

*Master Thesis*  
*Electrical Engineering*



# **SPECTRAL CLUSTERING FOR SEGMENTATION OF FINGER PRINT IMAGES**

**Vuppala Ranjith Kumar**  
**Vakkalanka Sri Harsha**

Department of Applied Signal Processing  
Blekinge Institute of Technology  
SE-371 39, Karlskrona  
Sweden

This thesis is submitted to the Department of Applied Signal Processing at Blekinge Institute of Technology in partial fulfilment of the requirements for the degree of Master of Sciences in Electrical Engineering with emphasis on Signal Processing.

**Contact Information**

Authors:

Vuppala Ranjith Kumar

E-mail: ranjithvuppala92@gmail.com

Sri Harsha Vakkalanka

E-mail: harshasree165@gmail.com

**External Advisor:**

Dr. Benny Sällberg

Sällberg Technologies

Email: benny@sallberg.at

**University Advisor:**

Dr. Josef Ström Bartunek

Department of Applied Signal Processing

Blekinge Institute of Technology

E-mail: josef.strombartunek@bth.se

**University Examiner:**

Dr. Sven Johansson

Department of Applied Signal Processing

Blekinge Institute of Technology

E-mail: sven.johansson@bth.se

Department of Applied Signal Processing  
Blekinge Institute of Technology  
SE-371 39, Karlskrona  
Sweden

[www.bth.se/eng/research/applied-signal-processing/](http://www.bth.se/eng/research/applied-signal-processing/)



# Abstract

---

Fingerprint recognition systems are one of the oldest biometric recognition systems, which are under constant development. Presence of noise in fingerprint severely affects performance of recognition systems. Fingerprint segmentation is one of the important part in recognition systems; they eliminate noisy background in the image improving performance of fingerprint recognition systems.

This thesis focuses on proposing a new algorithm for segmentation of fingerprint images using data clustering technique known as spectral clustering. The algorithm is based on block wise segmentation of fingerprint images. The criteria considered for segregating blocks into foreground and background are variance and double gradient of the image. The data in the blocks is clustered using spectral clustering technique.

Performance of the developed algorithm is evaluated using FVC2000, FVC2002, FVC2004 databases. Two sub-databases from each database are selected and relative sizes of the masks are compared to the hand annotated masks of the respective databases. From results, it is observed that double gradient is slightly better than variance in using as parameter for performing fingerprint segmentation.



# Acknowledgements

---

We would like to express our sincere gratitude to our supervisor Dr. Benny Sällberg for introducing us to this topic and for the encouragement and motivation to conduct research in the topic. We would also like to thank our co-supervisor Dr. Josef Ström Bartunek for his useful support and valuable information for progress of our research.

We will forever be grateful to friends and fellow students, who provide moral support during the years of our stay in Sweden.

Finally, we would like to thank our parents for supporting us throughout our studies and encouraging us to travel overseas from India.



# Table of Contents

Abstract.....	4
Acknowledgements.....	6
List of figures.....	10
1. Introduction .....	11
1.1 Motivation.....	12
1.2 Objectives and scope of work.....	12
1.3 Research questions .....	12
1.4 Survey of related works .....	13
1.5 Proposed Solution .....	13
1.6 Outline.....	13
2 Background .....	14
2.1 Fingerprints .....	14
2.1.1 Fingerprint representation.....	14
2.1.2 Fingerprint acquisition .....	15
2.1.3 Fingerprint Recognition systems.....	16
2.2 Fingerprint Image Segmentation .....	18
2.2.1 Types of segmentation.....	18
A. Pixel wise segmentation .....	19
2.3 Spectral Clustering .....	20
2.3.1 $k$ -means clustering.....	20
2.3.2 Spectral clustering.....	22
2.3.3 $k$ -Nearest neighbour similarity Matrix.....	24
3 Fingerprint Image Segmentation Model.....	26
3.1 Pre-processing of the Image .....	26
3.2 Splitting Fingerprint Image .....	28
3.3 Parameter selection .....	29
3.3.1 Using variance as a parameter.....	30
3.3.2 Using Double Image gradient as parameter .....	31
3.4 Spectral clustering.....	34
3.4.1 Algorithm .....	34
3.4.2 Implementation .....	35
3.5 Post-Processing .....	36
3.6 Segmentation of the Image .....	37
4 Implementation of the Model .....	40
4.1 Selection of the suitable block size .....	40

4.1.1	Block size 2 x 2 pixels .....	40
4.1.2	Block size of 8x8 pixels .....	41
4.1.3	Block size of 16 x 16 pixels .....	41
4.2	Number of clusters.....	42
5	Experimental Results .....	43
5.1	Input specifications .....	43
5.2	Performance evaluation.....	43
5.2.1	Proportion of foreground area to entire image area.....	43
5.2.2	Comparison parameters .....	44
5.3	Discussions .....	46
6	Conclusions and Future work.....	48
6.1	Conclusion.....	48
6.2	Future work.....	48
	Appendix .....	49
	Bibliography .....	52

## List of figures

---

Figure 2.1 Different type of ridge flows in fingerprint.....	14
Figure 2.2 Different type of minutiae present in fingerprint image.....	15
Figure 2.3 Block diagram of fingerprint recognition systems .....	17
Figure 2.4 Visualisation of k-means algorithm.....	21
Figure 2.5 Dataset where k-means algorithm yields bad results .....	21
Figure 2.6 Data clustered into two clusters using spectral clustering.....	24
Figure 3.1 original Image.....	26
Figure 3.2 Flow chart of the proposed Fingerprint Image segmentation.....	27
Figure 3.3 Enhanced Image .....	28
Figure 3.4 Image divided into non-overlapping blocks .....	29
Figure 3.5 Variance of each block shown as an image.....	30
Figure 3.6 Gradient of the Image in x-direction .....	32
Figure 3.7 Gradient of the image in y-direction .....	32
Figure 3.8 Gradient of the image in both directions combined. ....	33
Figure 3.9 Double gradient of the image .....	33
Figure 3.10 Mask generated from H .....	36
Figure 3.11 Mask obtained after post-processing removing small blobs .....	37
Figure 3.12 Images showing segmented mask over original image. ....	38
Figure 4.1 Segmented Image and Mask of the Image for block size of $N = 2$ .....	40
Figure 4.2 Segmented Image and Mask of the Image for block size of $N = 8$ .....	41
Figure 4.3 Segmented Image and mask of the image for $N = 16$ .....	41
Figure 5.1 Comparison paramters Illustrated.....	45

# 1. Introduction

---

Humans are by nature social beings. Humans tend to socialise with others more often than any other species in the nature. One of the main strides in socialising is recognising others and identifying others. Humans recognise others by face, voice etc. However, these features cannot be related to person's identity. Identity of the person lies in the features, which are not similar to any other person on the earth. Study of such identifiable features of people is known as biometrics. This has many applications in Security systems. Fingerprints, iris, retina, handprint, gait etc. are the features, which are studied in biometrics.

A fingerprint is a mark made by the pattern of ridges on the pad of human finger. Fingerprints are different for different people and these are used for individual identification, criminal identification etc. Once upon a time fingerprints were extensively used only for forensic analysis where they are detected manually. Now-a-days, fingerprints are detected without manual interference using Automatic fingerprint identification systems (AFIS). Automatic fingerprint identification is a process of automatically matching one or many unknown fingerprints against a database of known and unknown prints. This paved a way for usage of fingerprints in other fields such as personal identification, personal security etc.

Sometimes, AFIS identifies false finger print image, one of the main reasons for the false identification is due to addition of noise to fingerprint images during acquisition. A fingerprint image is a pattern, which consists of two regions namely Foreground and Background. Foreground contains all of the important information needed for AFIS. Background is a noisy region that contributes to extraction of false minutiae in system. To avoid this problem, we perform fingerprint image segmentation, which involves decomposition of image into various components. After fingerprint segmentation, the background region is separated from foreground region.

The common types of segmentation methods are block wise methods and pixel wise methods. In block wise method, fingerprint image is divided into blocks and each block is classified into foreground or background based on features calculated for block. In pixel wise methods, segmentation will be achieved on pixel-value. Segmentation of image can be performed using clusters.

Clusters are nothing but group of data points. Clustering is mostly used for data analysis. Clustering analysis divides the data into groups that are useful for many purposes. At present, task of finding good clusters has been very important in some fields such as machine learning, pattern recognition etc. Clustering is virtually used in every scientific field where the empirical data is dealt.

One of the methods used for data clustering is Spectral clustering. The main goal of spectral clustering is to find group of data points that are similar to each other. This is achieved by making use of Eigen values of similarity matrix of the given data and perform dimensional reduction before clustering in fewer dimensions. Compared to traditional algorithms such as k-means algorithm or single linkage algorithm; Spectral clustering has many fundamental advantages. Spectral clustering can be implemented easily, and can be solved efficiently using standard linear algebra methods.

## **1.1 Motivation**

Generally, Fingerprint image segmentation is performed using supervised learning techniques. This approach requires training of the algorithm before it can be implemented in the real world scenarios. In this way to train the algorithm correctly, we need sample data of various fingerprints of varied intensities and fingerprints acquired from vastly different acquisition systems. This process of training the algorithm with databases can be cumbersome. Therefore, by using unsupervised techniques the burden of training the algorithm can be reduced. Spectral clustering is a good technique, which is used to segment datasets in data engineering. Employing this technique for segmentation of fingerprint images can lead to good results.

## **1.2 Objectives and scope of work**

Main objective of the thesis is to develop a new algorithm for segmentation of fingerprint images. Although there are many algorithms for segmentation of fingerprint images, the goal of the thesis is to develop a novel unsupervised learning technique using spectral clustering. The additional feature of the thesis is to gain knowledge on methods required to increase computational speed of spectral clustering for image segmentation methods.

## **1.3 Research questions**

1. How can spectral clustering used majorly in data mining be implemented in Image segmentation applications?
2. How can developed algorithm be implemented for finger print images?
3. What are the challenges faced in implementation of the developed algorithm on fingerprint images?
4. How can the performance of developed algorithm be assessed?

## 1.4 Survey of related works

Fingerprint segmentation is an unfading research topic with improvements presented to present day. There are so many algorithms proposed in this topic. All categories can be majorly subdivided into pixel-based methods and block based methods. Akram et al. proposes a segmentation algorithm by calculating mean, variance and gradient deviation of the blocks and using linear classifier for segmentation of images [5]. Das et al. put forward a method of segmentation based on morphological operations and calculating block based statistics [6]. R.Suralkar et al. propose a method of fingerprint segmentation based on correlation. They divide image into blocks and subdivide it into sub-blocks and perform correlation on the sub-blocks and uses a threshold level for segmentation [7]. Yang et al. introduce a novel method by segmentation of fingerprint images by unsupervised learning technique using k-means algorithm [8] [9], they perform block wise segmentation by calculating coherence and mean of each block and separating blocks using k-means algorithm.

## 1.5 Proposed Solution

Most of the research in the area of fingerprint segmentation is carried out using supervised learning techniques, which require training of the algorithm. One of the researchers used k-means algorithm, which is an unsupervised learning technique. *k*-means algorithm performs data segmentation only if data is separated on distance metric. In fingerprint analysis, the ridges and valleys are inter-linked. Therefore, distance metric might not be a good indicator for segmenting image. In order to overcome this drawback, we would like to use Spectral clustering technique and perform fingerprint image segmentation.

## 1.6 Outline

The report has been organised into several chapters.

**Chapter 1** introduces the topic of research work.

**Chapter 2** helps in conceptual learning of fingerprints, recognition systems and gain basic knowledge on spectral clustering.

**Chapter 3** deals with the method proposed in the thesis.

**Chapter 4** explains the implementation of the designed algorithm.

**Chapter 5** discusses the results obtained.

**Chapter 6** concludes the research and directs possible research extensions in future.

## 2 Background

---

In this chapter the detailed explanation is given about the fundamental concepts and theory behind the fingerprint segmentation and clustering techniques. The knowledge obtained from this chapter is helpful for understanding of the future chapters discussed in this document.

### 2.1 Fingerprints

Fingerprint is an impression on the fingertips of humans. This mark is present on fingertip in the form of ridges and valleys on the epidermis layer of the skin. This pattern is unique to every individual and is unchangeable. Fingerprints can be used for identification of individuals. Although Fingerprints are unchangeable, they can be destroyed temporarily in case of injuries. In olden days, Fingerprints are acquired using ink. In present world, fingerprints are acquired using sensors.

#### 2.1.1 Fingerprint representation

When fingerprint is considered as an image, ridges are the dark part of the image except for the presence of noise. Valleys are the bright part of the image. Analysing the patterns of the fingerprint can be performed at three different levels namely, Global features, Local features and very fine features [1].

##### A. Global Features

These features show macro details about the shape in which ridges are flown. These features are visible to human eye. Mainly, ridges can be flown in three ways. They are

- Loop, similar to  $\cap$  shape.
- Delta, similar to  $\Delta$  shape.
- Whorl, similar to  $\bigcirc$  shape.



Loop

Whorl

Delta

*Figure 2.1 Different type of ridge flows in fingerprint.*

## B. Local features

These features are mainly used for fingerprint identification. They are highly individual and important feature for fingerprint matching. These features are known as minutiae of the fingerprints. There are about 150 different types of minutiae points based on their configuration [1]. Among all, ridge terminations and ridge bifurcations are the most common minutiae and are employed in fingerprint matching methods. All different type of minutiae can be seen as combinations of these configurations.

Ridge termination is a point where the ridgeline ends abruptly. Ridge bifurcation is a point where the ridge is split into two or more ridges. Some general type of minutiae points is given in Figure 2.2.

## C. Very fine features:

These are very fine details in a fingerprint including width of the ridge at each point, sweat pores on the fingerprint, ridge edge contour. General fingerprint identification systems are not used for recording very fine details. These features are used in high security and sensitive applications.









Minutiae	Example
Ridge Ending	
Bifurcation	
Dot	
Island	
Spur or hook	
Bridge	
Lake	
Independent Ridge	

Figure 2.2 Different type of minutiae present in fingerprint image.

### 2.1.2 Fingerprint acquisition

The process of getting a digitalised image of an individual's fingerprint is known as fingerprint acquisition. Fingerprint of the Individual can be digitised in two ways, they are:

## **A. Offline fingerprint acquisition**

In this technique, all the fingers of individuals are smeared in ink and they are collected on piece of paper to acquire offline print. Later, this paper is scanned by digital scanner to produce a digital finger print image. The standard resolution for scanning is 500 dpi. This is an older technique for collecting fingerprint images. In recent years, this technique is rarely used.

Latent fingerprint acquisition is also a form of offline fingerprint acquisition. This technique is mostly used and popular in forensics. When we touch a surface, the impression of fingerprint is deposited on the surface due to the oily nature of the skin. These fingerprints are photographed and saved or lifted onto a paper using chemical techniques.

## **B. Live-scan fingerprint acquisition**

As the world is being changed, technology also progressed to acquire the fingerprints quickly in seconds and save time, which gave a rise to fingerprint sensor technology. In this type of acquisition, fingerprints are acquired digitally using fingerprint scanners. There are two types of fingerprint scanners plate type sensors and sweep sensors.

In plate sensors, fingerprints are gathered by placing fingertips on flat surface of scanners. The sensor scans fingertips and provides digital image of the fingerprint of the individual. In sweep sensors, the parts of the fingerprint are scanned separately into different images and are stitched afterwards to form a finger print image.

General standard set by FBI for fingerprint acquisition is 500dpi. Some cheap and user-friendly scan devices are not designed for AFIS (Automatic fingerprint Identification System). Therefore, they do not meet specifications of FBI.

Fingerprint pattern and minutiae points of the fingerprint does not change with age. Colour of the skin and race of the individual do not play an important role in the fingerprint image analysis. Therefore, fingerprint images are always acquired on greyscale.

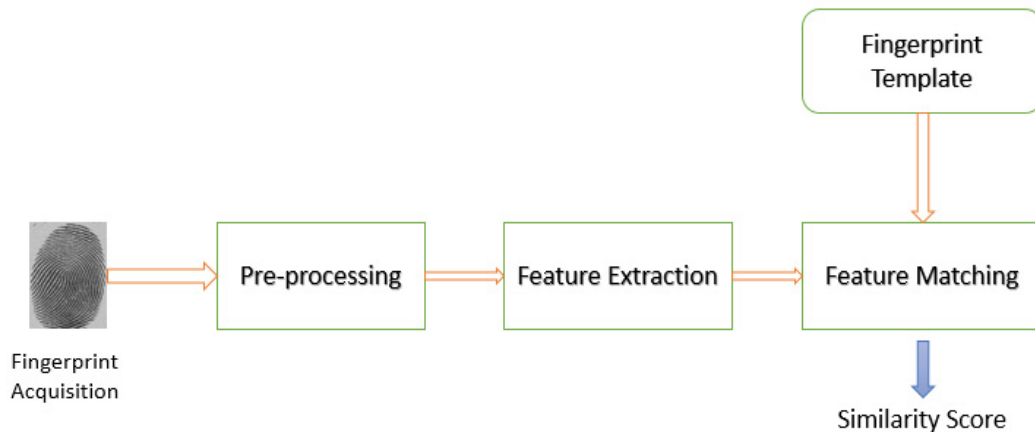
### **2.1.3 Fingerprint Recognition systems**

Fingerprint recognition is a term used for naming set of procedures, where fingerprint images are analysed to identify the individuals. Fingerprint recognition systems offer a reliable means of identification. Fingerprint recognition system can be used either for fingerprint matching or fingerprint verification. In fingerprint identification system, acquired fingerprint is matched against the database of fingerprints for identifying the individual. In fingerprint verification system, acquired fingerprint is matched against the saved copy of verified individual. These are used for verifying identity of the individual.

Different stages of fingerprint systems:

- Fingerprint Acquisition
- Pre-processing
- Feature extraction
- Feature matching

The overview of fingerprint recognition system with all stages is as shown below *Figure 2.3*



*Figure 2.3 Block diagram of fingerprint recognition systems*

### **A. Fingerprint acquisition**

In this step, fingerprints are acquired from the sensors. The acquisition method is discussed in detail earlier in this chapter. The acquired fingerprints undergo further processing in the next stages before they are matched.

### **B. Pre-processing**

The quality of fingerprint acquired by the scanner or the digital image input plays an important role in recognition systems. Fingerprint acquired may be of low quality, it may contain noise because of poor contact with sensor or wet fingers and many other physiological factors [2]. Minutiae extraction might be affected due to low image quality and noise. Extracting features from this poor quality images may lead to extraction of false positives, leading to false similarity scores and detection of false individual. So pre-processing step is an important process in recognition systems.

Pre-processing of fingerprint images consists of two steps, they are:

Enhancement: Enhancement step is performed to improve overall quality of the fingerprint image. This step improves quality of ridge structures in a noisy image. Enhancement is used in case of broken ridges and low contrast images. Some of the regions in the image cannot be enhanced or recovered due to presence of large amount of noise.

Segmentation: Segmentation is a process, which is used to separate noisy background region from foreground region, which contains fingerprint. Segmentation is very important step as it removes background-avoiding detection of any false minutiae. At the same time, segmentation should be performed in a way that the true detectable minutiae must not be segmented as background. There are various types of segmentation techniques, which are discussed in detail in the next section.

### **C. Feature extraction**

In this stage, features of the fingerprint i.e., minutiae points are extracted from the fingerprint. Minutiae extraction is generally done using thinning algorithms, where the ridges are thinned and minutiae are extracted. This is beyond the scope of the thesis.

### **D. Feature matching**

In this stage, features of the extracted fingerprint are matched with features of the fingerprints stored in the database. A similarity score is generated between the fingerprints and if the similarity score is above acceptable limit fingerprint is identified. This is also beyond the scope of the thesis.

## **2.2 Fingerprint Image Segmentation**

Separation of fingerprint foreground area from noisy background area is known as fingerprint segmentation. This step is a pre-processing step before extracting the minutiae of the fingerprint images. This step helps in removing false minutiae from the image. The main objective of segmentation is to remove all false minutiae and not remove any true ridges of fingerprint data.

### **2.2.1 Types of segmentation**

Segmentation of fingerprint images can be broadly classified into two categories

- i. Pixel wise segmentation
- ii. Block wise segmentation

### **A. Pixel wise segmentation:**

In this approach, features of each and every pixel are considered separately and pixel is classified as foreground or background. It is computationally intensive procedure because features of each and every pixel have to be calculated. The results achieved using pixel wise segmentation are usually better than block-wise segmentation methods for general images.

### **B. Block wise segmentation:**

In this approach, the image is divided into non-overlapping blocks. Based on the statistical characteristics of the block, it is classified as foreground block or background block. This approach is not as accurate as pixel wise approach but it is computationally faster.

## 2.3 Spectral Clustering

Clustering is a process of grouping set of objects in a manner in which the characteristics of objects in same group are similar to each other and differ widely from the characteristics of objects from other group. Clustering is majorly used in exploratory data analysis; it has applications virtually in any scientific field dealing with empirical data. There are various techniques for clustering the data. One of the most basic method for clustering data is  $k$ -means algorithm.

### 2.3.1 $k$ -means clustering

In Conventional Clustering analysis, there are various clustering algorithms. Each of the algorithms have their own advantages and disadvantages. Most commonly used technique is  $k$ -means algorithm.

Even though the concept of  $k$ -means algorithm is simple, it is one of the fastest algorithms in data clustering. It is one of the biggest advantages behind deploying  $k$ -means algorithm. On the other hand, the algorithm is limited in its performance and often does not result in good clustering.

$k$ -means refines the data in an iterative manner, that is the result is refined after each and every iteration.  $k$ -means algorithm is initiated by selecting number of clusters ' $k$ ' into which the data need to be separated. The algorithm can be visualised in *Figure 2.4*

Once number of clusters is chosen, the algorithm is as follows:

1. Randomly choose  $k$  centroids in the data.
2. Each data point is assigned to cluster with closest centroid.
3. Centroids for each cluster are recalculated.
4. Go to step2 until the algorithm meets exit condition.

Usually, exit condition for algorithm can be set as aborting the algorithm when new centroids do not differ from old centroids. The exit condition can also be set as maximum number of iterations the algorithm needs to be performed. There are many variations of this algorithm; where in the initial conditions are changed for obtaining better results.

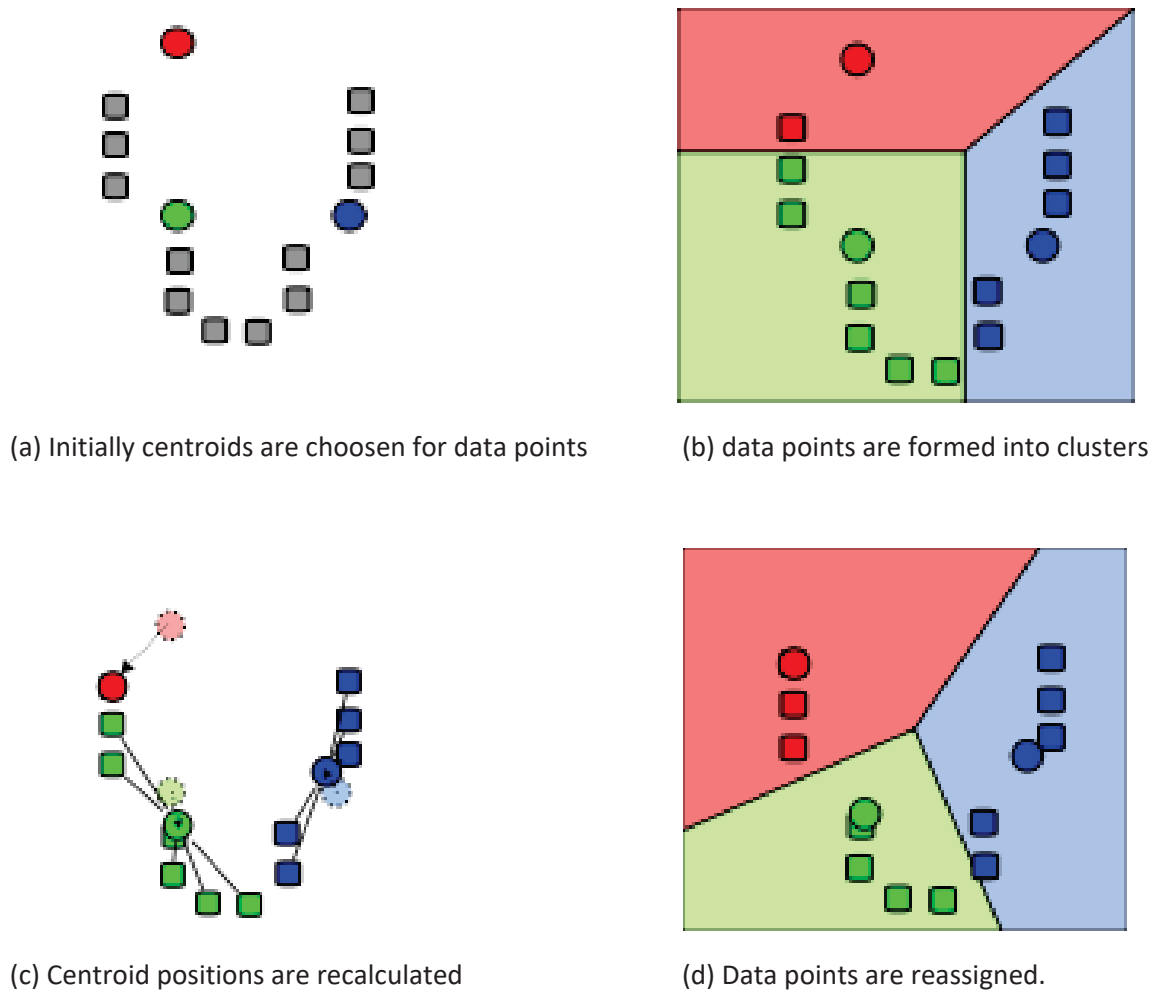


Figure 2.4 Visualisation of *k*-means algorithm

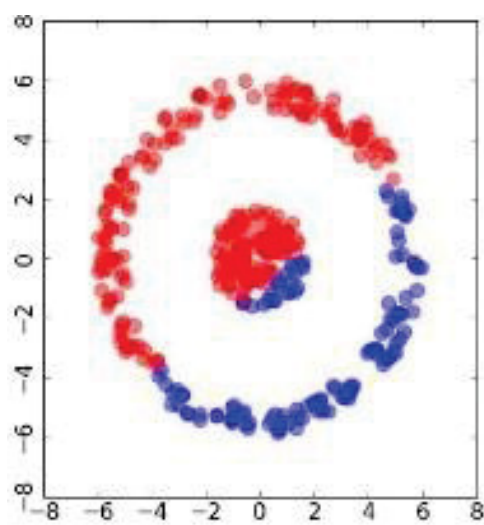


Figure 2.5 Dataset where *k*-means algorithm yields bad results

Since  $k$ -means algorithm works on separating data points from centroids, it does not work in some cases. When the similarity in the data is visible but data is not equidistant to centroid this algorithm fails. Therefore,  $k$ -means algorithm is used in the cases where the data is not connected but compact.

If the data is connected, this method is generally not preferred as shown in the *Figure 2.5*. In this figure, the data is spread in two concentric circles. Similarity of the data is to separate data into two concentric circles but employing  $k$ -means algorithm yields bad results.

### 2.3.2 Spectral clustering

Spectral clustering works on dimensionality reduction of the data, i.e., the number of dimensions in which data is spread is reduced by analysing spectral content (Eigen Vectors) of ‘similarity’ matrix of the data. To cluster the data into ‘ $k$ ’ clusters it is redistributed over ‘ $k$ ’ Eigen vectors of similarity matrix and a classic clustering algorithm is performed.

The algorithm for spectral clustering is as follows

- Let the set  $S$  represents the data of ‘ $n$ ’ points to be clustered. Given in eq (2.1)

$$S = \{x_i : i \in [1, n]\} . \quad (2.1)$$

- Calculate Similarity matrix ( $W$ ). It consists of similarity measures between data points. If number of data points is ‘ $n$ ’, the size of similarity matrix is ‘ $n \times n$ ’. The elements of similarity matrix are given by eq (2.2)

$$W = [S_{ij} : i \in [1, n], j \in [1, n] ] . \quad (2.2)$$

Where,  $S_{ij}$  is similarity between points  $x_i, x_j$  given in eq (2.3)

$$S_{ij} = \exp\left(\frac{-d(x_i, x_j)^2}{2\sigma^2}\right). \quad (2.3)$$

Here,  $d(x_i, x_j)$  is distance between points  $x_i, x_j$  and  $\sigma$  is parameter controlling size of neighbourhoods. After similarity matrix is calculated, we proceed to calculate degree matrix of the similarity matrix.

- Degree matrix (D) is a diagonal matrix with elements on the diagonal of each row given as  $d_i$  is in eq (2.4)

$$d_i = \sum_{j=1}^n S_{ij}. \quad (2.4)$$

- Compute the normalized Laplacian matrix 'L'. There are many variants of L. But within scope of image segmentation we calculate using eq (2.5)

$$L = D^{-\frac{1}{2}} (D - W) D^{-\frac{1}{2}}. \quad (2.5)$$

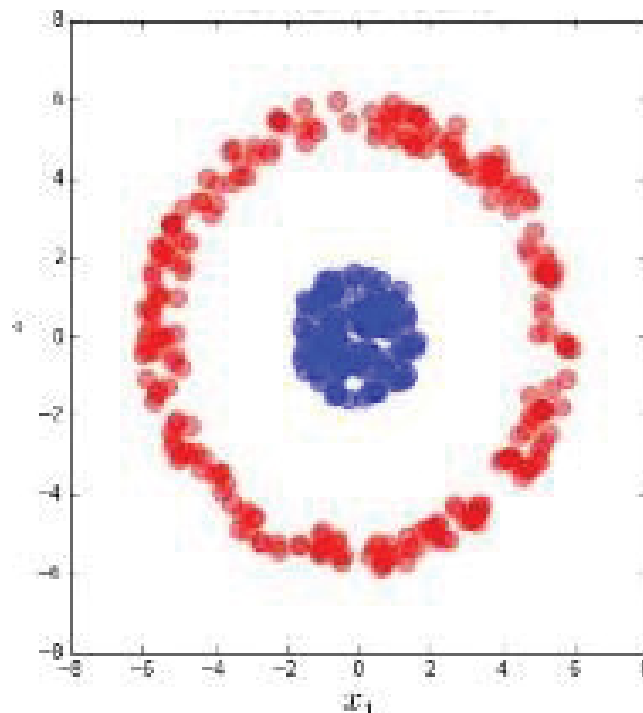
- To cluster the data into 'p' clusters, calculate 'p' smallest Eigen vectors of L.
- Using these 'p' smallest Eigen vectors cluster the data points into 'p' clusters using k-means algorithm.
- Thus, data points are divided into clusters using Spectral-clustering technique.

This variant of spectral clustering technique is proposed by Shi and Malik [3], It is especially useful in Image segmentation techniques.

The number of computations required for calculation of Eigen vectors is in the order of  $O(n^3)$  [4]. Therefore, for an image of size M x N number of data points is MN. Similarity matrix is of dimension MN x MN which implies number of computations required for clustering this data using spectral clustering algorithm is in the order of  $O((MN)^3)$ . For an image of size 500 x 500 pixels, dimension of similarity matrix is 250000 x 250000.

This requires huge amount of memory and performing operations on this matrix is computationally exhaustive. Therefore, memory efficiency of the processor can be increased by making similarity matrix (W) sparse. This goal can be achieved by considering k-nearest neighbours similarity matrix.

Spectral clustering clusters data, based on the similarity rather than on the distance from the data as shown in the *Figure 2.6*



*Figure 2.6 Data clustered into two clusters using spectral clustering*

### 2.3.3 k-Nearest neighbour similarity Matrix

Here similarities between data points is considered only if the data point  $x_j$  is one of the  $k$  nearest neighbours of  $x_i$ , else similarity between these points is considered to be 0. In similarity matrix, the similarity  $S_{i,j}$  is calculated only for the  $k$ -nearest neighbours of  $x_i$  as given in eq (2.6)

$$S_{ij} = \begin{cases} \exp\left(\frac{-d(x_i, x_j)^2}{2\sigma^2}\right) & \text{if } x_i \text{ is } k\text{-nearest neighbour of } x_j, \\ 0 & \text{Otherwise .} \end{cases} \quad (2.6)$$

By using above representation, we can make similarity matrix sparse. In MATLAB, using sparse functions the computational speed can be increased by achieving memory efficiency.

Computational speed of the system is in order of  $O(n^3)$ , where  $n$  is number of data points to be clustered. Therefore, Computational speed can be further increased by decreasing number of data points for calculating similarity matrix.

The size of similarity matrix can be reduced by representing entire dataset of ' $n$ ' data points with ' $l$ ' data points. All the data points which are very close to each other are represented by a unique point, whereby reducing size of the dataset. This algorithm is discussed in detail here [4].

Number of computations are reduced to  $O(l^3) + O(lnt)$  where ' $t$ ' is number of iterations for performing  $k$ -means algorithm.  $O(lnt)$  is operations required to map clustered data of  $l$  points on to dataset of  $n$  points.

## 3 Fingerprint Image Segmentation Model

---

This chapter discusses about the method implemented for segmentation of fingerprint images. Generally, fingerprint segmentation is performed using pixel wise methods or block wise methods. In this thesis, block wise segmentation approach is used. There are two main motives in selecting this approach. Spectral clustering is a computationally intensive technique. So using block wise segmentation, the data to be clustered reduces by  $N^2$ , where  $N$  is length of the block. This reduces number of computations to be performed.

In a fingerprint image, the intensities of pixels change rapidly in fore ground area from ridges to valleys. Therefore, in pixel wise approach similarity matrix in spectral clustering should also account for position of the pixel in the image. This also increases number of computations required. In block wise approach, even though intensity of pixels change rapidly the features considered as a block are uniform in foreground area. Therefore, position of block can be ignored for clustering of data reducing number of computations required. Thus, block wise segmentation is preferred over pixel wise segmentation.

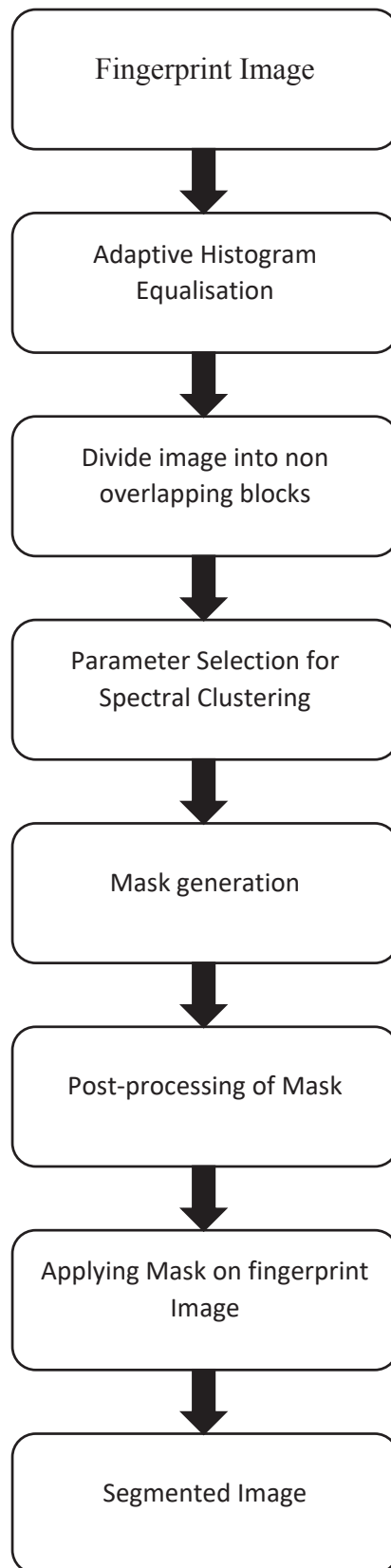
### 3.1 Pre-processing of the Image

Generally, fingerprints acquired vary differently depending on type of the scanner used for acquisition. The noise during acquisition from different sources are vastly different. This affects the final fingerprint acquired which may have high intensity areas and low intensity areas. In order for the method to be implemented correctly, the initial fingerprint image is pre-processed before implementing the segmentation method. General, fingerprint Image given in *Figure 3.1*



*Figure 3.1 original Image*

All the steps involved in Fingerprint Image are shown in flow chart in *fig 3.2*



*Figure 3.2 Flow chart of the proposed Fingerprint Image segmentation*

The pre-processing step known as adaptive histogram equalisation is performed. This algorithm is slightly different from Histogram equalisation method. In this method instead of computing histogram for entire image, histograms for distinct sections of the image are computed and these values are used to redistribute the intensity values in the image. This method improves local contrast and enhancing edges in each region of the image. However, in relatively homogenous image this method tends to over amplify the intensity values of noise resulting in noisier image. To overcome this effect, a variant of this algorithm known as Contrast Limited Adaptive Histogram Equalisation (CLAHE) is considered. It prevents over-amplification of noise by limiting the amplification of intensities. Thus, this method is employed for pre-processing of the fingerprint image. This ensures uniform contrast throughout the fingerprint image. The enhanced figure is given in *Figure 3.3*

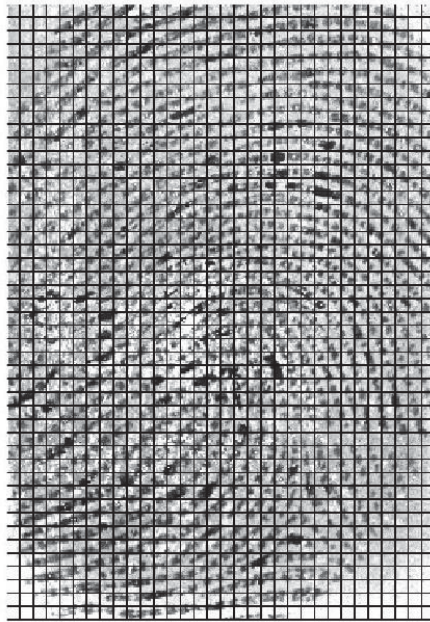


*Figure 3.3 Enhanced Image*

## **3.2 Splitting Fingerprint Image**

This step is important in the process of fingerprint image segmentation. This step involves splitting of image into blocks. The size of dataset for spectral clustering is determined in this step. Fingerprint image is split into non-overlapping blocks of height  $N$  pixels and width  $N$  pixels. Each block is considered as a data point used for generating data in clustering. Assuming the size of the image to be  $X \times Y$  pixels, without dividing the image into blocks the size of the dataset would have been  $X \times Y$ . By splitting into blocks of size  $N \times N$  pixels. The size of the data set is reduced to  $(X \times Y)/N^2$ . Thus this step is important in reducing computational complexity of the method.

If value of 'N' is small, the size of the dataset increases thereby increasing the size of the dataset. Value of N is inversely proportional to the computational complexity. If value of 'N' increases, block size increases. As block size increases, number of pixels in each block increases which result in pixels around the boundary area to be assigned incorrectly to the foreground or background. If value of 'N' increases proportion of the area of the fingerprint image segmented correctly reduces. Therefore, there is trade-off between computational complexity and proportion of area of fingerprint correctly segmented. Fingerprint blocks are split as shown in *Figure 3.4*



*Figure 3.4 Image divided into non-overlapping blocks*

### **3.3 Parameter selection**

This step involves in selection of an appropriate parameter for the block to perform spectral clustering. Each block divided in the previous step is assigned to either foreground or background by using spectral clustering. In order to apply spectral clustering, we have to have a good parameter for the block. The ideal parameter should have the following properties:

- i. The value of the parameter for the block should be closer to the value of other blocks if they both belong to same part of the image i.e., either foreground or background.
- ii. The value of the parameter should be vastly different from value of other blocks corresponding to different part of the image. i.e., if the block belongs to foreground then its value should be different from the block in the background.

The parameters satisfying the above mentioned conditions yields very good results in spectral clustering technique. There are many parameters of the block such as average intensities of the

pixels, variance of the pixels, gradient image, histogram etc., Even though, there are many parameters that can be attributed to the block, there are many less parameters that satisfy the ideal properties mentioned. In this thesis, variance and double gradient of the image are considered as parameters. They are discussed in detail here:

### 3.3.1 Using variance as a parameter

A fingerprint image can be separated into foreground area containing fingerprint and background area containing noise. The foreground area has ‘ridges’ of low grey value and ‘valleys’ of high grey value. Therefore, the intensity of the pixels in foreground area varies significantly. The background area is mostly plain or contains some noise. Therefore, the intensity of pixels in background area is almost similar. This variation in intensities can be used as a predicate for separation of fingerprint image into foreground and background [10]. Variance of each block is computed and is shown as greyscale image in *Figure 3.5*

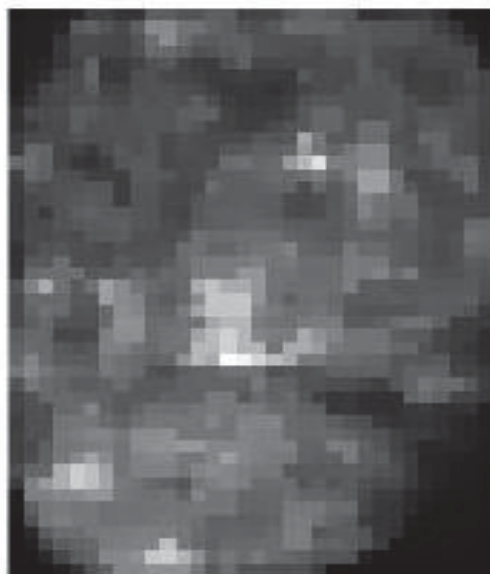
Variance of intensities of the pixels in block is calculated using eq (3.1).

$$Variance = \frac{\sum_{i=1}^N \sum_{j=1}^N (I(i,j) - m)^2}{N \times N} . \quad (3.1)$$

Where,  $I(i,j)$  is intensity of the corresponding pixel in the block,

$m$  is mean of intensities of pixels in the block given by equation (3.2)

$$m = \frac{\sum_{i=1}^N \sum_{j=1}^N I(i,j)}{N \times N} . \quad (3.2)$$



*Figure 3.5 Variance of each block shown as an image.*

### 3.3.2 Using Double Image gradient as parameter

Image gradient is a directional change in the intensity of the image. Magnitude of gradient at a point in the image is directly proportional to change in the intensities of pixel values at that point. Therefore, Image gradients are used to detect edges in the image.

Mathematically, Gradient vector is a derivative of the function which points in the direction of the largest possible change of magnitude of the function and length of vector corresponds to rate of change of magnitude [11].

Digital images consist intensities only at discrete levels; derivate of the image cannot be calculated directly. There are many approximations for calculating derivatives, which assume the pixel values as sample values of a continuous function. One of the common approximations for calculating gradients is Sobel operator.

Sobel operator uses two 3x3 matrices to obtain gradients in X-direction and Y-direction. The matrix 'S<sub>x</sub>' given in eq (3.3) is used to calculate gradient in X-direction ( $G_x$ ), similarly matrix 'S<sub>y</sub>' given in eq (3.4) is used to calculate gradient in Y-direction ( $G_y$ ). Gradients in both directions are calculated by convolving respective matrices with the Image. The magnitude of the gradient ( $G$ ) is obtained from  $G_x$  and  $G_y$ , it is given in eq (3.7)

$$S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad (3.3)$$

$$S_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}. \quad (3.4)$$

If fingerprint image is represented by  $I(i,j)$

$$G_x = S_x * I, \quad (3.5)$$

$$G_y = S_y * I, \quad (3.6)$$

$$G = \sqrt{G_x^2 + G_y^2}. \quad (3.7)$$



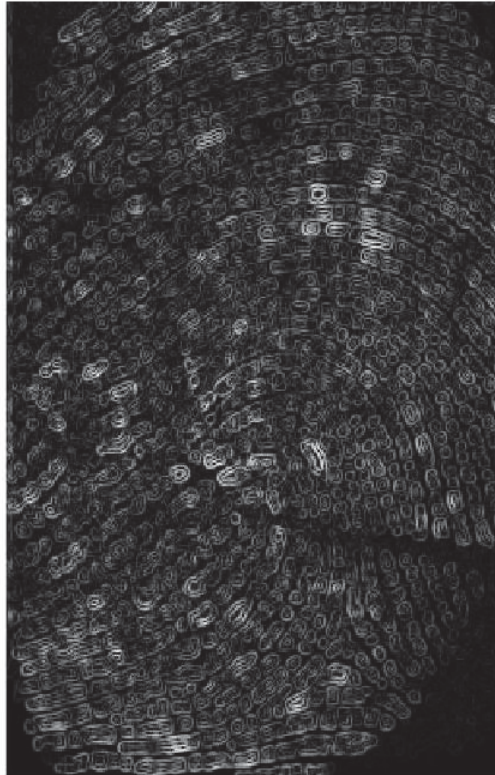
*Figure 3.6 Gradient of the Image in x-direction*



*Figure 3.7 Gradient of the image in y-direction*



*Figure 3.8 Gradient of the image in both directions combined.*



*Figure 3.9 Double gradient of the image*

In fingerprint image, foreground area contains ridges and valleys where change in pixel intensities is high. Therefore, there are so many edges in foreground area, which implies the magnitude of gradient is high in foreground area. In the background area, the change in intensities is not that rapid. Therefore, the magnitude of gradient image is small in this area. Gradient in both x-directions and y-directions and final gradient of the image is shown in *Figure 3.6*, *Figure 3.7* and *Figure 3.8*.

It is observed that by considering gradient ( $G'$ ) of this gradient image ( $G$ ), the magnitude of foreground area is much higher than magnitude in background area. Double gradient of the image is as shown in the *Figure 3.9*. Therefore, magnitude of double gradient of an image is considered as a predicate for separation of background and foreground areas.

## 3.4 Spectral clustering

The data generated from the parameters considered in the previous step is stored in matrix  $H$ . This matrix  $H$  is used as the input for the spectral clustering technique. The properties and general implementation of spectral clustering is studied in earlier chapters. Here the algorithm for spectral clustering used in this thesis and its implementation is discussed.

### 3.4.1 Algorithm

Within scope of the thesis, spectral clustering algorithm used to perform on matrix  $X$  to divide into 'p' clusters is as follows:

- Matrix  $X$  data of size  $M \times N$ , is converted into a vector  $V$  of size  $1 \times MN$ .
- The unique data values in the vector  $V$  is taken in vector  $U$ .  $U$  is taken as representative of entire dataset in  $V$ . This step is performed to reduce the dataset which reduces computational complexity. It is assumed that values of the predicates considered differ vastly from foreground to background, therefore if there are data values with same value then it is assumed that they belong to same part of the image. This helps in reducing the data size further. pixel intensities are represented using 8bits in Uint8 format, i.e there exists only 256 unique intensities. Spectral clustering is not performed on pixel values because intensity of the pixel is similar in both foreground and background of the image. This is another reason to avoid pixel wise segmentation in this thesis.
- $k$ -nearest neighbours similarity matrix  $W$  is calculated for the vector  $U$ .
- Calculate degree matrix ( $D$ ) for similarity matrix  $W$ .
- Calculate graph Laplacian matrix  $L$ .

- Calculate ' $p$ ' smallest Eigen vectors of  $L$  and perform  $k$ -means clustering to divide into ' $p$ ' clusters.
- Cluster entire data into ' $p$ ' clusters using clustered data of  $U$ .
- Each data point is given cluster number from 1 to  $p$ , depending on the cluster it belongs.
- The vector is converted back to matrix  $Y$  of size  $M \times N$ .

The Euclidian difference between the values is taken as metric for the measure of similarity is taken as values in the matrix  $X$ . Although position of data in matrix is also a good measure along with value, it requires two values for each data point vastly increasing the size of the dataset. In order to avoid complexity of the algorithm and reduce computational capacity, position of data in the matrix  $X$  is not taken into consideration as metric in this thesis.

### 3.4.2 Implementation

Spectral clustering as described above is performed on  $H$ . The the data in  $H$  is divided into two clusters. The output obtained from clustering of data is stored in matrix  $S$ . The dimensions of  $S$  are same as that of  $H$ . The output of  $S$  contains an element from  $\{0, 1\}$ . Elements of  $S$  determine nature of elements in  $H$  as shown in the eq (3.8)

$$S(i,j) = \begin{cases} 1 & \rightarrow H(i,j) \in \text{Foreground}, \\ 0 & \rightarrow H(i,j) \in \text{Background}. \end{cases} \quad (3.8)$$

If an element of  $H$  belongs to foreground, then corresponding block of  $N \times N$  pixels in the image also belongs to foreground region and vice versa. Therefore, a mask ( $M$ ) is generated for the image to differentiate foreground from the background.

The mask is generated in a simple way, a matrix ( $M$ ) of same size as of image ( $I$ ) is selected and initialised with zeros. If  $H(i,j)$  belongs to foreground then all elements in corresponding block of  $N \times N$  pixels is assigned to value 1. If the element in  $H$  belongs to background, then all elements in the block are assigned value 0. Thus, the generated mask is scaled version of  $H$  increased by  $N \times N$  to make the size of the mask same as the size of the image. The scaled mask image consists of 1's and 0's and is same as the size of the image. Thus the mask  $M$  is generated for the image and is shown in

Figure 3.10



*Figure 3.10 Mask generated from H*

### **3.5 Post-Processing**

This step is performed on the mask generated in the earlier step. The mask generated may contain small blobs. These blobs are created during the spectral clustering technique, if the block is classified as foreground but all neighbouring blocks are considered as background or vice versa.

Post processing is performed on the mask to remove small blobs. This technique is performed separately for background and foreground. At first, all unique blobs in the background are given numbers and area of each blob is calculated. Area of the blobs are normalised to zero and one. A predefined threshold is set and the blobs with normalised area less than threshold are inverted i.e., they are given value of '1' [2]. Thus the blobs below threshold are considered as part of the foreground. This step is repeated for foreground and blobs inside foreground are also removed.

Thus, post-processing also is an important step in image segmentation. This steps removes blobs which are classified incorrectly because of noise and helps in removing false minutiae and prevents from removal of correct minutiae in the foreground. The output of this post-processing step is shown in *Figure 3.11*



*Figure 3.11 Mask obtained after post-processing removing small blobs*

### **3.6 Segmentation of the Image**

The generated mask from the previous step consists of all 1's and 0's. Final segmented image is obtained by multiplying the mask (M) with fingerprint image (I). The boundary between foreground and background can be drawn by tracing the position in M where the value changes from 0 to 1. This boundary can be drawn onto image and the image with boundary separating background and foreground is obtained. General fingerprint image with boundary between foreground and background is shown in Segmented Image

Figure 3.12(a) and the final segmented image is shown in Segmented Image

Figure 3.12(b)



a) *Boundary of mask over segmented image*



b) *Segmented Image*

Figure 3.12 Images showing segmented mask over original image.

Thus, following all the steps described in this chapter fingerprint Image segmentation using Spectral clustering is achieved. The implementation of the model is discussed and shown in next chapter.

## 4 Implementation of the Model

---

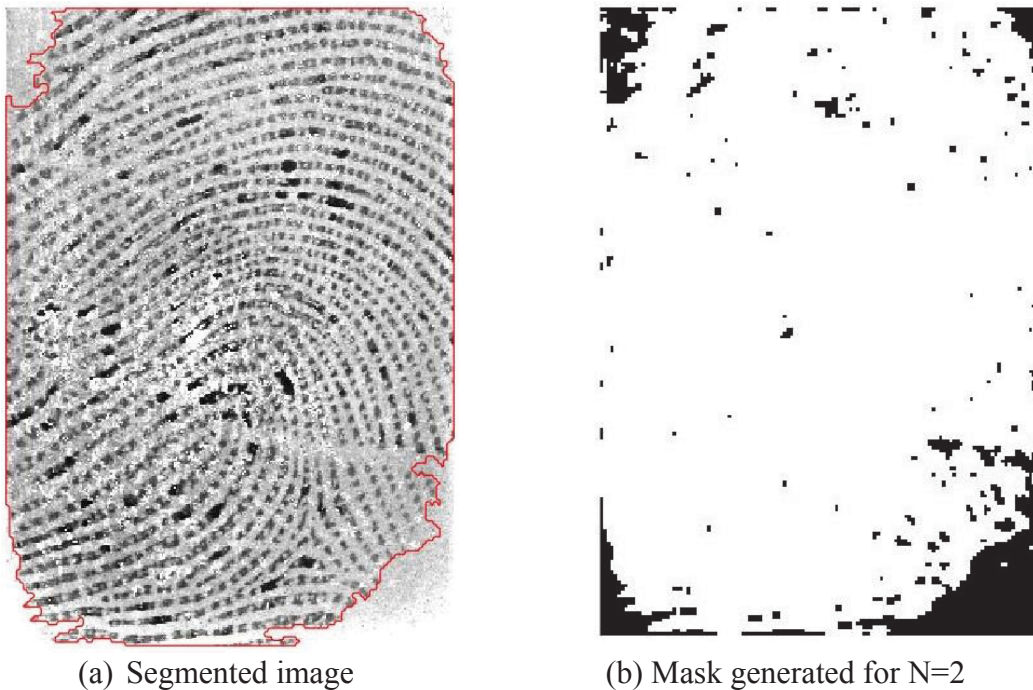
This chapter focuses on the implementation of the fingerprint segmentation model proposed in Chapter 3.

### 4.1 Selection of the suitable block size

In previous chapter it is discussed that there is trade-off between computational complexity and proportion of the image recognised as foreground near the boundary of the image. In this chapter, the fingerprint segmentation method is applied for different values of  $N$ .

#### 4.1.1 Block size 2 x 2 pixels

For the block size of 2 x 2 pixels the generated mask and final segmented image is shown in *Figure 4.1*

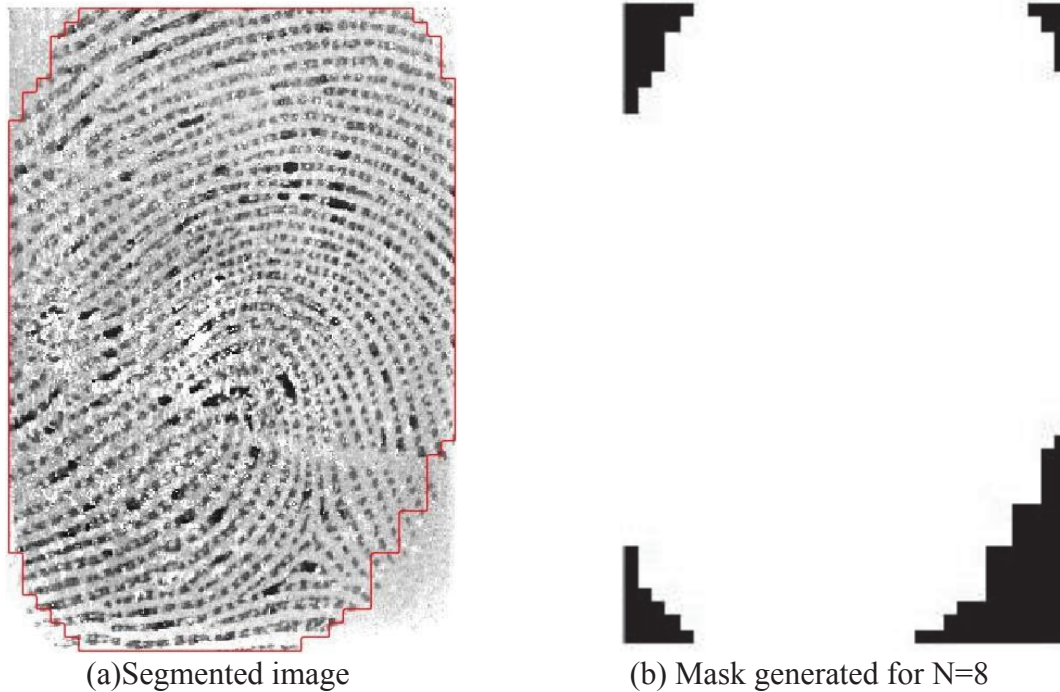


*Figure 4.1 Segmented Image and Mask of the Image for block size of  $N = 2$*

For the block size of 2x2 pixels, there are many areas in the foreground which are considered as background, because of the small size of the block. The parts of the image where intensity is too dark, variance is similar to that of background part of the image. This led to various small blobs in foreground. Even though, we post process the mask, in some cases it is impossible to post-process the mask.

### 4.1.2 Block size of 8x8 pixels

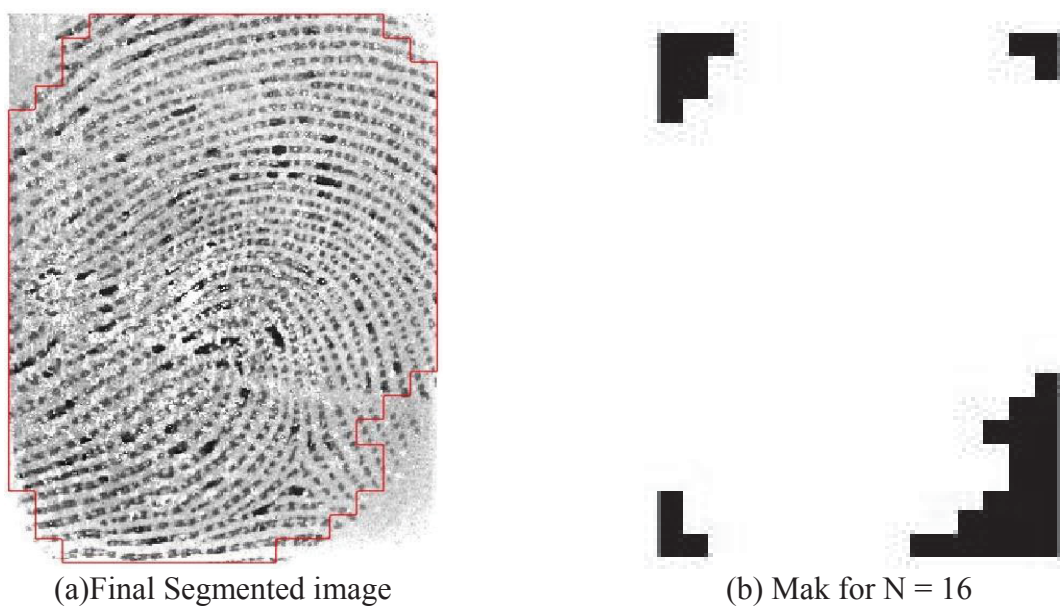
The generated mask and final segmented image for block size of 8 x 8 pixels is shown in *Figure 4.2*



*Figure 4.2 Segmented Image and Mask of the Image for block size of  $N = 8$*

### 4.1.3 Block size of 16 x 16 pixels

The generated mask and final segmented image for the block size of 16 x 16 pixels is shown in *Figure 4.3*



*Figure 4.3 Segmented Image and mask of the image for  $N = 16$*

From all the figures above, It can be seen that when block size is 2 x2 the segmented image obtained is highly accurate and computationally extensive. There is a limitation for it when there are so many small blobs in foreground. This can lead to problem in some of the cases where there is more noise. For  $N=8$ , the segmented image does not lose any information from the foreground and small blobs are also rarely present. For the case of  $N = 16$ , the segmented image shows loss of useful information from the foreground which compromise the performance of fingerprint recognition systems. Therefore,  $N = 16$  is never considered. By observing all these cases block size of  $N= 8$  is considered in this thesis.

## **4.2 Number of clusters**

Ideally, the number of clusters to be divided is 2, i.e., 1 for the foreground and 0 for the background. But in practical implementation it is found that the results from the spectral clustering are not accurate with splitting data into two clusters. This happens as the dataset for spectral clustering is drastically reduced by considering only unique values. Even though this step reduces computational complexity, it also compromised the result obtained. But dividing the data into three clusters and grouping top two clusters into foreground and last cluster to background yielded very good results. Therefore, within scope of the thesis, the data is clustered into three clusters and assigned to foreground and background.

## 5 Experimental Results

---

This section deals about results obtained by implementing the algorithms discussed in the earlier chapters. It also talks about the evaluation of performance of the proposed algorithm. This chapter also discusses about parameters required for implementation of the algorithms.

### 5.1 Input specifications

Algorithms are tested on FVC2000 B, FVC2002 B, FVC2004B databases. Each database consists of four sub-folders of 80 images each.

Algorithms are applied separately for each sub-database of 80 images each. The input parameters are taken as follows.

- ‘ $\sigma$ ’ value for spectral clustering technique is taken as 0.01 for variance and 0.1 for double gradient technique.
- Block size for performing algorithm is considered to be of 8 x 8 pixels.
- Number of clusters is divided into is 3. Cluster with lowest intensities is assigned 0 and rest two clusters are assigned 1. Thus forming S matrix.

### 5.2 Performance evaluation

Results are evaluated by comparing masks obtained from the proposed algorithms with the masks that are generated manually for all the databases used. This is done by calculating three parameters for comparison. Parameters are taken from [2] . They are,

#### 5.2.1 Proportion of foreground area to entire image area

This parameter calculates proportion of area of the fingerprint image considered as foreground by algorithm. i.e., this parameter is calculated by taking ratio of number of pixels considered as foreground in the algorithm to the total number of pixels of the fingerprint Image. This parameter is calculated to get an idea of the proportion of the area considered as foreground in the entire Image. This parameter is calculated separately for all algorithms using eq(5.1)

$$P = \frac{1}{M \times N} \sum_{i \in M} \sum_{j \in N} A(i, j) . \quad (5.1)$$

Where,

$A(i,j)$  is mask of the image  $I(i,j)$  of size M x N pixels.

### 5.2.2 Comparison parameters

In addition to P there are two more parameters that are evaluated they are:

$$P_I = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N I_{mask}(i,j) (1 - I_{man}(i,j)) \quad , \quad (5.2)$$

$$P_E = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N I_{man}(i,j) (1 - I_{mask}(i,j)) \quad . \quad (5.3)$$

Where,

$I_{mask}(i,j)$  is mask generated by algorithm,

$I_{man}(i,j)$  is manually segmented mask.

**P<sub>I</sub>**: P<sub>I</sub> is the measure of proportion of false pixels that are segmented as foreground by the algorithm. i.e., this parameter is calculated by taking ratio of number pixels of the image considered as foreground by the generated algorithm but are not considered as foreground by the manually annotated mask to that of total number of pixels of the image. This parameter can be used as comparison metric between algorithm generated mask and manually annotated mask. It is given using the eq(5.2)

**P<sub>E</sub>**: P<sub>E</sub> is the measure of proportion of true pixels that are not segmented as foreground by algorithm. i.e., the value of this parameter is calculated by taking ratio of the number of pixels of the image that are not considered as foreground by the generated algorithm but are actually considered as foreground by manually annotated mask to that of the total number of the pixels of the image. This parameter can also be used as comparison metric between algorithm generated mask and manually annotated mask. It is given using the eq(5.3). To get an Idea of the comparison parameters they are illustrated in the *Figure 5.1*.

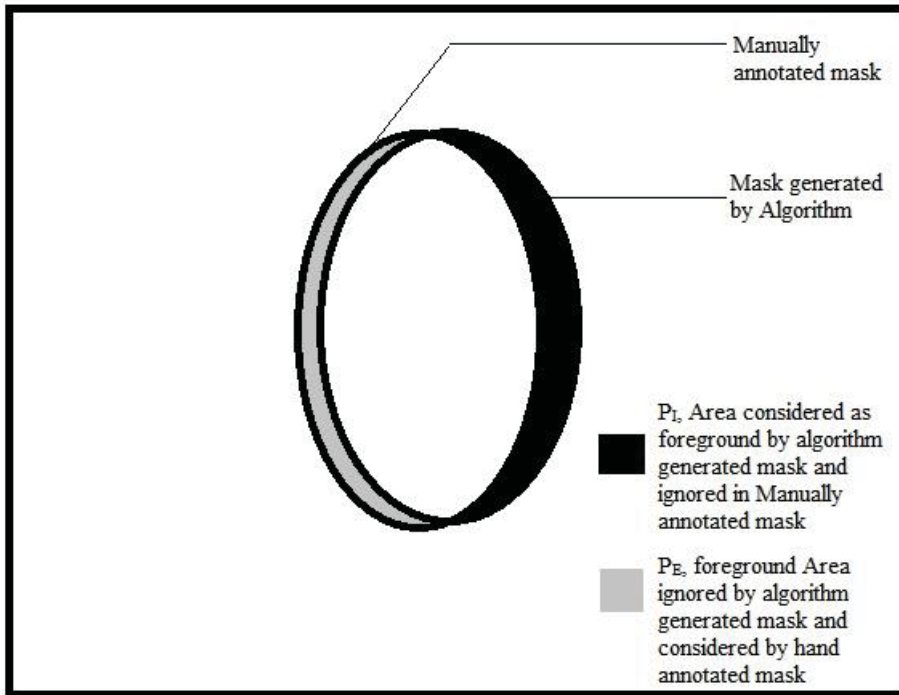


Figure 5.1 Comparison paramters Illustrated

Comparison parameters are used to compare algorithm generated masks with manually annotated mask. This helps in getting an estimate of the performance of the algorithm. Even though our algorithm is compared with manually annotated masks, it cannot be considered that manually annotated masks are the accurate measure for the foreground of fingerprint images. The generation of mask manually varies on different factors with human sight, visibility of the foreground, contrast of the image. Therefore, these parameters are strictly used only for comparison with manually annotated masks.

All the parameters discussed above are calculated for all the images in all the databases mentioned earlier. The values are averaged for each database and are tabulated in *Table 5-1*

Table 5-1: Comparison of parameters mention in equations (5.1) (5.2) (5.3), for generated masks and hand annotated masks

Database	$P_{man}$	Double Gradient			Variance		
		$P_{Proposed}$ [%]	$P_I$ [%]	$P_E$ [%]	$P_{Proposed}$ [%]	$P_I$ [%]	$P_E$ [%]
FVC 2000a	81.67	80.60	3.60	4.60	79.39	3.61	5.90
FVC 2000b	81.72	82.23	3.67	3.19	81.67	4.64	4.71
FVC 2000c	68.79	68.07	5.24	5.97	72.85	6.74	5.95
FVC 2000d	67.08	63.31	1.4	5.1	62.2	1.14	5.94
Average	74.82	73.55	3.48	4.72	74.03	4.03	5.63
FVC 2002a	53.34	49.27	0.77	4.85	48.045	0.63	5.95
FVC2002b	71.3	72.79	3.6	2.2	72.66	3.96	2.66
FVC2002c	53.11	55	8.9	7.09	55.63	10.5	8.06
FVC2002d	56.69	58.72	4.31	2.28	60	6	2.69
Average	58.61	58.945	4.395	4.105	59.084	5.272	4.84
FVC2004a	27.24	21.82	0.16	5.59	21.15	0.13	6.23
FVC2004b	65.10	63.98	3.4	4.59	64.04	3.93	5
FVC2004c	61.35	61.54	2.3	2.1	60.37	2.39	3.36
FVC2004d	63.37	53.73	0.29	9.9	51.47	0.28	12.18
Average	54.265	50.268	1.54	5.55	49.25	1.68	6.70
overall Average	62.56	60.92	3.13	4.78	60.789	3.66	5.72

### 5.3 Discussions

From the values in *Table 5-1*, it can be seen that using proposed algorithm area of masks generated with double gradient as parameter is 1.64% lesser than area of hand segmented masks. Similarly, area of masks generated using variance as parameter is 1.77% less than area generated from hand-segmented masks. From this, it can be said that the proposed algorithm generally produces masks of lesser area. This can be attributed to the fact that in block processing technique, blocks at the borders of the mask can be assigned to either background or foreground depending on parameter.

From the values it can also be seen that using double gradient, proportion of pixels wrongly assigned to foreground compared to hand annotated masks is 3.13%, Similarly, using variance the same value stands at 3.66%. It can also be noticed that double gradient technique outperforms variance method in each of the databases individually.

From the values, Proportion of pixels wrongly assigned as foreground using double gradient is 3.66% while compared to using variance where this metric is of 5.72%. It can also

be seen that double gradient technique outperforms variance technique for each of the database individually.

Therefore, from these values it can be concluded that both the methods used for segmentation of fingerprint images work efficiently. But comparing both the techniques used, it can be said that, using double gradient as a parameter over variance is optimal for fingerprint segmentation.

## 6 Conclusions and Future work

---

### 6.1 Conclusion

Segmentation of fingerprint images is performed using spectral clustering technique with gradient of gradient and variance as parameters. Segmentation is performed by splitting image into non-overlapping blocks. It is observed that the algorithm devised can be used as a novel unsupervised segmentation technique for fingerprint images. From this thesis, it can also be concluded that by approximation techniques, speed of spectral clustering algorithm is improved. This can be used for various image-processing applications.

Performance evaluation of devised algorithms reveal that gradient of gradient image is slightly better parameter than variance for segmentation. It can also be concluded that segmentation of images with uneven contrast improves significantly by enhancement of image before segmenting it. Comparing the algorithm with manually segmented images also proved that the algorithm is close to manual segmentation.

### 6.2 Future work

In future, this research work can be continued in many ways,

1. Position of data in the matrix is not considered for spectral clustering, considering this value increases performance of spectral clustering technique, which can result in better performance from segmentation.
2. Block size for splitting image into non-overlapping blocks is kept constant; altering these values behaviour of spectral clustering on fingerprint images can be studied.
3. Splitting data into multiple clusters instead of two clusters and analysis of redistribution of data to background and foreground can be analysed for better segmentation results for un-enhanced image.

# Appendix

---

In this section, we are attaching some of the fingerprint images before and after segmentation.



*Original Image*



*Final Segmented Image*



*Original Image*



*Final Segmented Image*



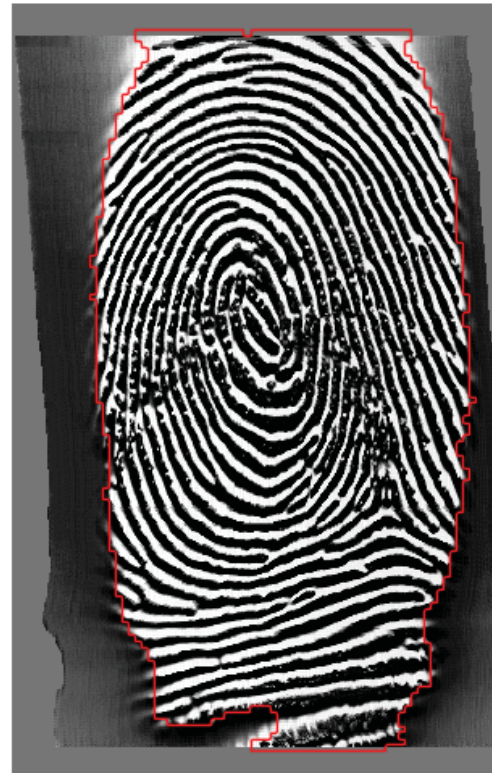
*Original Image*



*Final Segmented Image*



*Original Image*



*Final Segmented Image*



*Original Image*



*Final Segmented Image*

# Bibliography

---

- [1] Davide Maltoni, Dario Maio, Anil K. Jain and Salil Probhakar. *Handbook of Fingerprint Recognition*. Springer 2009. ISBN: 978-1-84882-253-5
- [2] J. Ström Bartůněk, *Fingerprint image enhancement, segmentation and minutiae detection*. Karlskrona: Blekinge Tekniska Högskola, 2016. ISBN: 978-91-7295-321-5
- [3] J. Shi and J. Malik, “Normalized cuts and image segmentation,” *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 22, no. 8, pp. 888–905, Aug. 2000.
- [4] D. Yan, L. Huang, and M. I. Jordan, “Fast Approximate Spectral Clustering,” in *Proceedings of the 15th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, New York, NY, USA, 2009, pp. 907–916.
- [5] M. U. Akram, A. Ayaz, and J. Imtiaz, “Morphological and gradient based fingerprint image segmentation,” in *2011 International Conference on Information and Communication Technologies (ICICT)*, 2011, pp. 1–4.
- [6] D. Das and S. Mukhopadhyay, “Fingerprint Image Segmentation Using Block-Based Statistics and Morphological Filtering,” *Arab. J. Sci. Eng.*, vol. 40, no. 11, pp. 3161–3171, 2015.
- [7] S. R. Suralkar and P. M. Patil, “Correlation based Fingerprint Image Segmentation,” *Int. J. Comput. Appl.*, vol. 68, no. 7, pp. 1–3, Apr. 2013.
- [8] G. Yang, G.-T. Zhou, Y. Yin, and X. Yang, “-Means Based Fingerprint Segmentation with Sensor Interoperability,” *EURASIP J. Adv. Signal Process.*, vol. 2010, no. 1, p. 729378, 2010.
- [9] G. Yang, Y. Li, Y. Yin, and Y.-S. Li, “Two-Level Evaluation on Sensor Interoperability of Features in Fingerprint Image Segmentation,” *Sensors*, vol. 12, no. 12, pp. 3186–3199, Mar. 2012.
- [10] F. Wang, X. Wang, and L. Xu, “An Improved Fingerprint Segmentation Algorithm Based on Mean and Variance,” in *International Workshop on Intelligent Systems and Applications, 2009. ISA 2009*, 2009, pp. 1–4.
- [11] Y. Mei, H. Sun, and D. Xia, “A gradient-based combined method for the computation of fingerprints’ orientation field,” *Image Vis. Comput.*, vol. 27, no. 8, pp. 1169–1177, Jul. 2009.

- [12] T. Xiang and S. Gong, “Spectral clustering with eigenvector selection,” *Pattern Recognit.*, vol. 41, no. 3, pp. 1012–1029, Mar. 2008.
- [13] F. Tung, A. Wong, and D. A. Clausi, “Enabling scalable spectral clustering for image segmentation,” *Pattern Recognit.*, vol. 43, no. 12, pp. 4069–4076, Dec. 2010.
- [14] J. Qi and M. Xie, “Segmentation of fingerprint images using the gradient vector field,” in *2008 IEEE Conference on Cybernetics and Intelligent Systems*, 2008, pp. 543–545.
- [15] H. Q. Liu, L. C. Jiao, and F. Zhao, “Non-local spatial spectral clustering for image segmentation,” *Neurocomputing*, vol. 74, no. 1–3, pp. 461–471, Dec. 2010.
- [16] Z. Li and J. Chen, “Superpixel segmentation using Linear Spectral Clustering,” in *2015 IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2015, pp. 1356–1363.
- [17] Z. Hou, W. Y. Yau, N. L. Than, and W. Tang, “Complementary Variance Energy for Fingerprint Segmentation,” in *Advances in Multimedia Modeling*, S. Satoh, F. Nack, and M. Etoh, Eds. Springer Berlin Heidelberg, 2008, pp. 113–122.
- [18] E. Zhu, J. Zhang, J. Yin, G. Zhang, and C. Hu, “Removing the remaining ridges in fingerprint segmentation,” *J. Zhejiang Univ.-Sci. A*, vol. 7, no. 6, pp. 976–983, Jun. 2006.
- [19] S. Zeng, R. Huang, Z. Kang, and N. Sang, “Image segmentation using spectral clustering of Gaussian mixture models,” *Neurocomputing*, vol. 144, pp. 346–356, Nov. 2014.
- [20] D. H. Thai, S. Huckemann, and C. Gottschlich, “Filter Design and Performance Evaluation for Fingerprint Image Segmentation,” *PLOS ONE*, vol. 11, no. 5, p. e0154160, May 2016.
- [21] A. Jurio, H. Bustince, M. Pagola, P. Couto, and W. Pedrycz, “New measures of homogeneity for image processing: an application to fingerprint segmentation,” *Soft Comput.*, vol. 18, no. 6, pp. 1055–1066, Jun. 2014.
- [22] X. Chen, J. Tian, J. Cheng, and X. Yang, “Segmentation of Fingerprint Images Using Linear Classifier,” *EURASIP J. Adv. Signal Process.*, vol. 2004, no. 4, pp. 480–494, 2004.