Fractionally Spaced Spatial Adaptive Equalization of S-UMTS Mobile Terminals

Jörgen Nordberg¹, Abbas Mohammed¹, Sven Nordholm² and Ingvar Claesson¹

¹ Blekinge Institute of Technology, Department of Telecommunications and Signal Processing, S-372 25 Ronneby, Sweden.

E-mail: jno@bth.se

² Australian Telecommunications Research Institute, Curtin University of Technology, Perth, Australia and Australian Telecommunications CRC.

Invited paper

Keywords

Blind equalization, Spatial Equalization, Fractionally Spaced Equalization, CMA

Abstract

In this paper we present fractionally spaced adaptive equalization techniques and space diversity combined receiver and evaluate their performance for the downlink of S-UMTS system. The conventional "training" (or non-blind) and the "unsupervised" (or blind) adaptive equalization algorithms are both investigated. Simulation results show that the equalizers are robust to Doppler shift and nonlinearity effects due to TWT amplifiers aboard the satellite. It is also shown that even with a moderate array size of two antenna elements, a significant improvement in terminal performance is achieved.

1 Introduction

The increasing demand for user capacity is today proven challenge for network operators. Since the number of radio channels allocated to the mobile system are strictly limited, the cellular concept was adopted to improve the

efficiency of mobile networks. In this approach the same channel can be reused in cells that are sufficiently separated (reuse distance), so that the co-channel interference is below a tolerable level. Terrestrial cellular networks and the mobile satellite system are converging towards an integrated satellite/terrestrial mobile communication network [1], in order to increase capacity and improve the performance of existing systems. This integrated system is called the Satellite-Universal Mobile Telecommunication System or S-UMTS for short.

The S-UMTS is expected to handle other forms of information than just speech, such as image, video and data. As a result, much higher data rates are required as compared to current systems like GSM, EDGE and GPRS. When the data rate is increased, the delay spread of the channel (resulting from multipath propagation) introduces significant amount of intersymbol interference (ISI) [2]. For this reason multipath fading compensation techniques must be implemented in order to realize such a broadband system. Due to the non-stationary behavior of the communication channel an adaptive, high performance and computationally efficient equalizers must be implemented. In the frame work of the European Advanced Communications Technologies and Services (ACTS) research program, the NEWTEST project has investigated several techniques for equalizing the S-UMTS channel; these included the conventional non-blind equalizers, the blind equalizers and neural network equalization techniques. In this paper we investigate the first two approaches. The reader is referred to [3, 4] for interesting results about neural network equalization for S-UMTS.

In conventional (non-blind) equalizers, the optimal adaptive weights of the equalizer are estimated by transmitting a predetermined data (training) sequence [5]. In many mobile communications standards such as GSM and IS-54, explicit training signals are inserted in the TDMA data bursts. The use of training sequences reduces the system capacity (number of users) which is a crucial issue in the design of current and future cellular mobile communication networks. For such applications, a blind equalizer which can adapt its tap weights without the aid of a training sequence seems to be an alternative and interesting option which needs further investigation.

By introducing several antenna elements at the receiving end it is possible to achieve spatial resolution. This array configuration of antennas has the ability to cancel unwanted signals (or jammers) that are not correlated with the desired signal, by directing nulls towards them (adaptive beamforming) [6, 7]. An additional advantage of an antenna array is its ability to avoid local fading minima; that is when one of the antenna elements cannot detect the desired signal, the other elements retain this capability as a result of appropriate spatial separation between the antenna elements [8]. The most

commonly used array configuration in practice is the linear array (see Figure 1). This array configuration has also been used in this paper. The receiver in this study was placed at the downlink, and thus the number of antenna elements must be restricted. We have opted to use two elements only (see Figure 2) in this study.

In this paper, the spatial channel equalization concept will be presented from the mobile terminal point of view. The paper is organized as follows. In Section 2, the NEWTEST¹ system model is presented. In Section 3, the different fractionally spaced equalization algorithms and concepts are introduced. The spatial adaptive version of the equalizers is also presented in Section 3. The simulation results and performance comparison between the different equalizers are presented in Section 4. Finally, the conclusions are drawn in Section 5.

2 The NEWTEST System Model

The S-UMTS model used in NEWTEST follows the project results [9] from the RACE 2 SAINT project, an European union research project. This model outlines a non-geostationary satellite system, in which the transmitting part of each satellite is constructed using a Beam Forming Network (BFN) consisting of 100 inputs and 100 outputs. The outputs from the BFN are applied to 100 power amplifiers (Traveling Wave Tube Amplifiers (TWTA) or Solid State Power Amplifiers (SSPA)). The output signals from the amplifiers are then filtered in order to suppress the spurious frequency components that have been generated by the non linearities in the amplifiers before the signals are transmitted.

The proposed frequency band for the NEWTEST downlink is 2170 MHz to 2200 MHz. The following frequency plan has been used: the bandwidth has been divided into four frequency bands, each cell having a bandwidth of 7.5 MHz. Each cell has seven channels with a bandwidth of 1.071 MHz. In each channel a Time Division Multiple Access (TDMA)² frame is transmitted; each frame hosts eight users with a rate of 64 kb/s. The NEWTEST transmitter is presented in Figure 3.

The transmitted signal usually reaches the receiving terminal antenna

¹NEWTEST is a collaborative research project of the 4th European Community Advanced Communications Technologies and Services (ACTS) Research Program.

²At the start of the NEWTEST project the standards for UMTS were not set. Therefore, we adopted a TDMA air interface to keep with the European GSM standard. At the beginning of 1998 a W-CDMA was chosen for UMTS. The principles and algorithms presented in this paper can be extended to similar TDMA nonlinear time-varying channels.

via several paths (or multipaths) due to reflections from different objects in the surroundings. Due to the multipath propagation phenomenon, the received signal energy level may vary significantly, thereby severely reducing the performance and capacity of the system. Since the received signal has a bandwidth of 1.071 MHz, a wideband channel model has been employed. The structure of the wideband channel model used in NEWTEST is presented in Figure 4. The values $\tau_1 \cdots \tau_M$ denote the time differences (or delays) between the line-of-sight (LOS) signal and the multipath components. The gain of the different parts of the received signal (i.e., the LOS and the multipath components) is given by $g_0 \cdots g_M$. The direct path gain g_0 is a discrete variable and it may take any value in the interval $0 \leq g_0 \leq 1$. The channel is named Rayleigh if $g_0 = 0$, otherwise it is called Rician. The different multipath component gains ($g_1 \cdots g_M$) are continuous complex random variables, which are used to represent the Rayleigh fading effect on the corresponding multipath signal. In order to simulate the delay spread of each path, each multipath signal is filtered with a low pass filter. Finally, at the receiver antenna the different signals are summed together with superimposed or Additive White Gaussian Noise (AWGN).

The channel settings used in this study are based on the SAINT project [9], where a suburban environment was selected and modeled as a 3 ray Rician fading channel. The specific channel parameters are presented in Table 1.

In Figure 5a a typical signal constellation is plotted for the received signal using the following parameters: the uplink SNR=15 dB, the downlink SNR=10 dB, $v=100$ km/h and the satellite amplifiers are working at the saturation point (that is the worst case for non-linearities is selected). It is clearly evident from the dispersion of the received signal points that the system is subject to errors which is an indication that the decision eye is completely closed. This situation can be combatted by employing an equalization technique to open the decision eye and reduce the errors in the system as shown in Figure 5b.

3 The Fractionally Spaced Spatial Adaptive Equalization Algorithms

The optimal receiver for a digital communication system, assuming linear AWGN channel, is a matched filter. This filter should be matched to both the transmitted signal and the channel, and thus good knowledge of the channel characteristics is required. However, in many time-varying mobile communications the exact knowledge of the channel characteristics is not

known *a priori*. Therefore, it is usual to have a receiver which is matched only to the signal waveform combined with a symbol rate clocked equalizer. This assumes that there is good timing between the transmitter and the receiver. These requirements may be relaxed when using a fractionally spaced equalizer [10], where the received signal is sampled r times per symbol. The fractionally spaced equalizer has the following major advantages over symbol rate equalizer [10]:

- There is no need for a matched filter.
- Sampling at a higher rate than the symbol rate makes it easier to extract timing and carrier information from the received signal.
- Fractionally spaced equalizers perform better than symbol rate equalizers in extremely noisy environments (minimum noise enhancement).

Other desirable properties of fractionally spaced equalization are also reported in [11]. The main drawback of the fractionally spaced equalizer as compared to the symbol rate equalizer is that the former has a higher computational complexity which is a direct result of the use of higher sampling rates.

The performance of the equalizer can be further improved by receiving the signal on two or more independent channels (i.e., by employing diversity). Diversity provides a powerful technique for combating signal fading in mobile communication systems. The basic principle of diversity is as follows: If several replicas of the same information-carrying signal are received over multiple channels, which exhibit *independent* or *uncorrelated* fading, then there is a good likelihood that at least one or more of these received signals will not be in a fade at any given instant of time, thus making it possible to deliver adequate signal level to the receiver. For spatial diversity, the key for obtaining minimum uncorrelated fading of antenna outputs is the adequate spacing of the antennas. This occurs when the diversity branches have antenna elements separated by with a distance greater than the coherence distance of the channel. It is shown in [8] that it is sufficient to separate the antennas by half a wavelength to achieve uncorrelated fading. Figure 6 illustrates the diversity combining principle for a downlink UMTS system employing two antenna branch diversity with antenna spacing of half the wavelength. It can be clearly seen from this figure that the combined signal from both antennas has much less severe fading than the individual antenna signals while also yielding higher average power.

In the following sub-sections two fractionally spaced spatial adaptive equalization schemes are presented, and in Section 4 their performance are evaluated and compared by using computer simulations.

3.1 Conventional (non-blind) fractionally spaced spatial adaptive equalization

One of the best known adaptive algorithms for channel equalization is the Least Mean Squared (LMS) algorithm or its Normalized version (NLMS). A spatial configuration of the LMS adaptive equalizer is shown in Figure 7. The performance of this algorithm is highly dependent on the quality of the desired signal [12]. Define $\mathbf{x}[k]$ and $\mathbf{w}[k]$ as a two-dimensional received signal vector and a weight vector, respectively. At time $t = kT_s$ ($k = 1, 2, \dots$), where T_s is the sampling period, $\mathbf{x}[k]$ and $\mathbf{w}[k]$ may be written as:

$$\mathbf{x}[k] = [x_1[k] \cdots x_1[k - L + 1] \ x_2[k] \cdots x_2[k - L + 1]]^T \quad (1)$$

$$\mathbf{w}[k] = [w_{11}[k] \ w_{12}[k] \cdots w_{1L}[k] \ w_{21}[k] \ w_{22}[k] \cdots w_{2L}[k]]^T \quad (2)$$

where L denotes the filter length and $[\cdot]^T$ is the transpose operation. For complex signals, both $\mathbf{x}[k]$ and $\mathbf{w}[k]$ are represented by a complex vector. The output signal of the LMS spatial equalizer $y[k]$ is then given by

$$y[k] = \mathbf{w}^H[k] \mathbf{x}[k] \quad (3)$$

where $[\cdot]^H$ denotes the transpose and complex conjugate.

The weight update equation for symbol rate NLMS is usually defined by

$$\mathbf{w}[k + 1] = \mathbf{w}[k] + \frac{\alpha \varepsilon_s[k]^* \mathbf{x}[k]}{\mathbf{x}^H[k] \mathbf{x}[k]} \quad (4)$$

where $[\cdot]^*$ denotes the complex conjugate and $\varepsilon_s[k] = d[k] - y[k]$ is the error signal which is defined as the difference between the known pilot reference signal and the output from the equalizer. Since the pilot signal is available once per symbol, the fractionally spaced coefficient update equation is formulated a little bit differently,

$$\mathbf{w}[k_s + 1] = \mathbf{w}[k_s] + \frac{\alpha \varepsilon_f[k_s]^* \mathbf{x}[k]}{\mathbf{x}^H[k] \mathbf{x}[k]} \quad (5)$$

where $k_s = \left(\left\lfloor \frac{k}{r} \right\rfloor \right)$ is the symbol count index which keeps track of the number of received symbols, and $\lfloor \cdot \rfloor$ denotes the truncation operator which converts a real number into an integer by truncating it. The fractionally spaced equalizer outlined in (5) performs the filtering at fractionally spaced rate (index k) while the filter weights are updated at symbol rate (index k_s). In order to update the filter weights at the symbol rate the $\varepsilon_f[k_s]$ in (5) is

defined as

$$\varepsilon_f[k_s] = \min_{0 \leq i_r \leq r-1} \left[d[k_s] - y[k_s + i_r] \right] \quad (6)$$

where $\varepsilon_f[k_s]$ is the minimum difference between the r fractionally spaced outputs during a symbol and the pilot symbol. Finally, the equalizer symbol rate output is the fractionally spaced rate output which corresponds to $\varepsilon_f[k_s]$.

3.2 Blind fractionally spaced spatial adaptive equalization

The reliance of an adaptive non-blind channel equalizer on a training sequence requires that the transmitter cooperates by re-sending the training sequence, lowering the effective data rate over the communication link. Blind equalization techniques avoid this disadvantage by exploiting some other *a priori* information about the signal characteristics instead of the training sequence. Among the various blind equalization algorithms, the Godard or Constant Modulus Algorithm (CMA) [5] is one of the simplest and best known blind adaptive equalization algorithms. This algorithm was also independently developed and extended by Treichler et. al. [13]. Figure 8 shows the basic architecture of the CMA spatial adaptive algorithm. The output signal of the CMA spatial equalizer $y[k]$ is given by

$$y[k] = \mathbf{w}^H[k] \mathbf{x}[k] \quad (7)$$

where the input vector to the equalizer $\mathbf{x}[k]$ and the equalizer weight vector $\mathbf{w}[k]$ are defined as for the NLMS algorithm (see eqs. 1 and 2). In many mobile communications standards, the transmitted signal has the constant envelope property. A typical example of a constant envelope (modulus) waveform is the GMSK modulation used in the GSM cellular system. In the S-UMTS model we adopted, a QPSK modulated signal is transmitted over the channel and, consequently, the array output $y[k]$ should have a constant envelope. However, the multipath fading problem causes amplitude fluctuations in the received signals. Thus, the objective of the CMA is to restore the array output $y[k]$ to the desired constant modulus characteristics. This is accomplished by adjusting the weight vector $\mathbf{w}[k]$ in order to minimize the cost function J , which provides a measure of the amplitude fluctuations. Since we have one signal $y[k]$ at the output of the spatial array, the cost function J is given by

$$J_{pq} = E \left[\left(|y[k]|^p - 1 \right)^q \right] \quad (8)$$

where E is an ensemble average operation. The cost function J can be minimized by using a gradient search algorithm. The parameters p and q control the sensitivity and convergence behavior of the CMA algorithm. Smaller p and q usually provide higher noise tolerance and stability but suffer from slower convergence. To ensure convergence and reasonably small tap-gain fluctuation, (p, q) in (8) are set to (1,2) or (2,2). By replacing the ensemble average operation with an instantaneous value in the cost function J , we obtain the update equation for $\mathbf{w}[k]$ as

$$\mathbf{w}[k+1] = \mathbf{w}[k] - \mu_a \varepsilon_p[k]^* \mathbf{x}[k] \quad (9)$$

$$\varepsilon_1[k] = y[k] - \frac{y[k]}{|y[k]|} \quad (10)$$

$$\varepsilon_2[k] = y[k] (|y[k]|^2 - 1) \quad (11)$$

where μ_a is the step-size constant and $\varepsilon_1[k]$ and $\varepsilon_2[k]$ replace the usual error signal in the NLMS algorithm. We will refer to $\varepsilon_1[k]$ and $\varepsilon_2[k]$ as the error term of CMA(1,2) and CMA(2,2) algorithm, respectively.

The update equation for $\mathbf{w}_k$ is similar to that used in the NLMS algorithm. There is one important difference, however, in that the CMA requires no reference signal for estimating $\varepsilon_p[k]$ while NLMS does. It is also clear from the definition of the cost function J and the CMA weight update equation that the algorithm is insensitive to phase fluctuations. Conventional (non-blind) algorithms such as LMS employ the mean square error (MSE) cost function to compensate both the amplitude and phase of the signal by the use of a reference signal which is a replica of the transmitted signal. However, due to the phase insensitivity of CMA, the carrier phase must be recovered after the adaptive processing. The phase recovering algorithm proposed by Godard [5] is a first order *phase locked loop* (PLL) algorithm.

The main drawback of the CMA algorithm is the relatively slow convergence rate. This becomes increasingly significant in wireless applications that are subject to rapid changes in their channel characteristics. In order to improve the convergence speed of the CMA equalizer, a normalized version of the CMA (NCMA) algorithm is usually employed [14]. In this paper, the input signals to the algorithm are divided (normalized) by their corresponding envelope (i.e., L_1 norm of the input vectors). This yields the following normalization factor

$$\eta_{k_i} = \frac{1}{L} \sum_k |\mathbf{x}_i[k]|, \quad 0 \leq k \leq L-1. \quad (12)$$

The symbol rate NCMA weight update equation is now given by

$$\mathbf{w}[k+1] = \mathbf{w}[k] - \mu'_a \varepsilon_p[k]^* \mathbf{x}'[k] \quad (13)$$

where $\mathbf{w}[k]$ is defined as before. The input vector $\mathbf{x}'[k]$, however, is now defined by

$$\mathbf{x}'[k] = \left[\frac{x_1[k]}{\eta_{k_1}} \dots \frac{x_1[k-L+1]}{\eta_{k_1}} \frac{x_2[k]}{\eta_{k_2}} \dots \frac{x_2[k-L+1]}{\eta_{k_2}} \right]^T \quad (14)$$

To construct a fractionally spaced NCMA equalizer one has to consider the fact that the received signal only has constant magnitude every r^{th} sample. Therefore, the fractionally spaced NCMA weights are only updated once per symbol, yielding the following update equation:

$$\mathbf{w}[k_{s'} + 1] = \mathbf{w}[k_{s'}] - \mu'_a \varepsilon_p[k_{s'}]^* \mathbf{x}'[k_{s'}] \quad (15)$$

where $k_{s'} = (r \lfloor \frac{k}{r} \rfloor + i)$, $0 \leq i \leq r-1$. The offset index i may be chosen arbitrarily between 0 and $r-1$, since every offset has a point-wise constant envelope every r^{th} sample. In this paper an offset index of half the over-sampling rate (i.e., $i = \frac{r}{2}$) has been selected.

4 Performance Evaluation of the Fractionally Spaced Spatial Adaptive Equalizers

In this section we present performance evaluation and comparison of the different fractionally spaced spatial adaptive equalizer algorithms presented in the previous section. In all simulations an over-sampling factor of 8 has been used for the fractionally spaced equalizer and they utilize an array of two antenna elements unless otherwise stated. In addition the uplink signal-to-noise ratio (SNR) has been fixed to 15 dB and the TWT amplifiers are set to be operate at their saturation point (that is the worst possible case for the nonlinearity effects has been simulated). The vehicle speed and the downlink SNR were varied in the different simulations.

The performance of the NCMA equalizer will also be compared with the NLMS equalizer for a multipath, noisy S-UMTS channel. In the NLMS case the whole transmitted block is used as a training sequence. Thus, the NCMA will be compared to the best NLMS performance possible. The performance measure used in the evaluation is the symbol error rate (SER) as a function of the downlink SNR. The SER values have been calculated after convergence of the different algorithms. The NCMA algorithm used in these simulations is for $p = 1$ or CMA(1,2) algorithm because it is less sensitive to noise than $p = 2$ or CMA(2,2) (see eqns. (10) and (11), respectively). Results will be provided for single-channel as well as dual-channel equalizers.

Figure 9 compares the performance of the NCMA equalizer for both fractionally spaced and symbol rate settings; the fractionally spaced equalizer achieves better SER performance, especially for SNR's above 10 dB. This improvement, however, is achieved at the cost of higher computational complexity.

Figure 10 plots the SER performance of the fractionally spaced NCMA and the NLMS equalizers for different SNR. From the curves, the NCMA requires an SNR of about 3-4 dB's higher than the NLMS to achieve the same SER. This is expected since the NCMA is a blind algorithm while the NLMS uses a predetermined training signal.

In Figures 11 and 12 the convergence rate of the NLMS, NCMA and CMA algorithms are investigated using a time-invariant channel, defined as $H(z) = 1/(1 - 0.9z^{-1})$. The results show that the convergence rate of the NCMA is roughly 22 times faster than the CMA but is still 5 times slower compared to the NLMS algorithm.

Next, the performance of the fractionally spaced NCMA equalizer is evaluated for different vehicle speeds. Figure 13 clearly shows that the equalizer is robust to the variation of vehicle speed (Doppler shift effect) even at low SNR's.

Finally, the performance improvement when using two antennas (i.e., when employing diversity) compared to a single antenna (no diversity) is shown in Figure 14. It is clear from this figure that for SNR's above 5 dB, an improvement by an order of magnitude or more in SER level can be achieved using two branch antenna diversity. This is the result of diversity gain; this gain is achieved due to the fact that a fading dip is less probable to occur at two elements at the same time as compared to the single antenna scenario (see Figure 6). Reducing fade margins directly translate to better reuse factors and, hence, increased system capacity.

5 Conclusions

In this paper we presented fractionally spaced adaptive equalization techniques and space diversity combined receiver and evaluated their performance and effectiveness in combating interference sources for S-UMTS/TDMA channels. The LMS provided lower error rates and faster convergence than the CMA, but is very costly in terms of data throughput. In this paper we were interested in investigating channel equalization at the terminal side (down-link), and so a two element linear array with $\lambda/2$ spacing was employed. Simulation results have shown that by including an extra antenna branch at the receiver, a substantial SNR diversity gain and better error rates can

be achieved. Comparison with neural network equalization and investigating techniques to improve the convergence speed comprise future research.

References

- [1] E. DelRe, A Coordinated European Effort for the Definition of a Satellite Integrated Environment for future Mobile Communications, *IEEE Communication Magazine*, February 1996.
- [2] R. Steel, "*Mobile Radio Communications*", Pentech Press, 1992.
- [3] M. Ibnkahla, J. Sombrin, F. Castanie and N. J. Bershad, Neural Network for Modelling Nonlinear Memoryless Communication Channels, *IEEE Trans. on Communication* July 1997, **45**(7):768-771.
- [4] S. Bouchired, M. Ibnkahla, D. Roviras and F. Castanie, Equalization of satellite mobile communication channels using combined self-organizing maps and RBF networks, *Proceeding IEEE ICASSP'98* April 1998.
- [5] D.N. Godard, Self-Recovering Equalisation and Carrier Tracking in Two Dimensional Data Communication Systems, *IEEE Trans. on Communication* November 1980, **28**(11):1867-1875.
- [6] O. Frost, An Algorithm for Linearly Constraint Adaptive Array Processing, *IEEE Proceeding* 1972, **60**.
- [7] L.J. Griffiths and C.W. Jim, An Alternative Approach Linearly Constrained Adaptive Beamforming, *IEEE Trans. ASSP* April 1985, **30**(2):459-471.
- [8] W.C. Jakes, *Microwave Mobile Communications*, John Wiley & Sons, New York, 1974.
- [9] SAINT, "*Deliverable No. 9, Air Interface*", Race Project 2217, R2117-AAU/CN DR P209-B1, December 1995.
- [10] J.R. Treichler, I. Fijalkow and C.R. Johnson J.R., Fractionally Spaced equalisers: How long should they really be ?, *IEEE Signal Processing Magazine* May 1996, 65-81.
- [11] Y. Li and Z. Ding, *Global Convergence of Fractionally Spaced Godard (CMA) Adaptive Equalizers*, *IEEE Trans. on Signal Processing* April 1996, **44**(4):818-826.

- [12] R.A. Monzigo and T.W. Miller, *"Introduction to Adaptive Arrays"*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1994.
- [13] J.R. Treichler and B.G. Agee, A New Approach to Multipath Correction of Constant Modulus Signals, *IEEE Trans. ASSP* April 1985, **31**(2):459-471.
- [14] D. L. Jones, A normalized constant-modulus algorithm, *Proceeding of 29th Asilomar Conference on Signals, Systems and Computers* October 1995, **2**:694-697.

Channel parameter	Relative attenuation (dB)	Delay (μs)
LOS	0	-
τ_1	-10	0.1
τ_2	-15	0.2
τ_3	-20	0.3

Table 1: The channel parameters for the simulated 3 ray Rician channel model.

Figure 1: Linear array configuration.

Figure 2: A two branch spatial adaptive equalizer.

Figure 3: The NEWTEST transmitter.

Figure 4: The NEWTEST channel model.

Figure 5: The received signal constellation (a) before equalization and (b) after the equalization, when downlink SNR=10 dB, uplink SNR=15 dB, ($v = 100$ km/h) and the amplifiers are working at their saturation point.

Figure 6: An illustration of spatial diversity principle. In the different sub-figures the magnitude of the following signals is plotted: (a) the signal from antenna 1, (b) the signal from antenna 2 and (c) the combined signal from both antennas.

Figure 7: Two branch spatial LMS adaptive equalizer.

Figure 8: Two branch spatial CMA adaptive equalizer.

Figure 9: SER performance for fractionally spaced and symbol rate NCMA spatial adaptive equalizers ($v = 100$ km/h).

Figure 10: SER performance for fractionally spaced NCMA and NLMS spatial adaptive equalizers ($v = 300$ km/h).

Figure 11: Comparison of the convergence rate of NCMA and CMA algorithms for a channel distortion filter given by $H(z) = \frac{1}{1-0.9z^{-1}}$.

Figure 12: Comparison of the convergence rate of NLMS and NCMA algorithms for a channel distortion filter given by $H(z) = \frac{1}{1-0.9z^{-1}}$.

Figure 13: SER performance of fractionally spaced NCMA spatial adaptive equalizer for different mobile speeds.

Figure 14: Comparison of SER of the spatial diversity (two antennas) equalization system with single channel for the NCMA fractionally spaced equalizer ($v = 100$ km/h).