FINITE FUZZY SETS IN THE MEDICATION DECISIONS CONCERNING THE RETREAT OF SYMPTOMS IN RADIATION CYSTITIS

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ABSTRACT

Radiation cystitis is a rare disease, appearing as the result of radiation of pelvic tumors. We support mathematically the recognition of the most efficacious treatments, which reduce the impact of symptoms typical of the illness. To permute the therapies in the ordering, commencing with the optimal therapy, we apply the fuzzy decision making model furnished with finite fuzzy sets. These act as measures of the treatment effectiveness-utility. In the solution, we adopt the older operations on fuzzy sets of type 1, which make the model simple to be easily converted into a computer program.

KEYWORDS

Finite fuzzy sets, fuzzy decision making, linguistic utilities, radiation cystitis, selection of therapy.

1. INTRODUCTION

Radiation cystitis occurs very rarely as a complication of the radiation, proved to reduce the growth of some cancer tumors in the pelvic organs. This fact of rareness makes the disease very difficult to study in a large group of clinical trials. Most available data about radiation cystitis treatment come from a small number of descriptive studies or from expert opinions (Denton et al., 2009; Martinez-Rodriguez, 2010). As clinical data are considered to have low quality then physicians, who are still facing patients with a disease hugely influencing the quality of life, mostly base on their own experience.

To provide some aid in selecting the best treatments, we test fuzzy decision-making model, which was already applied twice by us to this purpose for the same disease (Rakus-Andersson and Frey, 2012; 2014). This time, in order to extend the diversity of decisions, we have formulated two objectives of investigations. The first aim is to select the therapies, which highly influence the remission of the symptoms threatening the patient health state in the substantial grade (the most dangerous symptoms). Apart from this decision, we also wish to establish a sequence of therapies, showing their influence on all symptoms. In this way, we will be able to compare the conclusions made in this paper to previous results obtained for radiation cystitis by means of decision models based on fuzzy numbers in alpha-cut forms (Rakus-Andersson and Frey, 2012) and Choquet integrals (Rakus-Andersson and Frey, 2014). The mentioned models differ from the decision method, involving finite fuzzy sets prepared in the current paper.

Theoretical fuzzy decision-making models (Bellman and Zadeh, 1970; Jain, 1976; Yager, 2004a), possess the utility matrix filled with distinct utilities of pairs (decision, object-state). These models have given rise to own developments of applications, concerning the item of ranking treatments (Rakus-Andersson, 2006; 2008). We interpret pairs (decision, object-state) as (therapy, symptom) to prove decision-making, equipped with different fuzzy techniques as utilities. In the paper presented, the utilities are stated as finite fuzzy sets.

In computations, we insert Zadeh's operations on fuzzy sets (Zadeh, 1965; 1999) and Yager's OWA operators (Yager, 2004b).

There exist trials of using fuzziness to certain issues in medication like, e.g., in predicting some measures of parameters in radiation of cancer tumors (Papageorgiou, 2003), but the ranking of treatments can be mostly found in our own research works, already cited.

In Section 2, we present the basic data entering the model of fuzzy decision making. Section 3 provides weights of importance, which point out the symptoms' priorities to retreat. The discussion about the design of utilities is accomplished in Section 4. The effects of OWA operators are summarized in Section 5. Some conclusions are sampled in Section 6.

2. THE GENERAL OUTLINE OF FUZZY DECISION MAKING

The mathematical apparatus of a selection of the most efficacious treatment, concerning patients who suffer from radiation cystitis, is based on items originated from fuzzy set theory (Bellman and Zadeh, 1970; Dubois and Prade, 1980; Yager, 2004a). Let us thus acquaint with the conception of a fuzzy set.

The finite fuzzy set $F \subseteq \Phi = \{f_1,..., f_z\}$ is a collection $F = \{(f_1, \mu_F(f_1)),...,(f_z, \mu_F(f_z))\}$, where $\mu_F(f_c) \in [0,1]$, $c = 1,\ldots,z$. Each element f_c gets a membership degree $\mu_F(f_c)$, which expresses the strength of the relationship between f_c and F. Membership degrees equal to 1 inform about the total relation between the element and the set. The function μ_F is called "the membership function" of F. The finite fuzzy sets are sometimes symbolically denoted by $F = {}^{\mu_F(f_1)}\!/_{f_1} + ... + {}^{\mu_F(f_z)}\!/_{f_z}$ (Zadeh, 1965).

In the decision making model, we introduce a space of states $X = \{x_1,...,x_m\}$ and a decision space $A = \{a_1,..., a_n\}$ (Jain, 1976; Yager, 2004a). We consider a decision model, in which alternatives $a_1,..., a_n \in A$ act as therapies used to treat patients with radiation cystitis. The therapies should influence states $x_1,..., x_m \in X$, identified with m symptoms typical of the disease considered. The objective of the fuzzy decision making model, discussed in this paper, is an extraction of the therapy receding either the most destructive symptoms or reducing the impact of all symptoms.

When a decision maker applies therapy $a_i \in A$, $i = 1,\ldots,n$, to symptom $x_j \in X$, $j = 1,...,m$, then a utility u_{ij} of treating x_j by a_i is determined. In order to sample distinct utilities u_{ij}, assigned to pairs (a_i, x_j) for therapy a_i, $i = 1,\ldots,n$, we estimate the total utility U_{a_i} as

$$U_{a_i} = \text{aggregation of } u_{ij},\ j = 1,..., m\,. \tag{1}$$

The utilities u_{ij} are expected to be finite fuzzy sets of type 1 defined in space $Y = [1,\ldots,k,\ldots, p]$. The nature of Y will be explained in Section 4.

Let us associate with each symptom x_j, $j = 1,\ldots,m$, a weight w_j that indicates the importance of x_j's remission in accordance with the rule: the larger w_j is, the more important role of x_j's retreat will be considered.

The total utility U_{a_i} of a_i will be thus designed as

$$U_{a_i} = \bigcup_{j=1}^{m} u_{ij} \cdot w_j,\ i = 1,..., n. \tag{2}$$

Each U_{a_i}, as a union of fuzzy sets allocated in $Y = [1,\ldots,k,\ldots, p]$, will be placed in the same domain Y.

After this step of the fuzzy decision making algorithm, we propose adding a new operation. Let us add to each total utility U_{a_i} its cardinality $|U_{a_i}|$ expressed by (Zadeh, 1965)

$$|U_{a_i}| = \sum_{k=1}^{p} \mu_{U_{a_i}}(k)\,. \tag{3}$$

.

We suppose that the inequality $\left|(U_{a_i})\right| \geq \left|(U_{a_l})\right|$, i, l = 1,…,n, is interpreted as the outcome "therapy a_i has the stronger effect on symptoms than therapy a_l". We select the optimal treatment a^*, which reveals the largest value $\left|(U_{a^*})\right|$ of all $\left|(U_{a_i})\right|$.

3. IMPORTANCE WEIGHTS OF SYMPTOM REMISSION

We intend to furnish the model with extraction of the most efficacious treatment, provided that the particular emphasis is also concentrated on assigning differing degrees of importance to states-symptoms.

A procedure for obtaining a ratio scale of importance for a group of m symptoms is developed recently by the authors (Rakus-Andersson and Frey, 2014).

Generally, if we consider m symptoms x_j to find importance weights for them, then we want to place them in the sequence x_1>x_2>...>x_m due to the expert's opinion. The symbol ">" is used for the description "more important than". We wish the sum of all weights w_j, tied to xj, j = 1,...,m, to be 1. Hence,

$$m \cdot r + (m-1) \cdot r + ... + 2 \cdot r + 1 \cdot r = 1, \tag{4}$$

where r is a quotient depending on m. The weights w_j will be computed as

$$w_j = (m - j + 1) \cdot r, \tag{5}$$

for j = 1,…,m. The differences between two adjacent weighs are equal in this context, since the physician has not emphasized a particular distribution of w_j, j = 1,…,m. If we insert an extra variable in (5), then we can generate more asymmetric w_j due to the physician's recommendations.

Example 1 (symptoms in radiation cystitis and their weights of importance)
The symptoms, selected as the most decisive for radiation cystitis, are listed as x_1 = macrohaematuria > x_2 = urinary bladder pain > x_3 = urine retention > x_4 = dysuria > x_5 = urgency. The order of the symptom importance to disappear has been determined by a physician, who has engaged his own experience about the disease.

For m = 5, when solving (4) with respect to r, the equation $5 \cdot r + 4 \cdot r + 3 \cdot r + 2 \cdot r + 1 \cdot r = 1$ provides r = 0.066. We get: $w_1 = (5-1+1) \cdot 0.066$= 0.33, $w_2 = (5-2+1) \cdot 0.066 = 0.264$, $w_3 = (5-3+1) \cdot 0.066 = 0.198$, $w_4 = (5-4+1) \cdot 0.066 = 0.132$, $w_5 = (5-5+1) \cdot 0.066$.

4. MIN AND MAX OPERATORS IN TOTAL UTILITIES

The weights w_j, j = 1,…,m, as factors of the total fuzzy utility U_{a_i}, are already found. Let us now determine shapes of utilities u_{ij}, which constitute the other factor in (2).

We first want the utility u_{ij}, estimating the remission of symptom x_j after treating it by a_i, i = 1,…,n, j = 1,..,m, to be verbally expressed as the utility level in order to facilitate the communication with a professional adviser. We propose a list named L = "Verbal utility of applying a_i to x_j". L is a list

L = {N_1 = "*none*",…,N_k = "*utility level k*",…,N_p = "*best*"}.

Each expression N_k, k = 1,…,p, will be replaced by a fuzzy set, also named N_k. By linking the therapy to the symptom for pair (a_i, x_j), a physician selects expression N_k from the list due to his experience. It means that u_{ij} = N_k verbally. We suppose that N_k are fuzzy sets of utility levels in universe Y = {1,…,k,…,p}. To plan the construction of fuzzy set N_k, let us note that N_k = "*utility level k*" promotes k as the element of N_k characterized by the highest membership degree equal to 1. Other elements around k should have weaker relationship to N_k, which results in lesser membership degrees, assigned to them. Finally, we state fuzzy sets N_k as

$$N_k = {}^{\mu_{N_k}(1)}\!/_{1} + \cdots + {}^{\mu_{N_k}(k)}\!/_{k} + \cdots + {}^{\mu_{N_k}(p)}\!/_{p}$$
$$= \cdots + {}^{\frac{p-2}{p}}\!/_{k-2} + {}^{\frac{p-1}{p}}\!/_{k-1} + {}^{1}\!/_{k} + {}^{\frac{p-1}{p}}\!/_{k+1} + {}^{\frac{p-2}{p}}\!/_{k+2} + \cdots \; . \tag{6}$$

On the other hand, the intensity of utility u_{ij} is assimilated with intensity level codes forming the universe Y = "*Intensity levels of utility* u_{ij}" = $\{1,\ldots,k,\ldots,p\}$. Let us assume that level 1 does not emerge any utility, whereas level p indicates its best status. The set N, which corresponds to growing levels of the utility intensity, is proposed to be

$$N = {}^{\mu_N(1)}\!/_{1} + \cdots + {}^{\mu_N(k)}\!/_{k} + \cdots + {}^{\mu_N(p)}\!/_{p} = {}^{\frac{1}{p}}\!/_{1} + \cdots + {}^{\frac{k}{p}}\!/_{k} + \cdots + {}^{\frac{p}{p}}\!/_{p} \; . \tag{7}$$

Once again, we emphasize that space $Y = \{1,\ldots,k,\ldots,p\}$ is a common support for different fuzzy sets, standing for the symbolic utility levels, when treating symptom x_j by a_i.

We suggest creating sets

$$N_k` = N_k \cap N \tag{8}$$

to be representatives of utilities u_{ij} as computational utilities in practical applications. The intersection operator diminishes the highest membership degrees in sets N_k assigned to the lower utilities, which makes them more adaptable to express the weaker action of the therapy.

To perform the operation of a topological intersection of two fuzzy sets F_1 and F_2 in $\Phi = \{f_1,\ldots,f_z\}$, we adopt a general definition (Zadeh, 1965)

$$F_1 \cap F_2 = \left({}^{\mu_{F_1}(f_1)}\!/_{f_1} + \ldots + {}^{\mu_{F_1}(f_z)}\!/_{f_z} \right) \cap \left({}^{\mu_{F_2}(f_1)}\!/_{f_1} + \ldots + {}^{\mu_{F_2}(f_z)}\!/_{f_z} \right)$$
$$= {}^{\min\left(\mu_{F_1}(f_1),\, \mu_{F_2}(f_1)\right)}\!/_{f_1} + \ldots + {}^{\min\left(\mu_{F_1}(f_z),\, \mu_{F_2}(f_z)\right)}\!/_{f_z} \; . \tag{9}$$

Example 2 (computational utilities of treatments on symptoms in radiation cystitis)

The list L is designed for an arbitrary number of expressions. Let us decide L = {N_1 = "*none*", N_2 = "*little*", N_3 = "*moderate*", N_4 = "*large*", N_5 = "*best*"} for the effectiveness of therapies, tested in radiation cystitis. For $p = 5$, we establish verbal utilities as fuzzy sets

$N_1 = {}^{1}\!/_{1} + {}^{0.8}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.4}\!/_{4} + {}^{0.2}\!/_{5}, k = 1$, $N_2 = {}^{0.8}\!/_{1} + {}^{1}\!/_{2} + {}^{0.8}\!/_{3} + {}^{0.6}\!/_{4} + {}^{0.4}\!/_{5}, k = 2$,

$N_3 = {}^{0.6}\!/_{1} + {}^{0.8}\!/_{2} + {}^{1}\!/_{3} + {}^{0.8}\!/_{4} + {}^{0.6}\!/_{5}, k = 3$, $N_4 = {}^{0.4}\!/_{1} + {}^{0.6}\!/_{2} + {}^{0.8}\!/_{3} + {}^{1}\!/_{4} + {}^{0.8}\!/_{5}, k = 4$,

$N_5 = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.8}\!/_{4} + {}^{1}\!/_{5}, k = 5$,

in compliance with (6).

Set N is decided as $N = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.8}\!/_{4} + {}^{1}\!/_{5}$, when referring to (7).

The utilities u_{ij} will be computationally interpreted as sets $N_k`$ evaluated, after involving (8) and (9), by

$N_1` = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.4}\!/_{4} + {}^{0.2}\!/_{5}$ , $N_2` = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.6}\!/_{4} + {}^{0.4}\!/_{5}$, $N_3` = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.8}\!/_{4} + {}^{0.6}\!/_{5}$,

$N_4` = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.8}\!/_{4} + {}^{0.8}\!/_{5}$ , $N_5` = {}^{0.2}\!/_{1} + {}^{0.4}\!/_{2} + {}^{0.6}\!/_{3} + {}^{0.8}\!/_{4} + {}^{1}\!/_{5}$.

Example 3 (treatments recommended in radiation cystitis)

In the Blekinge County Hospital in Karlskrona, Sweden, the following therapies are tried to cause the remission of the radiation cystitis invasion: a_1 = alum irrigation, a_2 = formalin instillation, a_3 = oestrogens, a_4 = interruption of internal illiac arteries, a_5 = bilateral percutaneous nephrostomy, a_6 = ileal diversion (with cystectomy), a_7 = pentoxyfilline and a_8 = hyperbaric oxygen.

The judgments of verbal utilities $u_{ij} = N_k$ for pairs (a_i, x_j), $i = 1,\ldots,8$, $j = 1,\ldots,5$, $k = 1,\ldots,5$, are accomplished by physicians. The connections are collected in Table 1.

Table 1. Verbal utilities $u_{ij} = N_k$ of pairs (a_i, x_j) determined by a physician

$a_i \backslash x_j$	x_1	x_2	x_3	x_4	x_5
a_1	N_4=”*large*”	N_2=”*little*”	N_2=”*little*”	N_2=”*little*”	N_1=”*none*”
a_2	N_4=”*large*”	N_1=”*none*”	N_2=”*little*”	N_1=”*none*”	N_1=”*none*”
a_3	N_2=”*little*”	N_2=”*little*”	N_2=”*little*”	N_3=”*moderate*”	N_3=”*moderate*”
a_4	N_5=”*best*”	N_2=”*little*”	N_2=”*little*”	N_1=”*none*”	N_1=”*none*”
a_5	N_3=”*moderate*”	N_2=”*little*”	N_5=”*best*”	N_4=”*large*”	N_4=”*large*”
a_6	N_4=”*large*”	N_5=”*best*”	N_5=”*best*”	N_5=”*best*”	N_3=”*moderate*”
a_7	N_5=”*best*”	N_2=”*little*”	N_1=”*none*”	N_2=”*little*”	N_2=”*little*”
a_8	N_4=”*large*”	N_3=”*moderate*”	N_4=”*large*”	N_3=”*moderate*”	N_2=”*little*”

The computational utility $u_{ij} = N_k$`, from Example 2, assists N_k from Table 1.

In order to concatenate utilities u_{ij} for each a_i, we test the operation

$$U_{a_i} = \bigcup_{j=1}^{m} u_{ij} \cdot w_j = \max(\mu_{u_{i1}}(1)\cdot w_1,...\mu_{u_{ij}}(1)\cdot w_j,...,\mu_{u_{im}}(1)\cdot w_m)/1 + ...$$
$$+ \max(\mu_{u_{i1}}(k)\cdot w_1,...\mu_{u_{ij}}(k)\cdot w_j,...,\mu_{u_{im}}(k)\cdot w_m)/k + ... \tag{10}$$
$$+ \max(\mu_{u_{i1}}(p)\cdot w_1,...\mu_{u_{ij}}(p)\cdot w_j,...,\mu_{u_{im}}(p)\cdot w_m)/p,$$

in which u_{ij} are sets N_k` replacing N_k from row a_i of Table 1.

To build (10), we have chosen two operations on fuzzy sets, namely (Zadeh, 1965, 1999):

$$F \cdot c = \left(\mu_F(f_1)/f_1 + ... + \mu_F(f_z)/f_z \right) \cdot c = \mu_F(f_1)\cdot c/f_1 + ... + \mu_F(f_z)\cdot c/f_z \tag{11}$$

and

$$F_1 \cup F_2 = \left(\mu_{F_1}(f_1)/f_1 + ... + \mu_{F_1}(f_z)/f_z \right) \cup \left(\mu_{F_2}(f_1)/f_1 + ... + \mu_{F_2}(f_z)/f_z \right)$$
$$= \max\left(\mu_{F_1}(f_1), \mu_{F_2}(f_1)\right)/f_1 + ... + \max\left(\mu_{F_1}(f_z), \mu_{F_2}(f_z)\right)/f_z. \tag{12}$$

Example 4

For the computational utilities u_{1j}, $j = 1,\ldots,5$, given as N_4`, $N_2$`, N_2`, $N_2$` and N_1`, we test (10) to compute

$$U_{a_1} = N_4`\cdot w_1 \cup N_2`\cdot w_2 \cup N_2`\cdot w_3 \cup N_2`\cdot w_4 \cup N_1`\cdot w_5 = \left(0.2/1 + 0.4/2 + 0.6/3 + 0.8/4 + 0.8/5\right)\cdot 0.33$$
$$\cup \left(0.2/1 + 0.4/2 + 0.6/3 + 0.6/4 + 0.4/5\right)\cdot 0.264 \cup \left(0.2/1 + 0.4/2 + 0.6/3 + 0.6/4 + 0.4/5\right)\cdot 0.198 \cup \left(0.2/1 + 0.4/2 + 0.6/3 + 0.6/4 + 0.4/5\right)$$
$$\cdot 0.132 \cup \left(0.2/1 + 0.4/2 + 0.6/3 + 0.4/4 + 0.2/5\right)\cdot 0.066 = \max(0.2\cdot 0.33, 0.2\cdot 0.264, 0.2\cdot 0.198, 0.2\cdot 0.132, 0.2\cdot 0.066)/1 +$$
$$... + \max(0.8\cdot 0.33, 0.4\cdot 0.264, 0.4\cdot 0.198, 0.4\cdot 0.132, 0.2\cdot 0.066)/5 = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.264/5.$$

In the same way, we obtain the total utilities for rows 2-8 of Table 1. We provide:

$U_{a_2} = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.264/5$, $U_{a_3} = 0.06/1 + 0.132/2 + 0.198/3 + 0.198/4 + 0.132/5$,
$U_{a_4} = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.33/5$, $U_{a_5} = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.198/5$,
$U_{a_6} = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.264/5$, $U_{a_7} = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.33/5$
and $U_{a_8} = 0.06/1 + 0.132/2 + 0.198/3 + 0.264/4 + 0.264/5$.

We calculate $|U_{a_1}| = 0.918$, $|U_{a_2}| = 0.918$, $|U_{a_3}| = 0.72$, $|U_{a_4}| = 0.984$, $|U_{a_5}| = 0.852$, $|U_{a_6}| = 0.918$, $|U_{a_7}| = 0.984$, $|U_{a_8}| = 0.918$ due to (3). After reorganizing the ordering in compliance with descending values of the cardinalities, we build a sequence

$$a_4 = a_7 > a_1 = a_2 = a_6 = a_8 > a_5 > a_3 .$$

Comment 1

After the analysis of results, we conclude that the activity of the algorithm resulted in assigning the highest priority to these therapies, which have revealed the largest effect to liquidate the most dangerous symptoms, especially x_1. Medically, symptom x_1 has caused the greatest disturbance in the patient's health state.

We explain that the results have been affected by the operations of minimum and maximum, which act as a filter for numerical data, passing only the minimal and maximal values through the system. To let all data take part in the decision, exhibiting the action of treatments on all symptoms, we will suggest "softer" operations prevailing the essence of (9) and (10).

5. OWA-LIKE RESULTS AS ESTIMATES OF THE TOTAL UTILITIES

To involve all membership degrees of computational utilities u_{ij} into the decision making algorithm, we first suggest converting (9) to (Dubois and Prade, 2000)

$$F_1 \cap F_2 = \left(\mu_{F_1}(f_1)/f_1 + ... + \mu_{F_1}(f_z)/f_z \right) \cap \left(\mu_{F_2}(f_1)/f_1 + ... + \mu_{F_2}(f_z)/f_z \right) = \mu_{F_1}(f_1)\cdot\mu_{F_2}(f_1)/f_1 + ... + \mu_{F_1}(f_z)\cdot\mu_{F_2}(f_z)/f_z . \quad (13)$$

Example 5

Sets N_k, intersected by set N in compliance with (13), provide new N_k`

$N_1` = 0.2/1 + 0.32/2 + 0.36/3 + 0.32/4 + 0.2/5$ , $N_2` = 0.16/1 + 0.4/2 + 0.48/3 + 0.48/4 + 0.4/5$,
$N_3` = 0.12/1 + 0.32/2 + 0.6/3 + 0.64/4 + 0.6/5$ , $N_4` = 0.08/1 + 0.24/2 + 0.48/3 + 0.64/4 + 0.8/5$,
$N_5` = 0.04/1 + 0.16/2 + 0.36/3 + 0.64/4 + 1/5$.

Equation (10) will be transposed to the OWA-like operation (Mesiar, Calvo and Yager, 2000; Yager, 2004b)

$$U_{a_i} = \sum_{j=1}^{m} u_{ij}\cdot w_j = \frac{\mu_{u_{i1}}(1)\cdot w_1 + ... + \mu_{u_{ij}}(1)\cdot w_j + ... + \mu_{u_{im}}(1)\cdot w_m}{p}\Big/1 + ... + \frac{\mu_{u_{i1}}(k)\cdot w_1 + ... + \mu_{u_{ij}}(k)\cdot w_j + ... + \mu_{u_{im}}(k)\cdot w_m}{p}\Big/k + ... + \frac{\mu_{u_{i1}}(p)\cdot w_1 + ... + \mu_{u_{ij}}(p)\cdot w_j + ... + \mu_{u_{im}}(p)\cdot w_m}{p}\Big/P . \quad (14)$$

The total utilities, emerged by employing (14), constitute fuzzy sets:

$$U_{a_1(OWA)} = N_4`\cdot w_1 + N_2`\cdot w_2 + N_2`\cdot w_3 + N_2`\cdot w_4 + N_1`\cdot w_5 = \left(0.08/1 + 0.24/2 + 0.48/3 + 0.64/4 + 0.8/5\right)\cdot 0.33 + \left(0.16/1 + 0.4/2 + 0.48/3 + 0.48/4 + 0.4/5\right)\cdot 0.264 + \left(0.16/1 + 0.4/2 + 0.48/3 + 0.48/4 + 0.4/5\right)\cdot 0.198 + \left(0.16/1 + 0.4/2 + 0.48/3 + 0.48/4 + 0.4/5\right)\cdot 0.132 + \left(0.2/1 + 0.32/2 + 0.36/3 + 0.32/4 + 0.2/5\right)\cdot 0.066 = \frac{0.08\cdot 0.33 + 0.16\cdot 0.264 + 0.16\cdot 0.198 + 0.16\cdot 0.132 + 0.2\cdot 0.066)}{5}\Big/1 + ... + \frac{0.8\cdot 0.33 + 0.4\cdot 0.264 + 0.4\cdot 0.198 + 0.4\cdot 0.132 + 0.2\cdot 0.066}{5}\Big/5 = 0.026/1 + 0.067/2 + 0.093/3 + 0.103/4 + 0.102/5,$$

$U_{a_2 OWA} = 0.03/1 + 0.061/2 + 0.083/3 + 0.09/4 + 0.034/5$, $U_{a_3 OWA} = 0.028/1 + 0.073/2 + 0.105/3 + 0.107/4 + 0.095/5$,

$U_{a_4OWA} = {}^{0.025}\!/_1 + {}^{0.06}\!/_2 + {}^{0.082}\!/_3 + {}^{0.099}\!/_4 + {}^{0.11}\!/_5$, $U_{a_5OWA} = {}^{0.02}\!/_1 + {}^{0.058}\!/_2 + {}^{0.098}\!/_3 + {}^{0.118}\!/_4 + {}^{0.132}\!/_5$,
$U_{a_6OWA} = {}^{0.011}\!/_1 + {}^{0.039}\!/_2 + {}^{0.082}\!/_3 + {}^{0.126}\!/_4 + {}^{0.179}\!/_5$, $U_{a_7OWA} = {}^{0.025}\!/_1 + {}^{0.059}\!/_2 + {}^{0.084}\!/_3 + {}^{0.101}\!/_4 + {}^{0.113}\!/_5$
and $U_{a_8OWA} = {}^{0.02}\!/_1 + {}^{0.056}\!/_2 + {}^{0.105}\!/_3 + {}^{0.125}\!/_4 + {}^{0.137}\!/_5$.

The cardinalities $\left|U_{a_iOWA}\right|$ are still calculated as sums of membership degrees of fuzzy sets U_{a_iOWA}. We set together $\left|U_{a_1OWA}\right| = 0.2992$, $\left|U_{a_2OWA}\right| = 0.298$, $\left|U_{a_3OWA}\right| = 0.408$, $\left|U_{a_4OWA}\right| = 0.376$, $\left|U_{a_5OWA}\right| = 0.426$, $\left|U_{a_6OWA}\right| = 0.437$, $\left|U_{a_7OWA}\right| = 0.362$, $\left|U_{a_8OWA}\right| = 0.443$.

The new ordering of the treatments is determined as

$$a_8 > a_6 > a_5 > a_3 > a_4 > a_7 > a_1 > a_2 .$$

Comment 2

These therapies are prioritized, which affect all symptoms. This is a result of engaging all membership degrees, participating in computational utilities. The operations of multiplication and the modified weighted mean let all data values have their significance in the final results. The results are convergent to the effects already obtained in other papers on ranking the medication in radiation cystitis (Rakus-Andersson and Frey, 2012; 2014) and regarded by physicians as the optimal solutions. In addition to this, the physicians also positively evaluate the sequence, being an effect of applying maximum and minimum operators. The sequence can be considered as a relevant hint of the medicine choice, when an efficient therapy is needed to get rid of the most disturbing symptoms.

6. CONCLUSION

The basis of investigations has been mostly restricted to a judgment of the influence of different therapies on clinical symptoms, characterizing radiation cystitis.

One of the goals has been shaped as a selection of the therapies, which highly recede the most invading symptoms. Another purpose of the investigations should provide a hierarchy of treatments, when counting on their affection on all symptoms. In order to propose solutions to both issues, we have designed finite fuzzy sets to be the levels of utilities of the therapy actions on symptoms.

To solve the first problem, we have estimated the indices of the symptoms' importance to emphasize the essence of the symptom priority to recede. The emergence of the treatment, remitting the most threatening symptom, has demanded involving operations that have been mostly concentrated on pushing forward this symptom. Therefore we have adopted minimum and maximum in the computations of total utility of the treatment considered.

To see the effects of treatments on the whole collection of symptoms, we have created the average mean operator, which resembles the OWA operator. The presence of the contents of all data sets in computations of total utilities has helped to rank the therapies, which have power to treat all symptoms. To state the ordering, we have just used finite fuzzy sets as utility levels. In other papers (Rakus-Andersson and Frey, 2012, 2014) devoted to radiation cystitis, we applied alpha-cuts of fuzzy sets and Choquet integrals as alternative utility patterns. In spite of varying mathematical designs, the outcomes were similar.

Apart from own novelties like the design of finite fuzzy sets and weights of importance, we have not looked for new complicated operations on fuzzy sets. We think that the creators of fuzzy set theory made excellent contributions in the domain, and their results are still alive to use. Moreover, the simplicity of performed formulas allows converting the algorithm to a computer program.

The data, collected as the verbal levels of utilities of the treatments on single symptoms, have been prepared by a physician. The physician has observed the patients' reactions on different therapies, and his judgment has a very general character, which does not concern a single patient. The results, provided by fuzzy algorithms, have brought strings of hierarchies of therapies, already proved by physicians in the similar order. We thus want to extend our research to be able to prognosticate a patient-tailored treatment after inserting values of biological markers into the system, already created.

To sum up, we should emphasize that the treatment efficacies are mostly based on the personal experience and obviously can vary among the centers. We also note that the treatment of radiation cystitis is most often multimodal, when combining various methods. The final scale of therapy priorities from Section 5 should not be regarded as a guideline for future prognoses of treatments but the model itself, with dynamic input categories, seems to be a very valuable tool helping to determine the appropriate treatment path.

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