Master's Degree Thesis
ISRN: BTH-AMT-EX--2007/D-02--SE

Vibration-based Technique for Young's Modulus Determination and Permanent Deformation Assessment of Sheet Materials

Mukarram Hussain

Ramireddy Viswanath

Department of Mechanical Engineering
Blekinge Institute of Technology
Karlskrona, Sweden

2007

Supervisor: Etienne Mfoumou, Lic. Mech. Eng.

Vibration-based Technique for Young's Modulus Determination and Permanent Deformation Assessment of Sheet Materials

Mukarram Hussain

Ramireddy Viswanath

Department of Mechanical Engineering,

Blekinge Institute of Technology,

Karlskrona, Sweden.

2007

Thesis submitted for completion of Master of Science in Mechanical Engineering with emphasis on structural Mechanics at the Department of Mechanical Engineering, Blekinge Institute of Technology, Karlskrona, Sweden.

Abstract

This thesis work is devoted to low frequency vibration based method, with remote excitation and laser detection for nondestructive testing, characterization of sheet materials and also focus on the permanent deformation introduced by plastic deformation. By this method dynamic properties of different materials can be estimated from the measurement of the resonance frequency in flexural mode. Low Density Polyethylene (LDPE), Ethyl Acid Acrylic (EAA), Aluminium foil (Al) and Paperboard (PPR) are investigated in this work. It is observed that the method allows detection of permanent deformation in sheet materials loaded in tension, as well as determination of Young's modulus with some exceptions. This study also confirms the use of the presented method for health monitoring of materials having no bending stiffness, developed in a previous work.

Key words

Resonance frequency, material characterization, remote excitation, nondestructive testing, Mechanical Properties.

Acknowledgements

This work was carried at the Department of Mechanical Engineering, Blekinge Institute of Technology, Karlskrona, Sweden, under the supervision of Lic. Etienne Mfoumou.

We wish to express our sincere appreciation to Lic. Etienne Mfoumou for his guidance and professional engagement throughout the work.

We want to thank Kristian Haller Ph.D. student for his support and valuable discussion.

We want to thank our friends and all those who are not named above and have contributed to this work.

Finally, we would like to dedicate this work to our parents in INDIA for their support and inspiration.

Karlskrona, 19th Jan 2007.

Mukarram Hussain.

Viswanath Ramireddy.

Contents

1. Notations **5**

2. Introduction **7**

3 Previous work **9**

3.1. Determination of Young's Modulus from Static Deflection Data 9
3.2. Impulse Excitation of Vibration 10
3.3. Bulge Test 11
3.4. MAT-TEST 12
3.5. TA-ESPI Test 13

4. Theoretical Analysis **15**

4.1. Geometrical configurations and assumptions 15
4.2. Governing Equations 16

5. Experiment. **18**

5.1. Preparing the sample for Test 18
5.2. Experimental setup 19
5.3. Experimental Procedure 21

6. Simulation **23**

6.1 ABAQUS/CAE MODEL 23
6.1.1. Elements 23
6.1.2. Boundary Conditions and mesh 24
6.1.3. Modeling and Post-Processing 25
6.1.4. Assumptions 26

7. Results and Interpretation. **27**

7.1. Experimental Results Obtained for EAA. 27
7.2. Experimental Results Obtained for PPR 31
7.3. Experimental Results Obtained For Al. 34
7.4. Comparison of Simulation and Experiment. 36

8. Conclusion and Further Work. **40**

9.References. **41**

Appendix A. **43**

A.1.1. Permanent deformation assessment. 43

A.1.2. Experimental work. 46
A.2. Matlab Scripts 54
A.3. ABAQUS/CAE INPUT FILE. 65

1. Notations

A	*Amplitude[m]*
a	*Width[m]*
b	*Length [m]*
c	*Velocity[m/sec]*
F	*Frequency [Hz]*
H	*Thickness of Material[m]*
K	*Global stiffness matrix*
Kmn	*Stiffness matrix [-]*
Mmn	*Mass Matrix [-]*
Nc	*Number of elements [-]*
T	*Tensile force [N]*
T	*Time[s]*
u	*Displacement in an element in x-direction[m]*
ui	*Nodal displacement[m]*
v	*Displacement in an element y-direction[m]*
vi	*Nodal displacement[m]*
W	*Length[m]*
W	*Angular velocity [rad/sec]*
W	*Strain energy density [J/m3]*
E	*Displacement[m]*
P	*Density of material [Kg/m3]*
ξ	*Displacement of membrane*

Indices

I	*Node*
J	*Node*
Load	*fixed load conditions*
M	*Degree of freedom*
N	*Degree of freedom*
n	*Number of nodes in the element*
nm	*Mode numbers*

Abbreviations

NDT	Non Destructive Testing.
ASTM	American Society for Testing Materials.
PPR	Paper Board.
EAA	Ethyl Acid Acrylic.
Al	Aluminium foil
LDPE	Low Density Polyethylene

2. Introduction

This study aims is to determine the Young's modulus of sheet materials based on vibration technique and also focus on the permanent deformation introduced by plastic deformation. Permanent deformation assessment of sheet materials (considered as Membranes) using vibration analysis in flexural modes, and compare the values with those obtained with some other existing static and dynamic methods. In this work young's modulus determined from remote excitation of vibration in flexural mode and material response recorded using laser vibrometer.Vibration measurements are made for a variety of reasons including the verification of an analytical model of a system, and the determination of the resonance frequency for a system. Resonance frequencies are extremely important in predicting and understanding the dynamic behaviour of a system, but also in Non Destructive Estimation of material mechanical properties. We analyze a thin film, considered as an elastic body in which the mass and deformation properties are continuously distributed. Mechanical properties vary a lot for thin films which might be in processing. Deformation is occurred between the limit of elastic region and plastic region.

Chapter 3 describes the experiments which are co related with our thesis. The main aim of this study is to describe the start of gaining knowledge of fundamentals required for our thesis by analyzing from the past work. Some of the principals involved in these experiments are studied and is implemented in our thesis for finding a simple and easy way of solution to the problem in our thesis.

Chapter 4 describes the theory involved and deriving the governing equation, Estimation of Young's Modulus and solving the equation for permanent deformation. This chapter also describes what we analyze about permanent deformation.

Chapter 5 describes the experimental procedure and also in detailed about the material properties and specimen preparation. This chapter 5 also describes the experimental setup with diagrams and experimental procedure in steps involved in the experiment.

Chapter 6 describes about Simulation. Simulation is very important tool, when it comes to visualize results. The simulation model results are also useful in developing a NDT measurement technique or else correcting the existing results. This kind of cooperation between Simulation and

Experimental results form the basis of the approach used, permitting validation of the implemented model.

Chapter 7 describes about the results achieved and also an interpretation is made from the results achieved from our experiment.

Chapter 8 discusses about the conclusion made from the experiment and further work need if any with correspondence to the results which we achieved from our thesis experiment.

3 Previous work

3.1. Determination of Young's Modulus from Static Deflection Data

A technique for extracting Young's modulus from static deflection data was presented [1]. The technique was validated in experiments on thin film specimens of silicon nitride deposited on a silicon substrate (see figure3.1) under different conditions. The figure 3.1 represents the cantilever beam bending. In Cartesian coordinates relative to the root the tip location is (Xf, yf) and relative to the start of scan it is (Xf, yf).The distance of the profiler tip from the root measured along the beam is denoted by sf. Finite element analysis was used to assess the influence of factors affecting the bending of thin films, and thus guide the analysis of micro cantilever deflection data for reliable characterization of the material.

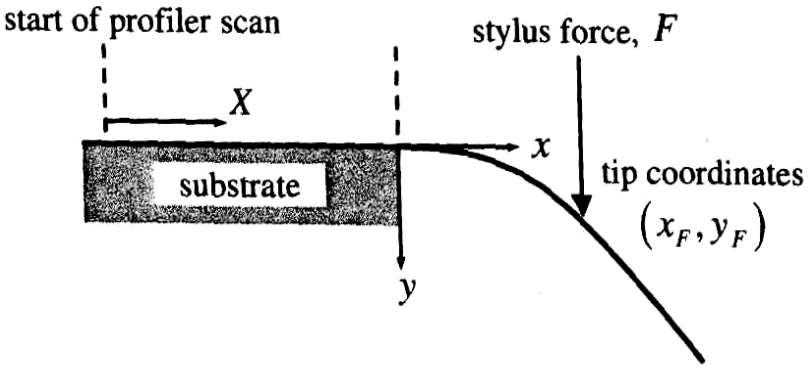


Figure 3.1: *Notation used to represent the cantilever beam bending* [1].

General procedure for testing cantilever beams fabricated from a thin film material deposited on a silicon substrate in order to extract Young's modulus is known from here. The testing and data analysis steps as followed in the experiments and based root effects and large deflections. The final value for young's modulus is obtained by taking an average over this range and is given for each material used for test.

Mechanical characterization techniques, based on the static bending of beams, are applicable to a wide range of materials. Test specimens, such as cantilevers or fully clamped bridges, can be fabricated using a variety of non specialist surface and bulk micromachining techniques from any material (insulating or conducting) with sufficient stiffness to permit a free standing structure. The approach of Tai and Muller theory [2] utilizes a scanning surface profiler to deflect transverse displacements relative to the horizontal.

3.2. Impulse Excitation of Vibration

Another standard test method for determining Young's Modulus of materials is by Impulse excitation of vibration. The process is explained in figure 3.2. This test method covers the measurement of the fundamental resonant frequencies for the purpose of calculating Dynamic Young's Modulus. The method is non destructive and is used for material characterization [3].

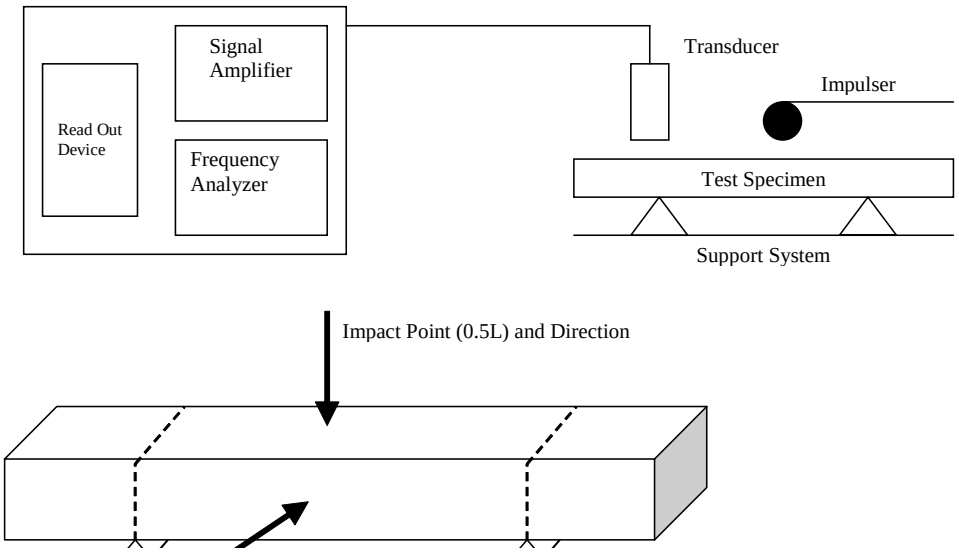


Figure 3.2 *Sketch of Diagram apparatus* [3].

$$E = 0.9475 \cdot \left(\frac{m \cdot f_f^2}{b} \right) \cdot \left(\frac{L^3}{t^3} \right) \cdot T_1$$

E : Young's modulus;

m : mass of the bar;

b : width of the bar;

L : length of the bar;

t : thickness of the bar;

f_f : fundamental resonant frequency in flexure;

T_1 : correction factor.

3.3. Bulge Test

One of the techniques to determine the properties of thin film materials is bulge test. Bulge test is a specialized form of deflection technique, where the substrate is removed to leave a thin membrane of the film, surrounded by the film still supported on the substrate. The manufacture of this device is well understood, and is the basis for many pressure sensor devices [4].The bulge test method is a miniaturization of the layer removal method where the bow of free standing film is measured to the residual stress. If the membrane is pressurized differentially and the bow is measured using interferometric techniques, a stress strain curve can be obtained, the slope of which gives the modulus of the film, and further more intercept gives the initial residual stress.

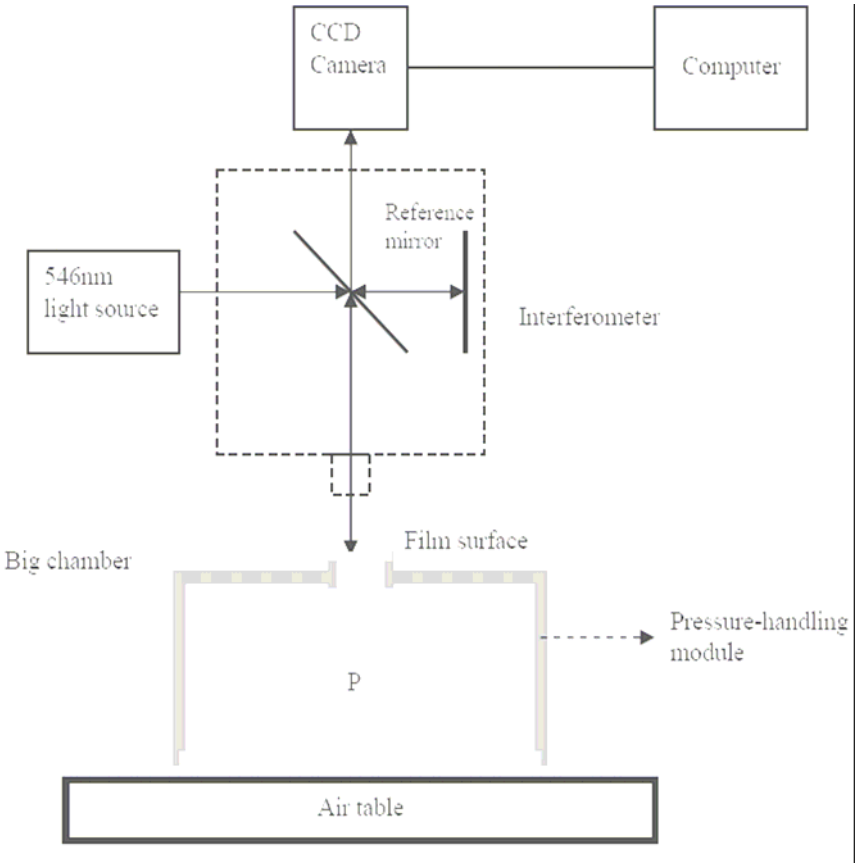


Figure3.3.*Setup of the interferometric displacement measurement system [4]*

3.4. MAT-TEST

One of the several testing techniques to determine the properties of film materials characterization is MAT –TEST. MAT TEST is widely applicable and this test requires common laboratory equipment [5].

MAT-TEST is a test method for evaluating the young's modulus and breaking stress of thin film materials. The MAT-TEST is based on beam-bending. The material under test also can be represented as MUT, is patterned into cantilever beams as shown in the figure 3.4.Substrate material

is removed, typical wet etched. Cantilevers are deflected with a surface profilometer.

The measured deflection is a function of the length of the beam, the force applied by the stylus and the stiffness. Young's Modulus of material under test can be obtained from the deflection data.

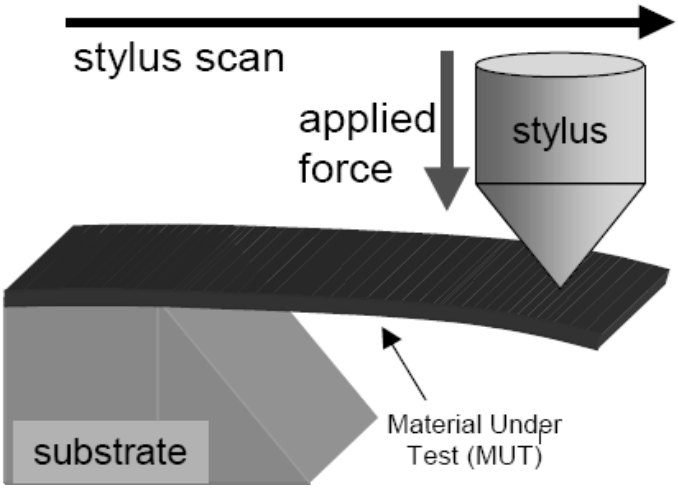


Figure3.4. *MAT TEST* [5] [1]

[1] http://micromachine.stanford.edu/~hopcroft/Research/MATTest_JUN2003.pdf

3.5. TA-ESPI Test

Some few other methods using a remote excitation and a non-contact sensing technique do exist such as TA-ESPI [6]. The method uses vibration analysis and Euler-Bernoulli method to determine the Young's modulus of a given material. The experimental setup is as shown in figure 3.5 and the measured parameter is the natural frequency as expressed in equation (4.1) below,

'n'th natural frequency is given by

$$\omega_n = \frac{c_n^2}{l^2}\sqrt{\frac{EI}{m}} \quad . \qquad (4.1)$$

Elastic modulus

$$E = \frac{m\,\omega_n^2\,l^4}{c^4\,I} \tag{4.2}$$

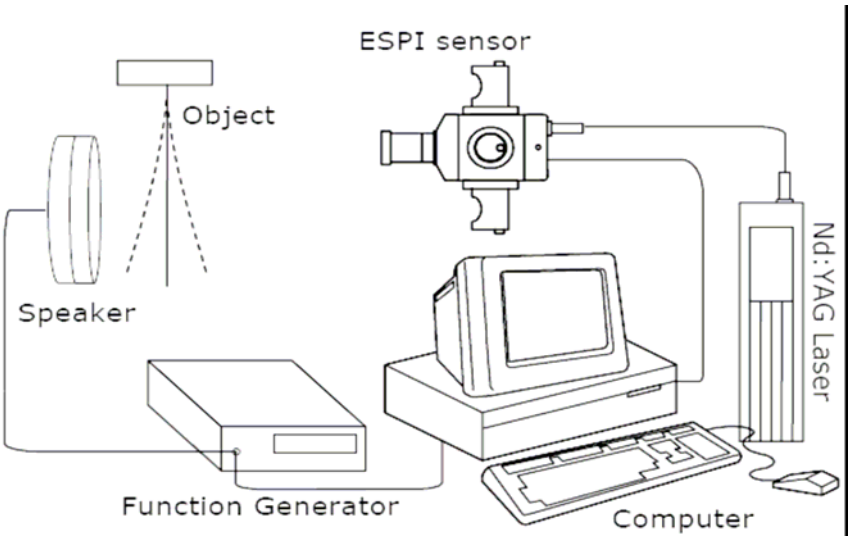


Figure 3.5.*TA-ESPI Setup* [6].

It follows from the previous equation that the Young's modulus can be extracted.

The specimens are excited by a loud speaker from back of the specimen, and the exciting frequency is controlled by a function generator. Dynamic response of the cantilevers is measured through TA-ESPI. Laser is subjected on to the specimen by an optical fibre into a sensor; the out of plane displacement sensitive interferometer is set up inside the sensor. Successive time averaged images are added to the reference image when the beam is vibrating and are displayed on a big screen. Vibration mode shape, according to the change of exciting frequency is noted and resonance frequency at the maximum fringe order and the maximum displacement of the object are defined. Resonance frequencies given at 2, 3, 4, 5,6th vibration modes are used to estimate the elastic modulus.

4. Theoretical Analysis

4.1. Geometrical configurations and assumptions

The relevant theory for conducting our investigation was developed in previous works [7, 8].

It mainly considers structural elements in form of rectangular strips clamped on two edges as shown in the figure 4.1.

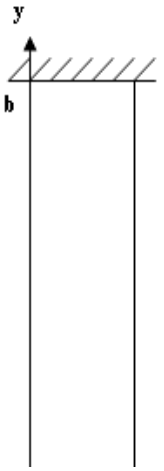


Figure 4.1.*Specimen clamped along width and free along the height.*

The materials investigated in the experiment are considered homogeneous and isotropic. The width of a specimen as shown in figure 4.1 is kept small enough, nearly 15mm, to neglect bending wave propagation in the x-direction. The tensile force F is assumed to remain constant during small vibrations in the y-z plane.

4.2. Governing Equations

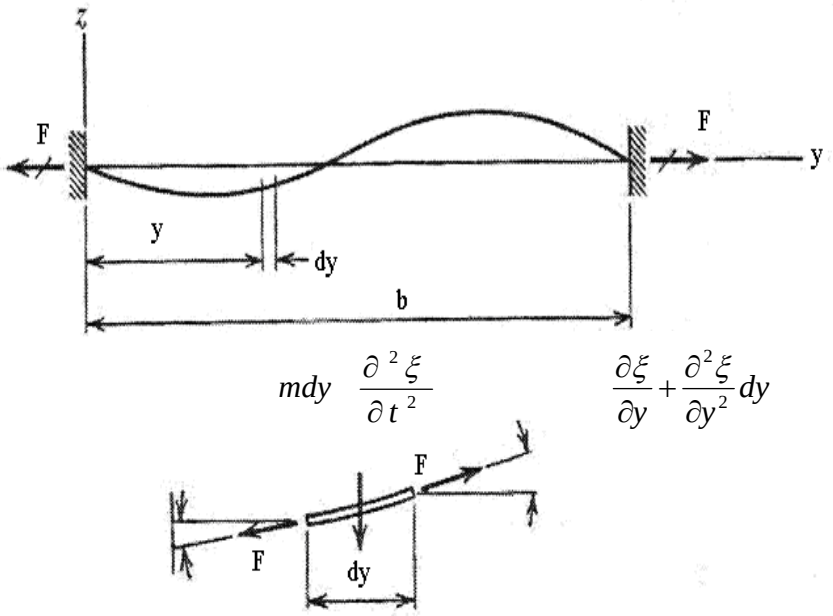


Figure 4.2.*Free body Diagrams.* [8]

The vibration of membrane which lies in the plane of Cartesian coordinate system and having intrinsic elasticity is governed by [7,8]

$$\frac{\partial^2 \xi}{\partial^2 t} - c^2\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right) + d^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \xi = \frac{p(x,y,t)}{\rho h.} \qquad (4.3)$$

Here ξ is the displacement of membrane along the z -axis from its equilibrium position. z = 0, c = $\sqrt{T/(\rho h)}$ is the velocity of propagation of bending wave at zero intrinsic elasticity which is determined by the tensile force T per unit length of boundary of membrane.' ρ ' is the Density and 'h' is thickness of the membrane.

The external pressure p(x, y, t) is a function of time and of spatial coordinates.

$$d^2 = \frac{Eh^2}{12\rho(1-v^2)}$$

Where, E is the young's modulus, v is the poisons ratio.

Assuming free vibrations of the membrane and for the given boundary condition, the solution was shown to be a sum of membrane modes as follows [7, 8].

$$\varepsilon = \sum_{m,n=0}^{\infty} \varepsilon_{mn} = \sum_{m,n=0}^{\infty} A_{mn} \cos\left(m\pi \frac{x}{a}\right) \sin\left(n\pi \frac{y}{b}\right) \sin\left(\omega_{mn} t + \varphi_{mn}\right) \qquad (4.4)$$

Here the constants A_{mn} , ψ_{mn} are amplitude and plane, ψ_{mn} is the natural frequency of mode mn .

The natural frequencies were expressed as:

$$\omega_{mn} = c\sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2 \left[1 + \frac{d^2}{2c^2}\left(\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2\right)\right]} \qquad (4.5)$$

Where, m=0, 1, 2, 3 …, n=1, 2, 3…, a, b, are the dimensions of the membrane.

It was also shown that the intrinsic elasticity can be neglected for the thicknesses under investigation in this study for small vibrations, which led to the following expression for bending modes in the length direction [8]

$$f_{0n}{}^2 = \frac{En^2}{4\rho b^2}\varepsilon \qquad (4.6)$$

'ε' is the longitudinal strain.

The square of the resonance frequency is a linear function of the strain, allowing extraction of dynamic Young's Modulus E from the slope of the curve.

5. Experiment.

5.1. Preparing the sample for Test

The various types of materials used for the experiment as specimen are Paper board, Aluminium Foil and EAA. For each material investigated, at least 6 pieces of 200mm x 15 mm size specimens (taken in cross machine direction) are cut using a paper cutter. The materials studied are presented in a table 5.1 below.

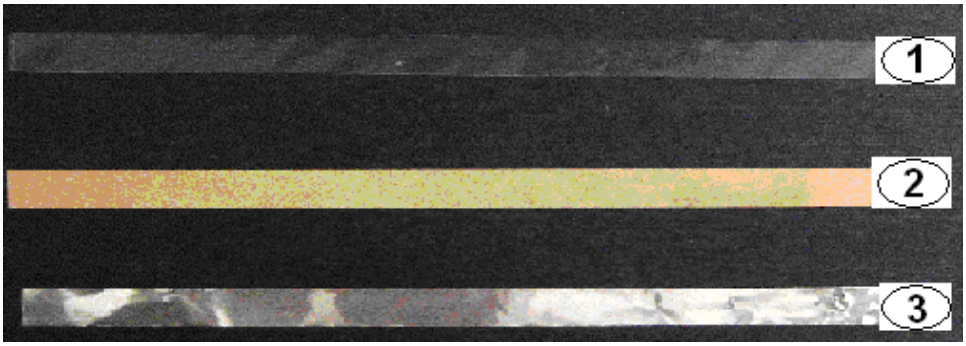


Figure 5.1. *Different specimen's EAA (1), Paper board (2), Al foil(3).*

Table 5.1 *Different material specimen properties.*

Material	Young's Modulus [M Pa]	Thickness (Microns)	Density (Kg/m3)
EAA	157.529	121	924
Al Foil	70000	9	2700
Paper Board	8000	100	684
LDPE*	132.1	33	905

* Obtained from [6].

Since LDPE layer was produced by extruding melted LDPE [6] on a PET layer with a table extruder, it is first pre-peeled (in one corner) from PET before cutting in the desired size. The 2 layers are then separated carefully by hands before the test.

The materials were placed in a conditioned room at 23 degrees centigrade and 50% relative humidity for atleast 3 days.

5.2. Experimental setup

The MTS Tensile Test Machine [4] is used for clamping and loading the specimen under test. This machine has a crosshead - a horizontal bar - that can be very accurately traversed up and down under digital control by means of two large jack screws. 100 KN load cell is used.

A loudspeaker is used for remote excitation of the specimen by a harmonic signal generated from a function generator. Laser vibrometer was used to record the response of the specimen. Unlike traditional contact vibration transducers, laser-based vibration transducers, or laser vibrometer, require no physical contact with the test object. Remote, mass-loading-free vibration measurements on targets that are difficult or impossible to access are typical examples of applications where a laser-based vibration transducer would be the natural choice. Primary advantage of using a laser vibrometer s the ability to conduct non-contact measurement of surface velocity or displacement. The sensitivity of materials studied to transducer gluing or to any other contact device is the main motivation of using laser vibrometer for the dynamic response measurement in this work.

The vibrations recorded from the laser vibrometer are monitored on the dashboard of an Oscilloscope .The time domain signal obtained allows determining peak responses of the specimen under vibration. Figure 5.2. shows the block diagram and a photo of the experimental arrangement.

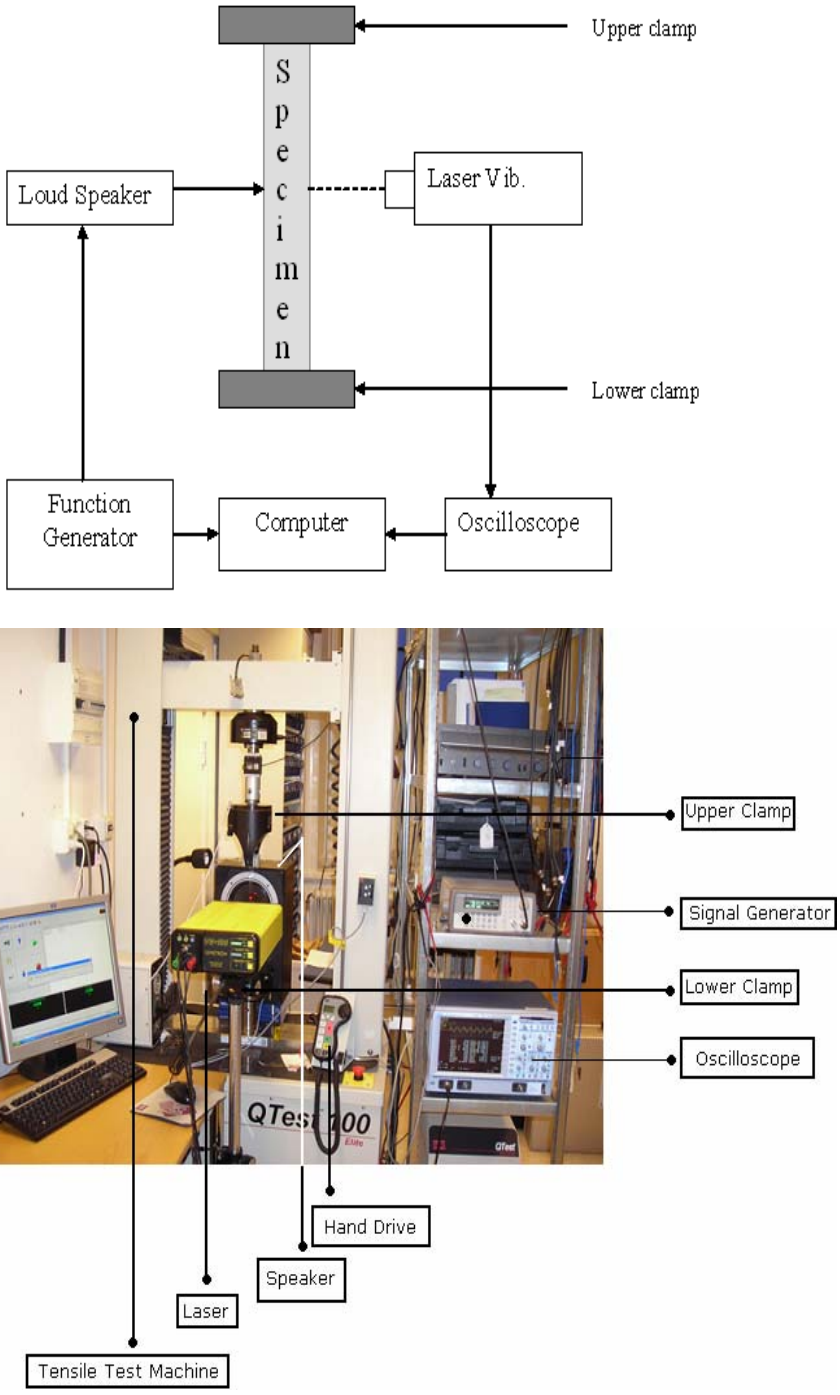


Figure 5.2. *Experimental Setup.*

5.3. Experimental Procedure

The MTS-software Test Works 4.08 is used for tensile testing. Input parameters such as grip separation, test speed, width and thickness are introduced. The Testing Machine controller is set to raise the crosshead at a strain rate of 5% per minute for tensile testing.

Specimens are tested by attaching them between the clamps. They are then submitted to increasing load until rupture or until the STOP button is pushed. A load-extension plot is obtained, and a data file is generated for possible future analysis.

For acoustic measurement, the specimen is submitted to increasing and decreasing manual loading steps, using the handset. In parallel, vibrations of the specimen are remotely excited through the loudspeaker. For each loading step, the vibrations as velocity measurement for an investigated point are recorded by the laser vibrometer. The resonance frequency of the system, corresponding to the peak amplitude, is monitored from the dashboard of the oscilloscope by tuning the signal from the function generator. Resonance frequency and extension of the specimen are the only measured parameters in this case, and will serve for further analysis. The detailed procedure is described in the following steps below for easy way of understanding.

The following Steps are followed during experimentation.

1. MTS machine is initiated and the software is activated on the computer.
2. Units are checked and changed before running the experiment in the test work software.
3. The inputs such as the length (grip separation), the width, the thickness and the test speed were set in Method>Edit>Method>Configuration –items>inputs.
4. The specimen is carefully mounted into the tensile test machine clamps without disturbing the material.
5. Before running the test, the fixtures are examined. The loop connection between laser Vibrometer, Oscilloscope, Signal Generator, Tensile Test Machine, Computer and Loud Speaker are examined.

6. To get the resonance frequency, Tuning is done to obtain a value of peak resonance .The value is noted and experimented by changing the length of the cross head. Using an Agilent function generator the excitation frequency can be tuned.
7. Approximately six to seven values are taken by changing the lengths of cross head.
8. Now the test can be started (For E) using the PLAY button either from the interface in the software or the headset.
9. After the tensile test is completed, there appears a prompt in the software informing the cross heads return to its original position, demounting the specimen is followed by acceptance of the prompt
10. The results are obtained from the review window and can be exported in the form of text files and can be used for further work in ABAQUS and MATLAB.

6. Simulation

Finite Element method can be considered as special case of variational methods with special emphasis on the systematic treatment of complicated geometry. Finite element method is a powerful tool to numerically solve mathematical problems. Finite element method has various aspects on a computational point of view efficiency should be sought in solving actual problems which results in large degree of freedom.

In finite element method the object is divided into smaller element for accurate results. This element along with the boundary conditions is termed as finite element model. each element is assigned a set of characteristic equations(describing mechanical properties, boundary conditions, imposed forces and other properties).this equations are then solved as a set of simultaneous equation to predict the behavior of the object.

To get a more accurate result the object is divided into much smaller elements.

1. Pre-processing of input data, to discrete function and equation.
2. Computing to solve the matrix equation.
3. Post-processing of output results, to retrieve the solution from discretization.

6.1 ABAQUS/CAE MODEL

It is decided that simulation part can be carried out in ABAQUS/CAE program. The specimen is modeled with given boundary condition. The result obtained from simulation is compared with experimental results.

6.1.1. Elements

The element type M3D4R is used to model the material. It is a three-dimensional four-node membrane element with reduced integration. Reduced integration provides accurate results at a significantly less running time. Membrane elements are sheets in space that can carry membrane force, so the only nonzero stress components in the membrane are those

components parallel to the middle surface of the membrane, therefore the membrane is in a state of plane stress [12]. M3D4R element has been selected for meshing. This type of element is appropriate for the thin shell elements with negligible shear flexibility. There were 8 elements in X-direction and 80 elements in Y-direction.

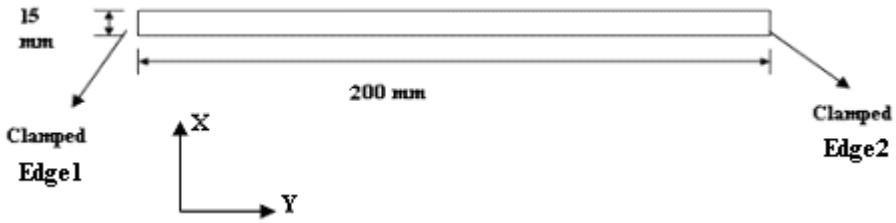


Figure 6.1. *Specimen*

6.1.2. Boundary Conditions and mesh

The specimen is clamped on both the ends within the elastic region. There is no movement in any direction.

Table 6.1. *Boundary conditions at the Edge 1 and Edge 2 as illustrated in table.*

Edges (1) & (2)	X	Y	Z
Translation	Fixed-0	Fixed-0	Fixed-0
Rotation	Fixed-0	Fixed-0	Fixed-0

Table 6.2. *Element type and solver used as illustrated in table.*

Element type	*Solver used*	*Element in X-direction*	*Element in Y-direction*
M3D4R	*Lanczos*	*80*	*8*

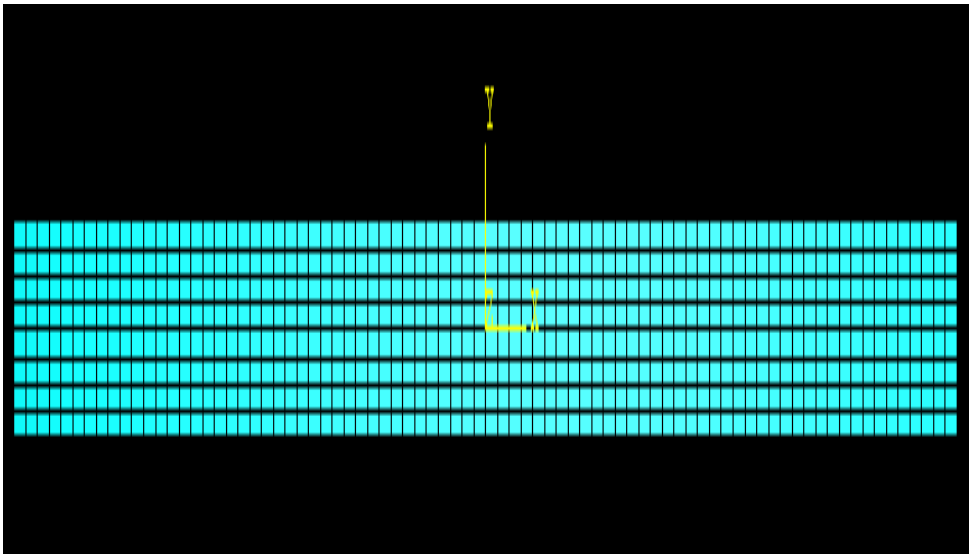


Figure 6.1. *Mesh of the model in ABAQUS/CAE*

6.1.3. Modeling and Post-Processing

Steps in construction:

1. A part is created using the part Module for analysis work. The specimen is modelled according to the dimensions used in experimental work.

2. Property model is used to create the material and to define its properties. Density, Young's modulus and Poisson's ratio is given as inputs and its section properties were defined and assigned. A shell homogeneous section was created and the thickness of the membrane is given as input.
3. To define the geometry of the assembly by creating instances of a part and then positioning the instances relative to each other in a global coordinates system, assembly module is used.
4. The boundary conditions were set as independent therefore steps have been defined in the analysis by using step module.
5. The load module is used to define prescribed conditions .boundary conditions described above is applied.
6. Mesh module is used to generate finite element mesh.M3D4R four node quadrilateral membrane reduced integration is used.
7. After defining all task of the model the Job module has been used to analyze the model.
8. Visualization module allows viewing the results graphically using different methods.
9. The resonance frequency of the fundamental mode is of utmost interest.

6.1.4. Assumptions

Some assumptions are made to keep the model simple.

1. Material is assumed to be isotropic and linear elastic.

2. Poisson's ratio is assumed to be 0.3 for the material.

7. Results and Interpretation

7.1. Experimental Results Obtained for EAA

Figure 7.1 shows the load versus extension curve obtained from tensile testing for EAA.

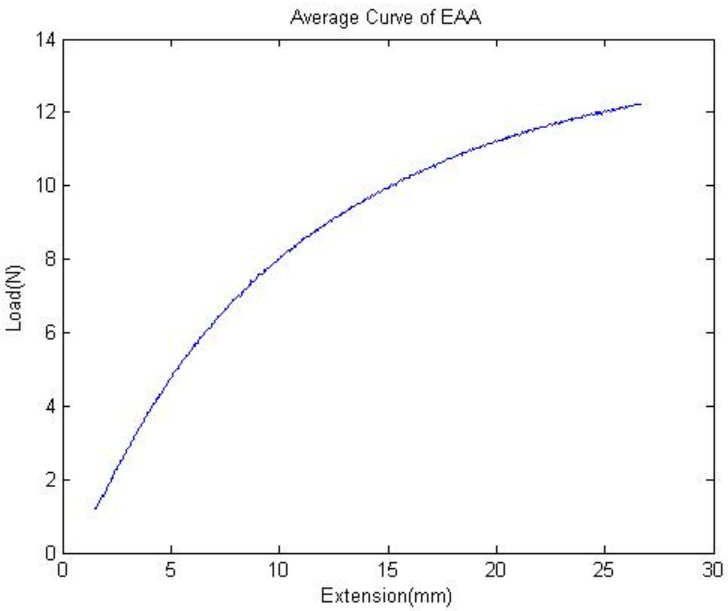


Figure 7.1.*Average curve of EAA.*

Figures 7.2 to 7.7 show acoustic measurements performed on EAA at different loading and unloading steps. It is observed that the results agree quite well at least with in plastic that is until the extension of around 7 mm , there is a small difference in frequency with loading and unloading steps and this difference becomes greater and greater when the increment of extension steps in plastic region with loading and unloading.

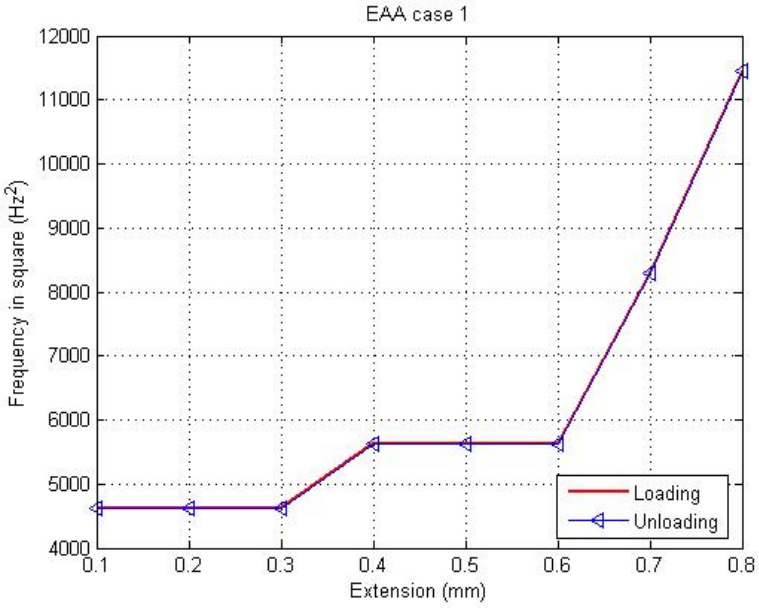


Figure 7.2 *Relation between frequency in square and extension for EAA. Case 1*

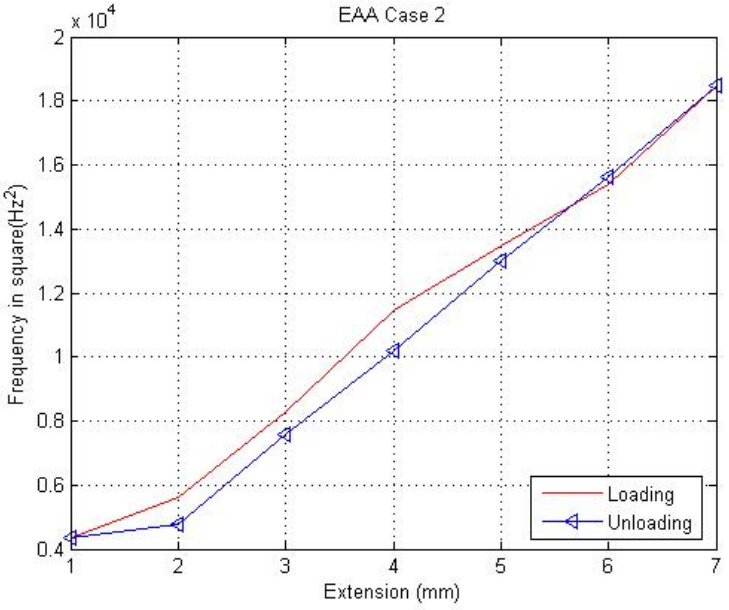


Figure 7.3 *Relation between frequency in square and extension for EAA, case 2.*

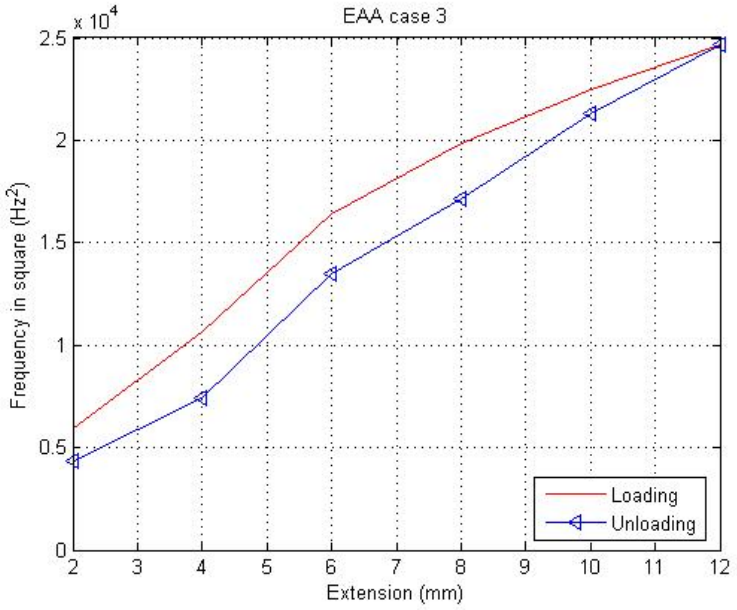


Figure7.4 *The relation between frequency in square and extension for EAA, case 3.*

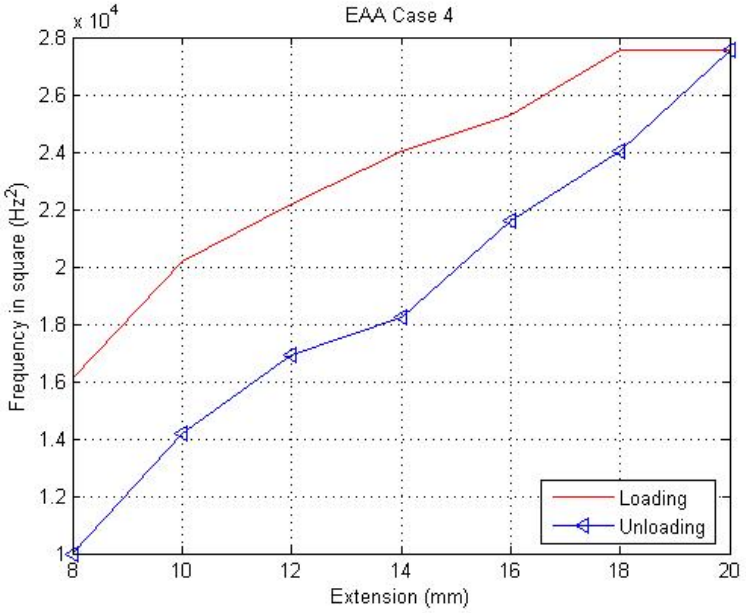


Figure 7.5 *The relation between frequency in square and extension for EAA, case 4.*

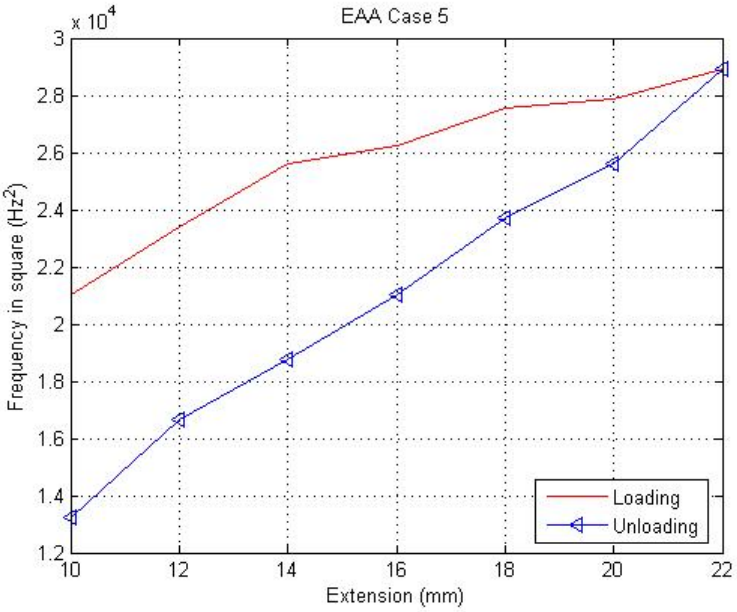


Figure 7.6 *The relation between frequency in square and extension for EAA, case 5.*

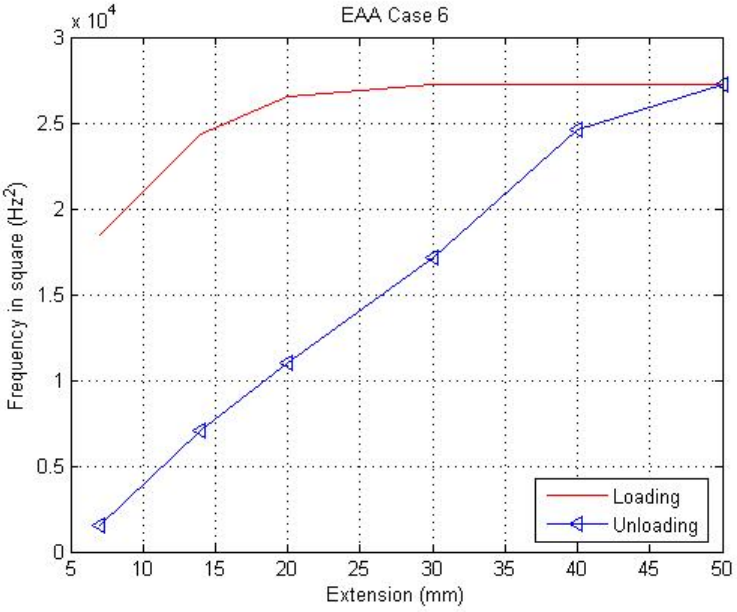


Figure 7.7 *The relation between frequency in square and extension for EAA, case 6.*

7.2. Experimental Results Obtained for PPR

Figure 7.8 shows results from tensile test of PPR.

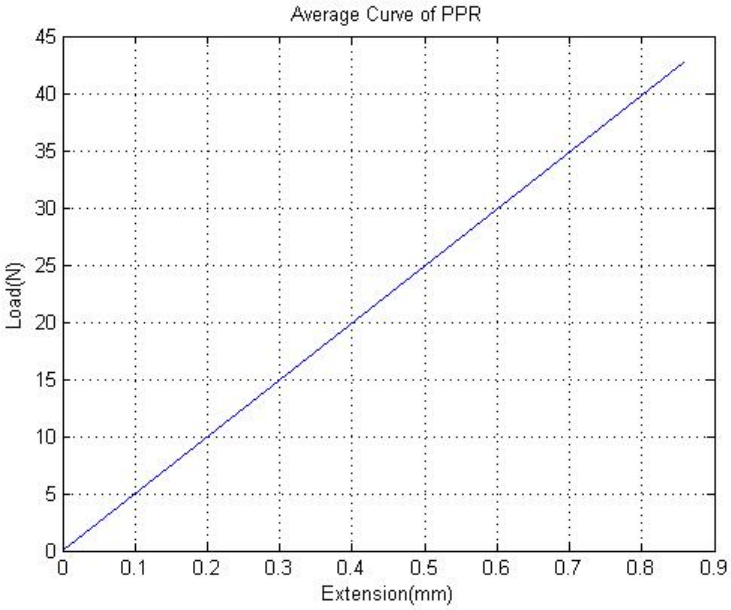


Figure7.8 *Average curve of PPR*

Figures 7.9 to 7.12 show acoustic measurements performed on PPR at different loading and unloading steps. It is observed that the results agree quite well at least with in plastic that is until the extension of around 1 mm , there is a small difference in frequency with loading and unloading steps and this difference becomes greater and greater when the increment of extension steps in plastic region with loading and unloading.

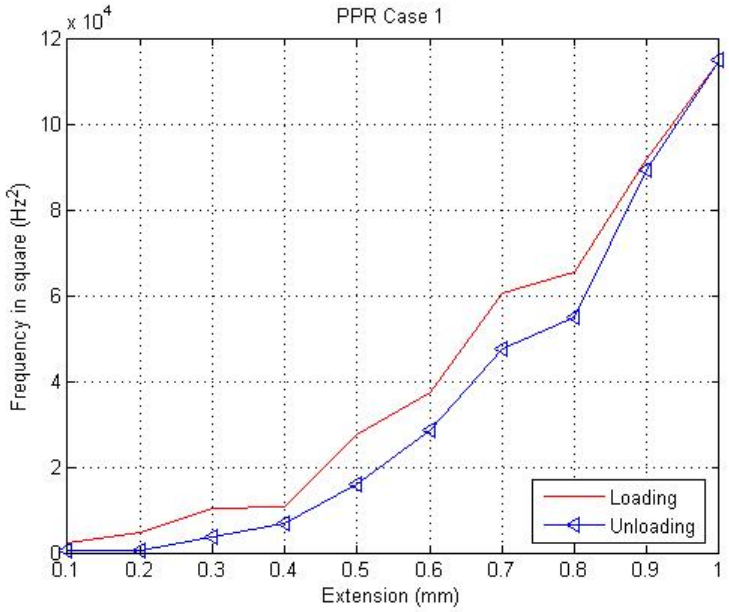


Figure7.9 *The relation between frequency in square and extension for Paper board, case 1.*

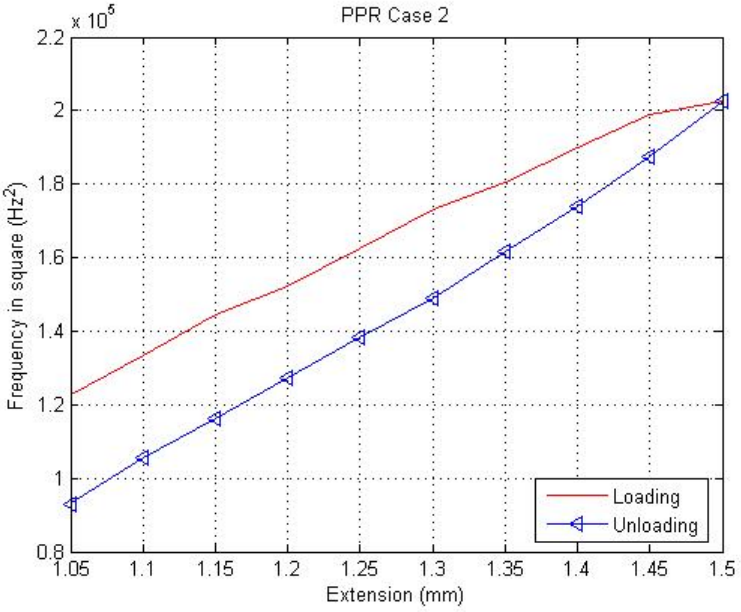


Figure 7.10 *The relation between frequency in square and extension for Paper board, case 2.*

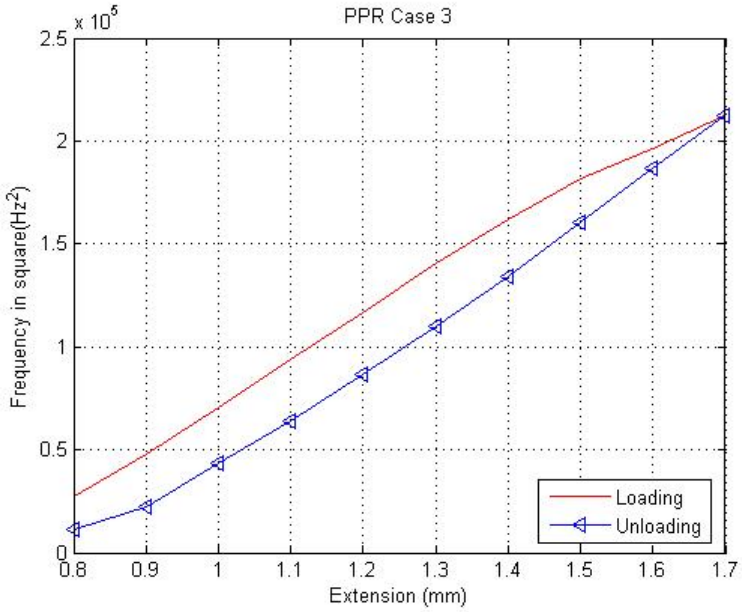


Figure 7.11 *The relation between frequency in square and extension for Paper board, case 3.*

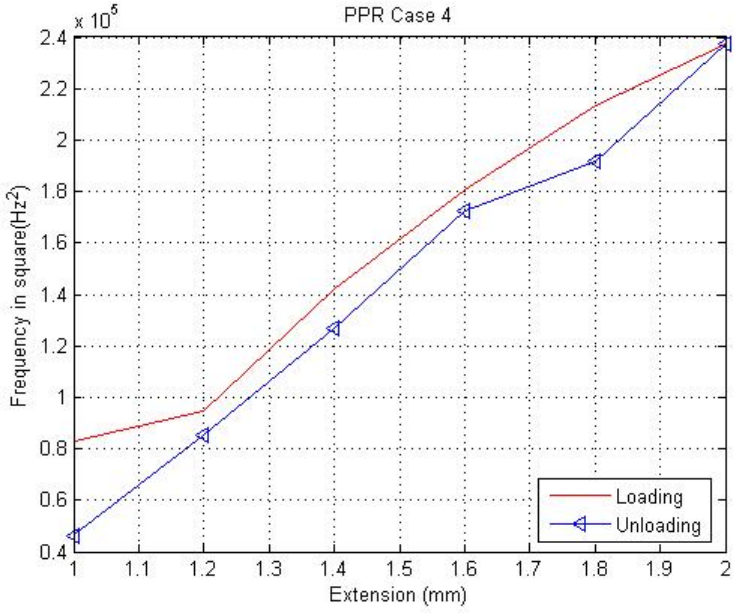


Figure 7.12 *The relation between frequency in square and extension for Paper board, case 4.*

7.3. Experimental Results Obtained For Al.

Form Tensile Test-Graphs for Aluminium foil.

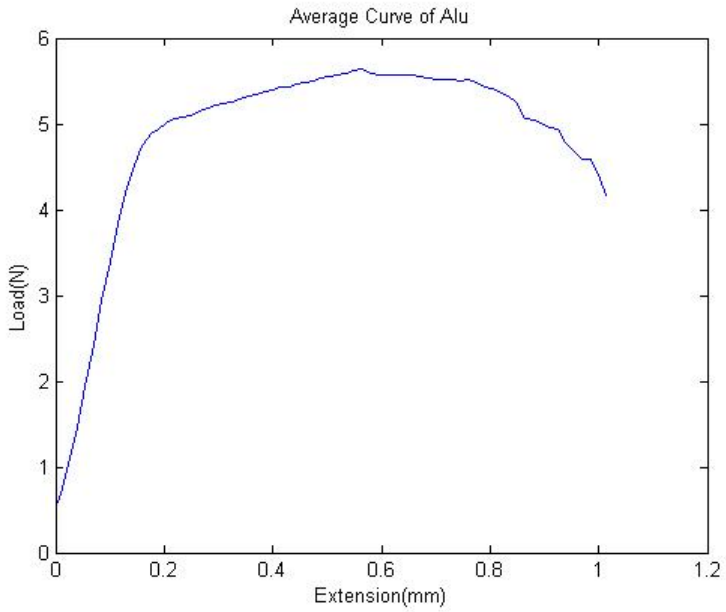


Figure 7.13 *Average curve of Aluminium.*

Figures 7.14 to 7.16 show acoustic measurements performed on Aluminium foil at different loading and unloading steps. It is observed that the results agree quite well at least with in plastic, there is a small difference in frequency with loading and unloading steps and this difference becomes greater and greater when the increment of extension steps in plastic region with loading and unloading.

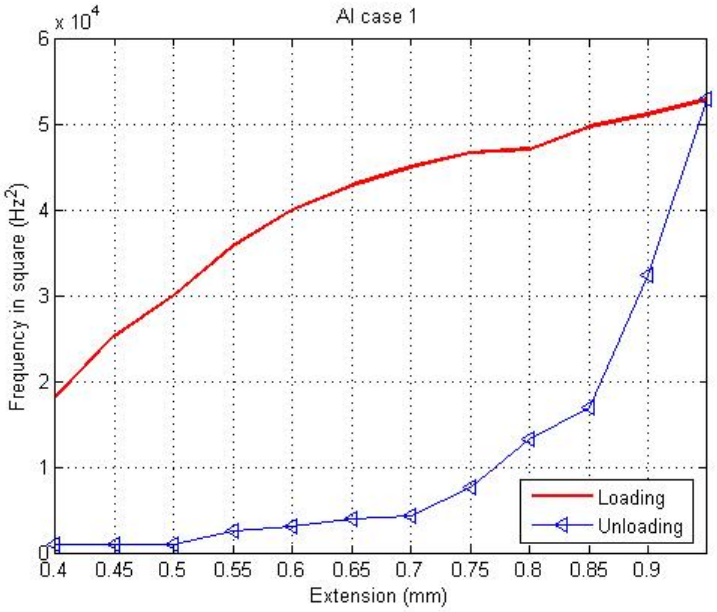


Figure 7.14 *The relation between frequency in square and extension for Aluminium foil, case1.*

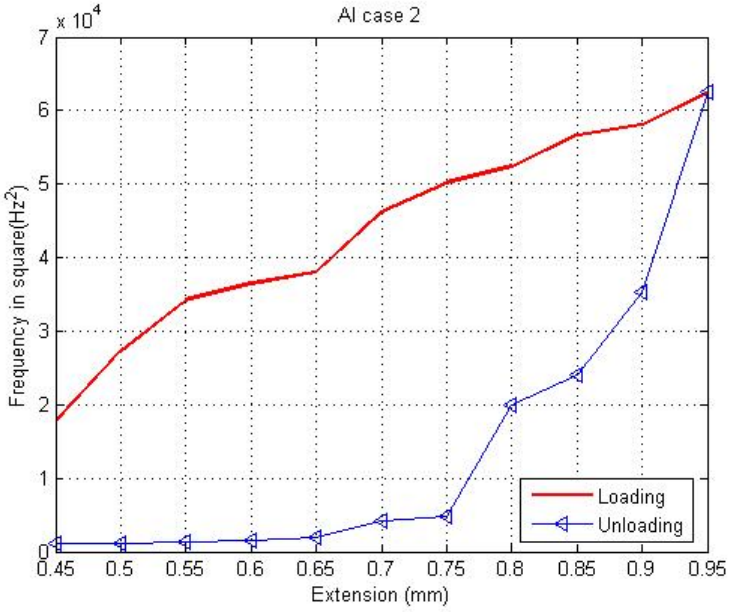


Figure 7.15 *The relation between frequency in square and extension for Aluminium foil, case2.*

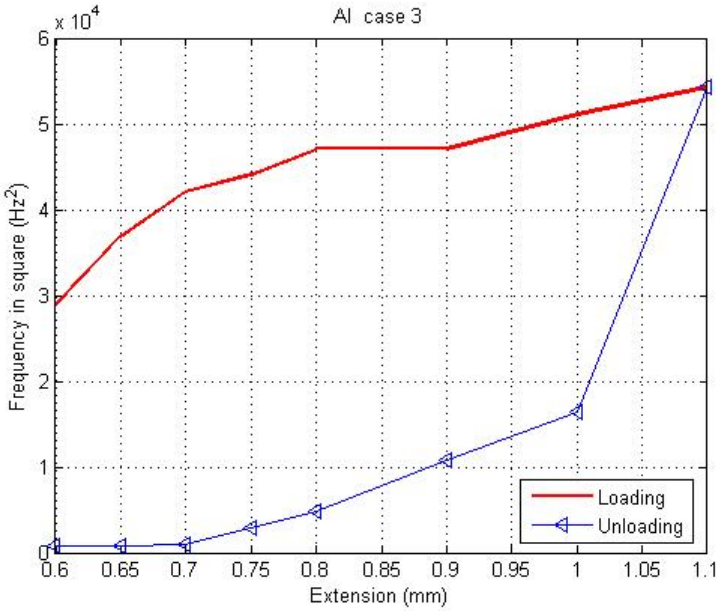


Figure 7.16 *The relation between frequency in square and extension for Aluminium foil, case3.*

7.4. Comparison of Simulation and Experiment.

Simulation results for the mode I is shown in figure 7.17.

In the experiments we have measured the frequency; it allows tracking the frequency in simulation (by varying E) and then extracting the corresponding value of Young's modulus.

A comparison of resonance frequencies for mode I is shown in table 7.1.

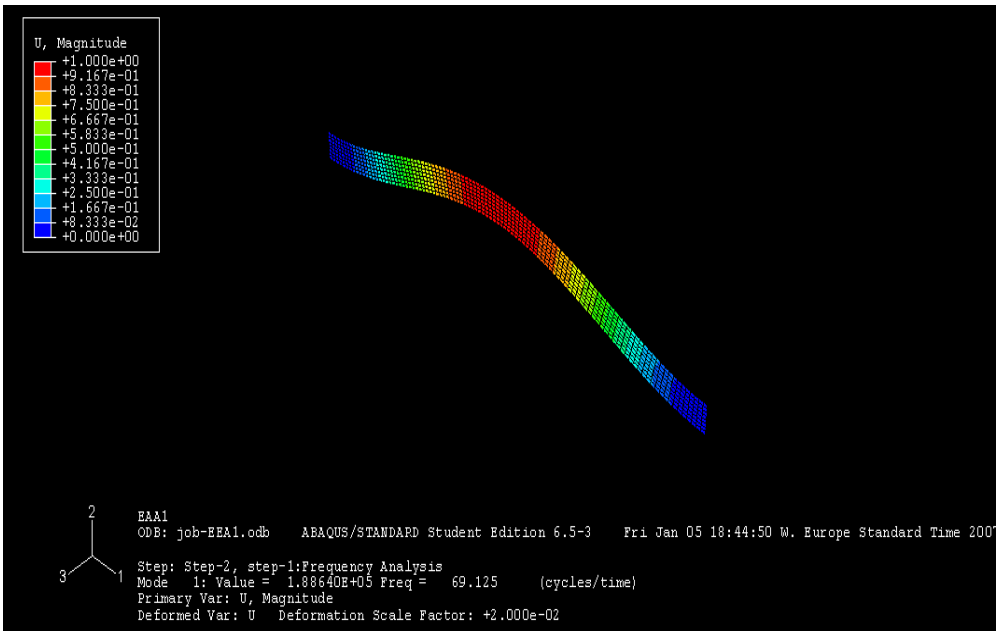


Figure 7.17. *The mode 1 of the specimen, material EAA.*

Table 7.1.*Comparision of Experimental and Model Frequency.*

Resonance frequency from Experiment at mode 1 in Hz.	68.8
Resonance frequency from simulation Model at mode 1 in Hz.	69.125

Table 7.2 below shows a comparison of Young's modulii from static and dynamic tests. Indeed, the dynamic Young's modulus appears lower than the static one, in the dynamic measurement, young's modulus is obtained from the dynamic behaviour of specimen, so it reflect the frequency dependence of the material and also This difference was explained to be due to that the modes are not purely bending along the length of the specimen, which is not taken into consideration in the derivation of equation used.Although this is a limitation ,the result allows to consider the

suggested method as an alternative for the purpose of structural health monitoring of materials of specific sizes. [7].

In this table 7.2.we observed that there is a huge difference in static and dynamic young's modulus of Aluminium foil. This may be explains from observation that non-homogenous deformation and buckles are present on the thin specimen [17].While this method work well for other materials.

Table 7.2. *Comparision of static and dynamic methods.*

MATERIAL	PAPER BOARD	EAA 220	ALUMINIUM FOIL	LDPE (*)
YOUNG'S MODULUS TENSILE TEST [M Pa]	6251.85	86	54776.68	132.10
YOUNG'S MODULUS ACOUSTIC AVERAGE [M Pa]	4281.84	71.456	9792	115.42
DIFFERENCE [%]	31.49	16.91	82.12	12.62

*Obtain from. [7]

Regarding the permanent deformation of the specimen due to plastic deformation, EAA was investigated. For a specimen of length 200mm, width 15mm and a thickness of 121 micron, the method allows determination of the change in length. Table 7.3 and 7.4. The change in length shows that the specimen enters into plastic region and there is a permanent deformation occurs, illustrates such application more can be seen in appendices.

Table 7.3. *Comparision of dynamic methods with loading and unloading*

Loading			Unloading		
Frequency [Hz]	Load [N]	Ext. [mm]	Frequency [Hz]	Load [N]	Ext. [mm]
66	1.6	1	212	19	25
90	3.3	2	169	17	20
125	6	4	153	14	15
145	8	6	131	4	10
170	10	10	102	3	6
182	13	15	82	2	4
192	14	20	66	2	2
212	15	25	40	0.7	1
Original length: 200mm, final length: 201mm.					

Table 7.4. *Comparision of dynamic methods with loading and unloading*

Loading			Unloading		
44	0.5	0.5	375	14.5	49
152	8.1	7	346	11.9	42
181	11.9	14	322	10	35
185	12.6	21	146	8.5	28
187	13.8	28	127	6.3	21
340	14.1	35	107	4.3	14
363	14.2	42	63	1.48	7
375	14.5	49	21	0.2	0.5
Original length: 200mm, final length: 202mm.					

8. Conclusion and Further Work

Young's modulus of sheet materials having no bending stiffness was investigated using

vibration-based technique.This method is based upon remote excitation of a rectangular strip in flexural mode with a harmonic signal from a loudspeaker, and vibration measurement with a laser vibrometer. Resonance frequencies are experimentally measured, which help extracting Young's modulus. Good results are obtained for the material EAA and PPR, while the method is shown to be not successful for aluminium foil. Subsequent investigations will clarify this issue.

Moreover, the material was successively loaded and unloaded, while resonance frequency and extension are recorded for each step. The loading and unloading curves are then plotted for tracking changes in the material properties. It is shown that the frequency measurement, as performed in this study allows accurate detection of changes in materials properties such as the permanent deformation occurred due to plastic deformation.

This method is suitable for condition monitoring of structures.

For further future work air damping should be taken into account for accurate results.

9. References

1. G.J Mcshane, M.Boutchich, A, Srikantha Phani, D.F.Moore, T.J.Lu. *Young's Modulus Measurement of Thin Film Materials using Micro Cantilevers,*Cambridge University ,UK,Published on 11 August 2006, www.stacks.iop.org/JMM/16/1926.
2. Carlos H.Mastrangelo, Yu-Vhong Tai, Richard S.Muller. *Thermo physical Properties of Low –residual stress, Silicon rich, LPCVD Silicon nitride films,* Department of Electrical and Computer Sciences and the Electronics Research Laboratory,University of California,USA,A 21-A23,pp 856-860,1990.
3. ASTM C 1548-02, *Standard Test method for dynamic Young's Modulus, Shear modulus and Poisson's ratio of Refractory materials Impulse Excitation Of vibration,*ASTM International.
4. Richard Gedney Norwood, *A practical guide to Testing metals under tension and common sources of Errors,* ADMET Inc 51, Norwood ,Massachutes,2001.www.metalsweekly.com.
5. MAT TEST.*A new method of thin film characterization*.Department of engineering,University of Cambridge,U.K,July 2003. http://micromachine.stanford.edu/~hopcroft/Research/mattest.html
6. Mfoumou E. *Variation of Mechanical Properties in EAA/Al Laminate with different coating Temperatures.* Master Degree Thesis 2003.ISRN: BTH-IMA-EX—2003/D-05—SE.Blekinge Institute of Technology,Karlskrona,Sweden.
7. Mfoumou.E, Rudenko, O.V., Hedberg, Kao-Walter. *Static versus Low frequency Dynamic Elastic Modulus measurement of thin film* , Electronic Journal "Technical Acoustics", http://www.ejta.org, 2006, 17.
8. Mfoumou.E, Rudenko, O.V., Hedberg, Kao-Walter. *Acoustical Measurement accompanying Tensile Test. New modality for NDT and characterization of Sheet metals.*Proceeding of 13(ICSV13),July 2-6, 2006,Vienna,University of Technology,Austria,ISBN:3-9501554-5-7
9. Morgon Stefik,Sarah Brum,Sean Yost,Tim lavery, *Trends of Young's Modulus Of a Metal and a Thermoplastic Polymer.* London.Feb 23,2004.

10. Richard Gedney. *Tensile Testing basics, tips and trends. Quality test and inspection.* ADMET Inc, Norwood Massachutes,January 2005.

11. Frederick Wilson, A.E.Lord,.*Young's Modulus determination Via Simple, Inexpensive Static and Dynamic Measurements,*American Journal Of Physics,Vol 41,Issue no 1,PP 653-656(1973).

12. Hibbitt, Karlsson, Sorensen, *ABAQUS tutorial*. Inc(HKS),Pawtucket,RI.1978.

13. Richard Gedney, *Do I Retrofit or replace my Universal Testing Machine*, ADMET,51,Norwood Massachutes 02062,as published in Metals weekly ,2005,www.tensiletesters.com,www.metalsweekly.com.

14. Kisso Kang,Koungsuk kim,Manyong Choi,Youngjune kang,Seongmo Yang,Dongpvo, *Evaluation of Elastic Modulus by Resonance frequency and Euler Bernoulli Equation,* Vienna,Austria,July 2-6,2006.

15. Casper Olsen, Ida Lykke Fabricius, *Static and Dynamic Young's Modulus of North Sea Chalk,* Technical University of Denmark,Institute of Environment and Resources,SEG/New Orleans 2006 Annual Meeting.

16. David Herman, Michael Gaitan,Don Devoe, *MEMS Test Structures for Mechanical Characterization of VLSI Thin Films*,Department of Mechanical Engineering,University of Maryland,College Park,MD 20742.

17. Johan Tryding, *In-Plane Fracture Of Paper*, Ph.D.Thesis LUTVDG/(TVSM-1008)/1-148/(1996),Division.of Structural Mechanics,Lund,Sweden,1996.

Appendix A.

A.1. Permanent deformation assessment.

Permanent deformation assessment experimental tables are shown below.

Specimen-1. Thickness -121 microns, **Initial Length**-200mm. **Final Length**-200mm, **Width**-15mm.**Load** -2.5 KN.					
Material	EAA	Loading	Material	EAA	Unloading
Frequency	Load	Extension	Frequency	Load	Extension
15	0.36	0.5	155	8.8	8
40	0.96	1	145	7.63	7
71	2.5	2	134	6.45	6
94	3.5	3	122	5.34	5
115	4.8	4	107	4.1	4
130	6	5	88	2.9	3
140	7.1	6	66	1.64	2
148	8	7	37	0.35	1
155	8.8	8	15	0	0.5

Specimen-2. Thickness -121 microns, **Initial Length**-200mm. **Final Length**-200mm, **Width**-15mm. **Load** -2.5 KN.					
Material	EAA	Loading	Material	EAA	Unloading
Frequency	Load	Extension	Frequency	Load	Extension
33	0.2	0.5	170	10.4	10
85	2.63	2	161	9.3	9
123	5.2	4	153	8.2	8
146	7.5	6	144	7.3	7
154	8.3	7	134	6.3	6
160	9	8	108	4	4
166	9.9	9	66	1.6	2
170	10.4	10			0.5

Specimen-3. Thickness -121 microns, **Initial Length**-200mm.
Final Length-200mm, **Width**-15mm. **Load** -2.5 KN.

Material	EAA	Loading	Material	EAA	Unloading
Frequency	Load	Extension	Frequency	Load	Extension
37	0.6	0.5	191	14.8	20
110	4.62	3	181	13.2	18
147	8.3	6	168	11.2	15
168	10.9	9	153	9.3	12
179	12.5	12	137	7.2	9
184	13.5	15	115	5	6
189	14.4	18	72	2.1	3
191	14.8	20			0.5

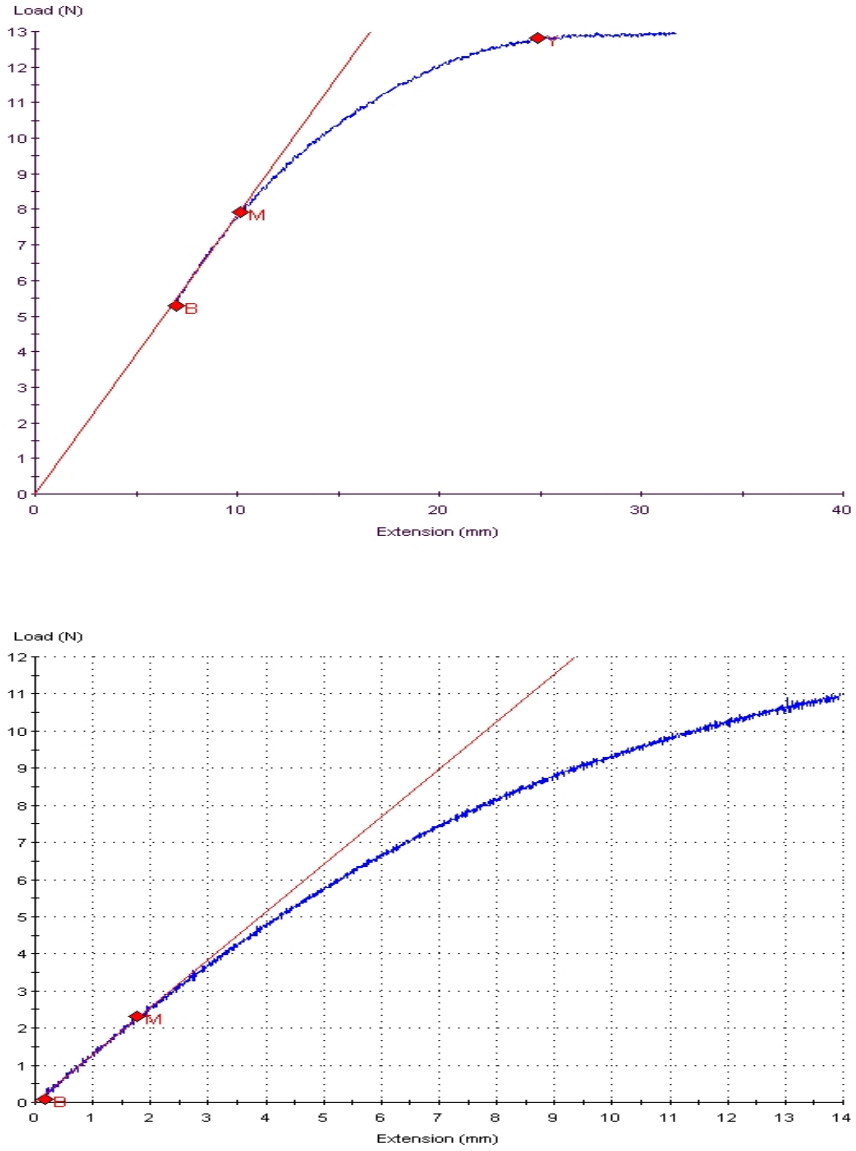
Specimen-4. Thickness -121 microns, **Initial Length**-200mm.
Final Length-201mm, **Width**-15mm, **Load** -2.5 KN.

Material	EAA	Loading	Material	EAA	Unloading
Frequency	Load	Extension	Frequency	Load	Extension
38	0.4	0.5	197	14.7	35
135	6.4	5	177	12.1	30
172	10.7	10	164	10.2	25
186	12.6	15	153	8.7	20
190	13.6	20	137	6.8	15
194	14.4	25	144	4.6	10
196	14.4	30	67	2.1	5
197	14.7	35			0.5

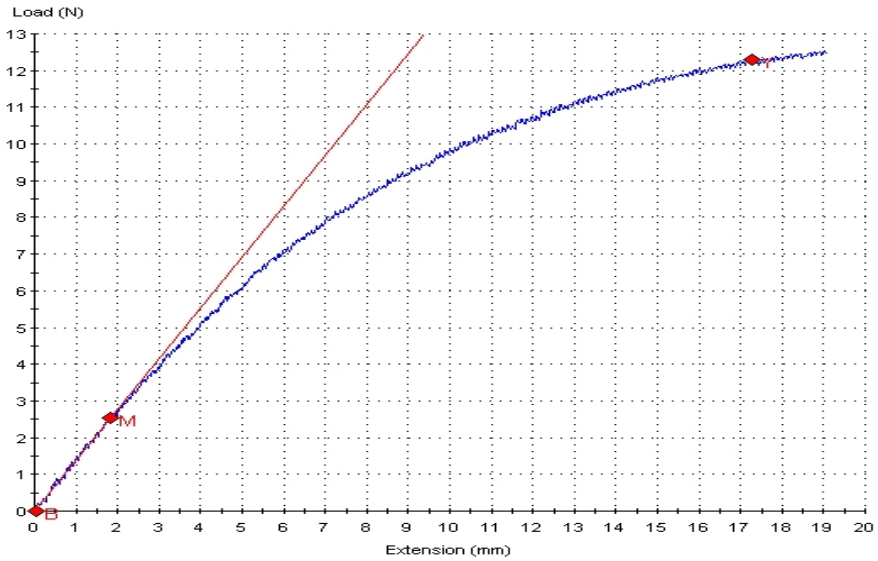
Specimen-5. **Thickness** -121 microns, **Initial Length**-200mm. **Final Length**-202mm, **Width**-15mm, **Load** -2.5 KN.					
Material	EAA	Loading	Material	EAA	Unloading
Frequency	Load	Extension	Frequency	Load	Extension
44	0.5	0.5	375	14.5	49
152	8.1	7	346	11.9	42
181	11.9	14	322	10	35
185	12.6	21	146	8.5	28
187	13.8	28	127	6.3	21
340	14.1	35	107	4.3	14
363	14.2	42	63	1.48	7
375	14.5	49			0.5

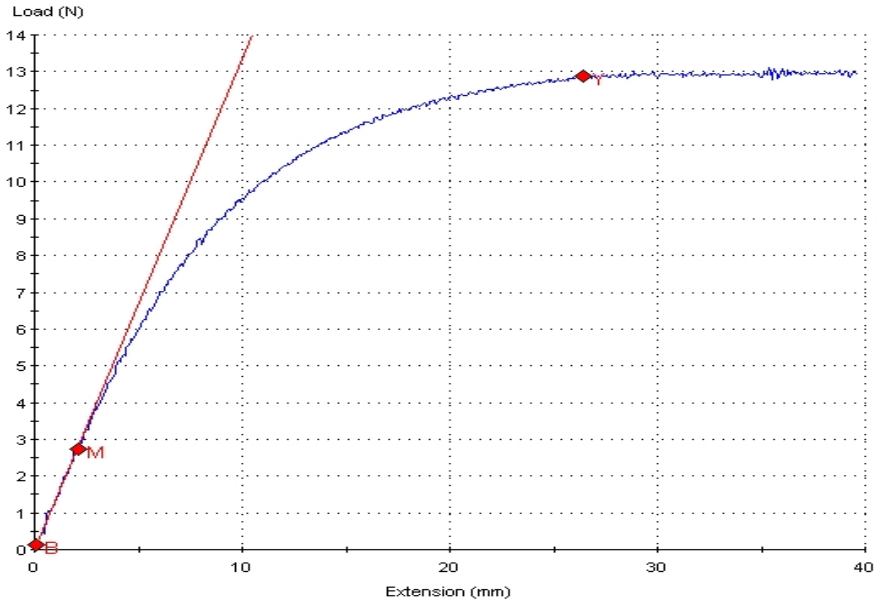
Specimen-6. Thickness -121 microns, **Initial Length**-200mm. **Final Length**-203mm, **Width**-15mm, **Load cell** 2.5KN.					
Material EAA		Loading	Material EAA		Unloading
Frequency	Load	Extension	Frequency	Load	Extension
31	0.6	0.5	196	21.3	56
160	12.5	8	174	17.4	48
190	18.2	16	161	14.5	40
192	18.3	24	148	11.8	32
194	20.2	32	133	9.3	24
196	20.3	40	112	6.4	16
196	20.7	48	65	2.14	8
196	21.3	56			0.5

A.2. Experimental work.

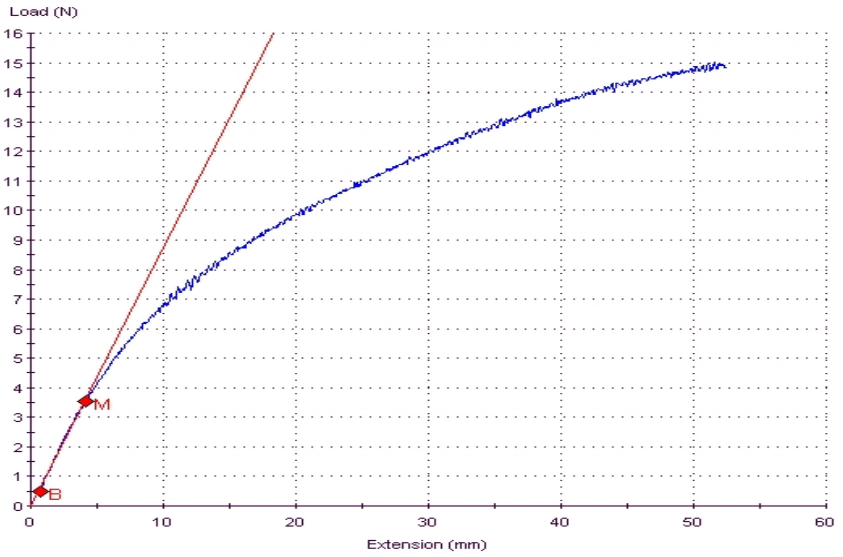


FigureA.2.1. Part I. *Load Vs extension graphs from MTS for the all cases of EAA material.*





FigureA.2.1. Part II. *Load Vs extension graphs from MTS for the all cases of EAA material.*



FigureA.2.1. Part III. *Load Vs extension graphs from MTS for the all cases of EAA material.*

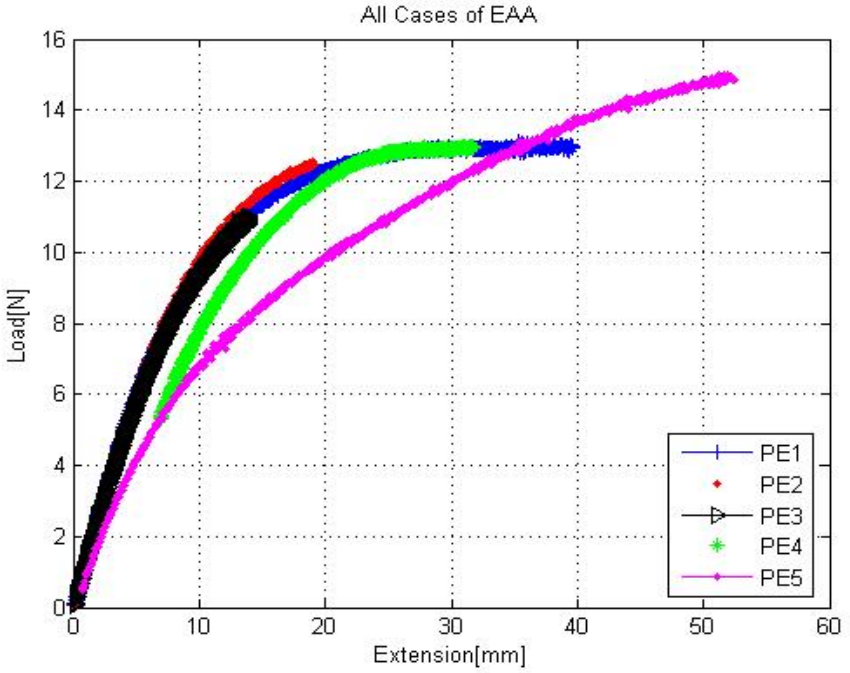
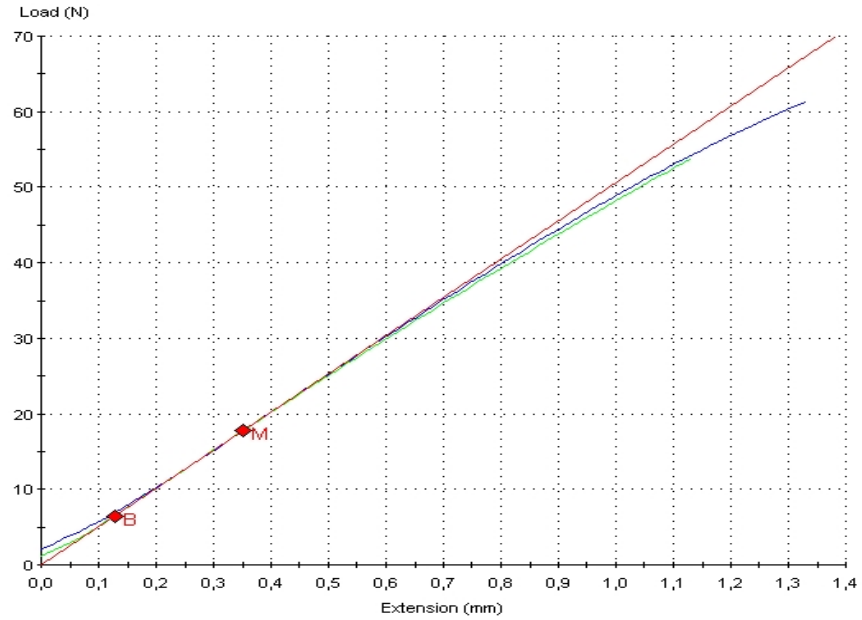


Figure A.2.2.*Average graph for all cases of Paper board.*



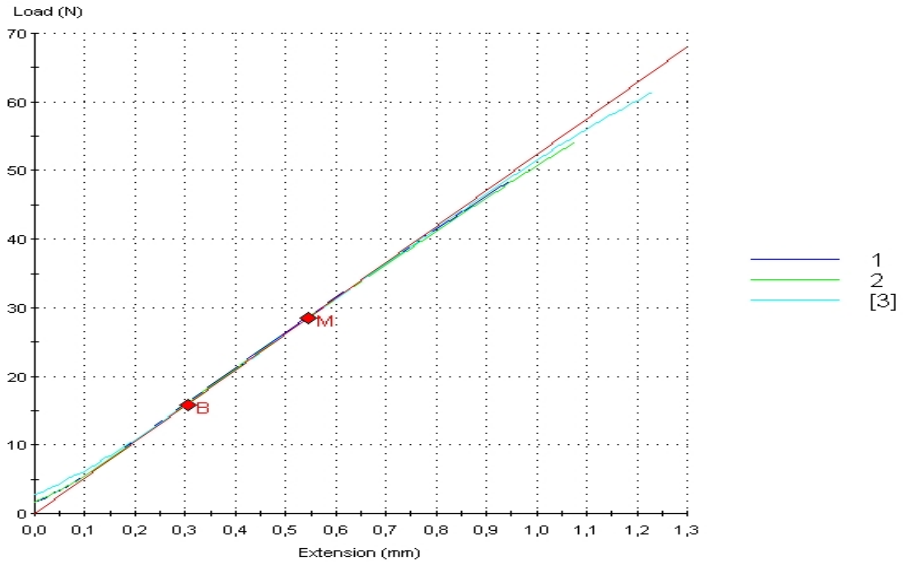
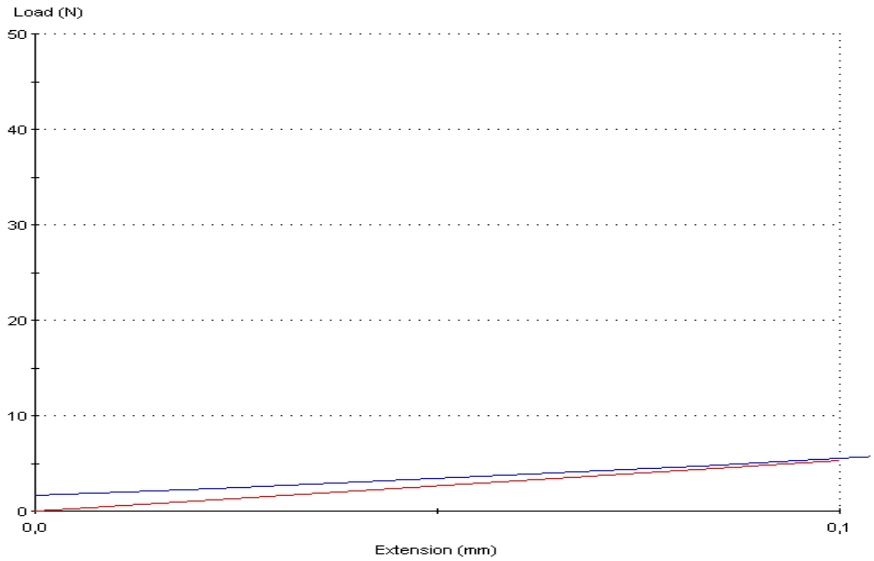


Figure A.2.3. Part I. *Load vs extension curve for all cases of Paper board.*



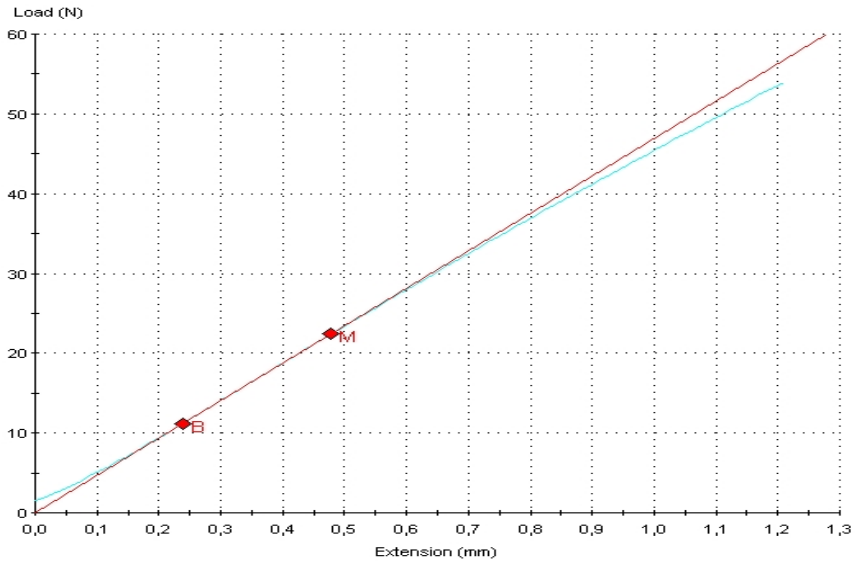


Figure A.2.3. Part II. *Load vs extension curve for all cases of Paper board.*

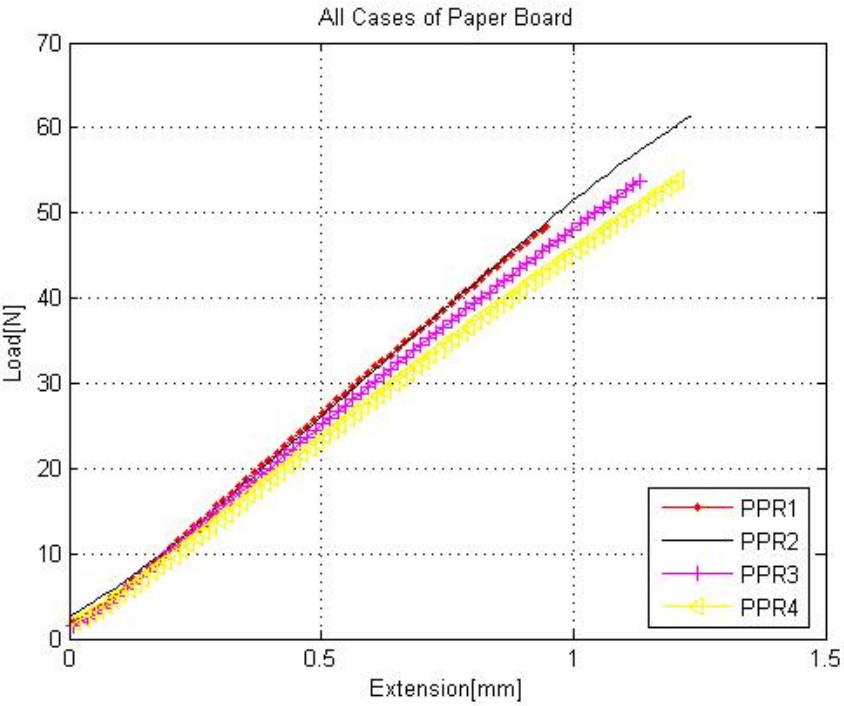


Figure A.2.4. *Average graph for all cases of Paper board.*

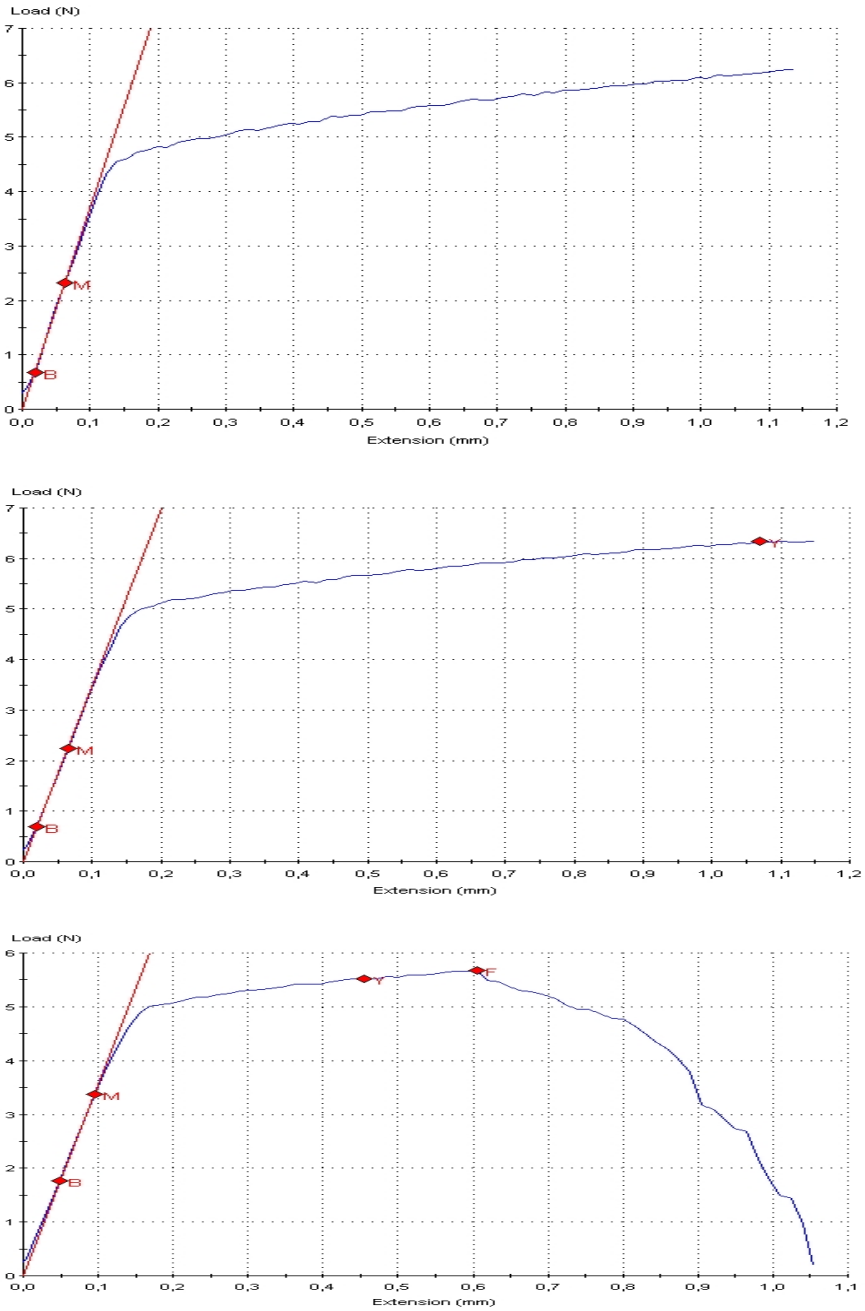


Figure A.2.5. *Load Vs extension graphs from MTS for the all cases of Aluminum foil.*

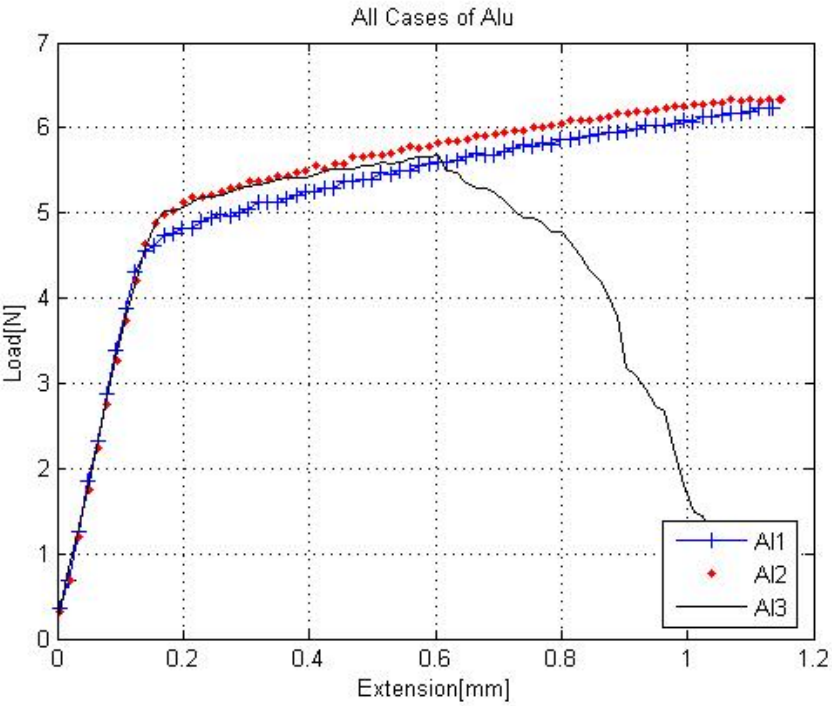


Figure A.2.6.*Average graph for all cases of Aluminium foil.*

A.3. Matlab Scripts

```
close all;
clear all;
freq=[101,120,167,187,204,221];%frequency in Hertz
ext=[0.1,0.2,0.5,0.6,0.7,0.8];%extension in mm
% ext=[0.0001,0.0002,0.0005,0.0006,0.0007,0.0008];
t=[34,69,118.2,118.2,118.2,118.2]/15;%load in N
b=0.25;%length in Meters
h=0.001;%Thickness in Meters
rho=0.3;%Pissions ratio
den=2700;%Density in Kg/m^3
 c=sqrt(t./(den*h));
% E=((2*freq)-(c/b))*((24*c*b^3)/(pi^2)*(den(1-rho^2)/(h^2)

%  x=(2.*freq)-(c./b);
%  y=(24*c.*b^3)/(pi^2);
%   z=(den(1-rho^2))/(h^2);
%  E=x.*(y.*z)
%c=sqrt(E*.ext)/den
E=t./(h*ext)
plot(ext, freq)
 xlabel('extension in mm');
 ylabel('frequency in hertz');
```

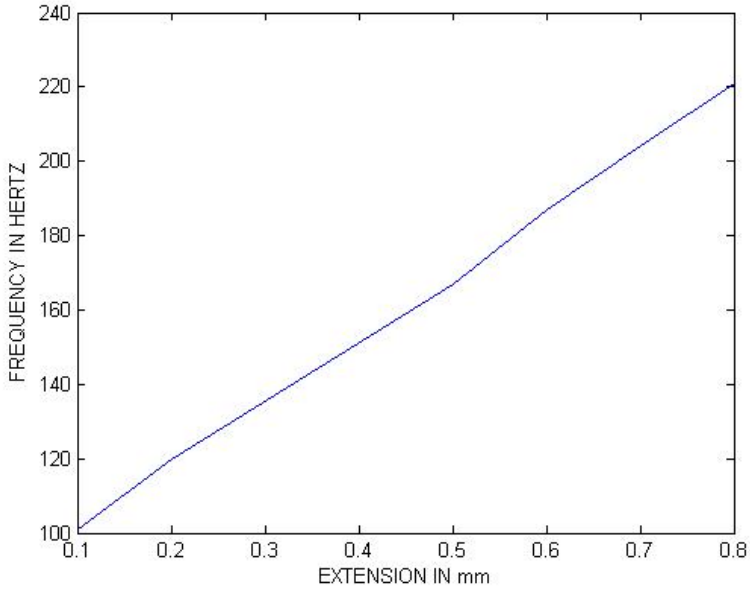


Figure A.3.1.

Version 2.

```
close all;
clear all;
freq=[101,120,167,187,204,221];%frequency in Hertz
ext=[0.1,0.2,0.5,0.6,0.7,0.8];%extension in mm
% ext=[0.0001,0.0002,0.0005,0.0006,0.0007,0.0008];
t=[34,69,118.2,118.2,118.2,118.2]/15;%load in N
b=0.25;%length in Meters
h=0.001;%Thickness in Meters
rho=0.3;%Pissions ratio
den=2700;%Density in Kg/m^3
 c=sqrt(t./(den*h));
% E=((2*freq)-(c/b))*((24*c*b^3)/(pi^2)*(den(1-rho^2)/(h^2)

 x=(2.*freq)-(c./b);
 y=(24*c.*b^3)/(pi^2);
  z=(den(1-rho^2))/(h^2);
 E=x.*(y.*z)
%c=sqrt(E*.ext)/den
plot(ext, freq)
```

```
xlabel('extension in mm');
ylabel('frequency in hertz');
```

Matlab script for loading text files.

```
clear all;close all;clc;
load PE1
Ext1=PE1(:,3);
Load1=PE1(:,1);
load PE2
Ext2=PE2(:,3);

Load2=PE2(:,1);
load PE3
Ext3=PE3(:,3);
Load3=PE3(:,1);
load PE4
Ext4=PE4(:,3);
Load4=PE4(:,1);
load PE5
Ext5=PE5(:,3);
Load5=PE5(:,1);
save all_PE12345.mat
```

Matlab script for for all average cases of EAA.

```
close all;clear all;clc;
load all_PE12345
maxpeak=max(Load1)+max(Load2)+max(Load3)+max(Load4)+max(Load
5);
disp('the average peak load of all the PE12345 is')
maxpeak
for i=1:length(Ext2)
   avgExt(i)=(Ext1(i)+Ext2(i)+Ext3(i)+Ext4(i)+Ext5(i))/5;
   avgLoad(i)=(Load1(i)+Load2(i)+Load3(i)+Load4(i)+Load5(i))/5;
end
plot(avgExt,avgLoad)

xlabel('Average Extension')
ylabel('Average Load')
title('Average Curve of EAA')
```

Matlab script for EAA case 1.

```
close all;close all;clc
f=[68 68 68 75 75 75 91 107];
fs1=f.*f;
ext=.1:.1:.8;
plot(ext,fs1,'r','linewidth',2)
hold on
grid on
title('EAA case 1')
f1=[107 91 75 75 75 68 68 68];
fs2=f1.*f1;
ext1=.8:-.1:.1;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for Aluminium foil for loading data.

```
clear all;close all;clc;
load Al1
Ext1=Al1(:,3);
Load1=Al1(:,1);
load Al2
Ext2=Al2(:,3);
Load2=Al2(:,1);
load Al3
Ext3=Al3(:,3);
Load3=Al3(:,1);
save all_Al123.mat
```

Matlab script for Aluminium foil for average curve.

```
close all;clear all;clc;
load all_Al123
maxpeak=max(Load1)+max(Load2)+max(Load3);
disp('the average peak load of all the PE12345 is')
maxpeak
for i=1:length(Ext3)
```

```
    avgExt(i)=(Ext1(i)+Ext2(i)+Ext3(i))/3;
    avgLoad(i)=(Load1(i)+Load2(i)+Load3(i))/3;
end
plot(avgExt,avgLoad)
axis([0 1.2 0 6])
xlabel('Average Extension')
ylabel('Average Load')
title('Average Curve of Alu')
```

Matlab script for EAA case 2.

```
close all;close all;clc
f=[66 75 91 107 116 124 136];
fs1=f.*f
ext=1:1:7;
plot(ext,fs1,'r');
grid on
title('EAA Case 2')
hold on
f1=[136 125 114 101 87 69 66];
fs2=f1.*f1;
ext1=7:-1:1;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for EAA case 3

```
close all;close all;clc
f=[77 103 128 141 150 157];
fs1=f.*f;
ext=2:2:12;
plot(ext,fs1,'r')
grid on
title('EAA case 3')
hold on
f1=[157 146 131 116 86 66];
fs2=f1.*f1;
ext1=12:-2:2;
plot(ext1,fs2,'<-')
```

```
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for EAA case 4

```
close all;close all;clc
f=[127 142 149 155 159 166 166];
fs1=f.*f;
ext=8:2:20;
plot(ext,fs1,'r')
grid on
title('EAA Case 4')
hold on
f1=[166 155 147 135 130 119 100];
fs2=f1.*f1;
ext1=20:-2:8;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for EAA case 5

```
close all;close all;clc
f=[145 153 160 162 166 167 170];
fs1=f.*f;
ext=10:2:22;
plot(ext,fs1,'r')
grid on
title('EAA Case 5')
hold on
f1=[170 160 154 145 137 129 115];
fs2=f1.*f1;
ext1=22:-2:10;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for EAA case 6.

```
close all;close all;clc
f=[136 156 163 165 165 165];
fs1=f.*f;
ext=[7 14 20 30 40 50];
plot(ext,fs1,'r')
grid on
title('EAA Case 6')
hold on
f1=[165 157 131 105 84 39];
fs2=f1.*f1;
ext1=[50 40 30 20 14 7];
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for loading matfiles of Paper board.

```
clear all;close all;clc;
% load PPR1
% Ext1=PPR1(:,3);
% Load1=PPR1(:,1);
 load PPR2
 Ext2=PPR2(:,3);
 Load2=PPR2(:,1);
load PPR3
Ext3=PPR3(:,3);
Load3=PPR3(:,1);
% load PPR4
% Ext4=PPR4(:,3);
% Load4=PPR4(:,1);
load PPR5
Ext5=PPR5(:,3);
Load5=PPR5(:,1);
load PPR6
Ext6=PPR6(:,3);
Load6=PPR6(:,1);
save all_PPR13456.mat
```

Matlab script for average of all cases of paper board.

```
close all;clear all;clc;
load all_PPR123456
maxpeak=max(Load1)+max(Load2)+max(Load3)+max(Load4)+max(Load
4)+max(Load5)+max(Load6);
disp('the average peak load of all the PPR123456 is')
maxpeak
for i=length(Ext2)
   avgExt(i)=(Ext2(i)+Ext3(i)+Ext5(i)+Ext6(i))/4;
   avgLoad(i)=(Load2(i)+Load3(i)+Load5(i)+Load6(i))/4;
end
plot(avgExt,avgLoad)
grid on
%axis([0 1.2 0 60])
xlabel('Average Extension')
ylabel('Average Load')
title('Average Curve of PPR')
```

Matlab script for paper board case 1.

```
close all;close all;clc
f=[48 68 102 104 166 193 246 256 303 339];
fs1=f.*f;
ext=.1:.1:1;
plot(ext,fs1,'r')
grid on
title('PPR Case 1')
hold on
f1=[339 299 234 218 169 126 82 62 22 22];
fs2=f1.*f1;
ext1=1:-.1:.1;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for paper board case 2.

```
close all;close all;clc
f=[350 365 380 390 403 416 425 436 446 450];
fs1=f.*f;
ext=1.05:.05:1.5;
```

```
plot(ext,fs1,'r')
grid on
title('PPR Case 2')
hold on
f1=[450 433 417 402 386 372 357 341 325 305];
fs2=f1.*f1;
ext1=1.5:-.05:1.05;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for paper board case 3.

```
close all;close all;clc
f=[165 218 265 307 341 375 402 426 443 461];
fs1=f.*f;
ext=[ 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7];
plot(ext,fs1,'r')
grid on
title('PPR Case 3')
hold on
f1=[461 432 401 366 332 294 253 209 150 106];
fs2=f1.*f1;
ext1=[1.7 1.6 1.5 1.4 1.3 1.2 1.1 1 0.9 0.8];
plot(ext1,fs2,'<-')
axis([.8 1.7 0 2.5e5])
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for paper board case 4.

```
close all;close all;clc
f=[288 308 377 425 462 487];
fs1=f.*f;
ext=1:.2:2;
plot(ext,fs1,'r')
grid on
title('PPR Case 4')
hold on
```

```
f1=[487 438 415 356 292 215];
fs2=f1.*f1;
ext1=2:-.2:1;
plot(ext1,fs2,'<-')
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for Aluminium foil case1.

```
close all;close all;clc
f=[135 159 173 189 200 207 212 216 217 223 226 230];
fs1=f.*f;%frequency in square
ext=[.4,.45,.5,.55,.6,.65,.7,.75,.8,.85,.9,.95];
plot(ext,fs1,'r','linewidth',2)
hold on
grid on
title('Al case 1')
f1=[230 180 130 115 87 66 63 56 50 31 30 30];
fs2=f1.*f1;  %frequency in square
ext1=[.95,.9,.85,.8,.75,.7,.65,.6,.55,.5,.45,.4];
plot(ext1,fs2,'<-')
axis([.4 .95 0 6e4])
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for Aluminium foil case 2

```
close all;close all;clc
f=[133 165 185 191 195 215 224 229 238 241 250];
fs1=f.*f;%frequency in square
ext=.45:.05:.95;
plot(ext,fs1,'r','linewidth',2)
hold on
grid on
title('Al case 2')
f1=[250 188 155 141 70 65 43 39 37 35 33];
fs2=f1.*f1;  %frequency in square
ext1=.95:-.05:.45;
plot(ext1,fs2,'<-')
```

```
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4)
```

Matlab script for Aluminium foil case 3

```
close all;close all;clc
f=[170 192 205 210 217 217 226 233];
fs1=f.*f;%frequency in square
ext=[0.6 0.65 0.7 0.75 0.8 0.9 1 1.1];
plot(ext,fs1,'r','linewidth',2)
hold on
grid on
title('Al  case 3')
f1=[233 128 104 70 54 32 27 27 ];
fs2=f1.*f1;  %frequency in square
ext1=[1.1 1 0.9 0.8 0.75 0.7 0.65 0.6];
plot(ext1,fs2,'<-')
axis([.6 1.1 0 6e4])
xlabel('Extension (mm)');
ylabel('Frequency in square (Hz)');
legend('Loading','Unloading',4).
```

A.4. ABAQUS/CAE INPUT FILE.

INPUT FILE OF ABAQUS MATERIAL EAA.

```
*Heading
 EAA1
** Job name: Job-EEA1 Model name: Model-1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Part-1
*End Part
**
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Part-1-1, part=Part-1
*Node
      1, -0.100000001, 0.00499999989,           0.
      2, -0.100000001, 0.00375000015,           0.
      3, -0.100000001, 0.00250000018,           0.
      4, -0.100000001, 0.00125000009,           0.
      5, -0.100000001,  3.88051084e-11,           0.
      ....
*Element, type=M3D4R
  1,   1,   2,  11,  10
  2,   2,   3,  12,  11
  3,   3,   4,  13,  12
  4,   4,   5,  14,  13
  5,   5,   6,  15,  14
  6,   6,   7,  16,  15
  7,   7,   8,  17,  16
  8,   8,   9,  18,  17
  9,  10,  11,  20,  19
  ...
799, 898, 899, 908, 907
800, 899, 900, 909, 908
*Nset, nset=_PickedSet2, internal, generate
   1,  909,    1
*Elset, elset=_PickedSet2, internal, generate
   1,  800,    1
** Region: (Section-1:Picked)
```

```
*Elset, elset=_PickedSet2, internal, generate
   1,  800,    1
** Section: Section-1
*Membrane Section, elset=_PickedSet2, material=Material-1
0.00121,
*End Instance
**
*Nset, nset=_PickedSet4, internal, instance=Part-1-1,
generate
 1,  9,  1
*Elset, elset=_PickedSet4, internal, instance=Part-1-1,
generate
 1,  8,  1
*Nset, nset=_PickedSet5, internal, instance=Part-1-1,
generate
 901,  909,    1
*Elset, elset=_PickedSet5, internal, instance=Part-1-1,
generate
 793,  800,    1
*End Assembly
**
** MATERIALS
**
*Material, name=Material-1
*Density
924.,
*Elastic
 7e+07, 0.3
** ----------------------------------------------------------
**
** STEP: Step-1
**
*Step, name=Step-1
EAA1
*Static
1., 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet4, ENCASTRE
** Name: BC-2 Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet5, ENCASTRE
**
** OUTPUT REQUESTS
**
```

```
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
*step
step-1:Frequency Analysis
*Frequency,eigensolver=lanczos
15,1.,10000.,1.
*ELPRINT
ELSE
**
*End step
```

Department of Mechanical Engineering, Master's Degree Programme
Blekinge Institute of Technology, Campus Gräsvik
SE-371 79 Karlskrona, SWEDEN

Telephone: +46 455-38 55 10
Fax: +46 455-38 55 07
E-mail: ansel.berghuvud@bth.se