91)$$

Since Eq. 91 contains nonlinear terms, several solutions may exist for $x(n)$ [42]. The value of $x(n)$ may be found using any of the zero searching algorithms such as the secant method, bisection method or Newton-Raphson [42] (which was used in this synthesis).
The models with a nonlinear function $g(x(n))$, simulated with the filter method were based on both the quadratic model in Eq. 80, and the cubed model in Eq. 81, both representing the softening spring.

$$g_{\text{Lin}}(n) - g_s(x(n)) = f(n), \text{ where } g_s(x(n)) = k_s |x(n)|$$  \hspace{1cm} (92)

$$g_{\text{Lin}}(n) - g_c(x(n)) = f(n), \text{ where } g_c(x(n)) = k_c x^3(n)$$  \hspace{1cm} (93)

The digital filter coefficients were based on the poles and residues estimated from experimental measured data.

### 2.5.4 Excitation Signal

True random was selected for the excitation signal so that the resolution and number of averages may be alter after the simulation results were produced. The estimation parameters used in the nonlinear simulations are presented in Table 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>True Random</td>
</tr>
<tr>
<td>Sampling Frequency $f_s$</td>
<td>$10000 , Hz$</td>
</tr>
<tr>
<td>Block Length $N$</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency Resolution $\Delta f$</td>
<td>$0.5 , Hz$</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>800</td>
</tr>
<tr>
<td>Window</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>-</td>
</tr>
<tr>
<td>Burst Length</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7: Spectral density estimation parameters and excitation signal used in the simulated nonlinear system.

### 3 Results

Firstly, this section presents results from the experimental modal analysis of boring bars, under different clamping conditions and excitation force levels. The results presented constitute a small part of an extensive investigation of the dynamic properties of boring bars for various configurations and setups;
however they represent the essence of the experimental results. Secondly, this section present results based on analytical Euler-Bernoulli models of boring bars with different spans, and simple models of boring bar clamping. Finally, results from simulations of the nonlinear models are given.

3.1 Experimental Modal Analysis

Shaker excitation was used for the experimental modal analysis of the boring bars. The utilized spectrum estimation parameters and excitation signals' properties are given in Table 3. A number of different phenomena were observed during the experimental modal analysis of the boring bars for various configurations and setups. For instance, large variations were observed in the fundamental bending resonance frequencies of the boring bar for different tightening torques of the clamp screws. Also, the order in which the clamp screws were tightened (first from the upper side of the boring bar or first from the under side of the boring bar) had a significant impact on, for example, the fundamental bending resonance frequencies. Fig. 15 illustrates typical frequency response function estimates based on the same measurement locations for input force and output response at the boring bar. These frequency response function estimates are produced using different clamp screw tightening torques and/or a different excitation level.

Based on the frequency function estimates, the modal parameters are then estimated using the poly-reference technique. A frequency range covering the significant part near the resonance frequencies was selected, i.e. ±100 to ±200 Hz around the resonance peaks. A Multivariate Indicator Function (MIF) was produced from all the driving point data and then overlayed on top of a stability diagram. The stability diagram consist of poles calculated using different model orders up to a given order (see Fig. 16 for a typical stability diagram with the corresponding indicator functions). When stable poles corresponding to the modes of interest have been selected, residues are estimated using the driving points. Finally, the mode shapes are estimated using all the measured FRF:s. In order to check whether the estimated parameters are functional or not, they are used to synthesize a number of FRF:s, among those are the driving points. Furthermore, orthogonality of the mass scaled mode shapes are checked using the MAC matrix. If any of the checks indicate on nonfunctional estimates, the maximum model order used to create the stability diagram is changed and new stable poles are selected. Also erroneous or strange FRF:s may be disregarded when the different models are calculated. These steps are
Figure 15: a) The acceleration of the boring bar driving point response in the direction of cutting speed (y-), using the standard boring bar, four screws of size M8 and tightened first from the top using five different tightening torques and four different excitation levels. b) The corresponding estimates zoomed in around the first resonance frequencies.
Figure 16: Typical stability diagram for experiments conducted on the boring bar. In order to identify the location and number of poles, two indicator functions are overlayed on the stability diagram.
performed until acceptable results are achieved, or until (what seems to be when) the best possible results given the data are achieved.

Results from six various setups, described by Table 8 are presented.

<table>
<thead>
<tr>
<th>Setup Number</th>
<th>Configuration</th>
<th>Boring bar</th>
<th>Number of Screws</th>
<th>Screw Size</th>
<th>Tighten first from</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Standard</td>
<td>four</td>
<td>M8</td>
<td>top</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Standard</td>
<td>six</td>
<td>M10</td>
<td>top</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Standard</td>
<td>six</td>
<td>M10</td>
<td>bottom</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Active</td>
<td>four</td>
<td>M8</td>
<td>top</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Active</td>
<td>six</td>
<td>M10</td>
<td>top</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Linearized</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: The configuration of the different setups from which experimental modal analysis results are presented.

### 3.1.1 Standard Boring Bar

When clamping the standard boring bar so that the bottom side of the boring bar is clamped against the clamping house (i.e. the screws are tightened from the topside first and subsequently from the bottom-side) the fundamental boring bar resonance frequencies increases with increasing tightening, see Fig. 17. In this setup, screws of size M8 were used; the spectrum estimation parameters and excitation signal is presented in Table 3. By changing the excitation levels, nonlinearities in the dynamic properties of the boring bar might be observable via changes in frequency response function estimates for the same input and output locations at the boring bar. Four different excitation levels were used with the proportion 1:2:3:4 for each of the torque configuration presented in section 3.1. As can be seen in Fig. 18 the fundamental boring bar resonance frequencies decreases slightly with increased excitation level. The estimated resonance frequencies and relative damping from all 20 measurements are presented in Table 9 and Table 10.

The clamp screws were replaced with M10 screws and the number of clamp screws was increased to six. Using these clamping conditions, experiments were performed which were identical to those carried out using a clamping house with four M8 screws.

When clamping the standard boring bar so that the bottom side of the boring bar is clamped against the clamping house, the fundamental boring
Figure 17: The accelerance of the boring bar response using the standard boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 18: The accelerance of the boring bar response using the standard boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bars, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
<table>
<thead>
<tr>
<th>Torque</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10Nm</td>
<td>509.52</td>
<td>507.87</td>
<td>506.61</td>
<td>505.51</td>
<td>540.86</td>
<td>540.15</td>
<td>539.33</td>
<td>540.07</td>
</tr>
<tr>
<td>15Nm</td>
<td>518.13</td>
<td>516.50</td>
<td>515.23</td>
<td>514.18</td>
<td>546.50</td>
<td>546.31</td>
<td>545.73</td>
<td>544.60</td>
</tr>
<tr>
<td>20Nm</td>
<td>523.84</td>
<td>522.97</td>
<td>522.13</td>
<td>521.55</td>
<td>553.01</td>
<td>552.86</td>
<td>552.49</td>
<td>552.13</td>
</tr>
<tr>
<td>25Nm</td>
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<td>526.05</td>
<td>525.50</td>
<td>525.10</td>
<td>556.07</td>
<td>555.84</td>
<td>555.66</td>
<td>555.42</td>
</tr>
<tr>
<td>30Nm</td>
<td>526.72</td>
<td>526.23</td>
<td>525.79</td>
<td>525.45</td>
<td>555.67</td>
<td>555.68</td>
<td>555.55</td>
<td>555.35</td>
</tr>
</tbody>
</table>

Table 9: Estimates of the fundamental boring bar resonance frequencies based on all the measurements using the setup with standard boring bar, clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Fig. 17 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions Fig. 18 a) and b) produced for the four different excitation levels.

<table>
<thead>
<tr>
<th>Torque</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
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<tr>
<td>10Nm</td>
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<td>1.04</td>
<td>1.08</td>
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<td>1.31</td>
<td>1.33</td>
<td>1.46</td>
<td>0.26</td>
</tr>
<tr>
<td>15Nm</td>
<td>0.97</td>
<td>1.00</td>
<td>1.03</td>
<td>1.08</td>
<td>1.26</td>
<td>1.32</td>
<td>1.28</td>
<td>1.23</td>
</tr>
<tr>
<td>20Nm</td>
<td>0.88</td>
<td>0.91</td>
<td>0.94</td>
<td>0.96</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>0.99</td>
</tr>
<tr>
<td>25Nm</td>
<td>0.87</td>
<td>0.88</td>
<td>0.90</td>
<td>0.92</td>
<td>0.97</td>
<td>0.93</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>30Nm</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
<td>0.93</td>
<td>0.97</td>
<td>0.95</td>
<td>0.92</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 10: The relative damping estimates for the fundamental boring bar bending modes based on all the measurements using the setup with standard boring bar, clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Fig. 17 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions Fig. 18 a) and b), produced for the four different excitation levels.
bar resonance frequencies increases with increasing tightening, see Figs. 19 a) and b). As can be seen in Fig. 20, the fundamental boring bar resonance frequencies decreases slightly with increasing excitation level.

Clamping by first tightening the clamp screws from the boring bar’s underside changes the frequency response functions significantly as compared with the case where the clamp screws were tighten first from the upper side. This might be observed by comparing Figs. 21 and 22 with Figs. 19 and 20.

3.1.2 Active Boring Bar

The active boring bar has a cavity, a milled space, onto which an embedded actuator was placed. This space constitutes a change in the dynamic properties of the boring bar in comparison to the standard boring bar. This is obvious since the material, steel, is removed from the boring bar and replaced partly with an actuator with a lower Young’s module, etc. The actuator was kept passive during the experiments, thus, no control authority was applied. The same experiments were conducted with the active boring bar as were performed with the standard boring bar. From the results presented in Figs 23 and 24 it is clear that the dynamic properties of the active boring bar have changed significantly, mostly with regard to cutting speed direction (compare with the results from the standard boring bar Figs. 17 and 18). However, we can observe the same phenomenon that occurred in results obtained with the standard boring bar; i.e. increasing resonance frequency with increasing torque and decreasing resonance frequency with increasing excitation force. The estimated resonance frequencies and relative damping from all the 20 measurements are presented in Table 11 and Table 12.

When the active boring bar is clamped with six, size M10 screws, results obtained resemble those derived from clamping the same bar with size M8 screws, see Figs. 23 and 25. The largest differences may be observed in the frequency function estimates in cutting speed direction, see Fig. 25 a).

3.1.3 Linearized Boring Bar

Finally, the results from the boring bar with a so-called ”linearized” clamping condition are presented. Since no screws were used in this setup, only the excitation levels were changed. The results are presented in Fig. 27 and Table 13, which are the driving point frequency response functions in both the cutting speed direction and the cutting depth direction. Thus, only a slight variation in the boring bar’s resonance frequencies and damping might be
Figure 19: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side of the boring bar, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 20: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side upper-side of the boring bar, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 21: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the underside of the boring bar, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 22: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the underside of the boring bar, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 23: The accelerance of the boring bar response using the active boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bar, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 24: The accelerance of the boring bar response using the active boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bar, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
### Part II

**Table 11:** Estimates of the fundamental boring bar resonance frequencies based on all measurements, using the setup in which the active boring bar is clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Figs. 23 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions in Figs. 24 a) and b), produced for the four different excitation levels.

<table>
<thead>
<tr>
<th>Torque</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10Nm</td>
<td>449.83</td>
<td>447.10</td>
<td>445.31</td>
<td>444.34</td>
<td>473.12</td>
<td>472.57</td>
<td>472.50</td>
<td>471.76</td>
</tr>
<tr>
<td>15Nm</td>
<td>466.62</td>
<td>464.55</td>
<td>463.10</td>
<td>462.37</td>
<td>478.55</td>
<td>477.36</td>
<td>476.75</td>
<td>475.98</td>
</tr>
<tr>
<td>20Nm</td>
<td>478.76</td>
<td>477.51</td>
<td>476.51</td>
<td>475.90</td>
<td>501.49</td>
<td>499.88</td>
<td>499.29</td>
<td>497.79</td>
</tr>
<tr>
<td>25Nm</td>
<td>482.29</td>
<td>481.62</td>
<td>480.81</td>
<td>480.36</td>
<td>510.13</td>
<td>508.39</td>
<td>507.35</td>
<td>506.96</td>
</tr>
<tr>
<td>30Nm</td>
<td>484.39</td>
<td>483.63</td>
<td>482.87</td>
<td>482.88</td>
<td>515.08</td>
<td>513.91</td>
<td>513.11</td>
<td>512.79</td>
</tr>
</tbody>
</table>

**Table 12:** The relative damping estimates for the fundamental boring bar bending modes based on all measurements, using the setup in which the active boring bar is clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Figs. 23 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions in Figs. 24 a) and b), level produced for the four different excitation levels.

<table>
<thead>
<tr>
<th>Torque</th>
<th>Relative Damping of Mode 1 [%]</th>
<th>Relative Damping of Mode 2 [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
<td>Level 2</td>
</tr>
<tr>
<td>10Nm</td>
<td>1.40</td>
<td>1.41</td>
</tr>
<tr>
<td>15Nm</td>
<td>1.22</td>
<td>1.20</td>
</tr>
<tr>
<td>20Nm</td>
<td>1.17</td>
<td>1.14</td>
</tr>
<tr>
<td>25Nm</td>
<td>1.16</td>
<td>1.22</td>
</tr>
<tr>
<td>30Nm</td>
<td>1.36</td>
<td>1.35</td>
</tr>
</tbody>
</table>
Figure 25: The accelerance of the boring bar response using the active boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
Figure 26: The accelerance of the boring bar response using the active boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).
observed. Unfortunately, both resonance frequencies coincide with periodic disturbances originating from the engines in the lathe producing the hydraulic pressure. One disturbance was at approximately 591 Hz and the other disturbance at approximately 600 Hz. These disturbances will have different influences on the estimates, depending on the excitation level, this may be observed near the peak in Fig. 27.

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>Modal Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>Level 2</td>
</tr>
<tr>
<td>Frequency [Hz]</td>
<td>583.82</td>
</tr>
<tr>
<td>Damping [%]</td>
<td>2.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode 2</th>
<th>Modal Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>Level 2</td>
</tr>
<tr>
<td>Frequency [Hz]</td>
<td>602.25</td>
</tr>
<tr>
<td>Damping [%]</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 13: The resonance frequency and the relative damping of the linearized boring bar, estimated with poly-reference technique.

3.1.4 Mode shapes

A boring bar mode shape shows the spatial deformation pattern of the bar for that particular mode and thus for each degree of freedom measured on the boring bar, in both amplitude and spatial phase. This section presents all the mode shapes estimated from the three different setups: the standard boring bar, the active boring bar and the linearized boring bar. The mode shapes were estimated in I-DEAS using the frequency poly-reference method. First, results are presented from the standard boring bar, with size M8 screws, tightening clamp screws firstly from the upper-side. The shapes are presented in zy-plane and xy-plane in Fig. 28, a) and b) respectively. The angle of rotation around z-axis (relative the cutting depth direction for each measurement) is presented in Table 14. The mode shapes in xy-plane illustrated in Fig. 28 b) and the corresponding values in Table 14 show an average rotation of approximately 20 degrees.

Measurements derived from the active boring bar differ somewhat from those obtained with the standard boring bar. Mode shapes are presented in Fig. 29 and the values of the angle of rotation in Table 15. The shapes are
Figure 27: The accelerance of the boring bar response using the linearized setup and with four different excitation levels. a) the driving point in cutting speed direction (Y-) and b) the driving point in negative cutting depth direction (X-).
Figure 28: The two first mode shapes of the standard boring bar clamped with four M8 screws, when the clamp screws were tightened firstly from the upper-side, for five different tightening torques and four different excitation levels. a) in the zy-plane and b) in the xy-plane.
Table 14: Angle of mode shapes for the standard boring bar, relative to cutting depth direction axis.

Table 15: Angle of mode shapes for the active boring bar, relative to cutting depth direction axis.

The results from the linearized setup are presented by Fig. 30 and in Table 16.

In this linear setup, the zy-plane shape is almost identical to those shapes produced from standard, and active boring bar measurements, see Figs. 28, 29 and 30. In the xy-plane the shapes only have a rotation of approximately 10 degrees.

3.1.5 Quality of Measurement

Since the frequency response function estimates are based on the linear H1 estimation method, a measure of the linear relation between input and output signals may be represented by the coherence function, or (as this case has several sources), the multiple coherence function. Typical multiple coherence function estimates obtained during the experiment are illustrated in
Figure 29: The two first mode shapes of the active boring bar clamped with four M8 screws, when the clamp screws were tightened firstly from the upper-side, for five different tightening torques and four different excitation levels. a) in the zy-plane and b) in the xy-plane.
Figure 30: The two first mode shapes of the linearized boring bar for four different excitation levels. a) in the zy-plane and b) in the xy-plane.
### Table 16: Angle of mode shapes for the linearized boring bar relative to cutting depth direction axis.

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>-7.98</td>
<td>-8.46</td>
<td>-8.05</td>
<td>-8.11</td>
</tr>
<tr>
<td>Mode 2</td>
<td>-99.72</td>
<td>-100.00</td>
<td>-99.82</td>
<td>-100.07</td>
</tr>
</tbody>
</table>

Fig. 31; observe that the level in Fig. 31 b) starts from 0.99. After parameter estimation the Modal Assurance Criterion (MAC) was used to measure the correlation between the estimated modes shapes. A typical MAC diagram is presented in Fig 32. Typical values of the off-diagonal elements are 0.000-0.001, few values reach 0.007.

#### 3.1.6 Mass-loading

As previously mentioned, all the sensors, cables and shakers etc. affect the structure and thus also the relationship between the frequency response function estimate the ”true” frequency response function. It is nice to have an estimate close to the true frequency response function, the purpose was, however, to examine the influence of different clamping condition on the boring bar’s dynamic system. In order to acquire information concerning the sensors’ influence on the resonance frequency and the damping of the boring bar, a measurement using an impulse hammer and two accelerometers was conducted. Both impedance heads and all the accelerometers were removed, with the exception of the accelerometers at the driving point. Thus, only two accelerometers were glued to boring bar providing the mass-loading when hitting the boring bar, (compared with 14 accelerometers and two impedance head connected to the shakers) see Fig. 33. Once again, the periodic disturbances are present but insignificant. Compare Fig. 33 with Fig. 27, that is, the estimates from the hammer excitation with shaker excitation, both for the linearized structure.

The estimates show a resonance frequency shift of approximately 20 Hz for the driving point in both directions. Mass-loading lowers the resonance frequency from approximately 604 Hz to 583 Hz in the cutting depth direction, and from approximately 620 Hz to 602 Hz in the cutting speed direction whilst still using two accelerometers.
Figure 31: The multiple coherence corresponding to typical frequency response function estimates, where solid lines represent cutting speed direction (y-) and dotted lines represent negative cutting depth direction (x-).
Figure 32: The modal assurance criterion matrix coefficients for the two estimated mode shapes at the resonance frequencies 526.72 Hz and 555.67 Hz, where the off-diagonal values are equal to 0.000. The modes are estimated using the standard boring bar (clamped with four screws tightened firstly from the top), the lowest excitation level and the highest tightening torque.
Figure 33: a) The accelerance of the boring bar response using the linearized setup, two accelerometers and an impulse hammer. The solid line represents the driving point in cutting speed direction (y-), the dashed line is the driving point in negative cutting depth direction (x-). b) The corresponding coherence functions; observe that the plot shows the coherence from 0.91 to 1.
3.1.7 Summary of the Estimated Parameters

A number of phenomenon may be observed in each individual experimental setup. The estimated parameters for max excitation level and max tightening torque, 30Nm, are presented in Table 17 in order that difference may be observed.

<table>
<thead>
<tr>
<th>Setup</th>
<th>Parameters of Mode 1</th>
<th>Parameters of Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. [Hz]</td>
<td>Damp. [%]</td>
</tr>
<tr>
<td>1</td>
<td>525.45</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>528.58</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>502.43</td>
<td>1.37</td>
</tr>
<tr>
<td>4</td>
<td>482.88</td>
<td>1.51</td>
</tr>
<tr>
<td>5</td>
<td>489.11</td>
<td>1.70</td>
</tr>
<tr>
<td>6</td>
<td>582.52</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Table 17: Eigenfrequencies, relative damping and mode shape angle relative to cutting depth direction for the six different boring bar setups. Clamp screw tightening torque, 30Nm and maximum excitation signal level.

3.2 Analytical Models of the Boring Bars

This section presents results from a number of different Euler-Bernoulli models of the boring bar, including different simple models of boring bar clamping.

The first model assumed rigid clamping of the boring bar by the clamping house. The second model assumes that boring bar clamping is pinned at the positions where the actual clamp screws clamp the boring bar inside the clamping house. Thus, at each pinned boundary condition, the boring bar model will be rigidly clamped in the cutting speed direction without any rotational constraints about the clamping position. All the Euler-Bernoulli models of the boring bar assume a homogenous constant cross-section, i.e. $E(z) = E$, $\rho(z) = \rho$, $A(z) = A$, $I_x(z) = I_x$ and $I_y(z) = I_y$.

3.2.1 Single-span Model

The simplest model is the single span model with rigid clamping at one end and no clamping (free) at other. The boring bar is assumed to be entirely contained by the clamping house, which has a length of 100mm, and clamping ends where the clamping house ends. Thus the model is a fixed-free beam with the length of 200mm. The first three resonance frequencies in the cutting speed direction and in the cutting depth direction are presented in Table 18,
Figure 34: The accelerance of the boring bar response for the six different setups, a) the driving point in cutting speed direction and b) the driving point in cutting depth direction.
and the three first mode shapes in Fig. 35. Depending on the direction of the

<table>
<thead>
<tr>
<th>Direction of mode</th>
<th>( f_1 ) [Hz]</th>
<th>( f_2 ) [Hz]</th>
<th>( f_3 ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting speed direction (y-)</td>
<td>698.33</td>
<td>4376.36</td>
<td>12253.94</td>
</tr>
<tr>
<td>Cutting depth direction (x-)</td>
<td>698.11</td>
<td>4374.98</td>
<td>12250.08</td>
</tr>
</tbody>
</table>

Table 18: The first three resonance frequencies of the Euler-Bernoulli fixed-free model with a length of \( l = 200 \text{mm} \).

Figure 35: The first three mode shapes in cutting speed direction and cutting depth direction of the Euler-Bernoulli fixed-free model.

transverse motion (cutting speed or cutting depth) assumed to be modeled by the Euler-Bernoulli beam, the shape of the boring bar cross-section will result in a slightly different moment of inertia, see Table 1. Thus, the resonance frequencies in respective direction will also differ.

### 3.2.2 Multi-span Model

Two multi-span Euler-Bernoulli boring bar models with pinned boundary conditions were considered: one corresponded to the boring bar clamped with four screws in the clamping house, and one corresponding to the boring bar
clamped with six screws in the clamping house. The eigenfrequencies and mode shapes (eigenfunctions) for the two models were calculated in Matlab by finding the roots to the characteristic equation produced using the boundary conditions presented in Appendix A, and using Eq. 39, etc. The results for the two different models are presented in Table 19. When the fixed clamping

<table>
<thead>
<tr>
<th>Direction of mode</th>
<th>Using four screws</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$ [Hz]</td>
<td>$f_2$ [Hz]</td>
<td>$f_3$ [Hz]</td>
<td></td>
</tr>
<tr>
<td>Cutting speed direction (y-)</td>
<td>527.47</td>
<td>3390.18</td>
<td>9539.40</td>
<td></td>
</tr>
<tr>
<td>Cutting depth direction (x-)</td>
<td>527.30</td>
<td>3389.11</td>
<td>9536.39</td>
<td></td>
</tr>
<tr>
<td>Using six screws</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cutting speed direction (y-)</td>
<td>566.92</td>
<td>3575.59</td>
<td>10058.76</td>
<td></td>
</tr>
<tr>
<td>Cutting depth direction (x-)</td>
<td>566.74</td>
<td>3574.46</td>
<td>10055.59</td>
<td></td>
</tr>
</tbody>
</table>

Table 19: The first three resonance frequencies in cutting speed direction and cutting depth direction for the Euler-Bernoulli models, with the following boundary conditions; free-pinned-pinned-free and free-pinned-pinned-pinned-free.

When the fixed clamping model is changed to the pinned model, the first resonance drops by approximately 170 Hz for the four-screw-clamped boring bar, and approximately 140 Hz for the six-screw-clamped boring bar. The first three mode shapes for the two models are presented in Fig. 36.

3.2.3 Multi-span Model on Elastic Foundation

Finally, the multi-span boring bar models with flexible boundary conditions (corresponding to the standard boring bar clamped using four clamp screws or six clamp screws) are considered. These two models were calculated in the same way as for the multi-span models with pinned boundary condition, but now for the elastic boundary condition, using the stiffness coefficients in Table 5. The length of the clamp screw overhang was selected to 1.5mm. Both eigenfrequencies and mode shapes were calculated for the two multi-span boring bar models with flexible clamping boundary conditions. The calculated eigenfrequencies are presented in Table 20, and mode shapes are shown in Fig. 37.
Figure 36: The first three mode shapes, for a) the free-pinned-pinned-free model of the boring bar and b) the free-pinned-pinned-pinned-free model of the boring bar.
Figure 37: The first three mode shapes for the Euler-Bernoulli boring bar model, with boundary conditions a) free-spring-spring-free (four clamp screws) and b) free-spring-spring-spring-free (six clamp screws).
### Analysis of Dynamic Properties of Boring Bars
Concerning Different Clamping Conditions

#### Table 20: The first three resonance frequencies in the cutting speed direction and in the cutting depth direction, for the two multi-span boring bar models, with flexible clamping boundary conditions.

<table>
<thead>
<tr>
<th>Clamping Conditions</th>
<th>Mode Direction</th>
<th>( f_1 ) [Hz]</th>
<th>( f_2 ) [Hz]</th>
<th>( f_3 ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using four M8 screws</td>
<td>Cutting speed direction (y-)</td>
<td>519.43</td>
<td>3303.79</td>
<td>9257.16</td>
</tr>
<tr>
<td>Using four M8 screws</td>
<td>Cutting depth direction (x-)</td>
<td>519.27</td>
<td>3302.81</td>
<td>9254.48</td>
</tr>
<tr>
<td>Using six M8 screws</td>
<td>Cutting speed direction (y-)</td>
<td>532.09</td>
<td>3335.17</td>
<td>9278.08</td>
</tr>
<tr>
<td>Using six M8 screws</td>
<td>Cutting depth direction (x-)</td>
<td>531.94</td>
<td>3334.20</td>
<td>9275.46</td>
</tr>
<tr>
<td>Using four M10 screws</td>
<td>Cutting speed direction (y-)</td>
<td>525.24</td>
<td>3346.84</td>
<td>9404.65</td>
</tr>
<tr>
<td>Using four M10 screws</td>
<td>Cutting depth direction (x-)</td>
<td>525.08</td>
<td>3345.83</td>
<td>9401.83</td>
</tr>
<tr>
<td>Using six M10 screws</td>
<td>Cutting speed direction (y-)</td>
<td>541.52</td>
<td>3398.74</td>
<td>9484.36</td>
</tr>
<tr>
<td>Using six M10 screws</td>
<td>Cutting depth direction (x-)</td>
<td>541.36</td>
<td>3397.74</td>
<td>9481.62</td>
</tr>
</tbody>
</table>

3.3 **Computer Simulations of Nonlinear Systems**

The simulation used a linear component of the models which was based on parameters derived from the experimental modal analysis of the standard boring bar, clamped with four screws, tightened firstly from the top. The tightening torque was 30Nm and the excitation level was lowest. For the purpose of simplification, only the mode in cutting speed direction, estimated from the driving point, was used, thus the linear part \( H_L(f) \) of the model only consists of one degree of freedom, which in terms of receptance may proximately be written as

\[
H_L(f) = \frac{A}{j2\pi f - \lambda} + \frac{A^*}{j2\pi f - \lambda^*}
\]

where

\[
\lambda = -\zeta 2\pi f_0 + j2\pi f_0 \sqrt{1 - \zeta^2}
\]

The values are: resonance frequency \( f_0 = 555.675 \text{ Hz} \), damping \( \zeta = 0.966 \% \) and the residue \( A = -j1.001 \cdot 10^{-4} \). Thus the linear system may be expressed
as

\[
H_L(f) = \frac{- j 1.001 \cdot 10^{-4}}{j 2 \pi f - (-33.727 + j3491.246)} + \frac{j 1.001 \cdot 10^{-4}}{j 2 \pi f - (-33.727 - j3491.246)} \quad (96)
\]

Fig. 38 displays a diagram if the synthesized SDOF system accelerance function, corresponding to the receptance in 94. This figure also presents an estimate of the driving point accelerance of the boring bar in the direction of cutting speed, and a synthesized two-degrees-of-freedom system accelerance function response for two fundamental boring bar modes. These parameters are directly applicable to the filter-method when calculating the filter coefficients, however, when using the ordinary differential equation solvers, the partial fractions are collected into one polynomial fraction, which may be expressed in terms of the mass, damping and stiffness coefficients \( m, c \) and \( k \).
Theses parameters were determined using the following relations

\[ A = \frac{1}{jm4\pi f_0} \quad (97) \]
\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (98) \]
\[ \zeta = \frac{c}{2\sqrt{mk}} \quad (99) \]

which yields a mass of \( m = 1.431 \) kg, a damping of \( c = 96.503 \) Ns/m and a stiffness of \( k = 17.440 \cdot 10^6 \) N/m.

### 3.3.1 Softening Spring Model

The nonlinear softening stiffness coefficients \( k_s \) and \( k_c \) in the signed squared and cubic model were not obtained by direct parameter estimation. The resonance frequency shifting phenomena always appears between accelerance function estimates for the standard boring bar clamped with screws for different excitation force levels. Typically, a resonance frequency shift of 5 Hz and, for example, an initial resonance frequency of 500 Hz renders a frequency deviation of 1%, which corresponds to a 10% deviation in the linear stiffness coefficient. By considering the stiffness deviation, the stiffness coefficient used in the linear model, the level of excitation force and the convergence rate in the simulation, the values for the nonlinear stiffness coefficients were selected as: \( k_s = 8 \cdot 10^{12} \) N/m\(^2\) and \( k_c = 4 \cdot 10^{19} \) N/m\(^3\) for the signed squared and cubic model, respectively. The levels of the excitation force were given the same ratios as for the experiments with the standard boring bar, and the signal type was normally distributed random noise, with peak levels 100, 200, 300, 400 mN.

Fig. 39 a) presents the frequency response function estimates that were produced based on simulations of the nonlinear model with a signed squared stiffness, using the filter method and the four different excitation levels. Fig. 39 b) presents the corresponding frequency response function estimates produced based on simulations of the nonlinear model system with a cubic stiffness, using the filter method and the four different excitation levels. Table 21 gives estimates of the resonance frequency and the relative damping for the frequency response functions based on the nonlinear models simulated with the filter method, for the four excitation force levels. The SDOF least square technique [18] was used to produce estimates of resonance frequency and relative
Figure 39: Frequency response function estimates based on simulations of the nonlinear models using the filter method and four different excitation levels, a) for the presented model with signed squared stiffness, and; b) for the model with cubic stiffness.
Analysis of Dynamic Properties of Boring Bars
Concerning Different Clamping Conditions

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Table 21: Resonance frequency and relative damping estimates for the frequency response functions based on the nonlinear models, simulated with the filter method.

damping. Fig. 40 a) presents the coherence functions for simulations of the nonlinear model with a signed squared stiffness, using the filter method and the four different excitation signals. Fig. 40 b) shows the coherence functions estimates for the simulations of the nonlinear model with a cubic stiffness, using the filter method and the four different excitation levels. The coherence function estimates are also presented for a narrow frequency range including the resonance frequency and are illustrated for the nonlinear model with a signed squared stiffness in Fig. 41 a) and for the nonlinear model with a cubic stiffness in Fig. 41 b).

If the ordinary differential equation solver ode45 in Matlab is used for simulations of the nonlinear model with a signed squared stiffness for the four different excitation levels, it results in the frequency response function estimates shown in Fig. 42 a). Fig. 42 b) presents corresponding frequency response function estimates, based on simulations of the nonlinear model system with a cubic stiffness, using the ordinary differential equation solver ode45 in Matlab and the four different excitation levels. Table 22 presents estimates of the resonance frequency and the relative damping for the frequency response functions based on the nonlinear models, simulated with the ordinary differential equation solver ode45 in Matlab, for the four excitation force levels. Also, in this case, the SDOF least square technique [18] was used to produce estimates of resonance frequency and relative damping. Fig. 43 a) gives the coherence functions for the simulations of the nonlinear model with a signed squared stiffness, using ode45 in Matlab and the four different excitation signals. Fig. 43 b) shows the coherence functions’ estimates for simulations of the nonlinear model with a cubic stiffness, using the ordinary differential
Figure 40: Coherence function estimates based on simulations of the nonlinear models, using the filter method and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.
Figure 41: Coherence function estimates based on simulations of the nonlinear models, using the filter method and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.
Figure 42: Frequency response function estimates based on simulation of the nonlinear models, using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.
Analysis of Dynamic Properties of Boring Bars
Concerning Different Clamping Conditions

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Squared Model

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<td>Level 4</td>
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Cubic Model

Table 22: Estimates of resonance frequency and relative damping for the frequency response functions based on the nonlinear models simulated with the differential equation solver ode45.

equation solver ode45 and the four different excitation levels. The coherence function estimates are also presented for a narrow frequency range (including the resonance frequency) and are illustrated for the nonlinear model with a signed squared stiffness in Fig. 44 a) and for the nonlinear model with a cubic stiffness in Fig. 44 b).

4 Summary and Conclusions

The results from the experimental modal analysis of boring bars demonstrate that a boring bar clamped in a standard clamping house with clamping screws has a nonlinear dynamic behavior. Also, the results indicate that the standard clamping house with clamp screws is the likely source of the nonlinear behavior.

The experimental modal analysis results from the boring bar clamped in a "linearized" standard clamping house with steel wedges and epoxy glue indicated a significant reduction in non-linear dynamic behavior. Thus, a boring bar clamped in a standard clamping house with clamping screws has significant, nonlinear dynamic properties. Different excitation force levels will not yield identical frequency response function estimates for the same transfer path in the boring bar (see any of the figures presenting boring bar FRF:s for different excitation force levels, i.e. Figs. 18, 20, 22, 24 and 26). Based on a large number of measurements, a trend may be observed; the fundamental boring bar resonance frequencies decrease with increasing excitation level; see Table 9 and 11 which summarize estimated resonance frequencies. However, with regard to the behavior of relative damping as a function of excitation
Figure 43: Coherence function estimates based on simulation of the nonlinear models, using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.
Figure 44: Coherence function estimates based on simulations of the nonlinear models using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness presented and b) for the model with cubic stiffness.
force level; the results from the standard boring bar indicate that damping for the first mode increases with increasing excitation force level, while damping for the second mode decreases with increasing excitation force level; see Fig. 10. Also, the results from the active boring bar give an ambiguous indication of the effects on damping properties; see Fig. 12.

The clamp screw tightening torque appears to affect the nonlinear behavior of the boring bar. Variation in the FRF:s which was introduced by the four different excitation force levels seems to be larger for a low tightening torque (10Nm) than for a high tightening torque (30Nm), see, for example, Fig. 18. Also, experimental modal analysis results involving the so-called "linearized" boring bar clamping, support the conclusion that clamping conditions influence the extent of nonlinearities in boring bar dynamics. By examining, for example, driving point accelerances in the boring bar with "linearized" clamping for the four excitation force levels (see Fig. 27), it can be seen that only insignificant differences are present. Thus, nonlinear behavior on the part of the boring bar seems to be almost removed (within the level of normal experimental uncertainty).

Another example of dynamic behavior on the part of the clamped boring bar is exhibited in the change in fundamental boring bar resonance frequencies with changing clamp screw tightening torques. Boring bar dynamics display increasing resonance frequency with increasing clamp torque; see Figs. 17, 19, 21, 23 and 25.

When changing the number of screws used for clamping, or when using "linearized" clamping, changes in dynamic properties of the boring bar (clamping system) are expected and obvious. Hence, new boundary conditions for the boring bar are introduced. Also, changing the standard boring bar to the active boring bar will alter the dynamic properties of the boring bar - clamping system, i.e. a structural part of the system is different. The order in which the clamp screws were tightened (first from the upper-side or first from the underside) had a major influence on the dynamic properties of the boring bar. This might be observed by comparing the boring bar driving point accelerances in Fig. 19 with the boring bar driving point accelerances in Fig. 21. If the clamp screws were tightened firstly from the upper side, the higher resonance frequency in Fig. 19 a) shows a variation from approximately 552 Hz to 562 Hz, for a change from the lowest to the highest clamp screws tightening torque. On the other hand, if the clamp screws were tightened firstly from the underside, the higher resonance frequency in Fig. 21 a) shows a variation from approximately 495 Hz to 530 Hz, for a change from the lowest to the highest clamp screws tightening torque. These results may have arisen
due to the difficulty in producing the exact same clamping conditions when tightening the clamp screws from the bottom first, compared to tightening the clamp screws from the top first.

Another interesting observation concerns mode shapes and, in particular angles of the different modes in the cutting depth - cutting speed plane (x-y plane). Assuming that the boring bar is rigidly clamped, and that the boring bar has a homogenous cross-section in the x-y plane, this would result in one set of mode shapes in the cutting speed direction (y-direction) and one set of mode shapes in the cutting depth direction (x-direction). However, this is not the case in the results presented - the transverse sensitivity of the accelerometers is not sufficient to explain the obtained deviations of mode shape angles in the x-y plane. Transverse sensitivity would only explain mode shape angles of up to 2-3 degrees. There are two possible explanations for this phenomenon. Firstly, the assumption of a constant cross-section may not be true. The major part of the boring bar in the length direction has a constant cross-section; however, this is not the case for the section of the boring bar head to which the tool is attached; see Table 1 and Fig. 5. Secondly, the clamping conditions may effect the mode shape rotation x-y plane. The standard boring bar clamped with four M8 screws, tightened firstly from the top has a first mode with an average mode shape angle or rotation of -20 degrees, relative to the cutting depth direction (x-direction). The second mode displays an average mode shape angle (or rotation) of -110 degrees relative to the cutting depth direction, see Table 14. It is also possible to notice a trend in the first mode for clockwise rotation with increasing torque; such a trend is not significant for the second mode, see Fig. 28. Changing the clamp screw size or the number of clamp screws affects the so-called "mode rotation", both for the standard boring bar and in the case of the active boring bar (see Figs. 28, 29 and 30). In addition, it should be noted that the boring bar in the linearized setup has rotated fundamental modes; the first mode has a mode shape angle or rotation of approx. -8 degrees relative to cutting depth direction (x-direction), and the second mode displays a mode shape angle or rotation of approx. -100 degrees relative to cutting depth direction (see Table 16). In comparison to the "linearized" clamping case, the use of six size M10 clamp screws (tightened firstly from the top) resulted in a similar rotation of the fundamental modes, constituting a difference of approx. 2 degrees; see Table 17.

The Euler-Bernoulli boring bar models provide rough approximations of the low-order resonance frequencies and the corresponding spatial shapes of the modes. Also, (in the x-y plane) the Euler-Bernoulli models will provide
one set of mode shapes in the cutting speed direction (y-direction) and one set of mode shapes in the cutting depth direction (x-direction). The first and simplest, fixed-free model overestimates the lower fundamental resonance frequencies by approx. 170 Hz, and the upper fundamental resonance frequency by approx. 140 Hz compared to the most rigidly clamped boring bar using six M10 clamp screws. Compared with the linearized boring bar setup the fixed-free Euler-Bernoulli model overestimates the lower fundamental resonance frequencies by approx. 115 Hz, and the upper fundamental resonance frequency by approx. 100 Hz. Furthermore, the Euler-Bernoulli model yields a 0.2 Hz difference in frequency between the two fundamental resonance frequencies, while, the experimental results from, for example, the linearized boring bar setup displays a 20-30 Hz difference in fundamental resonance frequencies. It may be assumed that the linearized setup will display a difference in fundamental resonance frequencies which adheres to the Euler-Bernoulli model. However, experimental results indicate a difference of approximately 20 Hz (see Fig 27 and Table 13). It is obvious the fixed-free model will overestimate the fundamental resonance frequencies since it assumes rigid clamping which is not the case in reality. In addition, the fixed-free model does not consider the influence of shear deformation and rotary inertia in the beam, meaning that resonance frequencies will be overestimated.

The multi-span models are assumed to be more realistic, yet simplified, models of boring bar clamping conditions. The results from the multi-span model (using the pinned boundary screw positions) illustrate that displacement of the boring bar is likely to occur between the screws, inside the clamping house. Since this configuration allows motion over a longer span than the simple fixed-free model, it also produces lower resonance frequencies. For this reason, the Euler Bernoulli model is more appropriate for the pinned boundary condition than for the fixed-free, since the length to diameter ratio has increased, even though this ratio is still below the recommended value of 10.

The effects of different screw dimensions and properties on the boring bar may be investigated by using elastic foundations. The results in Table 20 show that the fundamental resonance frequencies for the Multi-span model increase with an increasing number of clamp screws. An M8 screw features a 10 Hz increase, whilst an M10 screw features a 15 Hz increase. This increase in fundamental resonance frequency (for the Multi-span model) also occurs as clamp screw size is increased, so that (using four screws) yields an approximate 5 Hz increase, whilst changing from M8 to M10 (using six screws) yields approx. a 10 Hz increase. This result is confirmed by experimental results which yield a frequency change of approx. 5-15 Hz, depending on the clamp
screw tightening torque (see Figs 17 and 19). Mode shapes from the all the Euler-Bernoulli models are fairly similar. However, inside the clamping house, the mode shapes differ significantly between the fixed-free model and the multi-span models. In the case of the multi-span models, the mode shapes have a spatial deflection inside the clamping house, while, for the fixed-free model, the mode shapes have no deflection inside the clamping house.

Experimental results strongly indicate that the boring bar (clamped in the clamping house with screws) possesses nonlinear dynamic properties. Two different nonlinear single-degree-of-freedom models were simulated in order to investigate if they bear resemblance the dynamic behavior of the boring bar clamped in the clamping house with screws. In addition, two different simulation methods were used to provide redundancy due to the fact that there are no explicit analytical solutions for the two different nonlinear single-degree-of-freedom models which can be used as benchmark. Both the square with sign stiffness model and the cubic stiffness model show a similar trend in frequency response function estimates as the experimental results (see Tables 21 and 22). The trend is decreasing resonance frequency with increasing excitation level; see Figs. 39 and Figs. 42 (produced by the filter method and the ODE solver method, respectively). The coherence function estimates for the input and output signals of the nonlinear SDOF systems simulated with the filter method display an expected dip at the resonance frequency that increases with increasing excitation level, see Fig. 40. By using the ODE solver method to simulate the comparatively nonlinear systems, the coherence function estimates assume comparatively slightly higher levels in the resonance frequency range of the SDOF systems than the filter method.
5 Appendix A

The three span model without rotational springs and infinitely stiff transverse springs will have boundary conditions; Free-Pinned-Pinned-Free; yielding the equations as

\[
\begin{align*}
EI \frac{d^2 u_1(z)}{dz^2} \bigg|_{z=0} &= 0 \\
EI \frac{d^3 u_1(z)}{dz^3} \bigg|_{z=0} &= 0 \\
\left( \frac{d u_1(z)}{dz} + \frac{d u_2(z)}{dz} \right) \bigg|_{z=l_1} &= 0 \\
\left( EI \frac{d^2 u_1(z)}{dz^2} + EI \frac{d^2 u_2(z)}{dz^2} \right) \bigg|_{z=l_1} &= 0 \\
\left( EI \frac{d^3 u_1(z)}{dz^3} + EI \frac{d^3 u_2(z)}{dz^3} \right) \bigg|_{z=l_1} &= 0 \\
\left( \frac{d u_2(z)}{dz} + \frac{d u_3(z)}{dz} \right) \bigg|_{z=l_1+l_2} &= 0 \\
\left( EI \frac{d^2 u_2(z)}{dz^2} + EI \frac{d^2 u_3(z)}{dz^2} \right) \bigg|_{z=l_1+l_2} &= 0 \\
\left( EI \frac{d^3 u_2(z)}{dz^3} + EI \frac{d^3 u_3(z)}{dz^3} \right) \bigg|_{z=l_1+l_2} &= 0 \\
\frac{d u_3(z)}{dz} \bigg|_{z=l_1+l_2+l_3} &= 0 \\
EI \frac{d^2 u_3(z)}{dz^2} \bigg|_{z=l_1+l_2+l_3} &= 0 \\
EI \frac{d^3 u_3(z)}{dz^3} \bigg|_{z=l_1+l_2+l_3} &= 0
\end{align*}
\]
The three span model with transverse springs and rotational springs will have boundary conditions; Free-Elastic-Elastic-Free; yielding the equations as

\[
\begin{align*}
EI \frac{d^2 u_1(z)}{dz^2} & \bigg|_{z=0} = 0 \\
EI \frac{d^3 u_1(z)}{dz^3} & \bigg|_{z=0} = 0 \\
EI \frac{d^2 u_1(z)}{dz^2} + k_R \frac{d u_1(z)}{dz} + EI \frac{d^2 u_2(z)}{dz^2} & \bigg|_{z=l_1} = 0 \\
EI \frac{d^3 u_1(z)}{dz^3} - k_T u_1(z) + EI \frac{d^3 u_2(z)}{dz^3} & \bigg|_{z=l_1} = 0 \\
EI \frac{d^2 u_1(z)}{dz^2} + EI \frac{d^2 u_2(z)}{dz^2} & \bigg|_{z=l_1} = 0 \\
\left( EI \frac{d^2 u_2(z)}{dz^2} + k_R \frac{d u_2(z)}{dz} + EI \frac{d^2 u_3(z)}{dz^2} \right) & \bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^3 u_2(z)}{dz^3} - k_T u_2(z) + EI \frac{d^3 u_3(z)}{dz^3} \right) & \bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^2 u_2(z)}{dz^2} + EI \frac{d^2 u_3(z)}{dz^2} \right) & \bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^3 u_2(z)}{dz^3} + EI \frac{d^3 u_3(z)}{dz^3} \right) & \bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^2 u_3(z)}{dz^2} \right) & \bigg|_{z=l_1+l_2+l_3} = 0 \\
\left( EI \frac{d^3 u_3(z)}{dz^3} \right) & \bigg|_{z=l_1+l_2+l_3} = 0
\end{align*}
\]
The four span model without rotational springs and infinitely stiff transverse springs will have boundary conditions; Free-Pinned-Pinned-Pinned-Free; yielding the equations as

\[
\begin{align*}
EI \frac{d^2u_1(z)}{dz^2} igg|_{z=0} &= 0 \\
EI \frac{d^2u_1(z)}{dz^3} igg|_{z=0} &= 0 \\
\left. \left( \frac{du_1(z)}{dz} + \frac{du_3(z)}{dz} \right) \right|_{z=l_1} &= 0 \\
\left( EI \frac{d^2u_1(z)}{dz^2} + EI \frac{d^2u_3(z)}{dz^2} \right) igg|_{z=l_1} &= 0 \\
\left( EI \frac{d^3u_1(z)}{dz^3} + EI \frac{d^3u_3(z)}{dz^3} \right) igg|_{z=l_1} &= 0 \\
\left. \left( \frac{du_2(z)}{dz} + \frac{du_4(z)}{dz} \right) \right|_{z=l_1+l_2} &= 0 \\
\left( EI \frac{d^2u_2(z)}{dz^2} + EI \frac{d^2u_4(z)}{dz^2} \right) igg|_{z=l_1+l_2} &= 0 \\
\left( EI \frac{d^3u_2(z)}{dz^3} + EI \frac{d^3u_4(z)}{dz^3} \right) igg|_{z=l_1+l_2} &= 0 \\
\left. \left( \frac{du_3(z)}{dz} + \frac{du_4(z)}{dz} \right) \right|_{z=l_1+l_2+l_3} &= 0 \\
\left( EI \frac{d^2u_3(z)}{dz^2} + EI \frac{d^2u_4(z)}{dz^2} \right) igg|_{z=l_1+l_2+l_3} &= 0 \\
\left( EI \frac{d^3u_3(z)}{dz^3} + EI \frac{d^3u_4(z)}{dz^3} \right) igg|_{z=l_1+l_2+l_3} &= 0 \\
\left. \left( \frac{du_4(z)}{dz} \right) \right|_{z=l_1+l_2+l_3+l_4} &= 0 \\
\left( EI \frac{d^2u_4(z)}{dz^2} \right) igg|_{z=l_1+l_2+l_3+l_4} &= 0 \\
\left( EI \frac{d^3u_4(z)}{dz^3} \right) igg|_{z=l_1+l_2+l_3+l_4} &= 0
\end{align*}
\]
The four span model with transverse springs and rotational springs will have boundary conditions; Free-Elastic-Elastic-Elastic-Free; yielding the equations as

\[
\begin{align*}
EI \frac{d^2u_1(z)}{dz^2} &\bigg|_{z=0} = 0 \\
EI \frac{d^3u_1(z)}{dz^3} &\bigg|_{z=0} = 0 \\
\left( EI \frac{d^2u_1(z)}{dz^2} + k_R \frac{du_1(z)}{dz} + EI \frac{d^2u_2(z)}{dz^2} \right) &\bigg|_{z=l_1} = 0 \\
\left( EI \frac{d^3u_1(z)}{dz^3} - k_T u_1(z) + EI \frac{d^3u_2(z)}{dz^3} \right) &\bigg|_{z=l_1} = 0 \\
\left( EI \frac{d^2u_1(z)}{dz^2} + EI \frac{d^3u_2(z)}{dz^3} \right) &\bigg|_{z=l_1} = 0 \\
\left( EI \frac{d^2u_2(z)}{dz^2} + k_R \frac{du_2(z)}{dz} + EI \frac{d^2u_3(z)}{dz^2} \right) &\bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^3u_2(z)}{dz^3} - k_T u_2(z) + EI \frac{d^3u_3(z)}{dz^3} \right) &\bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^2u_3(z)}{dz^2} + EI \frac{d^3u_3(z)}{dz^3} \right) &\bigg|_{z=l_1+l_2} = 0 \\
\left( EI \frac{d^2u_3(z)}{dz^2} + k_R \frac{du_3(z)}{dz} + EI \frac{d^2u_4(z)}{dz^2} \right) &\bigg|_{z=l_1+l_2+l_3} = 0 \\
\left( EI \frac{d^3u_3(z)}{dz^3} - k_T u_3(z) + EI \frac{d^3u_4(z)}{dz^3} \right) &\bigg|_{z=l_1+l_2+l_3} = 0 \\
\left( EI \frac{d^2u_4(z)}{dz^2} + EI \frac{d^3u_4(z)}{dz^3} \right) &\bigg|_{z=l_1+l_2+l_3} = 0 \\
\left( EI \frac{d^2u_4(z)}{dz^2} \right) &\bigg|_{z=l_1+l_2+l_3+l_4} = 0 \\
\left( EI \frac{d^3u_4(z)}{dz^3} \right) &\bigg|_{z=l_1+l_2+l_3+l_4} = 0
\end{align*}
\]

(103)

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References


Part III

Vibration Analysis of Mechanical Structures over the Internet Integrated into Engineering Education
Parts of Part III is published as:

Vibration Analysis of Mechanical Structures over the Internet Integrated into Engineering Education


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Abstract

Experimental vibration analysis is one of the most important tools for analyzing dynamic properties of mechanical structures. Information derived from experimental vibration analysis is used in the development of products to obtain a required dynamic behavior or, for instance, to classify vibration problems in different public, industrial environments etc. In order to carry out such experiments efficiently, experience of different analysis methods is of great importance. As is the case in any field, reliable results are usually obtained by those with a high degree of experience conducting experiments. In engineering education, experiments using real mechanical structures and equipment is of high significance to the learning process. Traditionally, students gain this experience by conducting experiments in a university laboratory, under supervision of an instructor. However, as investment in laboratory maintenance decreases in educational institutes worldwide, simulations or theoretical experiments are increasingly utilized in place of hands on experiments, due to cost. However, Blekinge Institute of Technology (BTH), Sweden provides the opportunity for engineering students
to remotely access practical and theoretical knowledge advancement in experimental vibration analysis. This combination of practical and theoretical experience offered by BTH is highly attractive for the industry. Today, remote laboratory exercises are a reality at BTH, complementing on-campus laboratory experiments and increasing the availability of practical instruments. Previously at BTH, vibration experiments were conducted over the Internet using experimental hardware located in a small closed laboratory. However, BTH has now taken a step toward an increasingly user-friendly interface that increases the student’s sense of being in front of a real instrument, as opposed to a virtual front panel. This paper presents a new remote vibration laboratory and details the manner in which experimental vibration analysis has been integrated into engineering education at BTH as a complement to ordinary lessons and experiments in traditional laboratories.

1 Introduction

Today’s industry features a high demand for qualified personnel with experience of sound and vibration analysis. For example, in both the automobile and the aircraft industry, experimental vibration analysis is one of the most important tools in the development of more robust products. Information obtained from experimental vibration analysis provides detailed information required to develop and model real physical systems, for example, acoustic or mechanical systems. To provide students with the level of knowledge demanded by today’s industry, comprehensive theoretical and experimental studies are required. Theory is the basis for understanding the dynamics of different structures and systems and is required in order to predict the behavior of dynamic systems. Experiments should be used in order to verify different laws of physics and illustrate the limitations of models, etc., as well as to provide the students with experience of measurement affected by noise. However, the diminishing funding of universities by governmental institution has resulted in a trend toward a reduced number of laboratory experiments. Therefore, this paper proposes utilizing world wide expansion and development of the Internet in order to provide students with access to practical and theoretical knowledge in experimental vibration analysis.

Blekinge Institute of Technology (BTH) has recently introduced a remote laboratory within the field of experimental vibration analysis which allows students to perform experiments at any time, from any location in the world. BTH has previously established other, similar laboratories [1, 2], as have a
number of other educational institutions worldwide [3,4]. It is fundamental to both students, academics, and industry professionals with in the field of engineering to maintain and enhance their knowledge of experimental sound and vibration analysis. Thus, distance education in the form of, for example, remote laboratories stand to offer significant benefits to the industry.

The remote laboratory at BTH is designed to meet the demands of both students in undergraduate education and development engineers from industry; providing tasks and challenges ranging from basic to advanced level. The remote vibration analysis laboratory offers a mechanical structure for the purposes of study. This structure is connected to an electro-dynamic shaker which converts the electric signal from a signal analyzer to a mechanic, one-dimensional motion. Thus, the shaker applies a force as input signal to the mechanical structure. The test probes are accelerometers and force transducers connected to the signal analyzer. The signal analyzer then presents the results to the user via the Internet. The instruments provided by the remote laboratory are well known and commonly utilized within the field of vibration analysis. Students of the university have been familiarized with these tools during traditional laboratory experiments carried out as part of, for example, undergraduate signal processing courses.

The experiments in the remote laboratory are designed to reflect upon authentic vibration problems in the manufacturing industry, e.g. vibration in metal cutting processes [5–7]. The object under investigation is a boring bar used for metal cutting in a lathe. The vibration problem associated with this type of process is considered to be an important and critical factor with regard to the performance of, for example, tool life and surface finishing, which in turn impacts negatively on production costs. Due to the fact that vibration problems have been shown to originate from the lower order bending modes [5,6], it is of utmost importance to examine the dynamic properties of the boring bar itself. This may, for example, involve finding dominant bending modes. This paper focuses on the integration of the remote vibration analysis laboratory into a university course and its subsequent use by students to solve various problems and tasks.

The Sound and Vibration Analysis I course at BTH, presents students with their first experience of experimental vibration analysis. Prior to this course they have attended various signal processing courses involving experiments with systems based on analogue and digital electronic circuits. The experimental vibration analysis component of the new course features new sensors
and systems, and requires a large amount of time and practice on behalf of students in order to acquire sufficient knowledge to pass the course, and to be able to conduct simple vibration analysis by them self. Besides studying the theory of the subject and completing home assignments, the students also have to solve practical problems. These problems are solved in an experimental vibration analysis laboratory located in one of the rooms on campus. This room contains/provides all the equipment necessary for conducting the experiments, for example, data acquisition system, sensors, cables, glue, mechanical structures, amplifiers, shakers and so on. The students are first introduced to the dynamic system and the various setups by a supervisor, after the demonstration they have to book the laboratory to conduct the experiments in groups of two or three students. The required equipment is advanced and expensive, thus there is only equipment available for use by one group at a time. The separate sections of each experiments feature different pedagogical points. By allowing students to conduct experiments on a pre-configured experimental setup, they may focus in more easily on each of these points. For example, this may allow a student to experiment with settings related to data acquisition and estimators of vibration quantities; both of which are fundamental skills requires to collect quality data and produce reliable estimates. The result is a remote controlled experiment which is available to students at any time. The first prototype of a remote vibration laboratory [8] demonstrated the possibility of conducting vibration experiments over the Internet. However, the new version of the client-server structure has been improved substantially and is also integrated into an extensive booking system.

**Remote Experimental Vibration Analysis Laboratory**

The remote laboratory consists of a PC, a signal analyzer, an amplifier, two accelerometers, an impedance head, an electrodynamic shaker and a clamped boring bar attached rigidly to a heavy steel plate, see Fig. 1. The PC is the interface between the internet and the analyzer. The user controls all the settings necessary for performing the tasks through a virtual instrument panel. The instrument in the remote laboratory is designed and implemented to look as similar to the real front panel of the instrument as possible; the students will then recognize the instrument, handling it in the same way as they would in a traditional laboratory. Thus, the remote experiments carried out will
bear a close resemblance to the experiments carried out in a traditional, local laboratory.

Figure 1: To the left is the complete remote experiment setup presented with the HP35670A dynamic signal analyzer, an amplifier, the mechanical structure, the sensors and the screen belonging to the server. To the right is a close-up showing the mechanical parts of the remote experiment setup in detail.

1.1 Scheduling

The students must book a seat in the remote laboratory in much the same way as they would in the on-campus laboratory in order to be able to conduct the experiment. When logging on to the remote laboratory from the web page, the student will receive all information concerning the courses he/she is participating in. This may include everything from lecture notes to comments on the laboratory assignments, but at least an instruction manual for the assignment. The teacher predefines supervised sessions for students who are registered on the course, see Fig. 2.
Figure 2: An example of a schedule for supervised sessions, seen by a user (student) logged on to the remote laboratory.
1.2 The Front Panel

When the user begins an experiment, the front panel of the signal analyzer is displayed in the web-browser. The front panel is programmed using Macromedia Flash [9] which is supported on most systems and does not introduce security problems. In addition, Flash allows for smooth interaction with the interface so that users may, for example, rotate knobs on the virtual front panel using the mouse. Once the client has loaded, the web-browser displays the front panel (see Fig. 3) which is comparable to the real signal analyzer instrument (see Fig. 4). The user now has control of the instrument and no other users may use the instrument during this time.

![Figure 3: The virtual front panel of the signal analyzer, as displayed in a web-browser on a student client-pc during remote laboratory experiments.](image)

2 Laboratory Experiment Setup

The dynamic signal analyzer (model HP35670A from Agilent Technology) forms the main component of the experimental setup, producing the excita-
Figure 4: The front panel of the actual signal analyzer, this instrument is used by students in both the remote laboratory and in the traditional laboratory.

tion signal and collecting and analyzing the sampled version of the signals connected to its four inputs. The signal analyzer is connected to the server via a GP-IB interface. The analyzer source signal is amplified by an amplifier that powers an electrodynamic shaker. This shaker is mounted using four soft rubber strings in order to achieve an appropriate excitation of the structure [10], see Fig. 1. The shaker excites the boring bar via a slender stinger rod connected to the impedance head which is attached to the boring bar. The impedance head measures the force that is applied to the boring bar and the acceleration in the same point, i.e. the driving point. Two additional accelerometers are also attached to the boring bar with some distance between them. All four output signals, from the two accelerometers and the impedance head, are connected to the input channels of the analyzer, see Fig. 5. The data for the sensors is presented in Table 1. The boring bar is mounted in its holder under normal conditions and the holder is rigidly attached to a heavy steel construction in order to resemble the true case.
Figure 5: A block diagram of the experimental setup.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Type</th>
<th>S/N</th>
<th>Ch no.</th>
<th>Range [Hz]</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>8770A5</td>
<td>C191115</td>
<td>1</td>
<td>1-4000</td>
<td>226.62 mV/N</td>
</tr>
<tr>
<td>Acceleration</td>
<td>8770A5</td>
<td>C191115</td>
<td>2</td>
<td>10-7000</td>
<td>1028 mV/g</td>
</tr>
<tr>
<td>Acceleration</td>
<td>8704B25</td>
<td>2001535</td>
<td>3</td>
<td>10-7000</td>
<td>204 mV/g</td>
</tr>
<tr>
<td>Acceleration</td>
<td>8704B25</td>
<td>2001533</td>
<td>4</td>
<td>10-7000</td>
<td>201 mV/g</td>
</tr>
</tbody>
</table>

Table 1: Sensor description.
3 Examples of experimental vibration analysis carried out in the BTH remote laboratory

3.1 Modal Analysis

Modal analysis is defined as the process of characterizing the dynamic properties of a structure in terms of its modes of vibration. The experimental modal analysis is performed in order, either to confirm an analytical finite element model, or to characterize an unknown structure i.e. for the purpose of troubleshooting. The process of characterizing a structure or system is called modal parameter estimating, also referred to as curve fitting. There are many methods for modal analysis (e.g. peak picking and circle fitting) with different advantages and complexities [11]. The peak picking and circle fitting modal analysis methods rely on frequency response function (FRF) estimates, for extraction and determination of the modal parameter estimates for the structure undergoing analysis [11]. In experimental modal analysis, the selection of appropriate sensor locations (among other factors) will generally affect the results of the analysis. Whilst conducting, for example, an experimental modal analysis, it is important to dedicate sufficient time to preliminaries that will enable selection of appropriate excitation and response location. This will generally ensure higher quality in the resulting measurements and reliable modal parameter estimates.

However, the physical configuration (for example sensor location) of experimental setup in the remote vibration laboratory is already selected. Thus, making it suitable for students to examine the settings available on the instrument, such as; excitation signals, averaging, minimum leakage measurements by choosing various windows, adequate signal levels by steering the dynamic range of the A/D-converter etc. In the remote laboratory the goal of an experiment may be to extract the natural frequency, the relative damping and the mode shape of one of the low-order modes of the boring bar. An example of an experimental modal analysis of the boring bar is shown in Fig. 6. These results were captured from the screen during a conduction experiment via the client PC. Fig. 6 a) presents a Frequency Response Function (FRF) estimate, whilst Fig. 6 b) presents the corresponding coherence function.

3.2 Saving Data

As opposed to the previous version, the new client features the possibility to save the acquired data to the disk of a client PC. The procedure is similar to
saving the data on the signal analyzer, but instead of saving it to a diskette placed in the analyzer, a saving dialog will appear. It is therefore possible to download a variety of results and analyze them using any available tool. For example, students may currently export data from the data acquisition and analysis program "I-DEAS" into Matlab in order to compile results into a report. However, the new client allows student to use Matlab to the same effect. This allows the students to acquire accurate data and implement the mathematics behind the simplest estimators.

3.3 Partial Fraction Model

This method assumes that the Frequency Response Function (FRF) contains the local mode, and that any effects due to the other modes can be ignored. This method works adequately for structures whose FRF exhibit well separated modes [11]. An example of accelerances for such a structure -the boring bar in the remote experimental vibration laboratory- can be seen in Fig. 7, plotted in Matlab with data downloaded from the client.

The method is based on the modal model -expressed in terms of receptance-described by Eq. 1. Where $A_r$ and $\lambda_r$ is the residue and system pole, respectively, belonging to mode $r$. 

![Figure 6: Screen shots of the client PC presenting a) a frequency response function estimate after a frequency response measurement and b) the corresponding coherence function estimate.](image)
Figure 7: The estimate of the driving point, and between the driving point and respective accelerometer, a) the magnitude of boring bar accelerance functions and b) the corresponding phase functions.

\[ H(f) = \sum_{r=1}^{N} \frac{A_r}{j2\pi f - \lambda_r} + \frac{A^*_r}{j2\pi f - \lambda^*_r} \]  

(1)

Around the natural frequency \( f_r \) is the term \( \frac{A_r}{j2\pi f - \lambda_r} \) demenates, with negligible influence of all other terms in Eq. 1 are negligible [12].

Thus \( H(f) \) can be approximated by \( \hat{H}(f) \) as defined by Eq. 2;

\[ \hat{H}(f) = \frac{A_r}{j2\pi f - \lambda_r} \]  

(2)

An over-determined linear equation system such as Eq. 3 may be formed by selecting a number of frequencies close to the natural frequency and using Eq. 2. Preferably, these equations are solved by the least square method using (for example) the Moore-Penrose pseudo-inverse [13].

\[
\begin{bmatrix}
\hat{H}(f_0) & 1 \\
\hat{H}(f_1) & 1 \\
\vdots & \vdots \\
\hat{H}(f_K) & 1 \\
\end{bmatrix}
\begin{bmatrix}
\lambda_r \\
A_r \\
\vdots \\
A^*_r \\
\end{bmatrix}
= 
\begin{bmatrix}
j2\pi f_0 \hat{H}(f_0) \\
j2\pi f_1 \hat{H}(f_1) \\
\vdots \\
j2\pi f_K \hat{H}(f_K) \\
\end{bmatrix}
\]  

(3)
3.4 Circle fitting

The circle fitting method [12, 14] is another example of a modal analysis method. This method is also based on frequency domain data, but utilizes the fact that a mobility function forms a circle in the complex plane, close to each of its natural frequencies [12]. This can be observed in the Nyquist plots shown in Fig. 8.

![Circle fitting diagram](image)

Figure 8: a) Circle fitting diagram for a single degree of freedom system, b) a circle fitting diagram for a single degree of a multi degree of freedom system.

The aforementioned Single Degree of Freedom (SDOF) system has a physical interpretation; by describing this system in terms of mass $m$, spring $k$ and damper $c$, we yield the mobility function $H_m(f)$ as [12];

$$H_m(f) = \frac{j2\pi fX(f)}{F(f)}$$  \hspace{1cm} (4)

$$H_m(f) = \frac{2\pi f}{(k - (2\pi f)^2m) + j2\pi fc}$$  \hspace{1cm} (5)

where $X(f)$ is the displacement and $F(f)$ is the force. The real part $\Re(H_m(f))$ and imaginary part $\Im(H_m(f))$ will thus be defined as;

$$\Re(H_m(f)) = \frac{(2\pi f)^2c}{(k - (2\pi f)^2m)^2 + (2\pi fc)^2}$$  \hspace{1cm} (6)

$$\Im(H_m(f)) = \frac{2\pi f(k - (2\pi f)^2m)}{(k - (2\pi f)^2m)^2 + (2\pi fc)^2}$$  \hspace{1cm} (7)
By defining $A = \Re(H_m(f)) - 1/(2c)$ and $B = \Im(H_m(f))$ it can be seen that the Nyquist plot of the mobility is in fact a circle of radius $1/(2c)$ according to the following equation:

$$A^2 + B^2 = \left[ \Re(H_m(f)) - \frac{1}{2c} \right]^2 + [\Im(H_m(f))]^2 = \left[ \frac{1}{2c} \right]^2 \tag{8}$$

By approximate the influence from the other modes by a complex constant, the frequency response function near the the damped resonance frequency $f_{dr}$, can be expressed as [15];

$$H_m(f) \approx \left( jf_{dr} - f\zeta_r \right) (A_r) + R_r + jI_r \tag{9}$$

$$H_m(f) \approx \frac{U_r + jV_r}{f\zeta_r + j(f - f_{dr})} + R_r + jI_r \tag{10}$$

where $\zeta_r$ is the relative damping for mode $r$. The damped resonance frequency $f_{dr}$ is found at the maximum of change of angle, i.e. where the spacing between data points is largest in the complex plane. The relative damping $\zeta_r$ and the residue $A_r$ can be found as follows:

$$\zeta_r = \frac{f_2 - f_1}{f_{dr}(\tan(\frac{\theta_1}{2}) + \tan(\frac{\theta_2}{2}))} \tag{11}$$

selecting the half-power point frequencies $f_1$ and $f_2$ for which $\theta_1 = \theta_2 = 90^\circ$ yields

$$\zeta_r = \frac{f_2 - f_1}{2f_{dr}} \tag{12}$$

$$\Phi = \frac{\sqrt{U_r^2 + V_r^2}}{2\pi f_r \zeta_r} \tag{13}$$

$$\tan(\alpha) = \frac{U_r}{V_r} \tag{14}$$

$$A_r = U_r + jV_r. \tag{15}$$

where $\Phi$ is the diameter of the circle and $\alpha$ the angle of the damped resonance frequency $f_{dr}$, i.e. the angle between line $V_r$ and $CF_{dr}$ in Fig. 8 b). Fig. 9 presents an example of measured data downloaded from the client.
Figure 9: Three mobility functions presented as Nyquist plots. Note that the diagram is rotated 180° due to the fact that the positive direction of input force is defined in the opposite direction of acceleration by the transducers. This can be compared with the common analytical text book definition in which the positive direction of input force and acceleration by transducers are defined in the same direction.
3.5 Operating Deflection Shapes

Assume that we do not have access to either the input signal to the boring bar, nor the force signal - frequent problems in the experimental vibration of operating machines. If, however, output signals or vibration from different spatial locations of the structure are accessible, then the structure/system may be examined by observing the Operating Deflection Shapes (ODS). By considering the phase and amplitude of the response signals from \( N \) discrete points of an operating structure, it is possible to produce an estimate of \( \{ODS(f)\} \), where the cross power spectrum estimates and the power spectrum estimates may be used [16,17]. When estimating the spectral properties of a signal it is important to select an appropriate scaling of the spectrum estimator [16,17]. The spectrum estimates may be scaled for either the tonal components of a signal -power spectrum (PS) estimates- or the random part of a signal -power spectral density (PSD) estimates- [16]. In this experiment, the PSD estimate is preferable. The square root of the power spectra,
\[
\sqrt{\hat{P}_{nn}(f)}, \ n \in \{1, \ldots, N\}
\]
may be produced. By combining these spectra with the phase functions \( \hat{\theta}_{n1}(f), \ n \in \{2, \ldots, N\} \) of the cross power spectra \( \hat{P}_{n1}(f), \ n \in \{2, \ldots, N\} \), an estimate of the frequency domain operating deflection shape may be constructed as follows [6]:
\[
\{ODS(f)\}_{RMS} = \left\{\sqrt{\hat{P}_{11}(f)} \ \sqrt{\hat{P}_{22}(f)} e^{j\hat{\theta}_{21}(f)} \ \cdots \ \sqrt{\hat{P}_{NN}(f)} e^{j\hat{\theta}_{N1}(f)}\right\}^T
\]
\[ (16) \]

Here accelerometer 8770A5 has been used as reference signal and the result of the measurement is presented in Fig. 10 b).

4 Assessment

The evaluations aimed to determine whether the extra task contributed to and strengthened the student’s knowledge (regarding the experimental component of the course/assignment), revealed positive results. Evaluations where conducted through interviews with students, and comparatively, by contrasting the results obtained by students using the remote laboratory to complete the extra task, with a control group who did not. The interviews revealed substantial positive feedback; students considered the experimental component of the course/assignment to be the most important and found the remote lab to be a useful tool. In addition, students appreciated the opportunity to con-
duct the experiment at any time, in any location during the one-week period allotted. Feedback also provided suggestions for improving the administrative component of the assignment, some of which have already been implemented. Furthermore, comparative results demonstrated better performance on the part of the students using the remote laboratory than those who did not have this opportunity. Students using the remotely controlled laboratory also had fewer questions regarding setup parameters for vibration experiments in the traditional vibration laboratory.

5 Further Research

The next step for the remote laboratory is the implementation of a more flexible system in which it is possible to change the properties of a mechanical system - for example, mass, damp or stiffness. This step would also comprise the development of a more flexible measurement system, based on equipment provided by National Instruments.

In combination with a flexible mechanical system, equipment provided by National Instruments will enable more advanced and flexible analysis and increase the number of experiments possible to conduct. It is suggested that this also will incorporate sound and video transmission for some tasks, in
order to enhance the impression of conducting a real experiment.

6 Summary and Conclusions

The experiments detailed by this paper were successfully conducted by a group of students enrolled in the course *Sound and Vibration Analysis I* at BTH. Feedback from students was positive, they were surprised both by the resemblance of the remote controlled instrument and the real instrument and the simplicity and functionality of the remote controlled instrument. The scheduling was successful, however students also proposed a number of constructive ideas regarding enhancement of the reservation system.

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References


Vibration in internal turning is a problem in the manufacturing industry. Vibrations appear under the excitation applied by the material deformation process during the machining of a workpiece. In order for a lathe to perform an internal turning or boring operation, for example, in a pre-drilled hole in a workpiece, it is generally required that the boring bar should be long and slender; therefore extra sensitive to vibrations. These vibrations will affect the result of machining, in particular the surface finish, also the tool life may be reduced. As a result of tool vibration, severe acoustic noise frequently occurs in the working environment.

This thesis comprises three parts and the first part presents a method for active control of boring bar vibration. This method consists of an active boring bar controlled by, for example, an analog controller. The focus lies on the analog controller and the advantages that may be obtained from working in the analog domain. The controller is a lead-lag compensator with digitally controlled parameters, such as gain and phase. However, signals remain in the analog domain. In addition, the analog controller is compared with a digital adaptive controller and it is found that both controllers yield an attenuation of the vibration by up to 50 dB.

The second part of this thesis concerns the dynamic properties of a clamped boring bar used by the industry. In order to design a robust controller for a certain system, knowledge about the system’s dynamic properties is required. On the workshop floor, a boring bar is dismounted and remounted, and reconfiguration of boring bars will alter the dynamic properties of the clamped boring bar. The dynamic properties of a standard boring bar and an active boring bar for a number of possible clamping conditions, as well as for a linearized clamping have been investigated based on an experimental approach. Also simple Euler-Bernoulli modeling of clamped boring bars incorporating simple non-rigid models of the clamping and boring bar clamping are investigated. Initial simulations of nonlinear SDOF systems have been carried out; one with a signed squared stiffness and one with a cubic stiffness. The purpose of these simulations was to identify a nonlinearity that introduces a similar behavior in the SDOF system dynamics as the nonlinear behavior observed in the dynamic properties of a clamped boring bar.

The third and final part of this thesis focuses on vibration analysis methods in engineering education. A signal analyzer (which is a commonly used instrument in signal processing and vibration analysis) was made accessible via the Internet. Assignments were developed for students to learn and practice vibration analysis on real signals from a real setup of a relevant structure; a clamped boring bar. Whilst the experimental setup was fixed, the instrument and sensor configuration nonetheless enable a variety of experiment, for example: excitation signal analysis, spectrum analysis and experimental modal analysis.