Π-TRUNCATED FUNCTIONS AND ROUGH SETS IN THE CLASSIFICATION OF INTERNET PROTOCOLS

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ABSTRACT: The studies of Internet Protocol data give rise to the creation of polygons consisting of finite numbers of points tied together. Since the polygons are not formalized by some mathematical expressions, we suggest creating continuous functions, which approximate them thoroughly in spite of their irregular shapes. To warrant a high accuracy of approximating, otherwise impossible to obtain when using standard curves, we test a continuous function, which is composed of joined truncated π-class functions with seven parameters.

By operating with the functions representing polygons having unusual shapes, we attempt a classification of Internet traffic data, based on datagram sizes. We adopt rough sets to assign the members to an investigated Internet class even if their origin sometimes is unknown.

KEYWORDS: truncated π-functions, a polygon approximation, indiscernibility relation, rough sets in IP traffic characterization

1 INTRODUCTION

Some examinations of the behavior of two variables X and Y provide us with strings of values x and y, which can be included in the pairs (x, y), treated further as the coordinates of points in the two-dimensional system. We suppose that a finite set A consists of the points (x, y), thus it can be illustrated as a polygon with its points joined together by segments of straight lines.

Certain experiments, in which $y\in[0, 1]$, deliver the polygon (the set A) composed of parts looking like bells (or hills), e.g., like A, sketched in Fig. 1. The polygon, which ties a lot of straight-line bits, rather cannot constitute a piecewise interpolation of the points since a number of first degree polynomial equations is too large for further efficient analysis and, moreover, such interpolation is not smooth enough.

The most popular classical method of approximating applied to a set of points is known as the least-square regression with modern variants [2]. Other algorithms of approximating that we can mention adopt such technical tools as cubic polynomials based on four points [4], tangent curves [1], free algebras [3] or weighted approximations [11].

As the counterpart of the listed procedures we consider an approximation of multi-shapes from Fig. 1 by π-truncated functions used piecewise, since their y-values as well as y-coordinates of the points constituting the elements of A belong to the interval [0, 1]. The procedure forms the approximation of A by truncated π-functions tied by pieces of straight lines, if needed.

In some experimental domains of science, e.g., in the classification of Internet Protocol traffic we encounter polygons as results of the accomplished observations. We wish to explain that an Internet Protocol (IP) is regarded as a protocol, which provides the necessary functions to deliver Internet information units called datagrams.

We observe the density of the Internet datagram size in the universe $X = \{sizes\}$ to determine a finite set of pairs $A = \{(x, y)\} = \{(datagram\ size,\ density\ of\ datagram\ size)\}$, $y\in[0, 1]$. An average IP-datagram size experiment delivers the set A resembling the polygon from Fig. 1.

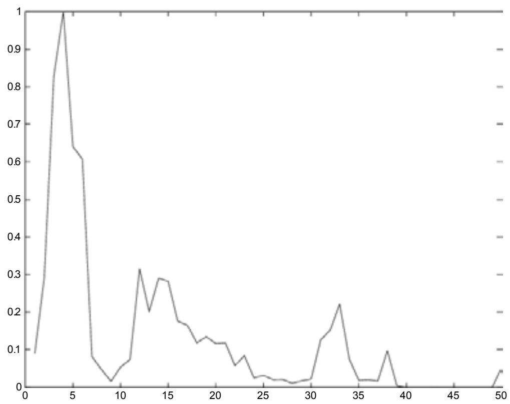

Figure 1: The polygon reflecting $A = \{(x, y)\}$

The procedure of IP-datagram approximation is developed in the second section of the paper. "The sampled truncated π", as we call the introduced approximation curve, consists of first and second degree-polynomials. It follows the polygon's shape very closely and it cumulates a very low error referring to deviations between the approximating curve and the polygon, which is regarded as an important advantage of this approximation method. We also assume that a continuous function, which provides us with y-values corresponding to regularly chosen x (not always appearing in the set of points), is more useful in further analysis of polygons, e.g., their comparison.

We should add that the proposed approach to the approximation of non-standard shapes contributes an original own solution of the problem.

The elements of Rough Set Theory appear in the third part of this work as important instruments of Internet [14871487]traf[1488]fic classification. The aim is to extract characteristic data for various application protocols such as HTTP (Hypertext Transfer Protocol), FTP (File Transfer Protocol), DNS (Domain Name Service) etc. We possess six polygons of the type $A = \{(\textit{datagram size, density of datagram size})\}$. Among the sets four belong to the same class DNS, one is a member of the HTTP class and one of them is unknown. By accepting the membership in the DNS class as a decision attribute in the indiscernibility relation, we are able to place the unknown polygon within the classes under consideration.

2 SAMPLED TRUNCATED Π IN THE APPROXIMATION OF CLOCK-LIKE POLYGONS

The approach to approximation of irregular polygons presented below constitutes the authors' own and new solution [10], which deviates from standard procedures of seeking approximation curves [1, 2, 3, 4, 11].

2.1 The adjustment of a π-function to the shape of a polygon

Let us first suppose that one part A_1 of the obtained polygon A, whose shape resembles a bell is determined by a set of pairs (x, y), which represent the finite fuzzy set $A_1 \subseteq A \subseteq X$.

Example 1
We examine the values of pairs included in the set A_1, which constitutes a part of A presented by Fig. 1 over the interval (30, 35). We find that A_1 is determined by $A_1 = \{(30, 0.03), (31, 0.06), (31.5, 0.17), (32, 0.20), (33, 0.25), (33.5, 0.23), (34, 0.14), (35, 0.06)\}$. The points corresponding to the pairs given above are tied together to build the polygon A_1 drawn in Fig. 2.

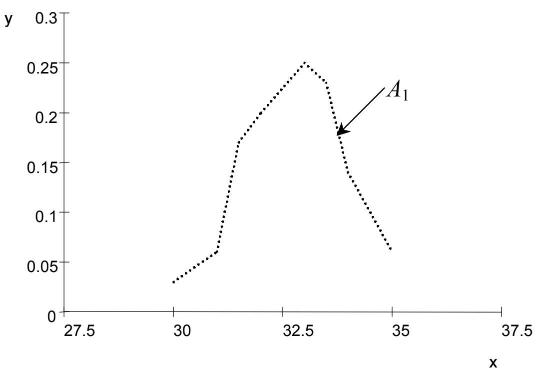

Figure 2: The polygon representing the set A_1

To an approximation of the pattern of points from Fig. 2 the π-function fits best because of its clock-wise shape and its range constituting the interval [0, 1].

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We quote the formula of π, in accordance with such sources as [5, 6, 10, 12, 13]

$$y = \begin{cases} (1)\ 0 & \text{for } x < \alpha_1 \\ (2)\ 2\varepsilon\left(\dfrac{x-\alpha_1}{\gamma_1-\alpha_1}\right)^2 & \text{for } \alpha_1 \le x < \beta_1 \\ (3)\ \varepsilon\left(1-2\left(\dfrac{x-\gamma_1}{\gamma_1-\alpha_1}\right)^2\right) & \text{for } \beta_1 \le x < \gamma_1 \\ (4)\ \varepsilon & \text{for } x = \gamma_1 = \alpha_2 \\ (5)\ \varepsilon\left(1-2\left(\dfrac{x-\alpha_2}{\gamma_2-\alpha_2}\right)^2\right) & \text{for } \alpha_2 < x < \beta_2 \\ (6)\ 2\varepsilon\left(\dfrac{x-\gamma_2}{\gamma_2-\alpha_2}\right)^2 & \text{for } \beta_2 \le x \le \gamma_2 \\ (7)\ 0 & \text{for } x > \gamma_2. \end{cases} \quad (1)$$

The function possesses six standard parameters α_1, β_1, γ_1, α_2, β_2, γ_2, and it has the additional parameter ε, added in (1), which accommodates the height of the function to the real data existing in the set A_1. The parameters β_1 and β_2 are estimated as

$$\beta_1 = \frac{\alpha_1+\gamma_1}{2},\ \beta_2 = \frac{\alpha_2+\gamma_2}{2}. \quad (2)$$

Example 2
We now intend to recall how the π-function inserted by (1) and (2) looks like. If we suppose that $\alpha_1 = 30$, $\gamma_1 = \alpha_2 = 32.5$, $\gamma_2 = 35$, $\beta_1 = \frac{\alpha_1+\gamma_1}{2} = 31.25$, $\beta_2 = \frac{\alpha_2+\gamma_2}{2} = 33.75$, $\varepsilon = 0.25$ then the function will have the graph depicted in Fig. 3.

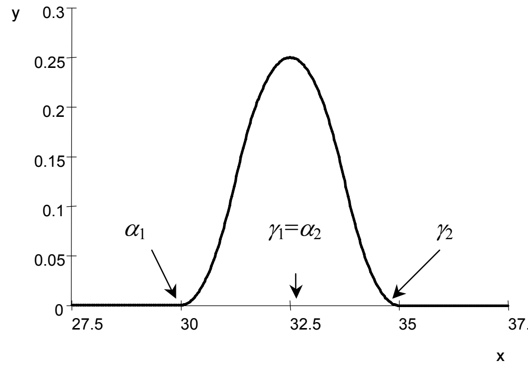

Figure 3: The π-function for $\alpha_1 = 30$, $\gamma_1 = \alpha_2 = 32.5$, $\gamma_2 = 35$ and $\varepsilon = 0.25$

The pairs in the set A_1 from Ex. 1 have no y-coordinates equal to zero, which means that the values of α_1, and γ_2 in the π-function, which is expected to approximate A_1, are unknown. By accepting the value of ε as the largest y-coordinate in A_1, corresponding to the x-coordinate taken as $\gamma_1 = \alpha_2$, we reconstruct the values of remaining parameters α_1, γ_2 according to the follow-

ing patterns:
* Denote by $A_1(X)$ all x-values, which belong to n points from A_1. If the pair $(x_{\min}, y(x_{\min}))$ begins the set A_1, which means that $x_{\min} = \min_{1\le k\le n} x_k, x_k \in A_1(X)$, then

a) $\alpha_1 = \dfrac{x_{\min} - \gamma_1\sqrt{\frac{y(x_{\min})}{2}}}{1-\sqrt{\frac{y(x_{\min})}{2}}}$ for $y(x_{\min}) < \dfrac{\varepsilon}{2}$. This case entails the changes in (1) in accordance with

$$y = \begin{cases} (1)\;\; 0 & \text{for } x < x_{\min} \\ (2)\;\; 2\varepsilon\left(\dfrac{x-\alpha_1}{\gamma_1-\alpha_1}\right)^2 & \text{for } x_{\min} \le x < \beta_1 \\ (3)-(7). \end{cases} \quad (3)$$

b) $\alpha_1 = \gamma_1 - \dfrac{\gamma_1 - x_{\min}}{\sqrt{\frac{1-y(x_{\min})}{2}}}$ for $y(x_{\min}) \ge \dfrac{\varepsilon}{2}$. Then the $\pi(x)$ formula appears as

$$y = \begin{cases} (1)-(2)\;\; 0 & \text{for } x < x_{\min} \\ (3)\;\; \varepsilon\left(1-2\left(\dfrac{x-\gamma_1}{\gamma_1-\alpha_1}\right)^2\right) & \text{for } x_{\min} \le x < \gamma_1 \\ (4)-(7). \end{cases} \quad (4)$$

** The pair $(x_{\max}, y(x_{\max}))$ ends the set A_1, which corresponds to $x_{\max} = \max_{1\le k\le n} x_k, x_k \in A_1(X)$. Hence

c) $\gamma_2 = \dfrac{x_{\max} - \alpha_2\sqrt{\frac{y(x_{\max})}{2}}}{1-\sqrt{\frac{y(x_{\max})}{2}}}$ for $y(x_{\max}) < \dfrac{\varepsilon}{2}$. We thus suggest the following changes in (1) to adapt it to the new assumptions

$$y = \begin{cases} (1)-(5) \\ (6)\;\; 2\varepsilon\left(\dfrac{x-\gamma_2}{\gamma_2-\alpha_2}\right)^2 & \text{for } \beta_2 \le x < x_{\max} \\ (7)\;\; 0 & \text{for } x \ge x_{\max}. \end{cases} \quad (5)$$

d) $\gamma_2 = \alpha_2 + \dfrac{x_{\max} - \alpha_2}{\sqrt{\frac{1-y(x_{\max})}{2}}}$ for $y(x_{\max}) \ge \dfrac{\varepsilon}{2}$. We adjust the $\pi(x)$ formula as

$$y = \begin{cases} (1)-(4) \\ (5)\;\; \varepsilon\left(1-2\left(\dfrac{x-\alpha_2}{\gamma_2-\alpha_2}\right)^2\right) & \text{for } \alpha_2 \le x < x_{\max} \\ (6)-(7)\;\; 0 & \text{for } x \ge x_{\max}. \end{cases} \quad (6)$$

The modified π constitutes a segment of the classical π-function; therefore we will name it a truncated π-function.

By selecting the minimal and the maximal x-values as well as the maximal y-value existing in the set A_1, we prepare the mathematical apparatus with (3)–(6) for computing the unknown parameters α_1 and γ_2. The point, in which the y-coordinate takes the ε-value and the x-coordinate – the $\alpha_2 = \gamma_1$ value, belongs both to the polygon and the function π. In spite of reconstructing the theoretical values of α_1 and γ_2 the approximating function is not intersected by the x-axis. The domain of π begins with the minimal x-value in $A_1(X)$ and is ended by the maximal x. This warrants that the polygon and the curve lie very close to each other.

Example 3
The adjustments, accomplished for the data describing A_1 from Ex. 1, should be done by applying both (3) and (5). Figure 4 shows the total effects of evaluating the finite point set A_1 by a continuous function $\pi_1(x)$ possessing the reconstructed parameters $\alpha_1 = 29.581$, $\beta_1 = 31.291$, $\gamma_1 = \alpha_2 = 33$, $\beta_2 = 34.210$, $\gamma_2 = 35.419$, and $\varepsilon = 0.25$.

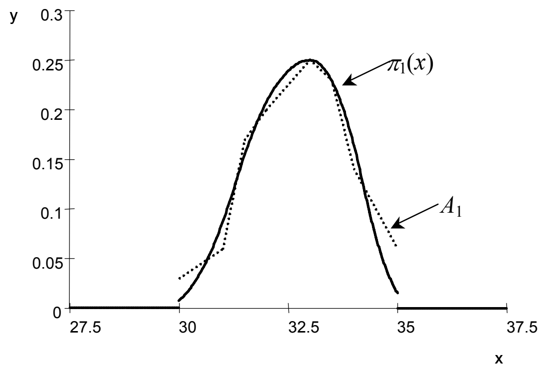

Figure 4: The approximation of A_1 by π-function

For instance, since $y(x_{\min}) = 0.03$ satisfies the assumed condition $y(x_{\min}) < \dfrac{\varepsilon}{2}$, then we will compute α_1 as $\alpha_1 = \dfrac{30-33\sqrt{\frac{0.03}{2}}}{1-\sqrt{\frac{0.03}{2}}} = 29.581.$

The truncated π_1-function, accommodated to the set A_1, has a full expansion as

$$y = \begin{cases} 0.25\cdot 2\left(\frac{x-29.581}{33-29.581}\right)^2, & 30 \le x < 31.291 \\ 0.25\left(1-2\left(\frac{x-33}{33-29.581}\right)^2\right), & 31.291 \le x < 33 \\ 0.25, & x = 33 \\ 0.25\left(1-2\left(\frac{x-33}{35.419-33}\right)^2\right), & 33 < x < 34.210 \\ 0.25\cdot 2\left(\frac{x-35.419}{35.419-33}\right)^2, & 34.210 \le x \le 35. \end{cases}$$

By using the same procedure to all "bells" visible in A in Fig. 1, we obtain other functions of the π type. We join the functions by inserting equations of straight lines to plot a full continuous curve $\pi(x)$ approximating A entirely. The final effect of approximation of the set, taking place in Fig. 1, is explained in the next subsection.

Since we adapt several π functions to truncated forms, then we will call a sampled approximation “sampled, truncated π”.

2.2 The sampled truncated π in the approximation of a DNS polygon

A central issue, regarding Internet traffic measurements and engineering today, is gaining enough information from measurements taken without compromising the integrity of users. These are important tasks for Internet Service Providers to perform in order to be able to provide users with the level of service that they expect. To overcome privacy issues and the unreliability of using Internet application protocol specific information, an approach to the classification of Internet traffic can be made by using information obtained from IP itself. Basing our observations on the presence of datagram size frequencies the objective is to extract characteristic data for various application protocols such as HTTP, FTP, DNS and the like. By using parameterized continuous functions instead of discrete values, in most cases the result will be a system with fewer parameters.

Example 4
Figure 5 is an example of a DNS traffic distribution, showing a characteristic mix of datagram sizes. These have been computed as results of our own measurements typical of the accomplished observations, with regards to the frequencies with which the Internet datagrams occur.

The DNS traffic distribution, already plotted in Fig. 1, is measured as the point set $A = \{(x, y)\}$. If the set is represented by a polygon $p(x)$ composed of segments of straight lines which tie (x, y), then Fig. 5 gives the image of an approximation of the shape from Fig. 1 by a collection of truncated π functions.

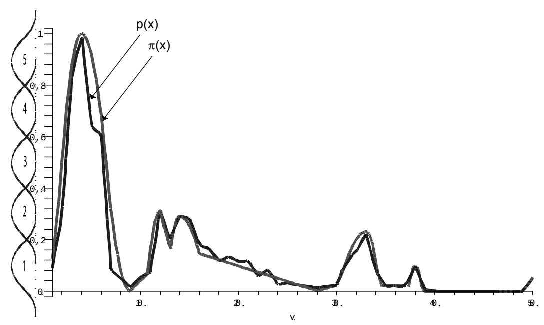

Figure 5: The sampled π in the approximation of a DNS datagram

It is worth noticing that the collective error that measures the deviations of $\pi(x)$ from $p(x)$ is not large, which is very important for the approximation of a composed polygon consisting of many “bells”.

The graph reflects the correlation between x and y values provided that the x values are determined as the sizes of IP datagrams carrying DNS traffic, while the y values describe the densities with which the sizes occur. The $p(x)$ graph is a 50 bin linear approximation of the obtained data, in which the original datagram sizes range from 0 to 1500 bytes. The $\pi(x)$ graph is the proposed sampled, truncated π-approximation of the same, save for the linear approximations used in order to make the function continuous. In the evaluation of the $p(x)$ polygon, when using the π-function, the following formulas have yielded satisfactory results:

$$y = \begin{cases} (1) & 0 & \text{for } x < 0 \\ (2) & 2\left(\frac{x}{4}\right)^2 & \text{for } 0 \le x < 2 \\ (3) & \left(1 - 2\left(\frac{x-4}{4}\right)^2\right) & \text{for } 2 \le x < 4 \\ (4) & 1 & \text{for } x = 4 \\ (5) & \left(1 - 2\left(\frac{x-4}{9-4}\right)^2\right) & \text{for } 4 < x < 6.5 \\ (6) & 2\left(\frac{x-9}{9-4}\right)^2 & \text{for } 6.5 \le x < 9 \\ (7) & 0.037x - 0.33 & \text{for } 9 \le x < 10.7 \\ (8) & 0.62\left(\frac{x-10}{2}\right)^2 & \text{for } 10.7 \le x < 11 \\ (9) & 0.31\left(1 - 2\left(\frac{x-12}{2}\right)^2\right) & \text{for } 11 \le x < 12 \\ (10) & 0.31 & \text{for } x = 12 \\ (11) & 0.31\left(1 - 2\left(\frac{x-12}{8}\right)^2\right) & \text{for } 12 < x \le 14 \\ & \ldots & \\ (n) & 0.052x - 2.55 & \text{for } 49 \le x \le 50. \end{cases}$$

A number of split functions, which are included in the sampled definition of $\pi(x)$ is substantially less than a number of linear functions that define short line pieces placed among the nodes (x, y) in the polygon $p(x)$. One π segment can surround a great many pairs (x, y). This reduces the number of piecewise definitions and the number of subintervals in the “sampled truncated π”. By introducing π we simplify a collective definition of the function approximating A, when comparing it to the linear splines taking place in the interpolation of $p(x)$. This property of π should be regarded as its advantage.

3 ROUGH SET THEORY IN THE POLYGON CLASSIFICATION

In order to include the unknown sets A within classes already possessing the declared members, we apply some elements of Rough Set Theory [6, 7, 8, 9], which have proven useful in the process of a polygon classification.

3.1 The theoretical background of classification

The y-axis in Fig. 5 is divided in five regions reflecting the growing density. We would like to assign codes associated with the densities by dividing the y-axis in five subintervals of the same length. Some scientists have a custom of applying fuzzy sets with their membership functions to accomplish the fuzzy determination of interval borders in the case of ascending values, which is discussed, e.g., in [6]. Anyhow, we do not want to discuss new elements of fuzzy set theory in this dissertation and we only announce another possibility of finding the boundaries for the considered intervals by drawing five membership functions along the *y*-axis in Fig. 5. Independently of the method, we list the density terms and their associations with the intervals and the codes in Table 1.

Table 1: The relationship between densities and codes

Density	Interval	Code
"very small"	(0.0, 0.2)	1
"small"	(0.2, 0.4)	2
"average"	(0.4, 0.6)	3
"big"	(0.6, 0.8)	4
"very big"	(0.8, 1.0)	5

Each considered point set has an envelope created by a continuous function that approximates it. When regarding any datagram size represented by a value placed on the *x*-axis, we are capable of establishing the association between the *x*-value and the code. To achieve this we should at first compute the $\pi(x)$ value and then place it in the appropriate interval from Table 1. We can thus accept the set $A = \{(x, y)\} = \{(x, code(x)\}$.

Let us introduce a universe set $U=\{H_1, \ldots, H_n\}$ composed of the "IP datagram" polygons. The objects of U are determined by two groups of attributes, so called condition and decision attributes presented by the sets B and D respectively. We assume that the set B consists of datagram sizes x_j, mapped into a set of values $code_{H_i}(x_j)$, $i = 1, \ldots, n, j = 1, \ldots, m$, which are equal to the integers 1, 2, 3, 4, 5. The set D has an attribute stated as "the membership of a polygon in the DNS class", where the membership is expressed as "yes", "no", "unknown".

The triple $I=(U, B, D)$ forms the decision table, which constitutes a data basis for an equivalence relation $I(B)$ called the indiscernibility relation and defined by a relationship

$$I(B) = \{(H_i, H_k) : code_{H_i}(x_j) = code_{H_k}(x_j)\} \text{ for each size } x_j, \qquad (7)$$

where $j = 1, 2, \ldots, m,\ i, k = 1, 2, \ldots, n$.

We find the equivalence classes of the relation $I(B)$, i.e., the blocks $IB(H_i)$ as the sets

$$IB(H_i) = \{H_k : (H_i, H_k) \in I(B)\}. \qquad (8)$$

By following a general rough set procedure we create a set $X = \{H_i$: which have the decision "yes" assigned$\}$.

The first decision set (the lower approximation of X)

$$B_*(X) = \{H_i : IB(H_i) \subseteq X\} \qquad (9)$$

reveals the polygons which surely match the DNS class. The other decision set (the upper approximation of X)

$$B^*(X) = \{H_i : IB(H_i) \cap X \neq 0\} \qquad (10)$$

contains the members of U, which may belong the considered class DNS.

The elements of a boundary set

$$B_{border}(X) = B^*(X) - B_*(X) \qquad (11)$$

are interpreted as members of DNS in a certain grade.

The membership degree of H_i, interpreted as a degree of being a member in DNS, is computed as

$$\mu_{DNS}(H_i) = \frac{|X \cap IB(H_i)|}{|IB(H_i)|}. \qquad (12)$$

3.2 DNS classification by means of rough sets

We collect the data concerning six IP point sets and approximate the obtained polygons by "sampled truncated π" as shown in Fig. 6.

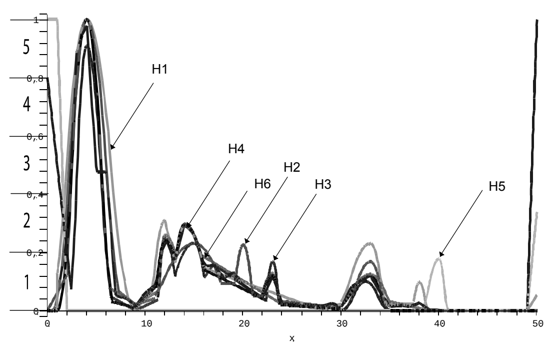

Figure 6: The approximation of polygons from different IP classes by a sampled truncated π

We state $U = \{H_1, H_2, H_3, H_4, H_5, H_6\}$.

The decision table $I = (U, B, D)$, made for the attributes listed in Subsection 3.1, has the properties expanded in Table 2.

Table 2: The decision table $I = (U, B, D)$

$H_i \backslash x_j$	0	4	8	12	16	20	24	28	DNS
H_1	1	5	1	2	1	1	1	1	yes
H_2	1	5	1	1	1	2	1	1	yes
H_3	5	5	1	2	1	1	1	1	yes
H_4	1	5	1	2	1	1	1	1	yes
H_5	5	1	1	1	1	1	1	1	no
H_6	1	5	1	2	1	1	1	1	unknown

The equivalence relation $I(B)$ provided in accordance with (7) is formed by a set of pairs

$$I(B) = \{(H_1, H_1), (H_2, H_2), (H_3, H_3), (H_4, H_4), (H_5, H_5), (H_6, H_6), (H_4, H_6), (H_6, H_4)\}.$$

The equivalence classes of $I(B)$ are created as the sets

$$IB(H_1)=\{H_1\},\ IB(H_2)=\{H_2\},\ IB(H_3)=\{H_3\},$$
$$IB(H_4)=\{H_4,H_6\},\ IB(H_5)=\{H_5\},\ IB(H_6)=\{H_6,H_4\}$$

according to (9).

The value of the decision attribute DNS = "yes" generates the set $X=\{H_1,H_2,H_3,H_4\}$, which in turn is an essential factor implementing $B_*(X)=\{H_1,H_2,H_3\}$, $B^*(X)=\{H_1,H_2,H_3,H_4,H_6\}$, $B_{border}(X)=\{H_4,H_6\}$.

The polygon membership degrees whose sizes confirm the membership in the DNS class are obtained as

$$\mu_{DNS}(H_1)=1,\ \mu_{DNS}(H_2)=1,\ \mu_{DNS}(H_3)=1,\ \mu_{DNS}(H_4)=\tfrac{1}{2},$$
$$\mu_{DNS}(H_5)=0,\ \mu(H_6)=\tfrac{1}{2}.$$

We can assume that H_1, H_2 and H_3 are the true members of DNS class in U while H_4 and H_6 may belong to the investigated class to certain degrees. We can also notice that H_6 affects a status of H_4 negatively and, on the contrary, we can see that H_4 upgrades an importance of H_6 in the DNS class.

4 CONCLUSIONS

The authors should emphasize that their approaches to the approximation and the classification of polygons are original contributions into the subjects.

Some finite sets of points are often interpolated by polygons, which seldom have comfortable equations mathematically expanded. If a number of points in the data set is very large and each pair of nodes obtains its linear equation, then a spline interpolation should consist of an enormous number of split definitions, which makes the further analysis of point sets rather impossible. We thus suggest applying continuous piecewise functions, originating from the standard π-functions, in a truncated form that smoothly approximate the irregular parts of the polygons. One segment of π approximates many points whose position resembles a bell shape, which simplifies the definition of an approximating function. The functions, called by us "the sampled, truncated π" are described by few simple equations in the form of split polynomials of the second and the first degrees. We admit that "the sampled, truncated π" sensitively follow the changes of the polygon patterns, which guarantees the thoroughness of approximation results.

By applying the Rough Set Theory we could classify polygons within the same class even if they have an unknown origin. Two introduced sets B_* and B^* act as a lower and an upper approximation of the investigated class. This makes possible to assign its sure members as well as such ones that have most of the properties characteristic of the class. Moreover, we can easily exclude the polygons, which do not satisfy the class's attributes.

REFERENCES

[1] A. Agadi and J-P. Penot (2005) "A Comparative Study of Various Notions of Approximation of Sets", Journal of Approximation Theory, in Press.

[2] V. Cherkassky, D. Gehring and F. Mulier (1996) "Comparison of Adaptive Methods for Function Estimation for Samples", IEEE Transactions on Neural Networks 7, 4:969-984.

[3] O. Haan (2005) "A Free Algebraic Solution for the Planar Approximation", Nuclear Physics B, 705: 563-575.

[4] G. Lavoue, F. Dupont and A. Baskurt (2005) "A New Subdivision Based Approach for Piecewise Smooth Approximation of 3D Polygonal Curves", Journal of Pattern Recognition Society, in Press.

[5] V. Novák and I. Perfilieva (1999) "Evaluating of Linguistic Expressions and Functional Fuzzy Theories in Fuzzy Logic", in Computing with Words in Information - Intelligent Systems 2, 33, Eds. L. A. Zadeh and J. Kacprzyk, 383-406.

[6] S. K. Pal and P. Mitra (2004) "Case Generation Using Rough Sets with Fuzzy Representation", IEEE Transactions on Knowledge and Data Engineering 16, 3:292-300.

[7] Z. Pawlak (1984) "On Rough Sets", Bulletin of the EATCS 24:94-108.

[8] Z. Pawlak (1997) "Vagueness – a Rough Set View", Structures in Logic and Computer Science:106-117.

[9] Z. Pawlak (2004) "Decision Networks", in Proceedings of Rough Sets and Current Trends in Computing, Uppsala, Sweden, 1-7.

[10] E. Rakus-Andersson and M. Salomonsson (2004) "The Truncated π-functions in Approximation of Multi-shaped Polygons", in Current Issues in Data and Knowledge Engineering, Eds. B. de Baets, R. de Caluwe, G, de Tré, J. Kacprzyk, 444-452.

[11] M. Salmeri, A. Mencattini and R. Rovatti (2001) "Function Approximation Using Non-normalized SISO Fuzzy Systems", International Journal of Approximate Reasoning, 26:211-231.

[12] L. A. Zadeh (1975) Calculus of Fuzzy Restrictions, Fuzzy Sets and Their Applications to Cognitive and Decision Processes, London, Academic Press.

[13] L. A. Zadeh (1973) "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", IEEE Trans. Systems, 3:28-44.