



**This report is submitted as partial fulfilment for  
the thesis entitled “Automatically measuring the  
losses of a transformer”**

**Bachelor thesis**

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## **Abstract:**

This thesis is an ongoing project of Alstom Power Sweden AB, Växjö with a collaboration of Blekinge Institute of Technology [BTH] to automatically measure the losses of a transformer. So far the company performed the measurements manually to get this loss, and they decided to automatize the measurement process. This paper presents the state space modelling approach of DC-DC Buck converter and simulation of the whole measurement circuit incorporated with the PID controller. The PID controller will control the outputs voltage of the buck converter and the output current of the resonant circuit (secondary current of the transformer) with PI controller and those will be displayed in graph. The switching frequency of the dc-dc converter (buck) is set to 50 kHz. This system is implemented in MATLAB Simulink software.

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## List of symbols

$L_b$	Back inductor
$I_{Lb}$	Buck inductor current
$C_1$	Back Capacitor 1
$C_2$	Back Capacitor 2
$R_{Lb}$	DC resistance of buck inductor
$R_{C1}$	ESR1 for the buck capacitor
$R_{C2}$	ESR2 for the buck capacitor
$V_{in}, v_g$	Input voltages
$D, d$	Duty cycle
$F_s$	Buck frequency
$F_{s1}$	Bridge frequency
$V_{sc}$	Resonance circuit capacitor voltage
$iL_{sc}$	Resonance circuit capacitor voltage
$R_{loss}$	Modelling the transitive loss
$L_{leak}$	Leakage inductance
$C_{load}$	Secondary capacitor
$RT_{on}$	Transistor turned on resistance
Set Point	Reference voltage/current
PWM	Pulse width modulation
DC	Direct current
ESR	Equivalent series resistance



# CHAPTER 1 INTRODUCTION

## 1.1 Thesis overview

This chapter describes the thesis background, objectives, scopes, and summary. In the chapter, it briefs the description of the buck converter as well as the objectives and the scopes. At the end, outline of this thesis is given in this chapter.

The dc-dc Buck is a device that uses to convert unregulated dc input to a controlled dc output with a desired voltage level [1]. The buck will step down the input voltage to an output voltage which is less than the input voltage with the switching frequency 50 kHz .Together with buck is PID controller that uses to control the behaviours of the system. This system is a close loop system with feedback. The software used to do simulation is MATLAB SIMULINK. This project consists of modelling, simulation and control of the buck converter subsystem and the whole measurement system.

## 1.2 Thesis Scopes:

The following are scopes of this thesis:

- Study the operation of Buck Converter.
- Design the mathematical model of the dc-dc buck converter.
- Design the Polynomial PD and PID Controller.
- Bode plots of the buck converter with continuous time transfer function and form stability analysis.
- To simulate the measurement and control circuit in MATLAB SIMULINK environment for testing the properties of the system.
- Resonant current control.
- Final conclusion from the simulation result.
- Reference and Appendix.

Table 1 shows the parameter values used in the simulation of the measurement circuit

<i>Parameter Name</i>	<i>Symbol</i>	<i>Value</i>
Buck inductor	$L_b$	1.2e-3H
Buck capacitor 1	$C_1$	66e-6F
Buck capacitor 2	$C_2$	66e-6F
Buck inductor equivalent resistance	$R_{Lb}$	5 $\Omega$
Buck capacitor 1 inductor equivalent resistance	$R_{C1}$	10e-3 $\Omega$
Buck capacitor 2 inductor equivalent resistance	$R_{C1}$	10e-3 $\Omega$
Input voltage	$R_{in}$	30 or 300 V
Duty cycle	D	varying
Buck frequency	$f_b$	50kHz
Half bridge frequency	$f_{hb}$	30859
Transformer turns ratio	Turns ratio	20 turns
Modelling the transformer losses	$R_{loss}$	10e-6 Henry
The leakage inductance	$L_{Leak}$	7.6e-6 Henry
The load capacitor	$C_{load}$	3.5e-6 Farad
Transistor on resistance	$R_{Ton}$	25e-3 Ohm

Table 1: The parameter values used in the simulation of the measurement circuit

## 1.4 Thesis outline

This thesis consists of seven chapters. In this first chapter, it discusses thesis background, objectives, and scopes.

Chapter 2 will explain about dc-dc converter (buck), pulse width modulation and proportional derivative integral (PID) that will use as controller.

Chapter 3 contains the design of mathematical system model and theories on buck converter incorporated with PID controller are displayed. It will also explain about the concept of buck converter and PID controller.

Chapter 4 shows the simulation implementation on Matlab and Simulink.

Chapter 5 discusses the simulation result and conclusions obtained. Finally, the last part in the thesis provides the references and appendices used in the thesis project.

Chapter 6 shows simulation implementation model and results of the whole measurement system.

Chapter 7 Contains conclusion, reference and appendix.

## CHAPTER 2 PRINCIPLE OF OPERATION OF BUCK CONVERTER

Figure 1 shows the circuit configuration of the pure buck converter with feedback controller. The converter is operated by turning the MOSFET ON and OFF at a high switching frequency.

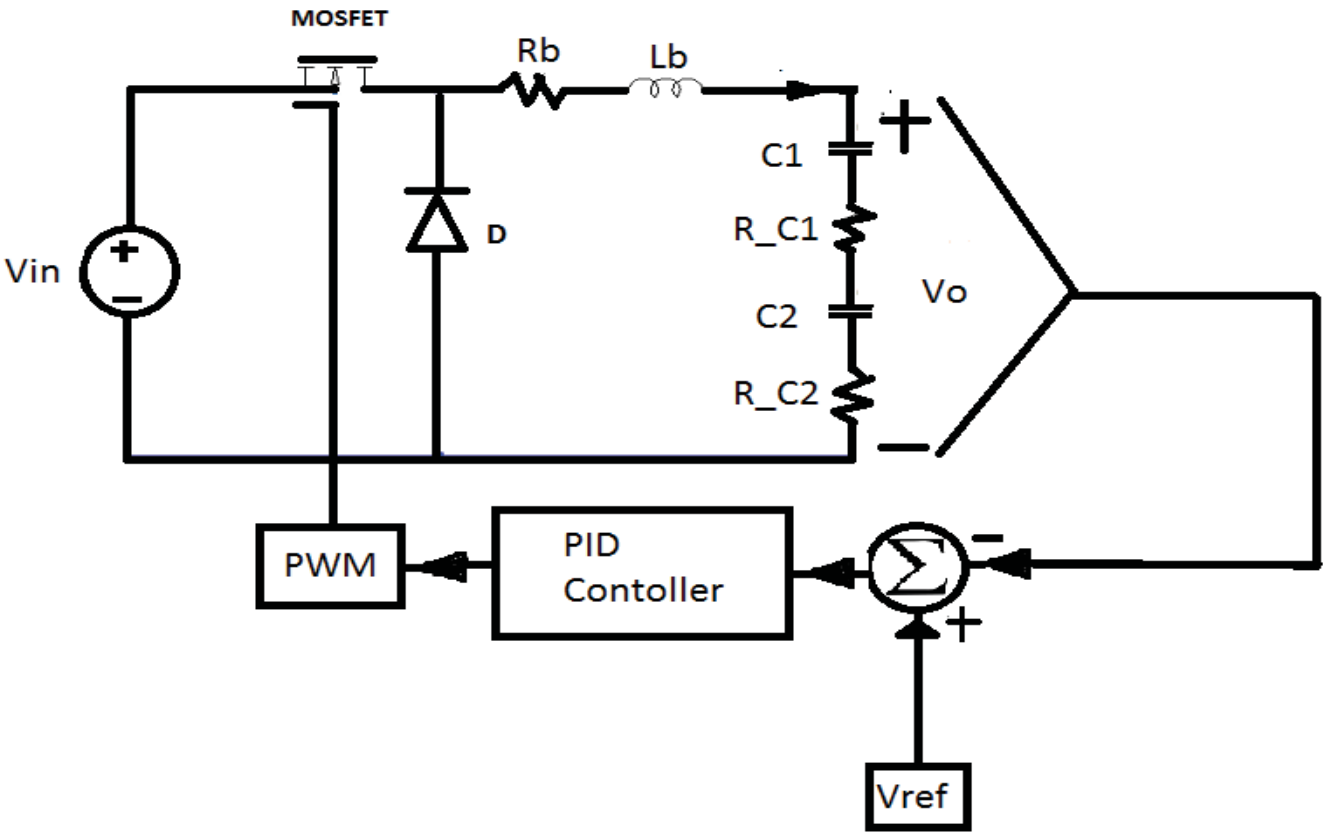


Figure 1: Buck converter with feedback controller

### 2.1 States of Operation:

The operation of the DC-DC converter works in two states i.e., ON and OFF states. When the MOSFET is turned ON, the voltage source is delivered to the RC circuit which results in transferring the energy from the source voltage [ $V_{in}$ ] to the inductor [ $L_b$ ], and

Capacitors C1 and C2. Thus, the diode becomes reverse bias. In this state, current through the inductor increases gradually [3]. During the OFF state, the switch is open, now the inductor acts as a source by maintaining constant energy transfer to the load. At this stage, the diode starts conducting and the current through the circuit decreases linearly as the energy in the inductor and capacitors are absorbed by the load, which is the primary side of the transformer [4].

The relation between the input voltage ( $V_{in}$ ), output voltage ( $V_o$ ) and the switching duty cycle ( $D$ ) of the buck converter when operating in continuous conduction mode is given by:

$$V_o = V_{in} \times D$$

Where:

$$D = \frac{T_{on}}{T_{on} + T_{off}}$$

Where:

$V_{in}$  = Input Voltage of the Buck Converter,  $V_o$  = Output voltage of the Buck Converter,  $D$  = Duty Cycle,  $T_{on}$  = on time of transistor and  $T_{off}$  = off time of transistor

The above voltage conversion relationship for  $V_o$  shows the fact that  $V_{out}$  can be adjusted by adjusting the duty cycle,  $D$ , and is always less than the input because  $D$  is a number between 0 and 1.

## 2.2. Modes of Operation

The buck converter operates in two modes i.e., continuous and discontinuous modes of operation [5]:

- **Continuous Conduction Mode (CCM):** A dc-dc buck converter is said to be operating in Continuous Conduction Mode, if inductor current never reaches to zero [5].
- **Discontinuous Conduction Mode (DCM):** A dc-dc buck converter is said to be operating in Discontinuous Conduction Mode, if inductor current reaches zero and remains there for certain period of time [5].

# CHAPTER 3. MODELLING OF BUCK CONVERTER:

In this subsection, the mathematical model of buck converter when the switch is ON and OFF and the method of state-space averaging are explained. Figure 2 shows the circuit diagram of the buck converter.

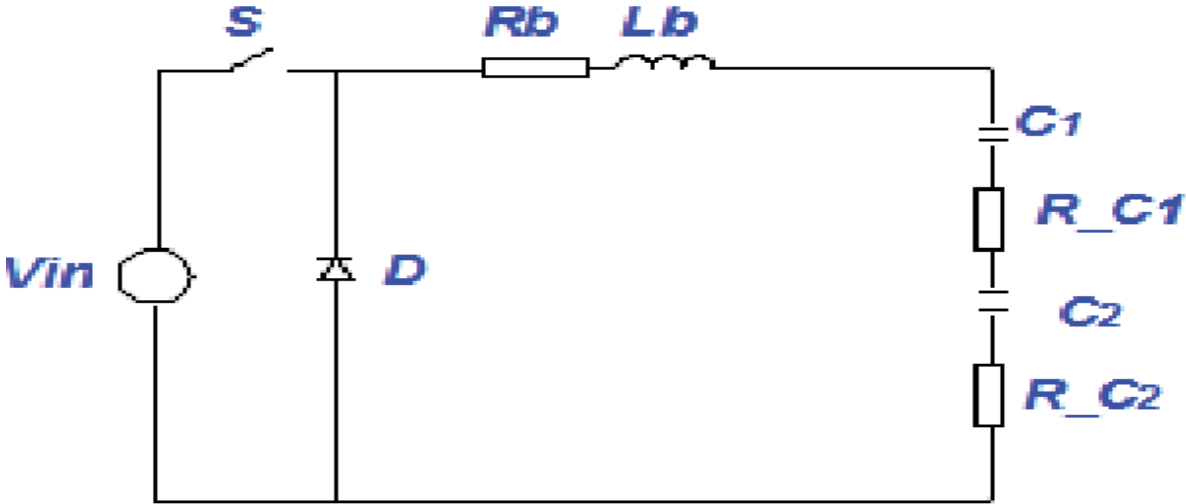


Figure 2: The circuit of the buck converter.

Where:

$V_{in}$ = voltage source,  $D$ =diode,  $L_b$ = Buck inductor,  $C_1$  and  $C_2$ = capacitors,  $R_b$ = the DC resistance for the buck inductor,  $R_{C1}$  and  $R_{C2}$  = the ESR for the buck capacitors.

## 3.1 PWM Generator Subsystem:

Figure 3 bellow shows the PWM subsystem diagram which accepts duty cycle in decimal form from the controller and outputs the required PWM waveforms of the corresponding duty cycle. This block is responsible for producing PWM pulses of varying duty cycle.

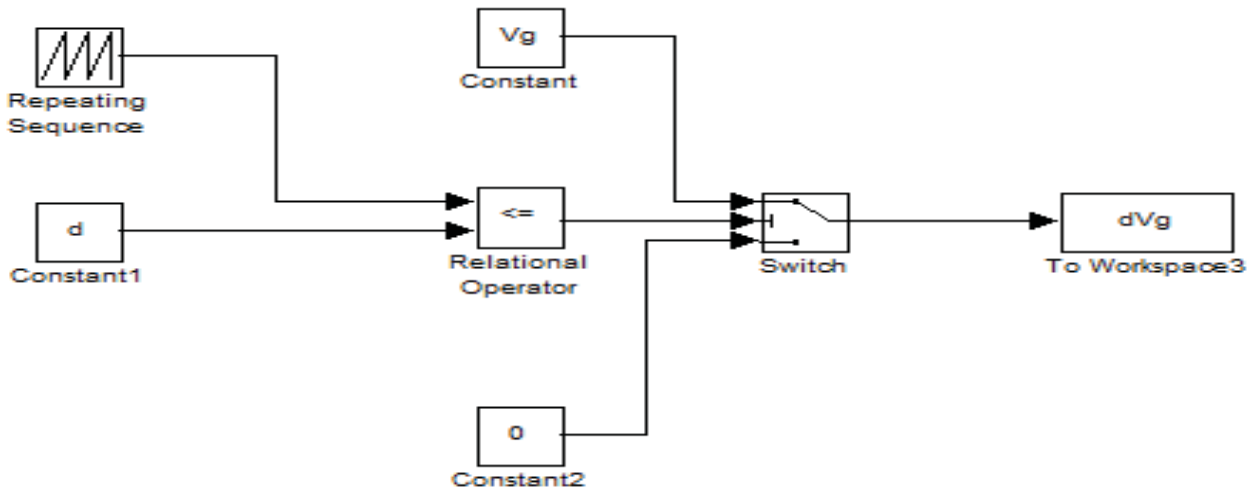


Fig 3: PWM Subsystem

As already explained, the circuit has two operating conditions, the Duty Interval ( $T_{on}$ ) when the Buck convert Switch is ON and the Duty Interval ( $T_{off}$ ) when the switch is OFF. The two inputs to the Subsystem are:

- The duty cycle from the controller in decimal form.
- Switching Frequency  $F_s$ ,

### 3.2 Mathematical equations of the buck converter when the switch is in ON state

At first, the voltage source is included in the circuit when the switch is on as shown in Fig. 4 bellow.

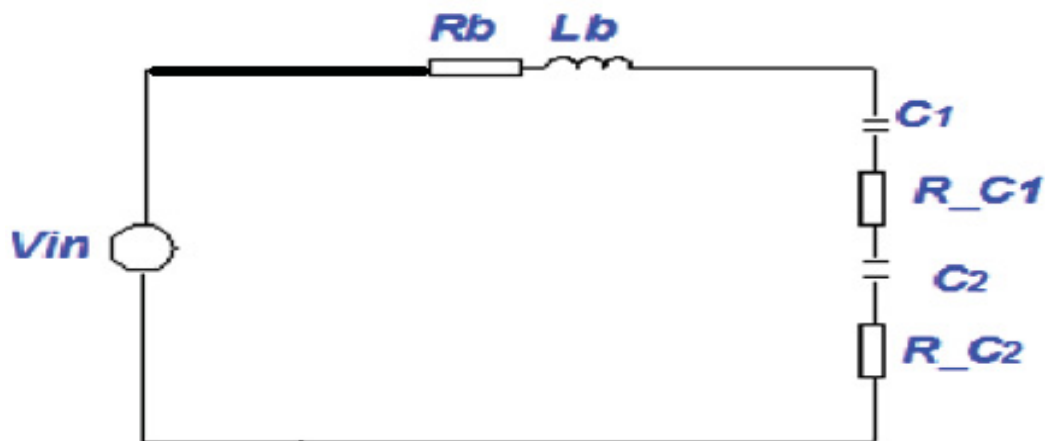


Figure 4: The circuit of the buck converter with voltage source

The simple Buck converter can be modelled as

$$\begin{pmatrix} i_U \\ i_{C1} \\ i_{C2} \\ i_{RL} \\ i_{RC1} \\ i_{RC2} \\ u_L \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} U_U \\ U_{C1} \\ U_{C2} \\ U_{RL} \\ U_{RC1} \\ U_{RC2} \\ i_L \end{pmatrix}$$

*Electrical networks*

Where  $U_U$  is the input voltage and  $i_L$  is the indicator current of the buck converter.

### 1. One network equation per element

$$i_U = i_L$$

$$i_{C1} = i_L$$

$$i_{C2} = i_L$$

$$i_{RL} = i_L$$

$$i_{RC1} = i_L$$

$$i_{RC2} = i_L$$

(1)

$$u_L = V - u_{C1} - u_{C2} - u_{RL} - u_L - u_{RC1} - u_{RC2}$$

## 2. One constitutive equation per element

$$\left( \begin{array}{l} u_U = V_g \\ i_{C_1} = C_1 \frac{d}{dt} u_{c1} \\ i_{C_2} = C_2 \frac{d}{dt} u_{C2} \\ i_{RL} = \frac{u_{RL}}{R_L} \\ i_{RC1} = \frac{u_{RC1}}{R_{C1}} \\ i_{RC2} = \frac{u_{RC2}}{R_{C2}} \\ u_L = L \frac{d}{dt} i_L \end{array} \right)$$

As the output voltage is the sum of capacitors voltage and resistors voltage, where resistors voltage is the product of current and resistor, then it has

$$u_0 = u_{C1} + u_{C2} + u_{R_{C1}} + u_{R_{C2}} \quad (2)$$

Where;

$$\begin{aligned} u_{R_{C1}} &= R_{C1} i_L \\ u_{R_{C2}} &= R_{C2} i_L \end{aligned} \quad (3)$$

So,

$$u_0 = \left( \begin{array}{cc|c} 1 & 1 & (R_{C1} + R_{C2}) \end{array} \right) \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} \quad (4)$$

Including the constitutive equations, the system is modelled by



$$\frac{d}{dt} \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} (V_g) \quad (5)$$

The *ABC*-differential equation system is given as

$$\begin{cases} \dot{x} = A_1 x + B_1 u \\ y = C_1 x + D_1 u \end{cases}$$

$$A_1 = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix}$$

$$C_1 = (1 \quad 1 \quad (R_{C1} + R_{C2})),$$

$$D_1 = 0$$

### 3.3 Mathematical equations of the buck converter when the switch is in OFF state

When the switch is off the voltage source is not included in the circuit as shown in Fig 5 bellow.

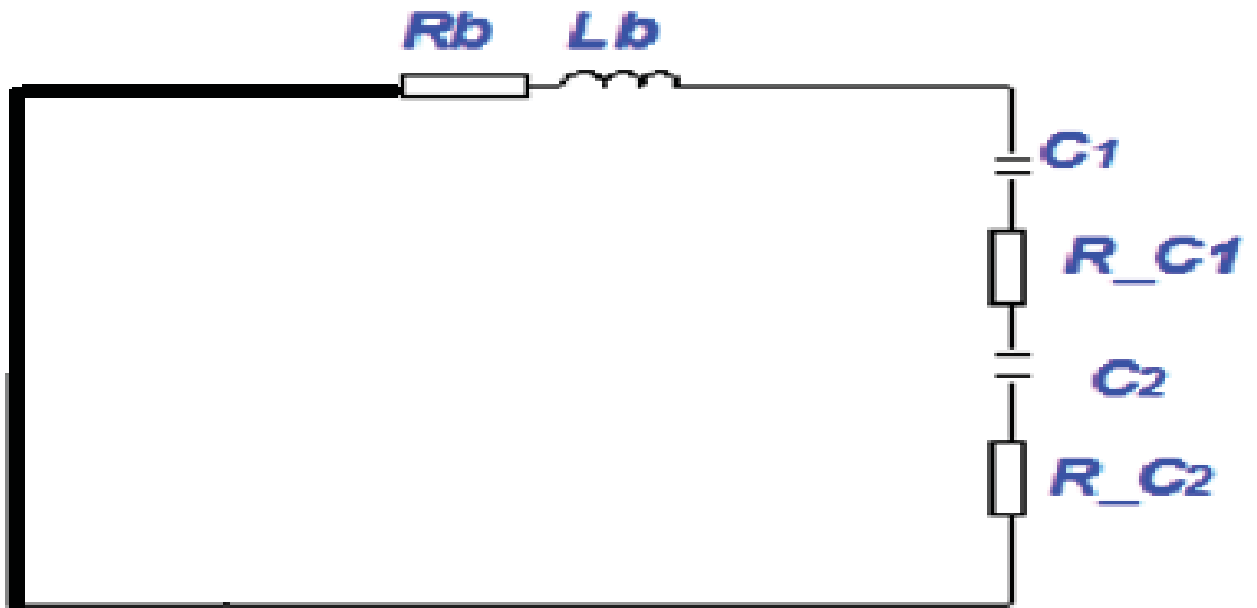


Figure 5: The circuit of the buck converter without voltage source

The simple Buck converter can be modelled as

$$\begin{pmatrix} i_{C1} \\ i_{C2} \\ i_{RL} \\ i_{RC1} \\ i_{RC2} \\ u_L \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} U_{C1} \\ U_{C2} \\ U_{RL} \\ U_{RC1} \\ R_{RC2} \\ i_L \end{pmatrix}$$

Electrical networks

## 1. One network equation per element

$$\begin{aligned}
 i_{C_1} &= i_L \\
 i_{C_2} &= i_L \\
 i_{RL} &= i_L \\
 i_{RC_1} &= i_L \\
 i_{RC_2} &= i_L
 \end{aligned} \tag{6}$$

$$u_L = -u_{C_1} - u_{C_2} - u_{RL} - u_L - u_{RC_1} - u_{RC_2}$$

## 2. One constitutive equation per element

$$\left( \begin{array}{l}
 i_{C_1} = C_1 \frac{d}{dt} u_{C_1} \\
 i_{C_2} = C_2 \frac{d}{dt} u_{C_2} \\
 i_{RL} = \frac{u_{RL}}{R_L} \\
 i_{RC_1} = \frac{u_{RC_1}}{R_{C_1}} \\
 i_{RC_2} = \frac{u_{RC_2}}{R_{C_2}} \\
 u_L = L \frac{d}{dt} i_L
 \end{array} \right)$$

Including the constitutive equations, the system is modelled by

$$\frac{d}{dt} \begin{pmatrix} u_{C_1} \\ u_{C_2} \\ i_L \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C_1} + R_{C_2}}{L}\right) \end{pmatrix} \begin{pmatrix} u_{C_1} \\ u_{C_2} \\ i_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} (V_g) \tag{7}$$

The  $ABC$ -differential equation system is given as

$$\begin{cases} \dot{x} = A_2 x + B_2 u \\ y = C_2 x + D_2 u \end{cases}$$

$$A_2 = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} 1 & 1 & (R_{C1} + R_{C2}) \end{pmatrix},$$

$$D_2 = 0$$

### 3.4 State-space averaging

In this subsection, the method of state-space averaging is explained. While the transistor is on, the voltage across the diode is equal to the input voltage. The circuit in Figure 4 can therefore be used as a model of the buck converter during ton. While the transistor is off, the voltage across the diode is equal to zero and the circuit in Figure 5 can be used as a model. Assume that the state-space descriptions of the circuits in Figure 4 and Figure 5 are:

$$\begin{cases} \frac{d(X_1)t}{dt} = A_1 X(t) + B_1 U(t) \\ y_1(t) = C_1 X(t) + D_1 U(t) \end{cases} \quad (8)$$

and respectively

$$\begin{cases} \frac{d(X_2)t}{dt} = A_2 X(t) + B_2 U(t) \\ y_2(t) = C_2 X(t) + D_2 U(t) \end{cases} \quad (9)$$

Weighted Average:

$$A = A_1d + A_2(1-d)$$

$$B = B_1d + B_2(1-d)$$

$$C = C_1d + C_2(1-d)$$

$$D = D_1d + D_2(1-d)$$

In state-space averaging, these two systems are first averaged with respect to their duration in the switching period:

Combining the above two models on fig 8 and 9 together by averaging we get:

The average system is:

$$\begin{cases} \frac{d(X)t}{dt} = AX(t) + BU(t) + [(A_1 - A_2)X(t) + (B_1 - B_2)U(t)]d(t) \\ y(t) = CX(t) + DU(t) \end{cases} \quad (11)$$

Where:

$d(t)$  is the control signal

The weighted Average models of the buck converter can be derived as bellow:

$$A = A_1d + A_2(1-d)$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} (d) + \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} (1-d) =$$

$$A = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix}$$

$$B = B_1 d + B_2 (1-d)$$

$$B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} d + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} (1-d)$$

$$B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix}$$

$$C = C_1 d + C_2 (1-d)$$

$$C = \begin{pmatrix} 1 & 1 & (R_{C1} + R_{C2}) \end{pmatrix} (d) + \begin{pmatrix} 1 & 1 & (R_{C1} + R_{C2}) \end{pmatrix} (1-d)$$

$$C = \begin{pmatrix} 1 & 1 & (R_{C1} + R_{C2}) \end{pmatrix}$$

$$D = [0]$$

Hence,

$$\frac{d(X)t}{dt} = AX(t) + BU(t) + [(A_1 - A_2)X(t) + (B_1 - B_2)U(t)]d(t)$$

With  $A = A_1 = A_2$

$$\begin{aligned}
\frac{d(X)t}{dt} &= \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} (Vg) + \\
&\left[ \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} - \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} \right] \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} + \left( \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right) (Vg) \quad (d)
\end{aligned}$$

Finally after adding and subtracting the above matrix and multiplying with (d) and [Vg], then simplifying we get the average model bellow:

Average models system:

$$\frac{d}{dt} \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{-1}{L} & \frac{-1}{L} & -\left(\frac{R_L + R_{C1} + R_{C2}}{L}\right) \end{pmatrix} \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{V_g}{L} \end{pmatrix} (d) \quad (11)$$

And the output equation is:

$$y(t) = CX(t) + DU(t)$$

$$u_0 = \begin{pmatrix} 1 & 1 & (R_{C1} + R_{C2}) \end{pmatrix} \begin{pmatrix} u_{C1} \\ u_{C2} \\ i_L \end{pmatrix} \quad (12)$$



# CHAPTER 4. P-I, P-D, PID CONTROLLER

Figure 6 below shows the block diagram of a PID controller in a feedback loop. Here the PID stands for P -Proportional, I - Integral, D - Derivative.

- Where
- Kp: Proportional gain
- Ki: Integral gain,
- Kd: Derivative gain,
- e: Error = SP-PV
- SP: Set Point
- PV: Process Variable

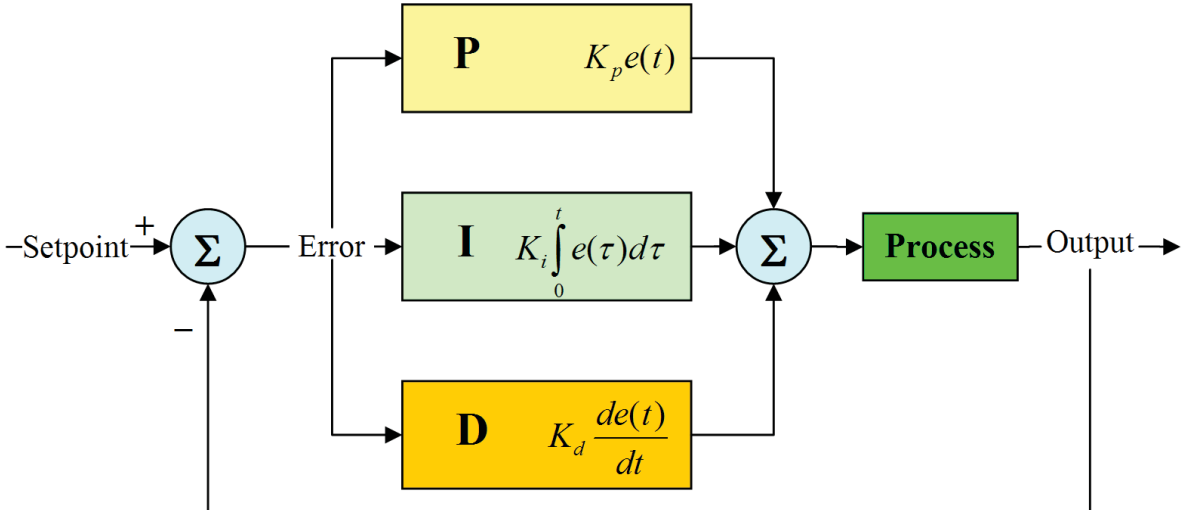


Fig 6. A block diagram of a PID controller

A PID controller calculates an error value as the difference between a measured process variable and a desired set point. A PID controller attempts to correct the error between a measured process variable (PV) and a desired set point (SP) by calculating and then outputting a corrective action that can adjust the process accordingly. [8]

In this chapter it is first intended to explain the usage of continuous time P-I-D controllers and show how the P-I, P-D, P-I-D controllers change the steady state response of the closed loop systems by controlling the output voltage of the Dc-Dc buck converter.

## 4.1 P-I Controller:

P-I controller is mainly used to eliminate the steady state error resulting from P controller. However, in terms of the speed of the response and overall stability of the system, it has a negative impact. This controller is mostly used in areas where speed of the system is not an issue. Since P-I controller has no ability to predict the future errors of the system it cannot decrease the rise time and eliminate the oscillations [8].

## 4.2 P-D Controller:

The aim of using P-D controller is to increase the stability of the system by improving control since it has an ability to predict the future error of the system response [8].

## 4.3 P-I-D controller:

P-I-D controller has the optimum control dynamics including zero steady state error, fast response (short rise time), no oscillations and higher stability. The necessity of using a derivative gain component in addition to the PI controller is to eliminate the overshoot and the oscillations occurring in the output response of the system. One of the main advantages of the P-I-D controller is that it can be used with higher order processes including more than single energy storage [8].

## 4.4 The effects of each of controller parameters $K_p$ , $K_i$ , $K_d$

The following table 2 shows the effects of each of controller parameters  $K_p$ ,  $K_i$ ,  $K_d$  to the system response.

CL response	Rise Time	OVERSHOT	Settling time	S-S ERROR
$K_p$	Decrease	Increase	Small change	Decrease
$K_i$	Decrease	Increase	Increase	Eliminate
$K_d$	Small change	Decrease	Decrease	No change

Table 2: The effects of each of controller parameters  $K_p$ ,  $K_i$ ,  $K_d$

In order to observe the basic impacts, described above, of the proportional, integrative and derivative gain to the system response, the simulations of the continuous time with a transfer function of the buck converter and step input on MATLAB is presented in the next chapter.

# CHAPTER 5. BODE PLOTS AND SIMULATION RESULTS OF THE BUCK CONVERTER WITH CONTINUOUS TIME TRANSFER FUNCTION.

## 5.1 Bode plot

A Bode plot is a graph of the frequency response of a system. It is usually a combination of a Bode magnitude plot, expressing the magnitude of the frequency response, and a Bode phase plot, expressing the phase shift. [13]

## 5.2 Gain and Phase Margins

The phase margin measures how much phase variation is needed at the gain crossover frequency to lose stability. Similarly, the gain margin measures what relative gain variation is needed at the gain crossover frequency to lose stability. Together, these two numbers give an estimate of the "safety margin" for closed-loop stability. The smaller the stability margins, the more fragile stability is. [13]

### 5.2.1 Bode and step response plot of Open-loop system

Figure 7 shows the bode plot of the open loop system without any controller. Let us first view the open-loop step response.

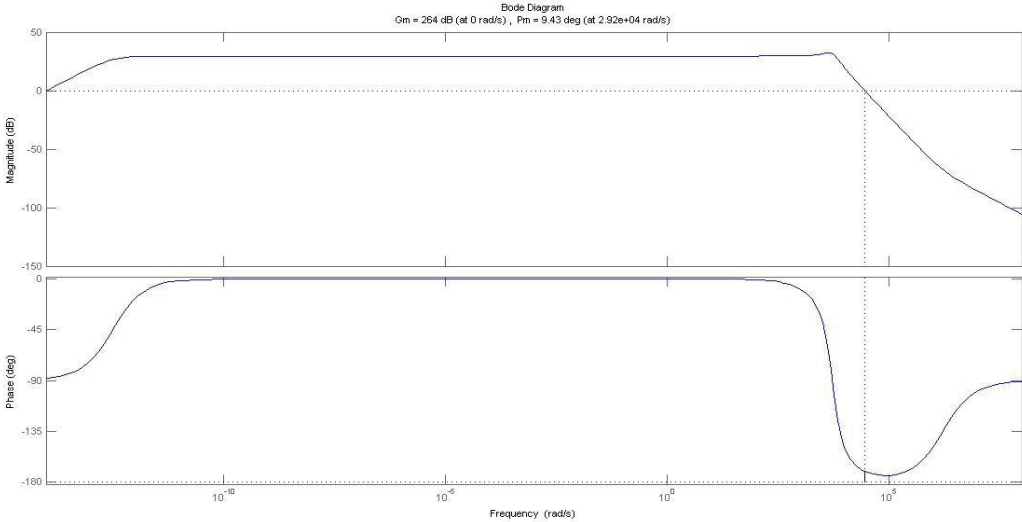


Figure 7: Bode plot of the open loop system without any controller

Figure 8 below shows the results of the step response of the open loop system without any controller.

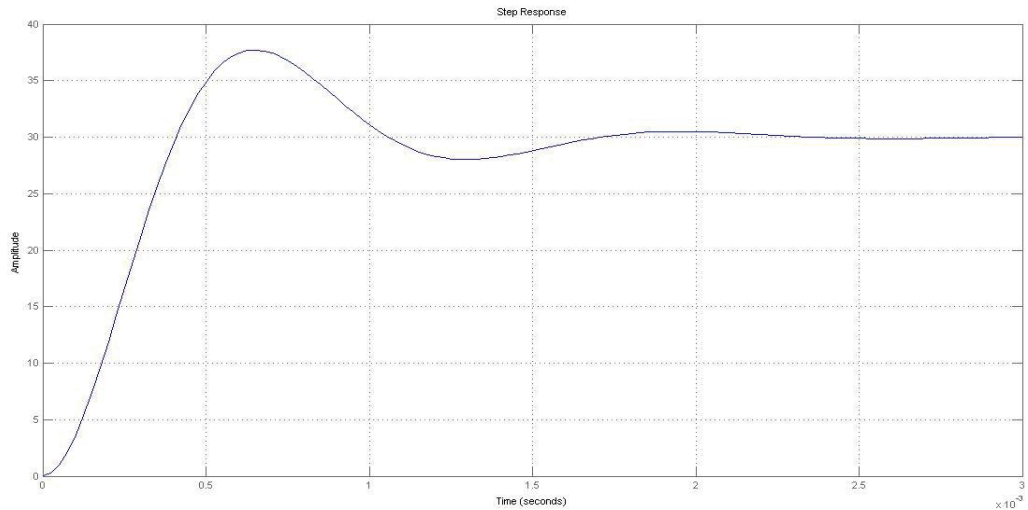


Figure 8: Step response of open loop system without any controller

The step input of the plant transfer function is 30v and the final value of the output to a step input is almost 30v. From the bode plot we can see that the Gain Margin is 264 dB (at a crossover freq. of 0rad/s) and Phase Margin = 9.43 degrees (at a crossover freq. of 161.6 rad/s). This shows the system is stable.

### 5.2.2 Bode and step response plot of P-I Control

From table 2, we see that an integral controller ( $K_i$ ) decreases the rise time, increases both the overshoot and the settling time, and eliminates the steady-state error.

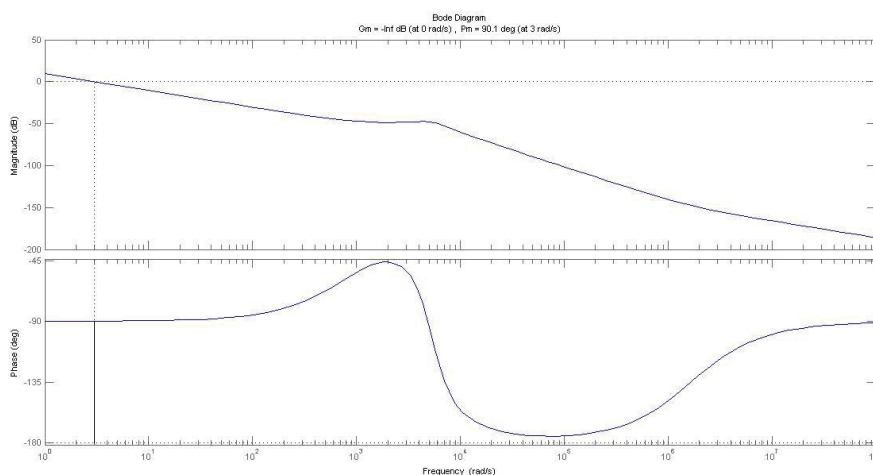


Fig 9: Bode diagram of the P-I controller

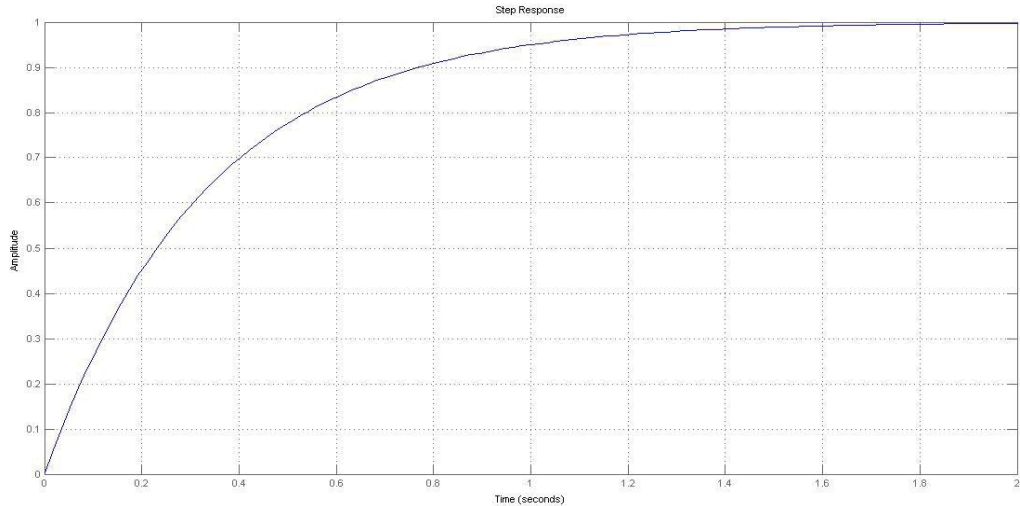


Figure 10: Step response of the P-I controller

**Result:**

Rise time = 0.734

Settling time= 1.31

Overshoot = 0

Steady state error = 0

Gain Margin = -inf dB (at a crossover freq. of 0 rad/s)

Phase Margin = 90.1 degrees (at a crossover freq. of 3 rad/s)

**Conclusion:**

The result of figure 10 shows that the integral controller reduced the rise time and eliminated the steady-state error. From the bode plot we can see that the Gain Margin is -inf dB and Phase Margin = 90.1 degrees (at a crossover freq. of 3 rad/s). The phase and gain margin shows the system is stable.

**5.3 Bode and step response plot of P-D Control**

From table 2, we see that the derivative controller ( $K_d$ ) reduces both the overshoot and the settling time

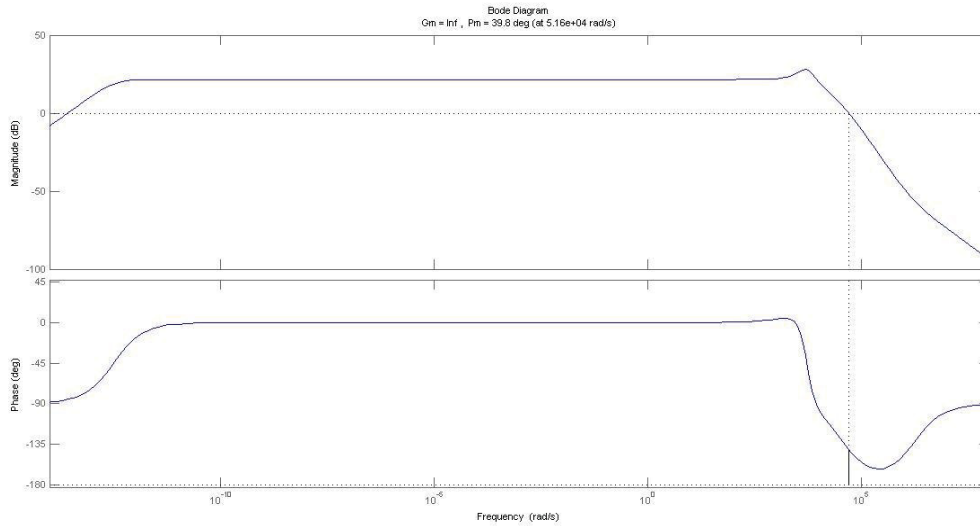


Fig 11: Bode diagram of the P-D controller

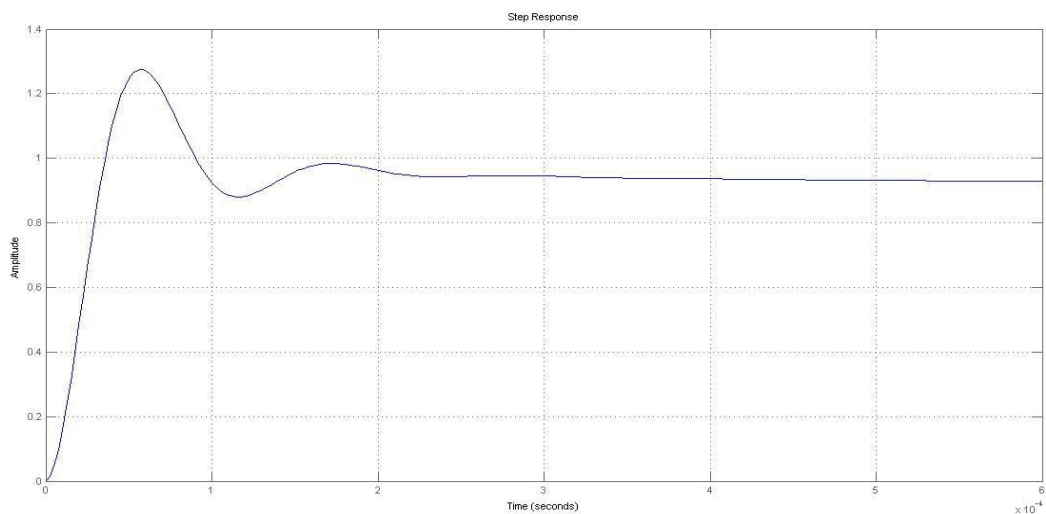


Figure 12: Step response of the P-D controller

**Result:**

Rise time = N/A

Settling time= 0.13

Overshoot = NaN

Steady state error = N/A

Gain Margin = inf dB (at a crossover freq. of 0 rad/s)

Phase Margin = 39.8 degrees (at a crossover freq. of 5 rad/s)

# Conclusion:

The result figure 12 shows that the derivative controller reduced the settling time. From the bode plot we can see that the Gain Margin is inf dB and Phase Margin = 39.8 degrees (at a crossover freq. of 281.73 rad/s). This shows the system is stable.

## 5.4 Bode and step response plot of P-I-D controller

As it is mentioned above the P-I-D controller has zero steady state error, fast response (short rise time), no oscillations and higher stability. The necessity of using a derivative gain component in addition to the PI controller is to eliminate the overshoot and the oscillations occurring in the output response of the system.

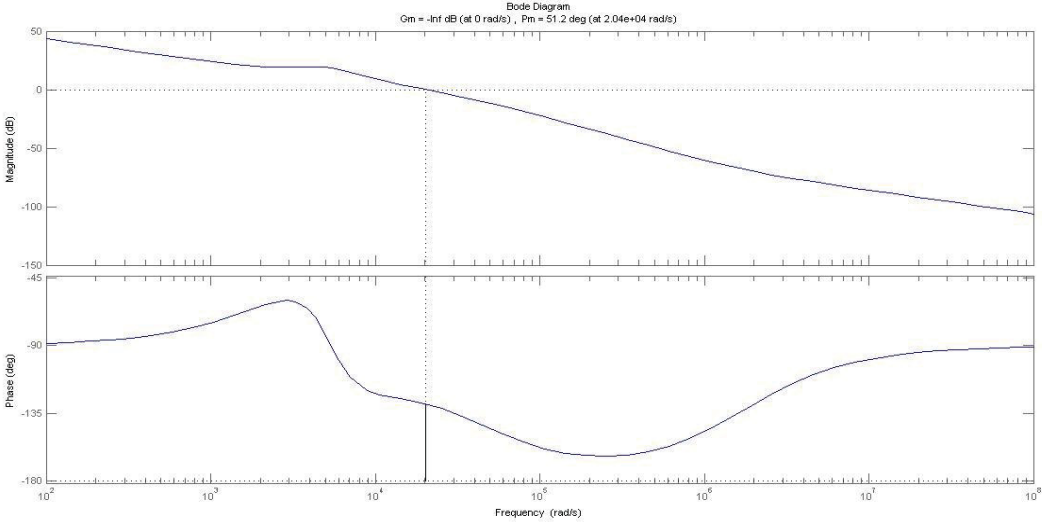


Fig 13: Bode diagram of the P-I-D controller

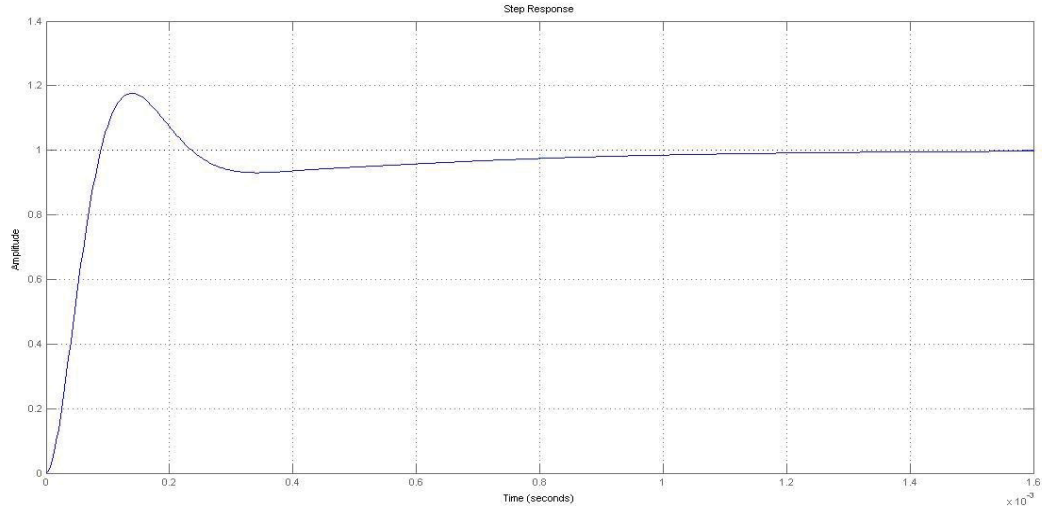


Figure 14: Step response of the P-I-D controller

**Result:**

Rise time = 0.0411015

Settling time= 0.00089

Overshoot = 0

Steady state error = 0

Gain Margin = -inf dB (at a crossover freq. of 0 rad/s)

Phase Margin = 51.2 degrees (at a crossover freq. of 111.4 rad/s)

Figure 14 shows now, we have obtained a closed-loop system with no overshoot, fast rise time, and no steady-state error. From the bode plot we can see that the Gain Margin is -inf dB and Phase Margin = 51.2 degrees (at a crossover freq. of 111.4 rad/s). This shows the system is stable.



# CHAPTER 6. SIMULATION MODEL AND RESULTS OF THE WHOLE MEASUREMENT SYSTEM

Figure 15 shows the complete simulation model for the measurement circuit with controller. The measurement circuit is simulated using the software MATLAB/SIMULINK.

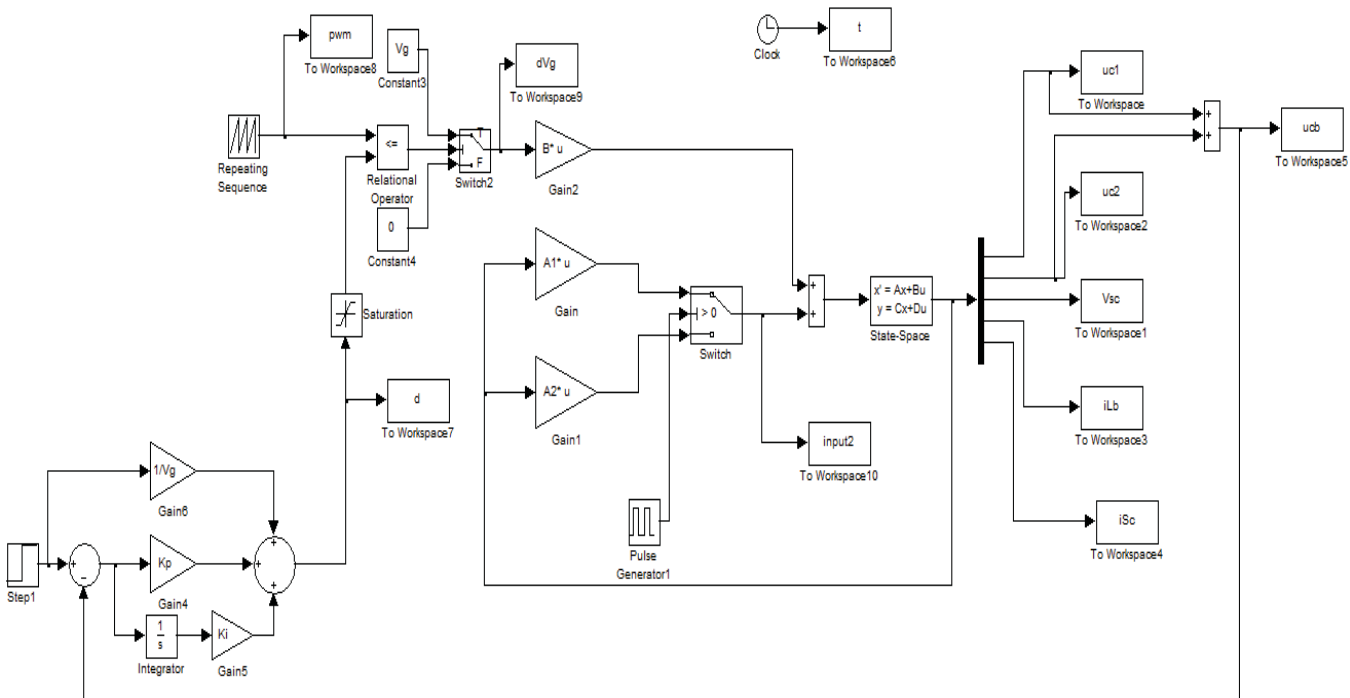


Figure 15: complete simulation model for the measurement circuit with controller

The PID controller Simulink block is utilized to generate the required duty cycle in decimal form based on the error signal. The P and I parameters are entered manually.

### 6.1 Simulation results of Continuous model

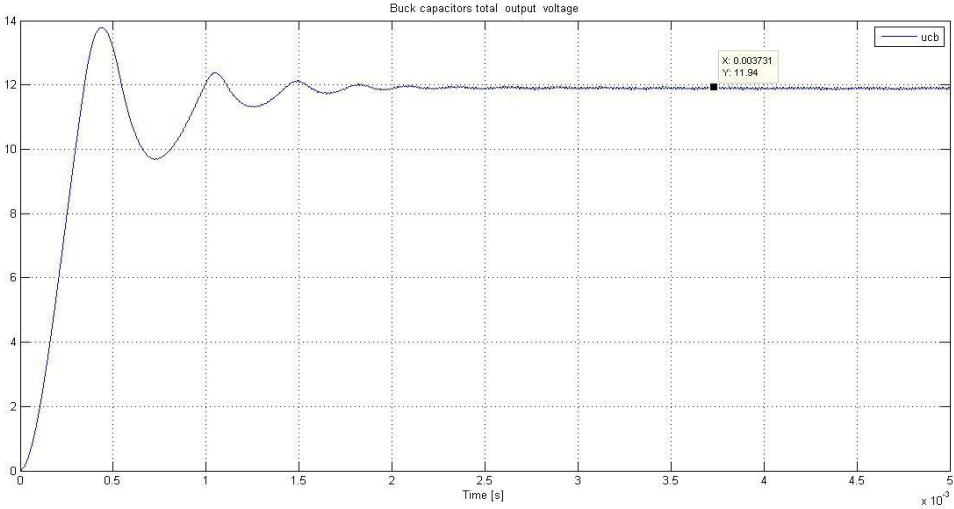


Fig16: Buck convert total capacitor voltage

From figure 16 we can see that as the reference voltage is 12V and the output voltage is 12.94 v which almost the same with the reference voltage. What is more, the output voltage remains stable closer at 12V. It shows that the continuous model with PID controller works well. The controller output can get stable quickly in 3ms) and remain stable.

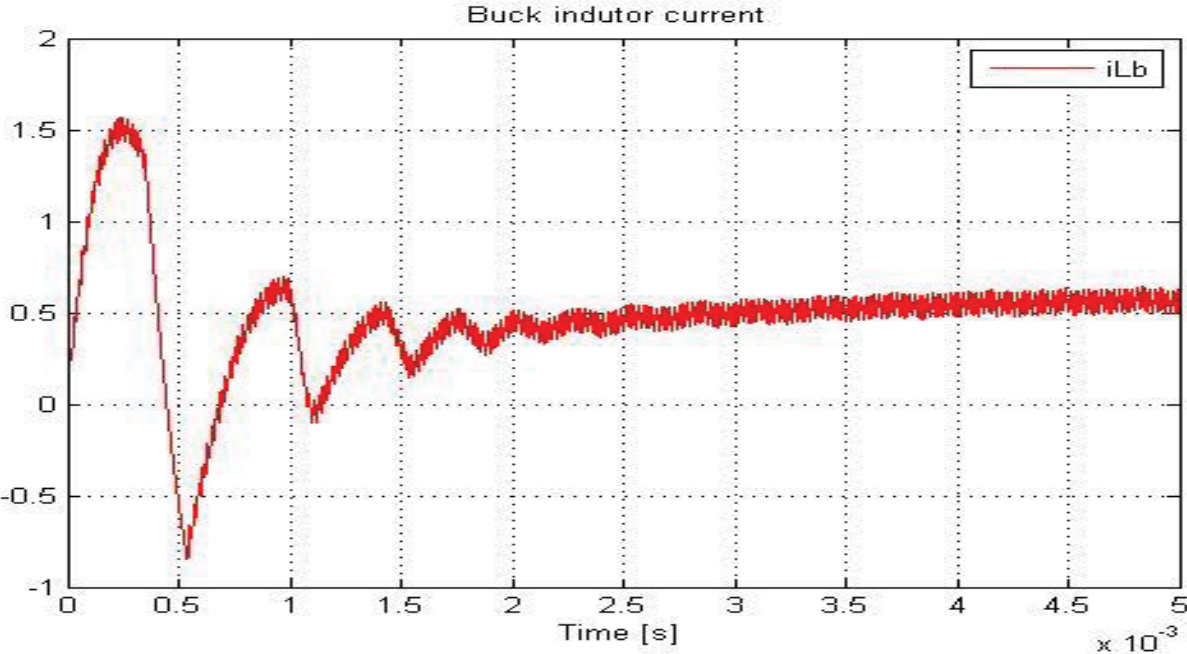


Fig17: Buck convert inductor current

Figure 17 shows the buck converter inductor current. From this figure it can be seen that the Period of the inductor current is 20μs. The graph agrees with the used switching parameters 50 kHz. What is more, the signal reaches below zero at around 0.5ms but doesn't remain there for certain period of time, so we can tell that it is working in continuous current

mode. This problem can be due to inappropriate selection of buck converter switching frequency or inductance value, or both.

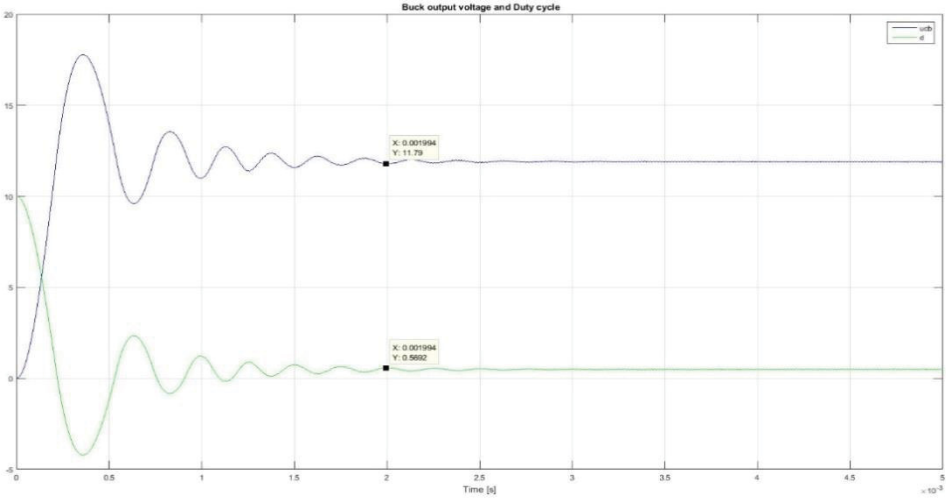


Figure 18: Simulation graph of buck converter output voltage and duty cycle

The simulation result of figure 18 shows that since the reference voltage is 12V, the output voltage can reach it very close. From the graph we can observe that the output voltage remains stable at 11.94V. The result shows that the continuous model of PI Controller works well. The controller attains the output voltage to be stable quickly [in 3 ms] and remain very closer to the desired output voltage [reference voltage 12V].

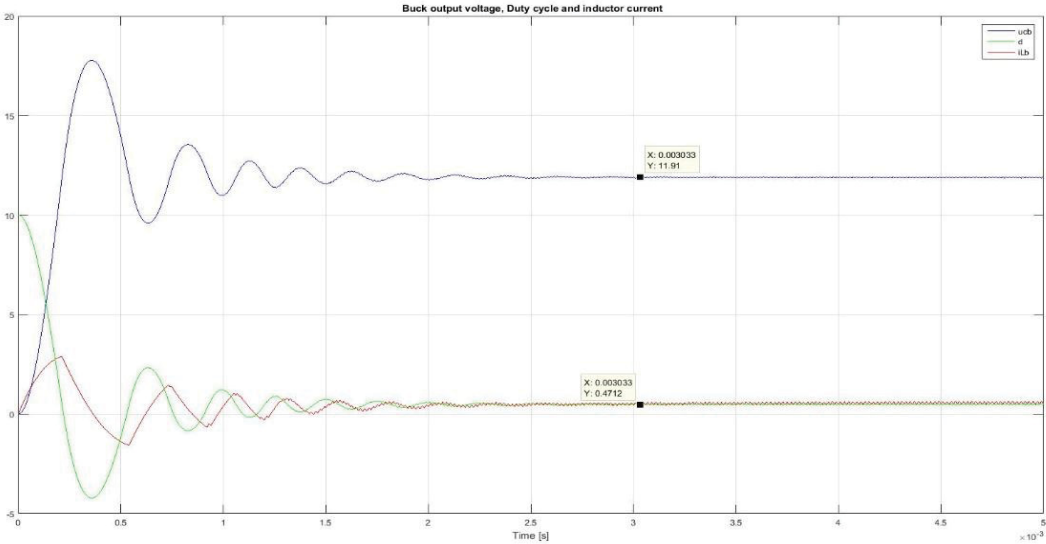


Figure 19: Simulation result of buck converter output voltage, duty cycle and inductor current

Figure 19 shows the red curve is the current over inductance  $L_b$ , while the blue graph is the total voltage over capacitors  $C_1$  and  $C_2$  and the green is the duty cycle. The period of the inductor current is about  $20 \mu\text{s}$ , which corresponds to the switching frequency of  $50 \text{ kHz}$  provided for buck transistor. It can be seen that the graph agrees with the used switching parameters of the buck converter.

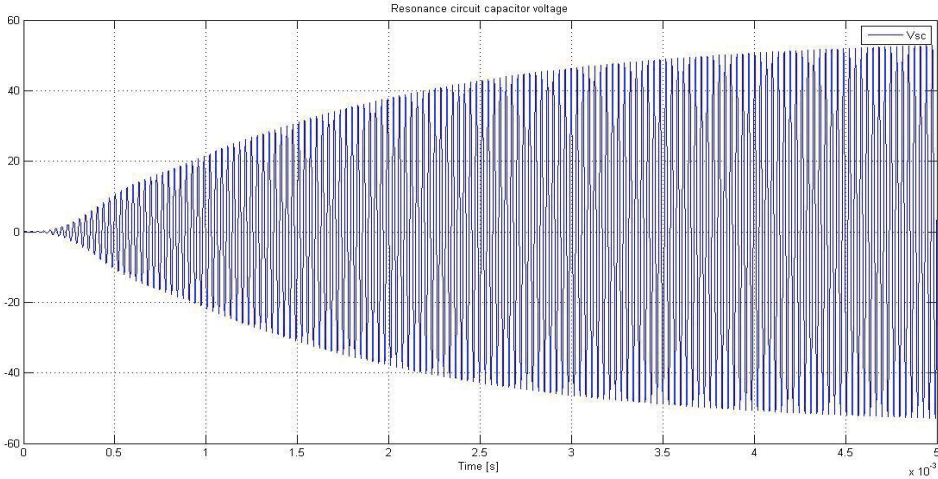


Fig 20: Resonant circuit capacitor voltage without controller

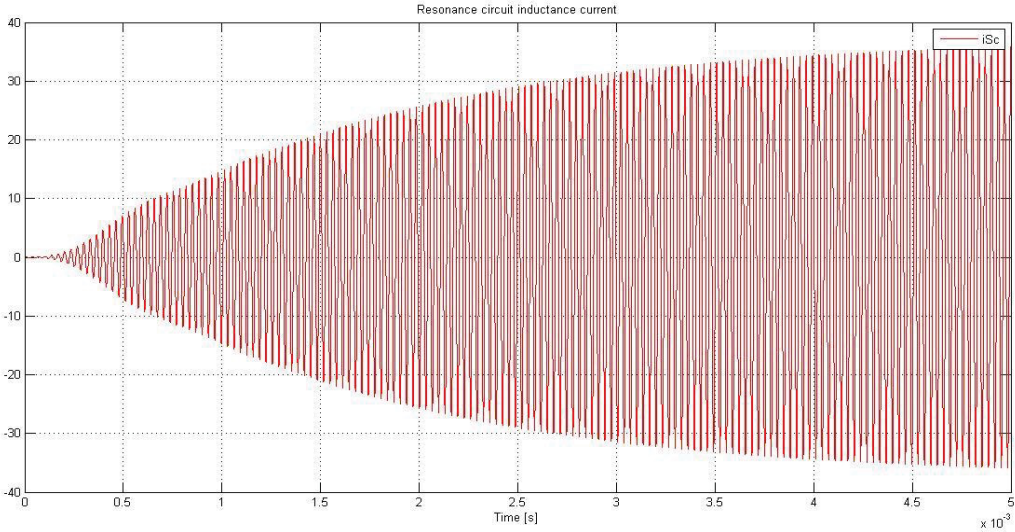


Fig 21: Resonant circuit inductor current without controller

Figure 20 and 21 shows the resonant circuit capacitor voltage and inductor current respectively. The period of both signals is approximately  $32 \mu\text{s}$ , which is equal to a frequency of  $31250 \text{ Hz}$ . This approximation based on the graph is close to the resonant frequency, and confirms the switching frequency of the half-bridge with only a difference of  $400 \text{ Hz}$ .

## 6.2 Resonant circuit current – Control

Figure 22 shows the block diagram for the resonant current controlled by feedback control loop. The resonant current amplitude value in each switching period is fed back to the PID controller.

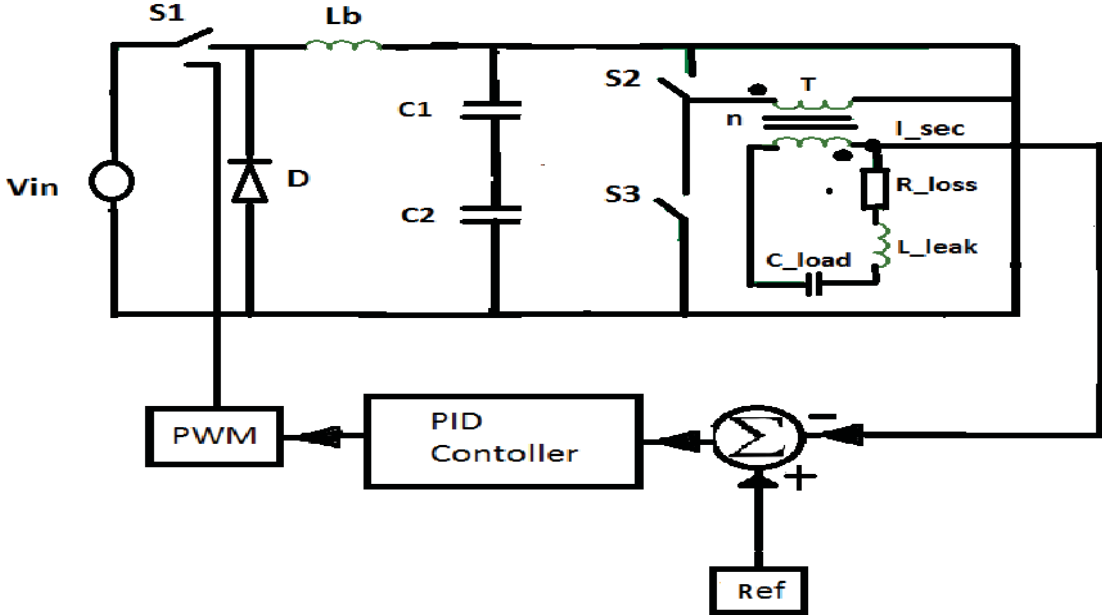


Fig 22: Measurement circuit with a resonant current controller.

### 6.2.1 Resonant current amplitude calculator model

Figure 23 shows the Simulink model which will exactly calculate the resonant current peak amplitude. The resonant output current is inputted to the current amplitude calculator block and is then used as the feedback signal to the buck converter controller.

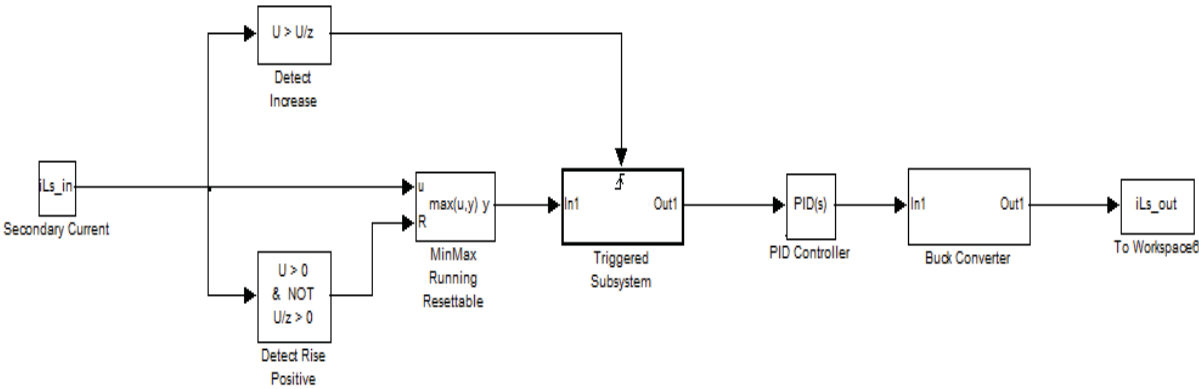


Fig 23: Model used to calculate the resonant current amplitude

### 6.2.2 Simulation results of resonant current control using PI controller

Fig 24 shows the Simulink block model for the measurement circuit with current controller. In this simulation the frequency of the half-bridge is the resonant frequency which is used in matlab simulation. With this simulation parameter the following result is obtained:

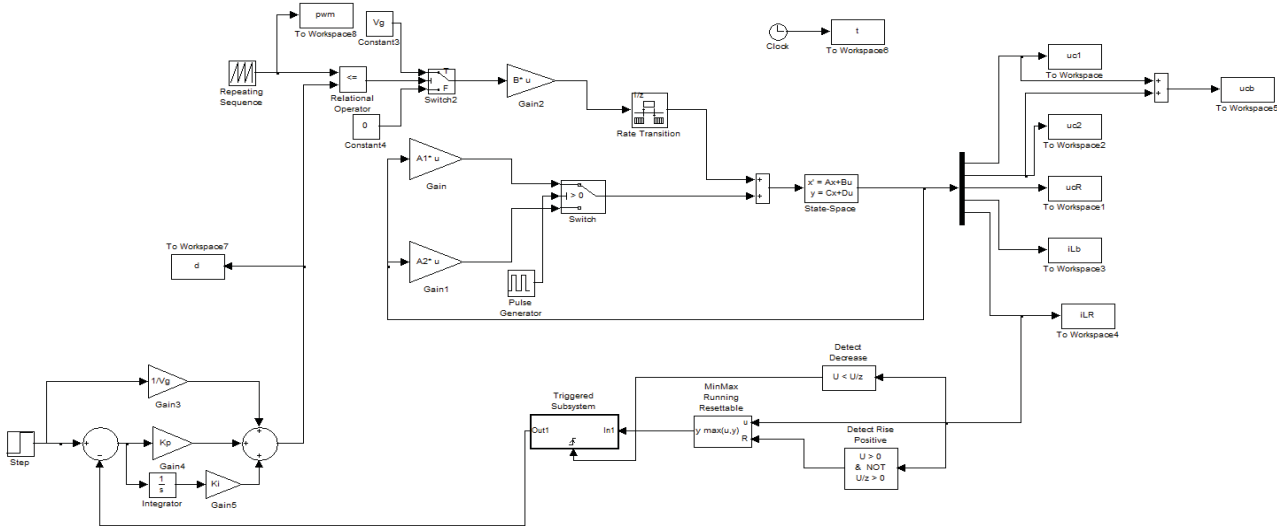


Fig 24: Simulink block model for the measurement circuit with current controller

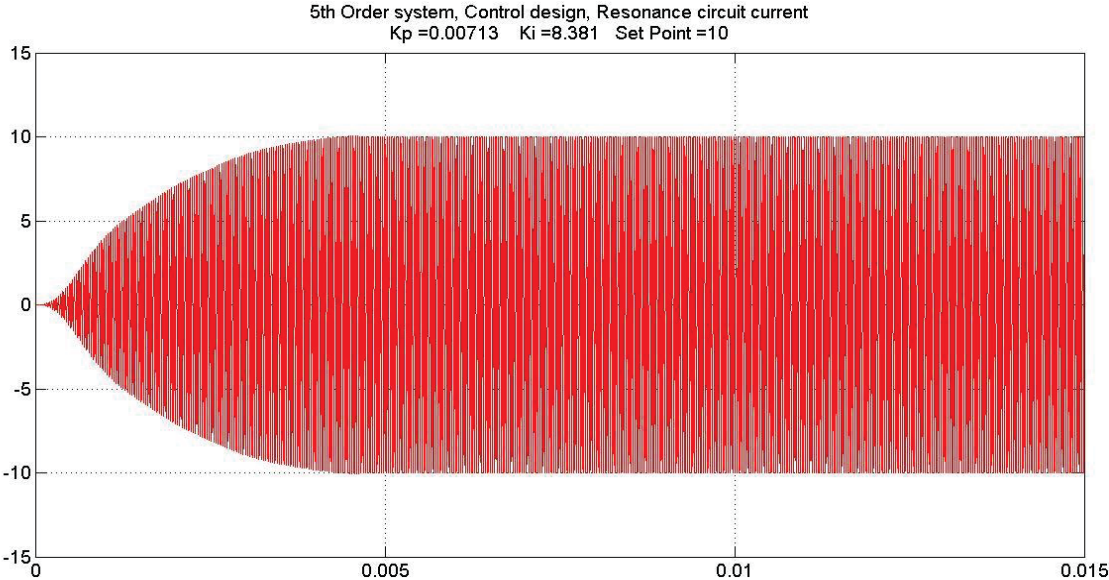


Fig 25: Controlled resonant circuit current

Fig 25 shows the controlled resonant current of the measurement circuit. It can be seen from the graph that the signal has an alternating waveform. The period of the signal is around  $33 \mu\text{s}$ , which is equal to a frequency of 30303 Hz. This result is very close with the used parameter of the resonant frequency. The set point of the resonant current is 10A and the output of the controlled resonant current satisfies the required result. We can observe from the graph that the resonant current remains stable at 4.48ms. The  $K_p$  and  $K_i$  parameter values of the PID controller are adjusted manually to give the expected response of the system. As it can be seen from the above figure it has no oscillations, very small overshoot around 0.3% and no steady-state error. This result shows that the continuous model of the PI Controller works well.

### 6.2.3 Effects of changing the gains of PID parameters

Figure 26 shows the simulation result of the controlled resonant circuit current with  $K_p$  turned 3 times up.

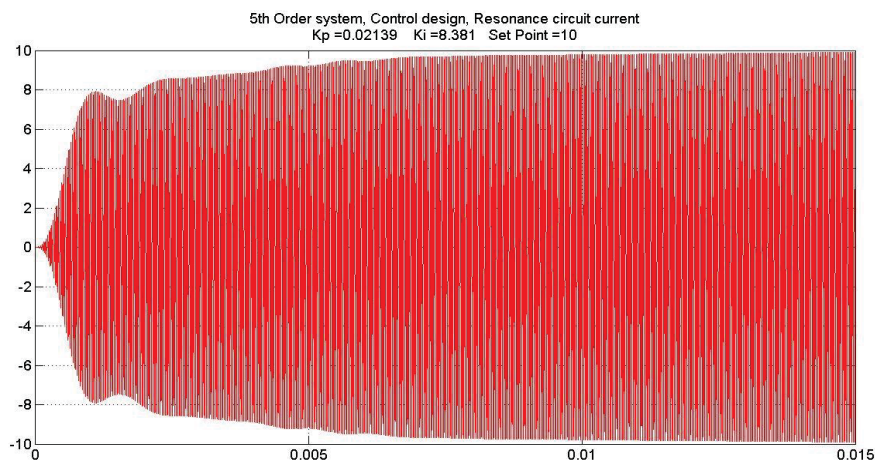


Figure 26: Controlled resonant circuit current with  $K_p$  turned 3 times up.

#### Results:

Overshoot: 1.05%

Undershoot: 25%

Peak: 9.895

Peak Time: 14.98 ms

Oscillation: Increase

Figure 27 below shows the simulation result of the controlled resonant circuit current with  $K_p$  turned 10 times up.

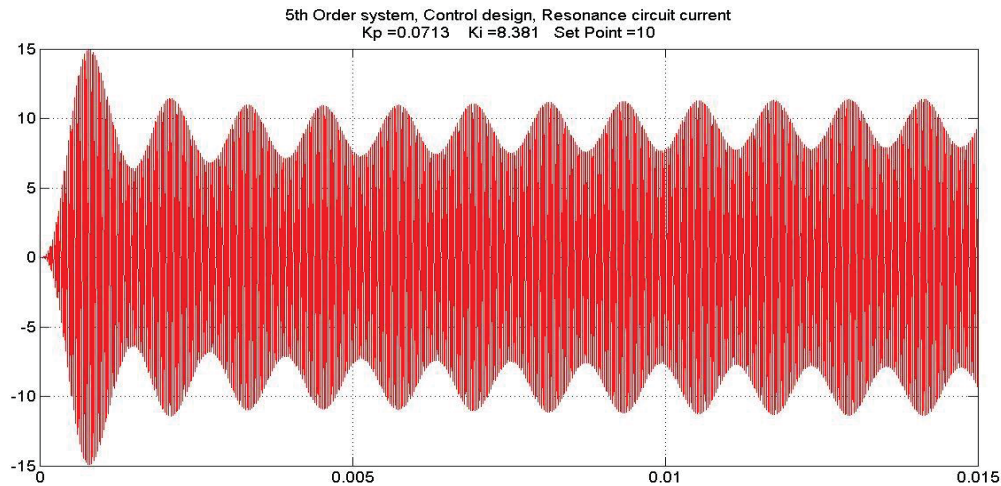


Fig 27. Controlled resonant circuit current with  $K_p$  turned 10 times up.

### Results:

Overshoot: 49.2%

Undershoot: 34.53%

Peak: 14.92

Peak Time: 0.7858 ms

Oscillation: Increase

### Conclusions:

- Increasing  $K_p$  will reduce the steady state error.
- After certain limit increasing  $K_p$  only causes overshoot.
- Increasing  $K_p$  will make the system to oscillate and become unstable.
- The system responds faster for higher  $K_p$  gain.

Figure 28 below shows the simulation graph of the Controlled resonant circuit current with  $K_i$  turned 3 times up.



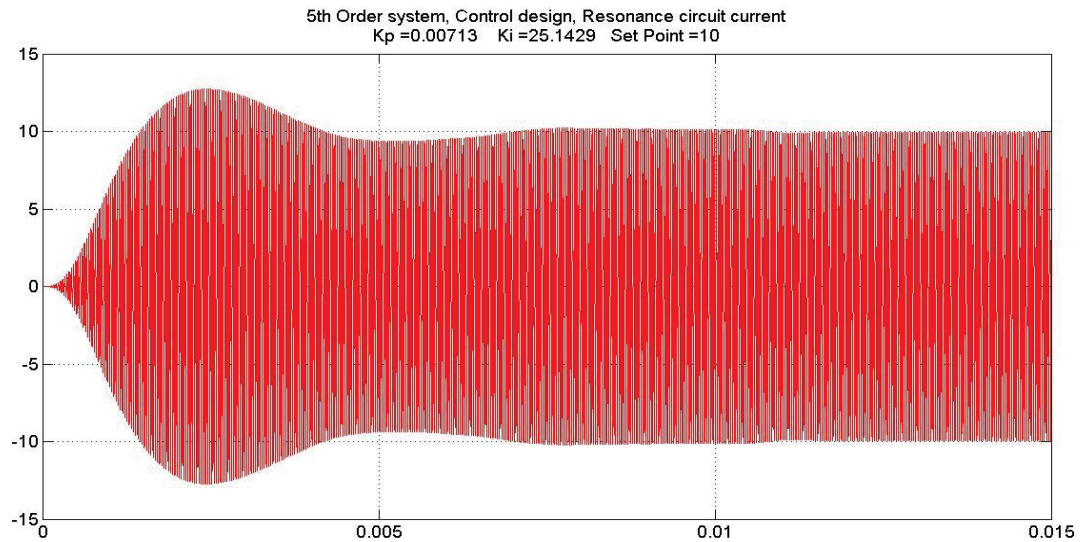


Fig 28. Controlled resonant circuit current with Ki turned 3 times up.

**Results:**

Overshoot: 27.3%

Undershoot: 6.29%

Peak: 12.73

Peak Time: 2.5 ms

Oscillation: Increase

Fig 29 below shows the simulation graph of the Controlled resonant circuit current with Ki turned 10 times up.

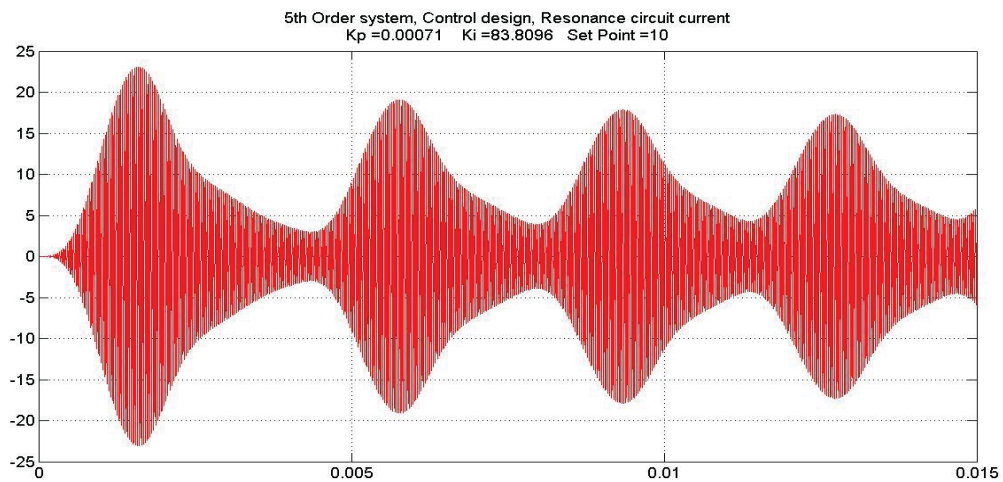


Fig 29. Controlled resonant circuit current with Ki turned 10 times up.

**Results:**

Overshoot: 106.6%

Undershoot: 58.76%

Peak: 20.66

Peak Time: 1.466 ms

Oscillation: Increase

**Conclusions:**

- Integral control eliminates the steady state error.
- After certain limit, increasing  $K_i$  will only increase overshoot.
- The system responds faster for higher  $K_i$  gain.
- Increasing  $k_I$  decreases rise time.

## 7. Discussion

In order to determine the stability of the system we need to analyse the bode plot and frequency response of the open loop and closed loop system of the measurement circuit model. And to represent the system in PID control system we need to use the averaged state-space model of the measurement circuit. If we carefully look in to the two state space models of the measurement circuit in Appendix 2, the equations are non-linear. The problem faced with PID controllers is that they are linear. So combining the two state space models with averaging method to use it with PID controller may not give an accurate result. My thesis partner and I have discussed about this issue and he suggested using the method of switching between the two state space models instead of using the averaging method in order to achieve a better result. Therefore I designed a mat lab Simulink model for the measurement circuit which switches between the two state space models and used it for the matlab simulation.

## 8. Conclusion:

A Simulink model of the measurement system including the buck converter has been designed to generate the output needed for the whole system. To get the required output from the buck and resonant system a reference voltage and current was used. Here the reference voltage and current is the key parameter for checking the performance of the controller and the controller block is implemented by using Matlab and showed through simulation results. The controller is good enough that it can keep both the output voltage and resonant current at the same level with the reference voltage and current respectively. The system responds faster for higher  $k_p$  and  $k_i$  gain, and that it responds with better accuracy for higher gain. Increasing  $K_i$  parameter results in faster response or short rise time. Increasing  $k_i$  decreases rise time, but may increase both overshoot and settling time.

## 9. References

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- [2] Robert W. Erickson and Dragan Maksimovic, Fundamentals of power electronics, 2nd ed., New York: Kluwer Academic Publishers, 2004.
- [3] [http://www.researchgate.net/publication/268425793\\_DC-DC\\_Converters\\_-\\_Dynamic\\_Model\\_Design\\_and\\_Experimental\\_Verification](http://www.researchgate.net/publication/268425793_DC-DC_Converters_-_Dynamic_Model_Design_and_Experimental_Verification) verified on 25-June-2015.
- [4] [https://en.wikipedia.org/wiki/Buck\\_converter](https://en.wikipedia.org/wiki/Buck_converter) verified on 25-June-2015
- [4] [http://www.slideshare.net/manish\\_nitte/voltage-mode-control-of-buck-converter](http://www.slideshare.net/manish_nitte/voltage-mode-control-of-buck-converter) verified on 25-June-2015
- [5] [http://en.wikipedia.org/wiki/DC-to-DC\\_converter](http://en.wikipedia.org/wiki/DC-to-DC_converter) verified on 25-June-2015
- [6] [http://en.wikibooks.org/wiki/Control\\_Systems/Polynomial\\_Design](http://en.wikibooks.org/wiki/Control_Systems/Polynomial_Design) verified on 25-June-2015
- [7] [http://en.wikipedia.org/wiki/Control\\_theory](http://en.wikipedia.org/wiki/Control_theory) verified on 13-July-2015
- [8] [http://en.wikipedia.org/wiki/PID\\_controller](http://en.wikipedia.org/wiki/PID_controller) verified on 13-July-2015
- [9] [http://www.ijirset.com/upload/2014/july/97\\_Modelling.pdf](http://www.ijirset.com/upload/2014/july/97_Modelling.pdf) verified on 25-June-2015
- [10] [http://www.mathworks.com/academia/student\\_version/2011-04-16](http://www.mathworks.com/academia/student_version/2011-04-16) verified on 15-August-2015
- [11] <http://edu.levitas.net/Tutorials/Matlab/Simulink/> verified on 15-August-2015
- [12] [https://en.wikipedia.org/wiki/Root\\_mean\\_square](https://en.wikipedia.org/wiki/Root_mean_square) verified on 2-September-2015
- [13] [https://en.wikipedia.org/wiki/Bode\\_plot](https://en.wikipedia.org/wiki/Bode_plot) verified on 25-June-2015

## 10. Appendix

### 10.1 Appendix 1: Matlab code for the PID Controller

```
% clear everything
close all
clear all
clc
% Back conveter parameters
Lb=1.2e-3;
C1=66e-6;
C2=66e-6;
RL = 5;
Rc1 = 10e-3;
Rc2 = 10e-3;
% input voltage
% input voltage
Vg=30;
%Model
A=[0 0 1/C1
   0 0 1/C2
  -1/Lb -1/Lb -(RL+Rc1+Rc2)/Lb];
B = [0 ;0 ;Vg/Lb ];
%output signals, the three states
C=[1 1 (Rc1+Rc2)];
D=0;
[n,d]=ss2tf(A,B,C,D);
%Transfer function
mySys_tf=tf(n,d);grid

%Open system transferfunction
G=tf(n,d);
figure(1)
bode(G)
figure(2)
margin(G)
figure(3)
step(G)

%PI controller
K=.0001;
TI=1/1000;
PI=tf(K*[TI 1],[TI 0]);
GPI=series(PI,G);
figure(11)
margin(GPI)
MPI=feedback(GPI,1);
figure(12)
step(MPI)

%PD-controller
K=.4;
TD=1/4000;
N=10;
PD=tf(K*[TD 1],[TD/N 1]);
GPD=series(PD,G);
figure(21)
margin(GPD)
MPD=feedback(GPD,1);
figure(22)
```

```
step(MPD)

%PID-controller
K=0.1;
TI=1/5000;
TD=1/4000;
D=tf([TD 1],[TD/N 1]);
GD=series(D,G);
PI=tf(K*[TI 1],[TI 0]);
GPID=series(GD,PI);
figure(31)
margin(GPID)
MPID=feedback(GPID,1);
figure(32)
step(MPID)
```

## 10.2 Appendix 2: Matlab code for the whole measurement circuit

```

clear all
close all

%% Parameters

%System parameters
RTon = 25e-3; %Ohm
Lb = 1.2e-3; %Henry
Rb = 5; %Ohm
R_C1 = 10e-3; %Ohm
R_C2 = 10e-3; %Ohm
C1 = 66e-6; %Farad
C2 = 66e-6; %Farad
turns_ratio = 20; %Turns
btheta = 1 / turns_ratio;
R_loss = 10e-3; %Ohm
L_leak = 7.6e-6; %Henry
C_load = 3.5e-6; %Farad
%Rsh=6.8;%Ohm
% load current parameters
%fs= 30077;
fs= 50000; % buck frequency
%fs1= 30859;
fs1=30850;% Brigge frequency
Vg = 30; % input Voltage
%K=32e-2;
hpwm=1/fs/100;
%hpwm1=1/fs1/100;
%Set point, desired output voltage
yRef=12;
%Sampling interval for the controller
%should be >= 1/fs
h=1/fs;
%% State space model

A1 = [0          0          0          1/C1
      -btheta/C1 0          0          1/C2
        0          0          0          0
        0          0          0          0
      1/C_load   -1/Lb      -1/Lb      0
      (btheta*R_C1)/Lb      0          -1/L_leak      (btheta*R_C1)/L_leak
      (-R_loss - (btheta^2 * (R_C1 + RTon)))/L_leak];

A2= [0          0          0          1/C1
      0          0          0          1/C2
      btheta/C2 0          0          0
        0          0          0          0
      1/C_load   -1/Lb   -1/Lb      0
        0          0          -btheta/L_leak  -1/L_leak
      (-R_loss - (btheta^2 * (R_C2 + RTon)))/L_leak];

```



```

B = [0
     0
     0
     1/Lb
     0];

C = eye(5);
D = zeros(5,1);
%Initial conditions
x0 = [0
      0
      0
      0
      0];

%% Code for controller part
Kp=0.00713;
Ki=8.38096;
SetPoint=10;;

%% Plot

%Simulation
[T,X,Y]=sim('AlstomTrafoLoss5',5e-3);

%% Plot
figure(1)
plot(t,ucb,t,d,t,iLb)
legend('ucb','d','iLb')
title('Controlled Buck converter output voltage ');
xlabel('Time [s]')
grid on

figure(2)
plot(t,iLb,'m')
legend('iLb')
title('Buck converter inductor current');
xlabel('Time [s]')
grid on

figure(3)
plot(t,ucR,'b')
legend('ucR')
title('Resonance circuit capacitor voltage ');
xlabel('Time [s]')
grid on

figure(4)
plot(t,iLR,'r')
legend('iLR')
title('Resonance circuit inductance current');
xlabel('Time [s]')
grid on

figure(5)

```

```
plot(t,pwm,'r')
legend('d')
title('Duty Cycle');
xlabel('Time [s]')
grid on
```

```
figure(6)
plot(t,pout,'b')
legend('pb')
title('pwm');
xlabel('Time [s]')
grid on
```