Identification of Dynamic Properties of Boring Bar Vibrations in a Continuous Boring Operation

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Contents

1	Introduction	3
2	Materials and Methods	5
2.1	Experimental Setup	5
2.2	Force Modulation in Rotating Machinery	8
2.3	Structural Dynamic Properties of the Boring Bar	8
2.4	Evaluation of the Dynamic Motion of the Boring Bar	10
3	Results	17
3.1	A Simple Model of a Boring Bar	17
3.2	Stationarity	18
3.3	Mean Square Value Property	18
3.4	Segmentation of the Boring Bar Vibration Records	20
3.5	Sample Probability Density Estimate	20
3.6	Spectral Properties	24
4	Discussion and conclusions	32
	Acknowledgement	35

Abstract

Vibrations in internal turning operations are usually a cumbersome part of the manufacturing process. This article focuses on the boring bar vibrations. Boring bar vibrations in alloyed steel, stainless steel and cast iron have been measured in both the cutting speed direction and the cutting depth direction with the aid of accelerometers. The dynamic response of a boring bar seem to be a time varying process that exhibits nonlinear behaviour. The process is influenced by nonstationary parameters that are not under the control of the operator or experimenter. The vibrations are clearly dominated by the first resonance frequency in one of the two directions of the boring bar. The problem with force modulation in rotary machinery, which appears as side band terms in the spectrum, is also addressed. Furthermore, the resonance frequencies of the boring bar are correlated to an Euler-Bernoulli beam model.

1 Introduction

A concern in the manufacturing industry today is the vibrations induced by metal cutting, e.g. turning, milling and boring operations. Turning operations, and especially boring operations, are facing tough vibration-related problems. To reduce the problem of vibration extra care must be taken in production planning and preparation with regard to the machining of a workpiece in order to obtain the desired shape and tolerance. Thus, the vibration problem in metal cutting has considerable influence on important factors such as productivity, production costs, etc. A thorough investigation of the vibrations involved is therefore an important step in challenging this problem.

During an internal turning operation the cutting tool and the boring bar are subjected to a prescribed deformation as a result of the relative motion between the tool and workpiece both in the cutting speed direction and feed direction. As a response to the prescribed deformation, the tool is subjected to traction and thermal loads on its faces that have interfacial contact with the workpiece or chip.

In the metal cutting process, during which chips are being formed, the workpiece material is compressed and subjected to plastic deformation. This involves considerable strain and strain rates in the primary deformation zone ($\dot{Q}_I$), see Fig. 1. Due to the

Figure 1: The primary, secondary and tertiary deformation zones in a metal-cutting operation.

friction and the plastic deformation in the secondary and tertiary deformation zones ($\dot{Q}_{II}$, $\dot{Q}_{III}$) the tool is subjected to both normal and shear traction loads on the workpiece-chip-tool interface. The traction loads vary as a result of the discontinuous deformation process. When a material yields, the direction and the magnitude of the dislocation are determined by the state of stress as well as the structure of the material lattice. Work materials, iron alloys in particular, have lattice errors. When alloying components have been added as interstitials, the lattice errors increase and the theoretical strength decreases. However, the alloying components will prevent or lock the dislocations during plastic deformation of a material. The dislocations themselves will also prevent plastic deformation. The grain size influences the material's yield strength [1].

In internal turning the metal cutting process is carried out in pre-drilled holes or holes in cast etc. The dimensions of the workpiece hole will generally determine the length and limit the diameter or cross sectional size of the boring bar. Usually a boring bar is long and slender and is thus sensitive to excitation forces introduced by the material deformation process in the turning operation. Commonly, the boring bar is the weakest link in the boring bar - clamping system of the lathe. The boring bar motion may vary with time. This dynamic motion originates from the deformation process of the work material. The motion or vibration of the boring bar will affect the result of the machining, and the surface finish in particular. Tool life is also likely to be influenced by the vibrations.

2 Materials and Methods

The internal turning operation is a very complex and cumbersome metal working process. In order to achieve a comprehensive knowledge of the cutting process in internal turning many experiments must be carried out. The magnitudes of cutting process parameters are determined from many other parameters, some of which are observable or measurable in experiments. The influence of a process parameter can only be examined if the other parameters are "frozen". However, some parameters may show linear dependence and it is thus important to select the right parameters to study. Even then, the number of parameters can be large and the number of experiments must then be reduced by careful planning of the trials.

2.1 Experimental Setup

The cutting trials have been carried out in a Mazak SUPER QUICK TURN - 250M CNC turning centre, see Fig. 2. This has 18.5 kW spindle power and a maximal machining diameter of 300 mm, with 1007 mm between the centers. The cutting operations were performed as external turning operation using a boring bar, the WIDAX S40T PDUNR15.

Figure 2: Mazak SUPER QUICK TURN - 250M CNC lathe used in the experiments.

2.1.1 Measurement Equipment and Setup

For measuring the following measurement equipment has been used:

- 2 PCB U353B11 Accelerometers

- TEAC RD-200T DAT-recorder

The two accelerometers were mounted as closely as possible to the cutting tool - one in the cutting speed direction and the other in the cutting depth direction as in Fig. 3. The accelerometers were mounted on the boring bar using threaded studs.

Figure 3: Vibration measurement setup.

2.1.2 Work Materials

In the cutting experiments, three different workpiece materials have been used:

- SS 0727-02, nodular graphite cast iron
- SS 2343-02, austenitic stainless steel
- SS 2541-03, chromium molybdenum nickel steel

The materials have different features from a production point of view. The machinability, or more specifically the cuttability, of the materials differs and the chemical composition of the three materials, which is shown in Table 1, is also different. In the experiments, solid cylindrical workpiece materials have been used. The diameter of them is chosen large (>180 mm). It is thus not necessary to consider the workpiece vibrations.

Work materials are usually classified according to the three production engineering application classes: P (alloyed steels), M (stainless steels) and K (cast irons), all standardized by ISO. For the experiments, one material has been selected from each class. Since the statistical properties of the cutting forces are to be investigated, the influence of the workpiece material can also be studied. The chemical composition and micro-structure of the materials as well as the strength and the thermo-dynamical properties determine behaviour in the cutting process. From a production engineering point of view, the optimal workpiece material should induce small cutting forces, be capable of producing a proper surface finish, have a fair chip breaking property, and not deteriorate the cutting tool. The selected materials will have different properties with respect to these four aspects [2].

Production Engineering Application Class	Swedish Standard	American Standard	C (%)	Si (%)	Mn (%)	Cr (%)	Ni (%)	Mo (%)
K	SS 0727-02	AISI 80-55-06	3.7	2.2	0.4			
M	SS 2343-02	AISI 316	0.05			18	12	2.7
P	SS 2541-03	AISI 4340	0.36	0.27	0.62	1.53	1.41	0.17

Table 1: The composition of the workpiece materials.

2.1.3 Cutting Tools

In the cutting experiments, standard 55° diagonal inserts have been used. These have the tool geometry ISO code DNMG 150608-SL, with chip breaker geometry for medium roughing. Different carbide grades have been used for different materials though the geometries were the same. For cast iron and alloyed steel, carbide grade TN7015 was used and for stainless steel, the carbide grade was TN8025.

2.1.4 Choice of Cutting Data

The selection of the cutting data parameter space must be based on thorough knowledge of the cutting process itself. Excessive wear, catastrophic failure or plastic deformation may result if too high cutting speeds or feed rates are selected. This may lead to results which are not characteristic of a turning operation during normal circumstances. After a preliminary set of trials the selection was made. The cutting data were chosen according to Table 2. No cutting fluid was applied during machining.

The aim was to find a parameter space which was the same for all materials. However, it proved to be impossible to exceed a 200 m/min cutting speed for stainless steel, SS 2343.

Material	Parameter	Range
SS 0727-02	Feed, s (mm/rev)	0.1-0.3 step 0.1
	Depth of cut, a (mm)	2
	Cutting speed, v (m/min)	50-300 step 25
SS 2343-02	Feed, s (mm/rev)	0.1-0.3 step 0.1
	Depth of cut, a (mm)	2
	Cutting speed, v (m/min)	50-200 step 25
SS 2541-02	Feed, s (mm/rev)	0.1-0.3 step 0.1
	Depth of cut, a (mm)	2
	Cutting speed, v (m/min)	50-300 step 25

Table 2: Cutting parameter space for the different materials.

2.2 Force Modulation in Rotating Machinery

Force modulation is inherent in certain types of mechanical systems and especially in rotating machinery [3, 4]. Rotating parts of mechanical systems introduce dynamic motion related to residual rotor mass unbalance [4, 5]. It is commonly known that rotation speed may vary due to imperfections in gears [3, 4], e.g. variation in tooth spacing, roundness errors, etc. associated with gear wheels [3, 4, 6]. All gear boxes give a varying torque over one turn given a uniform input torque. This causes a dynamic force (torque) by itself. These imperfections of rotating machines are known to introduce force modulation effects into the vibration signature of rotating machinery, e.g. amplitude modulation and/or frequency or phase modulation properties in the vibration signature of the rotating machinery [3, 4].

Based on the fact that rotating parts of mechanical systems introduce dynamic motion related to residual rotor mass unbalance, it is reasonable to assume that dynamic motion related to residual rotor mass unbalance also exists in the spindle - chuck - workpiece system in a lathe. Blankenship et al. [3] have suggested a simplified mathematical model to provide a plausible explanation of physical mechanisms involved in generating force modulation in a single gear pair system using a second order inhomogeneous differential equation. One of the mechanisms responsible for generating force modulation in the model is given by a spatially periodic displacement excitation function which excites the mass via a spatially periodic stiffness function.

2.3 Structural Dynamic Properties of the Boring Bar

In general, the dynamic response of a structure cannot be described adequately by a single-degree-of-freedom model. The response usually includes time variations in the displacement shape as well as its amplitude. The motion of a structure must then be described in terms of more than one degree of freedom, i.e. the motion or response must be represented by a multiple-degree-of-freedom system or a distributed parameter system. The motion of a structure is usually described by the displacement of a set of discrete points of the structure, or by a continuous function of the relative position along the structure.

In order to model the structural dynamic properties of a boring bar, a possible simple model is a Euler-Bernoulli beam. Most realistic structural systems are characterized by the ability to support transverse shear and exhibit internal stiffness. The Euler-Bernoulli beam model assumes that the deflection of the centerline is small and only transverse. While this theory assumes the presence of a transverse shear force, it neglects any shear deformation due to it. Also the rotary inertia is neglected by this model [7, 8]. In Fig. 4, a clamped-free model of a boring bar is shown. The Euler-Bernoulli differential equation describing the transversal motion in the y direction of the boring bar can be written as [7]

$$\rho A(x) \frac{\partial^2 u(x, t)}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[EI(x) \frac{\partial^2 u(x, t)}{\partial x^2} \right] = f(x, t) \quad (1)$$

where ρ is the density of the boring bar, $A(x)$ the cross-sectional area, $u(x, t)$ the deflection in the y direction, E the Young's elastic modulus, $I(x)$ the cross-sectional area moment

Figure 4: A clamped-free model of a boring bar.

of inertia and $f(x, t)$ the space- and time-dependent load. From the theory of mechanics of material, the beam sustain a bending moment, $M(x, t)$, which is related to the beam deflection, $u(x, t)$. This relation is given by [8]

$$M(x, t) = EI(x) \frac{\partial^2 u(x, t)}{\partial x^2}. \quad (2)$$

In this context it is assumed that both the cross-sectional area $A(x)$ and the flexural stiffness $EI(x)$ are constant along the boring bar. In other words, it is assumed that the geometric features of the tool end of the boring bar are negligible to the overall bar dynamics. The boundary conditions of the boring bar depend on the suspension of the boring bar ends, and in this case a clamped-free model is suggested; in this way, the boundary conditions will not be the same for the ends.

At the free end there is no bending moment nor shear force and consequently the boundary conditions at the free end are given by:

$$EI(x) \frac{\partial^2 u(x, t)}{\partial x^2} \Big|_{x=l} = 0, \quad (3)$$

$$\frac{\partial}{\partial x} \left(EI(x) \frac{\partial^2 u(x, t)}{\partial x^2} \right) \Big|_{x=l} = 0. \quad (4)$$

At the clamped end both the deflection and slope of the bar are zero, resulting in the following boundary conditions:

$$u(x, t) \Big|_{x=0} = 0, \quad (5)$$

$$\frac{\partial u(x, t)}{\partial x} \Big|_{x=0} = 0. \quad (6)$$

The solution to the boundary conditions yields the characteristic equation $\cos(\beta l) \cosh(\beta l) = -1$. Now the product of the eigenvalues β_n and the length of the boring bar can be calculated for $n = 1, 2, 3, \dots$. The first three values of β_n are [8] $\beta = \{1.8752/l, 4.6941/l, 7.8548/l\}$.

The resonance frequencies for these eigenvalues can be calculated using the relation

$$f_n = \frac{\beta_n^2}{2\pi} \sqrt{\frac{EI}{\rho A}} \quad (7)$$

To be able to solve the Euler-Bernoulli calculations the cross-sectional dimensions of the boring bar are needed in order to calculate the cross-sectional area moment of inertia, I . The cross section of the boring bar is given in Fig. 5. The values used in the Euler-Bernoulli calculation can be found in Table 3.

Figure 5: The cross section of the boring bar used in the experiments. The cutting speed direction corresponds to the y direction in Fig. 4.

Variable	Value		Unit
	Cutting speed direction	Cutting depth direction	
E	$206 \cdot 10^9$		N/m^2
l	0.2		m
I	$1.0958 \cdot 10^{-7}$	$1.1958 \cdot 10^{-7}$	m^4
ρ	7800		kg/m^3
A	$1.2386 \cdot 10^{-3}$		m^2

Table 3: Values used in the theoretical Euler-Bernoulli calculations.

2.4 Evaluation of the Dynamic Motion of the Boring Bar

The properties of the boring bar vibration may be investigated by applying the theory of time series analysis. It is obvious that there exists a time dependency, or correlation, in the cutting process. This time dependency is influenced by many parameters. In a

continuous boring operation, the influence of different work materials and cutting data on the boring bar vibrations has been studied.

When analyzing random data, some important characteristics are the stationarity of the data and the presence of periodicities in the data. Stationarity is important since analysis of nonstationary data is generally more complicated. To avoid erroneous interpretations in the analysis of the sampled data, periodicities must be identified.

2.4.1 Stationarity Test

Providing no alteration of cutting data takes place, the rate of deterioration of the tool is low and the basic structure of the work material is time invariant. The tool vibration can hence be hypothesized as a short time stationary random process. The stationarity tests procedure is non-parametric: no assumption needs to be made concerning the probability distribution of the random data. The stationarity can be examined by an investigation of a single time record of the boring bar vibration $u(t)$, as follows [9]:

1. Divide a sample record x into N equal time intervals with L data samples, where the data in each interval may be considered independent. Hence, the data sequence in each time interval n , $n \in \{1, 2, \dots, N\}$, is given by

$$\mathbf{x}_n = \{x(0+(n-1)L), x(1+(n-1)L), \dots, x(L-1+(n-1)L)\}.$$

2. Compute the mean square value for each interval and align these values in a time sequence as

$$\overline{\mathbf{x}^2} = \{\overline{x_1^2}, \overline{x_2^2}, \dots, \overline{x_N^2}\}. \quad (8)$$

3. Test the sequence of mean values for the presence of underlying trends or variations outside expected sampling variations.

Using the mean square for stationarity instead of running two tests - one for the mean and one for the variance - makes the test simpler. The test does not take into account that the mean square is stationary while the mean and the variance are nonstationary. However, this does not occur in practice. Furthermore, it is straightforward to apply this procedure to higher-order properties of a time record [9]. Stationarity can be tested with the *run test* and the *reverse arrangements test* [9].

The *run test* is described more comprehensively in [9]. The main idea of the test is to detect fluctuations or trends in the data. This is done by counting the number of so-called runs, r . A run is defined as a sequence of identical observations followed by a different observation or no observation at all. In the case of the stationarity test, the median of the sequence is first calculated. The median is preferred to the mean as there is less sensitivity to the outliers. An observation which is less than the median is identified by a minus (-), while an observation larger than the median is identified by a plus (+). The output of the

run test is the number of runs, r , in the sequence. For N independent observations of a stationary random variable, the number of runs has the mean

$$E(r) = \frac{N}{2} + 1 \quad (9)$$

and variance

$$V(r) = \frac{N(N-2)}{4(N-1)}. \quad (10)$$

If the observed number of runs r is in an interval between $r_{n;1-\alpha/2}$ and $r_{n;\alpha/2}$, where α is the chosen significance level and $n = N/2$, then the time record can be considered as stationary with the degree of significance α according to the run test.

Another test for stationarity is the *reverse arrangements test*. This test is also described in [9]. The reverse arrangements test is a common method for detecting monotonic trends in the data. Consider a sequence of N observations of a random variable $\mathbf{x} = \{x_1, x_2, \dots, x_N\}$. A reversed arrangement is defined as a subsequent observation that is smaller than the present observation. The number of reversed arrangements for the n th observation is denoted A_n . The total amount of reversed arrangements A is the sum of the reverse arrangements for all observations. Hence, for each $n_1 \in \{1, \dots, N-1\}$, the number of occurrences A_{n_1} is counted when the inequality

$$x_{n_2} < x_{n_1}, \text{ for } n_2 > n_1 \quad \text{where } n_2 \in \{n_1+1, \dots, N\} \quad (11)$$

holds. Such an inequality is defined as a reverse arrangement. The total number of reverse arrangements A is defined as

$$A = \sum_{n_1=1}^N A_{n_1} \quad (12)$$

For N independent observations of a stationary random variable, the number of reverse arrangements has the mean

$$E(A) = \frac{N(N-1)}{4} \quad (13)$$

and variance

$$V(A) = \frac{N(2N+5)(N-1)}{72}. \quad (14)$$

In the same way as in the run test, if the observed number of the reverse arrangements A is in an interval between $A_{N;1-\alpha/2}$ and $A_{N;\alpha/2}$, where α is the significance level, the time record can be considered as stationary to the degree of significance α according to the reverse arrangements test.

The reversed arrangements test tends to be more powerful in detecting monotonic trends than the run test. On the other hand, the run test is more powerful in finding fluctuations in the data. If the data record is accepted as stationary by both stationarity tests, the data record is considered stationary. The significance level used in the stationarity tests was $\alpha = 0.05$.

2.4.2 Mean Square Value Properties

Analysis of individual nonstationary time history records usually includes a time averaging operation. In statistical tests, one problem is to select an appropriate averaging time. It is not uncommon that the same time record can be classified as both stationary and nonstationary depending on the size of the averaging time. By using different lengths of the moving average it is possible to gain information about the time varying properties of the mean square value or power of the time record. Usually it is desired to follow an underlying curve and to smooth out the variations in the mean square value. One approach in selecting an average time is a trial and error based method [9], as follows:

1. Compute a moving average for the mean square value of the nonstationary data with an averaging time T_a that is too short to smooth out the variations in the mean square value versus time.
2. Continuously recompute the moving average with an increasing averaging time until it is clear that the averaging time is smoothing out variations in the mean square value versus time.

2.4.3 Sample Probability Density Estimate

The statistical properties of a real stochastic process are generally determined in terms of all its moments. The probability distribution of a stochastic process is closely related to its moments or cumulants. In fact, the moments or cumulants of a stochastic process are unambiguously determined by the probability distribution [10]. The probability distribution, i.e. the mean, the covariance and the higher-order cumulant functions, usually depend on process parameters, e.g. in metal cutting, these are feed rate and cutting speed. Also, the width of the wear land influences the probability distribution [2]. The probability density function $f_{X,t}(x)$, corresponding to a stationary stochastic process $\{X(t), t \in \mathbb{R}\}$, can be estimated. The estimation can be performed based on a sample record of the stationary stochastic process. The probability density function $p_{X,t}(x)$ may be estimated by [9]

$$\hat{p}_{X,t}(x) = \frac{N_x}{NW} \quad (15)$$

where W is a narrow interval centered at x , N_x is the number of data that falls within the range $x \pm W/2$, and N is the sample size. Let K denote the number of class intervals selected to cover the entire range of data values in the interval a, b , then the width of each interval is given by

$$W = \frac{b - a}{K} \quad (16)$$

and the interval I_k , $k \in \{0, 1, \dots, K + 1\}$ is given by:

$$I_k = \begin{cases}] - \infty, a], & k = 0 \\]a + (k - 1)W, a + kW], & k \in \{1, 2, \dots, K\} \\]b, \infty[, & k = K + 1 \end{cases} \quad (17)$$

If the probability density function to be estimated is a smooth function, i.e. it does not display abrupt or sharp peaks, then the normalized bias error is usually limited to less than 1% if the interval or window width W is selected as [9]

$$W \leq 0.2\sigma_x. \quad (18)$$

However, the statistical information required to evaluate the variance of an estimate $\hat{p}_{X,t}(x)$ for a random process is generally difficult to obtain [9].

2.4.4 Spectral Properties

The spectral properties of a signal express how the power or energy of the signal is distributed versus frequency. Typically, periodic and random signals have a power distribution, and transient signals have an energy distribution. If we discuss periodic signals, the power of the signal is distributed on the fundamental frequency of the signal and its harmonics, i.e. the power of a periodic signal will be distributed at discrete frequencies. On the other hand, both random signals and transient signals will usually have a spectral distribution with a continuous character. In order to estimate spectral properties of a signal it is important to select an appropriate spectrum estimator. The selection of a spectrum estimator is related to signal properties such as periodic, random and transient. If the signal is periodic it is usually preferable to estimate the spectrum using a power spectrum estimator $\hat{P}^{PS}(f)$. On the other hand, if the signal has a random nature the spectral properties are usually estimated by a power spectral density estimator $\hat{P}^{PSD}(f)$. In the case of transient signals it is preferable to use an energy spectral density estimator $\hat{P}^{ESD}(f)$.

Spectral density estimation can either be done by the Bartlett, Welch or Blackman-Tukey method. The Welch method is a further development of the Bartlett's method, where the data segments are allowed to overlap and the data are windowed prior to computing the periodogram [11]. This usually results in a better estimate, especially when only small amounts of data are available.

The Welch spectrum estimate is obtained by averaging a number of periodograms. Each periodogram is based on segments of a time sequence $x(n)$, each segment consisting of N samples. Thus the original time sequence of data must be divided into data segments as

$$x_l(n) = x(n+lD) \quad \text{where} \quad \begin{cases} n = 0, 1, \dots, N-1 \\ l = 0, 1, \dots, L-1 \end{cases}$$

where lD is the starting point for each periodogram and D is the overlapping increment. If $D = N$ there is no overlap, and if $D = N/2$ there is a 50% overlap between the consecutive time sequences $x_l(n)$ and $x_{l+1}(n)$.

Dividing the original time sequence into data segments is equivalent to multiplying the time series by a window. Actually, even the Bartlett method can be considered to be windowed with a rectangular window when the data sequence is limited to N samples. On the other hand, the Welch method allows the usage of arbitrary windows such as

kaiser, hanning, flattop, etc. Hence, each periodogram is based on a windowed sequence $w(n)x_l(n)$ where $n = 0, 1, \dots, N-1$. The Welch spectral density estimator $\hat{P}^{PSD}(f)$ is given by

$$\hat{P}_{xx}^{PSD}(f_k) = \frac{1}{F_s L N U_{PSD}} \sum_{l=0}^{L-1} \left| \sum_{n=0}^{N-1} x_l(n) w(n) e^{-j2\pi nk/N} \right|^2, \quad f_k = \frac{k}{N} F_s \quad (19)$$

where $k = 0, \dots, N/2$, L is the number of periodograms, N is the length of the periodogram, F_s the sampling frequency and

$$U_{PSD} = \frac{1}{N} \sum_{n=0}^{N-1} (w(n))^2$$

is the window-dependent effective analysis bandwidth normalization factor.

For lightly damped mechanical systems the normalized bias error in a spectral density estimate can, at the resonance frequencies f_r , be approximated using [9]

$$\varepsilon_b \approx -\frac{1}{3} \left(\frac{B_e}{B_r} \right)^2, \quad (20)$$

where B_r is the half power bandwidth of the smallest resonance peak and B_e is the window dependent resolution bandwidth of the spectral estimate [9], defined as

$$B_e = \frac{\sum_{n=0}^{N-1} w^2(n)}{\left(\sum_{n=0}^{N-1} w(n) \right)^2} F_s = \frac{\frac{1}{N} \sum_{k=0}^{N-1} |W(k)|^2}{W^2(0)} F_s,$$

where $W(k)$ is the Fourier transformed window function $w(n)$.

The normalized random error ε_r of the spectral estimator is dependent on the choice of time window $w(n)$ and the overlap between the periodograms. It is given by [9]

$$\varepsilon_r = \sqrt{\frac{V[\hat{P}(f)]}{P^2(f)}} \approx \frac{1}{\sqrt{B_e T_e}}, \quad (21)$$

where $V[\hat{P}(f)]$ is the variance of the spectral estimate and T_e is the effective record length.

The selection of data length, data segment length as well as the overlapping and averaging must be done carefully. There is usually a compromise between the variance and the bias. After preliminary trials, a selection of data length, data segment length L , number of periodograms K , digital window and sampling rate F_s was made. The selection is shown in Tables 4 and 5.

Parameter	Value
Total record length, T	60 s
Data segment length, N	16384
Number of periodograms, L	350
Digital window $w(\tau)$	Hanning
Overlapping	50 %
Effective data length, T_e	$T/1.08$
Sampling rate, F_s	48000 Hz

Table 4: Spectral density estimation parameters.

Parameter	Value
Total record length, T	60 s
Data segment length, N	65536
Number of periodograms, L	87
Digital window $w(\tau)$	Hanning
Overlapping	50 %
Effective data length, T_e	$T/1.08$
Sampling rate, F_s	48000 Hz

Table 5: Spectral density estimation parameters for the zoomed spectra.

3 Results

The results are discussed as follows: the Euler-Bernoulli calculations, the outcome of the stationarity tests, mean square value properties, segmentation of nonstationary data records into short time stationary segments, sample probability density estimates, and spectrum estimations. In Fig. 6 a typical plot of the boring bar vibration in the cutting speed direction is shown as a function of time.

Figure 6: Boring bar vibration in the cutting speed direction as a function of time, work-piece material SS 0727-02, $s = 0.2$ mm/rev, $a = 2$ mm and $v = 175$ m/min.

3.1 A Simple Model of a Boring Bar

The theoretical approach to the vibration analysis furnished the resonance frequencies for the first three modes of the boring bar. Using the values tabulated in Table 3 the frequencies for the first three modes were calculated. The results are presented in Table 6.

Direction of mode motion	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
Cutting speed direction	676	4238	11866
Cutting depth direction	708	4427	12369

Table 6: The first three resonance frequencies calculated using an Euler-Bernoulli approximation using the values given in Table 3.

The calculation of resonance frequencies of the boring bar clearly showed that the theoretical approach yielded too high estimates. However, as in subsequently demonstrated,

they give a good indication as to which frequency area to expect severe acceleration levels in.

3.2 Stationarity

The stationarity tests were first designed to filter out the influence of the lowest resonance frequency of the clamped boring bar. In order to accomplish this, the size of the interval used, where the sum of the mean square values was calculated, was 2400 samples or 0.05 s of continuous cutting operation. The stationarity tests have been carried out for all time records from the continuous cutting operations for all of the three materials SS 0727-02, SS 2343-02 and SS 2541-03. The cutting data used are given in Table 2. In Fig. 7 outputs from the run test and the reverse arrangements test are shown. Based on 1200 observations of mean square values, each calculated using 2400 samples, the number of boring bar vibration records considered as stationary was very poor, see Fig. 7 a). The number of vibration records considered as stationary was 2.3% in the cutting speed direction and 1.1% in the cutting depth direction of the total number of records. The significance level was $\alpha = 0.05$.

It was suspected that the boring bar vibration was influenced by low frequency periodicities deriving from the spindle engine vibration. The motion of the spindle - chuck - workpiece system due to the residual rotor mass unbalance was thus measured by an accelerometer mounted on the spindle engine. In Fig. 21 estimates of the power spectral density of the spindle engine vibrations are shown. It is obvious that these kinds of vibrations are present in the CNC lathe used in the experiments. The influence of the spindle engine vibration on the boring bar vibration may be filtered out by increasing the averaging time used to compute the mean values for the stationarity tests. The number of samples used to compute the mean square values for the stationarity tests was increased to 96000 samples or 2 s of continuous cutting operations. The number of mean square value observations used in the stationarity tests was 30. In Fig. 7 b) the outputs from run tests and reverse arrangements tests using the increased number of samples to compute the mean square values is shown. There was an increase in the number of vibration records considered to be stationary: in the cutting speed direction from 2.4% to 16.1% and in the cutting depth direction from 1.1% to 12.6% of the total number of vibration records. The significance level was $\alpha = 0.05$.

3.3 Mean Square Value Property

In order to facilitate the nonstationary mean square value or power of the boring bar vibration records the mean square value of the boring bar as a function of time was estimated by using sliding average. The mean square value was estimated by using three different averaging lengths. Fig. 8 a) and b) shows the mean square acceleration for three different averaging times T_a .

The periodic dynamic motion related to residual rotor mass unbalance in the spindle - chuck - workpiece system is clearly illustrated in Fig. 8. Furthermore, when the length of the moving average was increased to 96000 samples or 2 s, the periodic dynamic motion

Figure 7: The observed number of runs and reverse arrangements for boring bar vibration in the cutting speed direction , a) each mean square value observation extracted from 2400 samples, b) each mean square value observation extracted from 96000 samples. Workpiece material SS 0727-02, feed rate $s = 0.1$ mm/rev, cut depth $a = 2$ mm, cutting speeds $v = 50$ m/min - 300 m/min, cutting tool DNMG 150608-SL, grade 7015. If the observed number of runs is in the interval between $r_{n;0.975}$ and $r_{n;0.025}$ the boring bar vibration record is accepted as stationary according to the run test. If the observed number of reverse arrangements is in the interval between $A_{N;0.975}$ and $A_{N;0.025}$ the boring bar vibration record is accepted as stationary according to the reverse arrangements test.

Figure 8: a) The mean square value of boring bar vibration, averaging time $T_a = 0.05, 0.5, 2s$, and b) the boring bar vibration when cutting in SS 2541 in the cutting speed direction, feed rate 0.2 mm/rev, cutting speed 125 m/min and cut depth 2 mm. c) The mean square value of boring bar vibration, averaging time $T_a = 0.05, 0.5, 2s$, and d) the boring bar vibration when cutting in SS 0727 in the cutting speed direction, feed rate 0.3 mm/rev, cutting speed 250 m/min and cut depth 2 mm.

related to residual rotor mass unbalance in the spindle - chuck - workpiece system was filtered out. However, observe that the mean square value of the boring bar vibration produced by the sliding average 96000 samples varies slowly with time, as in Fig. 8 b). This probably indicates nonstationarities in the boring bar vibration related to the chip formation process.

3.4 Segmentation of the Boring Bar Vibration Records

The boring bar vibration records were divided into segments classified as stationary in accordance with the run test and the reverse arrangements test. A stationary segment, from an arbitrary starting point, is the largest number of consecutive observations classified as stationary. This results in short-time stationary segments of the boring bar vibration record. The stationarity tests used to divide the vibration record into short-time stationary segments were based on mean square values extracted from 96000 samples or 2 s of continuous cutting operation. Thus, the influence of the low-frequency spindle vibrations are neglected. Fig. 9 shows the power or mean square value of boring bar vibration records versus time and corresponding data segments accepted as stationary.

3.5 Sample Probability Density Estimate

Sample probability density estimates were produced based on the short-time stationary segmentation of the boring bar vibration records. The sample probability density estimate

Figure 9: a) The boring bar vibration power and b) the short-time stationary intervals, shaded areas, of a 60-second dynamic boring bar response in the cutting speed direction. c) The boring bar vibration power and d) the short-time stationary intervals, shaded areas, of a 60-second dynamic boring bar response in the cutting depth direction. Workpiece material SS 2541-03, $s = 0.3$ mm/rev, $a = 2$ mm, $v = 100$ m/min, cutting tool DNMG 150608-SL, grade 7015.

produces a time average of the underlying time-dependent boring bar vibration probability density. The sample probability density estimates of the boring bar vibration record usually showed different shapes for the different segments that were accepted as stationary. The observable shapes, see Fig. 10, were the sine wave probability density shape in a), the gaussian shape in b), and a combination of these in c). On occasions, similar shapes of the sample probability densities were observed in both the cutting speed and cutting depth directions during a continuous cutting operation. This is illustrated in Fig. 11. An essential feature of the vibration data was that the shape of the sample probability density estimates could be totally different in the cutting speed and in the cutting depth direction during a continuous cutting operation with constant cutting data, as in Fig. 12. Thus, the vibration might be dominated by an eigenmode in one measurement direction, but in the other direction there is little or no influence from this eigenmode or other eigenmodes. From the presented sample probability density estimates it is also obvious that the boring bar vibration is nonstationary. Furthermore, the sample probability density estimate presented as a waterfall in Fig. 13 a) clearly shows the nonstationary behavior of the boring bar vibration. There are differences between the shapes of the sample probability densities for the workpiece materials in the cutting speed direction. At high cutting speeds, generally higher than 150 m/min, the shape for the workpiece material SS 0727-02 is usually Gaussian, while the shape for the workpiece materials SS 2343-02 and SS 2541-03, tend to be more undetermined. However, all workpiece materials introduce sine wave probability density shape for lower cutting speeds.

Figure 10: The sample probability density estimates for the data segments considered as short-time stationary of the dynamic response of the boring bar in the cutting speed direction during a 60-second continuous cutting operation in: a) SS 2343-02, $s = 0.1$ mm/rev, $a = 2$ mm, $v = 50$ m/min, tool DNMG 150608-SL, grade 8025, b) SS 0727-02, $s = 0.3$ mm/rev, $a = 2$ mm, $v = 300$ m/min, tool DNMG 150608-SL, grade 7015, c) SS 0727-02, $s = 0.2$ mm/rev, $a = 2$ mm, $v = 225$ m/min, tool DNMG 150608-SL, grade 7015.

Figure 11: The sample probability density estimates of the short-time stationary data segments of a 60-second dynamic boring bar response in, a) the cutting speed direction, b) the cutting depth direction. Workpiece material SS 0727-02, feed rate $s = 0.2$ mm/rev, cut depth $a = 2$ mm, cutting speed $v = 250$ m/min, tool DNMG 150608-SL, grade 7015. Observe, the stationarity segments differ between the cutting speed and the cutting depth direction.

Figure 12: The sample probability density estimates of the short-time stationary data segments of a 60-second dynamic boring bar response in, a) the cutting speed direction, b) the cutting depth direction. Workpiece material SS 2343-02, $s = 0.2$ mm/rev, $a = 2$ mm, $v = 50$ m/min, tool DNMG 150608-SL, grade 8025. Observe the different shapes.

Figure 13: a) The sample probability density and b) the power spectral density of the short-time stationary data segments of a 60-second dynamic boring bar response in the cutting speed direction when cutting in SS 2541-03, $s = 0.2$ mm/rev, $a = 2$ mm, $v = 275$ m/min, tool DNMG 150608-SL, grade 7015.

3.6 Spectral Properties

The spectral density estimates are based on the short-time stationary segments of the boring bar vibration records, see section 3.4. The normalized bias error ε_b of the spectral density estimates was calculated using equation 20, where B_r was obtained in an estimate of the frequency response function of a hammer-excited boring bar, see Fig. 14. The normalized random error ε_r was calculated using equation 21 and was based on the length of the short time stationary segments in each continuous cutting operation. The calculated errors in the spectral density estimates are shown in Table 7. In the random error ε_r case the longest short-time stationary segment of each vibration record has been used.

Normalized bias error	Normalized random error
ε_b	ε_r
0.07	0.06-0.16

Table 7: Normalized bias error ε_b and normalized random error ε_r in the power spectral density estimates.

The reason for not presenting the power spectral density estimates in the same way as in the sample probability density estimate case, i.e. spectrum versus time, is that the nonstationarities are not clearly displayed in these spectrum estimates. This is obvious from Fig. 13 a) and b), which show the sample probability density estimate and power spectral density of a typical nonstationary data record. Fig. 15 shows the power spectral

Figure 14: Amplitude function of a frequency response function estimate for the boring bar in the cutting speed direction based on hammer excitation. The half power bandwidth of the narrowest peak is given.

density estimates in SS 0727-02, a) versus cutting speeds (50 - 300 m/min), b) versus feed rates (0.1 - 0.3 mm/rev). In Fig. 15, the resonance peaks appear to be independent of both cutting speed and feed rate. This is not the case, however, as in subsequently demonstrated. In order to facilitate observations of the influence of the parameter space, the spectral density estimates are presented as waterfall diagrams. If speed is used as a parameter, the growth of the wide-band vibration energy in the cutting process as a function of the cutting speed is obvious. Figs. 16, 17 and 18 show typical waterfall diagrams for the three materials used in the experiments.

Some differences between the spectral density estimates for the workpiece materials can be observed. The workpiece materials SS 0727-02 and SS 2541-03 (see Figs. 16 and 18) resulted in a similar vibration spectrum pattern, while SS 2343-02 (see Fig. 17) induces a smoother spectra at higher frequencies. Also, the workpiece materials SS 0727-02 and SS 2343-02 tend to be more wide-banded than the workpiece material SS 2541-03. On the other hand, SS 2541-03 causes most harmonics.

According to the results from the Euler-Bernoulli model of the boring bar, there exists two resonance frequencies below 800 Hz, one from the cutting speed direction and one from the cutting depth direction. Zooming the first resonance peak in the power spectral density estimates showed that the first peak is likely to be one of two peaks, see Figs. 19 and 20. The calculated errors in the zoomed spectrum can be found in Table 8.

A phenomenon evident at high frequency resolution was the sidebands in the power spectral density estimates in Figs. 19, 20 and 22. A spectrum of low-frequency spindle

Figure 15: The power spectral densities of the dynamic response of the boring bar in the cutting speed direction during a continuous cutting operation in SS 0727-02, a) at 11 different cutting speed $v = 50$ m/min to 300 m/min, $s = 0.3$ mm/rev, $a = 2$ mm, b) at three different feed rates $s = 0.1$ mm/rev to 0.3 mm/rev, $a = 2$ mm, $v = 50$ m/min, tool DNMG 150608-SL, grade 7015.

Normalized bias error ε_b	Normalized random error ε_r
0.004	0.13-0.31

Table 8: Normalized bias error ε_b and normalized random error ε_r in the zoomed power spectral density estimates.

Figure 16: The power spectral density estimate of the dynamic response of the boring bar in the cutting speed direction during a continuous cutting operation in SS 0727-02, at 11 different cutting speeds $v = 50 - 300$ m/min, $s = 0.3$ mm/rev, $a = 2$ mm, tool DNMG 150608-SL, grade 7015.

engine vibration is shown in Fig. 21. The spindle engine vibration was measured by an accelerometer mounted on the spindle engine when no cutting operation was performed. During continuous boring operations, these low-frequency spindle vibrations usually appear as sidebands of the dominating resonance frequency as shown in Fig. 22. At higher cutting speeds, the second resonance peak was active, and at lower cutting speeds the first resonance peak was active as illustrated in Figs. 19 and 20. Observe that the resonance frequencies of the boring bar vary both at different feed rates and at different cutting speeds. The first two resonance peaks were at approximately the same frequency for SS 0727-02 and SS 2541-03, but for material SS 2343-02 the frequency for the first two resonance peaks was slightly higher, about 20 Hz. Austenitic stainless steel SS 2343-02 produces higher mean cutting force as compared to SS 0727-02 and SS 2541-03 for the same chip cross section or the same cutting depth and feed rate [12]. Thereby causing a stiffer coupling between the cutting tool and workpiece.

Figure 17: The power spectral density estimate of the dynamic response of the boring bar in the cutting speed direction during a continuous cutting operation in SS 2343-02, at 7 different cutting speeds $v = 50 - 200$ m/min, $s = 0.1$ mm/rev, $a = 2$ mm, tool DNMG 150608-SL, grade 8025.

Figure 18: The power spectral density estimate of the dynamic response of the boring bar in the cutting speed direction during a continuous cutting operation in SS 2541-03, at 11 different cutting speeds $v = 50 - 300$ m/min, $s = 0.1$ mm/rev, $a = 2$ mm, tool DNMG 150608-SL, grade 7015. Observe the harmonics of the dominating resonance peak.

Figure 19: The power spectral densities, zoomed in frequency to the interval of the two dominating resonance peaks, in SS 2343-02 in the cutting speed direction, a) at seven different cutting speeds $v = 50$ m/min to 200 m/min, $s = 0.1$ mm/rev, $a = 2$ mm, b) at three different feed rates $s = 0.1$ mm/rev to 0.3 mm/rev, $v = 50$ m/min, $a = 2$ mm, tool DNMG 150608-SL, grade 8025. Observe the sideband terms in both waterfall diagrams.

Figure 20: The power spectral densities, zoomed in frequency to the interval of the two dominating resonance peaks, in SS 2541-03 in the cutting speed direction, a) at eleven different cutting speeds $v = 50$ m/min to 300 m/min, $s = 0.3$ mm/rev, $a = 2$ mm, b) at three different feed rates $s = 0.1$ mm/rev to 0.3 mm/rev, $v = 125$ m/min, $a = 2$ mm, tool DNMG 150608-SL, grade 7015. Observe the sideband terms in both waterfall diagrams.

Figure 21: The power spectral density of vertical spindle engine vibration, a) spindle speed = 3.95 rev/s, b) spindle speed = 2.667 rev/s.

Figure 22: Power spectral density estimate with sidebands or force modulation peaks in SS 2343-02 in the cutting speed direction, $a = 2$ mm, $s = 0.3$ mm/rev, $v = 200$ m/min, tool DNMG 150608-SL, grade 8025. Observe, the sidebands or force modulation peaks are approximately equally spaced with the speed of the spindle, 6.2 rev/s.

4 Discussion and conclusions

The above results demonstrate that the vibration pattern of a boring bar is complex. It has also been established that the boring bar vibrations usually exhibit nonstationary stochastic behavior, and that the dynamic response of the clamped boring bar has nonlinear properties. Furthermore, the energy of the boring bar vibration in a continuous boring operation is to a great extent dominated by the motion in the cutting speed direction. The vibration energy originates from the first two resonance frequencies of the clamped boring bar.

The estimates of the first resonance frequency in the cutting speed and in the cutting depth direction, 676 Hz and 708 Hz, obtained by the Euler-Bernoulli model of the boring bar, show of what frequency range to expect severe boring-bar vibration when machining. However, the estimates of these two eigenfrequencies are a little too high. A major reason for this is the clamped boundary conditions used in the Euler-Bernoulli model of the boring bar. They will differ from the actual boundary conditions applied by the clamping bolts in the clamping device, which are attached to a revolver of finite mass.

From the stationary tests it is obvious that the boring bar vibration may be considered to be nonstationary. However, revolution-dependent low frequency vibration related to the residual rotor mass unbalance in the spindle - chuck - workpiece system is present in the CNC turning center (see Fig. 21). By examining the mean square value of the boring bar vibration as a function of time, it is obvious that the mean square value usually exhibits a low-frequency periodic variation that is revolution dependent (see Fig. 8). In addition to this low-frequency revolution dependent periodic variation in the mean square value, the boring bar vibration mean square value generally exhibits trends that vary slowly compared to the spindle vibration related periodicities (see Fig. 8 b). These trends varying slowly over time or nonstationarities in the boring bar vibration are in all probability related to the chip formation process.

There is considerable variation in the shapes of the sample probability densities for the short-time stationary segments of the boring bar vibration. The observable shapes in the sample probability densities are the sine wave probability density shape, the Gaussian shape and a combination of these (see Fig. 10). The sample probability densities of short-time stationary segments within a boring bar vibration record are usually distinct (see Fig. 13 a). If the response of the boring bar is considered, one of the boring bar vibration directions may be dominated by a boring bar resonance frequency, but in the other direction there is little or no influence from this resonance frequency or other resonance frequencies. This may well explain some of the observed differences between the distribution in the cutting speed and in the cutting depth direction for the boring bar vibration (see Fig. 12).

According to the spectrum analysis, the major part of the boring bar vibration energy is usually in the cutting speed direction. The boring bar vibration is normally dominated by one of two resonance frequencies in the frequency interval between 500 Hz and 650 Hz (see Figs. 19 - 20) and several of its harmonics (see Figs. 18), irrespective of feed

rate and cutting speed. The acceleration level at these resonance frequencies is generally greatest in the cutting speed direction. At low cutting speeds below approximately 150 m/min, the first resonance peak, the resonance peak with the lowest frequency of the two dominating peaks, is active. At higher cutting speeds, the second resonance peak is active. If the feed rate or the cutting speed is changed, both resonance peaks exhibit variations in frequency. Different feed rates will introduce different boundary conditions for the cutting tool thereby resulting in different resonance frequencies. Also, at fairly low cutting speeds, approximately 50-100 m/min, effects of built up edge are likely to affect the boundary conditions of the cutting tool [12]. However, for cutting speeds above 150 m/min, the mean cutting forces in the cutting speed direction decrease with increasing cutting speed [12]. Now, assuming that the stiffness of the coupling between the workpiece and boring bar is proportional to mean cutting force, the decrease in frequency of the second resonance frequency with increasing cutting speed is obvious. Low frequency revolution dependent workpiece motion due to residual rotor mass unbalance in the spindle - chuck - workpiece system influences the motion of the tool holder via the relative dynamic motion between the cutting tool and workpiece. Thus, the motion of a tool holder is influenced by force modulation. This appears as sidebands, e.g. to the two dominating resonance peaks in the boring bar vibration power spectral density estimates. The second resonance peak is particularly affected by sidebands.

Both the harmonics of the dominating resonance frequency in the spectra and the frequently observed sine wave probability density shape in the sample probability densities indicate nonlinearities in the dynamic response of the boring bar [13, 14]. Tool shank vibrations corresponding to similar cutting operations in external turning are wide-band and usually Gaussian-distributed random processes [2]. Thus, it may be stated that the dynamic response of the boring bar has nonlinear properties. One reason for the nonlinear behavior of the boring bar may be derived from the fact that the motion of the cutting tool is limited by the workpiece in both the cutting depth direction and the reverse direction to the cutting speed. The results demonstrate an additional reason for the nonlinear response of the boring bar. Thus, the resonant motion of the boring bar will introduce cutting tool speeds that frequently exceed the cutting speed. Hence, intermittent contact between the workpiece and cutting tool is likely to occur. Where this is the case, a sinusoidal motion at the actual resonance frequency and several of its harmonics is likely to be introduced in to the boring bar response.

The following conclusions may be deduced from an analysis of the experiments:

- The boring bar vibrations in all three materials are usually nonstationary.
- The dynamic response of the clamped boring bar has nonlinear properties
- Three typical shapes in the sample probability density estimates have been observed: the sine wave probability density shape, the Gaussian shape and a combination of both.
- Sample probability densities from the cutting speed direction and the cutting depth direction may be of a totally different shape.

- In a continuous cutting operation with constant cutting data, the boring bar vibration sample probability density shape may vary significantly in time.
- The highest vibration level was in the cutting speed direction.
- The frequent sine wave probability shape observed in the sample probability density estimates indicated sinusoidal boring bar vibration.
- The resonance frequencies of the boring bar altered at different feed rates and cutting speeds.
- The boring bar vibration was often dominated by one of two resonance frequencies in the frequency interval between 500 Hz and 650 Hz.
- In the boring bar vibrations there were several harmonics of the dominating resonance frequency, especially in material SS 2541.
- The acceleration level of the dominating resonance frequencies was generally greatest in the cutting speed direction.
- Force modulation influences the dynamic motion of the boring bar vibrations.

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