Performance Improvements for Sector Antennas using Feature Extraction and Spatial Interference Cancellation

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Abstract — Effective utilization of the spatial domain enhances the capacity of a mobile radio network. A common technique is to use sector antennas, where the sectors are formed by weighting the outputs from the antenna elements. This results in spatial domain selectivity, which significantly improves the signal-to-noise and interference ratio in the received signals. However, the operation of the sector antenna will be limited by the sidelobes of the corresponding beam patterns. By introducing a blind spatial interference canceler that combines the six beamformers in the sector antenna with blind signal separation, a significant improvement in the multi-user interference suppression can be achieved. Thus, it will be able to efficiently handle the near-far problem, where the users are received with different power. The blind signal separation is performed by an independent component analysis algorithm. The convergence rate of the algorithm is significantly improved compared to the standard formulation by taking into account the modulation format. The algorithm is further improved by introducing a forgetting factor on the weight update. The blind spatial interference canceler is evaluated by simulations using the mean square error and the bit error rate as quality measures. The results show that the mean square error obtained from the blind spatial interference canceler is within 0.5 dB from the optimum Wiener solution for signal-to-noise ratios greater than 0 dB.

I. INTRODUCTION

The capacity in cellular communication systems is increased by the spatial diversity gained from the use of several antennas at the Base Station (BS) [1], [2], [3]. Spatial diversity gain can be achieved by the use of sector antennas (six beamformers), steerable antenna arrays, adaptive antenna arrays or by the use of interference cancellation schemes or multi-user detection schemes. Adaptive antenna arrays can be used to improve the Signal-to-Noise and Interference Ratio (SNIR) by suppressing the interfering signals. An adaptive antenna is either model based by using constraints to select the user or pilot signal based which has a unique training sequence for each user. A constrained adaptive antenna requires a thorough calibration and can suffer significant performance degradation when the array response vector deviates from the model [4]. The accuracy of the array response vector estimation depends on several factors such as the estimation of the Direction of Arrival (DOA), the traffic conditions and the sensor characteristics. The pilot trained adaptive antenna does not suffer from the above mentioned problems but some capacity needs to be used for the training.

The scope for this paper is a Blind Spatial Interference Canceler (BSIC). This BSIC combines \( K \) beamformers creating the sector antennas and blind signal separation for the interference cancellation. The output signals from the beamformers represent a spatially filtered version of antenna array input signals. These beamformer output signals will have improved SNIR compared to the antenna element signals. One of the beams is selected as the desired beam and its output signal is the Signal Of Interest (SOI). This SOI is filtered out by the corresponding beamformer that forms the sector of the cell. Due to non-perfect beamforming, interfering signals will also be presented in the SOI. To reduce the effect of the interfering signals, a post-processing (signal separation) of the outputs from the fixed beamformer has been used. The signal separation algorithm maps the \( K \) output signals from the beamform-
ers into $K$ independent signals. This algorithm is able to completely resolve the signals if the number of sensors is less than or equal to the number of source signals. When the number of source signals exceeds the number of sensors, the performance will gradually degrade. Since, the blind signal separation algorithms works on beamformed data the permutation problem inherent in all blind signal separation will be resolved as long as a dominant signal is found in each beamformers output. The blind signal separation is performed by an Independent Component Analysis (ICA) algorithm [5]. This algorithm has been substantially improved by taking into account the modulation format and noise model of the channel. Further improvement is made by using a forgetting factor on the weight update to give an unique weight solution with minimum norm. Since, the blind signal separation problem is scale invariant, the output signals from signal separation are normalized to recover the correct amplitude levels. By combining the beamforming and signal separation the receiver will be able to handle near-far problems efficiently. It will thus ease the requirements of the power control algorithms. An other important feature in the proposed BSIC scheme is that it does not require a priori knowledge of the transmitted signals. Consequently, there is no reduction in the system capacity, which is caused by transmission of training (pilot) signals.

This paper is organized as follows. Section II describes the system model. The proposed blind spatial interference cancellation scheme is presented in Section III, and evaluation examples are given in Section IV. Finally, Section V contains conclusions.

II. SYSTEM MODEL

Consider a communication system with $M$ users, transmitting $M$ signals $s_m(t), \ 0 \leq m \leq M - 1$. These signals have been discretized and thus, only a discretized model is used. Assume that all users utilize the same bandwidth and the transmitting signals are uncorrelated, 

$$E[s_m(n)s_l(n)] = 0, \ for \ m \neq l, \ 0 \leq m, l \leq M - 1,$$

where $E[\cdot]$ denotes the expected value and $n$ is the discrete time index. This can be achieved by using a modulation scheme with zero symmetric signal constellation. For a scheme with non-zero symmetric signal constellation, the transmitting signals are correlated due to the DC component and the performance of the BSIC reduces. A Quadrature Phase Shift Keying (QPSK) modulation scheme is used in this study.

The received signal $v_k(n)$ at the $k^{th}$ antenna element is obtained from the transmitted signals as follows

$$v_k(n) = \sum_{m=0}^{M-1} a_{km}(n)s_m(n) + \omega_k(n), \ 0 \leq k \leq K - 1, \ (1)$$

where $a_{km}(n)$ is the attenuation factor and phase shift of the channel between the $m^{th}$ source and the $k^{th}$ antenna and $\omega_k(n)$ denotes the Additive White Gaussian Noise (AWGN) component. In this study the quasi-static assumption is made where the relative speed of the sources to the receiver is very low. Thus, equation (1) is reduced to

$$v_k(n) = \sum_{m=0}^{M-1} a_{km}s_m(n) + \omega_k(n), \ 0 \leq k \leq K - 1. \ (2)$$

Synchronization, which is an essential part of all up-link schemes, is assumed to be perfect. Non-perfect synchronization generates a substantial study in itself and needs to be considered in the future.

III. THE BLIND SPATIAL INTERFERENCE CANCELER SCHEME

The proposed blind spatial interference cancellation scheme is shown in Fig. 1. The transmitted signals $s_m(n), \ 0 \leq m \leq M - 1$, are received by $K$ antenna elements. The received signals $v_m(n), \ 0 \leq k \leq K - 1$, are then processed by the beamforming weight matrix. The columns of the weight matrix are used at each of the antenna elements to create $K$ beams. The beamforming vector for beam 0 is denoted as

$$g_0 = [g(0), \cdots, g(K - 1)]^T, \ (3)$$

where $g(k), \ 0 \leq k \leq K - 1$, is the complex steering weight of the $k^{th}$ antenna element and $[\cdot]^T$ denotes the vector transpose. The beamforming vector for the $k^{th}$ beam is obtained by shifting the vector for beam 0 by $k$ steps,

$$g_k = [g(K - k), \cdots, g(0), \cdots, g(K - k - 1)]^T. \ (4)$$
The beamforming matrix is given by
\[ G = [g_0, \cdots, g_{K-1}]^T. \] (5)

The output signal \( z_k(n) \) from the \( k^{th} \) beamformer can now be expressed as
\[ z_k(n) = g_k^H v(n), \quad 0 \leq k \leq K - 1, \] (6)
where \([\cdot]^H\) denotes Hermitian operator and \( v(n) = [v_0(n), \cdots, v_{K-1}(n)]^T \) is the received signal vector. In this paper a least squares error is used to design the beamforming weights.

\[ \text{A. Least Square Design of the Fix Beamformer} \]

For a general array structure, the array response vector \( d(f_0, \theta) \) from a point source is determined by the carrier frequency \( f_0 \) and the angle of arrival \( \theta \). In a reflectionless medium, the response vector is given by
\[ d(f_0, \theta) = [e^{-j2\pi f_0 \Delta_0(\theta)/c}, \cdots, e^{-j2\pi f_0 \Delta_{K-1}(\theta)/c}]^T, \] (7)
where \( c \) denotes the wave propagation velocity and \( \Delta_k(\theta), \quad 0 \leq k \leq K - 1, \) denotes the orthogonal distance from the source wave front to the \( k^{th} \) array element. For a planar circular array with the \( K \) sensor elements, see Fig. 2, the orthogonal distance \( \Delta_k(\theta) \) is given by
\[ \Delta_k(\theta) = r \cos \left( \frac{k2\pi}{K} - \theta \right), \] (8)
where \( r \) denotes the radius of the array. It is assumed that the antennas are omni-directional.

The circular beamformer is designed by minimizing the least squares norm. The desired beamforming response for certain \( \theta \) is given by
\[ G_d(\theta) = \begin{cases} 1, & -\theta_p \leq \theta \leq \theta_p \\ 0, & \theta_p \leq \theta \leq 2\pi - \theta_s \end{cases}, \] (9)
where the positive angles \( \theta_p \) and \( \theta_s, \theta_p < \theta_s \), are the passband and stopband edges. The beamformer is designed over grid of \( I \) angular samples, \( \theta_1, \cdots, \theta_I \in [-\theta_p, \theta_p] \cup [-\theta_s, 2\pi - \theta_s]. \) Denote the array response matrix as \( D = [d(f_0, \theta_1), \cdots, d(f_0, \theta_I)] \) and the desired response vector as \( G_d = [G_d(\theta_1), \cdots, G_d(\theta_I)]^T \). The complex beamforming vector \( g_k \) is given by minimizing the least squares \( J(k) \),
\[ J(k) = \|G_d - g_k^H D\|_2^2, \quad 0 \leq k \leq K - 1, \] (10)
where \( \| \cdot \|_2 \) denotes the Euclidean norm. Since, all beamforming vectors can be obtained from \( g_0 \) it is enough to minimize
\[ J(0) = \|G_d - g_0^H D\|_2^2. \] (11)
Equation (11) can be formulated as quadratic programming problem, which can be solved easily for \( g_0 \).

\[ \text{B. Blind Signal Separation} \]

In the blind signal separation problem, \( M \) unknown source signals \( s(n) = [s_0(n), \cdots, s_{M-1}(n)] \) are transmitted through a medium so that an array of \( K \) sensors picks up a set of signals \( z(n) = [z_0(n), \cdots, z_{K-1}(n)] \) where \( z_k(n) \) is defined in (6).
Note that, for the identifiability of the considered mixing the number of sources should be less than or equal to the number of antennas, i.e. \( M \leq K \). Given the received mixed signals \( z(n) \), signal separation algorithms are used to estimate the unmixing matrix \( W \),

\[
W = \begin{bmatrix}
w_{0,0} & \cdots & w_{0,K-1} \\
\vdots & \ddots & \vdots \\
w_{K-1,0} & \cdots & w_{K-1,K-1}
\end{bmatrix},
\]

(12)

where \( w_{m,k}, \ 0 \leq k, m \leq K - 1, \) is complex unmixing weight connecting the signal \( z_k(n) \) to the output \( u_m(n) \). This output after the signal separation is given as

\[
u_m(n) = \sum_{k=0}^{K-1} w_{m,k} z_k(n), \quad 0 \leq m \leq K - 1.
\]

(13)

The unmixing weights \( w_{m,k} \) will be estimated by using leaky ICA algorithm.

B.1 Leaky Independent Component Analysis Algorithm

The independent component analysis algorithm, based on infomax rule [5], is derived by maximizing the joint entropy of an observation that has been linearly transformed and processed through a non-linearity. It is shown in [6] that the infomax rule is equivalent to the maximum likelihood estimation of the free vector components. The independent vector components are found by estimating the mixing matrix inverse, possibly rescaled and per mutated. Denote \( W_{ICA} \) as the unmixing matrix for the ICA algorithm. A general learning rule is derived from the maximum likelihood estimation to estimate \( W_{ICA} \). The probability density function of the observation vector \( z(n) \) is expressed as a function of modeled probability density function of the sources [7] as

\[
p(z(n)) = |\det(W_{ICA})| p(u(n)),
\]

where \( p(u(n)) = \prod_{i=0}^{K-1} p(u_i(n)) \) is the hypothesized distribution of the sources, \( |\det(\cdot)| \) is the absolute value of the matrix determinant and \( u(n) = [u_0(n), \cdots, u_{K-1}(n)] \) is the unmixed output vector. The log-likelihood function is then defined as

\[
L(u(n), W_{ICA}) = \log p(z(n)),
\]

where

\[
\log p(z(n)) = \log |\det(W_{ICA})| + \sum_{i=0}^{K-1} \log p(u_i(n)).
\]

The gradient vector of the log-likelihood function is also the original infomax gradient [5] and is given as

\[
\frac{\partial L(u(n), W_{ICA})}{\partial W_{ICA}} = \left(W_{ICA}^H - \varphi(u(n))z(n)^H\right),
\]

where

\[
\varphi(u) = \frac{\partial p(u)}{\partial u} = \left[\frac{\partial p(u_0)}{\partial u_0}, \cdots, \frac{\partial p(u_{K-1})}{\partial u_{K-1}}\right]^T.
\]

(14)

By multiplying the original infomax gradient with \( W_{ICA}^H \), a simplified algorithm, can be obtained [8] and [9]. This gradient is referred as the natural gradient infomax and is given as

\[
\frac{\partial L(u(n), W_{ICA})}{\partial W_{ICA}} W_{ICA}^H W_{ICA} = \Psi_\varphi W_{ICA}
\]

(15)

where \( \Psi_\varphi = [I - \varphi(u(n))u(n)^H] \)

where \( I \) is the identity matrix. This gradient gives a faster convergence behaviour [8] and does not involve a matrix inversion.

In general the source distributions are not known. For QPSK a bimodal distribution is assumed as the modeled distributions. Assume the disturbance is Gaussian, the source distribution model can be given as

\[
p(u) = \frac{1}{2}(N(1, \sigma^2) + N(-1, \sigma^2)),
\]

(16)

where \( N(\mu, \sigma^2), \mu = \pm 1, \) is the density function for the normal distribution with mean \( \mu \) and variance \( \sigma^2 \) that can be estimated from the mixtures. The update equation for the \((n+1)\)th iteration of the unmixing matrix \( W_{ICA} \) based on the gradient (15) can be derived [10] as

\[
W_{ICA}^{(n+1)} = W_{ICA}^{(n)} + \gamma \Psi_\alpha W_{ICA}^{(n)}
\]

\[
\Psi_\alpha = [I - \alpha(u(n))u(n)^H(n)]
\]

(17)

where \( \gamma \) is the learning parameter,

\[
\alpha(u(n)) = [\alpha(u_0(n)), \alpha(u_1(n)), \ldots, \alpha(u_{K-1}(n))]^T
\]
and

\[ \alpha(u_i(n)) = \frac{u_i(n)}{\sigma_i^2} - \frac{\tanh(u_i(n)/\sigma^2)}{\sigma^2}, \quad 0 \leq i \leq K - 1. \]

Since the ICA algorithm is scale invariant, the weights can get arbitrary large. Thus, the modified leaky-ICA is proposed to force the ICA algorithm to the minimum norm weights by introducing a leaky factor on \( W_{ICA} \). The modified update equation is given by

\[ W_{ICA}^{(n+1)} = W_{ICA}^{(n)} (1 - \zeta) + \Psi_\alpha W_{ICA}^{(n)}, \quad (18) \]

where \( \zeta \) denotes a small, constant leakage factor, e.g. in the range of \( 10^{-3} \) to \( 10^{-5} \).

B.2 Optimum Wiener Solution

The performance of the blind signal separation method will be compared to the optimum Wiener solution, where the source signals are assumed to be known. The optimum unmixing filters \( W_{opt} \) are obtained by using the Wiener correlation method. Denote \( R_{zi,zk} \) as the zero-lag correlation between the sensor signals \( z_i(n) \) and \( z_k(n) \) and \( r_{zi,zk} \) as the zero-lag cross-correlation between the sensor signal \( z_i(n) \) and the transmitted signal \( s_k(n) \). The unmixing filters are obtained by solving the Wiener equations

\[ R_{zz} W_{opt} = r_{zz}, \quad (19) \]

where

\[ R_{zz} = \begin{bmatrix} R_{z0,z0} & \cdots & R_{z0,zK-1} \\ \vdots & \ddots & \vdots \\ R_{zK-1,z0} & \cdots & R_{zK-1,zK-1} \end{bmatrix} \]

\[ r_{zz} = \begin{bmatrix} r_{z0,z0} & \cdots & r_{z0,zK-1} \\ \vdots & \ddots & \vdots \\ r_{zK-1,z0} & \cdots & r_{zK-1,zK-1} \end{bmatrix} \]

and

\[ W_{opt} = R_{zz}^{-1} r_{zz}. \quad (22) \]

The unmixing matrix is given by

These unmixing filters minimize the mean squared error between the output signals and the transmitted signals [11].

IV. SIMULATIONS RESULTS

In the simulations, the beamforming weight matrix is designed using the least squares method and eight antenna elements placed on a circle with a radius \( \frac{\lambda}{2} \) where \( \lambda \) denotes the wavelength of the transmitted signals. The beams are evenly distributed with the center of the eight main lobes have the directions of 0, 45, 90, 135, 180, 225, 270 and 315 degrees. The angle of arrival (AOA) of the signal of interest is at 10 degrees. The following channel situations are simulated: a) 6 sources and 8 receiving antennas (6,8); b) 8 sources and 8 receiving antennas (8,8); and c) 10 sources and 8 receiving antennas (10,8). Simulation results are presented based on the ensemble average of 30 independent Monte-Carlo simulations. The quality measures used in this paper are the Mean Square Error (MSE) and the Bit Error Rate (BER). Since, the SOI is at beam 0, the MSE is defined as the averaged squared difference between the transmitted signal \( s_0(n) \) and the received signal \( u_0(n) \) in dB

\[ MSE = 10 \log_{10} \frac{\sum_{n=1}^{N} |s_0(n) - u_0(n)|^2}{\sum_{n=1}^{N} |s_0(n)|^2}, \]

where \( N \) denotes the number of simulated bits. The near–far effect performance is evaluated based on the Near–Far Ratio (NFR) defined as

\[ NFR = 10 \log_{10} \frac{P_1}{P_2}, \]

where \( P_1 \) is the power of the SOI and \( P_2 \) is the power of the interfering signals. All the interfering signals are assumed to have equal power.

Fig. 3 shows the performance of the modified ICA algorithm with the update gradient designed for the QPSK modulation scheme and the conventional ICA based on standard Gaussian assumption. Both approaches converges to the same result but the modified ICA converges approximately 3 times faster than the standard approach. The improvement for the MSE performance presented in Fig. 4 shows a large improvement of the leaky ICA (18) over the ordinary ICA (17), even for moderate Signal–to–Noise Ratio (SNR). Both algorithms are using the bimodal source assumption. The leaky factor is set to \( 10^{-4} \) in all the simulations where the leaky ICA algorithm is used.
Figs. 5 and 6 present the MSE and BER for the different channel situations using the leaky ICA algorithm. It can be seen that when the number of sources is less than or equal to the number of antennas the MSE and BER performance is approximately the same. The MSE degrades when the number of sources is greater than the number of sensors. For BER $\geq 10^{-3}$, the degradation is less than 1 dB compared with the case when the number of sources equals to the number of antennas. Since, most coding schemes require uncoded BER between $10^{-1}$ and $10^{-3}$, it can be seen that the BSIC is robust for all the test cases.

Fig. 7 plots the performance of the BSIC using the leaky ICA and the optimum Wiener solution for the channel situation (8,8). The performance of the BSIC matches closely to the optimum solution.

In previous simulations the Near-Far-Ratio (NFR) is assumed to be 0 dB with all the transmitted signals received with equal power. This is not a realistic assumption in cellular systems since all the signals are received with different power. Fig. 8 presents simulation results with NFR varying from 0 dB to 20 dB. The results are compared for SNR levels of 0 dB and 12 dB. The MSE results show that for NFR $\leq$ 4 dB the BSIC is very robust and over 4 dB there is a gradual degradation of the results.

V. Conclusions

A blind spatial interference canceler has been presented that combines fix beamformers and blind signal separation controlled by a modified ICA algorithm. By tuning the ICA to the chosen modulation scheme (QPSK) the convergence rate of the algorithm is three times faster than the standard ICA. A leaky ICA algorithm was proposed to force the coefficients of the unmixing matrix to their minimum norm and hence make the scheme more robust. The resulting blind signal separation scheme has shown a good performance for the near–far interference problem.

REFERENCES

Fig. 4. MSE comparison of the leaky ICA and the standard ICA for 8 sources and 8 antennas for different SNR.

Fig. 5. MSE performance of the leaky ICA algorithm for different number of sources.

Fig. 6. BER performance of the leaky ICA algorithm for different number of sources.

Fig. 7. MSE comparison of the leaky ICA algorithm and optimum Wiener solution for 8 sources and 8 antennas for different SNR.
Fig. 8. MSE performance for different NFR.