Active Control of Propeller-Induced Noise in Aircraft

Algorithms & Methods

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Fighting Noise with Noise

\textit{Noise}

\textit{Residual}

\textit{Anti-Noise}
Preface

This doctoral dissertation deals with the topic of active noise control. The work focuses on methods, controller structures and adaptive algorithms for attenuating periodic low frequency noise produced by synchronized sources, or moderately synchronized sources generating beating sound fields. Theoretical work, off-line computer experiments and practical real-time experiments are presented. The computer experiments are principally based on real-life cabin noise data measured during flight in a twin-engine propeller aircraft and in a helicopter. The practical experiments were carried out in an acoustical test section, a full-scale fuselage section from a propeller aircraft.

Some of the work presented was performed within the BRITE/EURAM project ASANCA II, Advanced Study of Active Noise Control in Aircraft Part II. Several European aircraft manufacturers as well as university and research institutes were involved in the project. The aim of the project was to use active noise control technologies to reduce cabin noise in medium sized passenger propeller aircraft. The project covered new control concepts and the development of robust and high performance control systems. The University of Karlskrona/Ronneby\textsuperscript{1} was involved in the work concerning adaptive control algorithms and controller structures. The work presented in the dissertation has been sponsored by the ASANCA II project and the KK Foundation.

The thesis consists of seven papers. Parts I-VI deal with volumetric active noise control for propeller aircraft applications, while Part VII deals with active noise control in headset applications combined with intercom systems. The seven papers comprise:

Part I. Multireference Controller for Active Control of Noise and Vibration.

Part II. A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins.

Part III. Evaluation of Multiple-Reference Active Noise Control Algorithms on Dornier 328 Aircraft Data.

Part IV. Comparison of a Multiple- and a Single-Reference MIMO Active Noise Control Approach Using Data Measured in a Dornier 328 Aircraft.

Part V. Convergence Analysis of a Multiple-Reference Complex Least-Mean-Squares Algorithm.

Part VI. Experimental Performance Evaluation of an Active Noise Control system in an Aircraft Mock-up.

Part VII. A New Passive/Active Hybrid Headset for a Helicopter Application.

\textsuperscript{1}The University of Karlskrona/Ronneby became Blekinge Institute of Technology on October 12, 2000.
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Summary

Introduction

In the last decade acoustic noise has become increasingly regarded as a significant environmental problem. Noise problems in our homes have received considerable attention. In industry, for example, engines, fans, transformers and compressors radiate noise. In cars, boats, trains and aircraft, for example, noise reduces comfort. Lightweight materials and more powerful engines are used in high-speed vehicles, resulting in a general increase in interior noise levels.

The primary concern with noise in the low frequency range is not only the potential risk of damage to the hearing. Low frequency noise is annoying and during periods of long exposure it causes fatigue, discomfort and loss of concentration. Reduced concentration may also lead to an increased risk for accidents. The masking effect which low frequency noise has on speech also reduces speech intelligibility [1],[2]. Low speech intelligibility is perceived as disturbing and irritating.

Noise problems could be solved by redesigning. This is generally very costly, however. On the other hand, noise problems may be solved using traditional passive approaches and/or approaches based on the concept of active noise control [1]-[10]. The choice of technique is determined by the characteristics of the noise as well as of the application. However, this dissertation will not address the problems of attenuating noise using passive approaches. Instead it focuses exclusively on the field of active noise control.

Conventional passive approaches consist of absorbers and/or reflectors/barriers [1]-[3]. The absorbers convert the acoustic energy to thermal energy, while the reflectors/barriers prevent the noise from entering a space from another space by reflecting the incident wave field.

In terms of practical size passive methods are suitable when reducing noise in the frequency range over approximately 500 Hz. The thickness of the acoustical absorbers, or the distance between the absorber and the wall, must be large, approximately one quarter of a wavelength, to obtain reasonable low frequency reduction, e.g. a frequency of 200 Hz results in a wavelength of 1.7 meters. In addition, in order to reduce the sound transmission from one space to another, a heavy barrier between these spaces is required. Consequently, the use of passive methods to attenuate low frequency noise is often impractical since considerable extra bulk and weight are required. In all areas of transport large weight is associated with high fuel consumption.

In order to overcome the problems of ineffective passive suppression of low frequency noise, the technique of active noise control has become of considerable interest [4]-[10]. The fundamental principle of active noise control is based on the superposition of sound waves. Secondary sources produce an “anti-noise” of equal amplitude and opposite phase to the primary or unwanted noise. Superposition of the primary and generated noise results in destructive interference and reduced noise. The noise attenuation is determined by the accuracy of the amplitude and
the phase of the generated “anti-noise.”

Active noise control systems significantly increase the capacity for attenuating low frequency noise below approximately 1 kHz, resulting in potential benefits in volume and weight. The upper frequency for which active control is appropriate is determined by the application [4]-[10]. However, in enclosures whose dimensions are of the order of a few meters the upper frequency is limited to a few hundred hertz. The upper frequency limit is higher for smaller enclosures, e.g. in headsets. For noise above 1 kHz passive methods show higher potential, since neither great volume nor weight is required to achieve significant reduction. The active and passive approaches are thus complementary, and by combining these two techniques high noise attenuation over a wide frequency range is made possible, indeed over the entire audible frequency range (20–20 kHz).

Active noise control is applicable to a wide variety of noise problems in industrial applications, transport, and consumer products [5],[7],[9].

**Historical review of Active Noise Control**

The principle of superposition and the constructive and destructive interference of two wave fields were known since the seventeenth century. In the the nineteenth century Lord Rayleigh used practical acoustical experiments to demonstrate constructive and destructive interference [2].

In the thirties Paul Lueg first described the basic idea of generating a sound field to reduce an undesired sound field, and the concept of active noise control was introduced [4]. His description resulted in the first patent in the field of active noise control; in 1933 there was a German patent and in 1936 a US patent. In the patent Lueg described active control of sound propagating in a duct. A reference microphone detected the sound radiated by a noise source. The output signal of the microphone was filtered through a control unit before driving a loudspeaker, which generated the “anti-noise.” Figure 1 illustrates the principle of active noise control presented by Lueg. A properly adjusted controller produces a canceling sound field and reduced sound level downstream from the loudspeaker. This control strategy is named feedforward active noise control [4]-[10].

In the early fifties (1953) Harry Olson and Everet May introduced and experi-

![Figure 1: The concept of feedforward active noise control in a duct presented by Paul Lueg.](image-url)
mented with feedback active noise control, see Figure 2 [4]-[11]. This approach is not based upon a reference sensor giving a priori information of the radiated sound. Instead, a microphone located close to a loudspeaker detects the sound field. The output signal from the microphone is fed back to a controller designed to drive the loudspeaker to produce an output that results in an attenuation of the sound field in the vicinity of the microphone.

In 1956 William Conover worked with reduction of periodic noise radiated from transformers [5],[7],[12]. In this work he utilized the feedforward control approach described by Paul Lueg. Instead of using a reference microphone to observe the radiated sound from a noise source, a non-acoustic reference sensor was used to generate a periodic reference signal processed by the controller before driving the loudspeakers. In order to monitor the residual sound field Conover introduced the concept of utilizing error or control sensors. The output signals from these sensors were used to manually adjust the controller as high noise attenuation was achieved. The manually-adjusted feedforward controller is shown in Figure 3. Conover’s ideas form the basis of the concept of modern adaptive narrowband feedforward active noise control. An adaptive feedforward ANC system based on a non-acoustic reference signal is shown in Figure 4.

Figure 3: Manually-adaptive feedforward active noise control proposed by William Conover.
Due to the limited technology in the sixties the practical experiments on active noise control were limited and the work within this area was focused on theoretical studies [5]. In the seventies, however, the arrival of digital techniques made it possible to produce practical active noise control systems based on digital signal processing and digital devices. Both Kido and Chaplin carried out intensive work exploring the applications of digital signal processing techniques and active noise control devices.

In the early eighties (1980, 1981) Morgan, Burgess and Widrow worked independently of one another on algorithms compensating for the physical path between the control sources and sensors. This physical path often causes instability when using the conventional Least-Mean-Squares algorithm [14]. In 1981 Widrow introduced the so-called Filtered-x Least-Mean-Squares algorithm (Filtered-x LMS) [14]. The Filtered-x LMS algorithm is a cornerstone of adaptive algorithms used in active noise and vibration control. Modern feedforward active noise and vibration control systems are usually based on this algorithm or variants thereof. An extension of the basic Filtered-x LMS was introduced in 1987 by Elliott, et al. This is known as the multiple-error Filtered-x LMS algorithm or the multi-channel Filtered-x LMS algorithm [15]. The concept was based on the use of multiple control sources and sensors to control the sound field in a whole volume.

Over the last two decades much work has been done on the active control of sound as well as active control of vibrations [5],[7],[9]. Many companies, research institutes and universities are engaged in research and development in this field. Theoretical work, computer simulations and practical experiments carried out for a wide range of practical applications have been reported [16]-[26]. During the

**Active Noise Control in Aircraft**

During the nineties there was an increased interest in the use of propeller aircraft or turboprop aircraft carrying up to approximately 50 passengers. The new interest is the result of the fact that the new turboprop engines are more fuel efficient than jet engines. One disadvantage of the propeller aircraft is, however, that the interior cabin noise level is significantly higher than in jet aircraft [28]. The aim, however, is to reduce the noise level to that prevailing in jet aircraft. The noise level in propeller aircraft cabins is approximately 90 dB. Comfort would be considerably improved if this could be significantly reduced (approximately 20 dB).

Interior noise in aircraft is most frequently derived from two major external noise sources: the fuselage boundary layer, and the aircraft propulsion system, see Figure 5. [27],[28].

![Figure 5](image-url)

**Figure 5**: The dominant cabin noise is of two different types: boundary layer noise and noise originating from the propulsion system.

Boundary layer noise (wind noise) is generated by the shaking of the fuselage wall by external turbulence pressure fluctuations. Such noise is random and broadband and has mainly high frequency characteristics for propeller aircraft applications. Because of its properties, boundary layer noise is difficult to reduce...
using active methods. Today, passive insulation methods are frequently used to attenuate such noise.

The noise generated by the propulsion system is essentially composed of engine and propeller noise. It seems that propeller-generated noise is the most dominant form of cabin noise. Propeller-induced noise and vibrations consist of several low-frequency tonal components related to the propeller Blade Passage Frequency (BPF) and some of its harmonics. A typical cabin noise spectrum is shown in Figure 6. The BPF is usually in the range of 70 to 110 Hz, and varies with aircraft type. These noise/vibrations are caused by the periodic pressure fluctuations acting on the fuselage. These fluctuations are produced by the propeller blades each time they pass the fuselage. Figure 7 illustrates pressure variation on the fuselage caused by the rotating propeller. The direct airborne noise radiation from the propellers forcing the fuselage structure to vibrate. The fuselage vibrations are transmitted through the aircraft structure, the cabin walls radiating noise into the cabin.

![Figure 6: A typical cabin noise spectrum from a propeller aircraft.](image)

Commonly turboprop aircraft are equipped with Tuned Vibration Absorbers (TVAs), a common passive method for reducing propeller-induced noise inside the cabin [27],[28]. When properly used, this approach is quite efficient, but there are some important drawbacks. Each TVA is tuned to a specific frequency; if a broader absorption bandwidth is required, a low Q-value must be used, which results in less absorption. To compensate for the reduced absorption and enable energy to be absorbed at higher harmonics, a large number of absorbers must be used, significantly adding to the weight of the aircraft. The passive TVAs are unable to track variations in the BPF. The absorbers are only tuned to work well for cruise speed. By using an active control system several of the disadvantages
of passive tuned dampers can be overcome. The advantages of using ANC are: tracking variations in the propeller rotational speed during a whole flight envelope, reducing a large number of harmonics, and weight reduction.

The eighties and nineties saw much work on active noise control in aircraft. This included theoretical analyses, simulations, laboratory experiments and flight tests [29]-[35]. Figure 8 shows an ANC system in an aircraft for active control of propeller-induced cabin noise.

The two companies Ultra Electronics, England, and Saab Aircraft, Sweden, developed the first commercially available ANC system for reduction of propeller-induced noise in aircraft cabins. The first commercial aircraft in the world in which this technique was used is the SAAB 340 and its successor, the SAAB 2000. The first SAAB 340 was delivered in the spring of 1994, and the first SAAB 2000 was delivered later the same year. The ANC system in the SAAB 340 uses 48 control microphones and 24 loudspeakers. The system in the SAAB 2000 consists of 72 control microphones and 48 loudspeakers [35].

Today, most aircraft manufacturers are interested in ANC since comfort is a key issue. During the last ten years two European Community projects in the field of active control of aircraft interior noise have been performed. These projects were known as ASANCA I and ASANCA II (Advance Study for Active Noise Control in Aircraft) [32]. The projects covered concepts and systems for active noise and vibration control systems suitable for aircraft applications. Several European aircraft manufacturers, university and research institutes were involved in the two projects.
In recent years interest has also been shown in using ASAC technology (Active Structural Acoustic Control) [6]. The concept is based on controlling the structure, thereby reducing the sound radiated to the environment. Ultra has developed ASAC systems for reducing propeller-induced cabin noise [36]. This approach has also been used in jet aircraft applications for reducing frequency components originating from out-of-balance forces from jet engines. Today ASAC systems are used in commercial aircraft [36]. The use of silent seats has also excited considerable interest together with the use of active headsets [36]. A silent seat system gives a localized noise attenuation around the passenger head using loudspeakers incorporated in the headrest. Active headsets are, moreover, much cheaper than installing an ANC system inside the cabin.

Vibrations at low frequencies also cause passenger discomfort. These vibrations originate from engine and propeller shaft imbalance, and are transmitted through the wings into the fuselage. Since the vibrations are low frequency and the vibration sources and the transmission paths are known, active approaches also have the potential of being able to reduce such vibrations [36].

**Active Noise Control Systems**

Active Noise Control (ANC) is achieved by producing a sound field using secondary sources, e.g. loudspeakers [4]-[10]. The ANC system generates a canceling sound field with equal amplitude but with opposite phase to the noise field. The sources are driven by a control system which is generally based on adaptive digital signal processing. The system needs to be adaptive in order to track changes in the noise characteristics and in time-varying environments. The sound field under control is measured using control microphones. To adjust the controller the microphone
signals are used and a relevant optimization criterion is minimized, e.g. the sum of the squared control microphone signals.

The ANC system is either based on a feedback or feedforward control strategy, or a combination of the two [4]-[10]. Feedforward control is generally used in applications where the noise source(s) are well known, and where synchronization or reference signals derived from the sources are available. These signals contain a priori information about the radiated noise. When reference signals are not available feedback techniques are often used. Feedforward control implies that only noise components closely correlated with the reference signals will be significantly attenuated. Accordingly, in applications where several uncorrelated sources produce noise a reference signal from each is needed [17]. A convenient estimate of the maximum noise attenuation in decibels achieved by a multiple-reference ANC system is given by $-10 \log_{10}(1-\gamma_{d,x}^2)$ dB, where $\gamma_{d,x}^2$ is the multiple coherence function between the primary noise $d$, and all the reference signals, $x_r, r = 1, 2, \cdots, R$ [8],[25]. For a $\gamma_{d,x}^2$ of 0.99 an attenuation of 20 dB may potentially be achieved.

In order to obtain high noise attenuation a significant correlation between the reference signals and the noise is required. If the ANC controller process time is longer than the acoustical delay between the reference sensor and the secondary source the correlation decreases, and thereby the performance is degraded [8],[10]. In fact, the causality constraint is important in broadband ANC, and if the constraint is not fulfilled only periodic noise may be efficiently reduced. It is always possible to find a correlation between periodic signals of the same frequency.

In broadband feedforward ANC the reference signals are picked up by microphones situated close to the noise sources, while in narrowband control the microphones are usually replaced by non-acoustic sensors, e.g. a tachometer, optical or inductive sensor. A major advantage of using non-acoustic synchronization signals rather than reference microphone signals is that the reference signals can be internally generated by the ANC system. With reference signals generated in this manner, adaptive control becomes extremely selective. This makes it possible to determine which harmonics are to be controlled, and which are not [8],[10].

The use of reference microphones usually introduce an undesired feedback from the secondary loudspeakers back to the reference microphones. A complicated ANC system must generally be used in order to compensate for the acoustic feedback within the controller. By using non-acoustic reference sensors the feedback is eliminated, resulting in a simpler ANC system, a purely feedforward system [8]-[10]. This system can be described by an FIR filter structure instead of an IIR structure. Adaptive IIR filter structures are considerably more complex than adaptive FIR filter structures. Adaptive IIR filters are also much more difficult to use in practical applications, since such adaptive filters are often associated with instability problems [8].

In many applications the primary noise is mainly periodic and produced, for example, by rotating machines and propellers. A single periodic source produces noise containing essentially dominant harmonic components related to the rotational speed. However, several periodic sources running at almost the same rotational speeds give rise to beating sound fields. Control conditions with a slight rota-
tional speed difference are unavoidable in multi-engine applications in the absence of synchrophaser devices which synchronize the rotational speed of the engines. However, beating sound fields may also arise in vehicles fitted with an insufficient synchrophaser [16],[17].

The capacity of the ANC controller to handle beating sound fields depends on the structure of the controller, i.e. how the reference signals are processed. Different controller structures are presented and discussed in Part I [18]. The controllers are generally based on either a single-filter structure or a parallel-filter structure using several filters, see Figure 9.

Figure 9: (a) A single-controller structure using a composite reference signal. (b) A parallel controller structure using individually processed reference signals.

For the first configuration, all the reference signals are summed to form a composite reference signal which is processed by the controller to generate driving signals to the secondary sources. However, this technique does not make the best use of the information provided by the reference signals. Since the frequencies of the reference sinusoids are close together a long FIR filter is required, resulting in
slow convergence of the adaptive algorithm [8],[18]. For the parallel filter structure each reference signal is individually processed. This enables shorter filters and thereby better convergence performance. In narrowband ANC individually processed reference signals involve, however, individual harmonic control. If possible, the parallel structure should be used rather than the single-filter structure in order to achieve efficient and robust control of beating sound fields.

The ANC system configuration depends on whether noise within an enclored space or noise radiated into open space is to be controlled as well as whether local or global noise attenuation is desired. The configuration is also determined by the spatial distribution of the noise source, i.e. whether the source noise is a localized point source or a distributed source [4],[5],[10]. In general it is harder to reduce noise radiated by spatially distributed sources than noise radiated by spatial point sources. It is also much more complicated to obtain global noise reduction within an enclosures, for example, than local attenuation within a desired region inside the enclosure, particularly when a distributed sources produce the noise.

Efficient control of a point source in an enclosure or in a free space may be achieved if the secondary sources are properly placed close to the noise source compared with the wavelength. The required source spacing decreases with frequency. By combining primary and secondary noise sources a higher-order noise source (dipole, quadrupole, etc.) is produced. The mutual coupling between the primary source and the secondary sources reduces the radiation resistance seen by the combined system, thereby reducing the total acoustic power radiated to the environment. A globally reduced noise level is thus achieved. If the distance between the primary and the secondary sources increases only a local noise reduction is obtained in the vicinity to the control microphone(s). The size of the quiet zone depends on the frequency to be controlled, decreasing with increasing frequency.

Outside noise usually enters an enclosure in a complicated manner, which results in a distributed interior noise source. Noise within an enclosed space, generated by distributed noise sources, is not controlled in the same way as point source generated noise. An enclosed sound field is relatively complicated but can sometimes be described by several acoustic modes [2],[4],[6]. A mode describes the shape of the sound field, i.e. the spatial sound pressure distribution at the corresponding mode frequency. Figure 10 shows an example of a mode shape in a rectangular space.

The shape of the sound field is determined by the sum of the strength of the individual modes. At low frequency the sound field is dominated by a few well-defined modes, while for higher a frequency the number of significantly dominating modes increases, resulting in a more complex sound field. An excitation frequency close to a resonance frequency gives a single or a few dominant modes, while the number of excited modes increases for frequencies between resonances; that is the resonance curves may overlap. Since the mode density rapidly increases with frequency, the number of excited modes will increase with frequency [2],[4]. As a result, ANC at higher frequencies is particularly difficult. Hence, ANC techniques are only suitable for noise in the low-frequency range and for lightly damped enclosures.

Controlling a sound field consisting of several modes generally requires a multi-channel ANC system, i.e. a system using several control sources and sensors [4]-
The number of sources and sensors affects the number of acoustic modes which can be controlled. A global noise reduction in enclosures is not usually obtainable when the noise field is not harmonically excited. The intention with global control is to reduce the total radiated noise energy within the enclosure \([4],[6],[10]\). However, such noise reduction can be achieved if it is possible to arrange the secondary sources in such a way that the secondary sound field produced matches the spatial distribution of the primary sound field and also tracks its temporal variations. In this case, the secondary sources do not have to be close to the primary source to give high noise attenuation, provided it is able to couple into the most strongly excited modes. The effect of the secondary sources is to reduce the radiation impedance of the enclosure thereby reducing the acoustic energy radiated to the cavity. This approach has the effect of leveling out the spatial variation in the sound field; the dominant modes are suppressed. It will reduce the sound level where it was originally high, though the sound level may also increase where the level was originally low.

The acoustic energy within an enclosure is proportional to the sum of the squared sound pressure at all points in the space \([2],[4]\). In practice it is not possible to measure the total energy. An estimate of the interior acoustic energy can be obtained by spatially sampling the sound field using a discrete number of properly located control sensors \([4]\). The sum of the squares of the output signals is often used as a cost function to be minimized by the ANC system. The locations of these sensors have vital importance to the noise reduction obtained, although the location of the secondary sources is also crucial. Global control is easier to achieve at low frequencies where only a few modes dominate the sound field. It is important that the control sensors observe all the dominant modes, and that the secondary sources can excite or control these modes. It is thus usually necessary to perform an optimization procedure to achieve suitable positions for the secondary sources and control sensors \([26]\). Reduced observability and controlability of the modes are obtained if the sources and/or sensors are located at the nodes of the sound field. The number of sensors, \(M\), are often larger than the number of sources, \(L\), \((M > L)\). If \(M \leq L\) it has been shown that the noise levels between the sensors can be significant \([4]\).
Global control is usually desired but is in most cases difficult to achieve. Accordingly, the local control strategy is often used rather than attempting a global control. Using local control a number of control sensors are needed to achieve noise reduction within a large volume. The sensors are placed where the noise attenuation is desired. Local control strategy may cause an increase in the noise levels at other positions within the enclosure without control sensors, and the total interior acoustic energy might even increase [4].

In order to control globally or locally complex enclosed sound fields a large number of secondary sources and control sensors must generally be used [4]. The complexity of the ANC system is, for instance, related to the number of secondary sources and control sensors. In addition, if frequency components generated by several noise sources are also to be controlled, system complexity may increase significantly. The permitted complexity of the ANC system is limited by the processing power of the DSP system. The noise attenuation and the system performance is thus strongly influenced by number of used control sources and sensors, as well as the controller structure and the adaptive algorithm used; hence, the total computational complexity is crucial for large multi-channel systems in real-time DSP implementations.

An adaptive ANC system involves two main parts: a filtering process that generates driving signals to the secondary sources, and an adaptive algorithm that updates the filter weights in order to track variations in the noise characteristics. From a control point of view it is of great importance to use an algorithm with a high convergence rate, good tracking performance and robustness. A high performance algorithm is often associated with increased computational complexity. The filtered-x operations of the reference signals require, however, intensive computation for large multi-channel systems. Hence, there is a tradeoff between the control algorithm and the source/sensor configuration; using a control algorithm with high computational complexity allows fewer sources and sensors to be used, and/or a limitation in the number of reference signals to be processed. However, fewer control sources and sensors and fewer controlled frequencies generally cause deterioration in the noise attenuation. Consequently, in order to achieve high noise attenuation in both stationary and time-varying situations it is of great importance to use a high performance adaptive algorithm with low computational complexity.

The most widely used ANC systems are based on an FIR-based filter structure consisting of one or several filters, which uses the ordinary time-domain Filtered-X LMS (FX LMS) algorithm to update the filter weights [4],[8],[10],[15]. This dissertation presents a multiple-reference controller structure using parallel filtering of complex signals and an adaptive weight update scheme based on a complex-valued FX LMS algorithm [37],[38]. The complex representation reduces the number of computations by replacing the time-domain linear convolutions with complex scalar multiplications. For large multi-channel system a significant computational reduction can be obtained [8],[39].

The feedforward controller presented inherently exploits the narrowband assumption by using complex filtering and complex modeling of the control paths. The controller uses a single complex weight to steer the amplitude and phase of each
Figure 11: A multiple-reference system for active control of periodic noise.
reference signal. The structure of the overall ANC system is shown in Figure 11. **Part II** introduces the novel adaptive algorithm called the *actuator-individual normalized* FX LMS [39],[40]. The algorithm is of LMS-type, but owing to the novel normalization of the algorithm it can also be regarded as a Newton-type algorithm [14],[38]. The relationship to Newton’s algorithm results in high convergence rate and significant noise reduction.

One major advantage of this complex-valued algorithm is that it combines low numerical complexity with high performance. The fundamental reasons for this are the orthogonality of the quadrature components (sine and cosine pairs) constituting the complex reference signals, and the simplicity of complex representation. In fact, the complex algorithm requires a minimum of adaptive and acoustic path parameters, cf. a straightforward time-domain approach with ordinary FIR filters.

The complex representation and the structure of the controller implies that a Newton’s algorithm can be implemented with the same numerical complexity as LMS-type algorithms. The basis of a *fast* filtered-x Newton algorithm is therefore introduced [20]. The concept of the algorithm originates from the fact that the control path pre-filtering of the reference signals is performed by a complex scalar multiplication of the reference signal and the off-line measured control paths. An improvement of performance can be achieved by using the fast algorithm. The fast filtered-x Newton algorithm is addressed in **Part III**.

In several applications there is a need to control periodic noise from two sources that are strongly or moderately synchronized to each other, for example, in twin-engine aircraft and boats. In the development of new controllers for large multi-channel systems it is of great importance to reduce the complexity of the system while maintaining efficient noise control. A controller with lower complexity will reduce the computational power required.

Using a narrowband feedforward controller selective noise control is obtained, where the attenuation is strongly dependent upon the correlation between the reference signals and the noise [4],[8]. The question to be asked in such a case is whether it is really necessary to use a synchronization signal from each source or if a single synchronization signal from one of the sources would perform equally well. By using a single synchronization signal the complexity would be significantly reduced. The question above is of special interest in the case of new aircraft, where the two propellers are kept at almost identical speeds due to efficient synchrophasing. A comparison of the performance of a single- versus a twin-reference ANC controller for an aircraft application is addressed in **Part IV** [17]. The results demonstrate that the twin-reference controller performs better than the single-reference when there is a small difference in the rotational speed of the two propellers. To ensure the highest noise attenuation during the complete flight cycle, i.e. take-off, climb, cruise, descent and landing, the controller should be synchronized to both propellers.

A theoretical convergence analysis of a twin-reference complex LMS algorithm is presented in **Part V**. The results presented show that for small and large beat frequencies, i.e. slow and fast beating sound fields, high performance is obtained. On the other hand, in the transition region the system exhibits deterioration in
performance. System behaviors observed in practical experiments verify the theoretical results.

In narrowband feedforward ANC using non-acoustic reference sensors different approaches are applicable for generating reference signals [8]. The method applied depends on whether an FIR-based or complex-valued algorithm is used, the number of pulses per revolution in the synchronization signal, if fixed or synchronous sampling is used, and if a hardware or a software approach is used.

The reference generation method generally used is based on a software method using a table lookup scheme consisting of a table containing a single-sampled sinusoidal function. The synchronization signal is used to control a table pointer. A counter controlled by interrupts generated from the synchronization signal is used to move the pointer. At each sample time (fixed or synchronous) the value of the pointer represents the sample value of the reference signal. The generation of complex reference signals is straightforward and can be achieved either by using a pair of table pointers, where one pointer is suitably shifted, or by using a sine and a cosine table, see Figure 12(a).

![Figure 12: The generation of reference signals: (a) Table lookup scheme; (b) FFT-based reference generator.](image-url)
Part IV introduces an alternative approach for generating complex reference signals for fixed sampling [17]. This method was introduced since in the practical experiment presented in Parts V and VI, the table lookup method could not be used. The complex-valued algorithm was to be implemented on external hardware that was originally constructed for an FIR-based adaptive control scheme. This hardware delivered a composite reference signal. In order to split up the composite signal in separate, complex, harmonic reference signals for the algorithm, a sliding FFT operation was used [17],[41]. The FFT operation acts as a filter bank, which can extract selectively the narrowband reference signal corresponding to the desired harmonics [42]. A schematic figure of this reference generator is shown in Figure 12(b). The design of the FFT-based filter bank becomes a straightforward FIR-filter design problem. The choice of required FFT resolution, FFT length and window depends on the application [17],[43].

In the generation process of the reference signals an undesired time delay is introduced between the synchronization signal and the reference signals, which in turn degrades the tracking performance under non-stationary conditions. Hence, due to the inherent time delay induced by the FFT operation it is recommended, where possible, to use the table lookup scheme in which the time delay is much shorter. Accordingly, the time delay of the reference signal should be as short as possible to increase the noise suppression in real-time implementations [17],[20].

Performance evaluation for the total narrowband control system consisting of the actuator-individual normalized FX LMS algorithm and the FFT based reference generator has been performed in off-line computer experiments and real-time experiments for aircraft applications [17],[20],[44],[45]. The off-line experiments were based on real measured aircraft noise data, see Parts II-IV. The real-time acoustical experiments were carried out within a fuselage section of a SAAB 340 aircraft, see Part VI [44],[45]. To excite the structure and produce the propeller noise within the test section loudspeakers mounted in a ring around the fuselage were used. By using this noise generation system the airborne noise generated by the propellers can be simulated. A schematic figure of the experimental setup is shown in Figure 13. This figure also shows a photograph of the ring of loudspeakers mounted on the fuselage.

The system performance was evaluated for both stationary and non-stationary flight conditions as well as for beating and non-beating sound fields. The ANC system exhibited good performance with respect to convergence rate, tracking and robustness. The interior noise level was considerably reduced. The mean attenuation of the fundamental blade passage frequency (BPF) was approximately 18 dB in the control microphones, and 10 dB in the monitor microphones located at the passengers’ head positions. The noise reduction measured was in broad agreement with the optimum theoretical reduction, calculated by using the recorded primary noise and measured frequency responses.

The experiment shows that a feedforward control system based on the actuator-individual normalized FX LMS algorithm has high performance in practical applications with robust control of both beating and non-beating sound fields.
Figure 13: Schematic figure showing the experimental setup.
Active noise control techniques can be used in several applications to attenuate noise at lower frequencies. Often ANC is used to increase passenger comfort in different kinds of vehicles. The ANC system has proven to be a convenient approach for reducing low frequency noise in headset applications [8],[10],[21]. In communication situations the aim is to reduce the low frequency noise in order to enhance speech intelligibility. Active headsets are commercially available, and most of these are based on analog feedback control with broadband reduction. However, in order to achieve a better noise reduction a combination of feedback and feedforward ANC is proposed. In recent years there has been an increase in interest of such active headsets using ANC combined with an intercom system. This is particularly true in the case of military applications, where the characteristics of environmental noise are both narrowband and broadband, and high speech intelligibility in the intercom system is of great importance. Other interesting applications are in industry and in transport vehicles. Part VII presents a headset based on hybrid ANC combined with a communication system applied to a helicopter application [21]. The ANC system consists of a commercial analog feedback controller combined with an adaptive narrowband feedforward controller, see Figure 14. The digital feedforward controller is based on the actuator-individual normalized FX LMS algorithm addressed in Part II. The analog feedback controller attenuates low frequency broadband noise and the digital feedforward controller attenuates harmonic low frequency noise. The extension of the ANC system with the feedforward controller significantly increases the attenuation of the periodic noise.

Figure 14: The hybrid ANC headset based on feedforward and feedback control combined with the intercom system.
Outline

Part I. — Multireference Controller for Active Control of Noise and Vibration

Part I addresses some design aspects for multi-reference controllers based upon the feedforward strategy. The use of an adaptive feedforward controller has proven to be a very successful strategy for controlling noise and vibrations. In many applications, e.g., for boats and aircraft, there is a need to attenuate noise produced by several noise sources. In such cases, a reference signal from each source is usually required. The discussion in this paper is primarily focused on the question of selecting a controller structure for adaptive multiple-reference controllers.

There are different ways of implementing such controllers. Here, three different structures are presented and discussed. Advantages and disadvantages of the different structures are treated. One structure is based upon adding together the reference signals to form a single composite reference signal, which is fed to the controller. Another utilizes individual control, where each reference signal is filtered using a separate controller. A modulating controller is also evaluated. The main goal is for the controller to be able to handle the acoustic beating that occurs with rotating machinery running at slightly offset rotational speeds.

An individual controller for each reference signal results in a robust controller with rapid convergence. It is concluded that a single control filter will give the same performance as individual control, provided the reference signals are well separated in frequency.

Part II. — A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins

In many applications there are problems with periodic noise with low frequency characteristics. Such is the case with cabin noise in propeller aircraft, where the interior noise essentially consists of dominant tonal components at harmonics of the Blade Passage Frequency (BPF) of the propellers. An efficient method of achieving significant reduction in such periodic low frequency noise is the use of the active noise control system based upon adaptive narrowband feedforward techniques.

The configuration of the controller is based on parallel processing of single-frequency reference signals, where the amplitudes and phases of the signals are simultaneously adjusted before driving the secondary sources. This filter structure is recommended when reducing periodic noise produced by rotating machines with almost the same rotational speeds, see Part I.

This paper also introduces a novel algorithm called the actuator-individual normalized Filtered-X Least-Mean-Squares (FX LMS) algorithm, which is related to Newton’s algorithm. This algorithm is based on the approximation that the Hessian matrix of the cost function is diagonally dominant. The algorithm is derived
for a case minimizing the sum of the squared control sensor signals, and for a general case with $R$ reference signals, $H$ harmonics for each reference, $L$ secondary sources, e.g. loudspeakers, and $M$ control sensors, e.g. microphones. The proposed complex algorithm is advantageous in large multi-channel narrowband applications due to the high convergence rate and low numerical complexity.

The behavior of the actuator-individual normalized algorithm is compared with an ordinary LMS normalized variant. The ordinary LMS normalization is based on a single normalization factor for the whole system, while the actuator-individual normalized algorithm is normalized with respect to each reference signal and actuator or control source.

The results show that the actuator-individual normalized FX LMS algorithm exhibits a higher convergence rate and better tracking performance than the conventional normalized algorithm. The actuator-individual normalized algorithm works well in significantly reducing the controlled frequencies. The noise reduction obtained indicates that the algorithm shows a good potential for achieving close agreement with the optimum noise reduction, i.e. least squares solution. The comparison is performed off-line using real noise signals recorded within the passenger cabin of a twin-engine propeller aircraft during flight as well as real measured cabin frequency response functions between the loudspeakers and control microphones.

Part III. — Evaluation of Multiple Reference Active Noise Control Algorithms on Dornier 328 Aircraft Data

This paper presents a set of multiple reference adaptive algorithms combining low numerical complexity with high performance in narrowband ANC applications. Two LMS-type and a Newton-type algorithm are considered, and their performance has been evaluated using acoustical data from a Dornier 328 aircraft.

The algorithms presented are based upon the narrowband assumption by using complex-valued filtering, complex modeling of acoustic paths, and weighted complex gradient update of the adaptive weights, i.e. the filter structure introduced in Part II.

The special structure of the corresponding adaptive filtering problem suggests that the Newton-type algorithm can be implemented with the same numerical complexity as LMS-type algorithms. Hence, the concept of a fast filtered-x Newton algorithm is introduced.

Part IV. — Comparison of a Multiple- and a Single-Reference MIMO Active Noise Control Approach Using Data Measured in a Dornier 328 Aircraft

This part addresses the problem of controlling noise from two sources that are strongly or moderately synchronized. A typical application is the control of propeller-
generated noise within a twin propeller aircraft.

The controller used in the experiment is a narrowband feedforward controller. Since the narrowband feedforward controller is based on synchronization signals from the noise sources, such a controller is selective with respect to frequency components. In the development of new controllers for interior aircraft noise, it is of great importance to reduce the complexity of the control algorithm while maintaining efficient noise control. The question to be asked in such a case is whether it is really necessary to use a twin-reference controller, or if a single-reference controller may perform equally well. This question is of special interest in the case of new aircraft, where the two propellers are kept at an almost identical speed due to efficient synchrophasing.

To establish whether a multiple-reference controller really is necessary, or if a single-reference controller is sufficient, the performance of a single- versus twin-reference control algorithm is evaluated in a comparative study. The performance of each controller is evaluated using computer simulations for two different flight conditions: one flight condition with the propellers synchronized, and one with unsynchronized propellers. The study is performed as a computer simulation (off-line evaluation) using real-life data recorded in a Dornier 328 under different flight conditions. The results demonstrate that the twin-reference controller performs better than the single-reference whenever there is a slight deviation in the rotational speed of the two propellers.

Modern propeller aircraft are usually fitted with a synchrophaser unit, with the result that the rotational speed of the two propellers is similar, or almost similar, at all times. With a modern, efficient synchrophaser, it is possible that a single-reference controller would suffice. However, disturbances in air flow may cause transient speed slips that will cause acoustic beating inside the cabin which a single-reference controller cannot attenuate. To be able to reduce efficiently the propeller-induced cabin noise, the controller should be synchronized with both propellers. This will ensure the highest possible noise attenuation during the complete flight cycle, i.e. take-off, climb, cruise, descent and landing.

This part also treats the generation of reference signals, and problems related to the generation process of reference signals. The approach presented is based upon a fixed sampling rate and uses a sliding FFT filtering technique.

If there are periodic tachometer signals available with, for example, one pulse per propeller revolution, it is possible to obtain the desired complex reference signals using a fixed sampling rate and an FFT-filter bank. However, with only one pulse per propeller revolution, the required FFT length will be considerable and the filter bank will therefore introduce a significant time delay in the tachometer signals. This will not cause any problems in stationary flight conditions. However, when conditions are non-stationary, and the propeller rpm varies, the correlation between the reference signals and the noise harmonics may deteriorate due to the filter-bank-induced time delay. This may, in turn, result in reduced noise attenuation.
In order to perform adequately in non-stationary conditions of flight, it is important to have a reference signal generator with as short a time delay as possible. A reference signal generator with virtually no inherent time delay may be obtained by using look-up table techniques, and several tachometer pulses per propeller revolution.

Part V. — Convergence Analysis of a Multiple-Reference Complex Least-Mean-Squares Algorithm

Part V presents a theoretical analysis of the convergence performance of a complex Least-Mean-Squares (LMS) algorithm using multiple-reference signals. The single-frequency reference signals are filtered individually using an adaptive filter before driving a secondary source. An explicit expression of eigenvalues controlling the convergence behavior has been derived. The convergence in the filter weights and the decrease rate of the squared error (the learning curve) for noise control applications are also discussed. To validate the theoretical results obtained in the analysis, results from computer simulations are presented. Results from a computer experiment using real data recorded in an aircraft in flight are also given.

It is shown that the convergence and tracking performance depend both on the step-size parameter in the adaptive algorithm, and on the frequency separation between the reference signals, i.e. the beating frequency between tonal components. In cases of small frequency separations, convergence problems may occur independently of the value of the step-size. However, in such cases the steady-state noise reduction was improved by increasing the step-size parameter. On the other hand, for large frequency separations a small step-size is preferable in order to achieve high steady-state noise reduction. In practical noise control applications it is important to adjust the step-size parameter so that a desired behavior with respect to tracking performance and/or steady-state noise reduction is obtained for a particular control situation.

Part VI — Experimental Performance Evaluation of an Active Noise Control System in an Aircraft Mock-up

In the design of an adaptive control system the investigation or evaluation of its behavior is one of the important parts in development work. In some applications it is possible to install the system in a vehicle and carry out the evaluation under actual running conditions. In some cases it is difficult and expensive to do the evaluation under such conditions, e.g. in an aircraft application where in-flight tests are very expensive. A cheaper alternative and a compromise in the evaluation process is to perform experimental study using a full-scale model of the vehicle (mock-up), and within this model simulate the acoustical environment and conceivable running conditions.

The object of this paper as shown in Part VI is to present results from the performance evaluation of a feedforward-based ANC system using a real-time implementation of the actuator-individual normalized filtered-\(x\) LMS algorithm presented
in Part II. Convergence behavior and robustness were evaluated under varying conditions.

The experiment was performed in a fuselage section of a SAAB 340 aircraft. To excite the structure and produce the propeller noise within the test section loudspeakers mounted in a ring around the fuselage were used. The experimental setup and the noise generation system are described. The results of a series of experiments on the active control of propeller–induced noise are presented. Among the “flight” conditions evaluated were: stationary conditions where the “propellers” were completely synchronized, and non-stationary conditions with a frequency beat between “left and right propellers.”

Substantial attenuation was obtained together with robust behavior for both beating and non-beating primary sound fields. The mean attenuation of the BPF (82 Hz) with fully-synchronized propellers was approximately 18 dB measured at the control microphones. In cases with unsynchronized propellers the noise attenuation was reduced by typically 3–6 dB. The behavior observed in this practical experiment corresponded to that obtained earlier from computer simulations and theoretical analysis, see Parts IV and V. The steady-state mean noise attenuation measured in the experiment also corresponded well with the calculated optimum attenuation.

**Part VII. — A New Passive/Active Hybrid Headset for a Helicopter Application**

Part VII addresses active noise control in a headset application. The headset proposed is based on the ANC system used in combination with a communication system to enhance speech intelligibility and improved audibility within a noisy environment. Thus, this part is not focused on volumetric noise control using an MIMO ANC controller. Here a Single-Input, Single-Output (SISO) controller based upon a single loudspeaker/control microphone setup for each earcup is employed.

The headset combines several techniques. The passive earcups attenuate both broadband and narrowband noise at higher frequencies. The level of the remaining low-frequency noise inside the earcups is not normally harmful to the human ear, but such noise often masks and corrupts communication. For this reason one tends to turn up the volume of the intercom system to maximum, which leads to sound levels which are potentially damaging to the human ear. In order to reduce the volume of the intercom system and increase speech intelligibility, the level of the low frequency background noise inside the earcups must be reduced. To do this an active system was used. This system consists of a feedback and a feedforward controller. The analog feedback controller attenuates low frequency broadband noise, while the digital feedforward controller reduces harmonic low frequency noise. The control algorithm used is the complex filtered-x LMS algorithm presented in Part II. Furthermore, in order to increase speech intelligibility, spectral subtraction is also applied on the output signal from the communication microphone, resulting in a higher signal-to-noise ratio (SNR) in the intercom signal. The unprocessed microphone signal contains considerable helicopter noise, however, which is injected
into the communication system. During communication the noise picked up by the microphone from one pilot is fed directly to the other pilot’s headset.

This part presents results from an evaluation of the hybrid headset using data recorded in an AS332 Super Puma MKII helicopter. The data was recorded during flight. The evaluation shows that a combination of the different methods results in efficient noise attenuation. The attenuation is approximately 20 dB broadband in the frequency range 50–400 Hz, and further 20 dB attenuation is obtained of the controlled tonal components. At the same time, spectral subtraction improves the SNR in the outgoing speech signal by approximately 20 dB.
Publications

This part comprises a list of my publications.


Per Sjösten, Mathias Winberg, Sven Johansson, Thomas Lagö, Sven Nordebo, Ingvar Claesson, “Test Procedures and Requirements for Performance and Validation of Robustness of Self Tuning Regulator Algorithm and Algorithm Using Two Tacho Signals,” ASANCA-II Deliverable D84, November 1996.


Bibliography


Part I

Multireference Controllers for
Active Control of Noise and
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Multireference Controllers for Active Control of Noise and Vibration

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Abstract

The use of adaptive feedforward controllers has proven to be a very successful strategy for controlling noise and vibration in a variety of applications. One reason is that the feedforward controller is an open loop controller, which can be designed to cancel the undesired noise in one position with any accuracy. However, the feedforward controller requires an input signal, called a reference signal, correlated to the noise source. As a consequence, a single reference controller can only reduce noise radiated from a single noise source.

In many applications, there is a need to attenuate noise produced by several noise sources. This paper addresses some design aspects for multireference controllers based on the feedforward strategy.

1 Introduction

In this paper, we address the problem of using more than one reference signal to a feed forward adaptive controller. In applications concerning the control of sound or vibration, multiple reference controllers have gained an increased interest, especially in applications such as boats or aircraft. The discussion here is primarily focused on the question of selecting a controller structure for adaptive multiple reference controllers. Since there are different ways of implementing such controllers, it is of certain interest to explore the features of each of these implementations.

In noise and vibration control, a reference signal is an identification of a noise or vibration source. It is a signal that is either measured at or near the source, or derived from e.g. a synchronization signal. Either way, the reference signal must be correlated with the disturbance that is to be controlled. To put it the other way around, a feed forward controller can only control disturbances which are correlated to the reference signal. Different reference signals may be obtained from different sources or from different parts of the same source but with separated frequency contents. Different harmonics produced by a periodic source may, for instance, be considered as different reference signals.
2 Controller Structures

Three different structures for implementing a multiple reference controller will be discussed below. We direct our attention towards some advantages and disadvantages with the different structures and look at the convergence characteristics for two different cases: case 1 where the reference signals are pure tones with frequencies 100 Hz and 104 Hz, and case 2 where the frequencies are 100 Hz and 200 Hz. The sampling frequency is 1000 Hz and the controller is turned on after 200 samples.

It is important to note that the attenuations shown are obtained from simulations in MATLAB (except for figure 10) and do not in any way reflect the kind of attenuation that would be obtained in a real application. The purpose of these curves is to enable comparison between the three structures presented here.

2.1 Structure No 1: A single controller

One imaginative multiple reference controller is shown in figure 1. The two reference signals are just added together to form a single reference signal, which is fed to the controller. The benefit of this configuration is obvious: only one controller is necessary, which of course has a positive effect on the implementation cost of the controller. If $d(n)$ is a disturbance signal in figure 1, we obtain the error, $e(n)$, as

$$e(n) = d(n) + y(n) = d(n) + X^T_1(n)W(n) + X^T_2(n)W(n)$$  \hspace{1cm} (1)

where $X_1$ and $X_2$ are vectors containing finite histories of the two reference signals. The filter weight vector $W$ has a time index $n$, indicating that this vector is also altered on a per sample basis. The updating LMS algorithm for this structure would be

$$W(n+1) = W(n) - 2\mu e(n)[X_1(n) + X_2(n)]$$ \hspace{1cm} (2)

or just

$$W(n+1) = W(n) - 2\mu e(n)X(n)$$ \hspace{1cm} (3)

where $X = X_1 + X_2$.

Figure 1: Structure No 1. A single control filter is used to control the sum of all reference signals.
Problems with this configuration occur when pure tone reference signals are used with frequencies that are very close. The combined reference signal will contain a beating with a frequency equal to the difference between the two reference signal frequencies. Indeed, the acoustic sound field will contain the same beating, but since the controller cannot properly control the phase of the beating (which is given at the summation point), it is not possible to control the sound field properly.

Figure 2 shows the control error, e.g. as measured by a control microphone in an aircraft cabin. Although the controller seems to be able to control the two frequencies fairly well, the beating is still very strong and decays slowly. The decay rate can be increased by increasing the length of the control filter enough to contain information about the beating. In practice this means that the control filter length should cover a time slot approximately equal to 1/8 of a beat period. For a system with 1000 Hz sampling rate and a 1 Hz beat frequency, this leads to a filter length of 125 taps.

With this discussion in mind, it is clear that the controller configuration in figure 1 is not suitable for controlling sound from two sources with similar frequencies. If, however, the frequency components are well separated, such as e.g. the harmonics of a rotating machine, this controller structure may be used. Figure 3 shows the control error when reference No 1 is the fundamental frequency and reference No 2 is the first harmonic. With a control filter length of 8 taps, the convergence is quite acceptable.

Figure 2: Learning curve for structure No 1. Frequency difference: 4 Hz. Filter length: 8.
2.2 Structure No 2: A modulating controller

The controller structure above could not properly handle the beat frequency caused by the two reference signals of similar frequency. One solution to this problem would be to introduce a separate controller for the beat frequency. Such a controller structure is shown in figure 4. This structure is designed for twin-reference applications, where it can be expected that the frequency contents in the two reference signals are similar. A typical application would be a twin-engine propeller aircraft with a synchro-phaser to keep the two engines at the same, or almost the same rotational speed.

Assume that the primary goal is to control two reference signals, $x_1(n)$ and $x_2(n)$, which are single frequency references with frequencies $\nu_1$ and $\nu_2$ respectively. The reference signal $x_m(n)$ in figure 4 will then contain the mean frequency for $x_1(n)$ and $x_2(n)$ while $x_d(n)$ contains half the difference in frequency, i.e.

$$\nu_m = \frac{\nu_1 + \nu_2}{2}$$
$$\nu_d = \frac{|\nu_1 - \nu_2|}{2}$$

The frequency $\nu_d$ is equal to half the beat frequency, which is simply the difference $\nu_{beat} = |\nu_1 - \nu_2|$. By multiplying the output signals $y_m(n)$ and $y_d(n)$, the resulting output signal $y(n)$ will contain the frequencies $\nu_m + \nu_d$ and $\nu_m - \nu_d$ which evaluates to $\nu_1$ and $\nu_2$ respectively. Thus the lower controller in figure 4 is a separate controller for the beat frequency and the controller output is formed by modulating the mean frequency with (half) the beat frequency.
The multiplication of the two outputs results in a slightly different form for the adaptive algorithm. The control error is formed as

\[ e(n) = d(n) + y_m(n)y_d(n) = d(n) + X_m^T(n)W_mX_d^T(n)W_d \] (5)

where \( W_m \) and \( W_d \) are the adaptive filter weight vectors for the two reference inputs. The LMS algorithm is derived using a gradient estimate of the performance surface, see e.g. [1], according to

\[ \tilde{\nabla}(n) = 2e(n)\frac{\partial e(n)}{\partial W}. \] (6)

Using this expression on equation (5) results in the following gradient estimates for the two controllers,

\[ \tilde{\nabla}_m(n) = 2e(n)X_m(n)W_d^T X_d(n) = 2e(n)X_m(n)y_d(n) \] (7)

and

\[ \tilde{\nabla}_d(n) = 2e(n)X_d(n)W_m^T X_m(n) = 2e(n)X_d(n)y_m(n). \] (8)

The LMS update algorithm for this controller follows as

\[ W_m(n+1) = W_m(n) - \mu_m \tilde{\nabla}_m(n) = W_m(n) - 2\mu_m e(n)X_m(n)y_d(n) \] (9)

and

\[ W_d(n+1) = W_d(n) - \mu_d \tilde{\nabla}_d(n) = W_d(n) - 2\mu_d e(n)X_d(n)y_m(n). \] (10)

One may observe that in the update scheme for one controller, the output from the other is included as a factor. This is due to the multiplication of the output signals in equation (5). For this reason, the filter weights must not be initialized to zero since the controller will then be locked in this state.
The resulting error when this controller is used on the closely spaced reference signals is shown in figure 5. As expected, the error drops fairly rapidly although traces of the beating can still be shown. Since the two control filters are updated using different convergence factors, $\mu_m$ and $\mu_d$ respectively, and the fact that the output from one filter affects the update of the other, the behavior of this controller is rather difficult to predict. Some form of normalization should be applied.

This structure works well even when the two reference signals are widely separated, as shown in figure 6. With properly selected convergence factors, the error drops with approximately the same rate as for structure No 1 above.

### 2.3 Structure No 3: Individual control

A schematic view of the third structure to be presented here is shown in figure 7. With this structure, each reference signal is filtered with a separate controller.

The control error is calculated as

$$e(n) = d(n) + y(n) = d(n) + \mathbf{X}_1^T(n)\mathbf{W}_1 + \mathbf{X}_2^T(n)\mathbf{W}_2$$

which is similar to equation (1), except that now each input reference has an individual weight vector. The filter weights can be updated using e.g. a standard (single reference) LMS algorithm. Although the filters are updated using individual algorithms, the weight update schemes are not completely isolated, since they use the same control error, $e$. Still, when controlling signals with only a small frequency separation, the individual controller tends to have better convergence performance with fewer filter weights than the other two structures discussed above. Another advantage compared to the modulating controller is that this structure can have
any number of reference inputs, while the modulating controller is confined to two references.

One clear disadvantage with this configuration is that the implementation “cost” is effectively doubled. This is particularly important in multi-channel filtered-x implementations, where the filtered-x calculations are very time-consuming.

The error for the case with 4 Hz spacing is shown in figure 8. This curve was obtained with only two weights in the control filters and with a convergence factor set to approximately $1/10th$ of its maximum value. By choosing a larger value for the convergence factor, the beating will disappear more rapidly at the cost of a higher residual error. Figure 9 shows the resulting error for the fundamental and the first harmonic. The convergence properties are well in comparison with the other two control structures.

$$W(n) \leftarrow W(n) - \eta \cdot e(n) X(n)$$

$$y(n) = W_1(n) X_1(n) + W_2(n) X_2(n) + d(n)$$

Figure 7: An individual control filter sets the amplitude and phase for each reference signal. The controller output is the sum of the control filter outputs.

Figure 6: Learning curve for structure No 2: Frequency difference: 100 Hz. Filter length: 4.
Figure 8: Learning curve for structure No 3. Frequency difference: 4 Hz. Filter length: 2.

Figure 9: Learning curve for structure No 3. Frequency difference: 100 Hz. Filter length: 2.
3 Results From Practical Tests

A multiple reference controller with individual controllers for the references (structure No 3) has been used to control the noise in a boat cabin [2]. The noise was produced by two engines with individual throttle controls, so the rpms of the two engines could be anything from equal (highly unlikely) to very different. The controller was configured with 4 outputs, 8 inputs and 2 references, which was implemented on a single TMS320C31 floating point DSP.

![Sound pressure level in the boat cabin measured at one evaluation position.](image)

Figure 10: Sound pressure level in the boat cabin measured at one evaluation position. Solid curve: Control off. Dotted Curve: Control on.

The curves in figure 10 show the sound pressure level measured at one of six evaluation positions in the boat cabin. The linear SPL at the fundamental frequency (37 Hz) was 112 dB without control and approximately 20 dB lower with control. The attenuation extended throughout the whole cabin and the convergence of the controller was experienced as “instant”. The beat frequency between the two engine fundamentals was about 1 Hz and made the experience in the cabin quite unpleasant. With the controller on, the beating was completely removed.

4 Summary

There are several ways of implementing a multiple reference controller. In this paper we have studied three different structures. Which way to go is determined by several factors, such as the number of reference sources and their frequency content. Another important factor that has not been covered in this paper is how the control filters are implemented. The filters used in the simulations for this
paper were FIR filters. Other possibilities would be e.g. complex time domain filters or FFT-based frequency domain filtering.

It is our experience from practical applications that using an individual controller for each reference results in a robust controller with rapid convergence. From the experiments above it seems that a single control filter will give the same control as individual filters, if the reference frequencies are well separated. However, in practical applications, it may be necessary to use different convergence parameters for different harmonics due to the properties of the acoustic field, in which case the individual controller is to prefer.

References


Part II

A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins
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A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins

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Abstract

The cabin noise inside propeller aircraft is essentially dominated by strong tonal components at harmonics of the blade passage frequency of the propellers. In order to achieve an efficient reduction of such a periodic low frequency noise, it is advisable to use an active noise control system based on adaptive narrowband feedforward techniques. The feedforward controller presented in this paper exploits narrowband assumptions by using complex-valued filtering and complex modeling of control paths.

This paper introduces a multiple-reference controller based on the novel actuator-individual normalized Filtered-X Least-Mean-Squares (FX LMS) algorithm. This algorithm combines low computational complexity with high performance. The algorithm is of the LMS-type. However, owing to the novel normalization of the algorithm it can also be regarded as a Newton-type algorithm. A comparison between the actuator-individual normalized FX LMS algorithm and the ordinary normalized FX LMS algorithm is presented. The results demonstrate better performance in terms of convergence rate and tracking properties when the Newton-like actuator-individual normalized FX LMS algorithm is used as compared with the conventional normalized LMS algorithm. The evaluation was performed using noise signals recorded inside the cabin of a twin-engine propeller aircraft during flight. The paper also discusses variants of the actuator-individual normalized FX LMS algorithm.

1 Introduction

The Active Noise Control (ANC) technique is based on destructive interference between sound fields [1]-[3]. The ANC system generates a canceling sound field with equal amplitude but with an opposite phase to the noise field. The controlled noise is measured using control or error microphones. The output signals from these
microphones are then used by the controller to adjust the loudspeaker signals so that the noise level is minimized. In volumetric acoustic noise control inside large enclosures such as inside aircraft it is generally necessary to use a multiple-channel ANC system consisting of several loudspeakers and control microphones in order to control the relatively complicated interior sound field [1]–[5].

In many applications of noise control the greatest annoyance is caused by periodic low frequency noise. Successful reduction of such noise can often be achieved by using an active noise control system with narrowband feedforward control [1]–[3]. This control strategy uses reference signals from the noise sources which are correlated to the noise. The reference signals are then processed by the controller to produce driving signals to the loudspeakers. Where several noise sources contribute to the sound field a multiple-reference feedforward control system is usually required [6], [7].

The cabin noise inside propeller aircraft is essentially dominated by strong tonal components at the harmonics of the blade passage frequency of the propellers [8]. Propellers or periodic noise sources running with a slight rotational speed difference induce an acoustic beating. The capacity for the ANC system to handle beating sound fields is dependent on the structure of the controller [2], [9]. A structure based on a single filter and a single reference signal consisting of the sum of all reference signals does not make the best use of the information provided by the reference signals. Since the frequencies of the reference sinusoids are close together a long FIR filter is required, resulting in slow convergence of the adaptive algorithm [2], [9]. With the parallel filter structure each reference signal is individually processed, which in narrowband ANC involves individual harmonic control. The shorter filter can be used with better convergence [9]. If possible, the parallel structure is used rather than a single filter structure in order to achieve efficient and robust control of beating sound fields. The parallel structure has proven advantageous in the attenuation of propeller-generated noise and noise produced by rotating machines with almost the same rotational speeds [2], [10].

The most widely used algorithm in ANC applications is the well-known Filtered-X LMS algorithm owing to its simplicity and robustness [11]–[13]. In order to improve the controller performance with regard to convergence rate, noise attenuation and tracking performance, it could be possible to use faster and more efficient algorithms [2], [14], [15]. However, an increased performance often leads to more complex algorithms which demand greater computational capacity in the control system. In addition, since the computational power of the DSP hardware is limited, increased algorithm complexity allows fewer loudspeakers and error microphones to be used. Accordingly, in applications where a large multiple-channel system is needed, it is of great importance to keep down the computational complexity of the algorithms so that a large number of loudspeakers and microphones can be used.

The feedforward technique presented in this paper inherently exploits the narrowband assumption by using complex filtering and complex modeling of control paths [16], [18]. The proposed complex algorithms are advantageous in narrowband applications due to high convergence rate and low numerical complexity. These advantages are primarily the result of the orthogonality of the quadrature com-
ponents constituting the complex reference signals and the simplicity of complex representation. In fact, the complex algorithm requires a minimum of adaptive and control path parameters as compared to a straightforward time-domain approach using ordinary FIR filters [11],[16],[18].

The ANC system and the multiple-reference algorithm are presented in Section 2. In Section 3 the different normalization factors are described. The issue of stability is discussed in Section 4. A comparison of the computational complexity of the complex algorithm and the ordinary FIR-based FX LMS algorithm is presented in Section 5. Variants of the actuator-individual normalized FX LMS algorithm are presented in Section 6. Section 7 describes the evaluation and presents the results. The Appendix addresses the differentiation of a real-valued cost function with respect to a complex-valued parameter.

2 The Active Noise Control System

Consider the twin-reference feedforward active noise control system shown in Fig. 1. The controller presented is based on parallel filtering of the periodic reference signals. The system inherently exploits the narrowband assumption by using complex filtering. For each reference signal and loudspeaker a complex adaptive weight is used to control the amplitude and phase of the reference signal. The real parts of the filtered reference signals are then used to drive the loudspeakers. A general model of the narrowband feedforward ANC system in Fig. 1 is shown in Fig. 2.

The algorithm used to update the weights is based on the complex FX LMS algorithm. The advantage of such an algorithm is its low numerical complexity. The algorithm is described for a general case with $R$ reference signals and $H$ harmonics for each reference. Assume that $M$ control microphones and $L$ loudspeakers are used and that $M \geq L$. The loudspeaker signals $y_l(n), l = 1, 2, \cdots, L$ in Fig. 1 are generated according to

$$y(n) = \sum_{r=1}^{R} \sum_{h=1}^{H} \Re \{x_{rh}(n)w_{rh}(n)\}.$$  \hfill (1)

Here $y(n)$ is an $L \times 1$ vector of real-valued loudspeaker signals $y_l(n)$, $x_{rh}(n)$ is a complex scalar reference signal, $w_{rh}(n)$ is an $L \times 1$ vector of complex loudspeaker weights $w_{rh}(n)$, and $\Re \{\cdot\}$ denotes the real part. The real valued $M \times 1$ vector $e(n)$ of microphone signals $e_m(n), m = 1, 2, \cdots, M$, is modeled by

$$e(n) = d(n) + \sum_{r=1}^{R} \sum_{h=1}^{H} \Re \{F_{rh}x_{rh}(n)w_{rh}(n)\}$$  \hfill (2)

where the real $M \times 1$ vector $d(n)$ contains the primary noise $d_m(n)$ at the microphones $m$. The matrix $F_{rh}$ has the dimension $M \times L$ and contains the complex control paths, each associated with the $r$th reference and the $h$th harmonic. The control path $F_{rhl}$ includes the characteristics of the acoustic path between loudspeaker $l$ and microphone $m$ as well as such electronic equipment as amplifiers, D/A- and A/D-converters, anti-aliasing and reconstruction filters and the
Figure 1: Twin-reference, multiple-channel system for narrowband ANC of propeller-generated cabin noise.
A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins

...Primary Noise Paths
...Adaptive Algorithm

Figure 2: A model of the active noise control system in Fig. 1.

The cost function to be minimized by the adaptive algorithm is given by

\[ J = E \left\{ \sum_{m=1}^{M} e_m^2(n) \right\} = E \left\{ e^T(n)e(n) \right\} \]  

(3)

where \( E \{ \cdot \} \) denotes the expectation operation. The quadratic cost function can be minimized by using a gradient algorithm. As is customary in the derivation of stochastic gradient algorithms, we regard the instantaneous cost function \( J(n) \) as an estimate of the cost function (3), cf. [14], [15],

\[ J(n) = \sum_{m=1}^{M} e_m^2(n) = e^T(n)e(n). \]  

(4)

The complex derivative of the real-valued cost function \( J(n) \) with respect to the complex loudspeaker weights \( \mathbf{w}_{rh}^*(n) \) is given by

\[ \frac{\partial J(n)}{\partial \mathbf{w}_{rh}^*(n)} = x_{rh}^*(n)\mathbf{F}_{rh}^H e(n). \]  

(5)

where \((\cdot)^*\) and \((\cdot)^H\) denote the complex conjugation and the conjugate-transpose operation respectively, see Appendix [14]. Define the \( LRH \times LRH \) reference signal matrix \( \mathbf{X}(n) \) by

\[ \mathbf{X}(n) = \begin{pmatrix} x_{11}(n)\mathbf{I} & x_{12}(n)\mathbf{I} & \cdots & x_{1R}(n)\mathbf{I} \\ x_{21}(n)\mathbf{I} & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \cdots \\ x_{R1}(n)\mathbf{I} & \cdots & \cdots & x_{RR}(n)\mathbf{I} \end{pmatrix} \]  

(6)
where $I$ is the $L \times L$ identity matrix. Let the $LRH \times 1$ vector $w(n)$ and the $M \times LRH$ matrix $F$ be given by

$$w(n) = \begin{pmatrix} w_{11}(n) \\ w_{12}(n) \\ \vdots \\ w_{RH}(n) \end{pmatrix}, \quad F = \begin{pmatrix} F_{11} & F_{12} & \cdots & F_{RH} \end{pmatrix}. \quad (7)$$

The vector $e(n)$ is thereby given by

$$e(n) = d(n) + \Re \{FX(n)w(n)\} \quad (8)$$

and

$$\frac{\partial J(n)}{\partial w^*(n)} = X^H(n)F^He(n). \quad (9)$$

The adaptive weight vector $w(n)$ is updated in the direction of the negative gradient vector for the cost function

$$w(n+1) = w(n) - M\nabla J(n) \quad (10)$$

where $M$ is a matrix convergence factor of dimension $LRH \times LRH$. This matrix contains the step-size parameters. The relationship between the instantaneous gradient vector and the complex derivatives of the cost function is given by [14]

$$\nabla J(n) = 2\frac{\partial J(n)}{\partial w^*(n)}. \quad (11)$$

A compact matrix form for the weight updating scheme can now be written as

$$w(n+1) = w(n) - 2MX^H(n)F^He(n). \quad (12)$$

In (12) and throughout the text it is understood that the matrix $F$ should be interchanged for the corresponding matrix of control path estimates $\hat{F}$ whenever applicable. However, in order to simplify notation we will keep the plain notation $F$.

### 3 The Normalization Factors

The choice of matrix convergence factor $M$ is very important and affects the performance of the algorithm in terms of convergence rate and tracking ability. Either a steepest descent algorithm (LMS algorithm) or a Newton’s algorithm can be obtained [14],[15]. The matrix convergence factor for both algorithms is dependent upon the Hessian matrix $R$ of the cost function

$$J = E \{ \bar{e}^H(n)\bar{e}(n) \} = w^H(n)Rw(n) + w^H(n)p + p^Hw(n) + c. \quad (13)$$
Here \( \tilde{e}(n) \) is a complex vector of microphone signals given by

\[
\tilde{e}(n) = d(n) + FX(n)w(n)
\]

(14)

and

\[
R = E\left\{X^H(n)FFX(n)\right\}
\]

(15)

\[
p = E\left\{X^H(n)F^Hd(n)\right\}
\]

(16)

\[
c = E\left\{d^H(n)d(n)\right\}.
\]

(17)

Note that the Hessian matrix \( R \) denotes a spatial cross-correlation matrix of reference signals transmitted through different control paths.

The novel actuator-individual normalized FX LMS algorithm is compared with an ordinary normalized FX LMS algorithm. The ordinary normalized algorithm utilizes a single normalization factor for the whole system, while the actuator-individual normalized algorithm is normalized with respect to each loudspeaker (actuator) and reference signal.

### 3.1 LMS Normalization

If the ordinary normalized LMS algorithm is to be used the matrix convergence factor \( M \) may be chosen as a diagonal matrix given by

\[
M = \frac{\mu_0}{\text{trace}\{R\}}I = \frac{\mu_0}{\sum_{r=1}^{R} \sum_{h=1}^{H} \rho_{rh} \sum_{m=1}^{M} \sum_{l=1}^{L} |F_{rhm}|^2}I
\]

(18)

where \( \mu_0 \) denotes the step-size parameter, \( 0 < \mu_0 < 1 \), and \( \rho_{rh} \) the power of a reference signal \( x_{rh}(n) \) given by \( \rho_{rh} = E\{|x_{rh}(n)|^2\} \) [14],[15]. In this case, all the adaptive weights \( w_{rh}(n) \) are updated using the same convergence factor. The ordinary normalized LMS algorithm is based on a convergence factor which is dependent on the power of all reference signals and the control paths between all loudspeakers and all microphones.

### 3.2 Actuator-Individual Normalization

By using Newton’s method it is possible to produce an algorithm which is more efficient than the steepest descent algorithms [15]. In a Newton algorithm, the matrix \( M \) in (12) may be chosen as

\[
M = \mu_0 R^{-1}
\]

(19)

where \( 0 < \mu_0 < 1 \). If all the reference signals \( x_{rh}(n) \) are assumed to be mutually uncorrelated, the matrix \( R \) is block-diagonal and each block is given by

\[
R_{rh} = \rho_{rh} F_{rh}^H F_{rh}.
\]

(20)
The weighting matrix $M$ can now be chosen as a block diagonal matrix

$$
M = \begin{pmatrix}
M_{11} & M_{12} & \cdots & M_{1R^H} \\
& \ddots & \cdots & \\
& & \ddots & M_{R^H}
\end{pmatrix},
$$

(21)

where each diagonal block is given by

$$
M_{rh} = \mu_0 R_{rh}^{-1}.
$$

(22)

Note that the correlation matrix $R_{rh}$ involves no correlation over time. This characteristic is closely related to the fact that the reference signal is scalar without delays. The elements of $R_{rh}$ reflect the cross-correlation of the reference signals transmitted through different control paths as measured at the microphone positions:

$$
R_{rh} = \rho_{rh} \begin{pmatrix}
\sum_{m=1}^{M} |F_{rhm1}|^2 & \sum_{m=1}^{M} F_{rhm1} F_{rhm2}^* & \cdots & \sum_{m=1}^{M} F_{rhm1} F_{rhmL}^* \\
\vdots & \sum_{m=1}^{M} |F_{rhm2}|^2 & \ddots & \vdots \\
\sum_{m=1}^{M} F_{rhmL} F_{rhm1}^* & \cdots & \sum_{m=1}^{M} |F_{rhmL}|^2
\end{pmatrix}.
$$

(23)

Consider Figure 3. The cross-correlation of the reference signal $x_{rh}(n)$ is transmitted through control paths $F_{rhm}$ and $F_{rhm'}$ is thus given by

$$
E[x_{rh}(n) F_{rhm} F_{rhm'}^* x_{rh}(n)] = \rho_{rh} F_{rhm} F_{rhm'}^*.
$$

In the case of loudspeakers which are spatially separated (large value of $d$), the corresponding cross-correlation will be less significant at the microphone $m$ due to lower coupling between the loudspeaker $l'$ and the microphone $m$ than between $l$ and $m$.

![Figure 3: The reference signal $x_{rh}$ transmitted through two different control paths $F_{ml}$ and $F_{ml'}$.](image-url)
In applications of active noise control with a large number of loudspeakers and microphones as are in aircraft found the acoustic coupling between the loudspeakers will decrease as the distance between them increases. In our experience, this situation leads to a matrix that is usually diagonally dominant, since the diagonal elements contain magnitude squares of frequency responses, and the off-diagonal elements result from cross products of different responses, cf. (23). Hence, the diagonally dominant matrix $R_{rh}$ may be approximated by its diagonal elements

$$R_{rh} \approx \text{diag} \{R_{rh}\}. \quad (24)$$

The novel normalized FX LMS algorithm is based on the diagonally dominant properties of the correlation matrix $R_{rh}$. Actuator-individual normalization is introduced. Each actuator $l$ (loudspeaker) has an individual step-size parameter, $\mu_{rhl}$, for a given reference signal $x_{rh}(n)$. The convergence factor matrices are given by

$$M_{rh} = \mu_0 \text{diag} \{R_{rh}\}^{-1} = \begin{pmatrix} \mu_{rhl} & \mu_{rhl} & \cdots & \mu_{rhl} \\ \mu_{rhl} & \mu_{rhl} & \cdots & \mu_{rhl} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{rhl} & \mu_{rhl} & \cdots & \mu_{rhl} \end{pmatrix} \quad (25)$$

where each diagonal element is given by

$$\mu_{rhl} = \frac{\mu_0}{\rho_{rh} \sum_{m=1}^{M} |F_{rhm1}|^2}. \quad (26)$$

Note that the normalization factors for actuator-individual normalized algorithm are dependent on the control paths between one loudspeaker and all microphones for a given reference signal.

Although the approximation $R \approx \text{diag} \{R\}$ may be rather crude, it may be efficient to use the actuator-individual normalized algorithm given by (12) and (25). This is because in such cases this algorithm represents a sensible compromise between the LMS and the Newton’s algorithm. The advantage of using (25) instead of (22) is that no real-time matrix inversion and multiplications need be done: scalar multiplications in accordance with (25) are sufficient. This is advantageous in an ANC system based on on-line control path modeling.

The two variants of the normalized Filtered-X LMS algorithms are summarized in Table 1. The update scheme of the algorithms is motivated by the assumption that all reference signals are mutually uncorrelated.

<table>
<thead>
<tr>
<th>Control-Algorithm</th>
<th>$w_{rh}(n+1) = w_{rh}(n) - 2M_{rh}x_{rh}^*(n)F_{rh}^Te(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized FX LMS</td>
<td>$M_{rh} = \mu_0 (\text{trace} {R})^{-1}I$</td>
</tr>
<tr>
<td>Actuator-Individual Normal. FX LMS</td>
<td>$M_{rh} = \mu_0 \text{diag} {R_{rh}}^{-1}$</td>
</tr>
</tbody>
</table>

Table 1: Variants of complex-valued normalized Filtered-X LMS algorithms.
For the presented convergence factors the denominators include $\rho_{rh}$. To avoid division by zero if $\rho_{rh} = 0$ a small constant $\alpha$ can be added to the denominator, cf. (26) and $\mu_{rh} = \mu_0/(\alpha + \rho_{rh} \sum_{m=1}^{M} |F_{rm}|^2)$.

4 Stability Criterion

The following convergence analysis addresses the stability condition for the actuator-individual normalized LMS algorithm, i.e. the choice of the step-size parameter $\mu_0$ to obtain a stable control system. The analysis is based on the assumption that the reference signal matrix $X(n)$ in (6) and the filter weight vector $w(n)$ in (7) are independent. Consider the update scheme for the filter weights given in (12). From this equation it follows that $w(n)$ is only dependent on past reference signal vectors $X(n-1), X(n-2)$ and so on [14],[15].

By using the vector of complex microphone signals and equation (14) in the update scheme (12) and by taking the expected value of both sides of this equation the following expression is obtained

$$E[w(n+1)] = E[w(n)] - 2M\left( E[X^H(n)F^Hd(n)] - E[X^H(n)F^HFX(n)w(n)] \right).$$

(27)

The independent assumption and the expressions given in (15) and (16) enable the equation to be rewritten as

$$E[w(n+1)] = E[w(n)] - 2Mp - 2MR E[w(n)].$$

(28)

Using the optimum Wiener filter weight vector $w_{opt} = -R^{-1}p$ and subtracting $w_{opt}$ from both sides yields

$$E[w(n+1)] - w_{opt} = (I - 2MR)(E[w(n)] - w_{opt})$$

(29)

which also can be written as

$$v(n+1) = (I - 2MR)v(n)$$

(30)

where $v(n) = E[w(n)] - w_{opt}$. The Wiener filter weight vector $w_{opt}$ is obtained by differentiating the cost function in (13) with respect to $w(n)$ and equating to zero [14],[15].

Consider the matrix $MR$. The matrix $M$ contains the step-size parameter $\mu_0$ and elements associated with the different reference signals $x_{rh}(n)$, cf. (21),(25) and (26). A rewrite of this matrix is given by

$$M = \mu_0 M^\prime = \mu_0 \begin{pmatrix} M_{11}^\prime & M_{12}^\prime & \cdots \\ M_{21}^\prime & \cdots \\ \vdots & \ddots & \ddots \\ M_{RH}^\prime \end{pmatrix}$$

(31)

where each diagonal block is given by

$$M_{rh}^\prime = \text{diag } \{ R_{rh} \}^{-1}.$$
Now the matrix $M_0R$ is written as

$$M_0R = \mu_0M_0R$$

(33)

Hence, the convergence behavior of the algorithm is determined by the $LRH \times LRH$ non-Hermitian matrix $M_0R$. To decouple the filter weights this matrix is decomposed in its diagonal eigenvalue matrix $\Lambda = \text{diag}[\lambda_1, \lambda_2, \cdots, \lambda_{LRH}]$ and eigenvector matrix $S$. This is done using a decomposition of the matrix as

$$M_0' R = S \Lambda S^{-1}$$

(34)

provided that the inverse matrix $S^{-1}$ exists. By using $M_0R = \mu_0S \Lambda S^{-1}$ and the relation $v'(n) = S^{-1}v(n)$ the equation in (30) may be expressed as

$$v'(n + 1) = (I - 2\mu_0\Lambda)v'(n)$$

(35)

and the solution to this difference equation is given by

$$v'(n) = (I - 2\mu_0\Lambda)^n v'(0).$$

(36)

The update equation is stable provided that the magnitude of the elements in the matrix $I - 2\mu_0\Lambda$ is less than unity

$$|1 - 2\mu_0\lambda_j| < 1.$$ 

(37)

This stability criterion is satisfied when $\mu_0\lambda_j$ lies within a circle ($|\mu_0\lambda_j - \frac{1}{2}| < \frac{1}{2}$) in the complex plane as shown in Figure 4. As the eigenvalues $\lambda_j$ may be complex equation (56) can be rewritten in terms of the real ($\Re \{ \cdot \}$) and imaginary ($\Im \{ \cdot \}$) parts as

$$\sqrt{(1 - 2\mu_0\Re \{ \lambda_j \})^2 + (2\mu_0\Im \{ \lambda_j \})^2} < 1$$

(38)
or
\[(1 - 2\mu_0 \Re\{\lambda_j\})^2 + (2\mu_0 \Im\{\lambda_j\})^2 < 1.\] (39)

Expanding the above yields the expression
\[|\mu_0\lambda_j|^2 - \mu_0 \Re\{\lambda_j\} < 0\] (40)
which can also be rearranged as
\[\mu_0 < \frac{\Re\{\lambda_j\}}{|\lambda_j|^2}.\] (41)

The bounds on the step-size parameter \(\mu_0\) for algorithm stability are given by the criterion
\[0 < \mu_0 < \min\left\{\frac{\Re\{\lambda_j\}}{|\lambda_j|^2}\right\}.\] (42)

The matrix \(\mathbf{M}'\mathbf{R}\) has real and positive eigenvalues since it is a product of two positive definite Hermitian matrices [17]. Figure 5 shows the 256 eigenvalues of the matrix \(\mathbf{M}'\mathbf{R}\) for an ANC system based on 32 loudspeakers and 39 control microphones and where 4 harmonics from 2 noise sources were controlled. Real eigenvalues implies that the stability criterion in (42) can be simplified. A stable system is now obtained for step-size parameters satisfying the following condition
\[0 < \mu_0 < \frac{1}{\lambda_{\text{max}}}\] (43)
where \(\lambda_{\text{max}}\) denotes the largest eigenvalue to the matrix \(\mathbf{M}'\mathbf{R}\).

By introducing a more restrictive stability constraint than that given in (43) a simpler method can be used to compute the upper limit of the convergence factor. The restrictive stability constraint is based on that the largest eigenvalue cannot be larger than the sum of all eigenvalues. The sum is in its turn equal to the trace of matrix \(\mathbf{M}'\mathbf{R}\)
\[\lambda_{\text{max}} < \sum_{j=1}^{J} \lambda_i = \text{trace}\left\{\mathbf{M}'\mathbf{R}\right\}.\] (44)

If all the reference signals \(x_{rh}(n)\) are assumed to be mutually uncorrelated, the matrix \(\mathbf{R}\) is block-diagonal, and each block is given by \(\mathbf{R}_{rh} = \rho_{rh}\mathbf{F}_{rh}^H\mathbf{F}_{rh}\). As a consequence of the assumption of uncorrelated reference signals the matrix \(\mathbf{M}'\mathbf{R}\) is also block diagonal
\[
\mathbf{M}'\mathbf{R} = \begin{pmatrix}
\mathbf{M}'_{11}\mathbf{R}_{11} & \mathbf{M}'_{12}\mathbf{R}_{12} & \cdots \\
\mathbf{M}'_{21}\mathbf{R}_{21} & \mathbf{M}'_{22}\mathbf{R}_{22} & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{M}'_{J1}\mathbf{R}_{J1} & \mathbf{M}'_{J2}\mathbf{R}_{J2} & \cdots & \mathbf{M}'_{JJ}\mathbf{R}_{JJ}
\end{pmatrix}.
\] (45)
Each \(L \times L\) block is given by
\[\mathbf{M}'_{rh}\mathbf{R}_{rh} = \text{diag}\left\{\mathbf{R}_{rh}\right\}^{-1}\mathbf{R}_{rh}.\] (46)
The trace of $\mathbf{M}' \mathbf{R}$ is given by the sum of the trace of each block $\mathbf{M}'_{rh} \mathbf{R}_{rh}$. Each such matrix has unity on the diagonal, yielding a simple result

$$\text{trace} \{ \mathbf{M}' \mathbf{R} \} = \sum_{r=1}^{R} \sum_{h=1}^{H} \text{trace} \{ \mathbf{M}'_{rh} \mathbf{R}_{rh} \}$$

$$= \sum_{r=1}^{R} \sum_{h=1}^{H} \text{trace} \{ \text{diag} \{\mathbf{R}_{rh}\}^{-1} \mathbf{R}_{rh} \} = LRH.$$  

(47)

By combining the result above, (43) and (44) a simple stability criterion is obtained. Hence, the actuator-individual normalized algorithm is stable for step-size parameters within the range of

$$0 < \mu_0 < \frac{1}{LRH}. \quad (48)$$

The result demonstrates that the upper limit is given by a simple expression which is only dependent on the number of loudspeakers ($L$), the number of noise sources ($R$) and the number of controlled harmonics from each source ($H$). This is a more restrictive limit on $\mu_0$ than in (43), but is a much easier and more reliable in practical applications. The number of frequency components to be controlled as well as the number of loudspeakers are known.

![Figure 5: The eigenvalues of the matrix $\mathbf{M}' \mathbf{R}$. The matrix is based on control paths measured in a propeller aircraft.](image)

Eigenvalue

Eigenvalue Number
5 Computational Complexity

The most widely used ANC systems are based on the ordinary FIR-based FX LMS algorithm to update the filter weights. This algorithm is commonly used owing to its simplicity [11]-[18]. The controllers are also often based on a parallel filter structure. The computational complexity of such a controller is compared with the actuator-individual normalized FX LMS algorithm and the filter structure described in Section 2. The comparison is based on the computational complexity, i.e. real multiplications, for each of the two implementations. The complex algorithm requires a minimum of adaptive and control path parameters as compared to a straightforward time-domain approach with ordinary FIR-filters.

An adaptive system involves two main parts: the filtering process generating driving signals to the loudspeakers, and the update phase of the adaptive filter weights. For systems using a large number of loudspeakers and microphones, the pre-filtering of the reference signals (the filtered-x operation) requires high computational power.

In the complex case the pre-filtering of the complex reference signals with the control paths is performed by a complex scalar multiplication of the reference signal $x_{rh}(n)$ and the control path $F_{ml}$: $x'_{ml}(n) = F_{ml} x(n)$. On the other hand, for the FIR-filter, pre-filtering is given by $x'_{ml}(n) = f_{ml}^T x(n)$, where $x(n)$ is a real-valued reference signal vector, $x(n) = [x(n) \ x(n-1) \ \cdots \ \ x(n-Q+1)]^T$, and $f$ denotes a FIR-filter model, the impulse response function of the control path between loudspeaker $l$ and microphone $m$, $f_{ml} = [f_0 \ f_1 \ \cdots \ f_{Q-1}]^T$. The filter length $Q$ is chosen to ensure adequate modeling of the control paths.

Consider first the FIR-based controller with $I$ weights in each control filter $w_l(n) = [w_{l0} \ w_{l1} \ \cdots \ w_{lI-1}]^T$ and $Q$ weights in the control path impulse responses. The weight update scheme for the time-domain FX LMS algorithm is given by [2],[11]:

$$w_l(n+1) = w_l(n) - 2\mu \sum_{m=1}^{M} x'_{ml}(n) e_m(n),$$

where $x'_{ml}(n) = [x'_{ml}(n) \ x'_{ml}(n-1) \ \cdots \ x'_{ml}(n-I+1)]^T$. The driving signal to loudspeaker $l$ is given by

$$y_l(n) = w_l^T(n) x(n).$$

Assume that the system is based upon $L$ loudspeakers and $M$ microphones. For a single reference signal $IL$ multiplications are carried out to compute the $L$ loudspeaker signals, and $(Q+1)ML$ multiplications are performed to compute the weight update parts (the weight increments) for the $L$ control filters. Consider next the complex based controller (cf. Table 1):

$$w_l(n+1) = w_l(n) - 2\mu_l x^*(n) \sum_{m=1}^{M} \hat{F}_{ml}^* e_m(n).$$

According to the structure $(L+ML)$ complex multiplications or $4(L+ML)$ real multiplications are required to compute the weight increments and $2L$ real mul-
Multiplications are needed to compute the loudspeaker signals. The complexity ratio for the FIR-based controller to the complex based is defined by the number of real multiplications for the FIR-based algorithm to the number of real multiplications for the complex algorithm

\[
\text{Complexity ratio} = \frac{(Q + 1)IML + IL}{4(L + ML) + 2L}.
\]

For example, for \( I = 2, Q = 2, L = 32 \) and \( M = 39 \) the FIR-based controller requires 7552 real multiplications for each sample period, while the complex based control only requires 5184 real multiplications. For a case based on \( I = 2, Q = 2, L = 32 \) and \( M = 48 \) the number of real multiplications is given by 9280 and 6336 respectively. Accordingly, the number of multiplications can be significantly reduced if the complex algorithm is used instead of the ordinary FIR-based algorithm.

In addition, using the complex algorithm may also reduce the storage requirement. This algorithm is based upon present samples of the reference signals alone, while the FIR-based structure is based upon present and past samples. In general, the use of present samples alone reduces implementation complexity, resulting in a simpler implementation of the algorithm. The implementation can usually be made very compact, leading to fast execution of the code.

6 Algorithm Variants

6.1 Harmonic Individual Step-Size Parameters

To improve the performance of the ordinary actuator-individual normalized algorithm a modified variant of the algorithm can be used. In the ordinary variant a single step-size parameter \( \mu_0 \) is used for all harmonics. However, noise attenuation may be improved with the aid of a simple modification in the form of an individual step-size parameter \( \mu_{0,h} \) for each harmonic. The convergence factor matrices \( M_{rh} \) in (25) are modified to matrices given by

\[
M_{rh} = \mu_{0,h} \text{diag} \{ R_{rh} \}^{-1}.
\]

The individual step-size parameters speed up the convergence rate for each of the individual harmonics thereby increasing the noise attenuation. Care must, however, be taken in the choice of step-size parameters in order to avoid instability. In a practical situation it important that the ANC system can be adequately tested for different values of \( \mu_{0,h} \) for different operating conditions.

6.2 Control Effort Weighting

A variant of the actuator-individual normalized FX LMS algorithm is obtained by incorporating leakage factors or control effort weighting [2],[19],[20]. The leaky variant of the algorithm introduces constraints on the control signals and can therefore be used to limit the output power, thereby avoiding overloading of the control
Part II

source (loudspeaker) amplifiers. In practical applications it is desirable to keep the driving signals as small as possible, since clipping amplifiers may cause nonlinear distortion. The power in controller output signals can be controlled by using a cost function where the filter weights are included. The modified cost function to be minimized is given by

\[ J(n) = \sum_{m=1}^{M} e_m^2(n) + \sum_{r=1}^{R} \sum_{h=1}^{H} w_{rh}^H(n) \Gamma_{rh} w_{rh}(n) \]  

(54)

where \( \Gamma_{rh} \) denotes a diagonal matrix weighting the control effort and where its diagonal elements are given by \( \gamma_{rh} \), cf. (4). By including the control effort in the cost function the controller is forced to minimize both the control microphone signals and the loudspeaker signals. The higher the values in \( \Gamma_{rh} \), the more important is the contribution from the filter weights or the power of the control signals.

The complex derivatives of the modified cost function with respect to \( w_{rh}(n) \) are given by

\[ \frac{\partial J(n)}{\partial w_{rh}^*(n)} = x_{rh}^*(n)F_{rh}^H e(n) + \Gamma_{rh} w_{rh}(n). \]  

(55)

The leaky variant of the actuator-individual normalized FX LMS algorithm can now be written as

\[ w_{rh}(n+1) = \Theta_{rh} w_{rh}(n) - 2M_{rh} x_{rh}^*(n) F_{rh}^H e(n) \]  

(56)

where \( \Theta_{rh} = I - 2M_{rh} \Gamma_{rh} \). This matrix is diagonal and contains the leakage factors \( \theta_{rh} \) on its diagonal, \( 0 < \theta_{rh} < 1 \). The value of \( \theta_{rh} \) is usually slightly less than unity.

To simplify the analysis of the algorithm the real-value microphone signal vector \( e(n) \) in (56) is substituted by the complex microphone signals given by \( \hat{e}(n) = d(n) + \sum_{r=1}^{R} \sum_{h=1}^{H} x_{rh} F_{rh} w_{rh}(n) \). The update equation is now written as

\[ w_{rh}(n+1) = (I - 2M_{rh} \Gamma_{rh}) w_{rh}(n) - 2M_{rh} x_{rh}^*(n) F_{rh}^H [d(n) + \sum_{r=1}^{R} \sum_{h=1}^{H} x_{rh} F_{rh} w_{rh}(n)]. \]  

(57)

By taking the expected value \( E[\cdot] \) of both sides of (57), using the assumptions of uncorrelated reference signals as well as assuming that \( x_{rh}(n) \) and \( w_{rh}(n) \) are independent, the filter weight updating associated with the reference signal \( x_{rh}(n) \) is given by

\[ E[w_{rh}(n+1)] = E[w_{rh}(n)] - 2M_{rh} [(\Gamma_{rh} + R_{rh}) E[w_{rh}(n)] + p_{rh}]. \]  

(58)

Here \( R_{rh} = \rho_{rh} F_{rh}^H F_{rh} \) and \( p_{rh} = E[x_{rh}^*(n) F_{rh}^H (d(n))]. \)

By setting the term in the square brackets to zero the optimal weight solution for the leaky algorithm can be obtained as

\[ w_{rh}^{\text{leaky opt}} = -[\Gamma_{rh} + R_{rh}]^{-1} p_{rh}. \]  

(59)
Without control effort weighting $\Gamma_{rh} = 0$ the optimal weights are given by

$$w_{rh_{opt}} = -R_{rh}^{-1}p_{rh}. \quad (60)$$

Thus, the leaky algorithm seeks to reach the optimal weight solution in (59) instead of that in (60). The effect of control effort weighting can now be studied by comparing (59) and (60). The presence of the diagonal weight matrix $\Gamma_{rh}$ in (59) results in the magnitudes of the weights in $w_{leaky_{rh_{opt}}}$ being less than or equal to the magnitudes of the weights in $w_{rh_{opt}}$. The difference between the weights increases with increasing control effort weighting. However, small filter weights lead to low controller output signals, $y_{rh}(n) = x_{rh}(n)w_{rh}(n)$, and reduced noise attenuation. Accordingly, a limitation of the control signals ($\Gamma_{rh} \neq 0$) is obtained at the expense of reduced attenuation of the controlled noise components.

It is seen that the weights are strongly dependent on the control path matrices $R_{rh} = \rho_{rh}F_{rh}^H F_{rh}$. Inversion of an ill-conditioned matrix $F_{rh}^H F_{rh}$ leads to a solution with large filter weights, see (60). To attain such optimum weights large control signals are generally required. The algorithm, without leakage, seeks to minimize the mean squared error, resulting in the control signals possibly drifting until they reach a saturation limit. Hence, for ill-condition control path matrices the leaky variant is used rather than the unconstrained algorithm.

The convergence factor matrix $M_{rh}$ for the leaky variant of the actuator-individual normalized FX LMS algorithm can be obtained by considering the cost function, cf. (13) and (54)

$$J = w^H(n)(\Gamma + R)w(n) + w^H(n)p + p^Hw(n) + c. \quad (61)$$

where $\Gamma = \text{diag}(\Gamma_{11}, \Gamma_{12} \cdots \Gamma_{RH})$. The assumption of uncorrelated reference signals results in a block diagonal matrix $R$ with the diagonal block $R_{rh}$. Hence the diagonal block in $\Gamma + R$ is given by $\Gamma_{rh} + R_{rh}$. The convergence factor matrix $M_{rh}$ for the leaky algorithm can be obtained by following the steps outlined in Section 3.2. Accordingly the convergence factor matrix here is expressed as

$$M_{rh} = \mu_0 \text{diag} \{\Gamma_{rh} + R_{rh}\}^{-1} \quad (62)$$

where each diagonal element is given by

$$\mu_{rh} = \frac{\mu_0}{\gamma_{rh} + \rho_{rh} \sum_{m=1}^{M} |F_{rhml}|^2}. \quad (63)$$

A stability constraint of $\mu_0$ can now be obtained by considering trace $\{M'(\Gamma + R)\}$ where the block diagonal matrix $M'$ is associated with (62), cf. (47)

$$\text{trace} \{M'(\Gamma + R)\} = \sum_{r=1}^{R} \sum_{h=1}^{H} \text{trace} \{M'_{rh}(\Gamma_{rh} + R_{rh})\}$$

$$= \sum_{r=1}^{R} \sum_{h=1}^{H} \text{trace} \{\text{diag} \{\Gamma_{rh} + R_{rh}\}^{-1} (\Gamma_{rh} + R_{rh})\} = LRH. \quad (64)$$
The above result demonstrates that by using a normalization given in (62) the stability range of $\mu_0$ is the same as previously shown

$$0 < \mu_0 < \frac{1}{LRH}. \quad (65)$$

7 Computer Experiment

The performance evaluation of the two algorithms is based on a computer experiment using noise recorded inside a twin-engine propeller aircraft, a Dornier 328, during flight. All the control paths between the control sources and the control microphones were also measured. The evaluation was made for two different flight conditions: steady cruise flight, and transition from climb to cruise flight. In cruise flight the propellers were synchronized with a constant rotational speed and the Blade Passage Frequency (BPF) was 105 Hz. For the second flight condition, the engine rotational speed varied and an occasional slight rotational speed difference between the propellers was observed. The BPF dropped from 110 Hz to 105 Hz and the maximal BPF difference was approximately 1 Hz. These specific flight conditions were chosen in order to examine the stationary and the dynamic properties of the proposed control algorithms.

Primary noise was recorded by 39 microphones mounted at the passengers’ head rests. A tachometer signal from each engine with one pulse per revolution was available and was synchronously recorded with the noise. All the signals were recorded with a 1024 Hz sampling rate. The control paths between the 32 loudspeakers mounted on the trim panels to the 39 error microphones situated at the head rests were identified using white noise excitation and with a 1 Hz resolution.

Throughout the evaluation the control algorithms were set up to attenuate the BPF up to $4 \times$ BPF from the left and right propellers, respectively. The reference signals, which are in this case complex, were generated by filtering the tachometer signals through an FFT-filter bank [6],[21]. The step-size parameter $\mu_0$ was chosen as 1/10 of the value causing instability $\mu_{0_{\text{max}}}$. The value of $\mu_{0_{\text{max}}}$ was determined by an iterative experiment. The experiment was repeated many times with different values for the step-size parameter $\mu_0$. The first run used a small value for $\mu_0$. One each successive repetition of the experiment the step-size parameter was slowly increased until instability occurred.

The control algorithms were evaluated with respect to the convergence properties and by comparing the noise attenuation with the least square optimum attenuation. Convergence behavior is presented as the normalized mean Sound Pressure Level (SPL) versus time (learning curves), normalized to 0 dB at $t = 0$ seconds. The narrowband mean SPL was calculated as follows: the microphone signals $e_m(n)$ were filtered with a 256-point FFT-filter bank at the $h$th harmonic, yielding

$$E_{mh}(n) = \sum_{l=0}^{255} h_q e_m(n + q)e^{-j \frac{2\pi}{256} k_h q} \quad (66)$$

where $h_q$ is a Hanning window and $k_h$ is the FFT-bin corresponding to the $h$th
The mean SPL at time index $n$ and harmonic $h$ is the average power over all microphones:

$$\sum_{m=1}^{M} |E_{mh}(n)|^2.$$  \hfill (67)

The narrowband mean reduction in dB at time index $n$ and harmonic $h$ was calculated by the logarithmic ratio

$$10 \log_{10} \frac{\sum_{m=1}^{M} |D_{mh}(n)|^2}{\sum_{m=1}^{M} |E_{mh}(n)|^2}.$$  \hfill (68)

where $D_{mh}(n)$ was obtained by filtering the primary noise signals $d_m(n)$ as in (66).

The optimum least squares reduction at time index $n$ and harmonic $h$ was calculated by solving (2) in least squares sense over 256 data points [22]. Minimization was performed with respect to the complex vectors $w_{rh}$. Data was then filtered with the optimum filter weights, and the corresponding narrowband mean reduction was calculated as in (68).

### 7.1 Convergence Performance (Steady Cruise Flight)

In the steady cruise flight condition, the BPFs of the two propellers were stationary at 105 Hz. As a result, there was thus no need to change the control paths $F_{rh}$ in the algorithms.

Figure 6 shows the normalized mean SPL versus time at BPF for the primary noise and the two algorithmic variants given in Table 1. The adaptation is switched on after approximately 1/3 second. The overall result in the figure is that the actuator-individual normalized algorithm performs better than the ordinary normalized algorithm in terms of convergence rate and noise attenuation. It is obvious from the learning curves that the actuator-individual normalized algorithm exhibits a faster convergent initial mode which in turn results in higher noise attenuation.

The direct path movement is obtained since the convergence factor matrix $M_{rh}$ for the Newton’s algorithm is based on $R_{rh}^{-1}$. The expressions given in (22) and (25) show the weight matrices for the Newton’s algorithm and the actuator-individual algorithm respectively. As previously mentioned, when the correlation matrices $R_{rh}$ are diagonally dominant, these matrices can be approximated by their diagonals:

$$R_{rh} \approx \text{diag} \{R_{rh}\}.$$  \hfill (69)

The fast convergence behavior demonstrates that the actuator-individual normalized algorithm is more similar to Newton’s algorithm than to the ordinary LMS algorithm. The Newton algorithm traces the direct path to the optimal solution, i.e. the change is in the direction of the minimum of the cost function. On the other hand, the ordinary LMS algorithm follows the direction of the negative gradient, the steepest descent, of the cost function until it reaches the optimum.

Figure 7 shows the magnitude of the elements in the correlation matrices $R_{rh}$ for the harmonics BPF-4×BPF. The diagonally dominant characteristics of these
matrices are clearly shown. For these cases the actuator-individual normalized FX LMS algorithm may approximate the Newton’s algorithm quite well.

Table 2 summarizes the narrowband mean reduction of the BPF and three harmonics averaged over the 39 microphones at approximately 9 seconds. The table also shows the calculated optimum least squares reduction at the corresponding time. By using the actuator-individual normalized FX LMS algorithm a significantly improved noise attenuation can be obtained. A comparison with the calculated optimum attenuations demonstrates that the algorithm performed well. The differences between obtained and optimum attenuation for the different harmonics were in the interval 1-2.5 dB.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized FX LMS</td>
<td>17.1</td>
<td>11.6</td>
<td>5.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Actuator-Individual Normal. FX LMS</td>
<td>19.2</td>
<td>13.1</td>
<td>6.7</td>
<td>5.3</td>
</tr>
<tr>
<td>Optimum least squares</td>
<td>21.3</td>
<td>15.5</td>
<td>7.7</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 2: The narrowband mean reduction of the primary noise in the cruise flight condition.
A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins

Figure 7: The diagonally dominant $32 \times 32$ correlation matrices $R_{xh}$ at BPF-4BPF (linear scale).

In practical situations it is difficult to choose a suitable value for the step-size $\mu_0$ in order to avoid instability. The limitation $\mu_0 < 1$ does not generally guarantee convergence. The value of $\mu_{0\text{max}}$ was experimentally determined for different control conditions using the iterative approach described in Section 7. The maximum step-size, $\mu_{0\text{max}}$, was the largest step-size found that gave a stable result. The values of $\mu_{0\text{max}}$ compared to the predicted values calculated using (48) are presented in Tables 3 and 4. Table 3 presents the results from the control conditions: first controlling BPF from the two propellers, and then controlling the BPF and 2×BPF, and so on ($R=2$ and $H=1–4$). Table 4 shows the results from a condition where the BPF–4×BPF from a single propeller was controlled ($R=1$ and $H=4$) and the number of loudspeakers used varied.

The experimental and calculated instability values of $\mu_{0\text{max}}$ are clearly dependent on the number of controlled harmonics as well as the number of loudspeakers. It was observed that the maximum value of the step-size decreases with the number of controlled harmonics, as is clearly shown in the expression for the predicted upper limit in (48). The results demonstrate that if the criterion in (48) is satisfied the system will be stable. The stability limit in (48) actually lies well inside the true stability limit in all the cases examined. The values of the experimental observed $\mu_{0\text{max}}$ are approximately 5-10 times greater than the calculated values.

Although there are differences between the observed and the calculated values the stability criteria in (48) can be useful in practical applications. By using the simple criterion in (48) it is easy to obtain a value for the step-size that guarantees stability. However, an improved system performance can be achieved by increasing
the step-size. The calculated value of \( \mu_0 \) can thus be used as an initial value in an iterative process where the step-size value is slowly increased until the desired performance is obtained. This should cause no problem in a practical situation provided that the system can be adequately tested.

<table>
<thead>
<tr>
<th>Controlled harmonics</th>
<th>BPF</th>
<th>BPF-2×BPF</th>
<th>BPF-3×BPF</th>
<th>BPF-4×BPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental ( \mu_{0_{\text{max}}} )</td>
<td>0.093</td>
<td>0.068</td>
<td>0.048</td>
<td>0.039</td>
</tr>
<tr>
<td>Calculated ( \mu_{0_{\text{max}}} )</td>
<td>0.016</td>
<td>0.0078</td>
<td>0.0052</td>
<td>0.0039</td>
</tr>
</tbody>
</table>

Table 3: Comparison between the experimental and calculated maximum value of \( \mu_0 \) to guarantee stability. The ANC system controlled the BPF up to 4×BPF from both propellers \((R = 2, H = 1 – 4)\) using \( L=32 \) loudspeakers and \( M=39 \) control microphones.

<table>
<thead>
<tr>
<th>Number of Loudspeakers</th>
<th>( L=16 )</th>
<th>( L=24 )</th>
<th>( L=32 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental ( \mu_{0_{\text{max}}} )</td>
<td>0.097</td>
<td>0.071</td>
<td>0.065</td>
</tr>
<tr>
<td>Calculated ( \mu_{0_{\text{max}}} )</td>
<td>0.016</td>
<td>0.011</td>
<td>0.0078</td>
</tr>
</tbody>
</table>

Table 4: Comparison between experimental obtained and calculated maximum value of \( \mu_0 \) to guarantee stability. The ANC system controlled the BPF–4×BPF \((H = 4)\) from a single propeller \((R = 1)\) using \( L \) loudspeakers and \( M=39 \) control microphones.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Same step-size for all harmonics ( \mu_0 )</td>
<td>19.2</td>
<td>13.1</td>
<td>6.7</td>
<td>5.3</td>
</tr>
<tr>
<td>Individual step-size for each harmonic ( \mu_{0_{\text{rh}}} )</td>
<td>20.2</td>
<td>14.9</td>
<td>7.0</td>
<td>5.6</td>
</tr>
<tr>
<td>Optimum least squares</td>
<td>21.3</td>
<td>15.5</td>
<td>7.7</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 5: The narrowband mean reduction of the primary noise (cruise flight condition) using a single step-size parameter, \( \mu_0 \), for all harmonics or harmonic individual step-size parameters, \( \mu_{0_{\text{rh}}} \), cf. (25) and (53).

As discussed in Section 6.1 performance can be improved by using an individual step-size parameter \( \mu_{0_{\text{rh}}} \) for each harmonic. Table 5 shows the attenuation for BPF–4×BPF using a controller based upon a single \( \mu_0 \) for all harmonics and one based upon individual step-size parameters \( \mu_{0_{\text{rh}}} \) for each harmonic. Each controller simultaneously controlled the BPF–4×BPF from the two propellers. In this
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Figure 8: The normalized mean SPL versus time at BPF (105 Hz) when using a leakage variant of the actuator-individual normalized FX LMS algorithm where $\Theta_{rh} = \theta_0 I$: (a) $\theta_0 = 1$; (b) $\theta_0 = 0.99999$; (c) $\theta_0 = 0.9999$; (d) $\theta_0 = 0.999$.

The results demonstrate that the last dB of attenuation requires more control power than the first dB of reduction. Compare cases (a) and (d). In case (d) the output power was reduced by 3.8 dB and a noise attenuation of 15.3 dB was obtained, as compared to 19.2 dB without leakage. Although large leakage was used a significant noise reduction was obtained.

A comparison of the observed and calculated values of $\mu_{0 \text{max}}$ for the leaky algorithm is presented in Table 7. The results also shows for this case that the
Table 6: The relation between the mean noise attenuation at BPF and the output power reduction of the loudspeaker signals for different leakage factors, cf. Figure 8 (a)–(d).

<table>
<thead>
<tr>
<th>Leakage factor $\theta_0$</th>
<th>1</th>
<th>0.99999</th>
<th>0.9999</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Attenuation at BPF [dB]</td>
<td>19.2</td>
<td>19.1</td>
<td>18.2</td>
<td>15.3</td>
</tr>
<tr>
<td>Mean Attenuation at 2×BPF [dB]</td>
<td>13.1</td>
<td>13.0</td>
<td>12.2</td>
<td>8.7</td>
</tr>
<tr>
<td>Mean Attenuation at 3×BPF [dB]</td>
<td>6.2</td>
<td>6.7</td>
<td>6.6</td>
<td>5.1</td>
</tr>
<tr>
<td>Mean Attenuation at 4×BPF [dB]</td>
<td>5.3</td>
<td>5.4</td>
<td>5.2</td>
<td>4.7</td>
</tr>
<tr>
<td>Output Power Reduction [dB]</td>
<td>0</td>
<td>0.12</td>
<td>0.98</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 7: Comparison between the experimental and calculated maximum value of $\mu_0$ to guarantee stability. The ANC system is based on the leaky actuator-individual normalized FX LMS algorithm. The BPF–4×BPF from both propellers were controlled using $L=32$ loudspeakers and $M=39$ control microphones.

<table>
<thead>
<tr>
<th>Leakage factor $\theta_0$</th>
<th>1</th>
<th>0.99999</th>
<th>0.9999</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental $\mu_{0_{\text{max}}}$</td>
<td>0.0395</td>
<td>0.0395</td>
<td>0.0396</td>
<td>0.0400</td>
</tr>
<tr>
<td>Calculated $\mu_{0_{\text{max}}}$</td>
<td>0.00361</td>
<td>0.00361</td>
<td>0.00361</td>
<td>0.00361</td>
</tr>
</tbody>
</table>

In the above flight condition with time-varying rotational speed, the delay in reference generation caused by the FFT-filter bank leads to decreased correlation between the reference signals and the primary noise signals. The decreased reduction due to the deteriorated correlation is clearly shown in Figure 10. Note how the reduced noise attenuation coincides with the frequency variations visible at approximately 1.7 and 5.3 seconds as shown in Figure 9. In non-stationary flight conditions it is thus very important that delays in the reference signals are as short as possible. In stationary conditions, on the other hand, there is always enough correlation between narrowband (sinusoidal) signals, i.e. it is always possible to find
correlation between periodic signals of the same frequency, irrespective of delays.

Tracking behavior is shown in Figure 10, which shows the normalized mean SPL versus time. The actuator-individual normalized FX LMS algorithm exhibits the best tracking performance. Table 8 summarizes the narrowband mean reduction for the same time samples at approximately 9 seconds.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Normalized FX LMS</td>
<td>13.5</td>
<td>7.9</td>
<td>5.2</td>
<td>2.9</td>
</tr>
<tr>
<td>Actuator-Individual Normal. FX LMS</td>
<td>16.6</td>
<td>9.9</td>
<td>5.8</td>
<td>4.9</td>
</tr>
<tr>
<td>Optimum least squares</td>
<td>25.4</td>
<td>12.7</td>
<td>6.8</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Table 8: The narrowband mean reduction of the primary noise in the climb to cruise flight condition.
Figure 10: The normalized mean SPL versus time at BPF (105 Hz) the climb to cruise flight: (a) Primary noise; (b) Normalized FX LMS; (c) Actuator-individual normalized FX LMS.

8 Summary and Conclusions

This paper presents a comparison of multiple-reference, multiple-channel adaptive algorithms for active control of propeller-induced interior aircraft noise. The feedforward control algorithms inherently exploit the narrowband assumption by using complex filtering and complex modeling of control paths. The algorithms are based on the ordinary complex Filtered-X LMS algorithm using different normalization factors. The performance of the introduced actuator-individual normalized Filtered-X LMS algorithm was compared to the ordinary normalized Filtered-X LMS using a single normalization for the whole system. The evaluation was performed on cabin noise recorded in a twin-engine propeller aircraft in-flight.

The results indicate that the actuator-individual normalized Filtered-X LMS algorithm exhibits better performance than the conventional normalized Filtered-X LMS algorithm with respect to properties such as convergence rate, steady-state noise reduction and tracking behavior. The noise reduction obtained using the actuator-individual normalized algorithm was close to the optimum noise reduction, calculated by using the measured primary noise and control paths.

A stability criterion for the actuator-individual normalized algorithm has also been presented. The criterion is very simple and is based on the number of frequency components to be controlled and the number of loudspeakers. As a result, a step-size giving a stable control system can easily be calculated.
9 Acknowledgments

The authors wish to express their gratitude to the ASANCA consortium and to Dr. I. Borchers at Daimler-Benz Aerospace Dornier, Germany, for supplying an elaborate set of acoustic test data from a Dornier 328 aircraft. The availability of this data has been of invaluable support in the development of the control algorithms presented in this paper.
Appendix

The purpose of this appendix is to show the differentiation of a real-valued function with respect to a complex-valued parameter or vector [14]. Differentiation with respect to a complex variable \( w \) may be defined by

\[
\frac{\partial}{\partial w^*} = \frac{1}{2} \left( \frac{\partial}{\partial a} + j \frac{\partial}{\partial b} \right)
\]  

(70)

where \( w = a + jb \), cf. [14]. Using this definition, the following relationships may be verified

\[
\frac{\partial}{\partial w^*} (\Re \{g\})^2 = \Re \{g\} \frac{\partial}{\partial w^*} (g + g^*),
\]  

(71)

\[
\frac{\partial}{\partial w^*} w^* c = c,
\]  

(72)

\[
\frac{\partial}{\partial w^*} wc = 0
\]  

(73)

where \( g \) is a complex function of \( w \), \( c \) is a complex constant, \((\cdot)^*\) denotes complex conjugation and \( \Re \{ \cdot \} \) denotes the real part.

The real microphone signal \( e_m(n) \) is given by

\[
e_m(n) = d_m(n) + \Re \left\{ \sum_{l=1}^{L} F_{ml} x(n) w_l \right\}
\]  

(74)

and the complex microphone signal \( \tilde{e}_m(n) \) can now be written as

\[
\tilde{e}_m(n) = d_m(n) + \sum_{l=1}^{L} F_{ml} x(n) w_l.
\]  

(75)

The real-valued cost function to be minimized by the adaptive algorithm is given by

\[
J(n) = \sum_{m=1}^{M} e_m^2(n).
\]  

(76)

By using the relationships (71)–(75), the following derivatives are obtained

\[
\frac{\partial \tilde{e}_m(n)}{\partial \tilde{w}_l^*} = 0.
\]  

(77)

and

\[
\frac{\partial \tilde{e}_m^*(n)}{\partial \tilde{w}_l^*} = x^*(n) \sum_{m=1}^{M} F_{ml}^*.
\]  

(78)
The derivatives of the real function $J(n)$ with respect to the complex weight $w_i^*$ can now be obtained as

$$\frac{\partial J(n)}{\partial w_i^*} = x^*(n) \sum_{m=1}^{M} F_{ml}^* e_m(n).$$

(79)

Matrix notation is now introduced in order to simplify further development. Let $e(n)$ denote a real $M \times 1$ vector containing the elements $e_m(n)$, given by

$$e(n) = d(n) + \Re\{Fx(n)w\}$$

(80)

where $d(n)$ is a real $M \times 1$ vector containing the elements $d_m(n)$, $w$ is a complex $L \times 1$ vector with elements $w_l$ and $F$ is a complex $M \times L$ matrix with elements $F_{ml}$.

Let $\frac{\partial}{\partial w^*}$ denote the vector partial derivative operator with elements $\frac{\partial}{\partial w_l}$. The derivatives of $J(n) = e^T(n)e(n)$ with respect to the vector $w^*$ are then given by

$$\frac{\partial J(n)}{\partial w^*} = x^*(n)F^H e(n).$$

(81)

where $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the conjugate–transpose operation respectively.
References


A Novel Multiple-Reference Algorithm for Active Control of Propeller-Induced Noise in Aircraft Cabins


Part III

Evaluation of Multiple-Reference Active Noise Control Algorithms on Dornier 328 Aircraft Data
This part has been published as:

Evaluation of Multiple–Reference Active Noise Control Algorithms on Dornier 328 Aircraft Data

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Abstract

This correspondence presents an evaluation of multiple–reference adaptive algorithms. Two LMS–types and a Newton–type algorithm are considered. The special structure of the adaptive filtering problem implies that the Newton–type algorithm can be implemented with the same numerical complexity as LMS–type algorithms. The concept of a fast filtered–x Newton algorithm is thus introduced.

1 Introduction

This correspondence concerns the problem of active noise control (ANC) in propeller aircraft, cf. [1], and in particular on Dornier 328 aircraft data. Since the sound field inside such aircraft is usually dominated by narrowband noise harmonics generated by the propellers, and the aircraft are usually fitted with tachometer devices from which a tachometer signal corresponding to each engine can be obtained, the situation is well–suited for using a filtered–x feedforward ANC technique [2]–[4]. The tachometer signals can be used to generate narrowband reference signals containing the same frequency components as the propeller–induced noise. These reference signals are then filtered through an adaptive controller in order to generate the driving signals to the loudspeakers, generating the secondary sound field inside the aircraft cabin.

The feedforward technique presented inherently exploits the narrowband assumption by using frequency domain filtering and complex modeling of acoustic paths. The proposed complex algorithms are advantageous in narrowband applications due to high convergence rate and low numerical complexity [2],[5],[6]. The special structure of the adaptive filtering problem implies that the Newton–type algorithm [3],[7] can be implemented with the same numerical complexity as LMS–type algorithms.
2 The Multiple–Reference Complex Adaptive Algorithms

The multiple reference controller using \( L \) loudspeakers and \( M \) microphones is described below for a general case with \( R \) reference signals and \( H \) harmonics for each reference. The following notation is introduced: Let \( x_{rh}(n) \), \( w_{rh} \) and \( F_{rh} \) denote the complex scalar reference signal, the \( L \times 1 \) vector of complex loudspeaker weights and the \( M \times L \) matrix of complex transfer functions of acoustic paths respectively, each associated with the \( r \)th reference and the \( h \)th harmonic. It is assumed that the reference signals \( x_{rh}(n) \) are mutually uncorrelated.

The real \( M \times 1 \) vector \( e(n) \) of broadband microphone signals \( e_m(n) \), is given by

\[
e(n) = d(n) + \sum_{r=1}^{R} \sum_{h=1}^{H} \Re\{F_{rh}x_{rh}(n)w_{rh}\} \tag{1}
\]

where \( n \) is time index, \( d(n) \) is a real \( M \times 1 \) vector containing primary sound at the microphones, and \( \Re\{\cdot\} \) denotes the real part. A block diagram representation of this equation is shown in Fig. 1.

![Figure 1: A model of the feedforward active noise control system.](image)

The objective function to be minimized by the adaptive algorithm is given by

\[
J(n) = \sum_{m=1}^{M} e_m^2(n) = e^T(n)e(n). \tag{2}
\]

The derivative of (2) with respect to the complex weight vector \( w_{rh}^* \) is given by [6]

\[
\frac{\partial J(n)}{\partial w_{rh}^*} = x_{rh}(n)F_{rh}^H e(n) \tag{3}
\]
where \((\cdot)^*\) and \((\cdot)^H\) denote complex conjugation and conjugate–transpose respectively. The complex gradient in (3) is used to define the updating scheme of the adaptive algorithm, given by

\[
w_{rh}(n + 1) = w_{rh}(n) - 2M_{rh}x_{rh}^*(n)F_{rh}^He(n)
\]

where \(M_{rh}\) is a matrix weighting factor of dimension \(L \times L\). The algorithm in (4) is motivated by the assumption that all reference signals are mutually uncorrelated. Hence, the matrices that govern the convergence properties (in a simplified analysis) are given by the second differential (Hessian) of \(J = E[e^T(n)e(n)]\) with respect \(w_{rh}^*\)

\[
R_{rh} = \rho_{rh}F_{rh}^HF_{rh}
\]

where \(\rho_{rh} = E\{|x_{rh}(n)|^2\}\). If the well–known LMS–algorithm is to be employed, the weighting factor \(M_{rh}\) may be chosen as

\[
M_{rh} = \frac{\mu_0}{\text{trace}\{R_{rh}\}}I
\]

where \(0 < \mu_0 < 1\) and \(I\) is the identity matrix.

In a Newton–like algorithm, the matrix \(M_{rh}\) in (4) may be chosen as

\[
M_{rh} = \mu_0 R_{rh}^{-1}
\]

where \(0 < \mu_0 < 1\).

Note that the elements of \(R_{rh}\) reflect the spatial cross–correlation of the reference signal transmitted through different loudspeaker paths as measured at the microphone positions. For loudspeakers which are widely spatially separated, the corresponding cross–correlation will be small at all microphone positions. In many practical situations, such as in aircraft, this condition holds, and the matrix \(R_{rh}\) will become \textit{diagonally dominant}, see Fig. 2. In this case, the matrix factor \(M_{rh}\) can be chosen as

\[
M_{rh} = \mu_0(\text{diag}\{R_{rh}\})^{-1}
\]

The special structure of the algorithm in (4) originates from the fact that the acoustic path pre–filtering of the reference signal (the filtered–x operation) is performed by a complex \textit{scalar} multiplication of the reference signal with the matrix of acoustic path parameters. This means that the matrix factor \(M_{rh}\) in (4) may be incorporated with the matrix \(F_{rh}^H\) in advance, yielding a new matrix \(G_{rh} = M_{rh}F_{rh}^H\), which takes the role of \(F_{rh}^H\). The matrix \(G_{rh}\) has the same dimension as \(F_{rh}^H\). With \(G_{rh} = R_{rh}^{-1}F_{rh}^H\), the fast filtered–x Newton algorithm is obtained. The three different variants of filtered–x complex adaptive algorithms are summarized in Table 1. Note that all three algorithms in the table require approximately \(LM\) multiplications. Thus, the fast filtered–x Newton algorithm is fast in the sense that it has the same computational complexity as the ordinary filtered–x LMS–algorithm.
Figure 2: The diagonally dominant 32 × 32 correlation matrices $R_{rh}$ at BPF-4BPF, measured in a Dornier 328 (32 loudspeakers and 39 microphones).

<table>
<thead>
<tr>
<th>Control–Algorithm</th>
<th>$w_{rh}(n+1) = w_{rh}(n) - 2\mu_0 x_{rh}(n)G_{rh}\epsilon(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex Filtered–x LMS</td>
<td>$G_{rh} = (\rho_{rh}\text{trace}{F_{rh}^H F_{rh}})^{-1}F_{rh}^H$</td>
</tr>
<tr>
<td>Actuator–Individual Filtered–x LMS</td>
<td>$G_{rh} = (\rho_{rh}\text{diag}{F_{rh}^H F_{rh}})^{-1}F_{rh}^H$</td>
</tr>
<tr>
<td>Fast Filtered–x Newton</td>
<td>$G_{rh} = (\rho_{rh}F_{rh}^H F_{rh})^{-1}F_{rh}^H$</td>
</tr>
</tbody>
</table>

Table 1: Three different variants of filtered–x complex adaptive algorithms.
3 Evaluation Examples

The computer evaluations presented in this paper are based on noise recordings and acoustic measurements performed in a twin-engine propeller aircraft, a Dornier 328. Two different flight conditions (steady cruise flight and climb to cruise flight) are evaluated in order to investigate the asymptotic and the dynamic properties of the control algorithms proposed. The primary noise field was recorded at 39 microphones placed at the passenger head level, and using a sampling rate of 1024 Hz. A tachometer signal related to each engine was also synchronously recorded with the noise field. The control algorithm was set up to cancel the four noise harmonics of the blade passage frequency (BPF–$4 \times$BPF) corresponding to the left and right propellers respectively. The transfer functions (acoustic paths) were measured from 32 loudspeakers (mounted on the trim panels) to the 39 microphones using white noise excitation, and with a 1 Hz resolution. The complex reference signals were obtained by filtering the tachometer signals through a 512–point FFT filter bank (one for each reference). The entire ANC system is shown in Fig. 3.

High frequency resolution (long FFT length) was required since the tachometer signals consisted of one pulse only per propeller revolution. In order to compensate for the relatively long delay induced by the 512–point FFT filter banks, the primary noise field was delayed 256 samples prior to evaluation.

For both conditions of flight, the three different variants of filtered–x complex adaptive algorithms given in Table 1 were compared. The resulting mean sound pressure level (SPL) was calculated as follows: The microphone signals $e_m(n)$ were filtered with a 256–point FFT filter bank at the $h$th harmonic, yielding

$$E_{mh}(n) = \sum_{q=0}^{255} h_q e_m(n + q) e^{-j \frac{2 \pi}{256} k_h q} \tag{9}$$

where $h_q$ is a Hanning window and $k_h$ is the FFT bin corresponding to the $h$th harmonic. The narrowband mean reduction at time index $n$ and harmonic $h$ was calculated by the logarithmic ratio of the average power over all microphones

$$10 \log_{10} \frac{\sum_{m=1}^{M} |D_{mh}(n)|^2}{\sum_{m=1}^{M} |E_{mh}(n)|^2} \tag{10}$$

where $D_{mh}(n)$ was obtained by filtering the primary noise signals $d_m(n)$ as in (9).

The optimum reduction at time index $n$ and harmonic $h$ was calculated by solving (1) in least squares sense over 256 data points $n, n + 1, \cdots, n + 255$. The minimization was performed with respect to the complex vectors $w_{rh}$. Data was then filtered with the optimum filter weights, and the corresponding narrowband mean reduction was calculated as in (10). In the evaluation the normalized convergence factor $\mu_0$ of each algorithm was chosen as 1/10 of the value for which the algorithms became unstable.
Figure 3: Twin-reference, multiple-channel system for active noise control.
3.1 Evaluation of the Steady Cruise Flight Condition

In the steady cruise flight condition, the BPFs of the two propellers were almost stationary at 105 Hz. There was thus no need to change the acoustic paths $F_{rh}$ in the model (1) and in the algorithm update equation (4).

Figure 4 shows the normalized mean SPL versus time for the primary sound field and the three algorithmic variants given in Table 1. The fast Newton variant showed the highest rate of convergence and the best agreement with optimum reduction, as expected. The narrowband mean reduction of the harmonics at approximately 8.5 seconds are summarized in Table 2. Figure 5 shows the power spectrum of the primary and reduced sound field averaged over all microphones. The microphone signals have been added up, then analysed (using the Welch method [8] with 2048 last data, blocks of 256 samples, and Hanning window). The figure shows that the first two harmonics strongly dominate the primary noise field.

Figure 6 illustrates the SPL of the primary noise inside the cabin at the BPF and the SPL achieved at the BPF by using the actuator–individual filtered–x LMS algorithm.

Mutually uncorrelated reference signals were assumed in order to give a simplified development of the algorithms in Section 2. Note that the algorithms performed well even though the reference signals were correlated in the present example.

<table>
<thead>
<tr>
<th>Control–Algorithm/Method</th>
<th>BPF [dB]</th>
<th>2×BPF [dB]</th>
<th>3×BPF [dB]</th>
<th>4×BPF [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex Filtered–x LMS</td>
<td>18.1</td>
<td>12.4</td>
<td>6.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Actuator–Individual Filtered–x LMS</td>
<td>18.9</td>
<td>12.6</td>
<td>6.7</td>
<td>5.1</td>
</tr>
<tr>
<td>Fast Filtered–x Newton</td>
<td>21.1</td>
<td>14.6</td>
<td>7.4</td>
<td>5.8</td>
</tr>
<tr>
<td>Optimum least squares</td>
<td>21.3</td>
<td>15.5</td>
<td>7.7</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 2: The narrowband mean reduction of the primary noise in the steady cruise flight condition.
Figure 4: Simulation of the mean SPL versus time at BPF (105 Hz) in the steady cruise flight condition. (a) Primary sound field. (b) Complex filtered–x LMS. (c) Actuator–individual filtered–x LMS. (d) Fast filtered–x Newton. Adaptation is switched on after approximately 1/3 second. The optimum least squares reduction is indicated by the level in the lower right corner.

Figure 5: Simulation of the power spectrum of the primary and reduced sound field averaged over all microphones. Upper solid line: Primary sound field. Lower solid line: Complex filtered–x LMS. Dashed line: Fast filtered–x Newton.
Figure 6: The SPL at BPF (105 Hz) inside the cabin at passenger head level. (a) Primary sound field. (b) Reduced sound field (actuator–individual filtered–x LMS algorithm). Note that the levels are not absolute sound pressure levels.
3.2 Evaluation of the Climb to Cruise Flight Condition

In the climb to cruise flight condition, the BPFs of the two propellers were changed from 110 Hz to 106 Hz as shown in Fig. 7. This plot was produced by employing a 1024–point FFT analysis (1 Hz resolution) of the corresponding tachometer signals. The acoustic paths $F_{r\theta}$ in the model (1) and in the algorithm update equation (4) were changed simultaneously according to this FFT analysis.

![Blade Passage Frequency (BPF) vs Time](image)

Figure 7: The BPFs of the two propellers during the climb to cruise flight condition. Solid line: Left propeller. Dashed line: Right propeller.

<table>
<thead>
<tr>
<th>Control–Algorithm/Method</th>
<th>BPF</th>
<th>2×BPF</th>
<th>3×BPF</th>
<th>4×BPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex Filtered–x LMS</td>
<td>19.0</td>
<td>10.3</td>
<td>2.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Actuator–Individual Filtered–x LMS</td>
<td>19.9</td>
<td>10.7</td>
<td>2.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Fast Filtered–x Newton</td>
<td>22.3</td>
<td>12.4</td>
<td>2.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Optimum least squares</td>
<td>25.8</td>
<td>12.7</td>
<td>2.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3: The narrowband mean reduction of the primary noise in the climb to cruise flight condition.
Figure 8: Simulation of the mean SPL versus time at BPF (106–110 Hz) in the climb to cruise flight condition. (a) Primary sound field. (b) Complex filtered–x LMS. (c) Actuator–individual filtered–x LMS. (d) Fast filtered–x Newton. Adaptation is switched on after approximately 1/3 second. The optimum least squares reduction is indicated by the level in the lower right corner.

Figure 8 shows the normalized mean SPL versus time for the primary sound field and the three algorithm variants. Table 3 summarizes the narrowband mean reduction for these algorithms at approximately 8.5 seconds.

Figure 9 shows the effect of compensating for the delay introduced by the 512-point FFT filter bank which was used to pre-process the tachometer signals. The solid line shows the improved dynamic performance when delaying the primary noise field by 256 samples. When investigating this effect on the steady cruise flight condition, no difference could be detected (visually or aurally). This establishes the fact that this delay–problem is a problem related to dynamic and not to stationary properties of the ANC system.

A likely explanation for the decreased noise reduction when omitting delay compensation is as follows: The delay of the tachometer signal implies decreased coherence between the reference signals and the primary noise signals in non–stationary conditions. Note how the decreased noise reduction in Fig. 9 coincides with the frequency variations visible in Fig. 7 (at approximately 1.7 and 5.3 seconds). In stationary conditions, there is always enough coherence between narrowband (sinusoid) signals, irrespective of delays.
Figure 9: The effect of delay–compensation of the actuator–individual filtered–x LMS algorithm. Primary sound field and reduced noise field as mean SPL versus time at BPF (106–110 Hz) in the climb to cruise flight condition. Solid line: With delay–compensation. Dashed line: Without delay–compensation.

In conclusion, in order to perform well in non–stationary conditions of flight, the proposed complex filtered–x algorithms should be supplied with complex reference signals with inherent delays as short as possible. With the present application the situation would have been greatly improved if six tachometer pulses per propeller revolution had been used instead of one. In this case, a much shorter FFT filter bank could be designed since the first tachometer signal harmonic would correspond to the first noise harmonic.

A better approach to generate the complex reference signals may be to use a lookup table technique where some periods of the harmonics are stored in tables, and many pulses per propeller revolution are available [2]. A counter would then be used to transform the tachometer pulses to corresponding indices in the tables. Such a technique need not introduce any reference–delay at all.

4 Summary and Conclusions

This paper presents a set of complex multiple–reference adaptive algorithms for active noise control (ANC) in propeller aircraft. The algorithms combine low numerical complexity with high performance in narrowband applications. The feedforward technique presented inherently exploits the narrowband assumption by
using complex filtering and complex modeling of acoustic paths. The evaluation is performed on cabin noise and acoustic data from a Dornier 328. The set of adaptive algorithms presented is based on a weighted complex gradient update. The special structure of the corresponding adaptive filtering problem implies that the Newton-type algorithm can be implemented with the same numerical complexity as the LMS-type algorithms. Hence, the concept of a fast filtered-x Newton algorithm. The evaluations indicate that the filtered-x complex adaptive algorithms show good potential for achieving close agreement with the optimum noise reduction (in a short-time least squares sense) for both the conditions of flight considered here (one stationary and one non-stationary flight condition).

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References


Part IV

Comparison of Multiple- and Single-Reference MIMO Active Noise Control Approaches Using Data Measured in a Dornier 328 Aircraft
This part has been published as:

Comparison of Multiple- and Single-Reference MIMO Active Noise Control Approaches Using Data Measured in a Dornier 328 Aircraft

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Abstract

In many applications of noise control, the greatest annoyance is caused by periodic low frequency noise. Successful reduction of such noise can often be achieved by using an active noise control system with narrowband feedforward control. If several noise sources contribute to the sound field, a \textit{multiple-reference} control system is usually required. This type of system allows the reference signals from each noise source to be processed individually within the controller, thereby enabling individual control of the sound field from each noise source.

The present paper addresses the problem of controlling noise from two sources that are more or less synchronized. A typical application is the control of propeller-generated noise within a twin propeller aircraft. To find out whether a multiple-reference controller is necessary, or if a single-reference controller is sufficient, the performance of a single- versus twin-reference control algorithm is evaluated in a comparative study. The study is performed as a computer simulation (off-line evaluation) using real-life data recorded in a Dornier 328 under different flight conditions. The results demonstrate that the twin-reference controller performs better than the single-reference whenever there is a slight deviation in the rotational speed of the two propellers. The paper also treats the generation of reference signals. The approach presented is based on a fixed sampling rate and uses a sliding FFT filtering technique.

1 Introduction

In recent years, active noise control systems based on feedforward adaptive controllers [1]-[4] have been used with some success to control propeller-induced noise in the interior of propeller aircraft [5]-[9]. The dominating frequencies found in such noise are the Blade Passage Frequency (BPF), and a number of harmonics.
The Active Noise Control (ANC) technique is based on destructive interference of sound fields. The ANC system utilizes loudspeakers to generate a secondary sound field of equal amplitude and with opposite phase to the uncontrolled sound field produced by the noise sources, in this particular case, the two propellers. Using narrowband feedforward ANC, the loudspeakers are driven at each harmonic via an adaptive controller which continuously adjusts the amplitude and phase of the single-frequency reference signals in order to minimize noise level. The controlled noise is measured using control microphones distributed at strategic positions within the cabin, and the output signals from these microphones are used to adjust the controller.

The reference signals, which are generated from a synchronization signal originating from the noise source(s), contain the same frequencies as the noise components to be suppressed. The attenuation of these frequencies is dependent upon the correlation between the reference signals and the noise [1],[2].

Since the narrowband feedforward controller is based on synchronization signals from the noise sources, such a controller is selective with respect to frequency components. In the development of new controllers for interior aircraft noise, it is of great importance to reduce the complexity of the control algorithm while maintaining efficient noise control. The question to be asked in such a case is whether it is really necessary to use a twin-reference controller, or if a single-reference controller would perform equally well. This question is of special interest in the case of new aircraft, where the two propellers are kept at almost identical speed due to efficient synchrophasing.

The present paper presents a comparative study of two control strategies to determine whether a multiple-reference controller is necessary, or if a single-reference controller can do the job equally well. The performance of each controller is evaluated using computer simulations for two different flight conditions: one flight condition with the propellers synchronized, and one with unsynchronized propellers. Details of the flight conditions and the recordings of the interior aircraft noise inside the passenger cabin are described in Section 2. This section also describes the control system set-up and the arrangement of loudspeakers, control microphones and monitor microphones. The control algorithm and the reference signal generation are treated in Section 3. Section 4 describes the off-line evaluation and the evaluation criteria. Section 5 presents the results.

2 Data Collection and Control System Set-Up

The aircraft used in the experiment was a Dornier 328, a twin-engined propeller aircraft with 31 passenger seats. This type of aircraft is equipped with Tuned Vibration Absorbers (TVAs), which is a common passive method for reducing propeller-induced noise inside the cabin [10],[11]. When properly used, this approach is quite efficient, but there are some important drawbacks. Each TVA is tuned to a specific frequency; if a broader absorption bandwidth is required, a low Q-value must be used, which results in less absorption. To compensate for the
reduced absorption and enable energy to be absorbed at higher harmonics, a large number of absorbers must be used, which significantly adds to the weight of the aircraft. The passive TVAs are unable to track variations in the BPF. They are often tuned to operate under cruise flight conditions.

In the investigation presented in this paper, all of the TVAs were dismounted from the fuselage. The evaluation of the different control strategies (single- and twin-reference controllers) was based on computer simulations (off-line evaluation) using noise recorded inside the fully-trimmed passenger cabin, and simultaneously recorded synchrophaser signals from the right and left engines. The synchrophaser signals delivered one pulse per propeller revolution. All of the signals used in the evaluation were recorded while the aircraft was in flight.

2.1 Flight Conditions

Data sets were recorded from two different flight conditions: the steady-cruise flight, and the non-stationary transition from climb to steady cruise flight. The synchrophaser unit was activated under both flight conditions, and was able to maintain almost identical rotational speed for the two propellers.

In the steady-cruise flight condition, the two propellers were synchronized and the rotational speed of the propellers was held virtually constant at 1050 rpm. The propellers of a Dornier 328 are equipped with six blades, resulting in a BPF of 105 Hz.

In the non-stationary transition from climb to steady-cruise, the rotational speed of the propellers dropped from 1100 down to 1050 rpm (the BPF changed from 110 to 105 Hz). Furthermore, the synchrophaser was unable to keep the propellers synchronized at all times, resulting in a slight difference in the rotational speed of the two propellers. The maximum frequency difference in the BPF between the right and left propeller was approximately 1 Hz.

A difference in the rotational speed of the propellers causes an acoustic beating inside the cabin which is usually unpleasant, with a reduction in comfort. The goal should thus be for the control system to eliminate or reduce the beating in order to increase passenger flight comfort.

2.2 Control System Set-Up

Schematic drawings of the single- and the multiple-reference controllers are shown in Fig. 1. The single-reference controller utilizes one synchronization signal from either the right or the left engine to generate the harmonic reference signals, while the multiple-reference controller uses synchronization signals from both engines. The reference signals for the single-reference controller contains the fundamental frequency (BPF) as well as the three first harmonics (2×BPF–4×BPF) originating from one propeller only, while the multiple-reference controller uses reference signals containing the fundamentals and the three first harmonics from both propellers.

In order to achieve volumetric noise control of an enclosed sound field such as that found in an aircraft, several loudspeakers and control microphones are required
Figure 1: MIMO system for active noise control. (a) Single-reference controller. (b) Twin-reference controller.
Comparison of Multiple- and Single-Reference MIMO Active Noise
Control Approaches Using Data Measured in a Dornier 328 Aircraft

thereby necessitating the use of a Multiple-Input, Multiple-Output (MIMO) control system [1],[2]. The experimental MIMO system used in the present evaluation was set-up with two different configurations: one with 32 loudspeakers and 48 control microphones, and one with 32 loudspeakers and 39 control microphones.

2.2.1 Control Microphones

In this paper, we refer to control microphones as the set of sensors used as feedback to the control system. In the present project, the 48 control microphones were placed at passenger ear level. The six full rows closest to the propeller plane (rows 2 to 7) were equipped with six microphones each. In this aircraft, a full row has 1+2 seats. Two of these microphones were placed at the left and right trim panels, three at the middle of the headrests, and one between the double seats. The remaining 12 microphones were distributed with four in row 1 (a double seat), and four each in rows 8 and 9, see Fig. 2(a).

The main results in this paper were produced using the full microphone set. The Results section also contains results obtained with a subset consisting of 39 microphones, referred to as the subset, see Fig. 2(b). It is important to note that the subset was not obtained from an optimization procedure, but merely by removing 9 microphones containing strong disturbances. At an early stage in the project, it was suggested that these disturbances could have a deteriorating effect on the overall control, and should thus be excluded from the control loop.

2.2.2 Monitor Microphones

In most practical applications, it is not possible to put the control microphones exactly in the positions where the noise reduction is most desired. Due to construction, design or assembly factors, or simply because these positions are occupied, e.g. by the passengers. For this reason, it has become general practice in active control installations to use a set of microphones solely for evaluation purposes. These monitor microphones are not part of the control system and can be placed at those arbitrary positions where the control system is to be evaluated.

In the present project, however, the control microphones were actually placed at the desired positions for historical reasons. The evaluation was therefore made at the control positions and no separate monitor microphones were used. As mentioned previously, the control system was tested with different numbers of control sensors, but the evaluation was always made using the full set, i.e. 48 microphones, see Fig. 2(a).

2.2.3 Loudspeakers

The locations of the 32 loudspeakers were identical for both control microphone configurations. The loudspeakers were placed in the first ten-seat rows, and in five horizontal planes. They were mounted on the trim panels at floor level, beside the seats, at passenger head level, under the overhead luggage bins, and in the ceiling. Fig. 3 shows a cross-section of the aircraft with the possible positions for
Figure 2: The interior of the cabin of a Dornier 328, showing layout of the passenger seats and the locations of the control microphones; (a) 48 microphones (*full set*), (b) 39 microphones (*subset*). Note, the full set was also used as the monitor microphone set.
the loudspeakers and the five horizontal planes: floor level, seat level, head level, luggage bin level and ceiling. The locations of the loudspeakers for each individual plane are shown in Fig. 4.

In order to achieve the best possible noise reduction at passenger head level (i.e. at the monitor/control microphones), the placements of the loudspeakers were based on an optimization of these positions, given the positions of the 48 microphones. In order to ensure that the selected loudspeaker positions were realistic from an installation point of view, the optimization algorithm could only choose positions from a given list of possible loudspeaker locations.

Each loudspeaker unit was mounted into a closed cabinet with the dimensions $20 \times 20 \times 12 \text{ cm}^3$. The cabinet was designed to simulate the volume between the trim panel and the fuselage at many of the possible loudspeaker unit positions for installation of an ANC system [11].

2.2.4 Acoustic Paths

The acoustic paths (frequency response functions) between each of the 32 loudspeakers and all of the 48 microphones were measured under quiet, steady conditions, with the aircraft on the ground. The excitation was white noise, and the signal generator was connected in turn to each of the loudspeaker power amplifiers. The input signals to the power amplifiers and the signals from all the microphones were recorded in parallel for 40 seconds using a sampling frequency of 2048 Hz. Based on these data, the frequency response functions between all loudspeakers and microphones were calculated with a frequency resolution of 1 Hz for the range of interest (approximately 100–450 Hz).
Figure 4: Arrangement of the loudspeakers for the five horizontal planes, see Fig. 3.
3 The Control System

3.1 The Algorithm

The interior noise in propeller aircraft is usually dominated by narrowband harmonic components related to the blade passage frequencies of the propellers [5],[10]. It is assumed that for each propeller there is a periodic synchronization signal available which is correlated with the noise inside the cabin. For this reason, a control model with pure sinusoidal reference signals and complex notation was used, as described below. The algorithm is based on the complex filtered-x LMS algorithm [2], [12]–[14].

The multiple-reference controller [15] is described for a general control situation with $M$ control microphones, $L$ loudspeakers and $R$ reference signals. Each reference has $H$ harmonics. The following notation is introduced: $x_{rh}(n)$ and $w_{rh}(n)$ denote the complex scalar reference signal and the $L \times 1$ vector of complex loudspeaker weights respectively. Both are associated with the $r$th reference and the $h$th harmonic. We assume that the reference signals, $x_{rh}(n)$, $r = 1, 2, \cdots, R$, and $h = 1, 2, \cdots, H$, are mutually uncorrelated. The $M \times L$ matrix of complex frequency responses between all loudspeakers and microphones associated with a given reference signal, $x_{rh}(n)$, is denoted by $F_{rh}$.

A block diagram representation of the multiple-reference MIMO system is shown in Fig. 5. The real valued $M \times 1$ vector $e(n)$ of control microphone signals $e_m(n)$ (controlled noise), is given by

$$e(n) = d(n) + \sum_{r=1}^{R} \sum_{h=1}^{H} \Re\{F_{rh}x_{rh}(n)w_{rh}(n)\}$$

where $n$ is the discrete time index, $d(n)$ is an $M \times 1$ vector of real signals ($d_m(n)$ representing the uncontrolled noise at microphone $m$), and the operator $\Re\{\cdot\}$ denotes the real part. The cost function to be minimized is the sum of the squared output signals (the power) from the control microphones:

$$J(n) = \sum_{m=1}^{M} e_m^2(n) = e^T(n)e(n).$$

The adaptive weight vector, $w_{rh}(n)$, is updated in the negative direction of the gradient vector for the cost function, $\nabla J(n)$,

$$w_{rh}(n+1) = w_{rh}(n) - M_{rh} \nabla J(n)$$

where $M_{rh}$ is a convergence factor matrix (step-size matrix). The relationship between the gradient vector and the complex derivatives of the cost function is given by [12],[16]:

$$\nabla J(n) = 2 \frac{\partial J(n)}{\partial w_{rh}^*(n)} = 2x_{rh}^*(n)F_{rh}^H e(n)$$
Adaptive Algorithm

\[ w_{rh}(n+1) = w_{rh}(n) - 2M_{rh} x^*_{rh}(n) \hat{F}^H_{rh} e(n). \]  

(5)

The algorithm in Eq. (5) is justified by the assumption that the single-frequency reference signals, \( x_{rh}(n), r = 1 \ldots R, \) and \( h = 1 \ldots H \) are mutually uncorrelated, thereby enabling individual control of each frequency. Since only one adaptive complex coefficient is required for each reference signal and loudspeaker, the resulting multiple-reference algorithm described above is extremely efficient in the sense that it employs a minimum of adaptive coefficients.

The choice of convergence factor matrix, \( M_{rh} \), is very important and affects the performance of the algorithm in terms of convergence rate and tracking ability. Either a steepest descent algorithm (LMS algorithm) or a Newton’s algorithm can be used. The convergence factor matrix for both algorithms depends upon the Hessian matrix \( E \{ x^*_{rh}(n) \hat{F}^H_{rh} \hat{F}_{rh} x_{rh}(n) \} \) [1]. The ordinary normalized filtered-x LMS-algorithm [12],[17] is given by:

\[ M'_{rh} = \mu_0(\rho_{rh} \text{trace} \{ \hat{F}^H_{rh} \hat{F}_{rh} \})^{-1}I \]  

(6)

where \( I \) is an \( L \times L \) identity matrix, \( \mu_0 \) is a positive normalized convergence factor, and \( \rho_{rh} \) denotes the power of the reference signal, \( x_{rh}(n) \) (\( \rho_{rh} = E \{ |x_{rh}(n)|^2 \} \)). The convergence factor matrix, \( M''_{rh} \), for the spatially Newton-like algorithm [1],[17] is given by:

\[ M''_{rh} = \mu_0(\rho_{rh} \hat{F}^H_{rh} \hat{F}_{rh})^{-1}. \]  

(7)

The Newton-like algorithm given by (5) and (7) may be highly efficient with respect to convergence rate, but is very demanding from a calculation point of view since the algorithm requires a real-time matrix multiplication.
In applications of active noise control with a large number of loudspeakers and microphones, such as in aircraft, the acoustic coupling between the loudspeakers will decrease as the distance between them increases. According to our experience, this situation leads to a matrix $\hat{F}_rh^H\hat{F}_rh$ that is usually diagonally dominant, since the diagonal elements contain magnitude squares of frequency responses, and the off-diagonal elements result from cross products of different responses ($\hat{F}_rh^H\hat{F}_rh \approx \text{diag} \left\{ \hat{F}_rh^H\hat{F}_rh \right\}$) [16]. By using the diagonally dominant property of the Hessian matrix an approximation of the convergence factor matrix for the Newton’s algorithm could be written as:

$$M''_{rh} = \mu_0 (\rho_{rh} \text{diag} \left\{ \hat{F}_rh^H\hat{F}_rh \right\})^{-1}$$  \hspace{1cm} (8)

where the matrix $\text{diag} \left\{ \hat{F}_rh^H\hat{F}_rh \right\}$ is the diagonal matrix consisting of the diagonal elements of $\hat{F}_rh^H\hat{F}_rh$. The algorithm given by Eqs. (5) and (8), [18], requires only scalar divisions and has a complexity comparable to the ordinary normalized filtered-x LMS-algorithm.

Although the approximation given in Eq. (8) is rather crude, the algorithm given by Eqs. (5) and (8) has proven to be very efficient, the reason being that it represents a sensible compromise between the robustness of the normalized LMS and the speed of the Newton’s algorithm. In addition to being highly efficient, the implementation can usually be made very compact, leading to fast execution of the code.

Care must be taken, however, in the choice of the convergence factor, $\mu_0$. The limit $\mu_0 < 1$ does not generally guarantee convergence of the algorithm given by Eqs. (5) and (8). In a practical situation, the choice of $\mu_0$ is not usually a problem provided that the ANC system can be adequately tested under different operating conditions.

### 3.2 Generation of Complex Reference Signals

One major advantage with narrowband active control of periodic noise components is that the reference signals can be generated internally the controller, using synchronization signals [2]. With reference signals generated in this manner, adaptive control becomes extremely selective and stable. Furthermore, it is possible to determine which harmonics are to be controlled, and which are not.

The method typically used by the present research group was a table lookup scheme consisting of several tables, where each individual table contains a single sampled sinusoidal function. With one table for each harmonic the number of operations can be kept to a minimum, which is important for the implementation of an efficient controller in a real-time system. The generation of complex reference signals is straightforward and can be done either by using a pair of table pointers, where one pointer is suitably delayed, or by using a sine and a cosine table.

In the present application, the table lookup method could not be used since the algorithm was to be implemented on external hardware that was originally constructed for a FIR-based adaptive control scheme. This hardware delivered
a composite reference signal, \( s_r \), containing the sum of the four harmonics to be controlled in accordance with:

\[
s_r(n) = A \sin\left(\frac{2\pi f_{rn}}{f_s}\right) + B \sin\left(\frac{2\pi 2f_{rn}}{f_s}\right) + C \sin\left(\frac{2\pi 3f_{rn}}{f_s}\right) + D \sin\left(\frac{2\pi 4f_{rn}}{f_s}\right)
\]

(9)

with \( f_s \) being the sampling rate, and \( f_r \) the BPF for this reference. The amplitudes of the sinusoids are given by \( A, B, C \) and \( D \). In order to obtain separated, complex, harmonic reference signals for the complex adaptive algorithm, a sliding FFT-operation was used as a parallel filter bank [19]. The required FFT resolution, \( \Delta f \), for the filter bank was selected according to the system sampling frequency and the frequency separation, \( f_r \), of the harmonics at the lowest engine speed. The frequency bin number, \( k_h \), to be used as the output for harmonic \( h \) with frequency \( f_h \) was calculated using the relationship:

\[
k_h = \frac{f_h}{\Delta f} = \frac{f_h N}{f_s}
\]

(10)

where \( N \) is the size of the FFT.

Another parameter that also affects the required FFT size is the shape of the filter function produced by the FFT. The attenuation of out-of-band frequencies is rather low for the unwindowed FFT and it is advisable to weight the signal using a window function before computing the FFT. The windowing function increases the attenuation of signals outside the pass band centered at each frequency bin. This is generally referred to as reducing the leakage in the FFT. The problem with common windows, e.g. Hamming and Blackman [20],[21], is the lack of precise control of the critical passband and stopband frequencies. The values of these frequencies, in general, depend on the type of window and the window length. A window function with the proper filter shape can be constructed using linear-phase filter design techniques [20],[21], such as the Matlab remez function or Kaiser windows [21],[22].

For a Kaiser window of a particular length, the passband attenuation can be controlled with an extremely high attenuation as a result. The disadvantage with such a window is, however, the lack of control of the passband and stopband frequencies. The advantage using remez designed windows is the possibility of choosing arbitrarily the passband and stopband frequencies, and designing windows with a wide and flat passband zone, and high attenuation in the stopband. A wide passband is recommended in flight conditions with varying rpm since the lowest possible amplitude distortion of the generated reference signals is desired.

With a required suppression of 40 dB for out-of-band frequencies, the minimum harmonic separation for this particular case was 5 FFT-bins. To resolve harmonics separated by 100 Hz, the required FFT size was \( N = 64 \) (\( f_s = 1024 \) Hz). Figure 6(a) shows the frequency responses of some common window functions of the length of 64, and a remez designed window function of the same length and with passband and stopband frequencies of 30 and 95 Hz respectively. Figure 6(b) shows the input to and outputs from a filter bank using a 64-point FFT and remez-designed window functions. At the far back is the magnitude the input signal, \( s_r(n) \), containing the
BPF and three harmonics. In front of that is shown the outputs, \( x_{rh}(n) \), of the filters for the BPF and its harmonics. Especially in the output from the BPF-filter, it is possible to see the residual components from the harmonics in the input signal that are not completely attenuated.

Given the real, scalar synchronization signals, \( s_r(n) \), where \( r = 1, 2 \), the complex, scalar reference signals, \( x_{rh}(n) \), are generated by computing the FFT-operation on a sample-by-sample basis in accordance with:

\[
x_{rh}(n) = \sum_{q=0}^{N-1} h(q) s_r(n - q) e^{j2\pi/k_h q} \tag{11}
\]

where \( n \) is the sample index and \( h(q) \) is a window function.

The implication of Eq. (11) becomes evident when a synchronization signal of the form \( s_r(n) = 2 \cos(\omega_r n) \) is considered. In this case

\[
x_{rh}(n) = e^{j\omega_r n} H(\omega_r - \frac{2\pi}{N} k_h) + e^{-j\omega_r n} H^*(\omega_r + \frac{2\pi}{N} k_h) \approx e^{j\omega_r n} H(\omega_r - \frac{2\pi}{N} k_h) \tag{12}
\]

where \( H(\omega) \) is the frequency response of the window \( h(q) \) determined by

\[
H(\omega) = \sum_{q=0}^{N-1} h(q) e^{-j\omega q}. \tag{13}
\]

The approximation in Eq. (12) is valid provided that the FFT size \( N \), the window \( h(q) \), and the FFT-bin \( k_h \), are properly chosen. The argument \( (\omega_r - \frac{2\pi}{N} k_h) \) corresponds to the passband, and the argument \( (\omega_r + \frac{2\pi}{N} k_h) \) corresponds to the stopband of the lowpass filter \( H(\omega) \).

For a given situation, such as with a particular flight condition, the choice of the FFT size \( N \), the window \( h(q) \), and the FFT-bin \( k_h \) becomes a straightforward FIR filter design problem. If the conditions are stationary, or almost stationary, fixed values of \( k_h \) may be used. If, on the other hand, the engine speed varies significantly, it may be necessary to estimate continuously the frequency of the \( h \)th harmonic, and to recalculate the corresponding FFT-bin, \( k_h \).

If a dynamic change of FFT-bin \( k_h \) is necessary, it is important to add a phase correction to the complex reference signals as the bin number changes. With an appropriate phase correction, however, the bin-change can be made without phase jumps. It is assumed that the windowing sequence is a linear-phase FIR filter with the frequency response:

\[
H(\omega) = e^{-j\omega \frac{N-1}{2}} A(\omega) \tag{14}
\]

where \( A(\omega) \) is a real amplitude function. With a complex reference signal generated as in Eq. (11), the phase difference between the outputs of two adjacent FFT-bins (e.g. \( k_h \) and \( k_h + 1 \)) is \( \pi \frac{N-1}{N} \) radians. According to Eqs. (12) and (14), the complex reference signal can be approximated as:

\[
x_{rh}(n) \approx e^{j\omega_r n} H(\omega_r - \frac{2\pi}{N} k_h) = e^{j\pi \frac{N-1}{N} k_h} e^{j\omega_r (n - \frac{N-1}{N})} A(\omega_r - \frac{2\pi}{N} k_h). \tag{15}
\]
Figure 6: (a) Frequency responses of several window functions of length 64. (b) Filter bank using a 64-point FFT and a remez designed window function.
Even for small numbers of $N$, this phase shift almost corresponds to a sign change in the reference signal and will generate an audible click at the output from the adaptive filter as the FFT-bin is changed. A complex reference signal, $x'_{rh}(n)$, the phase of which is independent of FFT-bin $k_h$, is obtained as:

$$x'_{rh}(n) = x_{rh}(n)e^{-j\pi \frac{N-1}{N}k_h} = e^{j\omega_r(n-N^{-1})}A(\omega_r - \frac{2\pi}{N}k_h). \quad (16)$$

It is also possible to obtain a unit magnitude complex reference signal, $x''_{rh}(n)$, the phase and magnitude of which are independent of the FFT-bin $k_h$, as:

$$x''_{rh}(n) = \frac{x'_{rh}(n)}{|x'_{rh}(n)|}. \quad (17)$$

One significant disadvantage of the sliding FFT filtering technique, except for the fairly high implementation cost, is the delay caused by the linear-phase filter employed by this method, see Eq. (14). The group delay corresponds to one-half of the length of the FFT, which for the present case is 32 ms. As a result of this delay, very rapid changes in the engine speed will not be accurately traced by the controller. Fortunately, this is not a major problem in an aircraft application since most speed variations occur relatively slowly.

4 The Computer Simulations

The computer simulations were carried out on signals recorded in a twin-engined propeller aircraft during flight. The signals from all 48 microphones within the aircraft cabin were recorded as well as synchrophaser pulses from both engines.

A power spectrum of the cabin noise during steady-cruise flight is shown in Fig. 7. The spectrum is dominated by strong tonal components originating from the two propellers. The most dominant frequency components are the BPF and the first harmonic. In order to achieve a significant overall reduction in the interior noise, it is necessary to reduce the propeller BPF and two or three harmonics.

In the present evaluation the controllers were based on (5) and (8) and used zero initial weight vectors. The controllers were set up to suppress the BPF and up to three harmonics ($2 \times \text{BPF} - 4 \times \text{BPF}$). The principle for the twin-reference controller (Multiple Reference) using $L$ loudspeakers and $M$ control microphones is shown in Fig. 8. In the computer simulations, the number of loudspeakers is $L = 32$, and the number of microphones is $M = 48$ or $M = 39$. The configuration of the single-reference controller was identical, except that only one of the composite real reference signals, $s_1(n)$ or $s_2(n)$, was used. As mentioned previously, these composite signals consisted of four harmonics (the $\text{BPF} - 4 \times \text{BPF}$) from the left and right propellers respectively.

The single- and twin-reference controllers were based on the complex filtered-X LMS algorithm given by Eqs. (5) and (8). For each controller $\mu_0$ was chosen as $\mu_0 = \frac{1}{2} \mu_{\text{max}}$, where $\mu_{\text{max}}$ was obtained experimentally. The relationship between $\mu_{\text{max}}$ for the single- and twin-reference controllers is given by $\mu_{\text{max}}^{\text{single}} \approx 2 \mu_{\text{max}}^{\text{twin}}$. 
Figure 7: Power spectrum of the interior noise in a Dornier 328 (without tuned dampers) during steady-cruise flight and synchronized propellers. BPF=105 Hz, 2×BPF=210 Hz, 3×BPF=315 Hz and 4×BPF=420 Hz.

4.1 Reference Signal Emulation

In preparation for the computer simulations, the composite reference signals, \( s_1(n) \) and \( s_2(n) \), had to be derived from the recorded synchrophaser signals containing one pulse per propeller revolution. On the targeted platform, \( s_1(n) \) and \( s_2(n) \) were obtained in real-time by using a phase-locked loop to multiply the synchrophaser pulse frequency by six, thus setting the fundamental of the pulse signal as the same frequency as the BPF. The composite reference signals were then obtained by suppressing all frequency components above the fourth harmonic using a filter, see Fig. 9(a). For the computer simulations, this operation was emulated off-line, using the sliding FFT filtering technique described above for generating complex reference signals from the composite reference signals; an FFT size of 1024 was used, however, see Fig. 9(b).

The FFT operation introduced a delay of 512 ms in the reference signal with respect to the microphone signals. This delay, that only exists for the computer simulations and not in the real implementation, reduces the tracking capabilities of the controller. The steady state performance, however, is not affected. In the simulations below, this delay was compensated for in order to obtain a more accurate emulation of the real-time operation performed by the hardware part of the controller implementation for this particular project. Note that the delay of 32 ms in the generation of the complex reference signals was not compensated for since this operation was actually a part of the controller implementation.
Comparison of Multiple- and Single-Reference MIMO Active Noise Control Approaches Using Data Measured in a Dornier 328 Aircraft

4.2 Assessment of Attenuation from Simulations

The mean narrowband attenuation, $A$, for harmonic $h$ obtained from simulated control is given by:

$$A = 10\log_{10} \frac{\sum_{m=1}^{M} |D_{mh}|^2}{\sum_{m=1}^{M} |E_{mh}|^2} \text{ [dB]}$$  \hspace{1cm} (18)

where $|D_{mh}|$ and $|E_{mh}|$ are the magnitudes of the Fourier transforms of the uncontrolled and controlled noise respectively. The full microphone set was used for evaluation purposes. The magnitudes were taken from averaged autospectra, using blocks of 256 samples with Hanning-windowed data. The data were taken towards the end of the simulation sequence to ensure that the control algorithm had converged properly.

4.3 Optimum Least Mean Squares Attenuation

The mean noise attenuation obtained in the computer simulations was compared with the theoretically computed optimum reduction in a least squares sense [1],[23]. The predicted optimum solution is obtained by finding the least squares solution to the following equation for each harmonic $h$:

$$F_h'w_h + D_h' = 0.$$  \hspace{1cm} (19)

Here, $D_h'$ is an $M \times 1$ complex vector containing the uncontrolled noise in the control microphone subset, and $0$ is an $M \times 1$ null vector. The optimum LMS
weights are thus given by:

\[ w_{h,\text{opt}} = - \left[ (F_h')^H F_h' \right]^{-1} (F_h')^H D_h' \]

and the optimum LMS attenuation is obtained by calculating the ratio of the powers of the uncontrolled and controlled noise in the full set

\[ A_{\text{opt}} = 10 \log_{10} \frac{\|D_h\|^2}{\|F_h w_{h,\text{opt}} + D_h\|^2} \text{ [dB]}, \]

were \( \| \cdot \|^2 \) is the squared Euclidean norm.

### 4.4 Evaluation Criteria

The controllers were evaluated with respect to their convergence properties and by comparing the steady state attenuation with the least mean squares optimum attenuation. The convergence performance of the controllers in this study is presented as learning curves, i.e. curves showing the change in normalized mean sound pressure level versus time, normalized to 0 dB at \( t = 0 \) seconds. The average is taken for the full set of microphones. The comparison with the optimum least mean squares solution is presented in tabular form, where the simulated attenuation is obtained from the performance curves at \( t = 9 \) seconds, the end of the time span.
5 Results

5.1 Steady-Cruise Flight

During the steady-cruise flight condition, the synchrophaser can usually keep the propellers almost perfectly synchronized. In the present case, the rpm was held fairly constant during recording, and the BPF was 105 Hz.

The noise reduction obtained at the BPF for the full microphone set is shown in Fig. 10. The figure shows the learning curves for both the single- and twin-reference controllers and it is obvious from these curves that the two controllers behave identically under the same circumstances. Clearly, since the two reference signals are identical, no extra control is gained by using the twin-reference controller. It is also interesting to note that the twin-reference controller converges nicely, although in fact two individual (though identical) controllers use the same inputs to control the same signals. It was suspected that ill-behaved convergence was due to too many degrees of freedom for this particular case.

Table 1 summarizes the attenuation obtained from the simulations, and from the optimum least mean squares calculations. The attenuation is shown for the BPF and three harmonics, and shows that the agreement between the optimum attenuation and the attenuation obtained from the simulations is fairly good.

![Figure 10: Normalized mean SPL versus time at the BPF in steady-cruise flight condition, and perfectly synchronized propellers. Solid curve: Twin-reference controller. Dashed curve: Single-reference controller (right). Dotted curve: Single-reference controller (left).](image-url)
Table 1: Comparison between the attenuation from the simulations and the optimum least squares attenuation.

<table>
<thead>
<tr>
<th>Controller</th>
<th>BPF</th>
<th>2×BPF</th>
<th>3×BPF</th>
<th>4×BPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twin-/Single-reference</td>
<td>dB</td>
<td>dB</td>
<td>dB</td>
<td>dB</td>
</tr>
<tr>
<td>Attenuation (A_{\text{sim}})</td>
<td>17.1</td>
<td>10.4</td>
<td>6.6</td>
<td>5.7</td>
</tr>
<tr>
<td>Optimum Attenuation (A_{\text{lms}})</td>
<td>18.6</td>
<td>11.2</td>
<td>8.7</td>
<td>8.7</td>
</tr>
</tbody>
</table>

### 5.2 The Transition from Climb to Cruise Flight

In the non-stationary transition from *climb* to *steady-cruise* flight, the BPF for the propellers varied from 110 Hz to 105 Hz within the time frame covered by the simulations. Simulations were performed to investigate the dynamic properties (tracking performance) of the controllers. Due to the large variation in the BPF, the acoustic paths, \(F_{rh}\), in the model, see Eq. (1), and in the algorithm update equation (5) had to be altered to reflect the change in BPF, keeping in mind that the acoustic paths were analyzed with a frequency resolution of 1 Hz. The frequency bin number for the reference signal generation did not need to be changed since the analyze bandwidth was approximately 20 Hz, and the third harmonic, which varied between 440 Hz and 424 Hz, was contained completely within a single bin. Fig. 11 shows the variation in the BPF for the two propellers during the transition from climb to cruise.

![Figure 11: Variation in BPF of the right and left propeller during the climb to steady-cruise flight.](image-url)
Figs. 12 and 13 show the normalized mean SPL versus time at the BPF and its first harmonic (2×BPF) respectively when using the full microphone set. Contained in these figures are the learning curves from three different controllers: a twin-reference controller, a single-reference controller synchronized to the left engine, and a single-reference controller synchronized to the right engine. Although the synchrophaser was constantly engaged, there were occasions when it failed to keep the two propellers perfectly synchronized. The maximum difference in the BPF between the left and right propellers during this transition was approximately 1 Hz. The overall result demonstrated in Fig. 12 is that the twin-reference controller performs better than, or as well as, the single-reference controllers at all times.

The figures also show that there was a performance difference between the two single-reference controllers, using the left and right propeller respectively as a reference. This suggests that the choice of synchronization source is crucial for controller performance. It is even more complicated than this, since the most favorable reference source varies during a flight cycle. One hypothesis is that in flight conditions where there are differences in the rotational speed between the propellers, the best single-reference based noise reduction is probably obtained by using the reference which is most stationary. This is a subject for future research.

![Figure 12: Normalized mean SPL versus time at the BPF in climb to steady-cruise flight condition, and unsynchronized propellers. Solid curve: Twin-reference controller. Dashed curve: Single-reference controller (right). Dotted curve: Single-reference controller (left).]

The fact that the sound field is dominated alternately by the left and right propellers suggests that a twin-reference controller is preferable to a single. This kind of controller is capable of tracking variations in the rotational speed of the individual propellers; this is necessary in order to obtain a substantial noise reduction during the complete flight cycle.

Fig. 14 illustrates the schematic behavior of the single- and the twin-reference controllers when the BPFs from the two propellers are almost identical. As mentioned above, the twin-reference controller is able to track both propellers, while the single-reference controller can only track one, resulting in less overall noise reduction. As a result, the total single-reference attenuation is reduced when the frequency separation, $\Delta f$, is increased. This effect becomes more accentuated as the frequency separation, $\Delta f$, increases and should be taken in consideration in the design of control systems for attenuation of the harmonics from unsynchronized noise sources.

The decrease in noise attenuation at approximately 1.5 and 5.5 seconds shown in Fig. 12 is the result of the time delay introduced in the reference signals by the FFT-operation. The 64–point FFT was used to extract the complex reference signals, $x_{rh}(n)$, from the composite reference signals, $s_1(n)$ and $s_2(n)$.

However, in non-stationary conditions this delay implies decreased correlation between the reference signals and the noise, with reduced noise attenuation as a
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Figure 14: Behavior of the single- and the twin-reference controllers. The noise consists of two pure tones \( f_1 \) and \( f_2 \) close in frequency. The frequency separation (the beating) is given by: \( \Delta f = f_1 - f_2 \) \((\Delta f = 0.5–4 \text{ Hz})\).

result. The rapid variations in the BPFs at the corresponding times are clearly visible in Fig. 11.

Fig. 15 shows the effect of compensating for the delay introduced by the 1024-point FFT-filter bank which was used to pre-process the synchrophaser signals producing \( s_1(n) \) and \( s_2(n) \). The lower curve shows the increased dynamic performance when compensating for the delay of 512 ms. In order to perform well in non-stationary conditions of flight, the reference signals should be generated with as short inherent delays as possible, e.g. using lookup tables instead of the sliding FFT-operation in the generation process.

When investigating the effect of delay on the control during steady-cruise flight condition, no difference could be detected. This confirms the fact that the delay-problem is a problem related to the dynamic, and not the stationary properties of the ANC system. Under stationary conditions, however, there is always sufficient correlation between narrowband (sinusoidal) signals, irrespective of delays. In such cases the time delay of the reference signals will thus not affect the noise reduction.

5.3 The Subset Configuration

The results presented in this section were obtained by using the \textit{subset} consisting of 39 control microphones, see Section 2.2.1. Note that the attenuation and the optimum least mean squares attenuation were evaluated using the \textit{full} microphone
The learning curves for both the single- and twin-reference controllers at the BPF are shown in Fig. 16. The attenuation obtained from simulations and from the optimum least mean squares calculations is shown in Table 2. Again the controllers converge nicely for this configuration, and the agreement between the optimum attenuation and the attenuation obtained from simulations is fairly good. The drop in attenuation in Table 2 as compared to Table 1 illustrates the effect that a malfunction of control microphones could have on the sound field inside the cabin. The loss in performance due to the malfunction is also clearly shown in a comparison of Figs. 10 and 16.

<table>
<thead>
<tr>
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<td>3.8</td>
<td>-1.8</td>
<td>2.7</td>
</tr>
<tr>
<td>Optimum Attenuation ($A_{lms}$)</td>
<td>9.4</td>
<td>4.1</td>
<td>0.4</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 2: Comparison between the attenuation from the simulations and the optimum least squares attenuation. Note that the controller using the control microphone subset, and the evaluation of noise attenuation is based on the full microphone set, see Eqs. (18) and (21).
The results in this section illustrate another important feature of the narrowband controller described in this paper. The 9 microphones were originally removed from the control loop since these control signals contained very strong disturbances which were uncorrelated with the propeller tones. Nevertheless, the overall noise reduction was substantially increased when these microphones were included, which proves that the control system is extremely frequency-selective, and robust to disturbances uncorrelated with the propeller tones picked up by the control microphones [26].

6 Summary and Conclusions

This paper addresses the problem of controlling noise from two sources that are strongly or moderately synchronized. To find out whether a multiple-reference controller is necessary, or if a single-reference controller is sufficient, the performance of a single- versus a twin-reference controller was evaluated on signals recorded in a propeller aircraft. The simulations performed in this study were all based on measurements produced with the synchrophaser unit engaged.

To be able to reduce efficiently the propeller-induced noise inside the cabin of a twin-propeller aircraft, the controller should be synchronized with both propellers.
This will ensure the highest noise attenuation during the complete flight cycle, i.e. take-off, climb, cruise, descent and landing. Modern propeller aircraft are usually fitted with a synchrophaser unit, with the result that the rotational speed of the two propellers is similar, or almost similar, at all times. With a modern, efficient synchrophaser, it is conceivable that a single-reference controller would suffice. However, disturbances in the air flow may cause transient speed slips that will cause acoustic beating inside the cabin that a single-reference controller cannot attenuate. The simulations indicate that the deviations in propeller synchronization are significant in the transition from climb to cruise flight.

During flight conditions with identical propeller rotational speed no extra attenuation is obtained by using a twin-reference controller. Redundant information in these control modes will, however, not lead to reduced performance for the twin-reference controller since the controller converges robustly.

In conclusion, a multiple-reference controller is preferable to a single-reference since its will efficiently reduce the propeller-induced noise and the beat effect irrespective of the propeller synchronization, resulting in a lower noise level and increased passenger flight comfort. In non-stationary flight conditions, however, the reference signals should be generated with as short inherent delays as possible in order to increase tracking performance.

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References


Part V

Convergence Analysis of a Multiple-Reference Complex Least-Mean-Squares Algorithm
This part has been submitted as:

Convergence Analysis of a Multiple-Reference Complex Least-Mean-Squares Algorithm

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Abstract

In many noise control applications the noise is dominated by low frequencies and generated by several independent periodic sources. In such situations the tonal noise may be suppressed by using a narrowband multiple-reference feedforward controller. The performance characteristics of the control system, e.g. the convergence behavior and noise reduction are directly related to the controller adaptation rate as well as the frequency separation of the tonal components in the noise, i.e. the beat frequency.

This paper treats the convergence performance of a complex Least-Mean-Squares (LMS) algorithm using multiple-reference signals. An analysis of its convergence behavior is presented as well as the results from computer simulations validating the convergence behavior. The convergence of the filter weights and the decrease rate of the squared error (the learning curve) for noise control applications are also discussed.

1 Introduction

Active Noise Control (ANC) is a highly suitable method for reducing noise at low frequencies. This technique is based on destructive interference of sound fields, i.e. the primary sound field from the noise sources, and the anti-noise produced by the control system. In many applications the primary noise is generated by rotating machines and is therefore periodic. In such applications, the use of an adaptive feedforward control system has proved to be a successful means of suppressing the low-frequency noise [1]-[4].

A feedforward controller utilizes a synchronization signal, e.g. a tachometer signal from the noise source which provides information about the radiated noise. This signal contains information about the rotational speed of the machine and may therefore be used to generate sinusoidal reference signals that have the same frequency content as the noise components to be suppressed. In many noise control
applications several noise sources contribute to the sound field, e.g. in twin-engined propeller aircraft [5]-[8], vessels and motor boats [9],[10]. In such applications a synchronization signal from each noise source and a multiple-reference control system are usually required in order to achieve an efficient and substantial noise reduction [11].

A multiple-reference approach allows the reference signals from each noise source to be processed individually within the controller, thereby enabling individual control of the sound field from each source [12],[13]. The principle of a narrowband twin-reference system for active noise control is illustrated in Fig. 1. The single-frequency reference signals are filtered individually using an adaptive filter (parallel filtering) before driving a secondary source, e.g. a loudspeaker. Each filter adjusts the amplitude and the phase of the corresponding reference signal. The loudspeaker produces a secondary noise field of equal amplitude which is 180° out of phase with the primary noise field. An error microphone is used to measure the residual noise achieved, and the output signal from this error-sensor is then used to adjust the adaptive filters so that the level of the residual noise is minimized.

Periodic noise sources with a slight difference in the rotational speed induce an acoustic beating which may lead to considerable discomfort. However, comfort could be improved significantly if the level of the noise and beating is significantly reduced, e.g. by using an ANC system. The performance of an active noise control system such as convergence rate and noise reduction is a direct result of the running conditions of the engines (the beat frequency). The running conditions for several engines can generally be divided into three main types:
• The engines have the same rotational speed.
• The engines have almost the same rotational speeds.
• The engines have completely different rotational speeds.

Synchronization of the engines can be achieved either automatically or manually. Most twin-engined propeller aircraft are fitted with a synchrophaser unit to synchronize automatically the rotational speeds of the two engines. The synchrophaser is, however, generally unable to keep the engines synchronized at all times, resulting in a slight difference in rotational speed variations between the two engines. On the other hand, twin-engined vehicles and boats are not normally fitted with a synchrophaser unit. In such cases, it can be difficult to synchronize manually the two engines. Consequently, the difference in the rotational speeds in the different engines is often larger for manually synchronized engines than for automatically synchronized ones.

The feedforward adaptive algorithm analyzed in this paper is based on the complex Least-Mean-squared (LMS) algorithm [14],[15]. In practical active noise control applications there exists, however, an acoustic path between the loudspeaker and the control microphone necessitating modification of the standard complex LMS algorithm to ensure stability. The algorithm must compensate for the effect of this acoustic path. For this reason, the filtered-x LMS algorithm is normally used in practical ANC applications [1],[16]. In such applications the behavior of the algorithm is also determined by the estimate of the acoustic path.

The algorithm and the analysis of its convergence behavior are presented in Section 2. Section 3 describes the computer simulations. The results are presented in Section 4.

2 A Twin-Reference Complex LMS Algorithm

Consider the feedforward filter structure shown in Fig. 2. This figure depicts the filtering process described herein. The adaptive linear combiner weights a set of input signals to produce an output. In this paper the input signals $x_1(n)$ and $x_2(n)$ and the adaptive filter weights $w_1(n)$ and $w_2(n)$ are assumed to be complex-valued scalars.

In an active noise control application the input signals may correspond to single-frequency reference signals originating from several independent periodic noise sources. Hence, assume that the reference signals $x_1(n)$ and $x_2(n)$ originate from two rotating machines running at different rotational speeds generating noise at the frequencies $f_1$ and $f_2$:

$$x_1(n) = e^{j(\omega_1 n + \varphi_1)}$$
$$x_2(n) = e^{j(\omega_2 n + \varphi_2)}.$$  \hspace{1cm} (1)

Here $\varphi_1$ and $\varphi_2$ denote arbitrary phases and $\omega_i = 2\pi f_i$, $i = 1, 2$, where $f_i$ is the sampling rate. Throughout the analysis $\varphi_1$ and $\varphi_2$ are assumed to be zero. Let the
Figure 2: The filter structure of the adaptive twin-reference active noise controller.

The reference signal vector $\mathbf{x}(n)$ be defined by

$$\mathbf{x}(n) = \begin{pmatrix} x_1(n) \\ x_2(n) \end{pmatrix}$$

and the weight vector $\mathbf{w}(n)$

$$\mathbf{w}(n) = \begin{pmatrix} w_1(n) \\ w_2(n) \end{pmatrix}. \quad (3)$$

The output signal from the filter at time $n$ is given by

$$y(n) = \mathbf{w}^H(n)\mathbf{x}(n) \quad (4)$$

where $(\cdot)^H$ denotes the transpose-conjugate. Assume that the desired signal $d(n)$ (the primary noise to be reduced) is a composite signal consisting of two single-frequency sinusoidal signals with the same frequencies as the reference signals:

$$d(n) = H_1 e^{j\omega_1 n} + H_2 e^{j\omega_2 n} \quad (5)$$

where $H_1$ and $H_2$ are complex constants. The error signal $e(n)$ can now be written as

$$e(n) = d(n) + y(n) = d(n) + \mathbf{w}^H(n)\mathbf{x}(n). \quad (6)$$

The complex LMS adaptive algorithm minimizes the quadratic cost function:

$$J(n) = |e(n)|^2$$
\[ |d(n)|^2 + d(n)\mathbf{x}(n)^H w(n) + w^H(n)\mathbf{x}(n)d^*(n) + w^H(n)\mathbf{x}(n)\mathbf{x}^H(n)w(n) \quad (7) \]

where \((\cdot)^*\) denotes complex conjugation. The LMS algorithm is a stochastic gradient algorithm. The minimum of the cost function \((7)\) is achieved by following the steepest descent path of this surface by recursively updating the weight vector \(w(n)\) according to the expression:

\[ w(n+1) = w(n) - 2\mu \frac{\partial J(n)}{\partial w^*(n)} \quad (8) \]

Here, the step-size parameter \(\mu\) controls the convergence rate and the stability of the algorithm. Differentiation of the real-valued cost function in \((7)\) with respect to the complex weight gives us the instantaneous gradient vector [15]:

\[ \frac{\partial J(n)}{\partial w^*(n)} = \mathbf{x}(n)e^*(n). \quad (9) \]

With this gradient expression in the update equation \((8)\), the complex LMS adaptive algorithm [14],[15] is given by

\[ w(n+1) = w(n) - 2\mu \mathbf{x}(n)e^*(n). \quad (10) \]

### 2.1 Convergence Analysis

The convergence investigation is based on a deterministic analysis. Hence, no assumptions are made about the dependence between the input vector \(\mathbf{x}(n)\) and the weight vector \(w(n)\). On the other hand such assumptions are often used in analysis of algorithms [15],[16].

Consider the cost function given in \((7)\). By setting the differential of this function with respect to the weight vector to zero the following expression is obtained:

\[ \frac{\partial J(n)}{\partial w^*(n)} = \mathbf{x}(n)d^*(n) + \mathbf{x}(n)\mathbf{x}^H(n)w_o = 0 \quad (11) \]

where \(w_o\) is the optimum filter weight vector for which \(\frac{\partial J(n)}{\partial w^*(n)} = 0\). Note that the matrix \(\mathbf{x}(n)\mathbf{x}^H(n)\) does not have full rank, so that \(w_o\) is non-unique. However, we may choose \(w_o\) as: \(w_o = -(\mathbf{x}(n)\mathbf{x}^H(n))^+\mathbf{x}(n)d^*(n)\), where \((\cdot)^+\) denotes the pseudoinverse [17].

Now we define the weight-error vector \(v(n)\), as the difference between the adaptive weights and the optimal solution:

\[ v(n) = w(n) - w_o. \quad (12) \]

By substituting \((6)\) into \((10)\) and using the expressions in \((11)\) and \((12)\), the recursive relation of the adaptive weights can be rewritten as

\[ v(n+1) = [I - 2\mu \mathbf{x}(n)\mathbf{x}^H(n)]v(n) \quad (13) \]
where $I$ is the identity matrix. To simplify this equation a new time variable matrix $A(n)$ is introduced:

$$A(n) = I - 2\mu x(n)x^H(n). \quad (14)$$

Equation (13) may now be written as

$$v(n + 1) = A(n)v(n). \quad (15)$$

Equation (15) represents a first-order difference equation, the solution of which is

$$v(n) = \left[ \prod_{i=1}^{n} A(n-i) \right] v(0) \quad (16)$$

where $v(0)$ denotes the initial weight-error vector. The matrix $A(n)$ in (14) may be expressed in the expanded form as

$$A(n) = \begin{pmatrix} 1 - 2\mu & -2\mu e^{-j\Delta \omega n} \\ -2\mu e^{j\Delta \omega n} & 1 - 2\mu \end{pmatrix} \quad (17)$$

where $\Delta \omega = \omega_2 - \omega_1$. Accordingly, the matrix $A(n)$ depends both on the step-size parameter $\mu$ and the frequency separation $\Delta \omega$ between the reference signals $x_1(n)$ and $x_2(n)$.

By decomposing the Hermitian matrix $A(n)$ [17], the matrix can be written as

$$A(n) = Q(n)\Lambda Q^H(n) \quad (18)$$

where $Q(n)$ is a unitary matrix where the columns contain the eigenvectors of $A(n)$:

$$Q(n) = \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-j\Delta \omega n} & e^{-j\Delta \omega n} \\ 1 & 1 \end{pmatrix} \quad (19)$$

and $\Lambda$ is a diagonal matrix whose elements consist of eigenvalues of $A(n)$:

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 1 - 4\mu \end{pmatrix}. \quad (20)$$

Observe that the eigenvalues are determined by the step-size parameter $\mu$, while the eigenvectors are determined by the frequency separation $\Delta \omega$ and the time index $n$, resulting in a rotation of the eigenvectors where the rotation speed is connected to the frequency separation.

The quantity that is connected to the convergence performance of the algorithm is given by the matrix product of $A(n)$. By substituting (18) into (16) the matrix product can be expressed as

$$\prod_{i=1}^{n} A(n-i) = A(n-1)A(n-2)\cdots A(0) = Q(n-1)\Lambda Q^H(n-1)Q(n-2)\Lambda Q^H(n-2)Q(n-3)\cdots Q(0)\Lambda Q^H(0). \quad (21)$$
In defining a new matrix $B$ given by
\[ B = \Lambda Q^H(m)Q(m - 1), \tag{22} \]
a compact form of (21) is given by
\[ \prod_{i=1}^{n} A(n - i) = Q(n - 1)B^{n-1}\Lambda Q^H(0) \tag{23} \]
where
\[ B = \frac{1}{2} \begin{pmatrix} 1 + e^{j\Delta\omega} & 1 - e^{j\Delta\omega} \\ (1 - 4\mu)(1 - e^{j\Delta\omega}) & (1 - 4\mu)(1 + e^{j\Delta\omega}) \end{pmatrix}. \tag{24} \]
Note that the matrix $B$ is independent of time index $m$.

A decomposition of the non-Hermitian matrix $B$ can be written on the form:
\[ B = S\tilde{\Lambda}S^{-1} \tag{25} \]
where $\tilde{\Lambda}$ is a diagonal matrix of eigenvalues given by
\[ \tilde{\Lambda} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \tag{26} \]
and $S$ is a matrix where the columns contain the eigenvectors of $B$
\[ S = \begin{pmatrix} s_1 & s_2 \end{pmatrix}. \tag{27} \]
If the eigenvectors $s_1$ and $s_2$ correspond to different eigenvalues $\lambda_1$ and $\lambda_2$, the eigenvectors are linearly independent and the matrix $S^{-1}$ exists [17]. By substituting (25) into (23) the following equation is obtained:
\[ \prod_{i=1}^{n} A(n - i) = Q(n - 1)S\tilde{\Lambda}^{n-1}S^{-1}\Lambda Q^H(0) \tag{28} \]
where $\tilde{\Lambda}^{n-1}$ is given by
\[ \tilde{\Lambda}^{n-1} = \begin{pmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-1} \end{pmatrix}. \tag{29} \]
The eigenvalues $\lambda_1$ and $\lambda_2$ are obtained by solving $\text{det}(\lambda I - B) = 0$. An explicit expression for the complex eigenvalues of $B$ with respect to $\mu$ and $\Delta\omega$ is given by
\[ \lambda_i = e^{\frac{j\Delta\omega}{2}} \left[ (1 - 2\mu)\cos\left(\frac{\Delta\omega}{2}\right) \pm \sqrt{4\mu^2 - (1 - 2\mu)^2 \sin^2\left(\frac{\Delta\omega}{2}\right)} \right]. \tag{30} \]
The eigenvectors $s_i$ are suitably determined by numerically solving $(\lambda_i I - B)s_i = 0$ for a given eigenvalue $\lambda_i$. 
Finally, the expression for the weight-error vector (15) is given by
\[
v(n) = \left[ \prod_{i=1}^{n} A(n-i) \right] v(0) = Q(n-1) S A^{n-1} S^{-1} AQ^H(0) v(0). \tag{31}
\]
The expression given in (31) is convergent if and only if the absolute value of the eigenvalues is less than unity:
\[
|\lambda_i| < 1. \tag{32}
\]
If the condition in (32) is satisfied the weights converge to the optimum weights, hence it follows that
\[
\lim_{n \to \infty} v(n) = 0. \tag{33}
\]
A sufficient condition for convergence, the constraints in (32) are fulfilled, is that the step-size parameter is within the range of
\[
0 < \mu < \frac{1}{2}. \tag{34}
\]

2.2 The Learning Curve

An explicit expression of the squared error versus time (learning curve) with respect to the error weight vector may be written as
\[
J(n) = J_{min} + v^H(n)x(n)x^H(n)v(n) + e_o(n)x^H(n)v(n) + v^H(n)x(n)e_o^*(n) \tag{35}
\]
where \(e_o(n) = d(n) + w_o^H x(n)\) and \(J_{\text{min}} = |e_o(n)|^2\) (the minimum squared error obtained by calculating (7) for \(w(n) = w_o + v(n)\)). However, by studying (35) and (31) it is obvious that the decrease rate of the learning curve is connected to the eigenvalues given in (30), where eigenvalue \(\lambda_i\) is associated with the \(i\)th mode of the learning curve. Accordingly, the convergence performance of the algorithm is strongly connected to the step-size parameter \(\mu\) and the frequency separation between the reference signals \(\Delta \omega\). The sampling rate may also affect the performance, since \(\Delta \omega = 2\pi \frac{\Delta f}{f_s}\). The smaller the absolute value of \(\lambda_i\), the faster the decrease rate of the corresponding mode, and the closer to unity the absolute value of \(\lambda_i\) is, the slower the decrease rate of the mode will be. Thus, with proper eigenvalues the algorithm is convergent:
\[
\lim_{n \to \infty} J(n) = J_{\text{min}}. \tag{36}
\]

3 Computer Experiment

In order to evaluate the performance of the complex multiple reference algorithm computer simulations were performed using different frequency separations \(\Delta f\), between the reference signals and different step-size parameters \(\mu\). The frequency of reference signal \(x_1(n)\), see Fig. 2, was fixed at 100 Hz, while the reference signal
$x_2(n)$ was assigned a frequency in the range of 100-200 Hz ($0 < \Delta f \leq 100$ Hz). The sampling rate was $f_s = 1$ kHz. The desired signal $d(n)$ (the noise signal) consisted of two sinusoidal signals with exactly the same frequencies as the two reference signals and additive white noise $w(n)$ ($E[w^2(n)] = \sigma_w^2 = 10^{-10}$), $d(n) = H_1e^{j\omega_1n} + H_2e^{j\omega_2n} + w(n)$. Small frequency separations between the reference signals correspond to two sources running at almost the same rotational speeds, while large separations correspond to sources running at completely different rotational speeds, or harmonic frequencies generated by a single source. Throughout the simulations the initial filter weight vector $w(0)$ was zero and a normalized step-size parameter was used [15],[16]. Accordingly, in (10) $\mu$ was substituted by

$$\mu = \frac{\mu_0}{\text{trace}(R)} = \frac{\mu_0}{2}$$

where $R$ denotes the input correlation matrix $R = E[x(n)x^H(n)]$ and $E[\cdot]$ denotes the expectation value. The interval of $\mu_0$, for which the algorithm is stable, is given by (cf. (34))

$$0 < \mu_0 < 1.$$ (38)

4 Results

The eigenvalues of the matrix $B$, see (24) and (30), are the critical quantities in the weight iterative process, and influence the performance of the algorithm e.g. the convergence rate and tracking performance. If the condition in (38) is met the algorithm is stable and the magnitudes of the eigenvalues are less than or equal to unity ($|\lambda_i| \leq 1$) irrespective of the frequency separation. Hence, the stability of the algorithm is related to the step-size parameter only and not to the frequency separation.

For small $\Delta f$ the eigenvalues of the matrix $B$ are approximately real valued $\lambda_1 \approx (1 - 2\mu_0)$ and $\lambda_2 \approx 1$. Thus, one of the eigenvalues is very closely related the step-size parameter, while the other approximately is unity for all step-size parameters. The eigenvalue plot and typical learning curves of the multiple-reference algorithm for such cases are shown in Figs. 3 and 4. Here, $\Delta f = 0.5$ Hz and the step-size parameter is in the range of $\mu_0 = 0.1 - 0.9$.

Note that when the magnitude of both eigenvalues is less than unity, the rate of convergence increases as the magnitude decreases. As is clearly shown, small magnitude eigenvalues result in a fast decreasing initial mode, and the closer to zero the value is the faster the mode decays, reaching its maximum for $\lambda_1 = 0$ ($\mu_0 = 0.5$). For positive real parts of the eigenvalues less than unity the learning curves decay with no oscillations (overdamped), while for negative real parts larger than minus unity there are decaying oscillations (underdamped). The duration of the oscillation is determined by the negative real part of the eigenvalue; a value close to minus unity results in slowly decaying oscillations, while a value close to zero results in rapidly decaying oscillations. The effect of the step-size parameter choice is as follows:
0 < \mu_0 < 0.5 \quad \text{overdamped}
\mu_0 = 0.5 \quad \text{critically damped}
0.5 < \mu_0 < 1 \quad \text{underdamped}

It is obvious that an eigenvalue close to unity prevents the algorithm from converging completely, \(\lim_{n \to \infty} J(n) = J_{\min}\). The learning curves level out to steady-state levels larger than \(J_{\min}\), or very slowly decaying modes. However, the steady-state level and the slope of the modes are determined by the step-size parameter. Using (35) the instantaneous excess squared error \(J_{\text{ex}}(n)\) can be defined as:

\[ J_{\text{ex}}(n) = J(n) - J_{\min}. \quad (39) \]

In these simulations \(J_{\min} = \sigma_w^2 = 10^{-10} (-100 \text{ dB})\). In Fig. 4 it is clearly shown that the algorithm converges to a lower steady-state level with an increasing \(\mu_0\), \(J_{\text{ex}}(n)\) decreases and higher noise reduction is obtained. On the other hand, a slower initial mode is obtained for oscillating convergence. The results from the simulations also show that \(J_{\text{ex}}(n)\) decreases for a given step-size parameter when the frequency separation approaches zero (\(\Delta f \to 0\)).

Figures 5 and 6 show the eigenvalue plot and the convergence performance respectively for \(\Delta f = 100 \text{ Hz} \) (large \(\Delta f\)). In these cases there is counterclockwise rotation of the eigenvalues connected to \(\Delta f\) (cf. (30)). The radius of the circular arc on which the eigenvalues are located is also dependent on the frequency separation; the radius increases for increasing \(\Delta f\). When the two eigenvalues are located on this arc their magnitudes are equal (\(|\lambda_1| = |\lambda_2|\)), resulting in uniform and fast convergence. A smaller magnitude leads to faster convergence and vice versa. Compare the cases \(\mu_0 = 0.1\) and \(\mu_0 = 0.2\). Note that in these cases rippled convergence behavior occurs. However, by increasing the step-size parameter the magnitude of one of the eigenvalues approaches unity, which slows down the convergence speed. Furthermore, rippled behavior disappears as a result of different magnitudes of eigenvalues. An additional increase in the step-size causes convergence behavior with initial decreasing oscillations (underdamped) which then change to monotonic slowly decreasing modes or steady-state levels. In cases of large \(\Delta f\) it is seen that \(J_{\text{ex}}(n)\) increases with an increasing step-size parameter [15],[16]. On the other hand, by using small step-size parameters (\(\mu_0 < 0.1\)) to reduce the excess square error, a very slow rate of convergence and rippled convergence behavior is obtained as a result of eigenvalues with magnitudes quite close to unity. This is clearly shown in Fig. 7, which illustrates the change in the magnitude of the eigenvalues for different frequency separations and step-size parameters.

A summary of the noise attenuation obtained after 1000 sample iterations (1 second) versus \(\Delta f\) and using different \(\mu_0\) is shown in Fig. 8. High noise reduction is obtained for frequency separations up to a few hertz using large step-size parameters, while the reverse is true for large separations. In the transition region at approximately 10 Hz the system exhibits degraded performance and significantly reduced attenuation.

Sometimes the noise is composed of harmonic components generated from sources running at the same or almost identical rotational speeds. Such a case can be re-
Convergence Analysis of a Multiple-Reference Complex Least-Mean-Squares Algorithm

Considered as a combination of the two control cases described above. The convergence behavior of an algorithm using four reference signals is shown in Fig. 9. Here, the noise contains the frequencies 100, 100.5, 200 and 200.5 Hz ($\Delta f = 0.5$ and 100 Hz). This figure clearly shows the characteristics previously obtained (cf. Figs. 4 and 6), for instance, as a result of the close frequency components ($\Delta f = 0.5$ Hz). The learning curves level out to a constant level dependent on the step-size parameter.

The results presented above are based on computer-generated signals. The following results are, however, based on real-life signals recorded inside the passenger cabin of a twin-engined propeller aircraft during cruise flight with the two propellers synchronized ($\Delta f \approx 0$ Hz). The multiple-reference controller used was based on a multi-channel complex filtered-x LMS algorithm. The controller used 32 loudspeakers and 39 control microphones [12],[19],[20]. Figure 10 shows the convergence behavior for the controller when controlling the blade passage frequency (105 Hz). It is seen that the convergence behavior obtained in this test coincided with the behavior previously observed in Figs. 4 and 9; a rapid initial mode which levels out into a slowly decreasing secondary mode, and increased noise reduction with increased step-size parameter. In the figure the calculated optimum least squares noise reduction is indicated by a horizontal line. For two different step-size parameters following mean reduction of the blade passage frequency was obtained; 18.5 dB ($\mu_0 = 0.003$) and 20.8 dB ($\mu_0 = 0.03$) respectively averaged over the error microphones. The optimum least squares reduction was calculated to 24.0 dB.

5 Summary and Conclusions

In this paper a theoretical analysis has been performed to explain the convergence behavior of a multiple-reference complex LMS algorithm. An explicit expression of eigenvalues controlling the convergence behavior has been derived. To validate the analysis computer simulations have also been performed.

It is shown that the convergence performance is determined both by the step-size parameter $\mu_0$ and the frequency separation $\Delta f$ between the reference signals (the tonal components which will be reduced). The stability of the algorithm is, however, connected to the step-size parameter only and not to the frequency separation. The analysis shows that different eigenvalues contribute to different degrees to the convergence performance. Each mode of the learning curve has its own time constant, which is determined by the eigenvalue associated with that mode. A small eigenvalue results in a rapidly decreasing mode (rapid convergence), while an eigenvalue close to unity leads to a slowly decreasing mode (slow convergence). A rapid initial mode, however, results in good tracking performance.

Often no distinction is made between convergence in the filter weights and a decrease rate of the learning curve. Convergence is, however, important in adaptive signal processing, but its significance depends on the application. The convergence of the weights is less significant in active noise control and noise canceling applications, while it is of vital importance in inverse filtering problems. In inverse filtering problems all modes are equally important, with the result that the slowest
mode ultimately rules the convergence [18].

On the other hand, in noise control applications the weights in the filter are only of indirect interest. The main purpose is to reduce the noise (minimize the error signal). Accordingly, a complete convergence of the algorithm is not necessary: one is satisfied when a sufficient noise reduction is obtained (the error is small enough). However, in some applications the tracking performance of the algorithm is also of great importance.

In cases of small frequency separations convergence problems may occur independently of the value of the step-size parameter (one eigenvalue is very close to unity). However, in these cases the steady-state noise reduction is increased by increasing the step-size parameter. On the other hand, for large frequency separations a small step-size was shown to be preferable in order to achieve high steady-state noise reduction. In all the cases (both for small and large frequency separations) a large step-size parameter reduced the decrease rate of the initial mode and introduced a decreasing oscillation in the learning curves. Accordingly, in order to obtain good tracking performance it is advisable to choose a step-size parameter giving an eigenvalue close to zero rather than use a large step-size.

To sum up, in practical noise control applications it is important to adjust the step-size parameter so that a desired behavior with respect to tracking performance and/or steady-state noise reduction is obtained for a particular control situation. The noise reduction obtained plays an important role in contrast to convergence in the filter weights. However, in non-stationary noise control applications such as that found in different kinds of vehicles, good tracking performance of the control algorithm is often more desirable than high steady-state noise reduction.
Figure 3: Eigenvalue plot for $\Delta f = 0.5$ Hz, and $\mu_0 = 0.1 - 0.9$.

Figure 4: Learning curves for $\Delta f = 0.5$ Hz and $\mu_0 = 0.1 - 0.9$. 
Figure 5: Eigenvalue plot for $\Delta f = 100$ Hz and $\mu_0 = 0.1 - 0.9$. The arrows show the path of the eigenvalues for increasing $\mu_0$.

Figure 6: Learning curves for $\Delta f = 100$ Hz and $\mu_0 = 0.1 - 0.9$. 

Figure 7: The change of the absolute value of the eigenvalues versus frequency separation between the two reference frequencies.

Figure 8: Attenuation obtained after 1000 samples for different $\Delta f$ and $\mu_0$. Note the attenuation limit is set at 100 dB. (a) $\mu_0 = 0.05$, (b) 0.1, (c) 0.2, (d) 0.3, (e) 0.4, (f) 0.5, (g) 0.6, (h) 0.7, (i) 0.8, (j) 0.9, (k) 0.95.
Figure 9: Learning curves for the algorithm when using four reference signals. The noise contains the frequencies 100, 100.5, 200 and 200.5 Hz, (cf. Figs. 4 and 6).

Figure 10: Learning curves for a multiple-reference ANC controller in an off-line computer evaluation based on noise recorded in an aircraft in flight. The horizontal line denotes the optimum least squares reduction.
References


Part VI

Experimental Performance Evaluation of a Multi-Reference Algorithm for Active Control of Propeller-Induced Cabin Noise
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Experimental Performance Evaluation of a Multi-Reference Algorithm for Active Control of Propeller-Induced Cabin Noise

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Abstract

A noisy environment dominated by low frequency noise can often be improved through the use of active noise control. This situation arises naturally in propeller aircraft where the propellers induce periodic low frequency noise inside the cabin. The cabin noise is typically rather high, and the passenger flight comfort could be improved considerably if this level were significantly reduced.

This paper discusses the operation and robustness of a narrowband feedforward active noise control system in a practical installation. The ANC system used 8 control sources and 11 control microphones, and the control algorithm was the multi-reference actuator-individual normalized filtered-x least mean squares algorithm. The experiment was performed in a full-scale fuselage section of a SAAB 340 aircraft. To produce the propeller noise, loudspeakers mounted in a ring around the fuselage were used. Results are presented from a series of experiments on the active control of propeller-induced cabin noise. Among the “flight” conditions evaluated were: conditions where the “propellers” were completely synchronized, and conditions with constant as well as time-varying frequency beat between “left and right propellers.”

1 Introduction

One major source for aircraft interior noise is the propulsion system [1],[2]. In particular, for propeller aircraft the cabin noise is dominated by harmonic low frequency noise produced by the propellers. The most disturbing noise is typically the first three or four harmonics of the blade passage frequency (BPF). The noise is transmitted through several paths into the cabin: vibrations from the engines are transmitted through the engine mounts into the wing structure, which in turn
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excites the whole aircraft body; turbulence from the propellers excites the rear wing which in turn causes vibrations in the rear part of the aircraft [3]. Another important path is through the fuselage in the plane of the propellers; the propeller blades cause very high pressure fluctuations at the outside of the fuselage which are transmitted into the passenger cabin. The importance of the different transmission paths varies with frequency. At the BPF, the sound field is usually excited throughout the whole cabin, while the harmonics tend to be excited primarily in the plane of the propellers [4].

Due to the low frequency range, typically 80–450 Hz, the practical use of passive noise reduction methods is very limited. The aircraft fuselage is constructed as a light stiff wall with only a marginal low-frequency transmission loss. By using tuned dampers, the transmission loss can be significantly increased [1]-[3]. A tuned damper is a mechanical resonance system consisting of a mass and a spring with a fairly high mechanical loss factor. The damper is tuned to one frequency, typically the BPF at normal cruise speed or one of its harmonics. By using several dampers, it is possible to obtain a noise reduction over a broader frequency range. One major disadvantage with the tuned dampers is the added weight, which can be the equivalent of one passenger or more. This is significant for an aircraft designed for 20–30 passengers. Another disadvantage is that the performance normally is tuned to one flight condition, which implies that the vibration absorbing effect is reduced at other flight conditions.

An active noise control system (ANC) offers much more potential to the noise control engineer [5]-[9]. First of all, the overall attenuation is generally higher than what can be obtained with a passive installation. Since the controller is synchronized with the engines, the attenuation is maintained throughout the complete flight cycle, including cruise, climb and descent. If the controller is synchronized to both engines, the beating that appears as the engines become unsynchronized is also controlled. Even with many (more than 30) loudspeakers including cabinets, the active noise control system is lighter than a normal set of tuned dampers [2].

The active noise control system evaluated in this paper is based on a multiple-reference narrowband feedforward control approach and is designed to attenuate the tones generated from the propellers. The controller is based on the actuator-individual normalized filtered-x Least Mean Squares (LMS) algorithm [11]-[12]. This algorithm is synchronized to both propellers. The need for a synchronization signal from each propeller arises from the fact that the synchrophaser devices are unable to perfectly synchronize the two propellers during a complete flight cycle [11]. By using the synchronization signals, internal single-frequency reference signals are generated and instantaneously adjusted by adaptive weights before driving the control sources, e.g. loudspeakers. The residual noise is observed by control sensors and the output signals from these are used to adjust the adaptive weights so that the overall noise level is minimized.

The most commonly used sources for generating the secondary interacting sound field, the “anti-noise,” are loudspeakers. However, since the sound field is generally excited by vibrations in the bounding walls, another approach is to use vibration exciters attached to the wall surface. The technique using control inputs applied
directly to the structure in order to reduce the vibration distribution with the objective of reducing the sound radiation has been termed Active Structural Acoustic Control (ASAC) [8],[13]. To observe the interior noise reduction, microphones are used as control sensors.

In this report, both loudspeakers and shakers are used by the controller to reduce the noise. The focus is, however, not on the mixture of different types of control sources and sensors. The objective with the report is to present the performance of an active control system in an acoustical environment.

In development work of a control system, an important part is the evaluation of its performance and robustness. In some applications it is possible to install the control system in a practical application, e.g. in an aircraft, motor boat or another vehicle, and then evaluate its performance under actual running conditions. This report deals with the evaluation of a control system in an aircraft. In this particular application, it is difficult, time consuming and very expensive to do the performance evaluation under actual in-flight tests. An alternative approach could be to carry out the evaluation on grounded aircraft. Even in this case, the evaluation is expensive. Furthermore, the results would not be directly comparable with results obtained from in-flight tests due to the influence of the ground on the sound field inside the aircraft. Another alternative and a compromise in the evaluation process is to use a test section of a real aircraft, i.e. a mock-up, and use artificially generated propeller noise. A major advantage using a mock-up is that the system performance and robustness can easily be investigated in an acoustical environment. Another advantage is that the system can be tested for cases which are difficult to test during flight, such as different disturbances in the synchrophaser angle etc.. The system evaluation presented in this report is performed in a aircraft mock-up.

The experimental setup consisting of a mock-up and a noise generation system is described in Section 2. The control algorithm and the reference signal generation are treated in Section 3. Section 4 and Section 5 describe the evaluation conditions and criteria, respectively. The results are presented in Section 6.

2 The Experimental Setup

This section describes in detail the experimental setup, including the mock-up and the specially designed sound generation system.

2.1 The Mock-Up

The mock-up used in the experiments consisted of a full-scale fuselage section of a SAAB 340 aircraft. The length of the section was 2.5 meters, with 0.9 meters of its length forward of the plane of the “propellers.” The mock-up was fully trimmed, i.e. fitted with thermal insulation, interior panels and floor panels, but without seats or luggage compartments.

The ANC system consisted of a multi-channel system using a maximum of
8 control sources and 11 control microphones. In the present experiment the control sources (secondary sources) consisted of both acoustic and structural exciters, namely 5 loudspeakers and 3 structural shakers. Each loudspeaker unit was mounted into a closed cabinet with the dimensions $20 \times 20 \times 12 \text{ cm}^3$. The cabinet was designed to fit into the volume between the trim panel and the fuselage. The structural shakers were mounted on the inner side of the fuselage. The locations of the control sources and control microphones are displayed in Figure 1. Their positions were found as the result of an optimization procedure. In most practical applications it is not possible to put the control microphones exactly at the positions where the noise reduction is most desired. The objective of the actuator/sensor optimization is generally to find a configuration of actuators and control sensors from a given set of conceivable positions to obtain a high noise-attenuation in a desired space, not only at the control microphones [14]. The conceivable positions are often chosen from a practical installation point of view.

Figure 1: The locations of (a) the actuators and the control sensors, and (b) the monitor microphone positions for each cross section (see cross sections 1–5 in (a)).
The performance of the ANC system was not only evaluated in the control microphones. A separate set of 20 microphones was used solely for evaluation purposes, so-called monitor microphones. These microphones were not part of the control system and could be placed at arbitrary positions where the control system was to be evaluated. Figure 1(b) shows the locations of the monitor microphones.

2.2 The Propeller Noise Generation System

One major concern in mock-up tests is how the primary sound field is generated. The aim should of course be to excite a sound field within the mock-up that resembles as closely as possible the sound field present during flight.

In this setup, an artificial propeller noise simulator was used. The simulator consisted of 12 loudspeakers mounted in a single plane around the exterior of the fuselage, covering about $\frac{3}{4}$ of the fuselage circumference. The loudspeakers were mounted perpendicular to the fuselage. To avoid interference between the loudspeakers they were fitted as tightly as possible to the fuselage without direct contact. The excitation was acoustical, not mechanical. Each loudspeaker was driven individually by a noise generator. The noise generator basically consisted

Figure 2: The propeller noise generation system consisting of a ring of 12 loudspeakers.
of a computer with a digital signal processing board (TMS320C32) and power amplifiers (200 W for each channel). The loudspeakers were ordinary car audio woofers (4 Ω, 8 inches). A photograph of the loudspeaker setup is shown in Figure 2.

The purpose of the loudspeaker array was to recreate the sound pressure distribution on the outside of the fuselage in the plane of the propellers. This was achieved by generating loudspeaker signals with a relative amplitude and phase difference, which was implemented in the system as a loudspeaker specific complex coefficient. These coefficients were calculated by the Aeronautical Research Institute of Sweden (FFA) based on a vibroacoustic finite element model.

By exciting the fuselage with a loudspeaker array in this manner, only airborne propeller-induced noise originating from the pressure fluctuations on the fuselage could be simulated. Structure-borne noise originating from propeller shaft rotation or boundary layer noise could not be generated. This was not a significant problem since the major concern in this study was the airborne propeller-induced noise.

The synthesized propeller noise consisted of several purely sinusoidal components internally generated in the computer. The sound field consisted of contributions from the left and right “propellers.” Each propeller field was obtained as the sum of an arbitrary number of harmonics—the fundamental BPF and some of its harmonics. In this study the BPF was 82 Hz which corresponds to the BPF of a SAAB 340 during cruise flight. The sound field could be generated to simulate the propellers running with equal or different rotational speeds.

The driving signal for loudspeaker \( i \) is given by:

\[
p_{i}(n) = \sum_{r=1}^{R} \sum_{h=1}^{H} A_{rh,i} \sin(2\pi f_{rh}n + \delta_{rh,i})
\]

where \( A_{rh,i} \) and \( \delta_{rh,i} \) denote the amplitude and phase coefficients associated with the frequency \( f_{rh} \). \( R \) and \( H \) denote the number of sources (“propellers”) and the number of harmonics from each source, respectively. \( R \) equals 2 throughout this study. A schematic figure of the noise generation system is shown in Figure 3.

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**Figure 3:** Schematic figure of the noise generation system.
In the simulations of steady cruise flight, both “propellers” rotated with fixed and equal rotational speeds. All other flight conditions were generated by adjusting the rotational speed of the right “propeller.” The noise generation system also delivered a synchronization signal for each “propeller,” corresponding to one pulse per propeller revolution.

A schematic figure of the entire experimental setup is shown in Figure 4.
3 The Control System

The core of the control system is a multi-channel, narrowband feedforward controller using complex signals and complex filter-weights. The complex reference signals are individually processed by a single complex weight that adjusts the amplitude and phase for each actuator. The structure of a twin-reference, multi-channel feedforward active noise control system is shown in Figure 5.

![Diagram of Twin-reference, multi-channel system for narrowband active noise control of propeller-generated cabin noise.](image-url)

Figure 5: Twin-reference, multi-channel system for narrowband active noise control of propeller-generated cabin noise.
One major advantage with narrowband active control of periodic noise components is that the reference signals can be synthesized internally in the controller. In this study, the synchronization signals obtained from the noise generation system were used to generate the complex reference signals. With reference signals generated in this manner, the adaptive control becomes extremely selective and it is possible to determine which harmonics are to be controlled and which are not.

The controller update algorithm uses estimates of the acoustic paths between all control sources and all control microphones (control paths). In the present study, the control algorithm was implemented on hardware from another part in the project. This hardware was originally designed for an FIR-based control algorithm, where the control paths are modeled as 8-tap FIR filters. The source path identification procedure was performed sequentially, source by source, using an excitation signal containing the four harmonics (BPF, 2×BPF, 3×BPF and 4×BPF) to be controlled. The path was identified using an adaptive LMS algorithm [6],[15],[16].

The control paths were identified under quiet and steady conditions before the control system was started. With the complex LMS algorithm, only single frequency complex estimates are used. These values were obtained by performing an FFT on the estimated impulse responses.

### 3.1 The Multiple-Reference Control Algorithm

The control algorithm is a central part of the ANC system. In order to obtain a significant global noise reduction in a large cavity, such as in an aircraft, a large number of control sources and sensors are needed [4],[5]. In the case of a twin-propeller aircraft, the algorithm must also be able to handle two reference signals. Therefore, it is important to use an algorithm with low computational complexity to save computation power. The low-complexity, multi-reference control algorithm used in this study is called an actuator-individual, normalized filtered-x LMS algorithm [10],[11],[17]. The algorithm is described for a general control situation with M control microphones, L actuators and R reference signals, where each reference has H harmonics.

The objective of the active noise control system is to minimize the sound pressure at the control microphones. The quadratic cost function to be minimized is chosen to be the sum of the squared output signals from the M control microphones:

$$ J(n) = \sum_{m=1}^{M} e_{m}^2(n). \quad (2) $$

The updating scheme of the complex filter weights associated with the complex reference signal $x_{rh} \ (r = 1, 2, \cdots, R, \text{and} \ h = 1, 2, \cdots, H)$ can be written in a matrix notation as

$$ w_{rh}(n + 1) = w_{rh}(n) - 2M_{rh}x_{rh}^*(n)\hat{F}_{rh}^H e(n) \quad (3) $$

where $w_{rh}(n) = [w_{rh1}(n) \cdots w_{rhL}(n)]^T$ is a vector of controller weights, $e(n) = [e_1(n) \cdots e_M(n)]^T$ is a vector of control errors and $\hat{F}_{rh}$ is an $M \times L$ matrix.
of control paths. The convergence factor matrix, $M_{rh}$, is given by the actuator-individual normalization

$$M_{rh} = \mu_0 \left( \rho_{rh} \text{diag} \left\{ \hat{F}_{rh}^H \hat{F}_{rh} \right\} \right)^{-1} \mu_{rh} = \begin{pmatrix} \mu_{rh1} \\ \mu_{rh2} \\ \vdots \\ \mu_{rhL} \end{pmatrix}$$ \hspace{1cm} (4)

where each diagonal element (the step-size parameter for actuator $l$, reference $r$ and harmonic $h$) is given by

$$\mu_{rhl} = \frac{\mu_0}{\rho_{rh} \sum_{m=1}^M |\hat{F}_{rhm}|^2}$$ \hspace{1cm} (5)

where $\rho_{rh}$ denotes the power of the reference signal $x_{rh}$. This convergence factor matrix may also be written as $M_{rh} = \mu_0 (\rho_{rh} \text{diag} \left\{ \hat{F}_{rh}^H \hat{F}_{rh} \right\})^{-1}$. The actuator-individual normalized algorithm is based on the assumption that in ANC applications with a large number of loudspeakers and microphones, such as in aircraft applications, the acoustic coupling between the loudspeakers decreases as the distance between them increases. This situation leads to a matrix $\hat{F}_{rh}^H \hat{F}_{rh}$ that is diagonally dominant, since the diagonal elements contain magnitude squares of frequency responses and the off-diagonal elements result from cross products of different responses ($\hat{F}_{rh}^H \hat{F}_{rh} \approx \text{diag} \left\{ \hat{F}_{rh}^H \hat{F}_{rh} \right\}$) [11]. For such situations, this algorithm can be seen as an approximation of Newton’s algorithm where the step-size matrix is given by $M_{rh} = \mu_0 (\rho_{rh} \hat{F}_{rh}^H \hat{F}_{rh})^{-1}$. Figure 6 shows the magnitude of each element in the diagonally dominant $\hat{F}_{rh}^H \hat{F}_{rh}$ matrix obtained for the measured frequency response functions in the mock-up at the BPF (82 Hz).

The control signals $y_l(n)$, $l = 1, 2, \ldots, L$ in Figure 5 are generated according to

$$y(n) = \sum_{r=1}^R \sum_{h=1}^H \Re \{x_{rh}(n)w_{rh}(n)\}.$$ \hspace{1cm} (6)

Here, $y(n)$ is an $L \times 1$ vector of real-valued control signals $y_l(n)$, $x_{rh}(n)$ is a complex scalar reference signal, $w_{rh}(n)$ is an $L \times 1$ vector of complex loudspeaker weights $w_{rhl}(n)$, and $\Re \{\cdot\}$ denotes the real part.

### 3.2 Reference Generation

The reference generation method typically used by the present research group has been a table lookup scheme consisting of several tables, one table for each harmonic, where each table contains a single sampled sinusoidal function. The generation of complex reference signals can be done either by using a pair of table pointers, where one pointer is delayed 90°, or by using a sine and a cosine table.

In this study, the algorithm was to be implemented on external hardware that was originally constructed for an FIR-based adaptive control scheme. This hardware delivered a composite reference signal, one for each propeller, containing the sum of the harmonics to be controlled. In order to split this composite signal into
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Figure 6: The magnitude of the elements in the *diagonally dominant* matrix $\hat{F}_r \hat{F}_rh$ for the BPF of 82 Hz (linear scale). The matrix consists of control paths measured inside the mock-up. Sources 1–3 correspond to the structural exciters and sources 4–8 correspond to the loudspeakers.

separate, complex, harmonic reference signals suitable for the algorithm, a sliding FFT-operation was used as a parallel filter bank [11],[18]. The process of generating the complex reference signals from a synchronization signal is shown in Figure 7.

Figure 7: The reference generation of complex reference signals.

The synchronization signals derived from the noise generation system contained only one pulse per “propeller” revolution. On the targeted platform these signals were converted to contain as many pulses per revolution as the number of propeller blades on the propeller, which is four on the SAAB 340. This conversion was done by using a phase-locked loop. The frequency contents of the square wave thus produced contains the fundamental frequency (BPF) and all harmonics with decreasing amplitudes. The composite reference signals, $s_1(n)$ and $s_2(n)$, were
obtained by feeding the square wave through a low-pass filter, allowing only the four lowest frequency components (BPF to 4×BPF) to pass through. As an interface between the real composite reference signals and the complex algorithm a software splitter based on an FFT-filter bank was used.

Given the real, scalar synchronization signals, \( s_r(n) \), the complex, scalar reference signals, \( x_{rh}(n) \), are generated by computing the FFT-operation on blocks of \( N \) samples for every new sample (sample-by-sample basis) according to

\[
x_{rh}(n) = \sum_{q=0}^{N-1} h(q)s_r(n - q)e^{j2\pi \frac{q}{N}}
\]  

where \( n \) is the sample index and \( h(q) \) is a window function [11].

The required FFT resolution for the filter bank was determined by the system sampling frequency and by the frequency separation of the harmonics at the lowest engine speed. The frequency bin number to be used as output for the complex reference signal was determined by the harmonic \( h \) to be controlled. In the evaluation, variations in the propeller speed was small. For this reason, no shifts between frequency bins were carried out.

It has been shown that the length of the FFT affects the response time for the algorithm to rapid changes in the input signals [11],[17]. For this reason, as well as considering the calculation effort, it is desirable to keep the length of the FFT as short as possible. If the FFT is too short, on the other hand, the frequency resolution will suffer, leading to poor suppression of nearby harmonics. In the present application, the FFT size was chosen to 64 and a Blackman window was used. With a sampling frequency of 1 kHz, the filter resolution becomes 1024/64=16 Hz. For a Blackman window the suppression of the side lobes is at least 58 dB, compared to 13 dB when a rectangular window is used [11],[19].

One significant disadvantage with the sliding FFT filtering technique, besides the fairly high implementation cost, is the delay caused by the linear-phase filter employed by this method. The group delay equals one-half the length of the FFT, which for the present case corresponds to 32 ms. Due to this delay, very rapid changes in the rotational speed will not be accurately traced by the controller. Another disadvantage is that the calculation of the FFT requires a large amount of computational effort in the DSP.

## 4 Simulated Flight Conditions

The ANC system performance was evaluated for “flight” conditions with synchronized “propellers” with constant rotational speed as well as unsynchronized propellers with constant or sinusoidally varying rotational speed differences. The experiment was performed with a fundamental Blade Passage Frequency (BPF) of 82 Hz. The rotational speed of the “left propeller” was fixed (\( f_1 = 82 \) Hz). Different control situations were achieved by varying the BPF of the “right propeller” (\( f_2(t) \)). The relative phase angle (synchrophase angle) between the propellers is denoted by \( \Delta \varphi(t) \), see Figure 8.
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Figure 8: The difference in the synchrophase angle, $\Delta \varphi$.

With fully synchronized propellers the BPF was equal to 82 Hz and the synchrophase angle was constant and equal to zero ($\Delta \varphi(t) = 0$). The objective with this test was to verify the operation of the control system under ideal conditions corresponding to steady cruise flight. The objective also was to obtain a “benchmark” for conditions with unsynchronized sources.

Modern propeller aircraft are usually fitted with a synchrophaser unit which synchronizes the propellers. The synchrophaser is, however, often unable to keep the propellers fully synchronized at all times, resulting in a slight rotational speed difference between the propellers. The performance was evaluated for constant BPF differences $\Delta f$ and sinusoidally varying BPF differences $\Delta f_{\text{max}} \sin(2\pi t/T)$. Here $\Delta f_{\text{max}}$ denotes the maximum beat frequency and $T$ is the periodicity of the variation. The BPF of the “right propeller” was $f_2 = 82 + \Delta f$ Hz or $f_2(t) = 82 + \Delta f_{\text{max}} \sin(2\pi t/T)$ Hz, depending on the test being run.

The following constant beat frequencies were evaluated: $\Delta f = 0.5, 1$ and 2 Hz. For the time-varying case, $\Delta f_{\text{max}}$ was equal to 0.5 or 1 Hz and the period time, $T$, was either 1 or 5 s.

During flight it is not unusual that a variation in the relative phase angle between the propellers occurs. In order to test the algorithm in an environment that closely resembles such a flight situation, a case with a time-varying phase angle between the two propellers, $\Delta \varphi(t)$, was simulated. The relative phase angle varied sinusoidally in the range $\pm 1^\circ$ within one revolution of the propeller.

Usually, it is desired to attenuate not only the fundamental BPF in order to achieve significant noise attenuation. Interior propeller-induced noise consists of a number of dominant harmonics. The system performance for control of harmonics was also investigated in the experiment.
5 Evaluation Criteria

The controller was evaluated with respect to steady state noise attenuation, convergence behavior, and robustness for different non-stationary conditions. The noise attenuation is presented diagrammatically, and the convergence behavior is shown as learning curves. The learning curves show the change in mean sound pressure level versus time averaged both over the control and monitor microphones.

The mean narrowband attenuation, $A_h$, for harmonic $h$ is given by

$$A_h = 10\log_{10} \frac{\sum_{m=1}^{M} |D_{mh}|^2}{\sum_{m=1}^{M} |E_{mh}|^2} \quad [\text{dB}]$$

where $|D_{mh}|$ and $|E_{mh}|$ are the magnitudes of the Fourier transforms of the uncontrolled and controlled noise at the control microphones. The magnitudes were taken from averaged autospectra, using blocks of 256 samples with Hanning-windowed data. The data were taken towards the end of the recorded sequences to ensure that the control algorithm had converged properly.

The measured mean noise attenuation was compared with the theoretically computed optimum reduction in a least squares sense [5]. The optimum attenuation is obtained by calculating the ratio of the powers of the uncontrolled and controlled noise in the control microphones:

$$A_{opt} = 10\log_{10} \frac{||D_h||^2}{||F_h y_{hopt} + D_h||^2} \quad [\text{dB}]$$

where $||\cdot||^2$ denotes the squared Euclidean norm and $y_{hopt} = -[\hat{F}_h^H \hat{F}_h]^{-1} \hat{F}_h^H D_h$, where $D_h$ is an $M \times 1$ complex vector containing the uncontrolled noise in the control microphones.

6 Practical Control Results

High attenuation can be achieved by spatially and temporally matching the primary sound field with a sound field generated by the control sources. The temporal matching is mainly determined by the control algorithm, while the spatial matching mainly depends on the location of the control sources and sensors.

Figure 9 shows the sound pressure level of the uncontrolled and controlled sound fields at BPF (82 Hz), in the seated head height inside the mock-up, see also the Appendix. The controlled sound field was measured after the controller was allowed to converge. As shown in this figure the ANC system significantly reduced the overall noise level in seated head height. It is clear that the effect of turning on the ANC system was to level out the spatial variations in the sound pressure level. In the front region inside the mock-up where several control microphones were located a substantial noise attenuation was observed. The measured mean noise reduction averaged over all the monitor microphones was approximately 10.3 dB.
Figure 9: The interior sound pressure level distributions at the passenger head level measured by the 20 monitor microphones.
Figure 10 shows the convergence performance of the controller. The curves show the change in normalized mean sound pressure level versus time in the control microphones as well as the monitor microphones. The curves are normalized to 0 dB at $t = 0$ seconds. The controller was started after approximately 2 seconds. The control algorithm exhibited robust convergence to a constant level, approximately 18.4 dB attenuation at the control microphones and 10 dB at the monitor microphones. In this case with identical propeller rotational speed it is interesting to note that the twin-reference controller converges nicely, while in fact there is redundant information in this condition; two individual (but identical) controllers use similar inputs to control the same signals. Therefore, in this condition with fully synchronized propellers a single-reference controller is sufficient [11]. Such a controller uses a single synchronization signal from one of the propellers to obtain information about the noise to be reduced. In applications where a possible unsynchronization may occur between the noise sources, a multiple-reference controller is, however, more preferable to a single-reference in order to efficiently reduce the noise level under different conditions [11].

The difference of 8 dB between the attenuation at the monitor and control microphones depends significantly on the placement of the control sources and the control microphones. This result demonstrates the importance of the locations of these sources and sensors in order to obtain high overall noise reduction [5].

Figure 11 shows a diagrammatic representation of the sound pressure in the control microphones with the ANC system on and off. The diagram shows the noise attenuation obtained at each control microphone. The levelling out of the sound field when using ANC is also clearly seen in this figure. A mean attenuation
Mean attenuation 18.4 dB.

The optimal theoretically-calculated attenuation was 22.7 dB. This result indicates that the controller performs well under this ideal running condition. A broad agreement between the calculated and obtained noise reductions has also been observed in off-line computer evaluations based on recorded noise and synchronization signals as well as measured control paths [11],[17].

The convergence behavior showed above is compared with the convergence behavior obtained in an off-line computer experiment based on recorded data corresponding to the same control condition, see Figure 12. It is shown that the convergence behavior obtained in the computer experiment highly coincided with the behavior observed in the practical experiment. In the practical experiment a relatively small step-size parameter $\mu_0$ was used ($\mu_0 = 0.001$). An improved performance could, however, be obtained by using larger values of $\mu_0$ which is shown in Figure 13 [20]. A stability criterion for $\mu_0$ is presented in [10]. This criterion shows that the controller is stable for a step size within the range $0 < \mu_0 < \frac{1}{\sqrt{M}}$. In the present experiment of controlling the BPF from two sources the upper limit was calculated to $\frac{1}{16} = 0.0625$. The value of $\mu_{0\text{max}}$ was experimentally determined in an off-line computer experiment to $\mu_{0\text{max}} = 0.085$. The maximum step-size, $\mu_{0\text{max}}$, was the largest step-size found that gave a stable result in the off-line experiment.

Owing to the structure of the controller and the adaptive algorithm, rapid convergence was realizable. This ability for rapid convergence enables the control system to track changes in the sound. This type of controller is favorable for attenuating noise from periodic sources running at moderately different rotational speeds, as observed, for example, when the fields generated by the two propellers...
Figure 12: Convergence behavior from a practical experiment and an off-line computer experiment (synchronized propellers).

Figure 13: Convergence behavior for different step-size parameters $\mu_0$ (synchronized propellers).
beat together. Figure 14 presents a diagrammatic representation of the mean SPL for each control microphone averaged over a time period of 5 seconds for conditions with a constant beat frequency of 0.5 Hz. The levelling out of the sound field caused by the control system is also clearly seen here. Similar results was observed for beat frequencies of 1 and 2 Hz. Table 1 summarizes the mean noise attenuation averaged over all control microphones. The results demonstrate significant noise reduction. Note that the reduction decreases with increased beat frequency, which is in accordance with theoretical results [20]. A comparison with the fully synchronized case shows a degradation in the noise attenuation of approximately 4 dB, due to the beats. By using a larger step-size parameter an improved performance could also be obtained in conditions with beating sound fields [20].

![Figure 14: Performance of the active system for constant beat frequency (Δf = 0.5 Hz). Mean attenuation 15.3 dB.](image)

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</table>

Table 1: The mean noise attenuation in the control microphones of BPF for constant beat frequencies.

Figures 15-17 show the convergence behavior for a beat frequency of Δf =0.5, 1 and 2 Hz. Also shown in these figures is the convergence behavior from the fully synchronized case. Despite the beating the controller proved to be stable and converged robustly.
Figure 15: Learning curves: (a) constant beat frequency ($\Delta f = 0.5$ Hz); (b) two synchronized sources.

Figure 16: Learning curves: (a) constant beat frequency ($\Delta f = 1$ Hz); (b) two synchronized sources.
The noise attenuation in the control microphones obtained for conditions with a time-varying beat frequency excitation, in this particular case a sinusoidally varying, is shown in the Figures 18-21. Figures 22-25 show the convergence behavior. In these cases a mean noise attenuation of approximately 11–13 dB was obtained depending on the $\Delta f_{\text{max}}$ and $T$, see Table 2. A comparison with the constant beat frequency situation demonstrates a reduced noise attenuation of several dB for the cases of time-varying beat frequencies. The results show a better noise attenuation for slowly varying conditions than for rapidly changing conditions. The decreased noise attenuation results because the FFT-based reference generator introduces a time delay in the reference signals of approximately 32 ms. The controller remained stable and converged robustly to a constant mean level despite the delay in the reference signals and the time-varying beats. In non-stationary conditions this delay implies decreased correlation between the reference signals and the noise, resulting in degraded tracking performance and reduced noise attenuation. However, this delay will not cause any problems under stationary conditions. Under such conditions it is always possible to find a correlation between periodic signals of the same frequency, irrespective of delays [17]. In such cases the time delay of the reference signals will thus not reduce the noise attenuation.

The synchronization signals delivered by the noise generation system contained a single pulse per “propeller revolution.” Reference signals generated using such synchronization signals may have slightly time-varying phase angles (“phase jitter”), which may degrade the performance of the ANC system. In the cases of a time-varying synchrophase angle a larger amount of phase jitter was introduced in the reference signals. The phase jitter as well as the time-delay in the ref-
Figure 18: Performance of the active system for time-varying beat frequency ($\Delta f = 0.5$ Hz, $T = 1$ s). Mean attenuation 11.6 dB.

Figure 19: Performance of the active system for time-varying beat frequency ($\Delta f = 0.5$ Hz, $T = 5$ s). Mean attenuation 12.6 dB.
Figure 20: Performance of the active system for time-varying beat frequency ($\Delta f = 1$ Hz, $T = 1$ s). Mean attenuation 7.5 dB.

Figure 21: Performance of the active system for time-varying beat frequency ($\Delta f = 1$ Hz, $T = 5$ s). Mean attenuation 10.0 dB.
Figure 22: Learning curves: (a) sinusiodally-varying beat frequency ($\Delta f_{\text{max}} = 0.5$ Hz, $T = 1$ s); (b) two synchronized sources.

Figure 23: Learning curves: (a) sinusiodally-varying beat frequency ($\Delta f_{\text{max}} = 0.5$ Hz, $T = 5$ s); (b) two synchronized sources.
Figure 24: Learning curves: (a) sinusiodally-varying beat frequency ($\Delta f_{\text{max}} = 1$ Hz, $T = 1$ s); (b) two synchronized sources.

Figure 25: Learning curves: (a) sinusiodally-varying beat frequency ($\Delta f_{\text{max}} = 1$ Hz, $T = 5$ s); (b) two synchronized sources.
Part VI

<table>
<thead>
<tr>
<th>$\Delta f$ [Hz]</th>
<th>0.5</th>
<th>0.5</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period $T$ [s]</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$A_h$ [dB]</td>
<td>11.6</td>
<td>12.6</td>
<td>7.5</td>
<td>10.0</td>
</tr>
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</table>

Table 2: The mean noise attenuation of BPF for *time-varying* beat frequencies.

The reference signal introduced by the FFT-based reference generator implied that the controller never converged completely, since it constantly reacted to the variations. The performance of the controller is shown in Figure 26. The mean noise attenuation obtained in the control microphones was approximately 15 dB. In order to reduce the phase jitter it is recommended to use synchronization signals containing as many pulses as possibly per revolution, thereby increasing performance criteria such as tracking and noise reduction.

![Mean Sound Pressure Level (SPL)](image)

Figure 26: Learning curve: Time-varying synchrophase angle.

Figures 27 and 28 shows the results from a condition where the BPF and $2 \times$ BPF generated by two synchronized propellers were controlled. The results in these figures show a high reduction of both frequencies. The mean attenuation for BPF and $2 \times$ BPF was 15.8 dB and 10.6 dB, respectively, and the optimum attenuation for the BPF was calculated at 19.2 dB. The optimum attenuation for the $2 \times$ BPF could not be calculated since the control paths were unavailable for this frequency. It can be seen that the attenuation of the BPF in this case is lower than for the case of only controlling the BPF. Due to the fact that the primary noise level was lower. Due to the hardware limitations the level of the primary noise had to be decreased, and this affected the ability to control the $3 \times$ BPF and $4 \times$ BPF in which the peaks were too low compared to the level of the surrounding noise. Consequently, only the BPF and $2 \times$ BPF were controlled.
Figure 27: Performance of the active system controlling BPF generated by synchronized propellers. Mean attenuation 15.7 dB.

Figure 28: Performance of the active system controlling 2×BPF generated by synchronized propellers. Mean attenuation 10.6 dB.
7 Summary and Conclusions

This paper presents results from a practical experiment where the performance of the multi-reference actuator-individual normalized filtered-x LMS algorithm has been evaluated. The evaluation was performed within a fuselage section from a propeller aircraft. In order to simulate the propeller noise produced by two rotating propellers 12 loudspeakers mounted around the exterior of the fuselage were used to excite the structure. The ANC system used 8 control sources and 11 control microphones.

The controller exhibits good performance with respect to convergence rate, tracking and robustness, and the interior noise level was considerably reduced. The mean attenuation of the BPF (82 Hz), under stationary “flight” conditions with fully synchronized propellers, was approximately 18 dB in the control microphones, and 10 dB in the monitor microphones located at the passenger head positions. In cases with unsynchronized propellers, however, the attenuation achieved decreased typically 3–6 dB due to beating. In the different cases the controller converged robustly, and the behavior of the controller displayed in these tests agreed with the behavior obtained from off-line computer simulations. The noise reduction measured was in broad agreement with the optimum theoretical reduction, calculated by using the recorded primary noise and measured frequency responses.

The performance degradation under non-stationary conditions is partly due to the time delay introduced by the FFT-filter bank used to generate the internal complex reference signals. Increased performance in non-stationary conditions could be obtained if the reference signals are generated with as short inherent delay as possible, e.g. using lookup tables instead of the sliding FFT-operation in the generation process.

In conclusion, the multiple-reference actuator-individual normalized filtered-x LMS algorithm exhibited high performance in a practical acoustic environment. Significant noise attenuation was obtained. The system was robust and fast convergence was observed for stationary as well as non-stationary conditions.

Acknowledgments

The authors wish to express their gratitude to the partners in the BRITE/EURAM project ASANCA II for the possibility of carrying out the experiment. The authors would especially like to thank the collaborator partners in the experiment; the Aeronautical Research Institute of Sweden (FFA), SAAB AB in Sweden, and the Matra Cap Systems in France.
Appendix

Uncontrolled Sound Field

The plane of the propellers

Controlled Sound Field

The plane of the propellers

Figure 29: The interior sound pressure level distributions at the passenger head level measured by the 20 monitor microphones.
References


Part VII

A New Passive/Active Hybrid Headset for a Helicopter Application
This part has been published as:

A New Passive/Active Hybrid Headset for a Helicopter Application

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Abstract

In helicopters, the low frequency noise generated by the rotors and engines often masks and jeopardizes safe communication. In addition, pilots are likely to suffer from damage to their hearing due to the high sound levels in the headset produced to overcome the noise caused by increased speaker levels. A feasible approach is to reduce the low frequency noise by using active techniques combined with a method for reducing the noise in the intercom microphone signal, with lower speaker levels as a result. Helicopter noise consists of tonal components embedded in broadband noise. In order to achieve an efficient attenuation of the primary noise inside the headset, a combination of a digital feedforward controller and an analog feedback controller is employed. Spectral Subtraction is used to suppress the background noise in speech signals. This paper evaluates a combination of the two techniques and their application to real data.

1 Introduction

There are substantial noise levels in the Super Puma helicopter, especially at low frequency [1]. The noise level is not normally harmful to the hearing, although low frequency noise generated by the engines and rotors (main and tail rotor) masks and corrupts speech. Noise with dominant frequency distribution just below, or within the lower frequency range for speech, degrades speech recognition and intelligibility [2]. For helicopter pilots it is important to hear radio communication correctly during flight. Pilots thus tend to select the maximum hearing volume in the communication system. The sound levels produced are harmful to the ear, inducing fatigue and loss of hearing. This phenomenon is denoted Noise Induced Hearing Loss (NIHL), a condition in which high sound pressure levels (SPLs) are possible. The level of risk NIHL that occur depends on the frequency content of the noise.
In order to increase speech intelligibility the noise level inside the earcups must be reduced. Since the noise has low frequency characteristics, it cannot be substantially reduced by employing passive techniques since the passive earcups cannot be made large enough due to the fact that they must fit inside the helmet. The helicopter pilots and the rescue personnel are required to wear a helmet for safety reasons. Had there been no such stipulation, larger earcups such as the Bilsom 747 which provide good low frequency attenuation could be used. A more feasible approach is to reduce the noise by employing Active Noise Control (ANC). As figure 1 shows, noise up to 100 Hz inside the helicopter, is normally dominated by tonal components while the noise is more broadband between 100 Hz to 400 Hz. The total sound pressure level inside the helicopter is 103 dB(A).

The approach in this paper uses a hybrid ANC headset which combines both feedforward and feedback ANC techniques [3]. The adaptive feedforward controller is based on a digital system, while the feedback system is based on an analog system. The principle of the hybrid headset is depicted in figure 2. This type of ANC headset is used in order to improve noise attenuation. The feedback controller reduces broadband noise, while the feedforward controller reduces tonal components (harmonics of the main and tail rotor).

The feedback controller is based on a commercial analog headset. Pure analog feedback technique will not be discussed in the paper. Here, the approach focuses on the adaptive algorithm used in the digital feedforward controller, as well as the combined performance of the hybrid headset.

Even though the hybrid headset efficiently reduces the low frequency noise inside the earcups, high noise levels are still produced in the earcups through the
intercom microphones used for communication. It is thus desirable that the noise in
the speech signal also be reduced. Speech enhancement and speech recognition are
either model-based, where typical solutions include Markov Models, Predictors and
Lattice filters [4]; or non-parametric, based on a spectral description of speech such
as the Short-Term Spectrum or Cepstral techniques [5]. Spectral Subtraction (SS)
is a suitable method which uses a single microphone. This non-parametric method
is based on the additive properties of the magnitudes of the FFT coefficients of the
noise. SS has been combined with the hybrid headset as depicted in figure 2.

Figure 2: The principle of the hybrid ANC headset based on feedforward and
feedback control combined with spectral subtraction to reduce the noise in the
communication signal

2 Helicopter Noise

In order to analyze the frequency components of helicopter noise, several noise
measurements were performed during flight. The noise was recorded inside the
helicopter using a microphone situated between the passenger seats, in the vicinity
of the right rear door (see figure 3). When hovering, this door is open.

The Super Puma helicopter has one main rotor and one tail rotor, as well as
several gear boxes. There are four blades on the main rotor and five on the tail
rotor. Figure 4 shows a typical noise power spectrum inside the cabin during flight
for frequencies from 0 to 200 Hz. According to the frequency analysis, the Blade
Passage Frequency (BPF) should be 17.6 Hz for the main rotor. The BPF of the
main rotor induces high infra-sound levels inside the cabin. These components, though not audible, affect the human body. There are different subjective symptoms of annoyance: headache, vertigo and nausea [6],[7]. All these have a bad influence on a helicopter pilot, on a rescue mission.

The abbreviations in figure 4 indicate the components comprising the harmonics of the main rotor BPF. The tone at 82 Hz is not an exact order of the main rotor BPF. This is the tail drive shaft fundamental. This component is, however, related to the BPF, though with a fractional number. Note the 107 Hz component which is the BPF of the tail rotor. The 6th order of the main rotor BPF is at almost the same frequency as the 1st order of the tail rotor BPF. The main components in the 100 Hz range are, however, due to the main rotor as well as its first orders, and the BPF of the tail rotor.

Standardized SPL measurements recorded at normal cruise speed and in two different ventilation situations were made inside the headset, in the pilot’s ear canal with the aid of a tiny microphone. Figure 5 illustrates the 1/3 octave analysis of this measurement both during communication and when no communication was taking place. Figure 5 also presents the total sound pressure levels in dB, dB(A) and dB(C). The SPL inside the ear canal was approximately 88 dB(A) when no communication was taking place. However, when communication (internal and external) was taking place the A-weighted sound pressure level was increased by 5-13 dB, resulting in 93-101 dB(A). The exposure permitted (in Sweden) for the actual sound levels when no communication is taking place is 4 - 5 hours a day, which is more than the normal exposure time for the helicopter crews. However, the exposure allowed during communication will be decreased to less than 30 minutes a day.

Internationally, the standardized A-weighting function is used to express the integrated SPLs in the frequency range of hearing. In a number of states, in the USA, 85 dB(A) is an accepted maximum equivalent sound level for 8 hours a day, 5 days a week.

To summarize, there is a large increase in dB(A) levels when the communication system is activated. This is due to the fact that the speech is masked by the low frequency sound that consists primarily of tonal components (harmonics of the main and tail rotor). The BPF of the main rotor, 17.5 Hz, is below the hearing range and creates infra-sound. Even though this tone is reduced inside the headset,
it will still affect the human body through bone conduction. This infra-sound will significantly affect speech perception and speech intelligibility in particular [8]. The performance of the hybrid headset may not as a result be as good as expected. It is important to suppress the second, third and fourth order of the BPF for the main rotor. The BPF of the tail rotor, 105 Hz, must also be suppressed, since the passive mechanisms in the helmet are not able to attenuate this low frequency noise. Increased attenuation should be achieved with an active system enabling lower communication levels.

3 The Hybrid Headset

We now present the concept of a hybrid ANC headset that combines both feed-forward and feedback ANC techniques [3]. The adaptive feedforward controller is based on a digital system, while the feedback system is based on an analog system. The principle of the hybrid headset is depicted in figure 2. This type of ANC headset is used in order to improve noise attenuation. The feedback controller reduces broadband noise (boundary layer noise, noise from air conditioning, in other words noise not related to the rotor or the tail), while the feedforward controller reduces narrowband noise (harmonics of the main and tail rotor). Noise up to 100 Hz is normally dominated by tonal components while in the range 100-400 Hz the noise is more broadband.

The feedback controller is based on a commercial analog headset. We focus here on the adaptive algorithm in the digital feedforward controller, and the performance of the hybrid headset.
The feedforward controller utilizes a tachometer signal related to the main rotor to generate reference signals to the controller. Noise components that are correlated with the reference signals will be suppressed. The reference signals are inputs to the feedforward controller. The output of the controller is added to the output of the feedback controller the loudspeaker, generating a secondary sound field that is $180^\circ$ out of phase with the primary sound field. An error microphone inside the earcup which measures the residual noise is used to adjust the adaptive feedforward controller.

The feedback controller feeds the output signal of the error microphone back through an analog amplifier with a magnitude and phase response which is designed to produce an output that results in noise attenuation in the error microphone.

The adaptive algorithm employed in the feedforward controller is based on the complex filtered-X Least-Mean-Square (LMS) algorithm, [9], [10]. The proposed complex algorithm is advantageous in narrowband applications due to its properties.
of good convergence rate and low numerical complexity. These are the results of the orthogonality of the quadrature components (or Hilbert pairs) that constitute the complex reference signals, and the simplicity of complex representation. In fact, the complex algorithm requires a minimum of adaptive and acoustic path parameters as compared with a straightforward time domain approach using ordinary FIR filters.

3.1 The Feedforward Controller

Noise up to 100 Hz inside the helicopter consists essentially of narrowband harmonic components related to the rotation frequencies of the main and tail rotors. It is assumed that there is a periodic tachometer signal available which is correlated to the noise harmonics. For this reason, pure sinusoidal reference signals and complex notation will be used in the following description.


![Feedforward Controller Diagram](image)

Figure 6: Single reference, Single Input, Single Output (SISO) system for active noise control.

The controller is described for a general situation with $H$ harmonics. Each harmonic is individually controlled. Let $t(n)$, $x_h(n)$, $w_h(n)$ and $F_h$ denote the tachometer signal, the complex scalar reference signal, the complex scalar filter weight and the complex acoustic path from the loudspeaker to the error microphone respectively for the $h$th harmonic. The set of complex reference signals $x_h(n)$ is
generated from the tachometer signal $t(n)$ by using, for example, an FFT-filter bank,\cite{[14]}, or the lookup table technique,\cite{[3]}.

The real error microphone signal $e(n)$ is given by

$$e(n) = d(n) + \sum_{h=1}^{H} \Re\{F_h x_h(n) w_h(n)\}$$

where $d(n)$ is a real signal representing the primary sound field at the error microphone (at the discrete time index $n$). Here $\Re\{\cdot\}$ denotes the real part operation.

The objective function to be minimized is given by

$$J(n) = e^2(n)$$

where $(\cdot)^*$ denotes complex conjugate. The derivative of $J(n) = e^*(n)e(n)$ with respect to $w_h(n)$ is given by

$$\frac{\partial J(n)}{\partial w_h^*(n)} = x_h^*(n)F_h^* e(n).$$

The complex gradient in (3) is used to define the updating scheme of the adaptive algorithm, given by

$$w_h(n+1) = w_h(n) - 2\mu_h x_h^*(n)F_h^* e(n).$$

The convergence factor $\mu_h$ is given by

$$\mu_h = \frac{\mu_0}{\rho_h |F_h|^2}$$

where $\mu_0$ is a positive normalized convergence factor and $\rho_h = E\{|x_h(n)|^2\}$ (the power of the signal $x_h$). The power of the reference signal $x_h$ is estimated by using an exponential moving window technique as follows

$$\hat{\rho}_h(n) = (1 - \beta)\hat{\rho}_h(n-1) + \beta |x_h(n)|^2$$

where $\beta$ is a weighting factor.

In a practical application, the acoustic path $F_h$ is unknown and must be estimated. $F_h$ should thus be exchanged for the corresponding estimate $\hat{F}_h$ in (3),(4) and (5).

### 3.2 Evaluation

The evaluation has been made on data recorded in an AS332 “Super Puma” MKII helicopter during flight. The two engines in the helicopter always run with the same rpm. The sound field is thus relatively stationary. The noise inside the cabin contains strong tonal components originating from the main and tail rotors. In order to achieve an efficient noise reduction inside the earcups it is necessary to reduce the BPFs and their related harmonics. The feedforward controller presented
in this paper was set up to cancel the BPF to $5 \times \text{BPF}$ for the main rotor, and the BPF for the tail rotor respectively.

Figure 7 shows the performance of the feedforward controller only. These results for the feedforward controller are based on computer simulations within the environment of the Matlab software package; while the passive attenuation and the attenuation in the analog ANR system are derived from real experiments. The frequency range is 0 to 200 Hz. The following attenuation of the dominating tones was obtained:

<table>
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<tr>
<th>Frequency Component</th>
<th>Frequency</th>
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<td>BPF, Main rotor</td>
<td>17.7</td>
<td>23</td>
</tr>
<tr>
<td>2xBPF, Main rotor</td>
<td>35.3</td>
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<td>3xBPF, Main rotor</td>
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<td>22</td>
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<td>4xBPF, Main rotor</td>
<td>70.7</td>
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<td>5xBPF, Main rotor</td>
<td>88.3</td>
<td>8</td>
</tr>
<tr>
<td>BPF, Tail rotor</td>
<td>106.7</td>
<td>15</td>
</tr>
</tbody>
</table>

The BPF for the main rotor is 17.7 Hz and is outside the audible frequency range. The attenuation of the tones is satisfactory though the audible result is limited when the total sound pressure level is measured in dB(A) only. Note also that the 6xBPF for the main rotor coincides with the BPF for the tail rotor. This facilitates the generation of the reference signals.

Figure 8 illustrates the attenuation in the passive earcups only. The attenuation is relatively poor due to the fact that the earcups are restricted in size in order to fit into the helmet.

Figure 9 shows the performance of the feedback controller combined with the passive damping of the earcups. The analog system is a commercial headset fitted into a headset from Hellberg Safety AB and has closed earcups. The analog system only affects the spectrum up to approximately 400 Hz. The frequency range 0-400 Hz only is presented. The controller achieves a broadband noise attenuation of approximately 20 dB in the given frequency range. Note that the tonal components are still present. Since both passive damping material in the earcups and the analog feedback controller affect broadband noise reduction, it is useful to investigate the broadband reduction when the analog controller is switched on and off. The difference between passive damping and passive attenuation together with analog control is depicted in figure 9 and 8. When the analog controller is on, a more efficient broadband reduction is achieved. The feedforward controller does not affect the broadband attenuation of the noise. This controller reduces the tonal components only.

Finally, the result of the hybrid headset is shown in figure 10. A combination of feedforward and feedback control results in significant attenuation of both the tonal components and broadband noise.
Figure 7: Attenuation for the digital feedforward controller.

Figure 8: Sound pressure level of the primary sound field outside the earcups. Reduced sound pressure level inside the earcups when only passive damping is applied.
Figure 9: Sound pressure level of the primary sound field outside the earcups. Reduced sound pressure level inside the earcups after the analog feedback controller has been switched on.

Figure 10: Sound pressure level of the primary sound field outside the earcups. Reduced sound pressure level inside the earcups after the hybrid headset has been switched on.
4 Spectral Subtraction

When flying the helicopter the pilots use the intercom system almost continuously. During communication the microphone signal from one pilot is fed directly into the other pilot’s headset. The microphone signal contains considerable helicopter noise and this noise is again injected in the hybrid headset. An important task is thus to enable a noise reduction in the intercom system by supplying a better signal-to-noise ratio in the microphone signals. The headset is normally equipped with a unidirectional, close-talk microphone, which significantly reduces low frequency noise but offers little or no noise reduction above 800 Hz. To clean the noisy speech signal a single microphone nonlinear method, denoted “spectral subtraction” is proposed.

This method enhances speech by estimating a noise bias in the magnitude in the frequency domain and subtracting it. The speech is assumed to be random corrupted with uncorrelated random noise. The noise estimation must be performed during “talker inactive” (silent) time frames. The magnitude of the estimated noise power spectrum is then subtracted from the Fourier transform magnitude and inversely transformed back into time frames when the talker is active.

Spectral subtraction is based on the following assumptions: the background noise is acoustically added to the speech; the background noise environment remains stationary locally to the extent that its expected spectral magnitude value immediately prior to speech activity equals its expected value during speech activity. If the environment changes to a new stationary state, there must be sufficient time (400 ms) to estimate a new background noise spectrum. The algorithm requires a speech detector to signal to the program when the updating of the noise bias can be continued.

Let $s(n), v(n)$ and $x(n)$ be stochastic short-time stationary processes representing speech, noise and noisy speech, respectively. From

$$x(n) = s(n) + v(n)$$  \hspace{1cm} (7)

and the assumption that $s(n)$ and $v(n)$ are uncorrelated processes it follows that

$$R_x(f) = R_s(f) + R_v(f)$$  \hspace{1cm} (8)

where $R$ denotes the power spectral density of a random process. The power spectral density magnitude for $w(n)$ is estimated and subtracted according to

$$\hat{R}_s(f) = R_x(f) - \hat{R}_v(f)$$  \hspace{1cm} (9)

To estimate the power spectrum density we start with the simple periodogram which is evaluated using

$$\hat{R}_x(f) = P_{x,N}(f) = \frac{1}{N} |\hat{X}(f)|^2$$  \hspace{1cm} (10)

where $\hat{X}(f)$ is the short time Fourier transform of $x(n)$, and $N$ denotes the block
length. If we combine Eqs. (9) and (10) we obtain

$$|\hat{S}(f)|^2 = |X(f)|^2 - |\hat{V}(f)|^2$$

This is the conventional squared (power) form. Alternatively, the straight-forward amplitude spectrum is often used as follows.

$$|\hat{S}(f)| = |X(f)| - |\hat{V}(f)|$$

The phase must also be estimated, however. Speech is rather insensitive to phase distortion, thereby giving rise to certain simplifications [5]. There is no straightforward way to compute the clean speech phase spectrum $\phi_s$. It is often sufficient, however, to use the noisy speech phase spectrum $\phi_X$ as an estimate, which yields

$$\phi_s \simeq \phi_X$$

Finally, the estimate of the frame of speech resulting from the spectral subtraction is recovered from the FFT estimate:

$$Y(f) = |\hat{S}(f)| e^{i\hat{\phi}_s} = \left( |X(f)|^2 - k \cdot |\hat{V}(f)|^2 \right)^{\frac{1}{2}} \cdot e^{i\phi_x}$$

A more general expression is

$$\hat{S}(f) = \left( |X(f)|^a - k \cdot |\hat{V}(f)|^a \right)^{\frac{1}{a}} \cdot e^{i\phi_x}$$

This is the common general form of spectral subtraction with extra parameters added to make the subtractor more versatile. The power exponent $a$ is normally chosen to be one or two, and the degree of subtraction can be further varied by the parameter $k$. Some form of rectification is also common, implying that when the frequency bins in $\hat{S}(f)$ are negative they are either set at zero, or their sign is switched positive. The expression is rewritten as follows

$$\hat{S}(f) = \left\{ \frac{|X(f)|^a - k \cdot |\hat{V}(f)|^a}{|X(f)|^a} \right\}^{\frac{1}{a}} \cdot |X(f)| e^{i\phi_x}$$

The curly brackets term is merely interpreted as a multiplicative weighting factor for each frequency on the incoming spectrum $X(f) = |X(f)| e^{i\phi_x}$, thereby resembling conventional Wiener filtering.

Observe that the separation into phase and amplitude information can be avoided when writing spectral subtraction as a multiplicative term on the incoming spectrum. This saves calculation time and makes the algorithm faster. However, care must be taken to avoid problems when the denominator has a value close to zero.
When implementing, the original sequence $x(n)$ is divided into blocks of $N$ length. Putting the block length as an index in equation, 16 we obtain

$$Y_N = \hat{S}_N = \left\{ \frac{|X|_N^a - k \cdot |V|_N^a}{|X|_N^a} \right\}^{\frac{1}{2}} \cdot X_N$$ \hspace{1cm} (17)

Equation 17 has been rewritten giving a simple algorithm

$$Y_N = G_N \cdot X_N \quad \text{where} \quad G_{A,N} = \left(1 - \frac{k \cdot |V|_{A,N}^a}{|X|_N^a}\right)^{\frac{1}{2}}$$ \hspace{1cm} (18)

The block diagram in figure 11 depicts the algorithm.

![Block diagram](image)

Figure 11: Diagram showing how spectral subtraction works block by block.

The variance of the estimates limits the performance and thereby limits the SNR improvement on the output signal. The variance in the noise estimate can be reduced by averaging it over several blocks, usually with an exponential block number decay

$$V_L(f) = \alpha \cdot V_{(L-1)}(f) - (1 - \alpha) \cdot X_{new}(f)$$ \hspace{1cm} (19)

By choosing $\alpha$ closer to 1, a more robust noise estimate is obtained.

### 4.1 Evaluation

The following evaluation has been performed on data recorded in an AS332 ”Super Puma MKII helicopter during flight using a ordinary unidirectional, close-talk
microphone. In order to evaluate the performance of the algorithm, subjective evaluations are most valuable, but there raise problems when repeated, since repetitivity is not guaranteed. An objective measure is

\[
\text{Noise Suppression} = \frac{\sum y(n)^2}{\sum x(n)^2}
\]  

over a noise-only interval. The degree of speech distortion is left for subjective evaluations, since no good objective measure is obvious here. Figure 12 depicts first the original sequence of noise and speech and then the same sequence after the spectral subtraction algorithm.

The sampling frequency was 8 kHz for the original sequence, which was also filtered to the telephone bandwidth (300-3300 Hz) before entering the spectral subtraction. During this evaluation the following values of the parameters were chosen:

- Subtraction parameter, \( k = 0.95 \)
- Power factor, \( a = 0.5 \)
- Block length, \( N = 256 \)

The noise reduction using Eq. 20 was as high as 21 dB, and the speech distortion was moderate. Since the algorithm works with blocks, a slight delay from input to the output signal is introduced. The time delay is \( 2 \times 256/8000 = 8/125 = 0.064 \) seconds if \( N = 256 \).
5 Summary and Conclusions

The noise levels in helicopters are substantial, especially at low frequency. These levels are not normally harmful to the ear. However, the low frequency content masks speech. For this reason, pilots tend to turn the intercom system to maximum sound level, producing sound levels which are potentially damaging to the human ear. The sound levels inside the ear canal have been measured to almost 100 dB(A) when the intercom system is in use. Such high sound levels expose the ear to fatigue and loss of hearing. It is thus important to reduce background noise, and also to improve the SNR in the speech signal in the intercom system. A hybrid headset combined with spectral subtraction is proposed as a good solution to the problem. The headset consists of a digital feedforward controller based on a complex LMS-algorithm and an analog feedback controller. This combination results in an efficient noise reduction of approximately 20 dB broadband in the frequency range 50-400 Hz, and a further 20 dB on several tonal components in the frequency range 17-107 Hz. At the same time, spectral subtraction improves the SNR in the speech signal by about 20 dB. These two techniques allow a sound level of no more than 80 dB(A) inside the earcups even when the intercom system is in use.

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References


