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Masters Thesis on

Estimation of Direction of Arrival and Beamforming in Adaptive Array Antennas

By

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Abstract

In recent years there has been a rapid growth in the number of wireless users, particularly in the area of mobile communications. This rapid growth in mobile communications demands for more system capacity through efficient utilization of frequency spectrum and also keeping the Interference as low as possible.

In today's Radio resource management, Adaptive array antennas have an important role in increasing the system capacity and controlling the interference in mobile communications.

Adaptive array antenna is an antenna system, which uses spatially separated antennas called array antennas and processes the received signals with a digital signal processor. Simply speaking these array antennas can reduce the co channel interference and effectively utilize the bandwidth by steering a high gain in the direction of interest and low gains in the undesired directions, technically this is called adaptive beamforming. This adaptive beamforming enables the base station to form narrow beam towards desired user and nulls towards the interfering user, hence improving the signal quality.

Acknowledgement

First of all we would like express our immense gratitude and wholehearted thanks towards our supervisor Mr. Tommy Hult for his invaluable guidance and encouragement through out the thesis. We would also like to show our gratitude and love towards our parents for their constant support and encouragement with out whom, we wouldn't be at this stage. Our sincere thanks to all our friends who supported and involved in helpful discussions.

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List of Symbols

M	Isotropic directional antenna elements
d	Element spacing
N	Uncorrelated narrow band sources
S(n)	Plane wave signals
θ_i	i^{th} source arriving from direction
K	Samples
X(n)	$M \times K$ Array output vector
A(θ)	$M \times N$ Array steering matrix
W(n)	$M \times K$ Noise matrix
L	Number of instants
σ^2	Noise covariance matrix
$\mathbf{R}_{XX}$	Spatial correlation matrix
$E[.]$	Expectation
H	Complex conjugate transpose
P	Covariance matrix

e_i	Noise eigen values
$\mathbf{E_S}$	N signal eigen vectors
$\mathbf{E_N}$	Noise eigen vectors
$P_{MUSIC}(\theta)$	Angular spectrum
R	Circle with the radius
p	Equipowered sources
ϕ_k	Azimuth angle
θ_k	Elevation angle
$\mathbf{X}$	$N \times K$ element space data matrix
$\mathbf{A}$	$N \times p$ element space array manifold
M'	Total number of excited modes
$J_m(\varsigma)$	Bessel function of the first kind of order m
F_e^H	Beam forming matrix
$\boldsymbol{J_\zeta}$	*Bessel function*
α_i	Azimuthally rotation angles
$b(\varsigma, \phi)$	Real beamspace manifold
Δ	constant displacement vector

$\theta = (\varsigma, \phi)$	To represent the source arrival directions
M	Denotes the highest order mode
F_r^H	Real Beamforming Matrix
T	Real nonsingular matrix
τ	Delay

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1. INTRODUCTION

1.1 Adaptive Array Antennas

An antenna array is a set of antenna elements that are spatially distributed at known locations with reference to a common fixed point [1]. An adaptive array is an antenna system that can modify its beam pattern or other patterns, by means of internal feedback control while the antenna system is operating.

Adaptive array antennas can increase the coverage area and the capacity of a wireless communication system. The coverage area is simply the area in which communication between a mobile and the base station is possible. The capacity is a measure of the number of users in a system that can support in a given area. In sparsely populated areas, extending coverage is often more important than increasing capacity. In such areas, the gain provided by adaptive antennas can extend the range of a cell to cover a longer area and more users than would be possible with omni directional. In populated areas, increasing capacity is of prime importance. The main strategy for increasing capacity is interference reduction on the downlink and interference rejection on the uplink. Clearly speaking, we can use same frequency to different users at the same time with out interference. This concept could be explained using satellite communication system.

Satellite communication uses SDMA (Spatial Division Multiple Access) technique in which signals are transmitted within the same frequency to different receiving zones on earth in narrow beams (fig 1.1)

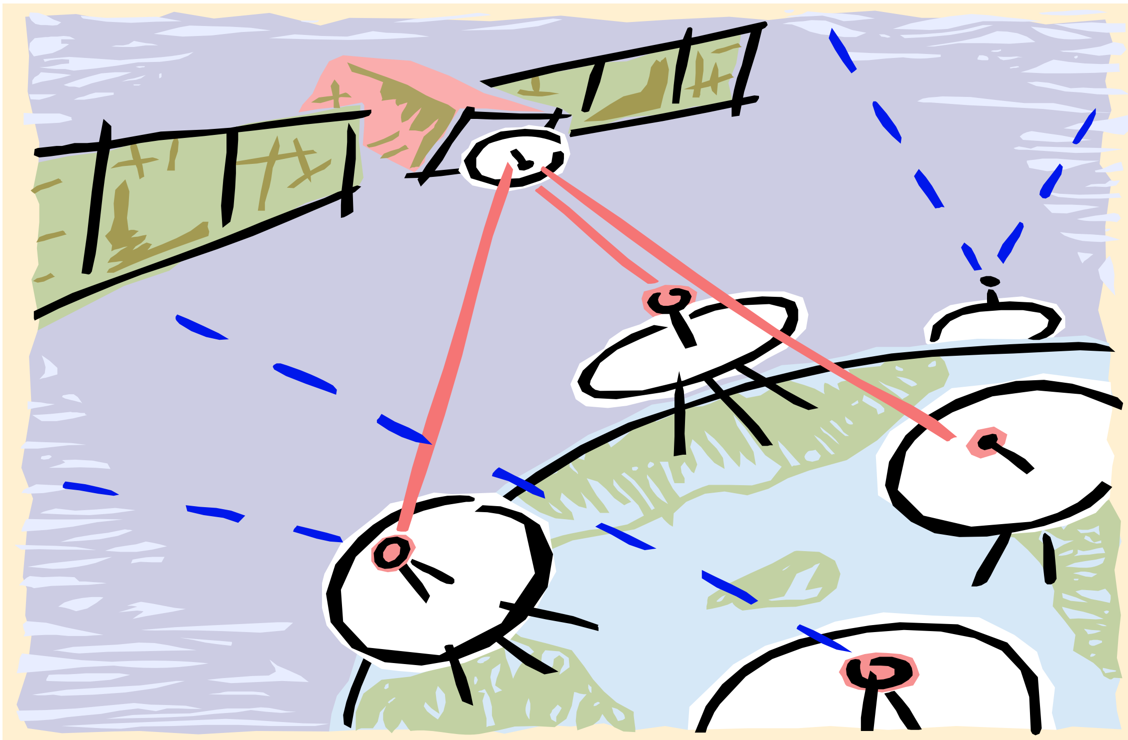

Fig 1.1. Satellite Communication Model

1.2 Array Geometry

The antenna elements can be arranged in various geometries, with linear, circular, planar and random arrays being very common. In linear array the centers of the antenna elements are aligned along a straight line. If the spacing between the array elements is equal, it is called a uniformly spaced linear array. A circular array is one in which the centers of the antenna elements lie on a circle. In planar array the centers of the antenna elements lie on a single plane. The linear array and circular array both are special cases of planar array. And in random array the antenna elements are arranged at random places in a plane or in three dimensions.

1.3 Adaptive Beamforming

Adaptive beamforming is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arrival from a desired direction while signals of the same frequency from other directions are rejected. This process is achieved by varying the weights of each of the sensors used in the array. This allows the antenna system to focus the maxima of the antenna pattern towards the desired mobile while minimizing the impact of noise, interference and other effects from undesired mobiles. The adaptive beamforming algorithms are categorized under two types according to whether a training signal is used or not. One type of these algorithms is the non-blind algorithm, in which a training signal is used to adjust the array weight vector. Another type is blind adaptive algorithm which does not require a training signal.

Spatially propagating signals encounter the presence of interfering signals and noise signals. If the desired signal and the interferers occupy the same temporal frequency band, then temporal filtering cannot be used to separate the signal from the interferers. However the desired and the interfering signals generally originate from different spatial locations. This spatial separation can be exploited to separate the signals from the interference using a beamformer. A beamformer consists of an array of sensors in a particular configuration

2. DOA Estimation Methods

The purpose of DOA estimation is to use the data received by the array to estimate the direction of arrival of the signal. The results of DOA estimation are then used by the array to design the adaptive beamformer, which is used to maximize the power radiated towards users, and to suppress interference. As a result, we can infer that a successful design of an adaptive array depends highly on the performance of the DOA algorithm. Direction of arrival algorithms are usually complex and their performance depend on many parameters such as number of mobile users and the spatial distribution, the number of array elements and their spacing and the number of signal samples.

A Number of algorithms have been developed for the purpose of determining the direction of arrival from the measurements of the signals received at the elements of array sensors. One of the simplest and popular algorithms used for DOA estimation is the MUSIC (MUltiple SIgnal Classification) algorithm. The MUSIC algorithm is a subspace algorithm aimed at eliminating the effect of noise. This can be done by splitting the M-dimensional space spanned by the antenna element outputs into a signal subspace and a noise subspace. The most used subspace based algorithms are MUSIC [1]-[6] and ESPRIT [7]-[13].

2.1 MUSIC Algorithm

The MUSIC algorithm which was introduced by Schmidth [1] is the high resolution Multiple Signal Classification algorithm that can determine the DOA of multiple radio waves arriving simultaneously at the antenna, which decomposes the autocorrelation matrix into signal space and noise space and exploits the characteristics of signal and noise spaces to estimate the DOA.

Data model:

Consider a uniform linear array(ULA) with M isotropic directional antenna elements with element spacing d and N uncorrelated narrow band sources emitting plane wave signals **S(n)** from different directions $\theta_1, \theta_2, ..., \theta_N$ where $M > N$ is assumed. The array model is shown in fig 2.1

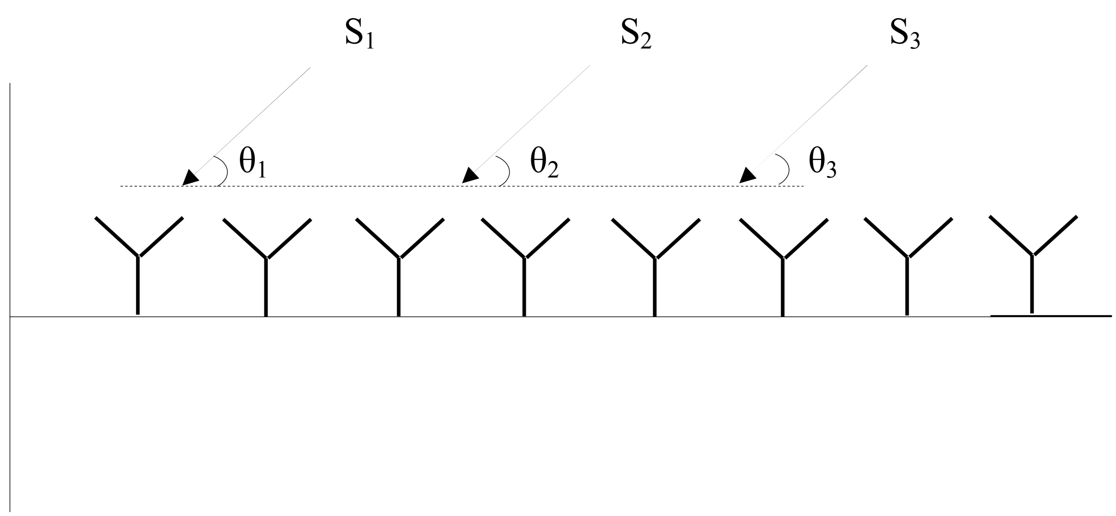

Fig 2.1 Linear Array Model

We assume that K samples are observed by the array, the array output vector **X(*n*)** is

$$\mathbf{X}(n) = \mathbf{A}(\theta)\mathbf{S}(n) + \mathbf{W}(n) \;, \quad n = 1,2,.....,K \tag{2.1}$$

X(*n*)is $M \times K$ matrix of array output signals at any given sampling time n which is called an instant, **A(*θ*)** is $M \times N$ steering matrix, **S(*n*)** is $N \times K$ signal matrix, **W(*n*)** is $M \times K$ noise matrix and L is number of instants. It is assumed that the signals and noise are weak sense stationary zero mean additive white Gaussian random processes and further, the noises are both spatially and temporally white with variance σ^2. The array steering matrix (array manifold) **A(*θ*)** is

$$\mathbf{A}(\theta) = \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ...\mathbf{a}(\theta_N)\right] \tag{2.2}$$

Where $\mathbf{a}(\theta_i) = \left[1, e^{j2\pi d \sin\theta_i/\lambda}, ..., e^{j2\pi(M-1)d\sin\theta_N/\lambda}\right], \qquad i = 1,2,3....,N$,

a(*θ*) is the response of the linear array to the i^{th} source arriving from direction θ_i.The array manifold is defined as the one dimensional manifold composed of all the steering vectors as θ ranges over all possible angles that is $\theta \in [0,2\pi]$.

The spatial correlation matrix $\mathbf{R}_{XX}$ of the observed array output matrix **X(*n*)** is then:

$$\mathbf{R}_{XX} = \mathbf{E}\left[\mathbf{X}(n)\mathbf{X}(n)^{H}\right] = \mathbf{APA}^{H} + \boldsymbol{\sigma}^{2}\mathbf{I} \tag{2.3}$$

Where $\mathbf{E}[.]$ is expectation and H is complex conjugate transpose, **P** is the covariance matrix $\mathbf{P} = \mathbf{E}\left[\mathbf{S}(n)\mathbf{S}(n)^{H}\right]$of the signal vector, $\boldsymbol{\sigma}^2$ is the noise covariance matrix $\mathbf{E}\left[\mathbf{W}(n)\mathbf{W}(n)^{H}\right]$ and **I** is $M \times M$ identity matrix. The correlation matrix $\mathbf{R_{XX}}$ will have N signal eigen values e_i, $i = 1,2,3,.....,N$ and $M - N$ noise eigen values e_i $i = N+1,....M$ ordered as:

$$e_1 \geq e_2 \geq e \ \geq ....e_N > e_{N+1} > ....e_M = \sigma^2 \qquad (2.4)$$

$\mathbf{E}_S$ is the matrix of the corresponding N signal eigen vectors $\mathbf{E}_s = [e_1, e_2, ...e_N]$ and $\mathbf{E_N}$ is the matrix noise eigen vectors $\mathbf{E}_n = [e_{N+1}, ...e_{2M}]$. The subspace spanned by the signal eigen vectors is denoted as signal subspace and subspace spanned by the noise eigen vectors is denoted as noise subspace. The Eigenvectors in the noise space are orthogonal to those in the signal space. As the signal space contains information about the angles of arrival from each plane wave, the steering vectors from those angles are also orthogonal to the vectors in the noise space. If the magnitude of the product between a steering vector from a plane wave's DOA and the noise-space vectors is zero, then $\mathbf{a}^H(\theta)\mathbf{E}_N\mathbf{E}_N^H\mathbf{a}(\theta) = 0$ for angles θ equal to the DOA of a signal direction. The inverse of the magnitude of the product between a steering vector from all possible angles and the noise-space vectors is known as the MUSIC spectrum. Using this principle the MUSIC spectrum is computed using noise subspace. The DOA of arriving signals is estimated by locating the peaks of the MUSIC spectrum $\mathbf{P}_{MUSIC}(\theta)$. These peaks in the MUSIC angular spectrum occur whenever the steering vector $\mathbf{A}(\theta)$ is orthogonal to the noise subspace. The angular spectrum is thus given by:

$$\mathbf{P}_{\mathrm{MUSIC}}(\theta) = \frac{1}{\mathbf{A}^{\mathrm{H}}(\theta)\mathbf{E}_{\mathrm{N}}\mathbf{E}_{\mathrm{N}}^{\mathrm{H}}\mathbf{A}(\theta)} \tag{2.5}$$

Simulation Results:

To analyze the performance of the MUSIC algorithm we compared in two cases. First case we observed MUSIC spectrum by changing the Number of array elements. Fig 2.1a shows the MUSIC spectrum generated with 12 array elements, where as Fig 2.1b shows the MUSIC spectrum generated with 8 array elements, from fig 2.1a and fig 2.1b using more array elements improves the resolution of the MUSIC spectrum.

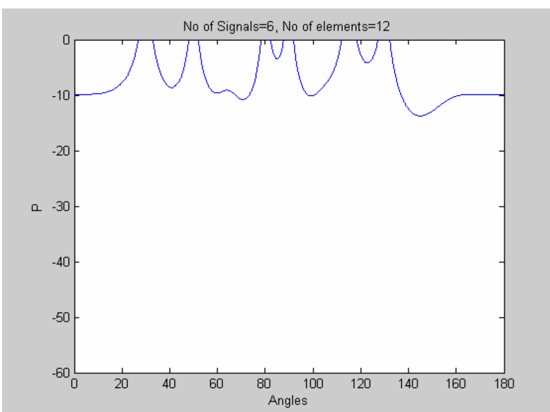

Fig 2.1a

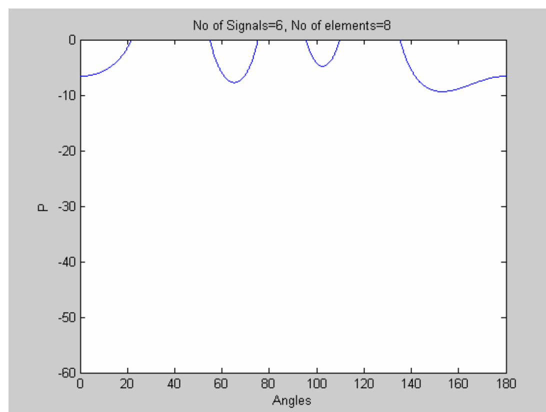

Fig 2.1b

Second case we observed the MUSIC spectrum by changing the element spacing. Fig 2.2a shows the MUSIC spectrum for 6 signal sources with array element spacing $d = 0.5\lambda$, fig 2.2b shows the MUSIC spectrum for 6 signal sources with array element spacing $d = 0.4\lambda$ .From fig 2.2a and fig 2.2b when the array elements are placed close to each other, mutual coupling occurs and this leads to a reduction of the accuracy of the DOA estimation.

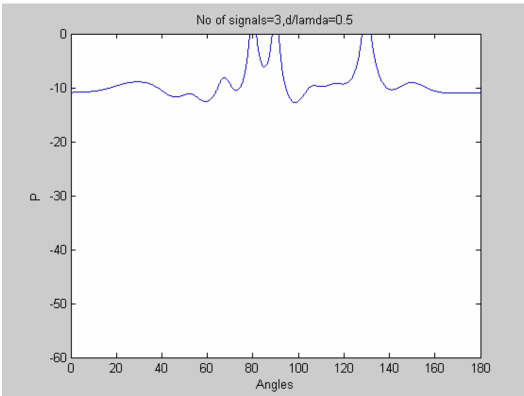

Fig 2.2a

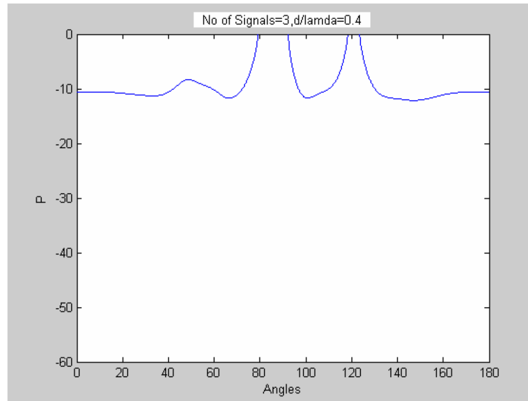

Fig 2.2b

2.2 UCA-RB MUSIC Algorithm

Uniform circular array (UCA) is a special planar array with many excellent properties over ULA it is able to provide 360^0 azimuthally coverage and a certain degree of source elevation information. The UCA Real Beamspace MUSIC introduced by Mathews and Zoltowski [3]. The N antenna elements are assumed to be omni directional, identical and uniformly distributed over the circumference of a circle with the radius r in the X-Y plane. p equipowered sources arriving at the center of the circular array of radius $r=\lambda$

from the far field with azimuth angle ϕ_k and elevation angle θ_k is shown in Fig 2.3.

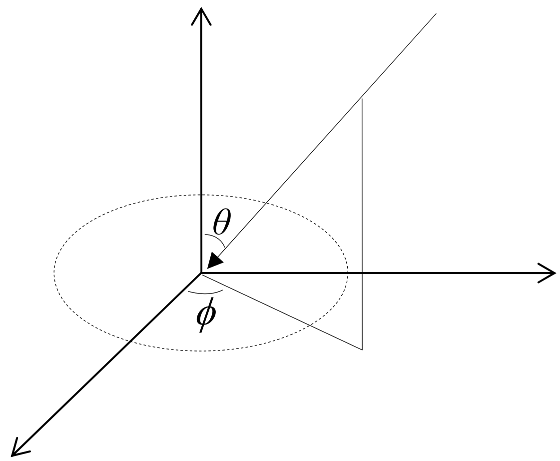

Fig 2.3

We assume K samples are observed by the array, the antenna array output matrix is

$$\mathbf{X} = \mathbf{AS} + \mathbf{W} \tag{2.6}$$

Where **X** is an $N \times K$ element space data matrix, **A** is an $N \times p$ element space array manifold, **S** is a $p \times K$ complex signal envelopes at the array centre, **W** is $N \times K$ matrix of noise complex envelopes. The signals and noise are assumed stationary zero mean, uncorrelated random processes. The element space array manifold matrix **A** is

$$\mathbf{A} = \left[\mathbf{a}_1(\varsigma,\varphi), \mathbf{a}_2(\varsigma,\varphi), \mathbf{a}_3(\varsigma,\varphi), \ldots\ldots., \mathbf{a}_\mathrm{p}(\varsigma,\varphi)\right] \tag{2.7}$$

The columns of matrix A are modeled as

$$\mathbf{a}(\theta) = \mathbf{a}(\varsigma,\varphi) = \left[e^{\mathrm{j}\varsigma\cos(\phi-\gamma_0)}, e^{\mathrm{j}\varsigma\cos(\phi-\gamma_1)}, \ldots\ldots., e^{\mathrm{j}\varsigma\cos(\phi-\gamma_{N-1})}\right] \tag{2.8}$$

Where the elevation θ dependence through parameter ς and the vector $\theta = (\varsigma, \phi)$ is used to represent the source arrival directions

Here $\varsigma = k_0 r \sin\theta$, $k_0 = \frac{2\pi}{\lambda}$ is wave number and *r* is the radius.

$\gamma_n = \frac{2\pi n}{N}$ $(n = 1,2,.....N)$ is the sensor location.

A circle is periodic with period 2π and can be represented in terms of a Fourier series, in such series each weight is called a phased mode. To the uniform circular array the normalized beamforming weight vector with phase mode *m* is

$$\mathbf{w}_{\mathrm{m}}^{\mathrm{H}} = \frac{1}{\mathrm{N}}\left[1, e^{\mathrm{j}2\pi\mathrm{m}/\mathrm{N}}, ...e^{\mathrm{j}2\pi\mathrm{m}(\mathrm{N}-1)/\mathrm{N}}\right] \tag{2.9}$$

Let *M* denotes the highest order mode that can be excited by the aperture at a reasonable strength $M = k_0 r = 2\pi$ and M' is the total number of excited modes, $M' = 2M + 1$ When $N > 2M$ the UCA array pattern for mode m is

$$\mathbf{f}_{\mathrm{m}}^{\mathrm{c}} = \mathbf{w}_{\mathrm{m}}^{\mathrm{H}}\mathbf{a}(\varsigma, \phi) \approx j^{|\mathrm{M}|}\mathbf{J}_{\mathrm{m}}(\varsigma)e^{\mathrm{j}\mathrm{m}\phi} \tag{2.10}$$

Where $\mathbf{J}_{\mathrm{m}}(\varsigma)$ is the Bessel function of the first kind of order m

The beam forming matrix $\mathbf{F}_{\mathrm{e}}^{\mathrm{H}}$ can be defined as

$$\mathbf{F}_{\mathrm{e}}^{\mathrm{H}} = \mathbf{C}\mathbf{Q}^{\mathrm{H}} \tag{2.11}$$

Where $\mathbf{C} = \mathrm{diag}\left\{\mathrm{j}^{-\mathrm{M}}, ...\mathrm{j}^{-1}, \mathrm{j}^{0}, \mathrm{j}^{-1}, ...\mathrm{j}^{-\mathrm{M}}\right\}$

$$\mathbf{Q} = \sqrt{\mathrm{N}}\left[\mathrm{w}_{-\mathrm{M}}, ..., \mathrm{w}_{0}, ..., \mathrm{w}_{\mathrm{M}}\right]$$

The beam space manifold synthesized by $\mathbf{F}_{\mathrm{e}}^{\mathrm{H}}$ is

$$\mathbf{a}_{\mathrm{e}}(\theta) = \mathbf{F}_{\mathrm{e}}^{\mathrm{H}}\mathbf{a}(\theta) = \mathbf{C}\mathbf{Q}^{\mathrm{H}}\mathbf{a}(\theta) = \sqrt{N}\mathbf{J}_{\varsigma}\mathbf{v}(\varphi) \tag{2.12}$$

The azimuthally variation of $\mathbf{a}_{e}(\theta)$ is through the vector $\mathbf{v}(\varphi)=\left[e^{-jM\phi},.....e^{-j\phi},e^{j0},e^{-j\phi},.....e^{jM\phi}\right]$, $\mathbf{J}_{\varsigma}=\mathrm{diag}\{\mathbf{J}_{M}(\varsigma),.....,\mathbf{J}_{0}(\varsigma),.....,\mathbf{J}_{M}(\varsigma)\}$ is composed of Bessel functions and it is implicitly depending on elevation angle.

The UCA-RB Music algorithm employs the beamformer $\mathbf{F}_{r}^{H}$ to make the transformation from element space to beam space and make it array manifold real, the beamformer $\mathbf{F}_{r}^{H}$ defined as

$$\mathbf{F}_{r}^{H}=\mathbf{V}^{H}\mathbf{C}\mathbf{Q}^{H} \tag{2.13}$$

Where $\mathbf{V}=\frac{1}{\sqrt{M'}}\left[\mathbf{v}(\alpha_{-M}),......,\mathbf{v}(\alpha_{0}),.......\mathbf{v}(\alpha_{M})\right]$,

$\alpha_{i}=2\pi i/M'$ $i\in\left[-M,M\right]$ are the azimuthal rotation angles.

Multiplying element space array manifold $\mathbf{a}(\varsigma,\varphi)$ by beam former $\mathbf{F}_{r}^{H}$ we obtain real beamspace manifold $\mathbf{b}(\varsigma,\varphi)$ is

$$\mathbf{b}(\boldsymbol{\varsigma},\boldsymbol{\varphi})=\mathbf{F}_{r}^{H}\mathbf{a}(\varsigma,\varphi)=\sqrt{N}\mathbf{V}^{H}\mathbf{J}_{\varsigma}\mathbf{v}(\varphi) \tag{2.14}$$

The beamformer $\mathbf{F}_{r}^{H}$ is applied over the element space data matrix **X,** and then the resulting Beamspace data matrix is

$$\mathbf{Y}=\mathbf{F}_{r}^{H}\mathbf{X}=\mathbf{F}_{r}^{H}\mathbf{A}\mathbf{S}+\mathbf{F}_{r}^{H}\mathbf{w}$$

$$\mathbf{Y}=\mathbf{B}\mathbf{S}+\mathbf{F}_{r}^{H}\mathbf{w} \tag{2.15}$$

Where $\mathbf{B}=\mathbf{F}_{r}^{H}\mathbf{A}$ is the real valued beamspace direction of arrival matrix containing vectors $\mathbf{b}(\varsigma,\varphi)$ as its columns.

The array output covariance matrix in the beamspace is

$$\mathbf{R}_{YY} = \mathbf{E}\left[\mathbf{Y}^{H}\mathbf{Y}\right] = \mathbf{BPB}^{H} + \boldsymbol{\sigma}^{2}\mathbf{I} \qquad (2.16)$$

where **P** is the signal covariance matrix. Let $\mathbf{R} = \mathrm{Re}\{\mathbf{R}_{YY}\}$ denotes the real part of the beamspace covariance matrix. The real valued eigen value decomposition of the matrix **R** yield the beamspace signal and noise subspaces. Let $\mathbf{E_S}$ and $\mathbf{E_N}$ be orthonormal matrices that spam the beam and noise subspaces respectively

$$\mathbf{E}_S = \left[\mathbf{e}_1, \mathbf{e}_2, \ldots.\mathbf{e}_p\right] \qquad \mathbf{E}_N = \left[\mathbf{e}_{p+1}, \ldots\ldots\mathbf{e}_N\right]$$

The UCA-RB MUSIC spectrum $\mathbf{P}(\varsigma, \varphi)$ is

$$\mathbf{P}(\varsigma, \varphi) = \frac{1}{\mathbf{b}^{T}(\varsigma, \varphi)\mathbf{E}_{n}\mathbf{E}_{n}^{H}\mathbf{b}(\varsigma, \varphi)} \qquad (2.17)$$

The spectrum has peaks corresponding to signal arrival directions.

Simulations:

We consider uniform circular array (UCA) with array elements chosen to N=15, circular radius $r=\lambda$ with maximum phase mode M=7, and signal samples K=10. We observed Direction of arrival of signals for the Elevation angle and Azimuthal angle as shown in Fig 2.4, Fig 2.5 respectively.

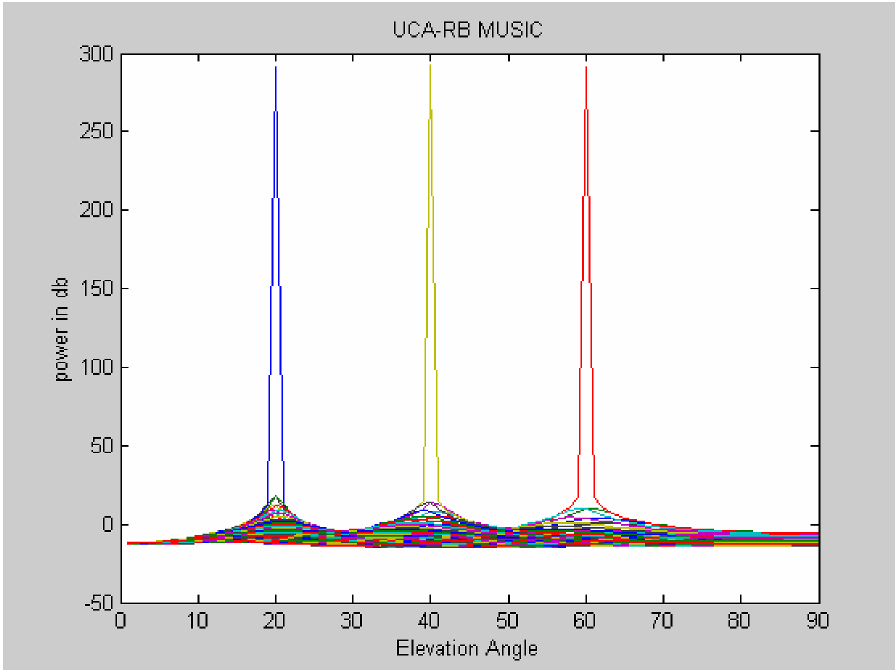
UCA-RB MUSIC
power in db
300
250
200
150
100
50
0
-50
0
10
20
30
40
50
60
70
80
90
Elevation Angle

Fig 2.4

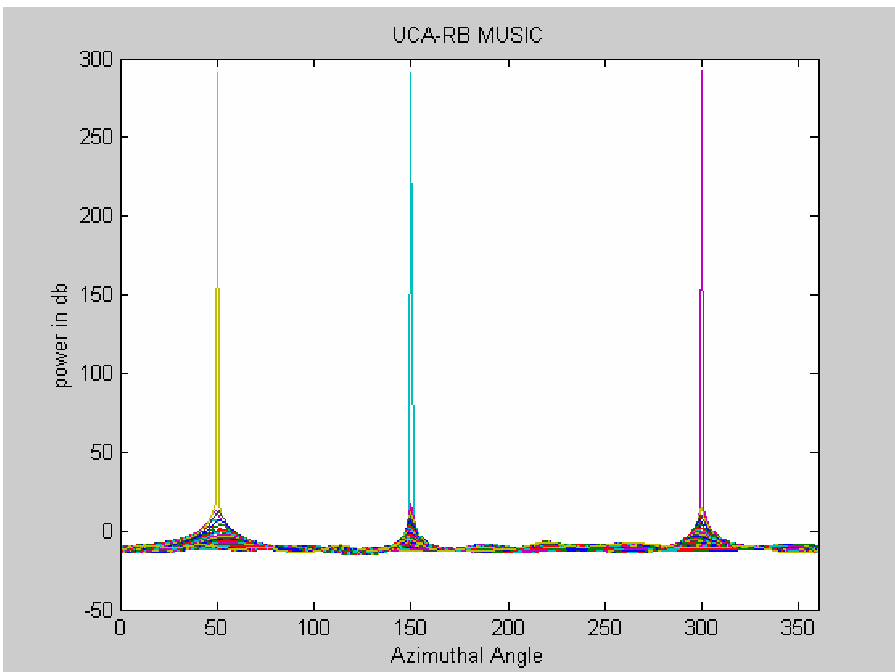
UCA-RB MUSIC
power in db
300
250
200
150
100
50
0
-50
0
50
100
150
200
250
300
350
Azimuthal Angle

Fig 2.5

2.3 ESPRIT Algorithm

ESPRIT method was proposed by R.Roy [7]. It is a subspace estimation method, in which parameter estimates are obtained by exploiting the rotational invariance structure i.e. two identical sub arrays are separated by a common displacement Δ of the signal subspace, induced by the translational invariance structure of the associated sensor array. This special structure allows the parameter estimates to be obtained without knowledge of the individual sensor responses and without computation or search of any spectral measure as in MUSIC. ESPRIT is a computationally efficient and robust method for estimating DOA.

Let us consider an array of 2*M* antenna elements that consists of two non-overlapping sub arrays of *M* elements each. The elements in each sub array have identical sensitivity pattern and are translationally separated by a known constant displacement vector Δ. The m^{th} element of the first sub array and the corresponding element of the second sub array are displaced by the same displacement vector. Assume that there are $N \leq 2M$ narrow band uncorrelated sources impinging on the array are planar and the sources assumed to be stationary zero-mean random processes additive noise is present in all 2*M* elements and is assumed to be weak stationary zero mean random process with a spatial covariance σ^2. We assume *K* samples are observed by the array. The signals received at the sub arrays can then be expressed as

$$\mathbf{X}_1(n) = \mathbf{AS}(n) + \mathbf{W}_{\mathrm{x}}(n) \qquad (2.18)$$

$$\mathbf{X}_2(n) = \mathbf{A\varphi S}(n) + \mathbf{W}_{\mathrm{y}}(n)\,, \quad n = 1,2,......K \qquad (2.19)$$

Where $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$ are $2M \times K$ matrices of the outputs of the sub arrays, $\mathbf{A}$ is $2M \times L$ array direction matrix, $\mathbf{S}(n)$ is $L \times K$ matrix of the source waveform and $\mathbf{W}_x(n),\mathbf{W}_y(n)$ is $2M \times K$ matrices of elements noise.. The matrix $\boldsymbol{\varphi}$ is a diagonal $L \times L$ matrix of the phase delays between sub array elements for the L signals.

$$\boldsymbol{\varphi} = \text{diag}\left\{e^{j2\pi\Delta\sin\theta_1}, \ldots\ldots\ldots\ldots, e^{j2\pi\Delta\sin\theta_L}\right\} \tag{2.20}$$

The total array output vector is

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1(n) \\ \mathbf{X}_2(n) \end{bmatrix} = \mathbf{GS} + \mathbf{W} \tag{2.21}$$

Where $\mathbf{G} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\boldsymbol{\varphi} \end{bmatrix}$ $\quad \mathbf{W} = \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \end{bmatrix}$

The array output covariance matrix $\mathbf{R}_{XX}$ is

$$\mathbf{R}_{XX} = \mathbf{E}\left[\mathbf{XX}^H\right] = \mathbf{GPG}^H + \boldsymbol{\sigma}^2\mathbf{I} \tag{2.22}$$

Where $\mathbf{P} = \mathbf{E}\left[\mathbf{S}(n)\mathbf{S}(n)^H\right]$ is covariance matrix of the signal. The covariance matrix $\mathbf{R}_{XX}$ will have N signal eigen values e_i, $i = 1,2,3,\ldots\ldots,N$ and $2M-N$ noise eigen values e_i, $i = N+1,\ldots\ldots 2M-N$ ordered as

$$e_1 \geq e_2 \geq e \geq \ldots\ldots\ldots\ldots\ldots e_N > e_{N+1} > \ldots\ldots\ldots e_M = \boldsymbol{\sigma}^2$$

The eigen vectors corresponding to the N largest eigen values are $\mathbf{E}_S = [e_1, e_2, \ldots\ldots e_N]$ and span the signal subspace and eigen vectors corresponding to the noise eigen values is $\mathbf{E}_N = [e_{N+1}, \ldots\ldots\ldots e_{2M}]$span the noise subspace. The invariance structure of the array implies $\mathbf{E}_S$ and it can be decomposed into $\mathbf{E}_{S1}$ and $\mathbf{E}_{S2}$.

$$\mathbf{E}_S = \begin{bmatrix} \mathbf{E}_{S1} \\ \mathbf{E}_{S2} \end{bmatrix} \tag{2.23}$$

By applying eigen decomposition on E_S then

$$\begin{bmatrix} \mathbf{E}_{S1}^H \\ \mathbf{E}_{S2}^H \end{bmatrix} \begin{bmatrix} \mathbf{E}_{S1} & \mathbf{E}_{S2} \end{bmatrix} = \mathbf{E} \Lambda \mathbf{E}^H \tag{2.24}$$

partition E into $N \times N$ sub matrices

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix} \tag{2.25}$$

Eigen values of $\mathbf{E}$ are $\mathbf{e}_n(-\mathbf{E}_{12}\mathbf{E}_{22}^{-1})$, $n = 1,2,.....N$

Then direction of arrival of the signals is

$$\theta_n = \sin^{-1}\left(\frac{-\mathbf{e}_n}{2\pi\Delta_n} \right) \tag{2.26}$$

Where $n = 1,2,.....N$ is number of signals.

2.4 UCA-ESPRIT Algorithm

UCA-ESPRIT is a closed-form algorithm for two-dimensional angle estimation with uniform circular arrays. It is represented a significant advance in the 2-D angle estimation, it provide automatically paired azimuth and elevation angles estimates for each incident signal. UCA-ESPRIT is fundamentally different from ESPRIT in that it is not based on a displacement invariance array structure but rather is based on phase mode excitation and hinges on a recursive relationship between Bessel functions.

The element space manifold is transformed into the beamspace manifold by phase mode excitation.

The N antenna elements are assumed to be omni directional, identical and are uniformly distributed over the circumference of a circle radius *r* in the xy plane $r=\lambda$, *p* equi powered sources in the far field of the array with azimuth ϕ_k and elevation θ_k arrive in the antenna array. We assume *K* samples are observed by the array, the antenna array output matrix is

$$\mathbf{X}(n) = \mathbf{A}\mathbf{S}(n) + \mathbf{W} \quad n = 1,2,......K \tag{2.27}$$

Where **X(*n*)** is $N \times K$ element space data matrix, **A** is $N \times p$ element space array manifold, **S(*n*)** is $p \times K$ complex signal envelopes at the array centre, **W** is $N \times K$ matrix of noise complex envelopes. The signals and noise are assumed stationary zero mean, uncorrelated random processes. The matrix **A** element space array manifold is

$$\mathbf{A} = \left[\mathbf{a}_1(\varsigma,\varphi), \mathbf{a}_2(\varsigma,\varphi), \mathbf{a}_3(\varsigma,\varphi), ..., \mathbf{a}_\mathrm{p}(\varsigma,\varphi)\right] \tag{2.28}$$

The columns of matrix **A** are modeled as

$$\mathbf{a}(\theta) = \mathbf{a}(\varsigma,\varphi) = \left[e^{\mathrm{j}\varsigma\cos(\phi-\gamma_0)}, e^{\mathrm{j}\varsigma\cos(\phi-\gamma_1)}, ..., e^{\mathrm{j}\varsigma\cos(\phi-\gamma_{N-1})}\right] \tag{2.29}$$

where the elevation θ dependence through parameter ς, and the vector $\theta = (\varsigma, \varphi)$ is used to represent the source arrival directions.

Where $\varsigma = k_0 r \sin\theta$, $k_0 = \dfrac{2\pi}{\lambda}$ is wave number *r* is the radius

$\gamma_n = \dfrac{2\pi n}{N}$ $(n = 1,2,.....N)$ is the sensor location

A circle is periodic with period 2π and can be represented in terms of a Fourier series, in such series each weight is called a phased mode. To the uniform circular array the normalized beamforming weight vector with phase mode *m* is

$$\mathbf{w}_{\mathrm{m}}^{\mathrm{H}} = \frac{1}{\mathrm{N}}\left[1, e^{\mathrm{j}2\pi\mathrm{m}/\mathrm{N}}, \ldots\ldots\ldots e^{\mathrm{j}2\pi\mathrm{m}(\mathrm{N}-1)/\mathrm{N}}\right] \tag{2.30}$$

Let M denotes the highest order mode that can be excited by the aperture at a reasonable strength $M = k_0 r = 2\pi$ and M' is the total number of excited modes, $M' = 2M + 1$ When $N > 2M$ the UCA array pattern for mode m is

$$\mathbf{f}_{\mathrm{m}}^{\mathrm{c}} = \mathbf{w}_{\mathrm{m}}^{\mathrm{H}}\mathbf{a}(\varsigma, \phi) \approx \mathrm{j}^{|\mathrm{M}|}\mathbf{J}_{\mathrm{m}}(\varsigma)e^{\mathrm{jm}\phi} \tag{2.31}$$

Where $\mathbf{J}_{\mathrm{m}}(\varsigma)$ is the Bessel function of the first kind of order m.

The beamforming matrix $\mathbf{F}_{\mathrm{e}}^{\mathrm{H}}$ can be defined as

$$\mathbf{F}_{\mathrm{e}}^{\mathrm{H}} = \mathbf{C}\mathbf{Q}^{\mathrm{H}} \tag{2.32}$$

where $\mathbf{C} = \mathrm{diag}\left\{j^{-\mathrm{M}}, \ldots. j^{-1}, j^{0}, j^{-1}, \ldots.. j^{-\mathrm{M}}\right\}$

$\mathbf{Q} = \sqrt{\mathrm{N}}\left[w_{-\mathrm{M}}, \ldots\ldots, w_{0}, \ldots.., w_{\mathrm{M}}\right]$

The beamspace manifold synthesized by $\mathbf{F}_{\mathrm{e}}^{\mathrm{H}}$ is

$$\mathrm{t}_{\mathrm{e}}(\theta) = \mathbf{F}_{\mathrm{e}}^{\mathrm{H}}\mathbf{a}(\theta) = \mathbf{C}\mathbf{Q}^{\mathrm{H}}\mathrm{a}(\theta) = \sqrt{\mathrm{N}}\mathbf{J}_{\varsigma}\mathbf{v}(\varphi) \tag{2.33}$$

where $\mathbf{v}(\varphi) = \left[e^{-\mathrm{jM}\phi}, \ldots.. e^{-\mathrm{j}\phi}, e^{\mathrm{j}0}, e^{-\mathrm{j}\phi}, \ldots.. e^{\mathrm{jM}\phi}\right]$

$\mathbf{J}_\varsigma = \mathrm{diag}\{J_M(\varsigma),.....,J_0(\varsigma),.....,J_M(\varsigma)\}$ matrix is composed of Bessel functions and it is implicitly depend on elevation angle.

The UCA-RB Music algorithm employs the beamformer $\mathbf{F}_r^H$ to make the transformation from element space to beam space and make it array manifold real, the beamformer $\mathbf{F}_r^H$ defined as

$$\mathbf{F}_r^H = \mathbf{V}^H \mathbf{C} \mathbf{Q}^H \tag{2.34}$$

Where $\mathbf{V} = \frac{1}{\sqrt{M'}}[\mathrm{v}(\alpha_{-M}),......, \mathrm{v}(\alpha_0),.......\mathrm{v}(\alpha_M)]$,

$\alpha_i = 2\pi i/M'$ $i \in [-M, M]$ are the azimuthally rotation angles.

Multiplying element space array manifold $\mathbf{a}(\varsigma,\varphi)$ by beamformer $\mathbf{F}_r^H$ we obtain real beamspace manifold $\mathbf{b}(\varsigma,\varphi)$ is

$$\mathbf{b}(\varsigma,\varphi) = \mathbf{F}_r^H \mathbf{a}(\varsigma,\varphi) = \sqrt{N}\mathbf{V}^H \mathbf{J}_\varsigma \mathbf{v}(\varphi) \tag{2.35}$$

The beamformer $\mathbf{F}_r^H$ is applied over the element space data matrix **X,** and then the resulting Beamspace data matrix is

$$\mathbf{Y} = \mathbf{F}_r^H \mathbf{X} = \mathbf{F}_r^H \mathbf{A}\mathbf{S} + \mathbf{F}_r^H \mathbf{w}$$

$$\mathbf{Y} = \mathbf{B}\mathbf{S} + \mathbf{F}_r^H \mathbf{w} \tag{2.36}$$

Where $\mathbf{B} = \mathbf{F}_r^H \mathbf{A}$ is the real valued beamspace direction of arrival matrix containing $b(\varsigma,\phi)$ as its columns,

The array output covariance matrix in beamspace is

$$\mathbf{R}_{YY} = \mathbf{E}[\mathbf{Y}^H \mathbf{Y}] = \mathbf{B}\mathbf{P}\mathbf{B}^H + \boldsymbol{\sigma}^2 \mathbf{I} \tag{2.37}$$

where **P** is the signal covariance matrix. Let $\mathbf{R} = \mathrm{Re}\{\mathbf{R}_{YY}\}$ denotes the real part of the beamspace covariance matrix. The real valued eigen value decomposition of the matrix **R** yield the beamspace signal and noise subspaces.

Let **S** and **W** are orthonormal matrices that spam the beamspace signal and noise subspaces respectively

$$\mathbf{S} = \lfloor e_1, e_2, ...e_\mathrm{p} \rfloor, \ \mathbf{W} = \lfloor e_{\mathrm{p}+1}, ...e_\mathrm{N} \rfloor$$

A phase mode excitation-based $\mathbf{F}_\mathrm{o}^\mathrm{H}$ is employed to synthesize a beamspace manifold having the form required by UCA_ESPRIT. The beamformer $\mathbf{F}_\mathrm{o}^\mathrm{H}$ is

$$\mathbf{F}_\mathrm{o}^\mathrm{H} = \mathbf{C}_\mathrm{o}\mathbf{F}_\mathrm{e}^\mathrm{H} \tag{2.38}$$

Where $\mathbf{C}_\mathrm{o} = \mathrm{diag}\{(-1)^\mathrm{M}, ..., (-1)^1, 1, 1, ...1\}$ and $\mathbf{F}_\mathrm{e}^\mathrm{H}$ is defined in (2.31)

Then the beamspace manifold is

$$\mathbf{a}_\mathrm{o}(\theta) = \mathbf{F}_\mathrm{o}^\mathrm{H}\mathbf{a}(\theta) = \sqrt{\mathrm{N}}\mathbf{J}_{\varsigma-}\mathbf{v}(\phi) \tag{2.39}$$

Where $\mathbf{J}_{\varsigma-} = \mathrm{diag}\{J_{-\mathrm{M}}(\varsigma), ...J_{-1}(\varsigma), J_{-0}(\varsigma), J_1(\varsigma), ...J_\mathrm{M}(\varsigma)\}$

Three vectors of size $M_e = M'-2$ are extracted from the beamspace manifold as follows $a^{(i)} = \Delta^{(i)} a_o(\theta)$, $i = -1,0,1$. Here $\Delta^{(-1)}, \Delta^{(0)}, \Delta^{1)}$ are selection matrices that respectively select from the first, middle and last M_e elements from $a_o(\theta)$. The phases of the vectors $a^{(0)}, e^{j\phi}a^{(-1)}$ and $e^{-j\phi}a^{(1)}$ are the same. The recursive Bessel function relationship $J_{m-1}(\varsigma) + J_{m+1}(\varsigma) = (2m/\varsigma)J_m(\varsigma)$ can now be applied to match the magnitude components of the three vectors. The resulting relationship is

$$\mathbf{\Gamma a}^{(0)} = \mu \mathbf{a}^{(-1)} + \mu \mathbf{a}^{(1)} \tag{2.40}$$

where $\mu = \sin\theta e^{j\phi}$, $\mathbf{\Gamma} = \frac{\lambda}{\pi \mathrm{r}}\mathrm{diag}\{-(\mathrm{M}-1), ...-1, 0, 1, ..., \mathrm{M}-1\}$

Let $\mathbf{A}_o = \mathbf{F}_o^H \mathbf{A}$ denote the beamspace DOA matrix and $\mathbf{S_o}$ is signal subspace matrix that spans *R{A}*. $\mathbf{S}_o = \mathbf{A}_o \mathbf{T}$, where **T** is real nonsingular matrix that relates $\mathbf{S_o}$ and $\mathbf{A_o}$. We have $\mathbf{A}^{(i)} = \Delta^{(i)} \mathbf{A}_o$ and $\mathbf{S}^{(i)} = \Delta^{(i)} \mathbf{S}_o$, $i = -1,0,1$, it is easy to verify the relationship

$$\mathbf{A}^{(1)} = \mathbf{DIA}^{(-1)*}$$

where $\mathbf{D} = \mathrm{diag}\{(-)^{M-2}, \ldots.(-1)^1, (-1)^0, (-1)^1, \ldots..(-1)^M\}$, the matrix **DI** is unitary and is its own inverse. We have $\mathbf{S}^{(i)} = \mathbf{A}^{(i)} \mathbf{T}$ this leads to

$$\mathbf{S}^{(1)} = \mathbf{DIS}^{(-1)*}$$

The equation (3.39) leads to relationship

$$\mathbf{\Gamma A}^{(0)} = \mathbf{A}^{(-1)} \mathbf{\Phi} + \mathbf{A}^{(1)} \mathbf{\Phi}^* \qquad (2.41)$$

Where $\mathbf{\Phi} = \mathrm{diag}\{\mu_1, \mu_2, \ldots\ldots\mu_p\}$, the above equation in terms of signal subspace matrix is

$$\mathbf{\Gamma S}^{(0)} = \mathbf{S}^{(-1)} \mathbf{\Psi} + \mathbf{DIS}^{(-1)*} \mathbf{\Psi}^* \qquad (2.42)$$

where $\mathbf{\Psi} = \mathbf{T}^{-1} \mathbf{\Phi T}$, rewriting the above equation is

$$\mathbf{E} \underline{\mathbf{\Psi}} = \mathbf{\Gamma S}^{(0)} \qquad (2.43)$$

$$\mathbf{E} = \left[\mathbf{S}^{(-1)} : \mathbf{DIS}^{(-1)*}\right], \quad \underline{\mathbf{\Psi}} = \left[\mathbf{\Psi}^T : \mathbf{\Psi}^H\right]^T$$

The above equation is over determined when $M_e = M' - 2 > 2p$ and has unique solution $\mathbf{\Psi}$. Now $\mathbf{\Phi} = \mathbf{T \Psi \Psi}^{-1}$ and eigen values of $\mathbf{\Psi}$ give the diagonal elements of Φ which are $\mu_i = \sin\theta_i e^{j\phi}$, $i = 1,2,\ldots p$.The eigen values $\mathbf{\Psi}$ therefore yield automatically paired source azimuth and elevation angles.

3. Beamforming

Adaptive beamforming is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arrival from a desired direction while signals of the same frequency from other directions are rejected. Beamforming allows the antenna system to focus the maxima of the antenna pattern towards the desired mobile while minimizing the impact of noise, interference and other effects from undesired mobiles.

The main concept in beamforming is to produce interference pattern in signals from the antenna array. By changing the delay and spacing between array elements, we can control the interference pattern and there by maximizing the signal energy in one direction.

Part of the concept below was modeled by us and this solely belongs to us.

Array model:

Consider a uniform linear array with 3 isotropic antenna array elements with element spacing d and signal delay in adjacent elements is ϕ. Assume center element has no delay, the element right to the center element will have delay ϕ, and the element left to the centre element will have delay $-\phi$. We assume polar coordinate system with middle element of array being at origin. In this system each point represent (θ, r), here θ is measured clockwise starting with zero at the axis arrow on the right.

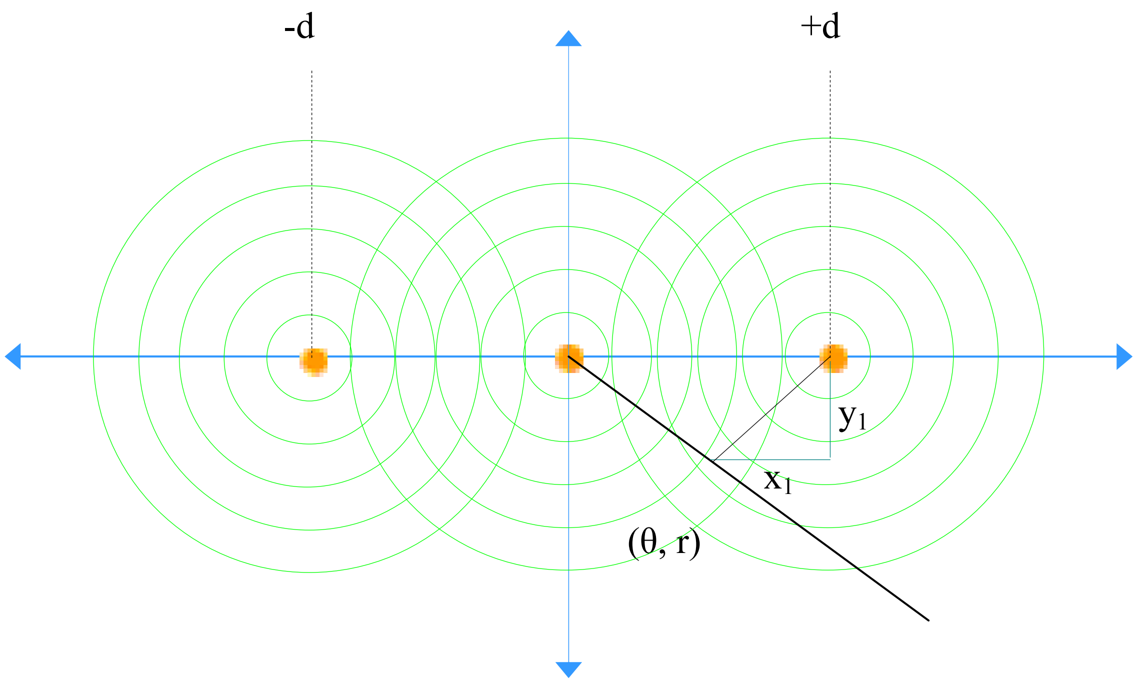

Fig.3.1

The total signal output of the array elements is

$$\mathbf{S}(\theta,r)=\sin(r)+\sin(\sqrt{x_1^2+y_1^2}+\varphi)+\sin(\sqrt{x_1^2+y_1^2}-\varphi) \qquad (3.1)$$

$$x_1=r\cos\theta-d \qquad y_1=r\sin\theta$$

$$\mathbf{S}(\theta,r)=\sin(r)+\sin(\sqrt{(r\cos\theta-\mathrm{d})^2+(\mathrm{r}\sin\theta)^2}+\phi)+\sin(\sqrt{(\mathrm{r}\cos\theta+\mathrm{d})^2+(\mathrm{r}\sin\theta)^2}-\phi)$$

(3.2)

In the above equation (3.2) the first term represents signal from center array element with no delay, second term represents signal from the right side array element with ϕ delay and last term represents signal from the left side array elements with -ϕ delay.

In the above equation (3.2) signal in the direction θ is maximum when $\frac{dS}{d\phi}=0$, then

$$\frac{dS}{d\phi}=\cos(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2}+\phi)-\cos(\sqrt{r\cos\theta+d)^2+(r\sin\theta)^2}-\phi)=0 \tag{3.3}$$

$$\Big[\cos(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2})\cos\phi-\sin(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2})\sin\phi$$
$$-\cos(\sqrt{(r\cos\theta+d)^2+(r\sin\theta)^2}\cos\phi-\sin(\sqrt{(r\cos\theta+d)^2+(r\sin\theta)^2})\sin\phi\Big]=0$$

$$\Big[\cos\phi\Big[\cos(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2})-\cos(\sqrt{(r\cos\theta+d)^2+(r\sin\theta)^2})\Big] -$$
$$\sin\phi\Big[\sin(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2})+\sin(\sqrt{(r\cos\theta+d)^2+(r\sin\theta)^2})\Big]\Big]=0$$

$$\tan\phi=\frac{\cos(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2})-\cos(\sqrt{(r\cos\theta+d)^2+(r\sin\theta)^2})}{\sin(\sqrt{(r\cos\theta-d)^2+(r\sin\theta)^2})+\sin(\sqrt{(r\cos\theta+d)^2+(r\sin\theta)^2})}$$

$$\phi = \tan^{-1}\left[\frac{\cos(\sqrt{(r\cos\theta - d)^2 + (r\sin\theta)^2}) - \cos(\sqrt{(r\cos\theta + d)^2 + (r\sin\theta)^2})}{\sin(\sqrt{(r\cos\theta - d)^2 + (r\sin\theta)^2}) + \sin(\sqrt{(r\cos\theta + d)^2 + (r\sin\theta)^2})}\right] \tag{3.4}$$

The signal strength in the direction of θ is maximum, when the delay in right element is ϕ and delay in left element is -ϕ. We can extend this for any number of elements. for example if there are 5 elements in the array, we can rewrite the equation (3.4) as follows.

$$\phi = \tan^{-1}\left[\frac{\cos(\sqrt{(r\cos\theta - 2d)^2 + (r\sin\theta)^2}) + \cos(\sqrt{(r\cos\theta - d)^2 + (r\sin\theta)^2}) - \cos(\sqrt{(r\cos\theta + d)^2 + (r\sin\theta)^2} - \cos(\sqrt{(r\cos\theta + 2d)^2 + (r\sin\theta)^2})}{\sin(\sqrt{(r\cos\theta - 2d)^2 + (r\sin\theta)^2}) + \sin(\sqrt{(r\cos\theta - d)^2 + (r\sin\theta)^2}) + \sin(\sqrt{(r\cos\theta + d)^2 + (r\sin\theta)^2}) + \sin(\sqrt{(r\cos\theta + 2d)^2 + (r\sin\theta)^2})}\right]$$

Simulation Results:

We consider uniform linear array (UCA) with array elements chosen to *N*=11, with array element spacing d=5 and r=1. From equation (3.4) we calculated phase delay ϕ for the direction θ=120. In this to form a beam in angle θ=120 the relative phase delay between elements is ϕ= -2.4977. The beamforming in angle of direction θ=120 is shown in Fig 3.2.

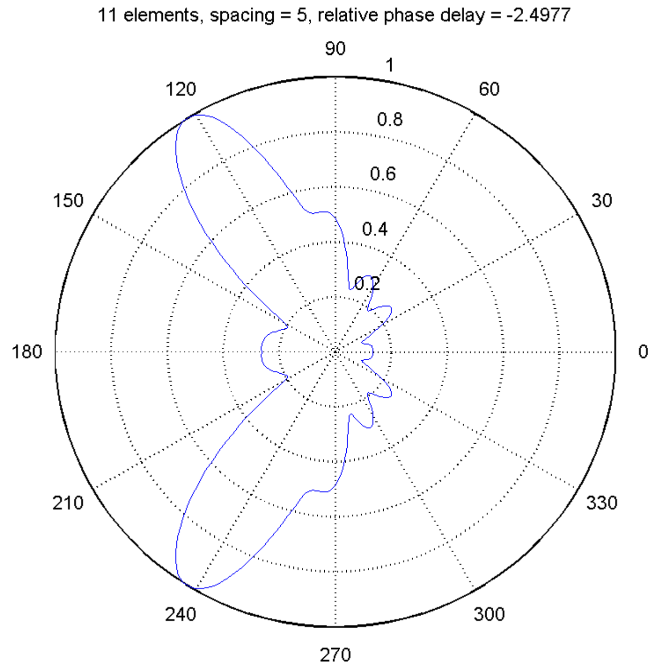
11 elements, spacing = 5, relative phase delay = -2.4977
90
1
120
60
0.8
0.6
150
30
0.4
0.2
180
0
210
330
240
300
270

Fig.3.2

4. Comparisons

Direction of Arrival Estimation Methods

- ESPRIT algorithm reduces computation and storage costs presented by that of MUSIC algorithm.
- ESPRIT algorithm has reduced sensitivity to array perturbations compared to MUSIC.
- But ESPRIT is not so accurate compared to MUSIC algorithm.

Beamforming Methods

The algorithm we have proposed is similar to Sum & Delay Method proposed in [17]. But the resultant beam produced was more accurate with our proposed algorithm.

The two algorithms defer in their delay angle estimation. Delay Expression with our idea was

$$\phi = \tan^{-1}\left[\frac{\cos(\sqrt{(r\cos\theta - d)^2 + (r\sin\theta)^2}) - \cos(\sqrt{(r\cos\theta + d)^2 + (r\sin\theta)^2})}{\sin(\sqrt{(r\cos\theta - d)^2 + (r\sin\theta)^2}) + \sin(\sqrt{(r\cos\theta + d)^2 + (r\sin\theta)^2})}\right]$$

and the delay expression proposed in [17] was

$$\tau_K(\theta) = k\frac{d\sin\theta}{c}$$

5. Conclusions and Future Work

- The primary objective to increase the capacity of a cell and reduce the interference has been attained.
- MUSIC is simple and accurate but requires faster DSPs compared to ESPRIT.
- Sum & Delay Beamforming method proposed by us and proposed in [17] should be optimized to reduce amount of memory needed to store signal strengths.

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Appendix

MatLab code for linear array MUSIC Algorithm:

```
clear all
close all
clc
Ne=12;          %number of Array elements
Nb=10;          %number of bits
dlamda=0.5;  % division of distance between two Array elements and wavelength
                (d/lamda=0.5)
D=3;              %number of signals
angles=[50 100 150]*(pi/180);


%generate a direction vector matrix for the three signals

X=zeros(Ne,Nb);
for k=1:D
   mu=2*pi*dlamda*cos(angles(1,k));
   A=exp(j*mu*(0:Ne-1));

%generate random bits of information

   temp=rand(1,Nb);
   Sr=ones(1,Nb);
   Sr(find(temp<0.5))=-1;
   Si=ones(1,Nb);
   temp=rand(1,Nb);
   Si(find(temp<0.5))=-1;
   S=Sr+j*Si;
%generate the signal with random bits and direction vector

   X=X+A.'*S;
end
Rxx=X*X';
[V,Z]=eig(Rxx);
signals=size(find(diag(Z)>1));%finding the number of signals
E=V(:,signals+1:Ne);

%compute MUSIC spatial spectrum using all noise eigenvectors
```

```
i=0;
for theta=0:.1:180
   i=i+1;
   E0=exp(j*pi*cos(theta*(pi/180))*(0:Ne-1));
   P(i)=10*log10(1/real(conj(E0)*E*E'*E0.'));
end
theta=0:.1:180;
plot(theta,P);
axis([0 180 -60 0]);
```

MatLab code for Uniform array MUSIC Algorithm:

```
clear all
close all
Ne=15; %No. of elements in the circular array
Nb=10; %No. of bits in a signal
dlamda=0.5;% division of distance between two Array elements and wavelength
(d/lamda=0.5)
D=3;%number of signals
ele_angles=[20 40 60]*(pi/180);%elevation angles
azi_angles=[30 80 170]*(pi/180);%Azimuthal angles
X=zeros(Ne,Nb);

for i=1:D
   zeta=2*pi*sin(ele_angles(i));
   k=1;
   for alpha=-(Ne-1)/2:(Ne-1)/2
      b(k)=sqrt(Ne/13)*(besselj(0,zeta)+2*sum(besselj(1:6,zeta) .*
cos((1:6)*(azi_angles(i)-(2*pi*alpha)/13))));
      k=k+1;
   end
   temp=rand(1,Nb);
   Sr=ones(1,Nb);
   Sr(find(temp<0.5))=-1;
   Si=ones(1,Nb);
   temp=rand(1,Nb);
   Si(find(temp<0.5))=-1;
   S=Sr+j*Si;
   X=X+b.'*S;
end
```

```
Rxx=X*X';
[V,Z]=eig(Rxx);
signals=size(find(diag(Z)>0.01));%finding the number of signals
G=V(:,signals+1:Ne);

e=1;
for ele_theta=0:1:90
   zeta=2*pi*sin(ele_theta*(pi/180));

   a=1;
   for azi_theta=0:1:180
      k=1;
      for alpha=-(Ne-1)/2:(Ne-1)/2
         b(k)=sqrt(Ne/13)*(besselj(0,zeta)+2*sum(besselj(1:6,zeta) .*
cos((1:6)*((azi_theta*(pi/180))-(2*pi*alpha)/13))));
         k=k+1;
      end
      P(e,a)=10*log10(1/(b*G*G'*b'));
      a=a+1;
   end
   e=e+1;
end
figure;
plot(real(P))
figure;
plot(real(P'))
```

MatLab code for linear array ESPRIT algorithm:

```
clear all
close all
clc
Ne=4;%nmber of Array doublets
Nb=3;%number of bits
dlamda=0.5;% division of distance between two Array elements and wavelength
(d/lamda=0.5)
D=2;%number of signals
c=330;
angles=[-20 30]*(pi/180);
X=zeros(Ne,Nb);
Y=zeros(Ne,Nb);
for k=1:D
```

```
    mu=2*pi*sin(angles(1,k));
    A1=exp(j*mu*dlamda*(0:Ne-1));
    A2=exp(j*mu*0.25).* A1;

%generate random bits of information
    temp=rand(1,Nb);
    Sr=ones(1,Nb);
    Sr(find(temp<0.5))=-1;
    Si=ones(1,Nb);
    temp=rand(1,Nb);
    Si(find(temp<0.5))=-1;
    S=Sr+j*Si;
%generate the signal with random bits and direction vector
    X=X+A1.'*S;
    Y=Y+A2.'*S;
end
X=X+0.01*randn(Ne,Nb);
Y=Y+0.01*randn(Ne,Nb);
Z=[X;Y];
Rzz=Z*Z';
[V,d]=eig(Rzz);
signals=size(find(diag(d)>0.01));%finding the number of signals
Es=V(:,1:signals);
Ex=Es(1:Ne,:);
Ey=Es(Ne+1:2*Ne,:);
[s,v,E]=svd([Ex Ey]);
E11=E(1:signals,1:signals);E12=E(1:signals,signals+1:2*signals);
E21=E(signals+1:2*signals,1:signals);E22=E(signals+1:2*signals,signals+1:2*sig
nals);
eigenvalues=eig(-E12*inv(E22));

    theta=asin((angle(eigenvalues))/(2*pi*0.25));
    theta=theta*180/pi
```

MatLab code for UCA-ESPRIT Algorithm:

```
%Uniform Circular Array ESPRIT
%No.Array Elements Ne=19
%Ne=2M+7, => M=6, => M'=2M+1=13
%radius of the circle r=lamda(wavelength)
clear all;clc;
```

```
Ne=19;M=6;
Nb=10;
s=2;%number of signals
ele_angles=[40 90]*(pi/180);%elevation angles
azi_angles=[30 260]*(pi/180);%Azimuthal angles
Y=zeros(2*M+1,Nb);
W=sqrt(Ne)*(exp(-j*2*pi*(0:Ne-1)/Ne).'*(exp(-M:M)));
C=diag([(-j).^(-M:0) j.^(1:M)]);
B_fcaH=C'*W';
K_H=(1/sqrt(2))*([eye(M) zeros(M,1) fliplr(eye(M));zeros(M,1)' sqrt(2)
zeros(M,1)';j*fliplr(eye(M)) zeros(M,1) -j*fliplr(eye(M))]);
F=diag([(-1).^((-M-1):1:-2) ones(1,7)]);
Br_H=K_H*F'*B_fcaH;
X=zeros(Ne,Nb);
for i=1:s
    zeta(i)=2*pi*sin(ele_angles(i));
    A=(exp(j*zeta(i)*cos(azi_angles(i)-2*pi*(0:Ne-1)/Ne))).';

    temp=rand(1,Nb);
    Sr=3*ones(1,Nb);
    Sr(find(temp<0.5))=-3;
    Si=3*ones(1,Nb);
    temp=rand(1,Nb);
    Si(find(temp<0.5))=-3;
    Si=Sr+j*Si;
    X=X+A*Si;
end
X=X+wgn(Ne,Nb,0,'complex');
Rxx=X*X';
Ryy=Br_H*Rxx*Br_H';
[U,Z,V]=svd(real(Ryy));
signals=size(find(diag(Z)>0.01));%finding the number of signals
Sr=V(:,1:signals(1,1));
S=F*K_H'*Sr;
delta_1=eye(2*M+1-2,2*M+1);delta0=[zeros(2*M+1-2,1) eye(2*M+1-2,2*M+1-
1)];delta1=[zeros(2*M+1-2,2) eye(2*M+1-2,2*M+1-2)];
S_1=delta_1*S;S0=delta0*S;S1=delta1*S;
Tau=(1/pi)*diag([-M:-2 0 2:M]);
C1=diag([ones(1,M-2) -1 -1 ones(1,M-1)]);
P=S_1'*S_1;Q=S_1'*C1*S_1;L=S_1'*Tau*S0;
sye_real_imag=inv([real(P+Q) imag(Q-P);imag(P+Q) real(P-
Q)])*([real(L);imag(L)]);
```

```
sye_real=sye_real_imag(1:signals,:);sye_imag=sye_real_imag(signals+1:signals
+signals,:);
sye=sye_real+j*sye_imag;
mu=eig(sye);
elevation=asin(abs(mu))*180/pi
azimuth=angle(mu)*180/pi
```

MatLab code for Beamforming algorithm proposed by us

```
% beamforming.m:
% Polar plot of angular average of sum of squares of amplitudes which
% interfere to create beam-formed response.  Exact same set-up here as in
% file polar_interference.m; the only difference is here the amplitudes are
% averaged at each angle to create a line plot rather than imagesc() style
% of plot.

function polar_ang_avg(angle)
angle=120;
theta=angle*pi/180;
N=11;   % number of transducer elements
d=5;  % spacing between elements (radians)
% phase delay calculation (radians)
p=pi*atan((cos(sqrt((100*cos(theta)-d).^2+(100*sin(theta)).^2)/pi)-
cos(sqrt((100*cos(theta)+d).^2+(100*sin(theta)).^2)/pi))...

/(sin((sqrt((100*cos(theta)+d).^2+(100*sin(theta)).^2)/pi))+sin((sqrt((100*cos(thet
a)-d).^2+(100*sin(theta)).^2))/pi)));


t=0:360;
t=t*2*pi/360;


z=S(N(1,1),d(1,1),p(1,1)); % Signal construction using subfunction
Z=sum(abs(z),2);
polar(t,Z'./max(Z'));
title([num2str(N) ' elements, spacing = ' num2str(d) ...
      ', relative phase delay = ' num2str(p)]);
filenm = ['n' num2str(N) 'd' num2str(d) 'p' num2str(p) '.polar.png'];
disp(['printing file ' filenm]);
eval(sprintf('print -dpng "%s"',filenm));
```

Signal construction subcode

```
function f = S(N,d,p)
% creates f(angle,radius) (of dim 361,101)
% takes args of N=numelements, d=spacing(rads), p=delay(rads)

for a=0:360;
  t=a*2*pi/360;  % convert to radians
  for r=0:100;
    g=0;
    for n = -(N-1)/2 : (N-1)/2
        g = g + sin( ((sqrt((r*cos(t)-n*d)^2+(r*sin(t))^2)) + n*p)/pi  );
    end;
    f(a+1,r+1)=g;
  end;
end;
```