NON–CAUSAL DELAYLESS SUBBAND ADAPTIVE EQUALIZER

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ABSTRACT

In wireless communication, multiple versions of the transmitted signals arrive at the receiver with different attenuations and time delays. Thus, an equalizer with long filter is required at the receiver to reverse the multipath effects. In this paper, a new non-causal delayless subband equalizer structure is proposed to reduce the computational complexity of the equalizer and to improve the channel tracking capability. This structure avoids the additional delays usually associated with the subband schemes. A design method is formulated to minimize the aliasing effect of the filter bank in the subbands. The significance of employing a non-causal equalizer is also emphasized. Simulation results show that the performance of the new structure is significantly improved by using the design method. Moreover, the complexity of the new subband equalizer is a fraction of the equivalent fullband at the expense of small degradation in the performance.

1. INTRODUCTION

Subband filtering has been studied as a means to reduce the computational complexity of high order filters. It has been mostly applied in echo cancelers for audio teleconferencing which requires FIR filters with thousands of coefficients. Recently, subband techniques have been considered for equalization [1, 2] in wireless communication systems where multiple versions of the transmitted signal are detected at the receiver with different attenuations and time delays. Since most communication systems operate in nonstationary surroundings, adaptive equalizers are employed to track the channel variations. These equalizers require long filters to ensure good equalization performance even for short delay channels.

Adaptive equalization in subbands is performed by decomposing the training sequence and the received signal to different frequency bands using a filter bank. In [1, 2] the least mean square (LMS) algorithm or the recursive least square (RLS) algorithm is used for adaptation in each subband. However, only conventional subband adaptive equalization including analysis and synthesis filter banks has been investigated.

In this paper, a new non-causal delayless subband structure is proposed for adaptive equalization. This structure combines the delayless structure [3] with a non-causal equalizer to improve the equalization performance for non minimum phase channels. The structure uses the advantage of multi-rate signal processing for the implementation of the algorithm but avoids the block processing delays by transforming the subband equalizer filters to a fullband equalizer. The performance of the subband adaptive equalizer depends on the filter bank and the subband to fullband transformation. The design of the filter bank with low aliasing between the subbands is presented to improve the subband performance.

Simulation results show that the new adaptive subband equalizer has approximately the same performance as the fullband adaptive equalizer but with a fraction of the computational complexity. This performance is significantly improved by using the filter bank design method. The mean square error (MSE), bit error rate (BER) and the computational complexity for different filter banks, different number of subbands and equalizer length are presented and compared.

2. CHANNEL MODEL AND NON-CAUSAL DELAYLESS SUBBAND EQUALIZER

Consider a communication channel given in Fig. 1. The transmitted bit $b(n)$, where n denotes the discrete time index, is independently generated with equal probabilities and modulated using quadrature phase shift keying (QPSK). The modulated signal $s(n)$ is transmitted through a channel which can be modeled as a time varying FIR filter with the impulse response $\mathbf{c} = [c(0), \cdots, c(N-1)]^T$, where N and $[\cdot]^T$ denote the length of the channel and the vector transpose, respectively.

Additive white gaussian noise $w(n)$ is added in the chan-

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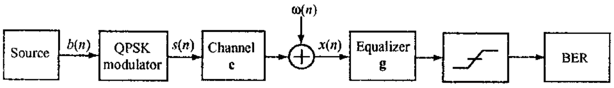

Fig. 1. Communication channel.

nel. The received signal $x(n)$ is given as

$$x(n) = \sum_{l=0}^{N-1} c(l)s(n-l) + w(n). \qquad (1)$$

The signal $x(n)$ is then presented to a non-causal delayless subband equalizer shown in Fig. 2. This equalizer is the estimated inverse of the channel which can have non-minimum or minimum phase response. If the channel has non-minimum phase then its inverting causal filter is not bounded. By introducing a non-causal equalizer, it is possible to approximate the inverse of the channel with a finite number of taps. This non-causal equalizer is implemented by introducing a delay τ_D at the receiver.

The training sequence and the received signal are divided into M frequency bands by using a uniformly modulated filter bank (UMF). This filter bank is formed by several modulated versions of a prototype filter [4, 5]. Thus, the design of the filter bank is reduced to the design of a prototype filter with the impulse response $\mathbf{h} = [h(0), \cdots, h(K-1)]^T$ and the transfer function

$$H(z) = \sum_{l=0}^{K-1} h(l)z^{-l} = \mathbf{h}^T \phi(z) \qquad (2)$$

where $\phi(z) = [1, \cdots, z^{-(K-1)}]^T$ and K is the filter length.

Polyphase decomposition is used to implement the filter bank efficiently where the decimation is done before the subband filtering [5]. To avoid aliasing, the filter bank is oversampled with a factor of two, i.e. the subband signals are decimated by a factor $D = \frac{M}{2}$.

The equalizer coefficients in each frequency band can be adapted by using the least mean square (LMS) or the recursive least square (RLS) algorithms [6]. Since the RLS algorithm converges faster than the LMS algorithm and the number of training symbols is limited, the RLS algorithm is chosen for the adaptation. The impulse response for the fullband equalizer is the IFFT of the fullband frequency response, which is obtained by stacking together the subband responses [3].

Denote $\mathbf{g} = [g(0), \cdots, g(L-1)]^T$ as the impulse response of the equalizer. The filter length L is chosen so that the signal $s(n)$ can be recovered. The output signals from the equalizer is given as

$$u(n) = \sum_{l=0}^{L-1} g(l)x(n-l). \qquad (3)$$

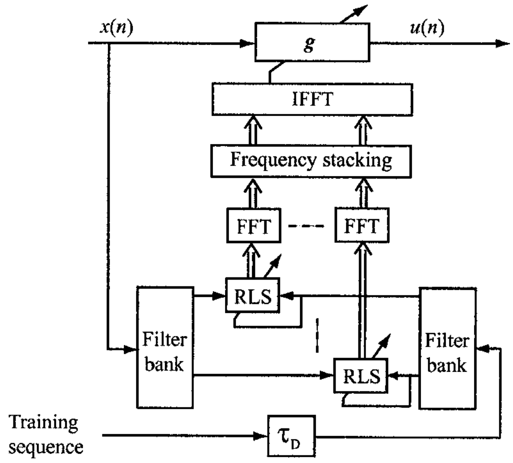

Fig. 2. Non-causal delayless subband adaptive equalizer.

3. FILTER BANK DESIGN

The transformation between subband and fullband depends on the performance of the filter bank. In this section, the design of the filter bank is presented to improve the subband equalizer performance.

Consider the design of a prototype filter with transfer function $H(z)$ and frequency response $H(e^{j\omega})$. The desired filter is lowpass with a normalized cut-off frequency $\omega_p = \pi/M$. Since it is desirable to have approximately linear phase in the passband, the desired complex frequency response is specified as

$$H_d(e^{j\omega}) = e^{j\omega\tau}, \quad \forall \omega \in [-\omega_p, \omega_p] \qquad (4)$$

where $\tau = (L-1)/2$.

The two most common criteria in filter design are the weighted least square and the weighted min-max [5]. Since it is important to have a flat passband response for the prototype filter, the weighted least square error is chosen as the cost function

$$e(\mathbf{h}) = \frac{1}{2\omega_p} \int_{-\omega_p}^{\omega_p} |H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega. \qquad (5)$$

It follows from (2) that $H(e^{j\omega})$ is a linear function of the coefficient $\mathbf{h}$. Thus, (5) is reduced to a quadratic function

$$e(\mathbf{h}) = \mathbf{h}^T\mathbf{A}\mathbf{h} - 2\mathbf{h}^T\mathbf{b} + 1$$

where $\mathbf{A}$ is a $K \times K$ matrix

$$\mathbf{A} = \frac{1}{2\omega_p} \int_{-\omega_p}^{\omega_p} \phi(e^{j\omega})\phi^H(e^{j\omega})d\omega \qquad (6)$$

and $\mathbf{b}$ is a $K \times 1$ vector

$$\mathbf{b} = \frac{1}{2\omega_p} \int_{-\omega_p}^{\omega_p} \mathcal{R}\{e^{j\omega\tau}\phi^H(e^{j\omega})\}d\omega. \qquad (7)$$

The operator $[\cdot]^H$ and $\mathcal{R}\{\cdot\}$ denote the Hermitian operation of a vector and the real part of a complex function, respectively.

The prototype filter is also designed to have a minimum aliasing effect. Since UMF is employed, it is sufficient to minimize the energy in the aliasing terms of the first subband. This aliasing error can be formulated as a quadratic function [4, 5]

$$\beta(\mathbf{h}) = \mathbf{h}^T\mathbf{Q}\mathbf{h} \tag{8}$$

where $\mathbf{Q}$ is a $K \times K$ matrix

$$\mathbf{Q} = \frac{1}{2\pi D}\sum_{d=1}^{D-1}\int_{-\pi}^{\pi}\phi(e^{j(\omega-2\pi d)/D})\phi^H(e^{j(\omega-2\pi d)/D})d\omega$$

where D is the decimation factor.

The problem becomes to minimize both the least square error $e(\mathbf{h})$ and the aliasing effect $\beta(\mathbf{h})$. A joint cost function is proposed to allow the tradeoff between the least square error in the passband and the aliasing effect

$$E(\mathbf{h}) = e(\mathbf{h}) + \gamma\beta(\mathbf{h}) = \mathbf{h}^T(\mathbf{A}+\gamma\mathbf{Q})\mathbf{h} - 2\mathbf{h}^T\mathbf{b} + 1 \tag{9}$$

where γ is a positive constant. The optimization problem becomes

$$\min_{\mathbf{h}} \mathbf{h^T}(\mathbf{A}+\gamma\mathbf{Q})\mathbf{h} - \mathbf{2h^T b} + \mathbf{1} \tag{10}$$

which can be solved using standard quadratic programming techniques. The weighting factor γ is used to emphasize either the passband or the aliasing requirements in a joint optimization problem. Depending on the channel, γ can be chosen accordingly to improve the performance.

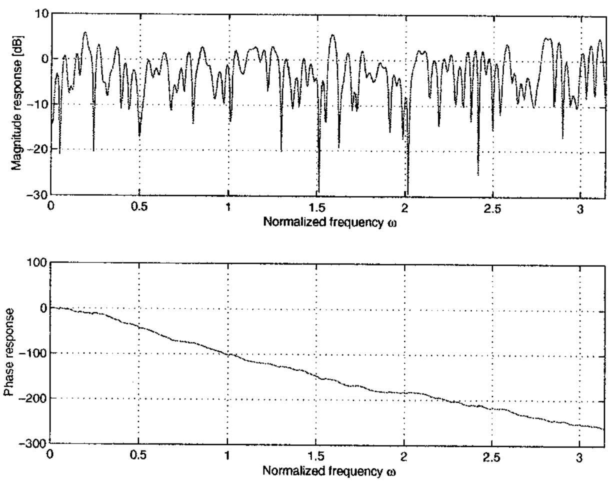

Fig. 3. Channel characteristics.

4. SIMULATION RESULTS

Consider a sparse frequency selective channel with 100 coefficients and 40 significant taps randomly generated and placed in the impulse response. The magnitude and phase responses of the channel are given in Fig. 3. This channel is used to evaluate and compare the performance of the non-causal delayless subband equalizer and the non-causal fullband adaptive equalizer. Both equalizers have the same delay τ_d, which equals to half of the filter length.

Performance comparison is based on a mean square error in dB defined as

$$\text{MSE} = 10\log_{10}\left(\frac{\sum_n |s(n)-u(n)|^2}{\sum_n |s(n)|^2}\right) \tag{11}$$

where the summation is taken over the total number of transmitted symbols. Bit error rate (BER) for different signal to noise ratio (SNR) and different number of subbands are compared. In the simulations, BER is calculated over 10^6 transmitted symbols. The length of the prototype filter is chosen as four times the total number of subbands and the weighting factor in (10) is one.

Comparison is also based on a relative number of floating point operations (RFLOP) which is normalized to fullband solution. The design of the prototype filter is excluded in the calculation of RFLOP.

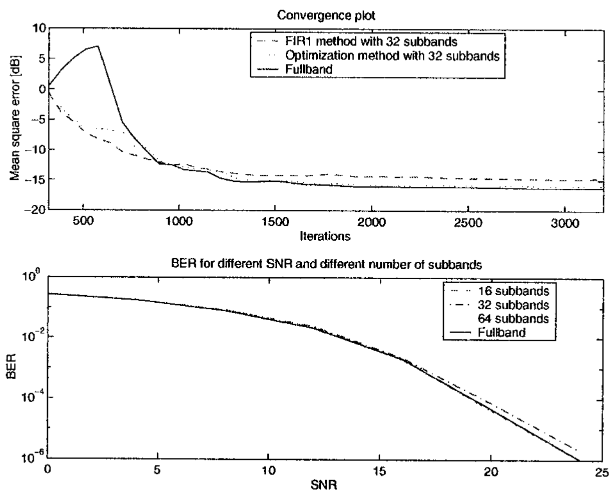

Fig. 4. Convergence and BER plot for 512 taps equalizer.

Fig. 4 plots the convergence for different methods using the MSE measure and a 512 tap equalizer. The training sequence increases from 0 to 3200 symbols. The subband approaches have 32 subbands. The convergence of the subband equalizer with the polyphase filters obtained using the $fir1$ routine in Matlab [7] and the optimization method in

Equalizer length	Mean square error [dB]	
	Prototype filter 1	Prototype filter 2
512	-12.56	-15.73
1024	-17.44	-20.55
2048	-22.64	-24.54

Table 1. Mean square error (MSE) for noise free channel and 64 subbands.

Section 3 are given in the dashed and the dotted lines, respectively. The MSE for the non-causal fullband equalizer is plotted in the solid line. The adaptation is much faster and smoother for both subband methods when compare to the fullband method. The subband adaptive equalizer with optimized prototype filter has approximately the same performance as the fullband adaptive equalizer.

The figure shows that the MSE for the optimized prototype filter is about 1 dB better than the one using $fir1$. The improvement is more significant when the total number of subbands is increased. Comparison between these two prototype filters for 64 subbands with different equalizer length is given in Table 1 for a noise free channel. It can be seen that the optimized prototype filter has 2-3 dB improvement in MSE when compares to the $fir1$.

Fig. 4 also shows the BER performance for equalizers with 512 taps with different number of subbands and different signal to noise ratio (SNR) [dB]. The corresponding MSE for different cases are given in Table 2. The BER and MSE for the subband method degrade slightly with an increased number of subbands. Table 2 also shows the relative floating point operation (RFLOP) which normalized to the fullband floating point operation for different number of subbands. The RFLOP is reduced by 99.66% when using 16 subbands. The computational complexity is reduced much further when the number of subbands increases to 32 and 64 at an expense of a small degradation in the BER and MSE performance.

5. CONCLUSIONS

In this paper, a new non-causal delayless subband equalizer is proposed to reduce the computational complexity and to increase the channel tracking capacity associated with long adaptive filters. A design method for the filter bank is presented to improve the subband performance. Simulations have been carried out for a sparse complex channel with 100 taps. The results show that the subband performance is significantly improved by optimizing the prototype filter. Moreover, the computational complexity is significantly reduced when the number of subband increases at an expense of a small degradation in the mean square error and bit error rate.

SNR [dB]	Mean square error [dB]			
	Fullband	Subband with Prototype filter 2		
		M=16	M=32	M=64
0	-1.158	-1.060	-1.052	-0.997
4	-2.550	-2.528	-2.554	-2.466
8	-4.669	-4.457	-4.509	-4.445
12	-7.091	-6.999	-6.865	-6.880
16	-9.629	-9.644	-9.562	-9.381
20	-12.190	-12.034	-11.818	-11.881
24	-14.055	-13.941	-13.759	-13.546
All	Relative floating point operation			
SNR	1	0.0034	$4.5 \cdot 10^{-4}$	$6.3 \cdot 10^{-5}$

Table 2. Mean square error (MSE) and relative floating point operation (RFLOP) for different SNR and different number of subbands.

6. REFERENCES

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