ANALYSIS OF THE ACCURACY OF BURSTY TRAFFIC MODELS

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1 Introduction

Traffic models are an essential component of performance analyses of telecommunication networks. Good models should be simple, accurate and applicable in both mathematical analysis and computer simulations.

In circuit switched networks, including narrowband ISDN, traffic is usually modelled as Poisson or characterised in terms of its first few moments. For packet switched networks, such as B-ISDN based on ATM, the Poisson model may still be applicable for some types of services on a higher level, such as call requests. However, when it comes to low level issues, for example dimensioning buffers in switches, a Poisson model is not likely to be valid. This is because most sources in such a network are known to produce information at a much more uneven rate than Poisson. Such traffic is commonly referred to as bursty, and it may even contain strong long term correlation. This has lead to the development of a large number of models emphasising different aspects of the packet arrival processes.

Strictly speaking, a complete model is not only the model itself but also a procedure to compute its parameters from some characteristic traffic data. It has been argued that different models and different traffic characteristics should be used for different sources [7]. Yet, to enable sufficiently general analysis and dimensioning of packet switched networks, it is most desirable to find characteristic parameters of traffics that are independent of sources, and tractable models that can reproduce these. Proposals for such a set of parameters, which we have not been able to verify as yet, are discussed in [1, 14, 31].

The present work, which is a part of a long term project, is an attempt to systematically collect and compare a number of commonly used models and fitting methods. Some earlier results are found in [3, 4, 6]. Our approach is to apply all models/fitting methods to the same data and examine differences in their performance. Having computed model parameters from given data using different techniques, the models are run in a computer simulator. The simulator consists of a server with a deterministic service time and an infinite queue, the performance of which is recorded and analysed. When several arrival processes are run in parallel, these are independent. Since all models are fitted to reproduce the same traffic, differences in the performance of the single server system reflect differences between models.

2 Models and Fitting Methods

2.1 Estimation of Traffic Characteristics

The important characteristics of most models presented in the literature are average rate, peak rate and burst length, although these are often used without explicit definitions.

The average rate is possibly the only metric for which a proper definition is relatively straightforward. Denoting the number of arrivals during a time t by $N(t)$, the average rate $\overline{r}$ can be defined as

$$\overline{r} = \lim_{t \to \infty} \frac{N(t)}{t} \tag{1}$$

The peak rate can intuitively be defined as the inverse of the shortest interarrival time that a given source is capable of producing, but this can lead to an almost infinite peak rate which is not useful in practice. A possibly more appealing approach is to relate the two concepts of peak rate and burst to each other: peak rate is the transmission rate during a burst and a burst length is the period during which a source transmits at the peak rate. Although peak rates and burst lengths defined in this way may be given numbers for the purpose of a theoretical analysis, this definition is not sufficient to estimate the characteristics from real data: When does a burst actually begin and where does it end?

Throughout our work, we have adopted the definition of a burst used by Solé et al., [27, 28]: A burst consists of arrivals that are not separated by more than a maximum allowed interarrival gap (MAIG), in an appropriate time unit, and must last at least for $L_{\min}$ time units. The last arrival within a burst is separated from the next arrival by at least an interval of MAIG time units. Also, a sequence of arrivals, each separated by less than the MAIG, is not considered a burst unless the time from the first arrival to the last one is at least $L_{\min}$. Like [27, 28], we use the mean interarrival time $1/\overline{r}$ as both MAIG and $L_{\min}$, though other values could be used as well: In [15] is $L_{\min} = 0$ and MAIG is set so that about 90% of all interarrival times are less than the MAIG.

2.2 The Autoregressive Markov Model

The autoregressive Markov model (AMM), due to Maglaris et al. [17], generates a Markov dependent number of arrivals within a frame. Denoting the number of arrivals in frame n by λ_n we write $\lambda_n = a\lambda_{n-1} + bw(n)$, where a and b are constants, $|a| < 1$, and w is a sequence of independent normally distributed random variables with mean η and variance σ^2, the latter is set to 1 for convenience.

The data available to us was not in terms of packets per frame but as successive interarrival times. We therefore defined a frame as the length of an average burst-silence cycle, [27, 28]. The parameters a, b and η of the model are matched to fit the mean, the variance and the first order correlation of the number of arrivals per frame.

2.3 The Switched Poisson Process

A switched Poisson process (SPP), is a Poisson process, the rate of which is determined by the state of an underlying two state Markov chain. The transition rates of the Markov chain, denoted by γ (from 1 to 2) and ω (from 2 to 1), and the arrival rates associated with the two states, λ_1 and λ_2 respectively, are independent.

We have tried six different approaches to estimate the parameters of an SPP which are due to Meier-Hellstern [19, 20], Rydén [25], Rossiter [23, 24], Heffes and Lucantoni [13], Gusella [12] and Okuda et al. [21] and in the sequel, these are referred to as KMH, TR, MR, HL, RG and OAS respectively.

The basic idea of the KMH approach is to find the parameters of the SPP that has the maximum likelihood of producing the observed interarrival sequence conditioned on an estimate of the behaviour of the underlying Markov regime. The method is to alternatively estimate the history of the process, that is the state in which each arrival respectively was generated, and the four parameters respectively. Starting from a basic estimate of the history a first set of parameters is estimated. A new set of parameters leads to a better estimate of the history which in turn lead to a better estimate of the parameters etc. in an iterative way.

The TR approach also employs a maximum-likelihood estimation but does not attempt to obtain a full history of the process. The idea is simply to find the four parameters that have the largest likelihood of producing an observed sequence of interarrival times. Comparing this method to the former, the TR method is consistent while KMH is not. We have used MR to provide the initial estimate required by the TR method.

The four other methods employ the index of dispersion $I(t)$ and the index of skewness $S(t)$ of the number of arrivals, viz:

$$I(t) = V\{N(t)\}/E\{N(t)\} \tag{2}$$
$$S(t) = T\{N(t)\}/E\{N(t)\} \tag{3}$$

where

$$V\{N(t)\} = E\{(N(t) - E\{N(t)\})^2\} \tag{4}$$
$$T\{N(t)\} = E\{(N(t) - E\{N(t)\})^3\} \tag{5}$$

for different values of t.

MR uses the mean m and the squared coefficient of variation C^2 of the interarrival times plus $I(t_1)$ and $I(t_2)$, where t_1 is of "medium length" and t_2 "substantially longer than t_1". In accordance with [23, 24], we have used $t_1 = 10m$ and $t_2 = 2.5t_1$.

The HL method uses the mean m, the index of dispersion at $t = t_0, t_1$ and the third moment of the number of arrivals, $T\{N(t_2)\}$. In accordance with [13] we have used $t_0 = t_2 = 10m$ and set t_1, which is "infinity", to $100m$.

RG uses four indexes of dispersion, at $t = t_0, t_1, t_2$ and the limit as $t \to \infty$. For t_0, which is to be of "medium length", we have used $t_0 = 10m$. Also, as suggested by [12], we have used $t_1 = t_0/2$ and $t_2 = 2t_0$. Finally, we have assumed that the properties of infinity are essentially valid for $t = 100m$.

The last method, OAS, uses the mean arrival rate $\bar{r}$, $\lim_{t\to 0} \frac{d}{dt}I(t)$, $\lim_{t\to\infty} I(t)$ and $\lim_{t\to\infty} S(t)$. We have not used the formulae in [21] to express the four metrics in terms of the distribution of interarrival times since they rely on the traffic being renewal. Instead, we have assumed infinity after $100m$ and, by estimating $I(t)$ at $t = 10^{-3}m$ and $t = m$, we take the slope as $\lim_{t\to 0} \frac{d}{dt}I(t)$.

2.4 The Switched Bernoulli Process

The switched Bernoulli process (SBP) is a discrete version of an SPP: To each of the two states in a discrete time Markov chain is associated a probability of a packet being generated at the end of a time slot. Transition probabilities are $1 - a$ from 1 to 2 and $1 - b$ from 2 to 1, while arrival probabilities are denoted by α and β for the two states respectively. It is convenient to associate arrivals with the state after a transition, i.e. the probability of an arrival during a transition from 1 to 1 is α, but β for a transition from 1 to 2, and so on. It is observed that the geometric model [11] is obtained as a special case of an SBP by setting $\alpha = 1$ and $\beta = 0$.

Due to the close relationship between SPPs and SBPs, fitting methods used for SPPs can be translated to SBPs too, see for example [16] for a maximum-likelihood approach. However, due to the very small significance of discrete or continuous time scales [2, 3], it was assumed this was not likely to add much of interest to our investigation. Instead we used three of the four approaches developed by Solé et al. [27, 28] specifically for SBPs. Hence the model is denoted by SDG in what follows.

The first approach is to estimate the mean burst length $E\{L\}$, the mean silence length $E\{I\}$ and the arrival rates A_a and A_s of the two states respectively, which are then fitted to the model.

The second approach relies on parallels with pulse trains. Four parameters, viz: the cycle length $C = L + I$, the difference in arrival rates $\Delta A = A_a - A_s$, the relative length of a burst in a cycle B and the mean traffic intensity over a complete cycle R, are measured and fitted. This strategy has not been used since it tends to result in an incorrect mean arrival rate and hence makes comparison with other models difficult.

The third approach is a modification of the second one. Having computed the parameters as in strategy 2, we set $A_s = 0$ and adjust the length of a stay in the silent state, I, so that the correct mean arrival rate is obtained.

Finally, in the fourth approach, we match to B and the mean and the variance of the interarrival times.

2.5 The Markov-Hyperexponential Model

The Markov-hyperexponential model [9, 22], in the sequel abbreviated to MH2, suggests that bursts last for a negative exponentially distributed time the mean of which is $1/\mu$, while silence periods have a hyperexponential-2 distribution with parameters λ_1, λ_2 and α. The two stages of the hyperexponential distribution are assumed to represent two different types of silence, viz. pauses between talk spurts and pauses due to the other party of a conversation being active.

In the original proposal, the authors used empirical data due to [30]. The latter reports on the hardware of a specific speech detector and voice discriminator, the output of which is measured, in dBm, and classified as burst or silence accordingly. Since we cannot adopt such an approach here, we have used the burst/silence definition of [27, 28]. We have also assumed that arrivals occur according to a Poisson process at a rate ρ while in the active state.

The model provides five parameters, but one is bounded by the constraint that a speaker on the average is active 50% of the time and is listening for the remaining 50%. This implies that the probability of finding the three state Markov chain in the listening state should equal the total probability of finding it in either the active state or the pause state. To solve for the four remaining parameters of the model, we use strategies 3 and 4 of [27, 28].

In strategy 3 we add the variance $V\{I\}$ of the silence lengths. When scaling the mean silence length, we also scale the variance so that the coefficient of variation C^2 remains constant and we adjust α to meet the speaking/listening constraint.

In strategy 4, the condition of fixed B gives μ. Next the mean and the variance of the interarrival times are used to find the arrival rate A_a and the mean of the hyperexponential silence distribution. (The variance of the latter is locked by the condition of constant C^2.) Again, α is selected to match the requirement of 50% activity.

2.6 The Threshold Model

The threshold model (TH) [8], is a two state model with an active state and a silent, passive, state. Following an arrival, the time to the next arrival is drawn from a distribution F_1. If, however, the value obtained is greater than a threshold T, the silent state is immediately entered for an independent time with distribution $F_2 \geq T$. F_1 and F_2 may in principle be chosen arbitrarily. In [8] different combinations of the uniform, exponential, sine distributions have been considered, however, we have chosen to restrict our investigations to the case where both F_1 and F_2 are uniform distributions.

Denoting the intervals of F_1 and F_2 by $(0, A_1)$ and (T, A_2) respectively, the three parameters of the model are A_1, A_2 and T. We determine these using strategies 3 and 4 of [27, 28], with a burst defined as a sequence of arrivals generated during one, uninterrupted, stay in the arrival state.

2.7 The Erlang-r model

The Erlang-r model [26], in the sequel denoted by ER, is a frame oriented model in which frames are supposed to last for a deterministic time which is approximated by an Erlang-r distribution. Depicting the model as a stage of r, $\{r : r \geq 1\}$ negative exponentially distributed times, arrivals are generated in the first k of these according to a Poisson process of rate λ while no arrivals are generated in the remaining $r - k$ stages. Leaving the rth state marks the end of one frame and a new one follows immediately as the first state of the stage is entered again.

Employing the same artificial frames as for the AMM-model, section 2.2, the parameters of the Erlang-r distribution, denoted by q and r, are fitted to the mean and the coefficient of variation of frame lengths. The number of arrival stages k is set to meet the relative length of active periods B and finally the arrival rate λ is chosen so that the mean interarrival time is correct. Our approach does not require k to be integer as we use a mixture of two Erlang-r distributions: Both have parameters q and r, but the number of arrival stages differ by one. At the end of one frame is the next one selected at random in such a way that the average number of arrival stages is k.

3 Results

A key aspect of our study has been to determine which, if any, of the models under investigation provides the best match to the performance of a real traffic stream. This is quite a difficult problem, since there are many possible "real traffics", many possible models and many different ways in which the parameters of these models can be fitted to a real data stream.

Our method is to apply all fitting methods to a sequence of real data, run the resulting models through a simulator consisting of a single deterministic server and record the first three moments for 5 suitably chosen metrics. The performance of the "real traffic" was recorded similarly by reading interarrival times directly from a data file and the resulting "model" is referred to in our results as "Dir.". In a second series of runs we used 50 statistically identical models in parallel, and a corresponding "real traffic" was created by superimposing the original data upon itself 50 times. Finally, we adopted a simple ranking scheme to assist in determining which models give the best match to the performance of the original traffic.

3.1 Observed Behaviour

Table 1 and 2 show, for a single source and multiple sources respectively, the observed behaviour in terms of interarrival times $\tilde{A}$, queue lengths at an arrival instant $\tilde{Q}$, time spent in system $\tilde{T}$, interdeparture times $\tilde{D}$ and server busy periods $\tilde{B}$ with the mean service time taken as the unit of time. The three rows show the mean, the coefficient of variation and the coefficient of skewness respectively. Denoting the rth central moment by μ_r, the two latter are defined as $\mu_2(\tilde{X})/E(\tilde{X})^2$ and $\mu_3(\tilde{X}^3)/\mu_2(\tilde{X})^{\frac{3}{2}}$ respectively. For each moment, we display the 95% confidence intervals. The relatively large intervals noted for the direct runs are caused by the limited amount of data available.

In table 1 the results for the single source case show that the mean interarrival and interdeparture times match exactly as we would have expected because the system is conserving packet flows and all models were fitted to yield the same average load. A study of the second moments for the interarrival times shows that the MH2 , SDG and AMM models (in that order) have the greatest degree of "burstiness". The pattern of server busy periods matches precisely the relativities in the variance peaks exhibited by the interarrival process. It is also approximately true that the larger the coefficient of

Model	$\tilde{A}$	$\tilde{Q}$	$\tilde{T}$	$\tilde{D}$	$\tilde{B}$
	1.235(±0.001)	15.39(±0.040)	16.95(±0.032)	1.235(±0.001)	33.41(±0.113)
AMM	13.45(±0.022)	0.666(±0.017)	0.547(±0.017)	1.922(±0.009)	3.351(±0.085)
	3.993(±0.015)	1.577(±0.235)	1.582(±0.270)	10.44(±0.133)	5.660(±0.401)
	1.261(±0.000)	12.50(±0.050)	13.99(±0.047)	1.261(±0.000)	11.04(±0.079)
KMH	1.957(±0.006)	1.330(±0.036)	1.061(±0.031)	0.762(±0.005)	13.52(±0.560)
	3.258(±0.020)	2.046(±0.231)	2.047(±0.257)	6.248(±0.076)	10.23(±1.099)
	1.260(±0.001)	9.918(±0.030)	11.42(±0.029)	1.260(±0.001)	21.69(±0.097)
TR	4.602(±0.016)	1.091(±0.029)	0.823(±0.023)	2.189(±0.010)	5.812(±0.160)
	7.243(±0.053)	1.991(±0.221)	1.993(±0.248)	11.48(±0.154)	6.524(±0.552)
	1.261(±0.001)	24.72(±0.120)	26.22(±0.113)	1.261(±0.001)	18.73(±0.129)
MR	4.061(±0.014)	1.131(±0.043)	1.005(±0.040)	1.441(±0.010)	14.71(±0.638)
	4.717(±0.031)	2.002(±0.318)	2.005(±0.332)	8.689(±0.138)	10.52(±1.226)
	1.261(±0.001)	37.67(±0.220)	39.17(±0.226)	1.261(±0.001)	17.45(±0.194)
OAS	3.361(±0.015)	1.173(±0.060)	1.085(±0.056)	1.328(±0.010)	26.71(±1.508)
	4.694(±0.044)	2.061(±0.435)	2.061(±0.448)	8.356(±0.174)	13.69(±1.962)
	1.260(±0.001)	36.85(±0.220)	38.35(±0.210)	1.260(±0.001)	26.65(±0.226)
RG	4.909(±0.024)	1.092(±0.059)	1.008(±0.055)	2.236(±0.019)	18.72(±0.808)
	6.831(±0.066)	2.043(±0.454)	2.045(±0.468)	10.99(±0.201)	10.47(±1.116)
	1.260(±0.001)	42.98(±0.310)	44.48(±0.307)	1.260(±0.001)	12.30(±0.159)
HL	2.222(±0.011)	1.338(±0.072)	1.249(±0.067)	0.855(±0.010)	43.59(±3.000)
	3.498(±0.038)	2.076(±0.435)	2.075(±0.445)	6.630(±0.166)	17.83(±2.883)
	1.287(±0.001)	13.77(±0.050)	15.27(±0.049)	1.287(±0.001)	11.96(±0.079)
SDG1	2.828(±0.007)	1.247(±0.036)	1.015(±0.029)	0.975(±0.007)	11.30(±0.446)
	3.360(±0.020)	2.040(±0.238)	2.038(±0.254)	6.557(±0.090)	9.666(±1.108)
	1.262(±0.001)	34.97(±0.180)	36.47(±0.178)	1.262(±0.001)	56.14(±0.275)
SDG3	16.23(±0.059)	1.030(±0.049)	0.946(±0.045)	4.723(±0.032)	5.790(±0.181)
	8.261(±0.061)	2.027(±0.400)	2.026(±0.410)	15.71(±0.259)	7.199(±0.639)
	1.262(±0.001)	24.02(±0.100)	25.52(±0.113)	1.262(±0.001)	43.59(±0.194)
SDG4	11.63(±0.038)	1.035(±0.039)	0.916(±0.035)	3.734(±0.022)	5.287(±0.158)
	7.685(±0.053)	2.014(±0.331)	2.018(±0.354)	14.03(±0.201)	6.779(±0.550)
	1.260(±0.001)	61.12(±0.410)	62.62(±0.421)	1.260(±0.001)	98.00(±0.501)
MH3	30.86(±0.164)	1.016(±0.063)	0.968(±0.060)	10.91(±0.105)	6.455(±0.202)
	14.48(±0.183)	2.005(±0.519)	2.004(±0.528)	25.51(±0.604)	7.264(±0.596)
	1.261(±0.001)	22.56(±0.090)	24.07(±0.097)	1.261(±0.001)	42.29(±0.194)
MH4	11.61(±0.041)	1.036(±0.041)	0.909(±0.035)	4.228(±0.026)	5.551(±0.163)
	8.907(±0.072)	1.977(±0.306)	1.985(±0.330)	15.51(±0.237)	6.804(±0.565)
	1.235(±0.001)	22.10(±0.090)	23.60(±0.081)	1.235(±0.001)	39.77(±0.178)
TH3	8.812(±0.019)	1.021(±0.038)	0.896(±0.035)	1.988(±0.010)	5.314(±0.161)
	4.719(±0.017)	1.961(±0.309)	1.961(±0.325)	8.661(±0.116)	7.253(±0.635)
	1.238(±0.001)	27.23(±0.120)	28.72(±0.113)	1.238(±0.001)	56.53(±0.226)
TH4	11.40(±0.032)	1.018(±0.040)	0.915(±0.039)	2.921(±0.017)	4.593(±0.131)
	6.120(±0.028)	1.982(±0.343)	1.979(±0.352)	10.39(±0.129)	6.641(±0.532)
	1.259(±0.001)	84.81(±0.660)	86.30(±0.647)	1.259(±0.001)	149.1(±0.663)
ER	41.20(±0.240)	1.015(±0.072)	0.980(±0.070)	13.43(±0.142)	5.463(±0.154)
	15.14(±0.181)	2.030(±0.612)	2.026(±0.613)	26.71(±0.654)	6.738(±0.511)
	1.261(±0.006)	93.76(±6.640)	95.26(±6.647)	1.261(±0.006)	22.66(±4.496)
Dir.	11.63(±0.605)	1.612(±0.641)	1.561(±0.623)	8.027(±0.628)	100.4(±85.27)
	31.97(±5.850)	1.476(±1.681)	1.476(±1.702)	46.74(±11.57)	34.15(±32.28)

Table 1: *Results for a single source.*

variance in the interarrival time, the longer the queue length and time spent in the system. In carrying out our studies we were amazed by the differences in performance from a given model which arose simply through changing the fitting method! These differences have previously been observed [27, 28] with respect to the SDG model and we have confirmed

Model	$\tilde{A}$	$\tilde{Q}$	$\tilde{T}$	$\tilde{D}$	$\tilde{B}$
	1.236(±0.001)	26.40(±0.050)	27.89(±0.049)	1.236(±0.001)	53.42(±0.162)
AMM	20.61(±0.017)	0.634(±0.013)	0.568(±0.012)	3.162(±0.013)	2.721(±0.051)
	5.613(±0.059)	1.241(±0.150)	1.252(±0.180)	13.09(±0.947)	3.925(±0.221)
	1.260(±0.000)	2.613(±0.016)	4.108(±0.016)	1.260(±0.000)	7.081(±0.061)
KMH	1.022(±0.005)	2.417(±0.129)	0.976(±0.054)	0.416(±0.004)	6.225(±0.444)
	2.064(±0.029)	4.763(±0.940)	4.776(±1.080)	4.463(±0.105)	16.50(±3.566)
	1.260(±0.000)	2.507(±0.012)	4.003(±0.012)	1.260(±0.000)	7.061(±0.057)
TR	1.022(±0.005)	2.098(±0.081)	0.821(±0.033)	0.416(±0.004)	5.639(±0.360)
	2.064(±0.026)	3.795(±0.534)	3.804(±0.647)	4.464(±0.095)	13.56(±2.805)
	1.262(±0.002)	4.166(±0.050)	5.662(±0.051)	1.262(±0.002)	7.200(±0.092)
MR	1.043(±0.005)	4.360(±0.368)	2.359(±0.193)	0.437(±0.005)	12.53(±1.270)
	2.137(±0.048)	6.050(±1.383)	6.050(±1.467)	4.668(±0.196)	25.79(±6.506)
	1.261(±0.002)	3.628(±0.071)	5.124(±0.071)	1.261(±0.002)	7.295(±0.103)
OAS	1.029(±0.007)	5.636(±0.977)	2.824(±0.476)	0.424(±0.006)	13.82(±3.065)
	2.093(±0.058)	9.298(±4.489)	9.306(±4.657)	4.570(±0.241)	58.89(±52.79)
	1.259(±0.001)	3.177(±0.054)	4.672(±0.053)	1.259(±0.001)	7.221(±0.125)
RG	1.027(±0.009)	4.613(±0.682)	2.131(±0.310)	0.420(±0.008)	10.93(±1.836)
	2.076(±0.059)	8.969(±3.587)	8.983(±3.783)	4.509(±0.231)	40.45(±17.81)
	1.260(±0.001)	3.341(±0.073)	4.836(±0.073)	1.260(±0.001)	7.150(±0.099)
HL	1.026(±0.009)	6.135(±1.392)	2.926(±0.650)	0.420(±0.007)	12.00(±2.150)
	2.073(±0.054)	13.10(±8.250)	13.12(±8.463)	4.498(±0.220)	53.36(±27.53)
	1.287(±0.000)	3.611(±0.026)	5.105(±0.026)	1.287(±0.000)	6.795(±0.052)
SDG1	1.059(±0.005)	3.343(±0.162)	1.672(±0.082)	0.476(±0.004)	9.576(±0.597)
	2.184(±0.026)	4.648(±0.653)	4.649(±0.728)	4.553(±0.094)	17.10(±3.064)
	1.262(±0.001)	8.970(±0.122)	10.46(±0.121)	1.262(±0.001)	7.745(±0.094)
SDG3	1.104(±0.006)	4.558(±0.349)	3.350(±0.251)	0.490(±0.006)	27.03(±2.322)
	2.362(±0.050)	4.582(±0.860)	4.581(±0.895)	5.166(±0.180)	25.70(±5.174)
	1.262(±0.001)	5.607(±0.062)	7.103(±0.063)	1.262(±0.001)	7.495(±0.079)
SDG4	1.070(±0.006)	4.169(±0.295)	2.599(±0.179)	0.460(±0.005)	17.99(±1.535)
	2.235(±0.042)	4.911(±0.950)	4.908(±1.001)	4.891(±0.166)	24.95(±5.571)
	1.260(±0.002)	10.88(±0.220)	12.37(±0.218)	1.260(±0.002)	7.772(±0.129)
MH3	1.108(±0.008)	5.490(±0.622)	4.246(±0.469)	0.489(±0.007)	36.74(±4.682)
	2.368(±0.069)	5.156(±1.383)	5.157(±1.421)	5.208(±0.266)	37.49(±12.25)
	1.261(±0.001)	4.948(±0.053)	6.444(±0.053)	1.261(±0.001)	7.496(±0.080)
MH4	1.072(±0.006)	3.836(±0.278)	2.262(±0.160)	0.458(±0.005)	14.95(±1.369)
	2.234(±0.042)	5.066(±1.046)	5.068(±1.113)	4.879(±0.156)	25.00(±6.981)
	1.235(±0.001)	6.451(±0.071)	7.947(±0.071)	1.235(±0.001)	7.872(±0.078)
TH3	1.021(±0.005)	4.448(±0.275)	2.930(±0.175)	0.398(±0.004)	20.12(±1.429)
	2.071(±0.038)	4.436(±0.641)	4.440(±0.685)	4.749(±0.169)	21.90(±3.702)
	1.238(±0.001)	5.514(±0.081)	7.010(±0.081)	1.238(±0.001)	7.631(±0.087)
TH4	1.010(±0.005)	5.680(±0.457)	3.516(±0.274)	0.390(±0.005)	20.13(±1.920)
	2.035(±0.043)	5.537(±0.991)	5.534(±1.043)	4.627(±0.182)	29.95(±8.352)
	1.259(±0.001)	11.74(±0.380)	13.23(±0.372)	1.259(±0.001)	7.551(±0.148)
ER	1.074(±0.010)	8.316(±1.386)	6.551(±1.076)	0.463(±0.009)	45.87(±7.448)
	2.238(±0.071)	6.502(±2.342)	6.499(±2.373)	4.912(±0.276)	51.99(±19.58)
	1.192(±0.013)	8.813(±1.910)	10.31(±1.908)	1.192(±0.013)	9.583(±1.375)
Dir.	1.026(±0.088)	6.634(±4.673)	4.846(±3.414)	0.335(±0.073)	28.42(±27.57)
	2.066(±0.640)	4.634(±4.318)	4.636(±4.455)	5.088(±2.992)	47.39(±37.16)

Table 2: *Results for multiple sources.*

their ordering of performance. This is clearly shown by the results in the table for each of the SPP, SDG and MH2 models. It is also worth observing that, in each model, the coefficient of variation has *decreased* significantly on the output (departure process) as compared to the input (arrival process) but, at the same time, the corresponding skewness coefficients have shown an *increase*.

Turning our attention to table 2, we note smoother arrival processes (the smoothness is, in fact, preserved in the departure process) and that many of our observations for the single source case can also be applied to the multiple source cases. Note however, that the multiple source cases generally have shorter queue lengths, less time spent in the system and shorter busy periods when compared with the single source models but that the variation and skewness are actually *increased* for all metrics. Finally, it is pointed out that the direct multiple source run is based on a limited set of data (due to restrictions in disk space) which has caused e.g. the differences in mean interarrival times between all models and the direct run.

3.2 Statistical Comparison

In order to rank the models, we have computed the absolute differences between the "Dir." results and the corresponding metric moments for each of the models and fitting methods. We have then ranked these absolute differences in order from the "closest match" to the furthest match on a scale of 1 to 15. Where two or more sets of results are very close, we rate the models equally and apportion equal rank values.

A complete set of rankings for the models was determined with respect to each of the three moments and five metrics. The next step in the analysis involved assigning a "weight" to these rankings within each of the metrics. We decided that $\tilde{Q}$ (queue length seen by an arriving packet) and the $\tilde{B}$ (server busy period) metrics were the most important and that the remaining metrics were the least important. By assigning suitable weights to the metrics, we were able to determine an overall ranking for the models with respect to each of the three moments.

Finally, we assigned weights to the three moments of the metrics. Once again we assigned the highest weight of 0.6 to the first moment, a weight of 0.3 to the second moment and 0.1 to the third moment on the basis that in any performance modelling of bursty traffics we would usually be more interested in the first and second moments.

Table 3 gives the results for the single source case. It will be seen that the first three models in the final rankings are based upon the Switched Poisson Process and that the final rankings with respect to the first moment are reasonably close to the final overall rankings with very few exceptions. (This is, of course, partly due to the fact that the first moment was afforded the highest weighting.) An interesting observation is that HL, OAS, RG and MR rank significantly better than KMH and TR. Indeed, all relate to the SPP model, but the four former fit general traffic metrics, e.g. indexes of dispersion, while the two latter employ a more direct fit to a sequence of interarrival times. Thus, although KMH and TR appear to provide better fit for observations of a *true* SPP, our traffic is not such a process and it is seems that traffic oriented methods are superior. This observation also holds when comparing the results for SDG and various approaches inspired by Solé et al. The most direct method is number 1, which ranks the worst, while the most traffic oriented one is number 3 (SDG3, TH3, TH3) and ranks the best. In other words we again observe that reproducing characteristics seem to be more successful than trying to recreate a presumed model.

Table 4 summarises the results for the multiple source case. The first major observation which can be made concerns the differences between tables 3 and 4. There has been a considerable shift in position between the models with respect to their accuracy in modelling multiple source systems. Perhaps the only consistent result has been the apparent poor showing of the Autoregressive Markov model (AMM) as it has appeared at the bottom of the table in both instances. This does not necessarily imply that this is a poor model of bursty traffic. In particular, it should be noted that this model attempts to match the production of packets within a frame structure, however, the simulation

Model	First Moment	Second Moment	Third Moment	Weighted Rank
HL — Heffes and Lucantoni	2.0	1.0	10.0	2.50
OAS — Okuda et. al.	2.0	2.0	10.0	2.80
RG — Gusella	2.0	6.0	10.0	4.00
MH3 — Markov-hyperexponential-3	4.5	7.5	1.0	5.05
MR — Rossiter	6.0	4.0	3.0	5.10
ER — Erlang-r	4.5	12.0	10.0	7.30
SDG3 — Solé-3	7.0	7.5	10.0	7.45
MH4 — Markov-hyperexponential-4	9.0	9.5	3.0	8.55
SDG4 — Solé-4	9.0	9.5	13.0	9.55
KMH — Meier-Hellstern	13.0	3.0	14.0	10.10
TR — Rydén	9.0	14.0	6.0	10.20
TH3 — Threshold-3	11.0	12.0	6.0	10.80
TH4 — Threshold-4	12.0	12.0	6.0	11.40
SDG1 — Solé-1	14.5	5.0	15.0	11.70
AMM — Autoregressive Markov	14.5	15.0	3.0	13.50

Table 3: *Final rankings for the single source systems.*

Model	First Moment	Second Moment	Third Moment	Weighted Rank
SDG3 — Solé-3	1.0	6.0	1.0	2.50
MH3 — Markov-hyperexponential-3	2.5	2.5	5.5	2.80
TH3 — Threshold-3	2.5	4.5	3.5	3.20
TH4 — Threshold-4	4.0	1.0	8.5	3.55
ER — Erlang-r	5.5	7.0	8.5	6.25
SDG4 — Solé-4	5.5	10.0	5.5	6.85
MH4 — Markov-hyperexponential-4	7.0	10.0	7.0	7.90
HL — Heffes and Lucantoni	10.0	2.5	13.5	8.10
OAS — Okuda et. al.	10.0	4.5	13.5	8.70
MR — Rossiter	8.0	10.0	10.5	8.85
RG — Gusella	10.0	8.0	12.0	9.60
SDG1 — Solé-1	12.0	13.0	2.0	11.30
KMH — Meier-Hellstern	13.5	13.0	3.5	12.35
TR — Rydén	13.5	13.0	10.5	13.05
AMM — Autoregressive Markov	15.0	15.0	15.0	15.00

Table 4: *Final rankings for the multiple source systems.*

studies have actually removed any frame considerations since the other models do not attempt to model frames, thus the comparison may be unfair in this case. Looking at the models in the top half of the table we see that the threshold model has moved up quite a number of places and has a similar rank to the Switched Poisson and Switched Bernoulli models in this case, possibly indicating that when multiple sources are present the threshold model may be more useful. Finally, the tendency of traffic oriented fitting methods being superior to model oriented ones still appears to hold.

4 Conclusions

¿From the above analysis, we are not able to conclude that we have discovered the "perfect model" for describing bursty traffic streams in either a single source or multiple source environment. Using our ranking scheme we have shown that, for the single source case, the

Switched Poisson Process models performed the best and that for the multiple source case, models based on the Switched Poisson Process, or its discrete version (Switched Bernoulli) together with the threshold model may give the closest match to the performance of the "real data". The results also seem to suggest that for traffics for which there is no known, ideal model, a traffic-oriented fitting method performs better than a model-oriented one.

This study has concentrated on a particular set of "real data" and it has not tried to consider the many different types of bursty traffic since, at the time of commencing this study, there was little real data available. This situation has begun to change and so future research will concentrate on considering more examples of live traffic streams so that a more rigorous comparison of the models can be performed.

Further research will also concentrate on multiple source scenarios as performance analysis of ATM networks may employ highly specific models on an upper multiplexing level [10] to study equivalent bandwidths [18] while general models still will be required on a lower level of multiplexing as cells on this level share the same physical channel.

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