# Adaptive Feedback Control of Machine-Tool Vibration based on The Filtered-x LMS-algorithm

# L. Håkansson\*, I, Claesson\*\* and P.-O. H. Sturesson\*\*\*

\* Department of Production and Materials Engineering
\*\* Department of Telecommunication Theory
Lund University, Sweden
\*\*\* Noise, Vibration & Harshness Center
Saab Automobile AB, Sweden

received 31 August 1997

# **ABSTRACT**

Adaptive feedback control of tool vibrations during metal cutting in a lathe has been investigated. The vibrations were controlled in the primary cutting direction and the control is based on the filtered-x LMS-algorithm. It was found that the adaptive feedback control can achieve a reduction of the tool vibrations with up to 35 dB at 1.7 kHz and simultaneously with approximately 30 dB at 3.1 kHz. A significant improvement of the workpiece surface was experienced together with a substantial reduction of the acoustic noise level with adaptive feedback control. Tool life is also expected to be extended and the material removal rate can probably be increased.

# 1 INTRODUCTION

In the turning operation the tool and tool holder shank are subjected to a dynamic excitation due to the deformation of work material during the cutting operation. The stochastic chip formation process usually induces vibrations in the machine-tool system. Energy from the chip formation process excites the mechanical modes of the machine-tool system. Modes of the workpiece may also influence the tool vibration. The relative dynamic motion between cutting tool and workpiece will affect the result of the machining, in particular the surface finish. Furthermore, the tool life is correlated with the amount of vibration and acoustic noise introduced, sometimes at almost unbearable levels.

The research in metal cutting is very intensive. With new cutting technologies, especially high speed machining, where the cutting force dynamics are significant in new "hard-to-cut" materials, such as aerospace alloys, there is an urgent need for methods which decrease the relative dynamic motion between cutting tool and workpiece. It is well known that the vibration problem is closely related to the dynamic stiffness of the structure of the machinery and workpiece material. The vibration problem may be solved in part by proper machine design which stiffens the machine structure. However, in order to achieve further improvements, the dynamic stiffness of the tool holder shank can be increased more selectively.

A solution to these problems is active control of the tool vibrations. The tool vibrations in a turning operation are mainly composed by vibrations in two directions, the cutting speed direction and the feed direction [1, 2]. Consequently, the control problem involves the introduction of two secondary

sources, driven such that the anti-vibrations generated by means of these sources interfere destructively with the tool vibration [1]. The statistical properties of the tool vibration imply a controller which is based on control employing the statistical correlation of the vibrations [3]. A classical, but yet useful statistical criterion is the mean square error criterion [4]. However, a controller based on this criterion can not generally solve this control problem, since such a controller is only "optimum" in a stationary environment [5]. Variations both in cutting data and in material properties influence the statistical properties of the tool vibrations [2]. A solution to the problem is an adaptive controller which is able to adjust its behaviour in a non-stationary environment [1, 5]. A complication in the turning operation is that the original excitation of the tool vibration, the chip formation process, cannot be observed directly and thus cannot be used as a feedforward control signal.

A single-channel feedback controller which is based on the well known filtered-x LMS- algorithm [5] has been used for the control of tool vibrations in the cutting speed direction. The single channel control system is illustrated in figure 1.

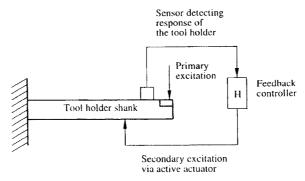


Figure 1. A machine-tool feedback control system.

The tool holder in this application is a construction with integrated actuators, i.e. secondary sources, which has been developed at DPME<sup>1</sup> [6]. The construction of the tool holder is shown in figure 2.

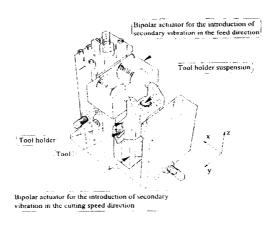


Figure 2. Tool holder with integrated actuators for the control of tool vibrations in the metal cutting process [6]

<sup>&</sup>lt;sup>1</sup>Department of Production and Materials Engineering, Institute of Technology, Lund University.

#### 2 MATERIALS AND METHODS

Active control of machine-tool vibrations is a fairly complex solution. It involves knowledge in metal cutting, mechanical vibrations, signal processing and control theory. Depending on workpiece material, cutting data and machine-tool-system, both the number of modes and the dominating mode involved in the motion of the tool, varies. The motion of the tool usually spans the space defined by the cutting speed direction and the feed direction. The difficulty of achieving good active control of machine-tool vibrations is essentially related to the machine-tool-system, the machining operation and the workpiece material.

## 2.1 Experimental Setup

The cutting experiments have been carried out on a Koping lathe with 6 kW spindle power. The equipment that has been used in the experiments is:

- 1. Tool holder construction with integrated actuators [6].
- 2. Accelerometer Brüel & Kjær model 4374
- 3. Charge amplifier Brüel & Kjær model 2635.
- 4. Current amplifier Techron 7700 series, 5kW power supply for the actuators.
- 5. Frequency Analyzer *HP 35665A Dynamic Signal Analyzer*, Bandwidth: 102 kHz one channel, or 51 kHz, two channels.
- 6. Signal processing unit *Burr Brown*, *PCI-20202C*, digital signal processor carrier with TMS320C25 signal processor.
- 7. A/D-converter Burr Brown. PCI-20023M-1, 180 kHz, 8 inputs with 12 bits resolution.
- 8. D/A-converter Burr Brown. PCI-20003M, 120kHz, 2 outputs with 12 bits resolution.
- 9. Programmable two channel low-pass filter Kemo VBF10M-Opt 25.

The accelerometer was mounted on the cutting tool in order to measure the vibrations in the cutting speed direction.

The tool holder construction with integrated actuators [6], is based on two bipolar actuators [6]. The bipolar design is motivated by an desire for linear behaviour and is composed of two actuators that work with 180° phase lag. The actuators are based on high magnetostrictive material.

# 2.1.1 Work Material - cutting data - tool geometry

The workpiece material SS 2541-03, chromium molybdenum nickel steel [2], was used in the experiments. This work material excites the machine-tool-system with a narrow bandwidth in the cutting operation. After a preliminary set of trials a suitable combination of cutting data and tool geometry was selected, see table I. The combination was selected to cause significant tool vibrations which resulted in an observable deterioration of the workpiece surface and severe acoustic noise.

# TABLE I

Cutting data and tool geometry.

Geometry Cutting speed, v (m/min) Depth of cut a (mm) Feed s (mm/rev) DNMG 150604-PF 4015 80 0.7 0.19

The diameter of the workpiece was chosen to be large, over 100 mm. The workpiece vibrations can therefore be neglected.

#### 2.2 Tool vibration

The obtained tool shank vibrations are random in nature, usually characterized by more than one mode except for the lowest feed rate and cutting speed [2]. If a linear model can be assumed, the time-domain dynamic response of the tool holder is determined by the mode superposition principle [2, 7]. Each modal displacement is determined by a convolution integral, a mechanical filtering, denoted Duhamel's integral [7], where the input is the dynamic generalized load. The modal response of the filter is determined by the modal damping ratio, the generalized modal mass, the generalized modal load as well as the eigenfrequencies for the damped and undamped system. Usually, machine-tool systems are classified as narrow-band systems. Hence, most structural systems have reasonably low damping and are therefore classified as narrow-band systems [7]. Consequently tool shank vibrations can usually be described as a superposition of narrow-band random processes at each modal frequency, which added together form a somewhat more wide-band random process [2, 7]. In figure 3 the spectral densities of the dynamic response of a tool holder shank are plotted. The densities are measured in the primary cutting direction during a continuous cutting operation in SS 0727-02. A constant feed rate, s = 0.3mm/rev, for varying cutting speeds between 50 m/min and 400 m/min with a step of 25 m/min was used. The modal behaviour of the tool shank vibrations is obvious from figure 3.

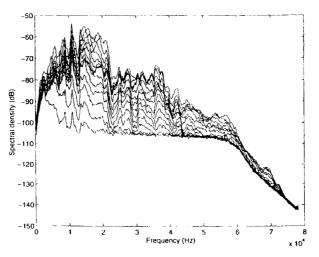


Figure 3. The spectral densities of the dynamic response of the tool holder shank in the primary direction during a continuous cutting operation. Workpiece material SS 0727-02, feed rate s=0.3 mm/rev, cut depth a=3mm, cutting speed v=50 m/min to 400 m/min, tool DNMG-QM 150612, grade 4025 [2].

It is likely that the tool holder construction with integrated actuators [6] have inherent nonlinearities. The tool holder is suspended on ball bearings. To emphasize that the narrow-band components in the tool vibration might be nonlinear, the notation resonance frequency is used instead of partly eigenfrequency.

The tool vibrations in the primary direction were characterized by a number of resonance frequencies. A typical estimated spectrum for the tool vibrations, is shown in figure 4.

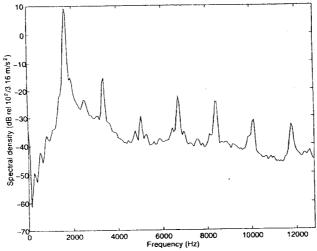


Figure 4. The spectral density of the dynamic response of the tool holder shank in the primary direction during a continuous cutting operation. Workpiece material SS 2541-03, cutting speed v = 80 m/min, cut depth a = 0.7 mm, feed rate s = 0.19 mm/rev, tool DNMG 150604-PF, grade 4015.

The estimation of the spectral density was performed on a HP 35665A dynamic signal analyzer with Welch's method [8]. The parameters used in the spectral density estimation are shown in Table II.

### TABLE II

Spectral density estimation parameters.

Parameter	Value
Data length	51200
Data segment length, L	1024
Number of periodograms, K	50
Digital window w (τ)	Hanning
Sampling rate	65536 Hz

# 2.3 Adaptive Feedback Control of Machine-Tool Vibration

The original excitation of the tool vibrations, originating from the material deformation process, cannot be directly observed. Consequently, the controller for the control of machine-tool vibration is based on a feedback approach. The response of the tool holder can be measured with a sensor mounted on the machine-tool. By the introduction of secondary anti-vibrations from secondary source, actuator, the response of the tool holder can be modified [1]. The actuator is steered by a controller which is fed with the accelerometer signal sensing the vibrations of the tool holder. A block diagram of the feedback control system is shown in figure 1.

Adaptive digital FIR filters based on the method of steepest descent are popular in various application areas, e.g. active control of sound [5, 3], active

control of vibration [9] and in other applications, such as electrical noise cancellation, system identification, adaptive beamforming, etc. [10, 11, 12]. This is due to the simplicity of the implementation and their unimodal error surface in the feedforward application. A feedforward active controller can easily be controlled to converge towards a feasible solution [13, 11, 5, 10]. Usually adaptive FIR filters are used in feedforward control [3, 14, 15] but can also be used in feedback control [16, 1], even though there is no guarantee that the error surface will be unimodal under these conditions [17]. Similar problems can also be observed in feedforward control systems, when the control problem is badly conditioned. A method to improve such systems is to add a leaky term to the adaptation algorithm [13, 18, 11, 19]. This will also prevent an accumulative build-up of bias in the coefficients of the adaptive filter [20].

# 2.3.1 The Adaptive FIR-filter for The Single Channel Control

The output signal from the adaptive filter is an estimate of the signal to be cancelled. In the case of active control, the adaptive filter works as a controller and controls a dynamic system containing actuators and amplifiers etc., so the anti-vibrations in this case can be seen as the output signal from a dynamic system, a forward path. Since there is a dynamic system between the filter output and the anti-vibrations, the selection of adaptive filter algorithms must be done with care. A conventional adaptive algorithm, such as the LMS algorithm, is likely to prove unstable in this application, due to the phase shift (delay) introduced by the forward path [3, 21]. However, the filtered-x LMS-algorithm is suitable for active control applications [3, 5, 10].

The filtered-x LMS-algorithm is developed from the LMS algorithm, where a model of the dynamic system between the filter output and the estimate, i.e. the forward path is introduced between the input signal and the algorithm for the adaptation of the coefficient vector [3, 5, 10]. Figure 5 shows an active control system with a controller based on the filtered-x LMS algorithm.

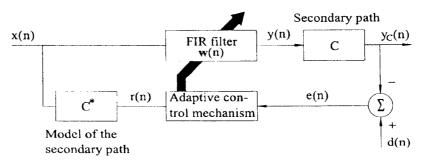


Figure 5. Active control system with a controller based on the filtered-x LMS-algorithm.

Assuming that the transfer function for the secondary path can be modelled with an I<sup>th</sup>-order FIR filter, with coefficients  $c_i$ ,  $i \in \{0,..., I-1\}$ , the error is given by

$$e(n) = d(n) - \sum_{i=0}^{I-1} c_i \sum_{m=0}^{M-1} w_m(n-i)x(n-i-m)$$
 (1)

The Wiener (Mean Square Error) solution of the coefficient vector is

obtained by minimizing the quadratic function [22, 3, 5, 10]

$$J_f(n) = E[e(n)^2] \tag{2}$$

The derivative of this function with respect to each coefficient is

$$\frac{\partial J_f(n)}{\partial w_m(n)} = 2E \left[ e(n) \frac{\partial e(n)}{\partial w_m(n)} \right]$$
 (3)

If we further assume that  $w_m$ ,  $m \in \{0,..., M-1\}$  is time invariant, the differential of the estimation error with respect to a coefficient is given by:

$$\frac{\partial e(n)}{\partial w_m(n)} = -\sum_{i=0}^{I-1} c_i x(n-i-m) \tag{4}$$

The assumption of time invariance is motivated by a slow coefficient change in comparison with the time scale of the response of the system to be controlled. In practice, the filtered-x LMS-algorithm shows stable behaviour even when the coefficients change within the time scale associated with the dynamic response of the secondary path [22, 3, 5, 10]. The filtered-x LMS-algorithm is given by the following four equations [22, 3, 5, 10];

$$y(n) = \mathbf{w}^{T}(n)\mathbf{x}(n) \tag{5}$$

$$e(n) = d(n) - y_C(n) \tag{6}$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{r}(n)e(n) \tag{7}$$

and

$$r(n) = \sum_{i=0}^{I-1} c_i^* x(n-i)$$
 (8)

where  $c_i^*, i \in \{0,..., I-1\}$  is an estimate of the impulse response of the forward (secondary) path.

# 2.3.2 The adaptive feedback controller

The controller used in the experiments reported here is a feedback controller based on the well known filtered-x LMS-algorithm [1, 16]. The block diagram of the feedback control system is shown in figure 6 (a). Also showing 6 (b) the block diagram of the filtered-x LMS-algorithm in the feedback application. In these diagrams C is the dynamic secondary system (forward path) under control, i.e. the electro-mechanic response. The estimate of this path is denoted C\*.

The secondary path was estimated in an initial phase with another FIR filter, which subsequently is used to prefilter the input signal to the algorithm for the adaptation of the coefficient vector.

In the experiments a 16 tap adaptive FIR filter was used together with three different estimates (4, 8 and 16 taps), of the secondary path. Typical secondary path estimates are shown in Fig. 7.

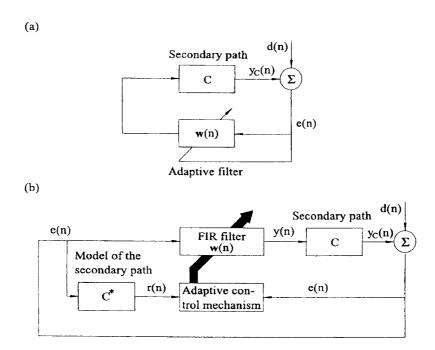


Figure 6. Equivalent block diagram of the feedback control situation (a) and the filtered-x LMS algorithm in the feedback application (b).

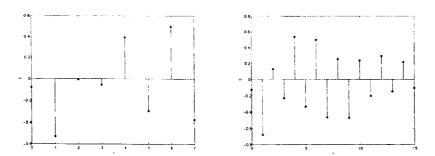


Figure 7. The coefficients of the 8 and 16 taps FIR filter estimates of the secondary path

The sampling rate of the digital filter was chosen as 15 kHz. In order to minimize the delay in the loop no anti-aliasing or reconstruction filters were used, which demands extra care be taken.

## **3 RESULTS**

In order to illustrate the effect of feedback control, the spectral densities of tool vibrations with and without feedback control are plotted. The spectral densities were estimated with an HP 35665A dynamic signal analyzer using Welch's method [8]. The parameters used for the spectral density estimation are shown in Table III.

Feedback control with a 16 tap FIR filter and only a 4 tap FIR filter estimate of the secondary path, reduced the tool vibration approximately by 25 dB at 1.7 kHz, see figure 8. With the 8 tap FIR filter model of the secondary path, the feedback controller managed to reduce the tool vibrations by an additional 10 dB, i.e. the tool vibrations were reduced by 35 dB at 1.7 kHz, according to figure 9.

# TABLE III

Spectral density estimation parameters.

Parameter	Value
Data length	51200
Data segment length, L	1024
Number of periodograms, K	50
Digital window w (τ)	Hanning
Sampling rate	8192 Hz

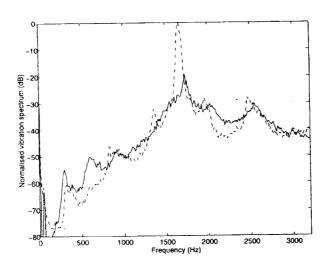


Figure 8. The spectral density of tool vibrations with 16 tap FIR filter feedback control (solid) and without (dashed). A 4 tap FIR filter was used to estimate the secondary path. Cutting speed  $\nu = 80$  m/min, cut depth a = 0.7 mm, feed rate s = 0.19 mm/rev, tool DNMG 150604-PF, grade 4015.

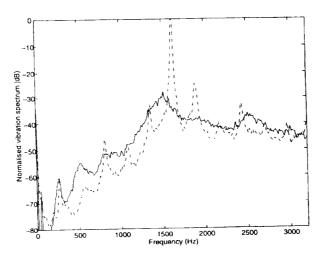


Figure 9. The spectral density of tool vibration with 16 tap FIR filter feedback control (solid) and without (dashed). An 8 tap FIR filter was used to model the secondary path. Cutting speed v=80 m/min, cut depth a=0.7 mm, feed rate s=0.19 mm/rev, tool DNMG 150604-PF, grade 4015.

Figure 10 a) shows the coefficients of the adaptive FIR filter, when the 4 tap FIR filter estimate was used in the controller. Further in figure 10 b) the coefficients of the adaptive FIR filter, when the 8 tap FIR filter estimate was used in the controller, are shown. Observe the phase differences originating from the different estimates of the secondary paths.

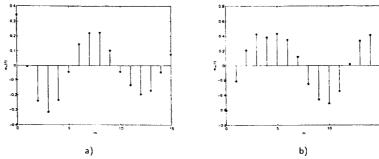


Figure 10. The coefficients of the adaptive FIR filter, a) 4 tap FIR filter used to model the secondary path and b) 8 tap FIR filter used to model the secondary path.

When the cutting proceeded, the number of revolutions of the workpiece was automatically increased to compensate for the reduction of the workpiece diameter, e.g. a constant cutting speed was kept. Consequently, the cutting conditions change and the first resonance frequency was shifted slightly downwards to approximately 1.5 kHz. The second resonance frequency also became more significant and was thereby automatically included in the vibration control.

The feedback control with a 16 tap FIR adaptive filter and a 16 tap FIR estimate of the secondary path, reduced the tool vibration by approximately 35 dB at 1.5 kHz and 30 dB at 3.1 kHz, see figure 11. Similar results were observed with the 8 tap FIR filter estimate of the secondary path, according to figure 12.

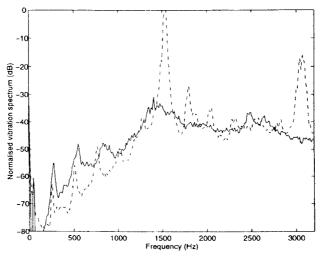


Figure 11. The spectral density of tool vibration with 16 tap FIR filter feedback control (solid) and without (dashed). A 16 tap FIR filter was used to model the secondary path. Cutting speed v = 80 m/min, cut depth a = 0.7 mm, feed rate s = 0.19 mm/rev, tool DNMG 150604-PF, grade 4015.

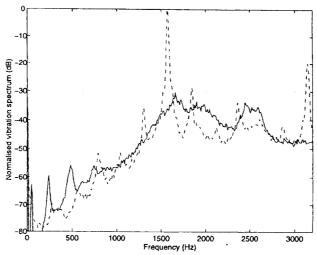


Figure 12. The spectral density of tool vibration with 16 tap FIR filter feedback control (solid) and without (dashed). A 8 tap FIR filter was used to model the secondary path. Cutting speed v = 80 m/min, cut depth a = 0.7 mm, feed rate s = 0.19 mm/rev, tool DNMG 150604-PF, grade 4015.

The coefficients of the adaptive FIR filter, when the 16 tap FIR estimate of the secondary path was used in the controller, are shown in figure 13 a). Figure 13 b) shows the coefficients of the adaptive FIR filter, when the 8 tap FIR estimate of the secondary path was used in the controller, controlling two resonance frequencies of the tool vibration.

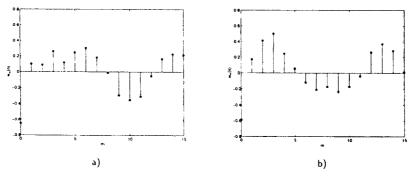


Figure 13. The coefficients of the adaptive FIR filter, a) 16 tap FIR filter used to estimate the secondary path and b) 8 tap FIR filter used to estimate the secondary path.

When the 4 tap FIR filter model of the secondary path was used, no significant reduction of the tool vibrations at the second resonance frequency were observed.

In the experiments, a significant improvement of the workpiece surface and a simultaneous reduction of the acoustic noise induced by the tool vibrations, were observed. In figure 14 a photo of the workpiece used in the experiments is given. Further, in figure 15 a microscope photo of the workpiece surface with and without control is shown.

The areas marked with a and b are results from the adaptive feedback control.

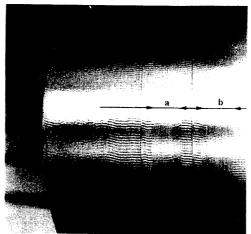


Figure 14. The workpiece used in the experiments.

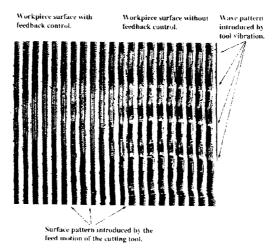


Figure 15. Workpiece surface with and without feedback control (magnification 20X), v = 80 m/min, a = 0.7 mm, s = 0.19 mm/rev.

# 4 DISCUSSION AND CONCLUSIONS

It is clear that tool vibrations in a lathe during metal cutting can be controlled with an active control system. The tool holder shank vibrations are fed into an actuator via a digital controller. The well known filtered-x LMS-algorithm, traditionally used as feedforward controller, seems to be very promising for the feedback control of tool vibrations in the turning operation.

The adaptive feedback controller achieved similar performance of the tool vibration with 8 or 16 taps FIR filter estimate of the secondary path. In both cases, a reduction of the tool vibration with approximately 35 dB at the first resonance frequency and 30 dB at the second resonance frequency were obtained, see figure 11 and figure 12. A 4 taps FIR forward path model is generally not sufficient. Less attenuation (10 dB less) at the first resonance frequency and no significant reduction of the tool vibration at the second resonance frequency was observed. It was also observed that the controller converged faster and tracked the spectral variations of the tool vibration better, when using the 8 or 16 taps FIR filter estimate of the secondary path.

Convergence speed and tracking performance of the filtered-x LMS algorithm are affected by the accuracy of the model of the secondary path [23, 24]. Errors within the model of the secondary path influence the gradient estimate [3], introducing bias in the estimate of the gradient vector [21, 3]. It is likely that a poor model of the secondary path causes a slow convergence, [3, 23, 24], which of course leads to a poor tracking performance of the algorithm [11, 3] and thereby limited performance, see figure 8.

The action of an adaptive FIR filter in the feedback application appears similar to the adaptive line enhancer widely used in adaptive signal processing. The sinusoidal shape of the FIR filter coefficients obtained in the experiments is typical, see figures 10 and 13. The spectral properties of the tool vibration are shown in figure 4, a broad-band background together with narrow-band components. It is likely that the delay introduced by the secondary path decorrelates the broad-band part of the tool vibration but not the narrow-band components.

From a manufacturing engineering point of view, the significant improvement of the work-piece surface, see figure 15, achieved with the adaptive feedback control of the tool vibration, is of great importance. The reduction of the noise introduced by the tool vibrations is an important feature. It is also interesting to note that the adaptive technique does not affect the cutting data, it may even allow an increase of the material removal rate. Further, it is well known that there exists a correlation between tool vibrations and tool life. Therefore, it is likely that the adaptive feedback control of the tool vibration extends the tool life.

From the analysis of the experiments the following conclusions can be drawn:

- Tool vibrations in a lathe can be reduced with adaptive feedback control based on the filtered-x LMS algorithm.
- Adaptive feedback control of the cutting tool enables a reduction of the tool vibrations with approximately 35 dB at the first resonance frequency and with 30 dB at the second resonance frequency.
- Adaptive feedback control is fairly insensitive to errors within the model of the secondary path.
- 4. The action of the adaptive FIR filter is similar to the adaptive line enhancer.
- The dynamic stiffness of the tool holder shank is increased with adaptive feedback control of the cutting tool.
- 6. The workpiece surface is improved with the adaptive feedback control of tool vibration.
- 7. The feedback control reduces acoustic noise induced by the tool vibration.
- 8. It is likely that the adaptive feedback control extends the tool life.
- It is likely that the adaptive feedback control allows an increased material removal rate.

Future work includes the influence of the adaptive FIR filter and the filtered-x LMS algorithm with a leakage factor. A theoretical foundation for the behaviour of the filtered-x LMS algorithm in this application is also urgent.

# REFERENCES

 L. Håkansson, P-O. Sturesson, and I. Claesson. Active control of machine-tool vibration. In 6th International Conference on Manufacturing Engineering, 1995.

#### ADAPTIVE FEEDBACK CONTROL OF MACHINE-TOOL VIBRATION

- P-O. H. Sturesson, L. Håkansson, and I. Claesson. Identification of the statistical properties of the cutting force dynamics in a continuous turning operation: Part 1. Journal of Mechanical Systems and Signal Processing, Academic Press, Submitted, 1996.
- 3. S.J. Elliott and P.A. Nelson. Active noise control. *IEEE signal processing magazine*, pages 12-35, October 1993.
- 4. A. Papoulis. *Probability, Random Variables, and Stochastic Processes*. McGraw-Hill, second edition, 1984.
- P.A. Nelson and S.J. Elliott. Active Control of Sound. Academic Press, Inc, 1992.
- P. Andersson. A tool holder construction with integrated actuators. Master's thesis, report 9054, Department of Production and Materials Engineering, Institute of Technology, Lund University, 1990. In Swedish.
- R.W. Clough and J. Penzien. Dynamics of Structures. McGraw-Hill, second edition, 1993.
- 8. P.D. Welch. The use of fast fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms. *IEEE Transactions on Audio and Electroacoustics*, pages 70-73, June 1967.
- 9. C.R. Fuller, S.J. Elliott, and P.A. Nelson. *Active Control of Vibration*. Academic Press, Inc, 1996.
- 10. B. Widrow and S.D. Stearns. Adaptive Signal Processing. Prentice-Hall, 1985.
- 11. S. Haykin. Adaptive Filter Theory. Prentice-Hall, third edition, 1996.
- 12. I. Claesson, S. Nordholm, B. Bengtsson, and P. Eriksson. A multi-dsp implementation of a broadband adaptive beamformer for use in a handsfree mobile radio telephone. *IEEE Trans. on Vehicular Technology*, February 1991.
- P.M. Clarkson. Optimal and Adaptive Signal Processing. CRC Press, Inc, 1993.
- S.J. Elliott, P.A Nelson, I.M. Stothers, and C.C. Boucher. In-flight experiments on the active control of propeller-induced cabin noise. *Journal of Sound and Vibration*, 140:219-238, 1990.
- 15. S.J. Elliott, I.M. Stothers, P.A. Nelson, A.M. McDonald, D.C. Quinn, and T.J. Saunders. The active control of engine noise inside cars. In *Proc. Inter-Noise* '88, pages 987-990, 1988.
- 16. I.M. Stothers, T.J. Saunders, A.M. McDonald, and S.J. Elliott. Adaptive feedback control of sun roof flow oscillations. *Proceedings of the Institute of Acoustics*, 15:383-393, 1993.

- 17. G. Billoud, M.A. Galland, C.H. Huu, and S. Candel. Adaptive active control of instabilities. In *Recent Advances in Active Control of Sound and Vibration*, pages 95-107, 1991.
- 18. S.J. Elliott, C.C. Boucher, and P.A. Nelson. The behaviour of a multiple channel active control system. *IEEE Transactions on signal processing*, 40(5):1041-1052, May 1992.
- 19. J.R. Treichler, C.R. Johnson, and M.G. Larimore. *Theory and Design of Adaptive Filters*. Wiley Interscience, New York, 1987.
- 20. J.M. Cioffi. Limited-precision effects in adaptive filtering. *IEEE Transactions on circuits and systems*, CAS-34(7):821-833, 1987.
- 21. D.R. Morgan. An analysis of multiple correlation cancellation loops with a filter in the auxiliary path. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-28(4):454-467, August 1980.
- 22. S.J. Elliott and P.A. Nelson. Multiple-point equalization in a room using adaptive digital filters. *Journal of audio engineering society*, 37(11):899-907, November 1989.
- 23. S.J. Elliott, I.M. Stothers, and P.A. Nelson. A multiple error lms algorithm and its application to the active control of sound and vibration. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-35(10):1423-1434, 1987.
- C.C. Boucher, S.J. Elliott, and P.A. Nelson. Effect of errors in the plant model on the performance of algorithms for adaptive feedforward control. In *Proceedings of Institution of Electrical Engineers*, volume 138, pages 313-319, 1991.

# **ACKNOWLEDGMENT**

The project has been sponsored by the Volvo Research Foundation and Volvo Educational Foundation, the National Swedish Board for Technical Development (NUTEK).

UK ISSN 0263-0923 VOLUME 17 NUMBER 4 1998

