Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions

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Abstract

The boring bar is one of the most widely used type of tool holders in metal cutting operations. The turning process subjects the tool to vibrations, and cutting in deep workpiece cavities is likely to result in high vibration levels. The consequences of such vibration levels generally results in: reduced tool life, poor surface finishing and disturbing sound. Internal turning frequently requires a long and slender boring bar in order to machine inside a cavity, and the vibrations generally become highly correlated with one of the fundamental bending modes of the boring bar. Different methods can be applied to reduce the vibrations, the implementation of the most efficient and stable methods require in depth knowledge concerning the dynamic properties of the tooling system. Furthermore, the interface between the boring bar and the clamping house has a significant influence on the dynamic properties of the clamped boring bar. This report focuses on the dynamic properties of a boring bar that arise under different clamping conditions of the boring bar and are introduced by a clamping house (commonly used in the manufacturing industry). The dynamic properties of a boring bar (for different cases of boundary condition of the boring bar) are presented partly analytically but also experimentally.

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Chapter 1

Introduction

In industry where metal cutting operations such as turning, milling, boring and grinding take place, degrading vibrations are a common problem. In internal turning operations vibration is a pronounced problem, as long and slender boring bars are usually required to perform the internal machining of workpieces. Tool vibration in internal turning frequently has a degrading influence on surface quality, tool life and production efficiency, whilst also resulting in severe environmental issues such as high noise levels. By applying, for example, an active control scheme, these vibrations can be significantly reduced, with the result of improved workpiece surface finish and increased tool life [1]. In order to successfully implement such a scheme, the dynamic properties of system (boring bar - clamping structure) be known, as must the nature of the disturbing vibrations.

A number of experimental studies have been carried out on mechanisms explaining tool vibration during turning operations [2, 3, 4] and on the dynamic properties of boring bars [5, 6, 7]. In 1946 the principles of the traditional theory of chatter in simple machine-tool systems were worked out by Arnold [8] based on experiments carried out on a rigid lathe, using a stiff workpiece but a flexible tool. In this way he was able to investigate chatter under controlled conditions. Later in 1965 Tobias [4] presented further investigations of the chatter phenomena, involving, for example, the chip-thickness variation and the phase lag of the undulation of the surface. Also, in the same year, Meritt et al. [2] discussed the stability of structures with n -degrees of freedom, assuming no dynamics in the cutting process; they also proposed a simple stability criterion. Parker et al. [9] investigated the stability behavior of a slender boring bar by representing it with a two-degree-of-freedom mass-spring-damper system and experimenting with regenerative cutting. They also investigated how the behavior of the vibration was affected by coupling between modes, by using different cutting speeds, feed rates and angles of the boring bar head rel-

ative to the two planes of vibrations. Pandit et al. [10] developed a procedure for modeling chatter from time-series by including unknown factors of random disturbances present in the cutting process, they formulated self-excited random vibrations with white noise as a forcing function. Kato et al. [11] investigated regenerative chatter vibration due to deflection of the workpiece, and introduced a differential equation describing chatter vibration based on experimental data. Furthermore, various analytical models/analysis methods relating to the boring bar/or cutting process have been continuously developed, assuming various conditions. For example, Zhang et al. [5] who's model is derived from a two-degree-of-freedom model of a clamped boring bar and four cutting force components. In addition, Rao et al. [6] includes variation of chip cross-sectional area in their model, whilst Kuster et al. [12] developed a computer simulation based on a three-dimensional model of regenerative chatter. Walter et al. [13] developed a model of the chuck-workpiece connection when the workpiece is considered to be weak, using Finite Element Method (FEM) and experimental studies of a ring shaped weak workpiece; this model focused on the influence from clamping forces when using jaw chucks. A time series approach was used by Andrén et al. [7] to investigate boring bar chatter and the results were compared with an analytical Euler-Bernoulli model. Also, Euler-Bernoulli beam modeling, experimental modal analysis and operating deflection shape analysis were used by Andrén et al. [14] to investigate the dynamic properties of a clamped boring bar. Results obtained demonstrate observable differences concerning the fundamental bending modes. They found that that the bending motion of the first two resonance frequencies is, to a large extent, in the direction of cutting speed. Scheuer et al. [15] investigated the dynamic properties of a boring bar, based on experimental modal analysis under different clamping conditions. Two different clamping houses were used: one clamping from two sides with clamp screws and one circular clamping sleeve; clamping along the circular surface of the boring bar. Results indicate that both the eigenfrequencies and the directions of the fundamental bending modes vary for different clamping pressures; in particular, for the circular clamping sleeve.

The problem of boring bar vibration can be addressed using conventional methods, such as redesigning the machine tool system, implementing tuned passive damping or implementing active control [16, 17]

However, the order of stability improvement achieved usually correlates to the quality and extent of knowledge of the dynamic properties of the tooling structure -the interface between the cutting tool, or insert, and the machine tool. Boring bar vibrations are usually directly related to the lower order bending modes and the dynamic properties of a boring bar installed in a lathe are directly influenced by the boundary conditions, i.e. the clamping of the

bar [14]. Following the literature review, it appears that little work has been done on the clamping properties' influence on the dynamic properties of a clamped boring bar. Thus, it is of significance to investigate the clamping properties' influence on the dynamic properties of the clamped boring bar in order to gain further understanding of the dynamic behavior of clamped boring bars in the metal cutting process. This report focuses on the variation in the dynamic properties of a clamped boring bar imposed by controlled discrete variations in the clamping conditions produced by a standard clamping house of the variety commonly used in industry today. The clamping house has a circular cavity that the boring bar fits easily into; the clamping is then carried out by means of screws on the tool side and on the opposite side of the boring bar. To investigate the influence of clamping properties on the dynamic properties of a clamped boring bar, experimental modal analysis have been conducted both for a clamped standard boring bar and a clamped active boring bar under different clamping conditions. Also, analytical Euler-Bernoulli beam models incorporating clamping flexibility through the use of transverse springs and rotational springs have been investigated for the modeling of a clamped boring bar. Finally, some simulations of nonlinear models have also been studied for observed nonlinear behavior of the clamped boring bar.

Chapter 2

Materials and Methods

Experimental modal analysis has been carried out on different boring bars for various clamping conditions in order to investigate the changes of the the clamped boring bar's dynamic properties.

2.1 Experimental Setup

The experimental setup and subsequent measurements were carried out in a Mazak SUPER QUICK TURN - 250M CNC turning center. The CNC lathe has 18.5 kW spindle power and a maximal machining diameter of 300 mm, with 1005 mm between the centers, a maximal spindle speed of 400 revolutions per minute (r.p.m.) and a flexible turret with a tool capacity of 12 tools. The lathe is presented by the photo in Fig. 2.1

Figure 2.1: a) Mazak SUPER QUICK TURN - 250M CNC lathe and b) the room in the lathe where machining is carried out.

Initially, a right-hand cartesian coordinate system was defined: z in the feed direction, y in the reversed cutting speed direction and x in the direction of cutting depth, see Fig. 2.1 b) (upper left corner). Subsequently a sign convention was defined for use throughout the report. The coordinate system and sign convention are based on the right-hand definition where the directions of displacements and forces in positive directions of the coordinate axes are considered positive. Moreover, moment about an axis in the clockwise direction (when viewing from the origin in the positive direction of the axis) is considered positive, see Fig. 2.2.

Figure 2.2: Right hand definition of the cartesian coordinate system and the sign convention, where direction shown by the arrows defines the positive direction of displacement, force and moment.

The boring bars were positioned in the operational position, mounted in a clamping house attached to a turret with screws, during all measurements. The turret may be controlled to move in the cutting depth direction, x -direction, and in the feed direction, z -direction, as well as to rotate about the z -axis for tool change. The turret, etc. is supported by a slide which in turn, is mounted onto the lathe bed. Even though the turret is a movable component, it is relative rigid, rendering the dynamic properties of the boring bars observable.

2.1.1 Measurement Equipment and Setup

The following equipment was used in the experimental setup;

- 12 Brüel & Kjær 333A32 accelerometer
- 2 Brüel & Kjær 8001 Impedance Head
- 1 Brüel & Kjær NEXUS 2 channel conditioning amplifier 2692.
- OSC audio power amplifier, USA 850
- Ling Dynamic Systems shaker v201
- Gearing & Watson Electronics shaker v4

- A custom designed amplifier for capacitive loads.
- Active boring bar with embedded piezo ceramic actuator.
- Hewlett Packard 54601B Oscilloscope.
- Hewlett Packard 35670A Signal Analyzer.
- Hewlett Packard VXI Mainframe E8408A.
- Hewlett Packard E1432A 4-16 Channel 51.2 kSa/s Digitizer
- PC with I-DEAS 10 NX Series

Twelve accelerometers and two cement studs for the impedance heads were attached onto the boring bars with X60 glue (a cold hardener two component glue). The sensors were evenly distributed along the centerline, on the under-side and on the back-side of the boring bar; six accelerometers and one stud on the respective side (see Figs. 2.3 and 2.4). To excite all the lower order bending modes, two shakers were attached via stinger rods to the impedance heads, one in the cutting speed direction (y -) and the other in the cutting depth direction (x -) see Fig. 2.3. The shakers were positioned relatively close to the cutting tool.

Figure 2.3: The experimental setup, in a) two shakers suspended from the ceiling are observable as well as a workpiece and the turret. b) shows a closeup of the sensors and the shaker configuration on the boring bar.

Figure 2.4: Drawings of the boring bar including clamp screws, cement studs and sensors. The sensors are attached along the underside and the backside of the boring bar. The threaded holes denoted B_1 , B_2 and B_1 are screw positions for clamping the boring bar from top and bottom. The dimension are in mm, where $l_1 = 10.7$, $l_2 = 18$, $l_3 = 101$, $l_4 = 250$, $l_5 = 17$, $l_6 = 100$, $l_7 = 18.5$, $l_8 = 25$ and $l_9 = 35$.

2.1.2 Boring Bars

Two different boring bars were used in the experimental setup in order to be able to analyze the changes of the eigenfrequencies and the mode-shapes in different cases. The first boring bar used in the modal analysis was a standard "non-modified" boring bar, WIDAX S40T PDUNR15F3 D6G, presented in Fig. 2.5. The second boring bar used in this experiment was an active bor-

Figure 2.5: a) Top-view of the standard boring bar "WIDAX S40T PDUNR15F3 D6G, b) the cross section of the boring bar where C_C is the center of the circle and M_C is the mass center of the boring bar."

ing bar, based on the standard WIDAX S40T PDUNR15 boring bar, with an accelerometer and an embedded piezo-stack actuator, see Fig. 2.6. The accelerometer was mounted 25 mm from the tool tip to measure the vibrations in the cutting speed direction (y -). This position was as close as possible to the tool tip, but at a sufficient distance to prevent metal-chips from the material removal process from damaging the accelerometer. The actuator was embedded into a milled space in the longitudinal direction (z -direction), below the centerline of the boring bar. By embedding accelerometers and piezo stack actuators in conventional boring bars, a solution was obtained for the introduction of control force to the boring bar with physical features and properties that fit the general lathe application. Assuming a constant cross-section along the boring bar, neglecting the head, the dimensions from Fig. 2.5 b) result in a cross-sectional area A and a moment of inertia I_x, I_y presented by Table 2.1. The standard WIDAX S40T PDUNR15 boring bar is manufactured in the material 30CrNiMo8, (AISI 4330) which is a heat treatable steel alloy (for high strength), see Table 2.2 for material properties.

2.1.3 Clamping Houses

The clamping house is a basic 8437-0 40mm Mazak holder, presented in Fig. 2.7 a) and b), and clamps the tool holder by means of either four or six screws:

Figure 2.6: The active boring bar with an accelerometer close to the tool tip and an embedded piezo-stack actuator in a milled space below the centerline.

Variable	Value	Unit
A	$1.19330295 \cdot 10^{-3}$	m^2
I_x	$1.13858895 \cdot 10^{-7}$	m^4
I_y	$1.13787080 \cdot 10^{-7}$	m^4

Table 2.1: Cross-sectional properties of the boring bar, illustrated in Fig. 2.5 b), where A is the cross-sectional area and I_x , I_y are the moments of inertia around the denoted axis.

Material composition besides Fe in percent							
C	Cr	Ni	Mo	Si	Mn	S	P
0.26-0.33	1.80-2.20	1.80-2.20	0.30-0.50	<0.40	<0.60	<0.035	<0.035
Material properties							
Young's Modulus		Tensile Strength		Yield Strength		Density	
205 GPa		1250 MPa		1040 MPa		7850 kg/m ³	

Table 2.2: Composition and properties of the material 30CrNiMo8.

two/three from the top and two/three from bottom. The basic holder itself is mounted onto the turret with four screws.

In addition to the screws, the clamping house also features a guide matching a track on the turret; this guide positions the clamping house along the z-axis on the xy plane, whilst the guide pin positions the clamping house on the z-axis, see Fig. 2.7 a). The clamping house has a default thread size of M8 for the

Figure 2.7: The clamping house. a) The guide and the guide pin may be observed on the one word of the clamping house, whilst the threaded holes for the screws clamping the boring bar are shown on the right side. b) The screw positions for attaching the clamping house to the turrets are shown from the top side.

screws clamping the boring bar. A second clamping house of the same model was rethreaded to the thread size M10. Furthermore, a third clamping house, also of the same type, was used in the construction of a so-called "linearized" clamping of the boring bar.

2.1.4 Clamping Conditions

A number of different setups were considered using different boring bars described in section 2.1.2 in conjunction with the different clamping houses. In the first setup, the reference boring bar was clamped using four M8 class 8.8 screws. The screws were tightened first from the top and then from the underside.

The recommended tightening torque for this class is 26.6Nm, however, evaluations of the screws revealed that threads remained intact and screws did not break for a tightening torque of 30Nm.

The second setup involved the same five torques as for the previous setup, but four screws of size M10 class 8.8, which were, again, tightened first from the top and then from the underside. As only four clamp screws were used, the clamping house center screw positions were not used.

The third setup involved the use of six screws of size M10, with the reference bar and same torques as previous. The use of six screws involved the use of all clamping house center screw positions. Setup one and two were then repeated, using the active boring bar.

In order to accomplish a linearized clamping condition, the standard clamping was modified. A boring bar WIDAX S40T PDUNR15F3 D6G, the same model as the standard boring bar, was used together with three steel wedges produced of the material SS 1650 (AISI 1148). The steel wedges were glued with epoxy on the flat surfaces of the boring bar along the clamping length of the bar end. The steel wedges were shaped geometrically to render a circular cross section on the boring bar along its clamped end. After the epoxy was set; the boring bar end with circular cross section was pressed into the clamping house and glued to it with epoxy to make the clamping rigid, see Fig 2.8.

Figure 2.8: The linearized boring bar-clamping house setup.

2.2 Experimental Modal Analysis

The primary goal of experimental modal analysis is to identify the dynamic properties of the system under examination, the modal parameters; i.e. determine the natural frequencies, mode shapes, and damping ratios from experimental vibration measurements. The procedure of modal analysis may be divided into two parts: the acquisition of data and the parameter estimation or parameter identification from these data, also known as curve fitting [18].

These procedures are often referred to as a discipline of art since the process of acquiring good data and performing accurate parameter identification is an iterative process, based on various assumptions along the way [18].

2.2.1 Spectral Properties

Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [19].

A non-parametric estimate of the power spectral density $P_{xx}(f)$, where f is frequency, for a signal $x(t)$ may be estimated using the Welch spectrum estimator [20], given by:

$$\hat{P}_{xx}(f_k) = \frac{1}{LNF_s} \sum_{l=0}^{L-1} \left| \sum_{n=0}^{N-1} w(n)x_l(n)e^{-j2\pi nk/N} \right|^2, \quad f_k = \frac{k}{N}F_s \quad (2.1)$$

where $k = 0, \dots, N-1$, L is the number of periodograms, N is the length of the data segments used to produce the periodograms, $x_l(n)$ is the sampled signal in segment l , F_s is the sampling frequency.

Thus, for each input signal $x(t)$ and output signal $y(t)$, a single-input-single-output system (SISO system) is simultaneously measured and the sampled signal $y(n)$ and $x(n)$ are recorded. By using, for example, the Welch spectrum estimator [20], the cross-power spectral density $\hat{P}_{yx}(f_k)$ between the input signal $x(n)$ and the output signal $y(n)$ and the power spectral density $\hat{P}_{xx}(f_k)$ for the input signal $x(n)$ may be produced [14, 19].

A least-squares estimate of a frequency response function between the input signal $x(n)$ and the output signal $y(n)$ may be produced according to [19]:

$$\hat{H}(f_k) = \frac{\hat{P}_{yx}(f_k)}{\hat{P}_{xx}(f_k)} \quad (2.2)$$

and the coherence function as [19]

$$\gamma_{yx}^2(f_k) = \frac{\hat{P}_{yx}(f_k)\hat{P}_{xy}(f_k)}{\hat{P}_{xx}(f_k)\hat{P}_{yy}(f_k)} \quad (2.3)$$

In the case of a multiple-input-multiple-output system (MIMO system) with P number of responses and Q number of references, an estimate of the cross spectrum matrix $[\hat{\mathbf{P}}_{xx}(f_k)]$ between all the inputs is produces as

$$[\hat{\mathbf{P}}_{xx}(f_k)] = \begin{bmatrix} \hat{P}_{x_1x_1}(f_k) & \hat{P}_{x_1x_2}(f_k) & \cdots & \hat{P}_{x_1x_Q}(f_k) \\ \hat{P}_{x_2x_1}(f_k) & \hat{P}_{x_2x_2}(f_k) & \cdots & \hat{P}_{x_2x_Q}(f_k) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{x_Qx_1}(f_k) & \hat{P}_{x_Qx_2}(f_k) & \cdots & \hat{P}_{x_Qx_Q}(f_k) \end{bmatrix} \quad (2.4)$$

where the diagonal elements is power spectral densities for the respective input signal. Also a cross spectrum matrix $[\hat{\mathbf{P}}_{yx}(f_k)]$ between all the inputs and outputs may be estimated as

$$[\hat{\mathbf{P}}_{yx}(f_k)] = \begin{bmatrix} \hat{P}_{y_1x_1}(f_k) & \hat{P}_{y_1x_2}(f_k) & \cdots & \hat{P}_{y_1x_Q}(f_k) \\ \hat{P}_{y_2x_1}(f_k) & \hat{P}_{y_2x_2}(f_k) & \cdots & \hat{P}_{y_2x_Q}(f_k) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{y_Px_1}(f_k) & \hat{P}_{y_Px_2}(f_k) & \cdots & \hat{P}_{y_Px_Q}(f_k) \end{bmatrix} \quad (2.5)$$

The least-square estimate for the (SISO) system in Eq. 2.2, can be rewritten for the (MIMO system) yielding the estimate of the system matrix $[\hat{\mathbf{H}}(f_k)]$ as [19]

$$[\hat{\mathbf{H}}(f_k)] = [\hat{\mathbf{P}}_{yx}(f_k)] [\hat{\mathbf{P}}_{xx}(f_k)]^{-1} \quad (2.6)$$

In the case of a multiple inputs, case, the multiple coherence is of interest as a quality of the measurements. The multiple coherence function is defined by the ratio of that part of the spectrum which can be expressed as a linear function of the inputs to the total output spectrum (including extraneous noise), and which is an extension of the ordinary coherence function from the SISO case. If the inputs are uncorrelated, the multiple coherence $\gamma_{y_p:x}^2(f)$ for the response in point p is given by [19]

$$\gamma_{y_p:x}^2(f) = \gamma_{y_px_1}^2(f) + \gamma_{y_px_2}^2(f) + \cdots + \gamma_{y_px_Q}^2(f) \quad (2.7)$$

where Q is the number of inputs and ":" denotes "linear dependent on". However, usually there is are some correlations between the inputs, then the multiple coherence is given by [19]

$$\gamma_{y_p:x}^2(f) = 1 - (1 - \gamma_{y_p:x_1}^2(f))(1 - \gamma_{y_p:x_{2..1}}^2(f)) \cdots (1 - \gamma_{y_p:x_{Q..(Q-1)!}}^2(f)) \quad (2.8)$$

where "." denotes "independent off" and "!" denotes factorial. The multiple coherence may also be expressed using an expanded spectral matrix $[\mathbf{P}_{y_pxx}(f)]$, who's determinant is a measure of extraneous on the output and is written as

$$[\mathbf{P}_{y_pxx}(f)] = \begin{bmatrix} P_{y_py_p}(f) & P_{y_px_1}(f) & \cdots & P_{y_px_Q}(f) \\ P_{x_1y_p}(f) & P_{x_1x_1}(f) & \cdots & P_{x_1x_Q}(f) \\ \vdots & \vdots & \ddots & \vdots \\ P_{x_Qy_p}(f) & P_{x_Qx_1}(f) & \cdots & P_{x_Qx_Q}(f) \end{bmatrix} \quad (2.9)$$

Based on the extended spectral matrix the multiple coherence may be expressed

$$\gamma_{y_p:x}^2(f) = 1 - \left(\frac{|[\mathbf{P}_{y_pxx}(f)]|}{P_{y_py_p}(f) |[\mathbf{P}_{xx}(f)]|} \right) \quad (2.10)$$

The normalized random error in frequency response function estimates for the amplitude function [19] is approximately given by

$$\varepsilon_r(|\hat{H}_{yx}(f_k)|) \approx \frac{\sqrt{(1 - \hat{\gamma}_{y:x_{Q \cdot (Q-1)!}}^2(f_k))}}{\sqrt{\hat{\gamma}_{y:x_{Q \cdot (Q-1)!}}^2(f_k)2(L_e + 1 - Q)}} \quad (2.11)$$

for the phase function it is approximately given by [19]

$$\varepsilon_r(|\hat{\Theta}_{yx}(f_k)|) \approx \arcsin\left(\varepsilon_r(|\hat{H}_{yx}(f_k)|)\right) \quad (2.12)$$

Finally, an estimate of the normalized random error for the multiple coherence function is given by [19]

$$\varepsilon_r(\hat{\gamma}_{yp:x}^2(f_k)) \approx \frac{\sqrt{2}(1 - \hat{\gamma}_{yp:x}^2(f_k))}{\sqrt{\hat{\gamma}_{yp:x}^2(f_k)(L_e + 1 - Q)}} \quad (2.13)$$

where L_e is the number of uncorrelated periodograms [19, 20] used in the average to produce the spectrum estimate.

2.2.2 Parameter Estimation

There are several different methods for identification of the modal parameters [18, 21]. There are two basic curve fitting methods: curve fitting in frequency domain using measured Frequency Response Function (FRF) data and a parametric model of the FRF; or curve fitting towards the measured Impulse Response Function (IRF) data using a parametrical model of the IRF [18]. Many methods use both domains, depending on which parameter is estimated [18]. A parametric model of an FRF, $H(f)$, expressed as the receptance between the reference point, input signal q , and the response, output signal in point p of a structure, may be written as [18],

$$H_{pq}(f) = \sum_{r=1}^N \frac{A_{pqr}}{j2\pi f - \lambda_r} + \frac{A_{pqr}^*}{j2\pi f - \lambda_r^*} \quad (2.14)$$

where r is the mode number, N the number of modes used in the model, A_{pqr} the residue belonging to mode r between reference point q and response p and λ_r is the pole belonging to mode r . The parametric model of the IRF, input force to output displacement impulse response may be expressed as

$$h_{pq}(t) = \sum_{r=1}^N A_{pqr} e^{\lambda_r t} + A_{pqr}^* e^{\lambda_r^* t} \quad (2.15)$$

Due to the fact that two sources (references) were used during data acquisition, a method capable of handling multi-references is required. One such method is the Polyreference least square complex exponential method developed by Vold [22, 23]. This method is defined for identification of MIMO-systems with the purpose of obtaining a global least squares estimates of the modal parameters. The estimated system matrix $[\hat{\mathbf{H}}(f)]$ is of size $P \times Q$, where P is the number of responses and Q the number of references, and is written as

$$[\hat{\mathbf{H}}(f)] = \begin{bmatrix} \hat{H}_{11}(f) & \hat{H}_{12}(f) & \cdots & \hat{H}_{1Q}(f) \\ \hat{H}_{21}(f) & \hat{H}_{22}(f) & \cdots & \hat{H}_{2Q}(f) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{H}_{P1}(f) & \hat{H}_{P2}(f) & \cdots & \hat{H}_{PQ}(f) \end{bmatrix} \quad (2.16)$$

The procedure of modal parameter estimation starts by determining the model order of the system under analysis. This can be done with the aid of a Mode Indicator Function (MIF) and a stability diagram [21]. The function used was the multivariate MIF and is expressed as [24]

$$\min_{\|\{\mathbf{F}(f)\}\|_2=1} \left(\frac{\{\mathbf{F}(f)\}^T [\mathbf{H}_{\Re}(f)]^T [\mathbf{H}_{\Re}(f)] \{\mathbf{F}(f)\}}{\{\mathbf{F}(f)\}^T ([\mathbf{H}_{\Re}(f)]^T [\mathbf{H}_{\Re}(f)] + [\mathbf{H}_{\Im}(f)]^T [\mathbf{H}_{\Im}(f)]) \{\mathbf{F}(f)\}} \right) = \lambda(f) \quad (2.17)$$

which yields a value $0 \leq \lambda(f) \leq 1$, where $[\mathbf{F}(f)]$ is a force vector, $[\mathbf{H}_{\Re}(f)]$ and $[\mathbf{H}_{\Im}(f)]$ is the real part and imaginary part, respectively, of the system matrix $[\mathbf{H}(f)]$ and T is the transpose operator. This minimization problem can be reformulated into an eigenvalue problem as [21]

$$[\mathbf{H}_{\Re}(f)]^T [\mathbf{H}_{\Re}(f)] \{\mathbf{F}(f)\} = ([\mathbf{H}_{\Re}(f)]^T [\mathbf{H}_{\Re}(f)] + [\mathbf{H}_{\Im}(f)]^T [\mathbf{H}_{\Im}(f)]) \{\mathbf{F}(f)\} \lambda(f) \quad (2.18)$$

where the smallest eigenvalue $\lambda(f)$ corresponds to the minimization problem in Eq. 2.17. Eq. 2.18 forms an eigenvalue problem of size $Q \times Q$, thus the problem yields the same number of solutions for each frequency as the number of sources. Plotting all solutions, repeated roots will be detected if the references excited those modes.

A stability diagram is constructed using estimates of systems poles and modal participation factors as a function of model order [21]. As the model order is increased, more and more modal frequencies are estimated but, hopefully, the estimates of the physical modal parameters will stabilize as the correct model order is found. From empirical evaluation of the stability diagram, the physical "true" poles seem to asymptotically go to the true values, whereas computational (nonphysical) poles which arise due to leakage, low signal to noise ratio (SNR), frequency shift etc, appear more unstructured [21]. Using the stability diagram with the multivariate MIF overlayed, stable poles which

appear to have physical correspondence are selected. Along with the poles and a driving point, real or complex residues are estimated. Mode shapes were estimated using the frequency polyreference method [25].

As quality assessment of the estimated parameters the FRF's were synthesized using the estimated parameters and overlayed with the estimated FRF's. Furthermore the Modal Assurance Criterion (MAC)[18] defined by Eq. 2.19.

$$MAC_{kl} = \frac{(\{\psi\}_k^H \{\psi\}_l)^2}{\{\psi\}_k^H \{\psi\}_k \{\psi\}_l^H \{\psi\}_l} \quad (2.19)$$

was used as a measure of correlation between mode shape $\{\psi\}_k$ belonging to mode k , and mode shape $\{\psi\}_l$ belonging to mode l , where H is the Hermitian transpose operator.

2.2.3 Excitation Signal

For the experimental modal analysis, burst random was used as the excitation signal. Based on initial experiments concerning suitable burst length and frequency resolution (data segment time or data block length time), a burst length of 90% of the data block length time was selected, see Table 2.3. Coherence function estimates and magnitude functions of frequency response function estimates were utilized for the selection of burst length and frequency resolution. Basically, the frequency resolution was tuned to provide high overall coherence in the analysis bandwidth and the burst length was tuned to provide high coherence at resonance frequencies. In other words, the time of the dead period of the burst random signal was set so as to be sufficiently long enough to enable the structural response to decay in order to render influences from leakage negligible. The data block length was set so as to maintain a sufficient signal to noise ratio.

Furthermore, four different excitation levels with the proportion $\{1, 2, 3, 4\}$ were applied for each of the boundary conditions of the boring bars. By using a number of different excitation levels and carrying out system identification for each of the excitation levels, differences between the estimates of the system may indicate nonlinear behavior of the system and might provide information concerning the structure of the nonlinearity or the nonlinearities involved.

2.3 Analytical Models of the Boring Bars

The boring bar has a cross section $A(z)$ and a length of l . Also associated with the beam is a flexural (bending) stiffness $EI(z)$, where E is Young's

Parameter	Value
Excitation signal	Burst Random
Sampling Frequency fs	10240 Hz
Block Length N	20480
Frequency Resolution Δf	0.5 Hz
Number of averages L	200
Window	Rectangular
Overlap	0%
Frequency Range of Burst	0-4000 Hz
Burst Length	90%

Table 2.3: Spectral density estimation parameters.

elastic modulus for the beam and $I(x)$ is the cross-sectional area moment of inertia about the "z axis." From mechanic theory, the beam sustains a bending moment $M(z, t)$, which is related to the beam deflection, or bending deformation $u(z, t)$, by the conservation of momentum. This can be derived as follows [26]: when the beam deflects, see Fig. 2.9, the longitudinal displacement

Figure 2.9: Deflection model of a beam undergoing pure bending.

$\theta(z, y, t)$ will cause a strain (illustrated by the arrow-field in the figure) due to the rotation of the cross-sectional plane which is parallel with s in Fig. 2.9. Hooke's law states that the uniaxial stress σ (or axial force per unit sectional area) applied to a bar in the z direction is proportional to the strain ε (or elongation per unit length) within the elastic limit according to

$$E = \frac{\sigma}{\varepsilon} \quad (2.20)$$

Assuming that the cross-section-plane remains flat after deformation; then the relation of strain ε and stress σ is given by

$$\varepsilon_{zz}(z, y, t) = \frac{\partial \theta(z, y, t)}{\partial z} = -y \frac{\partial^2 u(z, t)}{\partial z^2} \quad (2.21)$$

$$\sigma_{zz}(z, y, t) = -yE \frac{\partial^2 u(z, t)}{\partial z^2} \quad (2.22)$$

The total moment about an axis parallel with the x-axis in the mid-section, where strain is zero, will cause to stress become

$$M_x(z, t) = \iint_{y(z) \ x(z)} \sigma_{zz}(x, y, t) y dx dy \quad (2.23)$$

using the relation in Eq. 2.22, we get

$$M_x(z, t) = -E \frac{\partial^2 u(z, t)}{\partial z^2} \iint_{y(z) \ x(z)} y^2 dx dy \quad (2.24)$$

where

$$I_x(z) = \iint_{y(z) \ x(z)} y^2 dx dy \quad (2.25)$$

and we get

$$M_x(z, t) = -EI_x(z) \frac{\partial^2 u(z, t)}{\partial z^2} \quad (2.26)$$

A model of bending vibration may be derived by examining the force diagram of an infinitesimal element of the beam[27] as indicated in Fig. 2.10. Assuming the deformation is small enough so that the shear deformation is much smaller than displacement $u(z, t)$ (i.e., so that the sides of the element dz do not bend), a summation of forces in the y direction yields

$$V(z, t) + \frac{\partial V(z, t)}{\partial z} dz - V(z, t) + f(z, t) dz = \rho A(z) dz \frac{\partial^2 u(z, t)}{\partial t^2} \quad (2.27)$$

Here $V(z, t)$ is the shear force at the left end of the element dz , $V(z, t) + \partial V(z, t)/\partial z dz$ is the shear force at the right end of the element dz , $f(z, t)$ is the total external force applied to the element per unit length, and the term on the right side of the equality sign is the inertial force of the element. The assumption of small shear deformation used in the force balance is true if

Figure 2.10: Simple beam with transverse vibration and a free-body diagram of a small element of the beam as it is deformed by a distributed force per unit length, denoted $f(z, t)$.

length divided by the smallest radius of the beam is less than 10 (i.e., for long slender beams). Next, the moments acting on the element dz about the x axis through point Q are summed. This yields

$$M_x(z, t) - \left[M_x(z, t) + \frac{\partial M_x(z, t)}{\partial z} dz \right] + \left[V(z, t) + \frac{\partial V(z, t)}{\partial z} dz \right] dz + [f(z, t) dz] \frac{dz}{2} = 0 \quad (2.28)$$

Here, the left-hand side of the equation is zero since the rotary inertia of element dz is assumed to be negligible, simplifying the expression yields

$$\left[V(z, t) - \frac{\partial M_x(z, t)}{\partial z} \right] dz + \left[\frac{\partial V(z, t)}{\partial z} + \frac{f(z, t)}{2} \right] (dz)^2 = 0 \quad (2.29)$$

Since dz is assumed to be very small, $(dz)^2$ is assumed to be almost zero, thus the moment expression becomes

$$V(z, t) = \frac{\partial M_x(z, t)}{\partial z} \quad (2.30)$$

This states that the shear force is proportional to the spatial change in the bending moment. Substitution of this expression for the shear force into Eq. 2.27 yields

$$\frac{\partial^2}{\partial z^2} M_x(z, t) dz + f(z, t) dz = \rho A(z) dz \frac{\partial^2 u(z, t)}{\partial t^2} \quad (2.31)$$

Further substitution of Eq. 2.26 into Eq. 2.31 and dividing by dz yields

$$\rho A(z) \frac{\partial^2 u(z, t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left[EI(z) \frac{\partial^2 u(z, t)}{\partial z^2} \right] = f(z, t) \quad (2.32)$$

Eq. 2.32 is often referred to as the Euler-Bernoulli beam equation. The assumptions regarding the beam, used in formulating this model are:

- Uniform along its span, or length, and slender (diameter to length ratio > 10).
- Composed of a linear, homogenous, isotropic elastic material, without axial loads.
- Plane section remains plane.
- The plane of symmetry of the beam is also the plane of vibration, so rotation and translation are decoupled.
- Rotary inertia and shear deformation can be neglected.

Assuming that the cross-sectional area is constant $A(z) = A$, the beam equation can be rewritten as

$$\frac{\partial^2 u(z, t)}{\partial t^2} + c^2 \frac{\partial^4 u(z, t)}{\partial z^4} = 0, \quad c = \sqrt{\frac{EI}{\rho A}} \quad (2.33)$$

The solution for Eq. 2.33 is subjected to four boundary conditions and two initial conditions, however, in order to calculate the resonance frequencies and mode shapes we only need the boundary conditions. A separation-of-variables solution of the form $u(z, t) = u(z)u(t)$ is assumed, thus the equation of motion to yields

$$c^2 \frac{\partial^4 u(z)}{\partial z^4} \frac{1}{u(z)} = - \frac{\partial^2 u(t)}{\partial t^2} \frac{1}{u(t)} = (2\pi f)^2 \quad (2.34)$$

The spatial equation results from rearranging Eq. 2.34, which yields

$$\frac{\partial^4 u(z)}{\partial z^4} - \left(\frac{2\pi f}{c} \right)^2 u(z) = 0 \quad (2.35)$$

By defining

$$\beta^4 = \frac{(2\pi f)^2}{c^2} = \frac{\rho A (2\pi f)^2}{EI} \quad (2.36)$$

the general solution of Eq. 2.35 can be written as [27]

$$u(z) = a_1 \cos \beta z + b_1 \sin \beta z + c_1 \cosh \beta z + d_1 \sinh \beta z \quad (2.37)$$

where a_1 , b_1 , c_1 and d_1 are constants of integration determined by the boundary conditions. The general solution produces infinity solution for β (an infinite number of resonance frequencies and mode shapes), we denote each solution

as β_r , belonging to mode r . The equation for calculating the natural frequency is [27]

$$f_r = \frac{\beta_r^2}{2\pi} \sqrt{\frac{EI}{\rho A}} \quad (2.38)$$

The boundary conditions required in order to solve the spatial equation from the separation-of-variables solution of Eq. 2.33 are obtained by examining the deflection $u(z, t)$, the slope of the deflection $\partial u(z, t)/\partial z$, the bending moment $EI\partial^2 u(z, t)/\partial z^2$ and the shear force $EI\partial^3 u(z, t)/\partial z^3$ at all boundaries. From examining these boundary conditions, four equations should be found and may be written in matrix form as:

$$[\mathbf{C}] \{\mathbf{a}\} = \{\mathbf{0}\} \quad (2.39)$$

where $\{\mathbf{a}\} = \{a_1 \ b_1 \ c_1 \ d_1\}^T$ is the vector with unknown constants of integration and $[\mathbf{C}]$ the coefficient matrix determined from the boundary conditions. By equating the determinant of the coefficient matrix to zero, the characteristic equation and the eigenfrequencies may be found [27]. For each eigenfrequency f_r , three of the four unknown constants of integration can be found or expressed in terms of the fourth. This is sufficient in order to determine the mode shape $\{\Psi_r\}$. The fourth constant is found using the initial conditions and determines the participation of each mode in temporal solution. However, since we only consider the dynamic properties of the system is considered, the mode shape $\{\Psi_r\}$ is normalized.

2.3.1 Multi-span beam

The previous discussion concerned a beam with constant cross-section properties and boundary conditions at each end. In order to apply Euler-Bernoulli modeling to more complex beam structures with boundary conditions along the beam at discrete points and/or beam segments with different properties, the beam may be divided into several sub-beams, also referred to as a multi-span beam [28]. Each sub-beam will have the same general solution as the single span beam in Eq. 2.37, thus the mode shape for each sub-beam may be expressed as

$$\begin{aligned} u_j(z) = & a_j \cos \beta(z - z_j) + b_j \sin \beta(z - z_j) + \\ & + c_j \cosh \beta(z - z_j) + d_j \sinh \beta(z - z_j) \end{aligned} \quad (2.40)$$

where j is the sub-beam number, J the number of sub-beams and z_j the local coordinate offset. The local coordinate is expressed as

$$z_j = \sum_{k=1}^{j-1} l_k \quad (2.41)$$

where l_k is the length of section $1 \leq k \leq J$. The equation system will now consist of 4 times J coupled equations. Thus the coefficient matrix $[\mathbf{C}]$ will have the size $4J \times 4J$ and the vector $\{\mathbf{a}\}$ the size $4J \times 1$. The eigenfrequencies and mode shapes are found in the same way as for the single span beam, i.e. by first finding the solutions to the characteristic equation and then the corresponding eigenvectors.

2.3.2 Linearized Model

The simplest and most straightforward model of a boring bar is the Euler-Bernoulli model, which consists of a homogenous single span beam with constant cross-sectional area $A(z) = A$ and constant cross-sectional moment of inertia $I(z) = I$. The beam has four boundary conditions, two at each end. One end is clamped and the other is free, see Fig. 2.11.

Figure 2.11: Model of a Clamped - Free beam, where E is the elasticity modulus (Young's coefficient), ρ the density, A the cross-sectional area, I the moment of inertia and l the length of the beam.

The clamped side of the beam will be fixated, thus the displacement and the slope of the displacement in this point $z = 0$ will equal zero and the two first boundary conditions become

$$u(z, t)|_{z=0} = 0 \quad (2.42)$$

$$\left. \frac{\partial u(z, t)}{\partial z} \right|_{z=0} = 0 \quad (2.43)$$

The other end is free, thus no bending moment or shear force constrains the beam at the coordinate $z = l$ when the beam vibrates, this yielding the other

two boundary condition as

$$EI \frac{\partial^2 u(z, t)}{\partial z^2} \Big|_{z=l} = 0 \quad (2.44)$$

$$EI \frac{\partial^3 u(z, t)}{\partial z^3} \Big|_{z=l} = 0 \quad (2.45)$$

The general solution for $u(z)$ is then combined with the boundary condition, which yields

$$u(z) \Big|_{z=0} = a_1 + c_1 = 0 \quad (2.46)$$

$$\frac{du(z)}{dz} \Big|_{z=0} = b_1 + d_1 = 0 \quad (2.47)$$

$$EI \frac{d^2 u(z)}{dz^2} \Big|_{z=l} = EI (-a_1 \beta^2 \cos \beta l - b_1 \beta^2 \sin \beta l + c_1 \beta^2 \cosh \beta l + d_1 \beta^2 \sinh \beta l) = 0 \quad (2.48)$$

$$EI \frac{d^3 u(z)}{dz^3} \Big|_{z=l} = EI (a_1 \beta^3 \sin \beta l - b_1 \beta^3 \cos \beta l + c_1 \beta^3 \sinh \beta l + d_1 \beta^3 \cosh \beta l) = 0 \quad (2.49)$$

and in matrix form

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\beta^2 \cos \beta l & -\beta^2 \sin \beta l & \beta^2 \cosh \beta l & \beta^2 \sinh \beta l \\ \beta^3 \sin \beta l & -\beta^3 \cos \beta l & \beta^3 \sinh \beta l & \beta^3 \cosh \beta l \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.50)$$

Setting the determinant of the coefficient matrix equal to zero, i.e. $\det([\mathbf{C}]) = 0$, yields the characteristic equation as

$$2\beta^5 \cos \beta l \cosh \beta l + \beta^5 \cos^2 \beta l + \beta^5 \sin^2 \beta l + \beta^5 \cosh^2 \beta l - \beta^5 \sinh^2 \beta l = 0 \quad (2.51)$$

since $\cos^2 x + \sin^2 x = 1$ and $\cosh^2 x - \sinh^2 x = 1$ Eq. 2.51 can be simplified into

$$\cos \beta l \cosh \beta l + 1 = 0 \quad (2.52)$$

and β_r is determined by finding the roots of this equation.

2.3.3 Multi-span Boring bar with Elastic Foundation

The boring bar was clamped with either two screws on the top and two on the underside or three screws on the top and three on the underside. In addition, two different bolts were used: M8 and M10. If we consider the clamping

house to be a rigid body, and the screws to be rigid in the transverse direction, a number of boundary conditions are yielded, i.e. approximated as pinned. The pinned boundary condition assumes an infinitely stiff spring in the transverse direction but no rotational stiffness. Letting the screws assume more realistic properties as deformable bodies will yield "elastic supports" [29] as a boundary condition, instead of the pinned condition. The elastic support can be seen as two springs in one point, with one spring in the transverse direction; thus, transverse stiffness resistance and one rotational spring exhibits rotational stiffness resistance. The configurations of the "elastic support" condition are presented in Fig. 2.12.

Figure 2.12: a) A model of a three span beam with elastic support, b) a model of a four span beam with elastic support, where E is the elasticity modulus (Young's coefficient), ρ the density, A the cross-sectional area, I the moment of inertia, k_T the transverse spring coefficient, k_R the rotational spring coefficient the length of the different spans in mm are $l_1 = 35$, $l_2 = 50$, $l_3 = 215$ and $l_4 = 25$.

Two types of boundary conditions may be categorized from the models presented in Fig. 2.12, where z_{pos} denotes the position of the boundary condition. One is the "free" boundary condition, previously expressed as

$$EI \frac{\partial^2 u(z, t)}{\partial z^2} \bigg|_{z=z_{pos}} = 0 \quad (2.53)$$

$$EI \frac{\partial^3 u(z, t)}{\partial z^3} \bigg|_{z=z_{pos}} = 0 \quad (2.54)$$

where there is no bending or shear forces present. The other boundary conditions derive from the "elastic support" condition and may be expressed as [29]

$$EI \frac{\partial^2 u(z, t)}{\partial z^2} \Big|_{z=z_{pos}} = -k_R \frac{\partial u(z, t)}{\partial z} \quad (2.55)$$

$$EI \frac{\partial^3 u(z, t)}{\partial z^3} \Big|_{z=z_{pos}} = k_T u(z, t) \quad (2.56)$$

where the transverse spring produces a transverse force proportional to the displacement, and the rotational spring produces a bending moment proportional to the slope. However, if we let the rotational spring coefficient equal zero $k_R = 0$, and the transverse spring coefficient go to infinity $k_T = \infty$, we will have a third boundary condition termed "pinned". The pinned boundary condition can be expressed as

$$u(z, t) \Big|_{z=z_{pos}} = 0 \quad (2.57)$$

$$EI \frac{\partial^2 u(z, t)}{\partial z^2} \Big|_{z=z_{pos}} = 0 \quad (2.58)$$

The coefficient matrix may now be formulated in order to find the characteristic equations for the different models. The three span model will yield a 12x12 coefficient matrix and the four span model will yield a 16x16 coefficient matrix. Calculating the determinate of these matrices by hand is fairly time-consuming and the roots of the characteristic equation are often not possible to express explicitly [30, 31]. The solutions produced using these models were found using Matlab and by numerically finding the roots for the characteristic equations. The boundary conditions for the four models, (three span model with rotational and transverse springs; three span model without rotational spring and infinitely stiff transverse spring; four span model with rotational and transverse springs and four span model without rotational spring and infinitely stiff transverse spring) are presented in appendix A.

2.3.4 Screws - Elastic Foundation

The boring bar was clamped using screws. Since the screws are not rigid bodies they may, for example, be modeled as flexible bodies using springs. A model based on a transverse spring and a rotational spring was assumed. Thus, the two different stiffness coefficients may be calculated using very simplified models of what is going on whilst clamping the boring bar. Hence, one of the coefficients corresponds to a transverse spring and the second coefficient corresponds to a rotational spring. The screws' end surfaces, which are in

contact with the boring bar, apply pressure to the bar. This screw pressure on the boring bar is related to the screws' tightening torque.

The screws used to clamp the boring bar were of type MC6S norm "DIN 912, ISO 4762", presented in Fig. 2.13. The screws are zinc-plated, steel socket,

Figure 2.13: a) Dimensions of the ISO 4762/DIN 912 socket head cap screw
b) Theoretical metric ISO-thread profile for inside thread and outside thread without tolerances added, see Table 2.4 for symbol definitions.

head cap screws with the strength class 8.8, with a tensile yield strength of $R_{p02} = 660 \text{ MPa}$. Two various sizes were used: first M8 and then M10; see Table 2.4 for dimensions.

Symbol	M8	M10	Description of symbols in Fig. 2.13 a)
d	8 mm	10 mm	Diameter of the screw without head
D	13 mm	16 mm	Diameter of the head
s	6 mm	8 mm	Size or minimal diameter of the hexagon
k	8 mm	10 mm	Hight of the head
L	50 mm	50 mm	Length of the screw without the head
b	28 mm	36 mm	Length of thread part
Symbol	M8	M10	Description of symbols in Fig. 2.13 b)
d	8 mm	10 mm	Outer diameter of outside thread
d_1	6.65 mm	8.38 mm	Inner diameter of outside thread ($d_1 = \frac{5\sqrt{3}}{8}P$)
d_2	7.19 mm	9.03 mm	Average diameter of outside thread ($d_2 = \frac{3\sqrt{3}}{8}P$)
P	1.25 mm	1.5 mm	Pitch
H	1.08 mm	1.30 mm	Hight of base-triangle ($H = \frac{\sqrt{3}}{2}P$)
β	60°	60°	Basic profile angle for metric screw threads.

Table 2.4: Dimensions of the ISO 4762/DIN 912 socket head cap screw

The stiffness constants of the screws were calculated by modeling the screws as a beam rigidly clamped at one end, and free at the other. The beam was considered to be homogenous, having a constant cross-sectional area A , a

constant cross-sectional moment of inertia I and a length of l . When a screw is threaded in the clamping house and is clamping the boring bar, a part of the screw's tip will not be in contact with the clamping house; thus yielding both transverse, lateral and bending elasticity. This is due to the fact that the inside of the clamping house is circular with a radius of 40 mm plus tolerance and the boring bar has a thickness of 37 mm, plus tolerance where the boring bar is clamped. Two different lengths of the screw beam models were selected. The shortest length represents rigid clamping of the screws within the clamping house and the longer length represents the case of flexible clamping of the screw by the clamping house thread.

The screw clamping model is presented in Fig. 2.14, where a) shows the clamping configuration, b) illustrates the beam model of transverse vibrations and the transverse spring coefficient, and c) illustrates the beam model of the rotational spring coefficient.

Figure 2.14: a) Sketch illustrating screw clamping of the boring bar, via the clamping house, b) the transverse stiffness model, and c) the rotational stiffness model

The spring coefficients from a beam with one end fixed and the other end subjected to axial (vertical) loading and bending moment are the transverse spring constant k_T and rotational spring constant k_R , respectively. These constants are calculated from beam bending theory [27, 32] as

$$k_T = \frac{EA}{l} \quad (2.59)$$

$$k_R = \frac{EI}{l} \quad (2.60)$$

where E is the elasticities modulus, cross-sectional area A , cross-sectional moment of inertia I , and l the length of the beam model. These spring constants are used in the elastic support models.

Other important factors which affect clamping conditions include the coupling properties between the screws and the boring bar, and coupling properties between the screws and the clamping house. The force applied by the clamp screws in the axial direction of the screws on the boring bar surface may be related to the tightening torque M of the screws, and is identical to the prestressing force F_p . The tightening torque can be divided into two parts, one moment M_T (arising due to friction in the contact surfaces between the threads and the geometric relation from the prestressing force) and a second moment, M_C (due to the friction in the contact surface between the screw and boring bar). The moment due to the threads may be expressed as [33]

$$M_T = F_p r_m \tan(\varphi + \varepsilon) \quad (2.61)$$

where r_m is the average radius of the screw equal to $\frac{d_2}{2}$ in Table 2.4, ε is the angle of friction force component and φ is the pitch angle. The moment M_C , due to the friction between the boring bar and the screw, may be expressed as

$$M_C = F_p \mu_c r_c \quad (2.62)$$

where μ_c is friction coefficient between the surfaces, and r_c is the average radius of the contact surface [33], with regard to a contact surface with an inner and an outer circle. Then the average radius of contact equals

$$r_c = \frac{r_m}{2} \quad (2.63)$$

thus, the total moment M equals [33]

$$M = M_T + M_C = F_p r_m \tan(\varphi + \varepsilon) + F_p \mu_c r_c \quad (2.64)$$

Thus, the expression for the force F_p (which the screw is enacts upon the boring bar when applying the moment M) may be expressed as

$$F_p = \frac{M}{r_m \tan(\varphi + \varepsilon) + \mu_c r_c} \quad (2.65)$$

The pitch angle φ is calculated from the geometry of the screw as

$$\tan \varphi = \frac{P}{\pi d_m} \quad (2.66)$$

where P is the pitch and d_m is the average diameter of the screw, equal to d_2 in Table 2.4. The angle of the friction force component ε is related to the pitch angle φ and friction in the thread as

$$\tan \varepsilon = \frac{\mu_T}{\cos \frac{\beta}{2}} \quad (2.67)$$

where μ_T is the friction coefficient between the surfaces of the inner and outer threads, (i.e. the threads in the clamping house and the threads of the screws) and β is the profile angle, equal to 60° from the ISO-standard presented in Fig. 2.13.

2.3.5 Spring Coefficients and Clamping Forces

Materials and properties are required in order to model the screws as the transversal spring and the rotational spring (both previously presented). The screws' material is steel (zinc-plated). Whilst the exact type of steel is not specified, this is in general not relevant. Types of steel vary for different screw suppliers; usually, more important is the tensile yield strength. However, according to the supplier of the screws used in this experiment, the elasticity modulus E is approximately $200 \cdot 10^9 \text{ N/m}^2$. The dimensions are specified by the standard and are presented in Table 2.4. The tensile stress area A , according to the standard ISO 724, is given by

$$A = \frac{\pi}{4} \left(\frac{d_2 + d_1 - \frac{H}{6}}{2} \right)^2 \quad (2.68)$$

and the moment of inertia I was calculated as

$$I = \frac{\pi}{4} \left(\frac{d_2 + d_1 - \frac{H}{6}}{4} \right)^4 \quad (2.69)$$

where d_1 , d_2 and H are given in Table 2.4. The length l is the length of the "overhang" of the screw; an example was used in which the overhang length was considered to be 1.5 mm. This length was selected, because the circular inner diameter of the clamping space of the clamping hose is 40mm and the height of the boring bar is 37mm, thus resulting in a space from each side of the boring bar to the circular clamping house boundary of 1.5 mm. Furthermore, since the boring bar is clamped with screws from both the upper-side and the underside at the same positions along the z-axis, each axial spring coefficient includes the longitudinal stiffness from two screws, i.e. two springs in parallel.

The calculated spring coefficients and the spring parameters used in the spring models are presented in Table 2.5 together with the dimensions and elasticity modulus.

Estimates of the forces applied by the clamp screws on the boring bar were calculated using the torque-force relation presented in section 2.3.4, and are presented in Table 2.6. The screw force was calculated using the dimension previously presented in Table 2.4, assuming identical friction coefficients for

Size	A [m ²]	I [m ⁴]	E [N/m ²]	l [m]	k_T [N/m]	k_R [Nm]
M8	$3.661 \cdot 10^{-5}$	$1.067 \cdot 10^{-10}$	$200 \cdot 10^9$	$1.5 \cdot 10^{-3}$	$4.881 \cdot 10^9$	$1.422 \cdot 10^4$
M10	$5.799 \cdot 10^{-5}$	$2.676 \cdot 10^{-10}$			$7.732 \cdot 10^9$	$3.568 \cdot 10^4$

Table 2.5: The longitudinal and rotational spring coefficients and the spring parameters used in the spring models.

the contact surface between the boring bar and the screws, and also the contact surface between the threads in the clamping house and the screw threads, i.e. $\mu_c = 0.14$ and $\mu_T = 0.14$.

	Screw size	Tightening torques and clamping forces				
Torque	M8 & M10	10 Nm	15 Nm	20 Nm	25 Nm	30 Nm
Force	M8	9.81 kN	14.71 kN	19.61 kN	24.52 kN	29.42 kN
	M10	7.88 kN	11.81 kN	15.75 kN	19.69 kN	23.63 kN

Table 2.6: The clamp screw torque-force relation based on the presented clamp screw model.

2.4 Nonlinear Model

A linear model may not always be sufficient to explain different results from the experimental data. Nonlinearities may be caused by several different factors. A common source of nonlinearity is the contact problem, in which elements of the system come into contact with the surrounding environment due to "large" displacement, which, in turn, create a new set of boundary conditions. Another form of the contact problem is friction in joints, or sliding surfaces. This problem also involves large forces and/or deformation which may cause the properties of the material to behave in a nonlinear manner.

All of these nonlinearities are likely to exist to some extent in the boring bar - clamping house system, perhaps some of them more than others. The question is if, or to what extent they influence the dynamic behavior of the boring bar and how they can be determined (if relevant). In these simulations, nonlinearities regarding stiffness was examined, a so-called "softening spring" [18, 34, 35]. This nonlinearity was investigated as empirical data has shown that the fundamental boring bar resonance frequencies display a tendency to move towards lower frequencies as a result of increasing excitation force level.

The softening spring may be modeled in two different ways, yielding different properties with respect to the displacement. The first of these models

yields a force proportional to a nonlinear stiffness coefficient, multiplied by the displacement, squared with sign. The equation describing a SDOF system with this type of a softening spring nonlinearity is given by [36]

$$m \frac{d^2 x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) - k_s x|x|(t) = f(t) \quad (2.70)$$

where m , c and k are the mass, damping and stiffness coefficients of the underlying linear system, $x(t)$ - the displacement, $f(t)$ - the force, and k_s the nonlinear stiffness coefficient. The second model yields a force proportional to a nonlinear stiffness coefficient multiplied by the displacement cubed, and inserted into the equation of motion describing a SDOF system results in [36]

$$m \frac{d^2 x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) - k_c x^3(t) = f(t) \quad (2.71)$$

where k_c is the nonlinear stiffness coefficient.

In order to see if any of the proposed nonlinearities may explain the different results from the experimental modal analysis, a number of different simulations were carried out using different parameters.

2.4.1 Nonlinear Synthesis

There are different ways of simulating linear and nonlinear systems: Probably the most common method to solve ordinary differential equations (ODE) is the Runge-Kutta method, implemented in Matlab [37]. Another method is the digital filter method [38].

There are multiple advantages with using ODE solvers: firstly, they are rather straightforward to use and secondly they are well known. The disadvantage, however, is that they are relatively time consuming if large amounts of data are involved. The filter method, on the other hand, is significantly faster than the ODE solvers [38]. But is not as well documented as the ODE solvers with regarding to, for example, accuracy and the ability to handle nonlinear systems [39]. However, for linear systems, the limitations of the filter method are known [40] and depends on the sampling frequency and the transformation method used to convert the continuous time parameters to discrete time parameters [40].

2.4.2 Ordinary Differential Equation Methods

The simulation method used for simulating the nonlinear system is based on explicit Runge-Kutta of order (4,5) formula, the Dormand-Prince pair [37, 41],

referred to as ode45 in Matlab. The ode45 method combines a fourth order method and a fifth order method, both of which are similar to the classical fourth order Runge-Kutta [41, 42]. The numerical technique solves ordinary differential equations of the form

$$\frac{dx(t)}{dt} = f(x(t), t), \quad x(t_0) = x_0 \quad (2.72)$$

The Runge-Kutta 4th order method is based on the following expressions

$$\begin{aligned} x(t + \Delta t) &= x(t) + (a_1 k_1(x(t), t) + a_2 k_2(x(t), t) + \\ &+ a_3 k_3(x(t), t) + a_4 k_4(x(t), t)) \Delta t \end{aligned} \quad (2.73)$$

where Δt is the step size, $\{a_1, \dots, a_4\}$ and $\{k_1(x(t), t), \dots, k_4(x(t), t)\}$ are constants and functions respectively, determined based on the first five terms of the Taylor series [42]:

$$\begin{aligned} x(t + \Delta t) &= x(t) + f(x(t), t) \Delta t + \frac{1}{2!} f'(x(t), t) \Delta t^2 + \\ &+ \frac{1}{3!} f''(x(t), t) \Delta t^3 + \frac{1}{4!} f'''(x(t), t) \Delta t^4 \end{aligned} \quad (2.74)$$

Rewriting Eq. 2.74 into Eq. 2.73 yields [42]

$$x(t + \Delta t) = x(t) + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \Delta t \quad (2.75)$$

where

$$k_1(x(t), t) = f(x(t), t) \quad (2.76)$$

$$k_2(x(t), t) = f\left(x(t) + \frac{\Delta t}{2} k_1, t + \frac{\Delta t}{2}\right) \quad (2.77)$$

$$k_3(x(t), t) = f\left(x(t) + \frac{\Delta t}{2} k_2, t + \frac{\Delta t}{2}\right) \quad (2.78)$$

$$k_4(x(t), t) = f(x(t) + \Delta t k_3, t + \Delta t) \quad (2.79)$$

The Runge-Kutta method only solves first order differential equations, thus this requires that the second order differential equations in Eqs. 2.70 and 2.71 are rewritten to coupled first order differential equations as in Eqs. 2.82 and 2.83.

The nonlinear models simulated with the differential equation solvers were based on the softening spring using the quadratic model in Eq. 2.80, and the cubed model in Eq. 2.81

$$g_q(x(t)) = k_s x |x|(t) \quad (2.80)$$

$$g_c(x(t)) = k_c x^3(t) \quad (2.81)$$

where $g_q(x(t))$ and $g_c(x(t))$ replaces $g(x_1(t))$ in Eq. 2.83 for respective model.

$$\frac{dx_1(t)}{dt} = x_2(t) \quad (2.82)$$

$$m \frac{dx_2(t)}{dt} = -cx_2(t) - kx_1(t) + g(x_1(t)) + f(t) \quad (2.83)$$

where $x(t)$ is the response of the system, and $f(t)$ is the driving force.

2.4.3 Filter Method

The filter method is a time-discrete method for extracting digital filter coefficients from the analog system using an appropriate transformation method. Thus, the differential equation is transformed into a difference equation, represented by a digital filter [38]. The continuous time filtering may be expressed in terms of a convolution integral as [43]

$$x(t) = \int_{\tau=-\infty}^{\infty} h(\tau)f(t-\tau)d\tau \quad (2.84)$$

where $x(t)$ is the response or output signal and $f(t)$ is the input to the system with impulse response $h(t)$. The corresponding filtering procedure in the discrete time domain is given by

$$x(n) = \sum_{k=-\infty}^{\infty} h(k)f(n-k) \quad (2.85)$$

where, again, $x(n)$ is the response and $f(n)$ is the input to the system with the impulse response $h(n)$, but in discrete time domain.

The transformation may be performed by first dividing the total system (multiple degree of freedom) into subsystems using the modal superposition theorem and transforming each subsystems parameter into the filter coefficients. The frequency response function for a dynamic system may be expressed in terms of modal superposition as [18]

$$H(f) = \sum_{r=1}^R \frac{A_r}{j2\pi f - \lambda_r} + \frac{A_r^*}{j2\pi f - \lambda_r^*}, \quad (2.86)$$

where R is the number of modes, A_r is the system's residues belonging to mode r , and λ_r is the pole belonging to mode r . The poles and residues may be extracted from a lumped parameter system, a Finite Element Model,

distributed parameter system, or estimated from experimental modal analysis [27]. Another approach is to directly express the system as in Eq. 2.87 and transform the analog filter coefficients into digital filter coefficients. The frequency function for an analog filter can be expressed as [43]

$$H(s) = \frac{D(s)}{C(s)} = \frac{d_0 + d_1s + \dots + d_{M_a}s^{M_a}}{1 + c_1s + \dots + c_{K_a}s^{K_a}} \quad (2.87)$$

where M_a is the order of the polynomial $D(s)$ in the numerator, and K_a is the order of the polynomial $C(s)$ in the denominator. Transforming the analog filter yields a digital filter whose z-transform may be expressed as [43]

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1z^{-1} + \dots + b_Mz^{-M}}{1 + a_1z^{-1} + \dots + a_Kz^{-K}} \quad (2.88)$$

where M is the order of the polynomial $B(z)$ in the numerator, and K is the order of the polynomial $A(z)$ in the denominator. In the discrete time domain, the difference equation describing the filter may be written as [43]

$$x(n) = \sum_{m=0}^M b_m f(n-m) - \sum_{k=1}^K a_k x(n-k) \quad (2.89)$$

One of the most common transformation methods is the so-called "impulse invariant" method, which allows the digital signal to represent the analog signal by an impulse at sampled intervals, i.e. $x(t) \rightarrow Tx(nT)$, where T is sampling period [44]. Other methods include the step invariant, ramp invariant, centered step invariant, cubic spline invariant and Lagrange method; each with different properties [40]. The ramp invariant method was used in my simulations. This transform method introduces zero error at DC, low error at Nyquist frequency and low phase distortion [40]. However, the ramp invariant method introduces a large error at a resonance frequency than, for example, the impulse, step and cantered step invariant methods [40]. However, the error introduced in the area of a resonance frequency only becomes large as the resonance frequency approaches the Nyquist frequency. Therefore, an over-sampling of 20 times the highest resonance frequency was used in the simulations.

The filter method for simulation of nonlinear systems is performed by using the digital filter coefficient for the linear system, and finding the solutions for the nonlinear difference equation. A nonlinear, single degree of freedom system may be described by differential equation

$$m \frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) + g(x(t)) = f(t) \quad (2.90)$$

the corresponding equation in time discrete domain may be written using a difference equation as [38]

$$x(n) = \sum_{m=0}^M b_m (x(n-m) - g(x(n-m))) - \sum_{k=1}^K a_k f(n-k) \quad (2.91)$$

Since Eq. 2.91 contains nonlinear terms, several solutions may exist for $x(n)$ [42]. The value of $x(n)$ may be found using any of the zero searching algorithms such as the secant method, bisection method or Newton-Raphson [42] (which was used in this synthesis).

The models with a nonlinear function $g(x(n))$, simulated with the filter method were based on both the quadratic model in Eq. 2.80, and the cubed model in Eq. 2.81, both representing the softening spring.

$$g_{Lin}(n) - g_s(x(n)) = f(n), \quad \text{where} \quad g_s(x(n)) = k_s x(n) \quad (2.92)$$

$$g_{Lin}(n) - g_c(x(n)) = f(n), \quad \text{where} \quad g_c(x(n)) = k_c x^3(n) \quad (2.93)$$

The digital filter coefficients were based on the poles and residues estimated from experimental measured data.

2.4.4 Excitation Signal

True random was selected for the excitation signal so that the resolution and number of averages may be alter after the simulation results were produced. The estimation parameters used in the nonlinear simulations are presented in Table 2.7.

Parameter	Value
Excitation signal	True Random
Sampling Frequency f_s	10000 Hz
Block Length N	20480
Frequency Resolution Δf	0.5 Hz
Number of averages L	800
Window	Hanning
Overlap	50%
Frequency Range of Burst	-
Burst Length	-

Table 2.7: Spectral density estimation parameters and excitation signal used in the simulated nonlinear system.

Chapter 3

Results

Firstly, this section presents results from the experimental modal analysis of boring bars, under different clamping conditions and excitation force levels. The results presented constitute a small part of an extensive investigation of the dynamic properties of boring bars for various configurations and setups; however they represent the essence of the experimental results. Secondly, this section presents results based on analytical Euler-Bernoulli models of boring bars with different spans, and simple models of boring bar clamping. Finally, results from simulations of the nonlinear models are given.

3.1 Experimental Modal Analysis

Shaker excitation was used for the experimental modal analysis of the boring bars. The utilized spectrum estimation parameters and excitation signals' properties are given in Table 2.3. A number of different phenomena were observed during the experimental modal analysis of the boring bars for various configurations and setups. For instance, large variations were observed in the fundamental bending resonance frequencies of the boring bar for different tightening torques of the clamp screws. Also, the order in which the clamp screws were tightened (first from the upper side of the boring bar or first from the under side of the boring bar) had a significant impact on, for example, the fundamental bending resonance frequencies. Fig. 3.1 illustrates typical frequency response function estimates based on the same measurement locations for input force and output response at the boring bar. These frequency response function estimates are produced using different clamp screw tightening torques and/or a different excitation level.

Based on the frequency function estimates, the modal parameters are then estimated using the poly-reference technique. A frequency range covering the

a)

b)

Figure 3.1: a) The accelerance of the boring bar driving point response in the direction of cutting speed (y-), using the standard boring bar, four screws of size M8 and tightened first from the top using five different tightening torques and four different excitation levels. b) The corresponding estimates zoomed in around the first resonance frequencies.

significant part near the resonance frequencies was selected, i.e. ± 100 to ± 200 Hz around the resonance peaks. A Multivariate Indicator Function (MIF) was produced from all the driving point data and then overlayed on top of a stability diagram. The stability diagram consist of poles calculated using different model orders up to a given order (see Fig. 3.2 for a typical stability diagram with the corresponding indicator functions). When stable poles corresponding

Figure 3.2: Typical stability diagram for experiments conducted on the boring bar. In order to identify the location and number of poles, two indicator functions are overlayed on the stability diagram.

to the modes of interest have been selected, residues are estimated using the driving points. Finally, the mode shapes are estimated using all the measured FRF:s. In order to check whether the estimated parameters are functional or not, they are used to synthesize a number of FRF:s, among those are the driving points. Furthermore, orthogonality of the mass scaled mode shapes are checked using the MAC matrix. If any of the checks indicate on nonfunctional estimates, the maximum model order used to create the stability diagram is changed and new stable poles are selected. Also erroneous or strange FRF:s may be disregarded when the different models are calculated. These steps are performed until acceptable results are achieved, or until (what seems to be when) the best possible results given the data are achieved.

Results from six various setups, described by Table 3.1 are presented.

Setup Number	Configuration			
	Boring bar	Number of Screws	Screw Size	Tighten first from
1	Standard	four	M8	top
2	Standard	six	M10	top
3	Standard	six	M10	bottom
4	Active	four	M8	top
5	Active	six	M10	top
6	Linearized	-	-	-

Table 3.1: The configuration of the different setups from which experimental modal analysis results are presented.

3.1.1 Standard Boring Bar

When clamping the standard boring bar so that the bottom side of the boring bar is clamped against the clamping house (i.e. the screws are tightened from the topside first and subsequently from the bottom-side) the fundamental boring bar resonance frequencies increases with increasing tightening, see Fig. 3.3. In this setup, screws of size M8 were used; the spectrum estimation parameters and excitation signal is presented in Table 2.3. By changing the excitation levels, nonlinearities in the dynamic properties of the boring bar might be observable via changes in frequency response function estimates for the same input and output locations at the boring bar. Four different excitation levels were used with the proportion 1:2:3:4 for each of the torque configuration presented in section 3.1. As can be seen in Fig. 3.4 the fundamental boring bar resonance frequencies decreases slightly with increased excitation level. The estimated resonance frequencies and relative damping from all 20 measurements are presented in Table 3.2 and Table 3.3.

The clamp screws were replaced with M10 screws and the number of clamp screws was increased to six. Using these clamping conditions, experiments were performed which were identical to those carried out using a clamping house with four M8 screws.

When clamping the standard boring bar so that the bottom side of the boring bar is clamped against the clamping house, the fundamental boring bar resonance frequencies increases with increasing tightening, see Figs. 3.5 a) and b). As can be seen in Fig. 3.6, the fundamental boring bar resonance frequencies decreases slightly with increasing excitation level.

Figure 3.3: The accelerance of the boring bar response using the standard boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

Figure 3.4: The accelerance of the boring bar response using the standard boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bars, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

	Resonance Frequency, Mode 1 [Hz]				Resonance Frequency, Mode 2 [Hz]			
Torque	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
10Nm	509.52	507.87	506.61	505.51	540.86	540.15	539.33	540.07
15Nm	518.13	516.50	515.23	514.18	546.50	546.31	545.73	544.60
20Nm	523.84	522.97	522.13	521.55	553.01	552.86	552.49	552.13
25Nm	526.64	526.05	525.50	525.10	556.07	555.84	555.66	555.42
30Nm	526.72	526.23	525.79	525.45	555.67	555.68	555.55	555.35

Table 3.2: Estimates of the fundamental boring bar resonance frequencies based on all the measurements using the setup with standard boring bar, clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Fig. 3.3 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions Fig. 3.4 a) and b) produced for the four different excitation levels.

	Damping of Mode 1 [%]				Damping of Mode 2 [%]			
Torque	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
10Nm	0.99	1.04	1.08	1.14	1.31	1.33	1.46	0.26
15Nm	0.97	1.00	1.03	1.08	1.26	1.32	1.28	1.23
20Nm	0.88	0.91	0.94	0.96	1.04	1.04	1.01	0.99
25Nm	0.87	0.88	0.90	0.92	0.97	0.93	0.91	0.90
30Nm	0.86	0.88	0.90	0.93	0.97	0.95	0.92	0.90

Table 3.3: The relative damping estimates for the fundamental boring bar bending modes based on all the measurements using the setup with standard boring bar, clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Fig. 3.3 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions in Fig. 3.4 a) and b), produced for the four different excitation levels.

a)

b)

Figure 3.5: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side of the boring bar, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

Figure 3.6: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side upper-side of the boring bar, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

Clamping by first tightening the clamp screws from the boring bar's under-side changes the frequency response functions significantly as compared with the case where the clamp screws were tighten first from the upper side. This might be observed by comparing Figs. 3.7 and 3.8 with Figs. 3.5 and 3.6.

3.1.2 Active Boring Bar

The active boring bar has a cavity, a milled space, onto which an embedded actuator was placed. This space constitutes a change in the dynamic properties of the boring bar in comparison to the standard boring bar. This is obvious since the material, steel, is removed from the boring bar and replaced partly with an actuator with a lower Young's module, etc. The actuator was kept passive during the experiments, thus, no control authority was applied. The same experiments were conducted with the active boring bar as were performed with the standard boring bar. From the results presented in Figs 3.9 and 3.10 it is clear that the dynamic properties of the active boring bar have changed significantly, mostly with regard to cutting speed direction (compare with the results from the standard boring bar Figs. 3.3 and 3.4). However, we can observe the same phenomenon that occurred in results obtained with the standard boring bar; i.e. increasing resonance frequency with increasing torque and decreasing resonance frequency with increasing excitation force. The estimated resonance frequencies and relative damping from all the 20 measurements are presented in Table 3.4 and Table 3.5.

	Resonance Frequency, Mode 1 [Hz]				Resonance Frequency, Mode 2 [Hz]			
Torque	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
10Nm	449.83	447.10	445.31	444.34	473.12	472.57	472.50	471.76
15Nm	466.62	464.55	463.10	462.37	478.55	477.36	476.75	475.98
20Nm	478.76	477.87	476.51	475.90	501.49	499.88	499.29	497.79
25Nm	482.29	481.62	480.81	480.36	510.13	508.39	507.35	506.96
30Nm	484.39	483.63	482.87	482.88	515.08	513.91	513.11	512.79

Table 3.4: Estimates of the fundamental boring bar resonance frequencies based on all measurements, using the setup in which the active boring bar is clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Figs. 3.9 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions in Figs. 3.10 a) and b), produced for the four different excitation levels.

When the active boring bar is clamped with six, size M10 screws, results obtained resemble those derived from clamping the same bar with size M8 screws, see Figs. 3.9 and 3.11. The largest differences may be observed in the frequency function estimates in cutting speed direction, see Fig. 3.11 a).

a)

b)

Figure 3.7: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the underside of the boring bar, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

a)

b)

Figure 3.8: The accelerance of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the underside of the boring bar, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

a)

b)

Figure 3.9: The accelerance of the boring bar response using the active boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bar, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

a)

b)

Figure 3.10: The accelerance of the boring bar response using the active boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bar, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

Figure 3.11: The accelerance of the boring bar response using the active boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side, using five different tightening torques. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

a)

b)

Figure 3.12: The accelerance of the boring bar response using the active boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side, using two different tightening torques and four different excitation levels. a) the driving point in cutting speed direction (y-) and b) the driving point in negative cutting depth direction (x-).

	Relative Damping of Mode 1 [%]				Relative Damping of Mode 2 [%]			
Torque	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
10Nm	1.40	1.41	1.43	1.51	1.61	1.72	1.85	1.89
15Nm	1.22	1.20	1.25	1.35	1.25	1.24	1.25	1.34
20Nm	1.17	1.14	1.06	1.15	1.67	1.73	1.86	1.62
25Nm	1.16	1.22	1.23	1.27	1.18	1.23	1.25	1.33
30Nm	1.36	1.35	1.42	1.51	0.99	1.02	1.03	1.05

Table 3.5: The relative damping estimates for the fundamental boring bar bending modes based on all measurements, using the setup in which the active boring bar is clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Figs. 3.9 a) and b) respectively. The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions in Figs. 3.10 a) and b), level produced for the four different excitation levels.

3.1.3 Linearized Boring Bar

Finally, the results from the boring bar with a so-called "linearized" clamping condition are presented. Since no screws were used in this setup, only the excitation levels were changed. The results are presented in Fig. 3.13 and Table 3.6, which are the driving point frequency response functions in both the cutting speed direction and the cutting depth direction. Thus, only a slight variation in the boring bar's resonance frequencies and damping might be observed. Unfortunately, both resonance frequencies coincide with periodic disturbances originating from the engines in the lathe producing the hydraulic pressure. One disturbance was at approximately 591 Hz and the other disturbance at approximately 600 Hz. These disturbances will have different influences on the estimates, depending on the excitation level, this may be observed near the peak in Fig. 3.13.

Mode 1	Modal Parameters			
	Level 1	Level 2	Level 3	Level 4
Frequency [Hz]	583.82	584.15	583.13	582.52
Damping [%]	2.12	2.04	2.16	2.16
Mode 2	Modal Parameters			
	Level 1	Level 2	Level 3	Level 4
Frequency [Hz]	602.25	602.07	601.92	601.79
Damping [%]	0.76	0.75	0.75	0.74

Table 3.6: The resonance frequency and the relative damping of the linearized boring bar, estimated with poly-reference technique.

a)

b)

Figure 3.13: The accelerance of the boring bar response using the linearized setup and with four different excitation levels. a) the driving point in cutting speed direction (Y-) and b) the driving point in negative cutting depth direction (X-).

3.1.4 Mode shapes

A boring bar mode shape shows the spatial deformation pattern of the bar for that particular mode and thus for each degree of freedom measured on the boring bar, in both amplitude and spatial phase. This section presents all the mode shapes estimated from the three different setups: the standard boring bar, the active boring bar and the linearized boring bar. The mode shapes were estimated in I-DEAS using the frequency poly-reference method. First, results are presented from the standard boring bar, with size M8 screws, tightening clamp screws firstly from the upper-side. The shapes are presented in zy-plane and xy-plane in Fig. 3.14, a) and b) respectively. The angle of rotation around z-axis (relative the cutting depth direction for each measurement) is presented in Table 3.7. The mode shapes in xy-plane illustrated in Fig. 3.14 b) and the corresponding values in Table 3.7 show an average rotation of approximately 20 degrees.

	Angle of Mode 1, [Degree]				Angle of Mode 2, [Degree]			
Torque	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
10Nm	-17.55	-17.10	-16.33	-17.81	-107.13	-106.89	-106.68	-106.43
15Nm	-21.42	-20.84	-20.44	-20.13	-110.30	-109.80	-109.45	-109.22
20Nm	-22.09	-21.81	-21.33	-21.08	-110.96	-110.30	-109.92	-109.76
25Nm	-21.31	-20.90	-20.66	-20.48	-109.90	-109.65	-109.42	-109.32
30Nm	-22.54	-22.27	-22.04	-21.84	-110.92	-110.68	-110.45	-110.35

Table 3.7: Angle of mode shapes for the standard boring bar, relative to cutting depth direction axis.

Measurements derived from the active boring bar differ somewhat from those obtained with the standard boring bar. Mode shapes are presented in Fig. 3.15 and the values of the angle of rotation in Table 3.8. The shapes are almost identical in the yz-plane, but in the xy-plane the shapes rotate around the z-axis. Table 3.8, demonstrates a trend of clockwise rotation with increasing torque, as well as counterclockwise rotation with increasing excitation level; this applies to both modes. The angles lies between 20 and 73 degrees for the first mode, thus the first mode shifts from being most significant in cutting depth direction, to being most significant in cutting speed direction.

The results from the linearized setup are presented by Fig. 3.16 and in Table 3.9.

In this linear setup, the zy-plane shape is almost identical to those shapes produced from standard, and active boring bar measurements, see Figs. 3.14, 3.15 and 3.16. In the xy-plane the shapes only have a rotation of approximately 10 degrees.

a)

b)

Figure 3.14: The two first mode shapes of the standard boring bar clamped with four M8 screws, when the clamp screws were tightened firstly from the upper-side, for five different tightening torques and four different excitation levels. a) in the zy -plane and b) in the xy -plane.

a)

b)

Figure 3.15: The two first mode shapes of the active boring bar clamped with four M8 screws, when the clamp screws were tightened firstly from the upper-side, for five different tightening torques and four different excitation levels. a) in the zy -plane and b) in the xy -plane.

a)

b)

Figure 3.16: The two first mode shapes of the linearized boring bar for four different excitation levels. a) in the zy -plane and b) in the xy -plane.

	Angle of Mode 1, [Degree]				Angle of Mode 2, [Degree]			
Torque	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
10Nm	-20.98	-19.20	-18.28	-18.50	-112.06	-110.52	-109.67	-110.00
15Nm	-33.78	-29.55	-28.04	-27.58	-116.87	-116.89	-115.53	-114.23
20Nm	-60.47	-56.97	-55.28	-55.76	-149.03	-145.72	-145.56	-145.77
25Nm	-69.59	-68.21	-67.65	-67.69	-155.52	-154.13	-153.93	-154.03
30Nm	-72.25	-71.41	-70.93	-70.50	-156.19	-155.54	-155.19	-154.52

Table 3.8: Angle of mode shapes for the active boring bar, relative to cutting depth direction axis.

Angle of Mode, [Degree]				
Excitation	Level 1	Level 2	Level 3	Level 4
Mode 1	-7.98	-8.46	-8.05	-8.11
Mode 2	-99.72	-100.00	-99.82	-100.07

Table 3.9: Angle of mode shapes for the linearized boring bar relative to cutting depth direction axis.

3.1.5 Quality of Measurement

Since the frequency response function estimates are based on the linear H_1 estimation method, a measure of the linear relation between input and output signals may be represented by the coherence function, or (as this case has several sources), the multiple coherence function. Typical multiple coherence function estimates obtained during the experiment are illustrated in Fig. 3.17; observe that the level in Fig. 3.17 b) starts from 0.99. After parameter estimation the Modal Assurance Criterion (MAC) was used to measure the correlation between the estimated modes shapes. A typical MAC diagram is presented in Fig 3.18. Typical values of the of-diagonal elements are 0.000-0.001, few values reach 0.007.

3.1.6 Mass-loading

As previously mentioned, all the sensors, cables and shakers etc. affect the structure and thus also the relationship between the frequency response function estimate the "true" frequency response function. It is nice to have an estimate close to the true frequency response function, the purpose was, however, to examine the influence of different clamping condition on the the boring bar's dynamic system. In order to acquire information concerning the sensors' influence on the resonance frequency and the damping of the boring bar, a measurement using an impulse hammer and two accelerometers was conducted. Both impedance heads and all the accelerometers were removed,

a)

b)

Figure 3.17: The multiple coherence corresponding to typical frequency response function estimates, where solid lines represent cutting speed direction (y-) and dotted lines represent negative cutting depth direction (x-).

Figure 3.18: The modal assurance criterion matrix coefficients for the two estimated mode shapes at the resonance frequencies 526.72 Hz and 555.67 Hz, where the of-diagonal values are equal to 0.000. The modes are estimated using the standard boring bar (clamped with four screws tightened firstly from the top), the lowest excitation level and the highest tightening torque.

with the exception of the accelerometers at the driving point. Thus, only two accelerometers were glued to boring bar providing the mass-loading when hitting the boring bar, (compared with 14 accelerometers and two impedance head connected to the shakers) see Fig. 3.19. Once again, the periodic disturbances are present but insignificant. Compare Fig. 3.19 with Fig. 3.13, that is, the estimates from the hammer excitation with shaker excitation, both for the linearized structure.

The estimates show a resonance frequency shift of approximately 20 Hz for the driving point in both directions. Mass-loading lowers the resonance frequency from approximately 604 Hz to 583 Hz in the cutting depth direction, and from approximately 620 Hz to 602 Hz in the cutting speed direction whilst still using two accelerometers.

3.1.7 Summary of the Estimated Parameters

A number of phenomenon may be observed in each individual experimental setup. The estimated parameters for max excitation level and max tightening torque, 30Nm, are presented in Table 3.10 in order that difference may be observed.

Setup	Parameters of Mode 1			Parameters of Mode 2		
	Freq. [Hz]	Damp. [%]	Angle [°]	Freq. [Hz]	Damp. [%]	Angle [°]
1	525.45	0.93	-21.84	555.35	0.90	-110.35
2	528.58	0.97	-17.74	560.83	1.00	-108.26
3	502.43	1.37	-6.40	526.87	1.42	-95.51
4	482.88	1.51	-70.93	512.79	1.05	-154.52
5	489.11	1.70	-59.47	508.22	1.10	-141.06
6	582.52	2.16	-8.11	601.79	0.74	-100.07

Table 3.10: Eigenfrequencies, relative damping and mode shape angle relative to cutting depth direction for the six different boring bar setups. Clamp screw tightening torque, 30Nm and maximum excitation signal level.

3.2 Analytical Models of the Boring Bars

This section presents results from a number of different Euler-Bernoulli models of the boring bar, including different simple models of boring bar clamping.

The first model assumed rigid clamping of the boring bar by the clamping house. The second model assumes that boring bar clamping is pinned at the positions where the actual clamp screws clamp the boring bar inside the clamping house. Thus, at each pinned boundary condition, the boring bar

Figure 3.19: a) The accelerance of the boring bar response using the linearized setup, two accelerometers and an impulse hammer. The solid line represents the driving point in cutting speed direction (y-), the dashed line is the driving point in negative cutting depth direction (x-). b) The corresponding coherence functions; observe that the plot shows the coherence from 0.91 to 1.

Figure 3.20: The accelerance of the boring bar response for the six different setups, a) the driving point in cutting speed direction and b) the driving point in cutting depth direction.

model will be rigidly clamped in the cutting speed direction without any rotational constraints about the clamping position. All the Euler-Bernoulli models of the boring bar assume a homogenous constant cross-section, i.e. $E(z) = E$, $\rho(z) = \rho$, $A(z) = A$, $I_x(z) = I_x$ and $I_y(z) = I_y$.

3.2.1 Single-span Model

The simplest model is the single span model with rigid clamping at one end and no clamping (free) at other. The boring bar is assumed to be entirely contained by the clamping house, which has a length of 100mm, and clamping ends where the clamping house ends. Thus the model is a fixed-free beam with the length of 200mm. The first three resonance frequencies in the cutting speed direction and in the cutting depth direction are presented in Table 3.11, and the three first mode shapes in Fig. 3.21. Depending on the direction of the

Direction of mode	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
Cutting speed direction (y-)	698.33	4376.36	12253.94
Cutting depth direction (x-)	698.11	4374.98	12250.08

Table 3.11: The first three resonance frequencies of the Euler-Bernoulli fixed-free model with a length of $l = 200$ mm.

Figure 3.21: The first three mode shapes in cutting speed direction and cutting depth direction of the Euler-Bernoulli fixed-free model.

transverse motion (cutting speed or cutting depth) assumed to be modeled by the Euler-Bernoulli beam, the shape of the boring bar cross-section will result in a slightly different moment of inertia, see Table 2.1. Thus, the resonance frequencies in respective direction will also differ.

3.2.2 Multi-span Model

Two multi-span Euler-Bernoulli boring bar models with pinned boundary conditions were considered: one corresponded to the boring bar clamped with four screws in the clamping house, and one corresponding to the boring bar clamped with six screws in the clamping house. The eigenfrequencies and mode shapes (eigenfunctions) for the two models were calculated in Matlab by finding the roots to the characteristic equation produced using the boundary conditions presented in Appendix A, and using Eq. 2.39, etc. The results for the two different models are presented in Table 3.12. When the fixed clamping model

Using four screws			
Direction of mode	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
Cutting speed direction (y-)	527.47	3390.18	9539.40
Cutting depth direction (x-)	527.30	3389.11	9536.39
Using six screws			
Cutting speed direction (y-)	566.92	3575.59	10058.76
Cutting depth direction (x-)	566.74	3574.46	10055.59

Table 3.12: The first three resonance frequencies in cutting speed direction and cutting depth direction for the Euler-Bernoulli models, with the following boundary conditions; free-pinned-pinned-free and free-pinned-pinned- pinned-free.

is changed to the pinned model, the first resonance drops by approximately 170 Hz for the four-screw-clamped boring bar, and approximately 140 Hz for the six-screw-clamped boring bar. The first three mode shapes for the two models are presented in Fig. 3.22.

3.2.3 Multi-span Model on Elastic Foundation

Finally, the multi-span boring bar models with flexible boundary conditions (corresponding to the standard boring bar clamped using four clamp screws or six clamp screws) are considered. These two models were calculated in the same way as for the multi-span models with pinned boundary condition, but now for the elastic boundary condition, using the stiffness coefficients in

Figure 3.22: The first three mode shapes, for a) the free-pinned-pinned-free model of the boring bar and b) the free-pinned-pinned-pinned-free model of the boring bar.

Table 2.5. The length of the clamp screw overhang was selected to 1.5mm. Both eigenfrequencies and mode shapes were calculated for the two multi-span boring bar models with flexible clamping boundary conditions. The calculated eigenfrequencies are presented in Table 3.13, and mode shapes are shown in Fig. 3.23.

Using four M8 screws			
Direction of mode	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
Cutting speed direction (y-)	519.43	3303.79	9257.16
Cutting depth direction (x-)	519.27	3302.81	9254.48
Using six M8 screws			
Cutting speed direction (y-)	532.09	3335.17	9278.08
Cutting depth direction (x-)	531.94	3334.20	9275.46
Using four M10 screws			
Cutting speed direction (y-)	525.24	3346.84	9404.65
Cutting depth direction (x-)	525.08	3345.83	9401.83
Using six M10 screws			
Cutting speed direction (y-)	541.52	3398.74	9484.36
Cutting depth direction (x-)	541.36	3397.74	9481.62

Table 3.13: The first three resonance frequencies in the cutting speed direction and in the cutting depth direction, for the two multi-span boring bar models, with flexible clamping boundary conditions.

3.3 Computer Simulations of Nonlinear Systems

The simulation used a linear component of the models which was based on parameters derived from the experimental modal analysis of the standard boring bar, clamped with four screws, tightened firstly from the top. The tightening torque was 30Nm and the excitation level was lowest. For the purpose of simplification, only the mode in cutting speed direction, estimated from the driving point, was used, thus the linear part $H_L(f)$ of the model only consists of one degree of freedom, which in terms of receptance may proximately be written as

$$H_L(f) = \frac{A}{j2\pi f - \lambda} + \frac{A^*}{j2\pi f - \lambda^*} \quad (3.1)$$

Figure 3.23: The first three mode shapes for the Euler-Bernoulli boring bar model, with boundary conditions a) free-spring-spring-free (four clamp screws) and b) free-spring-spring-spring-free (six clamp screws).

where

$$\lambda = -\zeta 2\pi f_0 + j2\pi f_0 \sqrt{1 - \zeta^2} \quad (3.2)$$

The values are: resonance frequency $f_0 = 555.675$ Hz, damping $\zeta = 0.966$ % and the residue $A = -j1.001 \cdot 10^{-4}$. Thus the linear system may be expressed as

$$H_L(f) = \frac{-j1.001 \cdot 10^{-4}}{j2\pi f - (-33.727 + j3491.246)} + \frac{j1.001 \cdot 10^{-4}}{j2\pi f - (-33.727 - j3491.246)} \quad (3.3)$$

Fig. 3.24 displays a diagram if the synthesized SDOF system accelerance function, corresponding to the receptance in 3.1. This figure also presents an estimate of the driving point accelerance of the boring bar in the direction of cutting speed, and a synthesized two-degrees-of-freedom system accelerance function response for two fundamental boring bar modes. These parameters

Figure 3.24: The accelerance function for the driving point of the boring bar in the cutting speed direction, the synthesized sdof system and the synthesized two-degrees-of-freedom system.

are directly applicable to the filter-method when calculating the filter coefficients, however, when using the ordinary differential equation solvers, the partial fractions are collected into one polynomial fraction, which may be expressed in terms of the mass, damping and stiffness coefficients m , c and k .

These parameters were determined using the following relations

$$A = \frac{1}{jm4\pi f_0} \quad (3.4)$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (3.5)$$

$$\zeta = \frac{c}{2\sqrt{mk}} \quad (3.6)$$

which yields a mass of $m = 1.431$ kg, a damping of $c = 96.503$ Ns/m and a stiffness of $k = 17.440 \cdot 10^6$ N/m.

3.3.1 Softening Spring Model

The nonlinear softening stiffness coefficients k_s and k_c in the signed squared and cubic model were not obtained by direct parameter estimation. The resonance frequency shifting phenomena always appears between accelerance function estimates for the standard boring bar clamped with screws for different excitation force levels. Typically, a resonance frequency shift of 5 Hz and, for example, an initial resonance frequency of 500 Hz renders a frequency deviation of 1%, which corresponds to a 10% deviation in the linear stiffness coefficient. By considering the stiffness deviation, the stiffness coefficient used in the linear model, the level of excitation force and the convergence rate in the simulation, the values for the nonlinear stiffness coefficients were selected as: $k_s = 8 \cdot 10^{12}$ N/m² and $k_c = 4 \cdot 10^{19}$ N/m³ for the signed squared and cubic model, respectively. The levels of the excitation force were given the same ratios as for the experiments with the standard boring bar, and the signal type was normally distributed random noise, with peak levels 100, 200, 300, 400 mN.

Fig. 3.25 a) presents the frequency response function estimates that were produced based on simulations of the nonlinear model with a signed squared stiffness, using the filter method and the four different excitation levels. Fig. 3.25 b) presents the corresponding frequency response function estimates produced based on simulations of the nonlinear model system with a cubic stiffness, using the filter method and the four different excitation levels. Table 3.14 gives estimates of the resonance frequency and the relative damping for the frequency response functions based on the nonlinear models simulated with the filter method, for the four excitation force levels. The SDOF least square technique [18] was used to produce estimates of resonance frequency and relative damping. Fig. 3.26 a) presents the coherence functions for simulations of the nonlinear model with a signed squared stiffness, using the filter method and the four different excitation signals. Fig. 3.26 b) shows the coherence

a)

b)

Figure 3.25: Frequency response function estimates based on simulations of the nonlinear models using the filter method and four different excitation levels, a) for the presented model with signed squared stiffness, and; b) for the model with cubic stiffness.

Squared Model				
Excitation	Level 1	Level 2	Level 3	Level 4
Frequency [Hz]	552.06	548.47	544.87	541.30
Damping [%]	1.00	1.06	1.17	1.31
Cubic Model				
Frequency [Hz]	555.03	553.31	550.52	546.98
Damping [%]	0.98	1.01	1.14	1.42

Table 3.14: Resonance frequency and relative damping estimates for the frequency response functions based on the nonlinear models, simulated with the filter method.

functions estimates for the simulations of the nonlinear model with a cubic stiffness, using the filter method and the four different excitation levels. The coherence function estimates are also presented for a narrow frequency range including the resonance frequency and are illustrated for the nonlinear model with a signed squared stiffness in Fig. 3.27 a) and for the nonlinear model with a cubic stiffness in Fig. 3.27 b).

If the ordinary differential equation solver ode45 in Matlab is used for simulations of the nonlinear model with a signed squared stiffness for the four different excitation levels, it results in the frequency response function estimates shown in Fig. 3.28 a). Fig. 3.28 b). presents corresponding frequency response function estimates, based on simulations of the nonlinear model system with a cubic stiffness, using the ordinary differential equation solver ode45 in Matlab and the four different excitation levels. Table 3.15 presents estimates of the resonance frequency and the relative damping for the frequency response functions based on the nonlinear models, simulated with the ordinary differential equation solver ode45 in Matlab, for the four excitation force levels. Also, in this case, the SDOF least square technique [18] was used to produce estimates of resonance frequency and relative damping. Fig. 3.29 a) gives the coherence functions for the simulations of the nonlinear model with a signed squared stiffness, using ode45 in Matlab and the four different excitation signals. Fig. 3.29 b) shows the coherence functions' estimates for simulations of the nonlinear model with a cubic stiffness, using the ordinary differential equation solver ode45 and the four different excitation levels. The coherence function estimates are also presented for a narrow frequency range (including the resonance frequency) and are illustrated for the nonlinear model with a signed squared stiffness in Fig. 3.30 a) and for the nonlinear model with a cubic stiffness in Fig. 3.30 b).

a)

b)

Figure 3.26: Coherence function estimates based on simulations of the non-linear models, using the filter method and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.

Figure 3.27: Coherence function estimates based on simulations of the non-linear models, using the filter method and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.

Figure 3.28: Frequency response function estimates based on simulation of the nonlinear models, using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.

a)

b)

Figure 3.29: Coherence function estimates based on simulation of the nonlinear models, using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.

a)

b)

Figure 3.30: Coherence function estimates based on simulations of the nonlinear models using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness presented and b) for the model with cubic stiffness.

Squared Model				
Excitation	Level 1	Level 2	Level 3	Level 4
Frequency [Hz]	553.73	551.97	550.18	548.36
Dampamping [%]	0.98	1.00	1.02	1.06
Cubic Model				
Frequency [Hz]	555.35	554.64	553.44	551.82
Damping [%]	0.98	0.98	1.00	1.06

Table 3.15: Estimates of resonance frequency and relative damping for the frequency response functions based on the nonlinear models simulated with the differential equation solver ode45.

Chapter 4

Summary and Conclusions

The results from the experimental modal analysis of boring bars demonstrate that a boring bar clamped in a standard clamping house with clamping screws has a nonlinear dynamic behavior. Also, the results indicate that the standard clamping house with clamp screws is the likely source of the nonlinear behavior.

The experimental modal analysis results from the boring bar clamped in a "linearized" standard clamping house with steel wedges and epoxy glue indicated a significant reduction in non-linear dynamic behavior. Thus, a boring bar clamped in a standard clamping house with clamping screws has significant, nonlinear dynamic properties. Different excitation force levels will not yield identical frequency response function estimates for the same transfer path in the boring bar (see any of the figures presenting boring bar FRF:s for different excitation force levels, i.e. Figs. 3.4, 3.6, 3.8, 3.10 and 3.12). Based on a large number of measurements, a trend may be observed; the fundamental boring bar resonance frequencies decrease with increasing excitation level; see Table 3.2 and 3.4 which summarize estimated resonance frequencies. However, with regard to the behavior of relative damping as a function of excitation force level; the results from the standard boring bar indicate that damping for the first mode increases with increasing excitation force level, while damping for the second mode decreases with increasing excitation force level; see Fig. 3.3. Also, the results from the active boring bar give an ambiguous indication of the effects on damping properties; see Fig. 3.5.

The clamp screw tightening torque appears to affect the nonlinear behavior of the boring bar. Variation in the FRF:s which was introduced by the four different excitation force levels seems to be larger for a low tightening torque (10Nm) than for a high tightening torque (30Nm), see, for example, Fig. 3.4. Also, experimental modal analysis results involving the so-called "linearized" boring bar clamping, support the conclusion that clamping conditions influence the extent of nonlinearities in boring bar dynamics. By examining, for example,

driving point accelerances in the boring bar with "linearized" clamping for the four excitation force levels (see Fig. 3.13), it can be seen that only insignificant differences are present. Thus, nonlinear behavior on the part of the boring bar seems to be almost removed (within the level of normal experimental uncertainty).

Another example of dynamic behavior on the part of the clamped boring bar is exhibited in the change in fundamental boring bar resonance frequencies with changing clamp screw tightening torques. Boring bar dynamics display increasing resonance frequency with increasing clamp torque; see Figs. 3.3, 3.5, 3.7, 3.9 and 3.11.

When changing the number of screws used for clamping, or when using "linearized" clamping, changes in dynamic properties of the boring bar (clamping system) are expected and obvious. Hence, new boundary conditions for the boring bar are introduced. Also, changing the standard boring bar to the active boring bar will alter the dynamic properties of the boring bar - clamping system, i.e. a structural part of the system is different. The order in which the clamp screws were tightened (first from the upper-side or first from the underside) had a major influence on the dynamic properties of the boring bar. This might be observed by comparing the boring bar driving point accelerances in Fig. 3.5 with the boring bar driving point accelerances in Fig. 3.7. If the clamp screws were tightened firstly from the upper side, the higher resonance frequency in Fig. 3.5 a) shows a variation from approximately 552 Hz to 562 Hz, for a change from the lowest to the highest clamp screws tightening torque. On the other hand, if the clamp screws were tightened firstly from the underside, the higher resonance frequency in Fig. 3.7 a) shows a variation from approximately 495 Hz to 530 Hz, for a change from the lowest to the highest clamp screws tightening torque. These results may have arisen due to the difficulty in producing the exact same clamping conditions when tightening the clamp screws from the bottom first, compared to tightening the clamp screws from the top first.

Another interesting observation concerns mode shapes and, in particular angles of the different modes in the cutting depth - cutting speed plane (x-y plane). Assuming that the boring bar is rigidly clamped, and that the boring bar has a homogenous cross-section in the x-y plane, this would result in one set of mode shapes in the cutting speed direction (y-direction) and one set of mode shapes in the cutting depth direction (x-direction). However, this is not the case in the results presented -the transverse sensitivity of the accelerometers is not sufficient to explain the obtained deviations of mode shape angles in the x-y plane. Transverse sensitivity would only explain mode shape angles of up to 2-3 degrees. There are two possible explanations for this phenomenon. Firstly, the assumption of a constant cross-section may not be true. The major part

of the boring bar in the length direction has a constant cross-section; however, this is not the case for the section of the boring bar head to which the tool is attached; see Table 2.1 and Fig. 2.5. Secondly, the clamping conditions may effect the mode shape rotation x-y plane. The standard boring bar clamped with four M8 screws, tightened firstly from the top has a first mode with an average mode shape angle or rotation of -20 degrees, relative to the cutting depth direction (x-direction). The second mode displays an average mode shape angle (or rotation) of -110 degrees relative to the cutting depth direction, see Table 3.7. It is also possible to notice a trend in the first mode for clockwise rotation with increasing torque; such a trend is not significant for the second mode, see Fig. 3.14. Changing the clamp screw size or the number of clamp screws affects the so-called "mode rotation", both for the standard boring bar and in the case of the active boring bar (see Figs. 3.14, 3.15 and 3.16). In addition, it should be noted that the boring bar in the linearized setup has rotated fundamental modes; the first mode has a mode shape angle or rotation of approx. -8 degrees relative to cutting depth direction (x-direction), and the second mode displays a mode shape angle or rotation of approx. -100 degrees relative to cutting depth direction (see Table 3.9). In comparison to the "linearized" clamping case, the use of six size M10 clamp screws (tightened firstly from the top) resulted in a similar rotation of the fundamental modes, constituting a difference of approx. 2 degrees; see Table 3.10.

The Euler-Bernoulli boring bar models provide rough approximations of the low-order resonance frequencies and the corresponding spatial shapes of the modes. Also, (in the x-y plane) the Euler-Bernoulli models will provide one set of mode shapes in the cutting speed direction (y-direction) and one set of mode shapes in the cutting depth direction (x-direction). The first and simplest, fixed-free model overestimates the lower fundamental resonance frequencies by approx. 170 Hz, and the upper fundamental resonance frequency by approx. 140 Hz compared to the most rigidly clamped boring bar using six M10 clamp screws. Compared with the linearized boring bar setup the fixed-free Euler-Bernoulli model overestimates the lower fundamental resonance frequencies by approx. 115 Hz, and the upper fundamental resonance frequency by approx. 100 Hz. Furthermore, the Euler-Bernoulli model yields a 0.2 Hz difference in frequency between the two fundamental resonance frequencies, while, the experimental results from, for example, the linearized boring bar setup displays a 20-30 Hz difference in fundamental resonance frequencies. It may be assumed that the linearized setup will display a difference in fundamental resonance frequencies which adheres to the Euler-Bernoulli model. However, experimental results indicate a difference of approximately 20 Hz (see Fig 3.13 and Table 3.6). It is obvious the fixed-free model will overestimate the fundamental resonance frequencies since it assumes rigid clamping which

is not the case in reality. In addition, the fixed-free model does not consider the influence of shear deformation and rotary inertia in the beam, meaning that resonance frequencies will be overestimated.

The multi-span models are assumed to be more realistic, yet simplified, models of boring bar clamping conditions. The results from the multi-span model (using the pinned boundary screw positions) illustrate that displacement of the boring bar is likely to occur between the screws, inside the clamping house. Since this configuration allows motion over a longer span than the simple fixed-free model, it also produces lower resonance frequencies. For this reason, the Euler Bernoulli model is more appropriate for the pinned boundary condition than for the fixed-free, since the length to diameter ratio has increased, even though this ratio is still below the recommended value of 10.

The effects of different screw dimensions and properties on the boring bar may be investigated by using elastic foundations. The results in Table 3.13 show that the fundamental resonance frequencies for the Multi-span model increase with an increasing number of clamp screws. An M8 screw features a 10 Hz increase, whilst an M10 screw features a 15 Hz increase. This increase in fundamental resonance frequency (for the Multi-span model) also occurs as clamp screw size is increased, so that (using four screws) yields an approximate 5 Hz increase, whilst changing from M8 to M10 (using six screws) yields approx. a 10 Hz increase. This result is confirmed by experimental results which yield a frequency change of approx. 5-15 Hz, depending on the clamp screw tightening torque (see Figs 3.3 and 3.5). Mode shapes from all the Euler-Bernoulli models are fairly similar. However, inside the clamping house, the mode shapes differ significantly between the fixed-free model and the multi-span models. In the case of the multi-span models, the mode shapes have a spatial deflection inside the clamping house, while, for the fixed-free model, the mode shapes have no deflection inside the clamping house.

Experimental results strongly indicate that the boring bar (clamped in the clamping house with screws) possesses nonlinear dynamic properties. Two different nonlinear single-degree-of-freedom models were simulated in order to investigate if they bear resemblance the dynamic behavior of the boring bar clamped in the clamping house with screws. In addition, two different simulation methods were used to provide redundancy due to the fact that there are no explicit analytical solutions for the two different nonlinear single-degree-of-freedom models which can be used as benchmark. Both the square with sign stiffness model and the cubic stiffness model show a similar trend in frequency response function estimates as the experimental results (see Tables 3.14 and 3.15). The trend is decreasing resonance frequency with increasing excitation level; see Figs. 3.25 and Figs. 3.28 (produced by the filter method and the

ODE solver method, respectively). The coherence function estimates for the input and output signals of the nonlinear SDOF systems simulated with the filter method display an expected dip at the resonance frequency that increases with increasing excitation level, see Fig. 3.26. By using the ODE solver method to simulate the comparatively nonlinear systems, the coherence function estimates assume comparatively slightly higher levels in the resonance frequency range of the SDOF systems than the filter method.

Chapter 5

Appendix A

The three span model without rotational springs and infinitely stiff transverse springs will have boundary conditions; Free-Pinned-Pinned-Free; yielding the equations as

$$\begin{aligned} EI \frac{d^2 u_1(z)}{dz^2} \Big|_{z=0} &= 0 \\ EI \frac{d^3 u_1(z)}{dz^3} \Big|_{z=0} &= 0 \\ u_1(z) \Big|_{z=l_1} &= 0 \\ \left(\frac{du_1(z)}{dz} + \frac{du_2(z)}{dz} \right) \Big|_{z=l_1} &= 0 \\ \left(EI \frac{d^2 u_1(z)}{dz^2} + EI \frac{d^2 u_2(z)}{dz^2} \right) \Big|_{z=l_1} &= 0 \\ \left(EI \frac{d^3 u_1(z)}{dz^3} + EI \frac{d^3 u_2(z)}{dz^3} \right) \Big|_{z=l_1} &= 0 \\ u_2(z) \Big|_{z=l_1+l_2} &= 0 \\ \left(\frac{du_2(z)}{dz} + \frac{du_3(z)}{dz} \right) \Big|_{z=l_1+l_2} &= 0 \\ \left(EI \frac{d^2 u_2(z)}{dz^2} + EI \frac{d^2 u_3(z)}{dz^2} \right) \Big|_{z=l_1+l_2} &= 0 \\ \left(EI \frac{d^3 u_2(z)}{dz^3} + EI \frac{d^3 u_3(z)}{dz^3} \right) \Big|_{z=l_1+l_2} &= 0 \\ EI \frac{d^2 u_3(z)}{dz^2} \Big|_{z=l_1+l_2+l_3} &= 0 \\ EI \frac{d^3 u_3(z)}{dz^3} \Big|_{z=l_1+l_2+l_3} &= 0 \end{aligned} \tag{5.1}$$

The three span model with transverse springs and rotational springs will have boundary conditions; Free-Elastic-Elastic-Free; yielding the equations as

$$\begin{aligned}
 & EI \frac{d^2 u_1(z)}{dz^2} \Big|_{z=0} = 0 \\
 & EI \frac{d^3 u_1(z)}{dz^3} \Big|_{z=0} = 0 \\
 & \left(EI \frac{d^2 u_1(z)}{dz^2} + k_R \frac{du_1(z)}{dz} + EI \frac{d^2 u_2(z)}{dz^2} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^3 u_1(z)}{dz^3} - k_T u_1(z) + EI \frac{d^3 u_2(z)}{dz^3} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^2 u_1(z)}{dz^2} + EI \frac{d^2 u_2(z)}{dz^2} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^3 u_1(z)}{dz^3} + EI \frac{d^3 u_2(z)}{dz^3} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^2 u_2(z)}{dz^2} + k_R \frac{du_2(z)}{dz} + EI \frac{d^2 u_3(z)}{dz^2} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^3 u_2(z)}{dz^3} - k_T u_2(z) + EI \frac{d^3 u_3(z)}{dz^3} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^2 u_2(z)}{dz^2} + EI \frac{d^2 u_3(z)}{dz^2} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^3 u_2(z)}{dz^3} + EI \frac{d^3 u_3(z)}{dz^3} \right) \Big|_{z=l_1+l_2} = 0 \\
 & EI \frac{d^2 u_3(z)}{dz^2} \Big|_{z=l_1+l_2+l_3} = 0 \\
 & EI \frac{d^3 u_3(z)}{dz^3} \Big|_{z=l_1+l_2+l_3} = 0
 \end{aligned} \tag{5.2}$$

The four span model without rotational springs and infinitely stiff transverse springs will have boundary conditions; Free-Pinned-Pinned-Pinned-Free; yielding the equations as

$$\begin{aligned}
 EI \frac{d^2 u_1(z)}{dz^2} \Big|_{z=0} &= 0 \\
 EI \frac{d^3 u_1(z)}{dz^3} \Big|_{z=0} &= 0 \\
 u_1(z) \Big|_{z=l_1} &= 0 \\
 \left(\frac{du_1(z)}{dz} + \frac{du_2(z)}{dz} \right) \Big|_{z=l_1} &= 0 \\
 \left(EI \frac{d^2 u_1(z)}{dz^2} + EI \frac{d^2 u_2(z)}{dz^2} \right) \Big|_{z=l_1} &= 0 \\
 \left(EI \frac{d^3 u_1(z)}{dz^3} + EI \frac{d^3 u_2(z)}{dz^3} \right) \Big|_{z=l_1} &= 0 \\
 u_2(z) \Big|_{z=l_1+l_2} &= 0 \\
 \left(\frac{du_2(z)}{dz} + \frac{du_3(z)}{dz} \right) \Big|_{z=l_1+l_2} &= 0 \\
 \left(EI \frac{d^2 u_2(z)}{dz^2} + EI \frac{d^2 u_3(z)}{dz^2} \right) \Big|_{z=l_1+l_2} &= 0 \\
 \left(EI \frac{d^3 u_2(z)}{dz^3} + EI \frac{d^3 u_3(z)}{dz^3} \right) \Big|_{z=l_1+l_2} &= 0 \\
 u_3(z) \Big|_{z=l_1+l_2+l_3} &= 0 \\
 \left(\frac{du_3(z)}{dz} + \frac{du_4(z)}{dz} \right) \Big|_{z=l_1+l_2+l_3} &= 0 \\
 \left(EI \frac{d^2 u_3(z)}{dz^2} + EI \frac{d^2 u_4(z)}{dz^2} \right) \Big|_{z=l_1+l_2+l_3} &= 0 \\
 \left(EI \frac{d^3 u_3(z)}{dz^3} + EI \frac{d^3 u_4(z)}{dz^3} \right) \Big|_{z=l_1+l_2+l_3} &= 0 \\
 EI \frac{d^2 u_4(z)}{dz^2} \Big|_{z=l_1+l_2+l_3+l_4} &= 0 \\
 EI \frac{d^3 u_4(z)}{dz^3} \Big|_{z=l_1+l_2+l_3+l_4} &= 0
 \end{aligned} \tag{5.3}$$

The four span model with transverse springs and rotational springs will have boundary conditions; Free-Elastic-Elastic-Free; yielding the equations as

$$\begin{aligned}
 & EI \frac{d^2 u_1(z)}{dz^2} \Big|_{z=0} = 0 \\
 & EI \frac{d^3 u_1(z)}{dz^3} \Big|_{z=0} = 0 \\
 & \left(EI \frac{d^2 u_1(z)}{dz^2} + k_R \frac{du_1(z)}{dz} + EI \frac{d^2 u_2(z)}{dz^2} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^3 u_1(z)}{dz^3} - k_T u_1(z) + EI \frac{d^3 u_2(z)}{dz^3} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^2 u_1(z)}{dz^2} + EI \frac{d^2 u_2(z)}{dz^2} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^3 u_1(z)}{dz^3} + EI \frac{d^3 u_2(z)}{dz^3} \right) \Big|_{z=l_1} = 0 \\
 & \left(EI \frac{d^2 u_2(z)}{dz^2} + k_R \frac{du_2(z)}{dz} + EI \frac{d^2 u_3(z)}{dz^2} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^3 u_2(z)}{dz^3} - k_T u_2(z) + EI \frac{d^3 u_3(z)}{dz^3} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^2 u_2(z)}{dz^2} + EI \frac{d^2 u_3(z)}{dz^2} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^3 u_2(z)}{dz^3} + EI \frac{d^3 u_3(z)}{dz^3} \right) \Big|_{z=l_1+l_2} = 0 \\
 & \left(EI \frac{d^2 u_3(z)}{dz^2} + k_R \frac{du_3(z)}{dz} + EI \frac{d^2 u_4(z)}{dz^2} \right) \Big|_{z=l_1+l_2+l_3} = 0 \\
 & \left(EI \frac{d^3 u_3(z)}{dz^3} - k_T u_3(z) + EI \frac{d^3 u_4(z)}{dz^3} \right) \Big|_{z=l_1+l_2+l_3} = 0 \\
 & \left(EI \frac{d^2 u_3(z)}{dz^2} + EI \frac{d^2 u_4(z)}{dz^2} \right) \Big|_{z=l_1+l_2+l_3} = 0 \\
 & \left(EI \frac{d^3 u_3(z)}{dz^3} + EI \frac{d^3 u_4(z)}{dz^3} \right) \Big|_{z=l_1+l_2+l_3} = 0 \\
 & EI \frac{d^2 u_4(z)}{dz^2} \Big|_{z=l_1+l_2+l_3+l_4} = 0 \\
 & EI \frac{d^3 u_4(z)}{dz^3} \Big|_{z=l_1+l_2+l_3+l_4} = 0
 \end{aligned} \tag{5.4}$$

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