ACTIVE CONTROL OF CHATTER IN TURNING - THE ORIGIN OF CHATTER

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ABSTRACT

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> In the turning operation chatter or vibration is a frequent problem, which affects the result of the machining, and, in particular, the surface finish. Tool life is also influenced by vibration. Severe acoustic noise in the working environment frequently occurs as a result of dynamic motion between the cutting tool and the workpiece. By proper machine design, e.g. improved stiffness of the machine structure, the problem of relative dynamic motion between cutting tool and workpiece may be partially solved. To achieve a further reduction of the dynamic motion between cutting tool and workpiece one solution is the active control of machine-tool vibration. However, to successfully apply active control of tool vibration in the external turning operation the response of the tool holder shank has to be investigated in order to enable the proper introduction of secondary vibration in the tool holder shank and to select a suitable controller. The investigation of the dynamic response of the tool holder shank relies on a stochastic approach while the structural dynamic properties have been evaluated by a normal mode analysis. This resulted in active control of tool vibration in a lathe that enables a reduction of the vibration by up to 40 dB at 1.5 kHz and by approximately 40 dB at 3 kHz.

NOMENCLATURE

$[\mathbf{M}]$	Mass matrix
[C]	Damping matrix
$[\mathbf{K}]$	Stiffness matrix
$\{\mathbf{v}\}$	Displacement column vector
$\{{f p}({f x},t)\}$	The space and time dependent
	load column vector
$\{\mathbf{y}\}$	Modal amplitude column vector
$y_n(t)$	Modal displacement of mode n
m_n	Modal mass of mode n
ω_{Dn}	The damped system's eigenfrequency
	for mode n

$p_n(\mathbf{x},t)$	The generalised load
	for mode n .
ξ_n	The modal damping ratio for mode n
ω_n	The undamped system's eigenfrequency for mode
$[\Psi]$	Mode shape matrix
$\{\psi_n\}$	Mode shape column vector for mode n
$\stackrel{\circ}{N}$	Number of degrees of freedom
R(f)	Spectral density
ψ_{i_j}	i^{th} element of j^{th} mode shape
$H_{j}^{"}(f)$	Frequency response function for mode j
j,n,k,τ	Integer
$R^*(f_n)$	Spectral density estimate
f_n	Normalized discret frequency
K	Number of periodograms
L	Periodogram length
w(au)	Digital window
U^{\uparrow}	The window dependent effective analysis
	bandwidth normalization factor
$\kappa^2_{\mathbf{w}_v,\mathbf{w}_f}(f)$	squared coherence function,

1 INTRODUCTION

In the turning operation the tool and tool holder shank are subjected to a dynamic excitation due to the deformation of work material during the cutting operation. The relative dynamic motion between cutting tool and workpiece will affect the result of the machining, in particular the surface finish. Furthermore, the tool life is correlated with the amount of vibrations and the acoustic noise introduced. The noise level is sometimes almost unbearable.

Vibrations in the cutting process is often termed as "chatter". However, the definition of chatter is not clarified properly. Extensive research has been carried out concerning the mechanisms that control the induction of vibrations in the cutting process. There are many models that explain the induction of vibrations in the cutting process. The major part is derived from either a deterministic single degree of freedom or from empirical models. The driving force of vibrations or chatter is explained with either "the regenerative effect", "the velocity dependent effect", "mode coupling" or the scatter in cutting resistance $^{[1-5]}$.

However, it is well known that vibration problem is closely related to the dynamic stiffness of the structure of the machinery and workpiece material. The vibration problem may be solved in part by proper machine design which stiffens the machine structure. To achieve a further reduction of the dynamic motion between cutting tool and workpiece one solution is the active control of machine-tool vibration. However, to successfully apply active control of tool vibration in the external turning operation the response or vibration of the tool holder shank in the turning operation has to be investigated. Both placement and choice of secondary sources or actuators are based on dynamic properties of the response or vibration of the tool holder shank also the selection of controller relay on this information. Hence, the performance of the control of tool vibration relay on the understanding of the tool vibration. In order to get a basis for the understanding of tool vibration in the external turning operation the spectral properties was investigated and the structural dynamic properties have been evaluated by a normal mode analysis.

Based on this results the active control of tool vibrations was developed and it is based on a adaptive controller that relays on the wellknown leaky filtered-x LMS-algorithm to control the response of the FIR filter controller.

2 MATERIALS AND METHODS

The cutting trials have been carried out in a SMT 500 CNC turning centre with 70 kW spindle power, 650 mm swing, 1295 mm between the centres. Some modifications have been done on the turning centre, compared with the commercial version. The maximum revolutions per minute have been increased to 4200 rpm and the tool post has been changed in order to increase the stiffness.

2.1 EXPERIMENTAL SETUP

The following measurement system has been used:

- Force dynamo meter CTS-PDJNL-5032-E15 (cutting edge angle $\kappa=93^\circ$, cutting edge inclination angle $\lambda=-6^\circ$, rake angle $\gamma=-6^\circ$).
- Syminex SGA 02 programmable two channel strain gauge amplifier module.
- Syminex XFM 82 programmable low-pass filter module.
- Hewlett-Packard E1430A, 23 bits digitiser module.

The force dynamo meter, which has been developed at the Department of Production and Materials Engineering, Lung University, is based on strain gauges [6]. Two full bridges measure the force components in primary and feeding direction while a half bridge measures the force in the thrust direction. The dynamo meter is designed to achieve as high a bandwidge as possible with minimum cross influence.

2.2 Structure dynamic properties of the tool holder shank

In general, the dynamic response of a structure cannot be described adequately by a single degree of freedom model. Usually, the response includes time variations of the displacement shape as well as its amplitude. The motion of a structure has then to be described in terms of more than one degree of freedom, i.e. the motion or response must be represented by a multiple degrees of freedoms system. The motion of a structure is usually described by the displacements of a set of discrete points of the structure. If matrix notation is introduced, the differential equation that describes the motion of a N degrees of freedom system, can be written as N

$$[\mathbf{M}]\{\ddot{\mathbf{v}}\} + [\mathbf{C}]\{\dot{\mathbf{v}}\} + [\mathbf{K}]\{\mathbf{v}\} = \{\mathbf{p}(\mathbf{x}, t)\}$$
(1)

where $[\mathbf{M}]$ is the $N \times N$ mass matrix, $[\mathbf{C}]$ the $N \times N$ damping matrix, $[\mathbf{K}]$ the $N \times N$ elastic stiffness matrix, $\{\mathbf{v}\}$ displacement column vector and $\{\mathbf{p}(\mathbf{x},t)\}$ the space and time dependent load. Usually, the mass and stiffness matrices are computed by e.g. introducing a finite element formulation. The mass matrix can then be represented by either a consistent or a lumped matrix, see [7-9].

The response of a tool holder can be assumed to be linear, if small motions are assumed [10]. Then, the time-domain dynamic response of the tool holder is determined by the mode superposition principle and each modal displacement $y_n(t)$ is determined by a convolution integral or mechanical filter, denoted Duhamel's integral [7]:

$$y_n(t) = \frac{1}{m_n \omega_{Dn}} \int_0^t p_n(\mathbf{x}, t) e^{-\xi_n \omega_n (t - \tau)} \sin(\omega_{Dn} (t - \tau)) d\tau$$

where $m_n, \omega_{Dn}, p_n, \xi_n, \omega_n$ are the modal mass, the damped system's eigenfrequency, the modal damping ratio, and the undamped system's eigenfrequency. The time-domain dynamic response of the tool holder is determined by;

$$\{\mathbf{v}\} = \sum_{n=1}^{N} \{\psi_n\} y_n = [\Psi] \{\mathbf{y}\}$$
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where the mode shape matrix $[\Psi]$ is given by:

$$[\Psi] = [\{\psi_1 \cdots \{\psi_N\}] \tag{4}$$

Further the spectral density, $R_{v_i}(f)$, for the dynamic response $v_i(t)$ of a narrow-banded system with well separated eigen-

modes can be approximated with [7];

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$$R_{v_i}(f) \approx \sum_{j=1}^{N} R_{v_{i_j}}(f) = \sum_{j=1}^{N} \psi_{i_j}^2 |H_j(f)|^2 R_{p_j}(f)$$
 (5)

where ψ_{ij} is the corresponding mode shape coefficient to $v_i(t)$ at the jth eigenmode, $H_j(f)$ is the frequency response function for mode j and $R_{p_j}(f)$ is the spectral density of the generalised load for mode j. Hence, if the machine-tool-system is classified as narrow-band system, i.e $\xi < 0.1$ [7], it is likely that the tool holder shank will act as a filter, which has a passband at each eigenfrequency.

2.3 Spectral Properties

The spectral density, R(f), expresses how the power of a random process is distributed in terms of the frequency. Here, the spectral densities of the dynamic response in the primary cutting direction, $R_{\mathbf{w}_r}(f)$, and in the feeding direction, $R_{\mathbf{w}_f}(f)$, have been examined.

The spectral density estimation can either be done by Welch's method [11] or Blackman-Tukey's method [12]. The latter method results in somewhat lower variance but the computational efforts are much higher. Therefore Welch's method is usually preferred. The Welch spectral estimator is given by;

$$R^{*}(f_{n}) = \frac{1}{KLU} \sum_{k=1}^{K} \left| \sum_{\tau=0}^{L-1} x(\tau)w(\tau)e^{-i2\pi\tau n/L} \right|^{2}, \ f_{n} = \frac{n}{L}$$
(6)

where $n=0,\ldots,L/2$, K the number of periodograms, L the length of the periodogram and;

$$U = \frac{1}{L} \sum_{\tau=0}^{L-1} (w(\tau))^2$$

is the window dependent effective analysis bandwidth normalization factor. The correlation of the motion in the primary and feeding direction can be examined in terms of the cross spectral density $R_{\mathbf{w}_v,\mathbf{w}_f}(f)$. However, since this is a complex entity it is more convenient to study the squared coherency spectrum, which is given by:

$$\kappa_{\mathbf{w}_{v},\mathbf{w}_{f}}^{2}(f) = \frac{|R_{\mathbf{w}_{v},\mathbf{w}_{f}}(f)|^{2}}{R_{\mathbf{w}_{v}}(f)R_{\mathbf{w}_{f}}(f)}$$
(7)

All spectral densities are also here estimated by Welch's method.

3 RESULTS

3.1 Result - Normal Mode Analysis

The analysis of the dynamic properties of the tool holder shank was carried out in the general purpose finite element

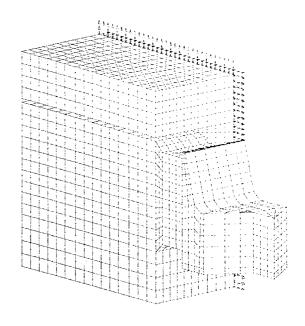


Figure 1: Finite element model of tool holder [13].

software package MSC/Nastran [14], using Lanczos method to extract the eigenvalues [15]. The model consists of 3550 solid 20 and 15 node brick elements and 17057 nodes or 51171 unknowns or degrees of freedom and is shown in figure 1. The result of the normal mode analysis is summarised in table 1.

Mode	f_e (Hz)	M_n	Γ_v	Γ_f	Γ_p
1	5530	0.162	-2.12E-10	6.93E-09	-1.10E-09
2	7310	0.164	3.15E-09	-3.55E-10	1.97E-09
3	8650	0.679	2.71E-10	-4.33E-10	-5.37E-10
4	9740	0.421	8.75E-10	1.46E-09	3.35E-10
5	11070	0.229	5.71E-10	6.70E-10	-7.31E-11
6	12810	0.188	2.52E-10	5.60E-10	-6.17E-10
7	13660	0.202	3.33E-10	-4.55E-10	1.31E-10
8	15500	0.480	-3.40E-10	1.12E-10	-2.56E-10
9	16760	0.209	1.14E-10	-8.31E-10	-3.05E-10
10	16980	0.372	3.46E-10	-1.33E-10	4.83E-10

TABLE 1: Eigenfrequency f_e with generalized mass, M_n and modal participation factors Γ_n [13].

3.2 Result - Spectral properties

In order to emphasize the correlation between the peak of the spectral density estimates and the result of the normal mode analysis the corresponding frequency to each normal mode is plotted as vertical line in a spectral density estimate plot, see figure 2. In order to facilitate observations of the influence of

the parameter space, the spectral density estimates are presented as waterfall diagrams. Especially, if speed is used as

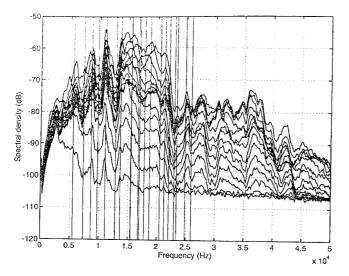


Figure 2: The first twenty calculated eigenfrequencies of the tool holder shank ,the vertical lines and the spectral densities in SS 0727-02 in primary direction, s=0.3 mm/rev, a=3mm, v=50 m/min to 400 m/min, tool DNMG 150612-QM, grade 4025 $^{[13]}$.

a parameter, the growth of the vibration energy in the cutting process as a function of the cutting speed is obvious. To illustrate that the eigenfrequencies are constant at each feed rate and speed, the spectral density estimates were first plotted for all of the cutting speeds at constant feed rate and then for all of the feed rate at constant cutting speed, see Figs. 3 and 4.

In Fig. 5 the spectral density estimate for the dynamic response in the primary direction during a continuous cutting operation in SS 0727-02, s=0.4 mm/rev is shown as a waterfall and in Fig. 5 the corresponding spectral density estimate for the dynamic response in the feed direction is shown as a waterfall.

The covariance between the dynamics of the tool motions in the primary cutting direction and the feeding direction can be expressed in terms of the cross covariance function $r_{\mathbf{w}_v,\mathbf{w}_f}(\tau),$ or its Fourier transform, the cross spectrum $R_{\mathbf{w}_v,\mathbf{w}_f}(f).$ Usually, it is more convenient to use the squared coherency spectrum $\kappa^2_{\mathbf{w}_v,\mathbf{w}_f}(f)$ to express the collinearity between the tool motions in the primary and feeding directions. In figure 7 the squared coherency spectrum during a continuous cutting operation in SS 0727-02 is shown. When the squared coherency spectrum $\kappa^2_{\mathbf{w}_v,\mathbf{w}_f}(f)$ is close to one we have a linear relationship. It can be noticed that there exists only a linear relationship at some of the eigenfrequencies. This relationship is determined by the modal participation factor. It shows in which directions in the global coordinate system the eigenmode participates.

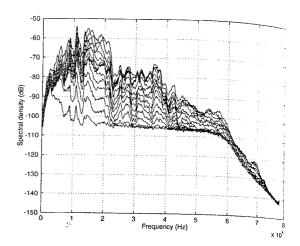


Figure 3: The spectral densities in SS 0727-02 in primary direction s=0.3 mm/rev, a=3mm, v=50 m/min to 400 m/min, tool DNMG-QM 150612, grade 4025 [13]

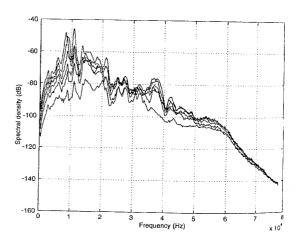


Figure 4: The spectral densities in SS 0727-02 in primary direction, s=0.1 mm/rev to 0.6 mm/rev , v=275 m/min, tool DNMG-QM 150612, grade 4025 $^{[13]}$.

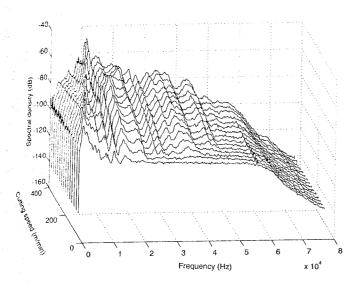


Figure 5: The spectral density estimate of the dynamic response of the tool holder shank in the primary direction during a continuous cutting operation in SS 0727-02, $s=0.4~\mathrm{mm/rev}$, $a=3~\mathrm{mm}$, v=50 - 400 m/min, tool DNMG 150612-QM, grade 4025 $^{[13]}$.

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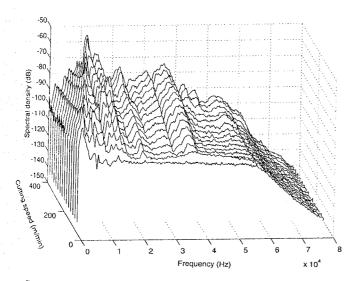


Figure 6: The spectral density estimate of the dynamic response of the tool holder shank in the feed direction during a continuous cutting operation in SS 0727-02, $s=0.4~\mathrm{mm/rev}$, $a=3~\mathrm{mm}$, v=50 - 400 m/min, tool DNMG 150612-QM, grade 4025 [13].

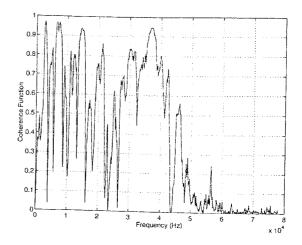


Figure 7: The squared coherency spectrum during a continuous cutting operation in SS 0727-02, $s=0.5,\ a=3$ mm, v=200 m/min, tool DNMG-QM 150612, grade 4025 [13].

3.3 Result-Adaptive Control of Tool Vibration

The original excitation of the tool vibrations, originating from the material deformation process, cannot be directly observed. Consequently, the controller for the control of machine-tool vibration is based on a feedback approach. The response of the tool holder can be measured with a sensor mounted on the machine-tool. By introduction of secondary anti-vibrations with a secondary source, actuator, the response of the tool holder can be modified [16]. The actuator is steered by a controller which is fed with the accelerometer signal sensing the vibrations of the tool holder, see Fig. 8. The

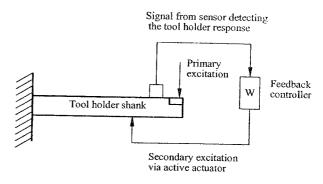


Figure 8: A machine-tool feedback control system^[16].

tool holder in this application is a construction with integrated actuators, i.e. secondary sources [17]. The construction of the tool holder is shown in Fig. 9. The controller used for the control of tool vibration in the cutting speed direction was based on the wellknown filtered-x LMS-algorithm. To illustrate the effect of feedback control of tool vibration in the cutting speed



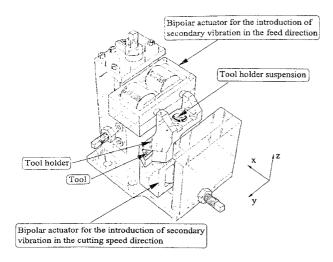


Figure 9: Tool holder with integrated actuators for the control of tool vibrations in the metal cutting process [17],

direction, the spectral densities of the tool vibrations with and without feedback control are given. Fig. 10 shows a typical result obtained with adaptive feedback control of tool-vibration. It performs a broad-band attenuation of the tool-vibration and manage to reduce the vibration level with up to approximately 40 dB simultaneously at 1.5 kHz and 3 kHz.

4 CONCLUSIONS AND FUTURE WORK

It is obvious that holder shank motion is characterized by more than one eigenmode except for the lowest feed rate and cutting speed. This emphasizes that a single degree of freedom approach can not describe the vibrations in the cutting process. A multiple degree of freedom system must be used in order to describe the tool holder shank motion. Also, each machine-tool-system will have individual vibration characteristics due to mass, damping and stiffness. The spectral density as well as the covariance property of the tool holder shank motion can easily be described in terms of the input load if the system is assumed to be linear. The mode superposition can then be applied to a random response. This means that the driving force of the tool holder vibrations are influenced by the spectral properties of the applied load as well as the structural dynamic properties of the tool holder. Each modal load can be assumed to be a random process [7]. Usually, for systems that are lightly damped and have separated eigenfrequencies. the modes are statistically independent. This means that the cross spectral density contribute very little to the mean square response. Hence, the term mode coupling is most often irrelevant in tool vibrations. By analyzing the modal participation factor for each eigenmode of a structure, critical excitations of the tool holder can be avoided. Also, the damping of the tool holder can be estimated by the half power bandwidth by utilizing the spectral density estimates for the most dominant

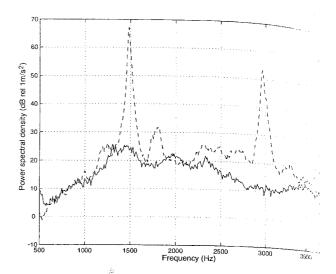


Figure 10: The power spectral density of tool vibration with 20-tap FIR filter feedback control (solid) and without (dashed). Step length $\mu=0.05$, cutting speed v=80 m/min, cut depth a=0.9 mm, feed rate s=0.25 mm/rev, tool DNMG 150604-PF, grade 4015 [18]

modes of the response.

If there exists a linear mapping between the measured signals and the dynamic response of the tool tip, i.e. the Duhamel's integral and the mode superposition is valid, then the squared coherency spectrum will be preserved during a linear transformation [19]. This means that the linear relationship between the coupling of the tool tip motion in the primary and feeding directions can be studied at all frequencies by an examination of the squared coherency spectrum of the measured signals.

The tool vibrations in a turning operation mainly comprise vibrations in two directions: the cutting speed direction and the feed direction $^{[13,\ 16]}$. Usually, the vibrations in the cutting speed direction and the feed direction are linearly independent, except at some of the eigenfrequencies $^{[13]}$. Consequently, the control problem involves the introduction of two secondary sources, driven in such a way that the antivibrations generated by means of these sources interfere destructively with the tool vibration $^{[20]}$. However, in external longitudinal turning, most of the vibration energy is usually induced in the cutting speed direction $^{[13,\ 16]}$. It is thus likely that the control of tool vibration in the cutting speed direction is an adequate solution the vibration problem $^{[16,\ 21]}$

The statistical properties of the tool vibration imply a controller which utilizes the statistical correlation of the vibrations [22]. A classical statistical criterion is the mean square error criterion [23]. However, a controller based on this criterion cannot generally solve the control problem, since a such controller is

only "optimum" in a stationary environment [24]. The statistical properties of the tool vibrations may vary during the machining process. Changes in cutting data and material properties influence the statistical properties of tool vibrations [13, 16]. Variation within the allowed cutting data interval may also influence the structural response of the tool holder [16]. A solution to the control problem is the adaptive feedback control of machine-tool vibration based on the wellknown filtered-x LMS-algorithm [16]. It performs a broad-band attenuation of the tool-vibration, and is able to reduce the vibration level by up to approximately 40 dB simultaneously at 1.5 kHz and 3 kHz (see Fig. 10).

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